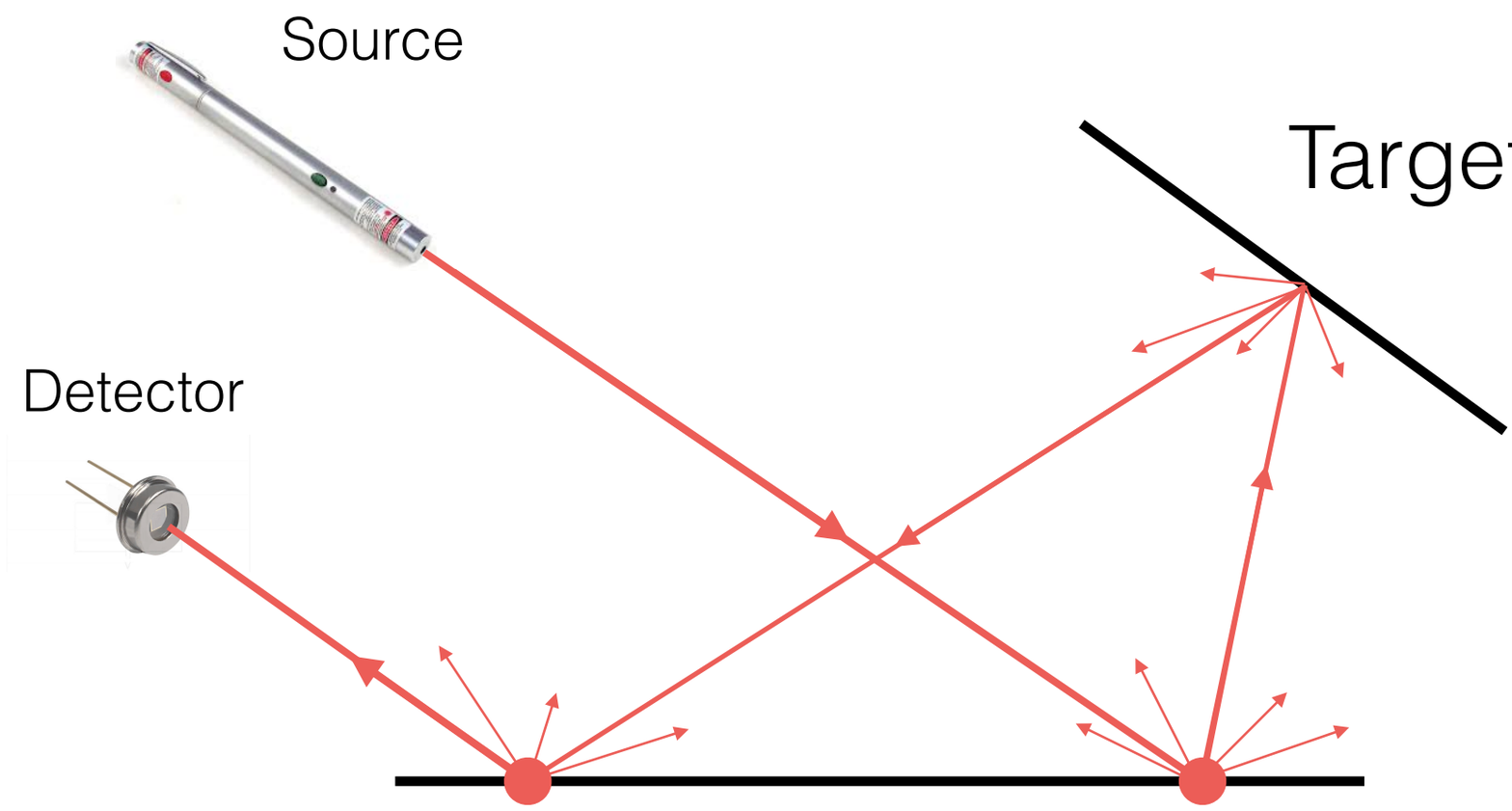
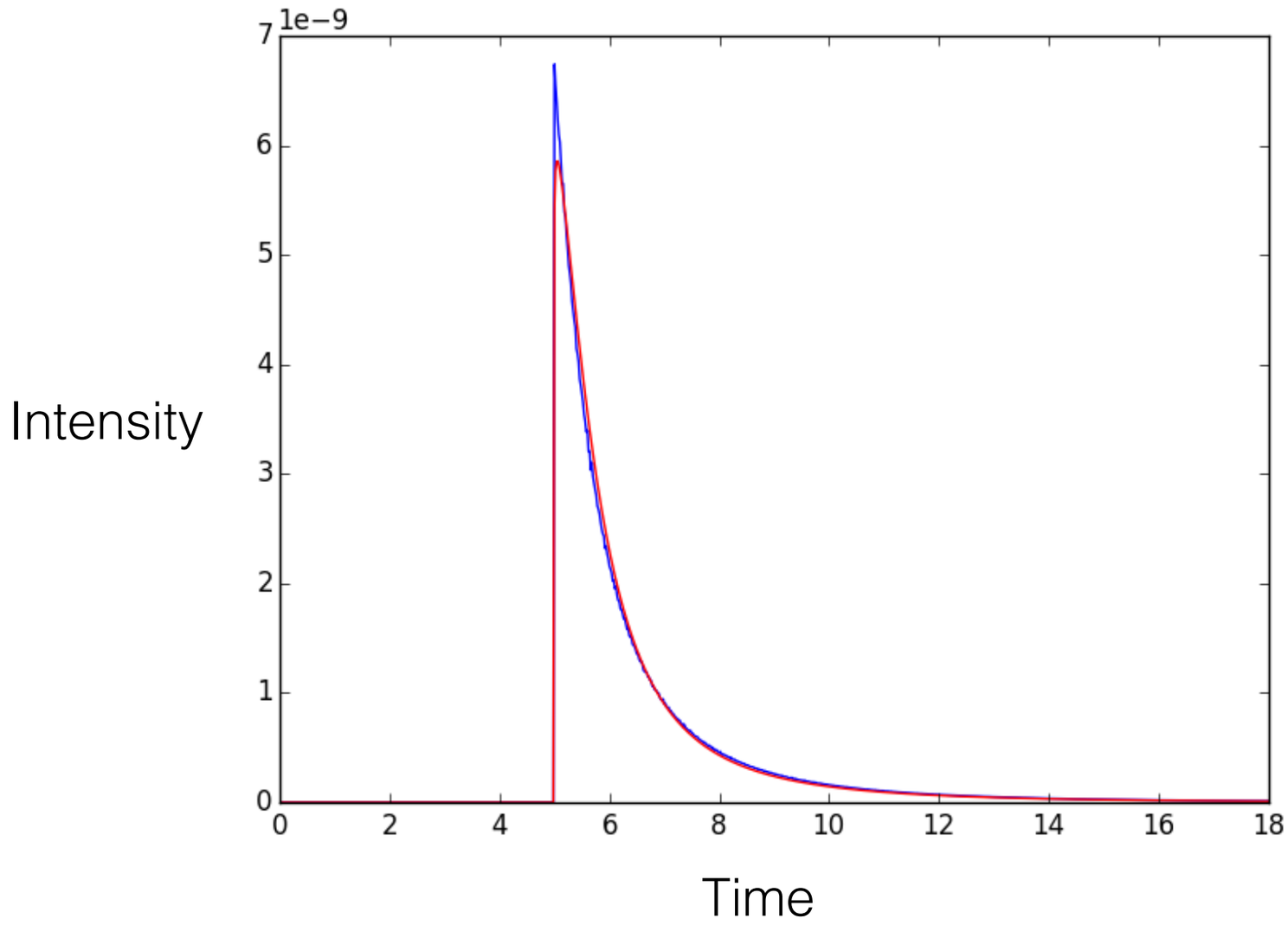
Scenario: Directional source and detector



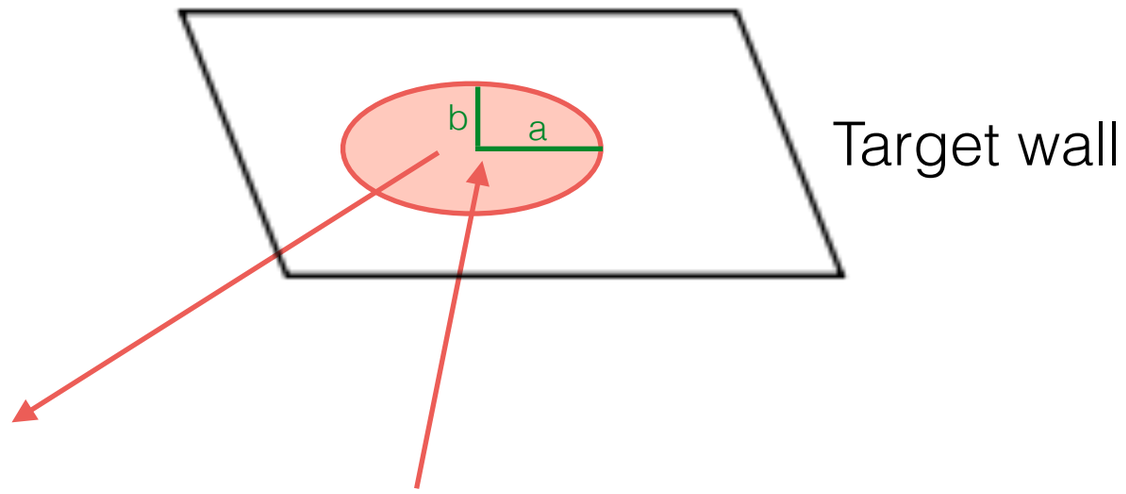
Can we locate the target from a single observation?

We found a mathematical formula (red) that closely matches simulation (blue) and is very fast to evaluate.



This will make it possible to perform a gradient-descent search for the location of the target much more quickly.

The mathematical formula is derived by considering the size ellipse of collisions which forms on the target wall.



This ellipse will grow with time, and the intensity of the returning light will grow with it.

$$I(t) = N_{\text{paths}} I_{\text{paths}}$$

$$N_{\text{paths}} = d N_{\text{total}}$$

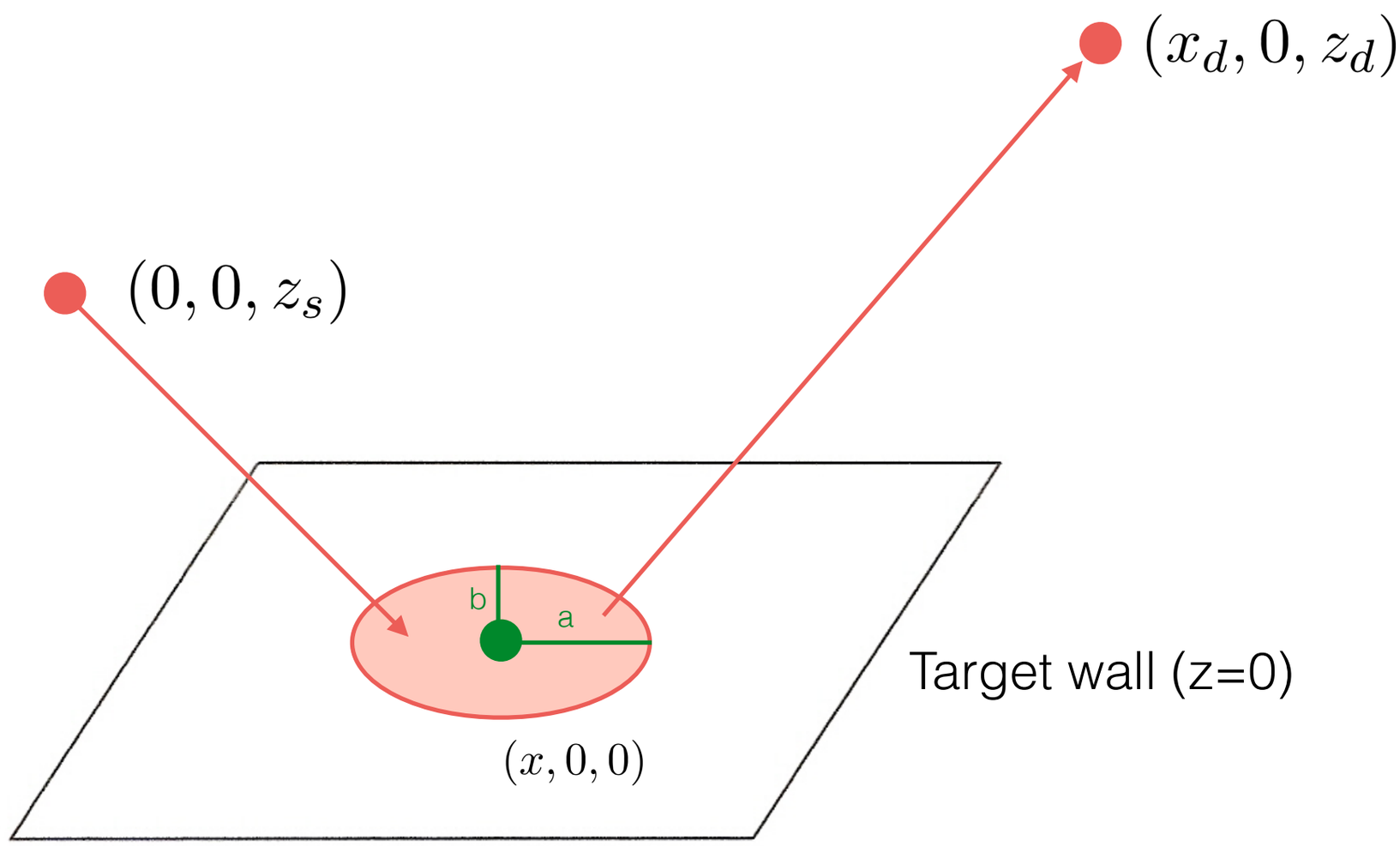
Area of ellipse

Instead of imagining an unknown wall location and known detection points, we can imagine a known wall location and unknown detection points.

$$x = x_d \frac{zs}{zs+zd}$$

$$N_{\text{total}} = \frac{\pi ab}{dx^2}$$

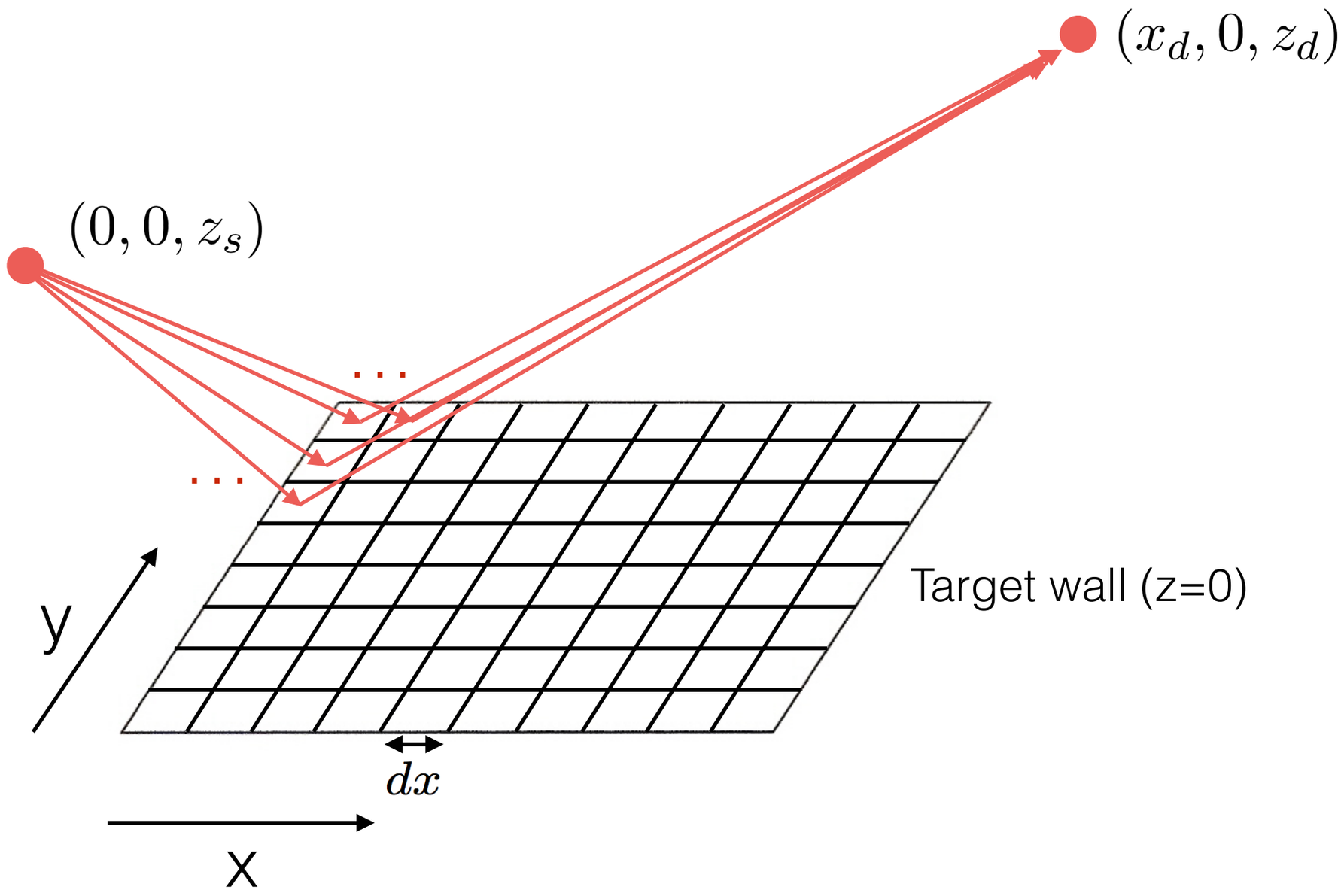
Plug in ↑



$$t = \sqrt{(x+a)^2 + z_s^2} + \sqrt{(x+a-x_d)^2 + z_d^2}$$ ← Solve for a (Average both solutions)

$$t = \sqrt{x^2 + b^2 + z_s^2} + \sqrt{(x-x_d)^2 + b^2 + z_d^2}$$ ← Solve for b

The simulation works by discretizing the wall into small chunks, and considering the contribution from each.



$$I(t) = \sum_{\text{chunks}} \left(\frac{z_s dx^2}{4\pi(x^2 + y^2 + z_s^2)^{\frac{3}{2}}} \frac{dx^2}{2\pi((x - x_d)^2 + y^2 + z_d^2)} \right)$$