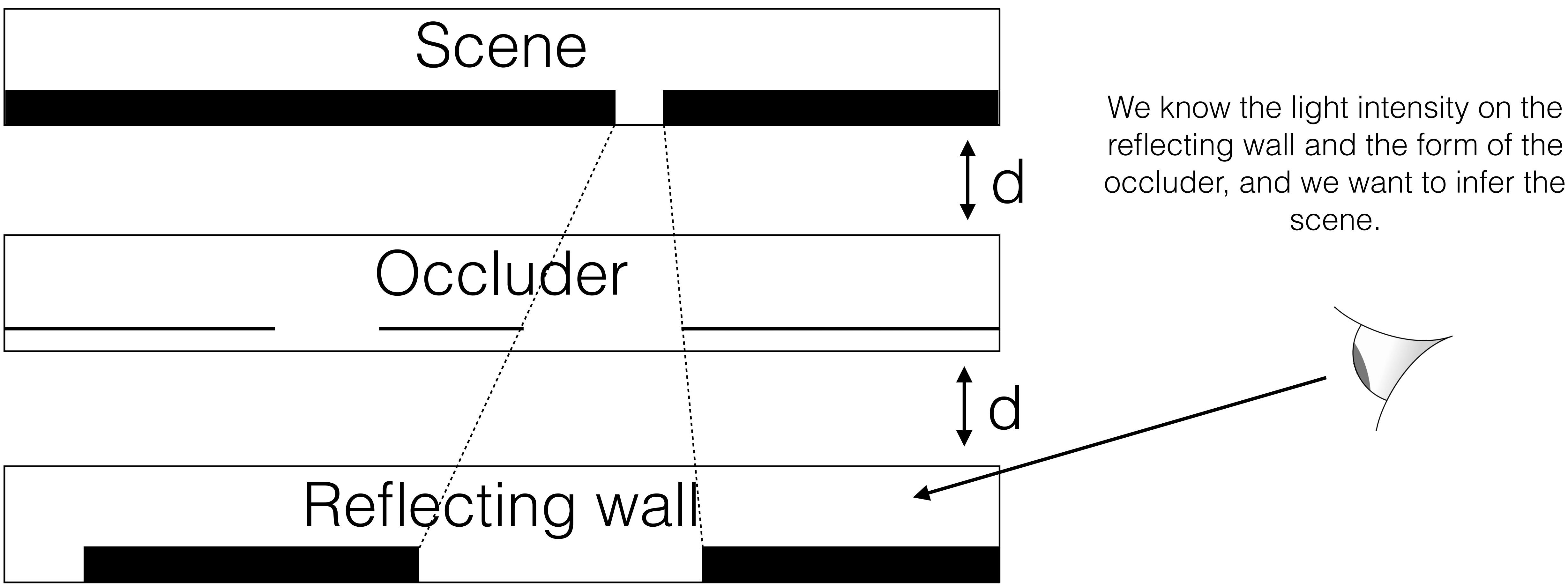
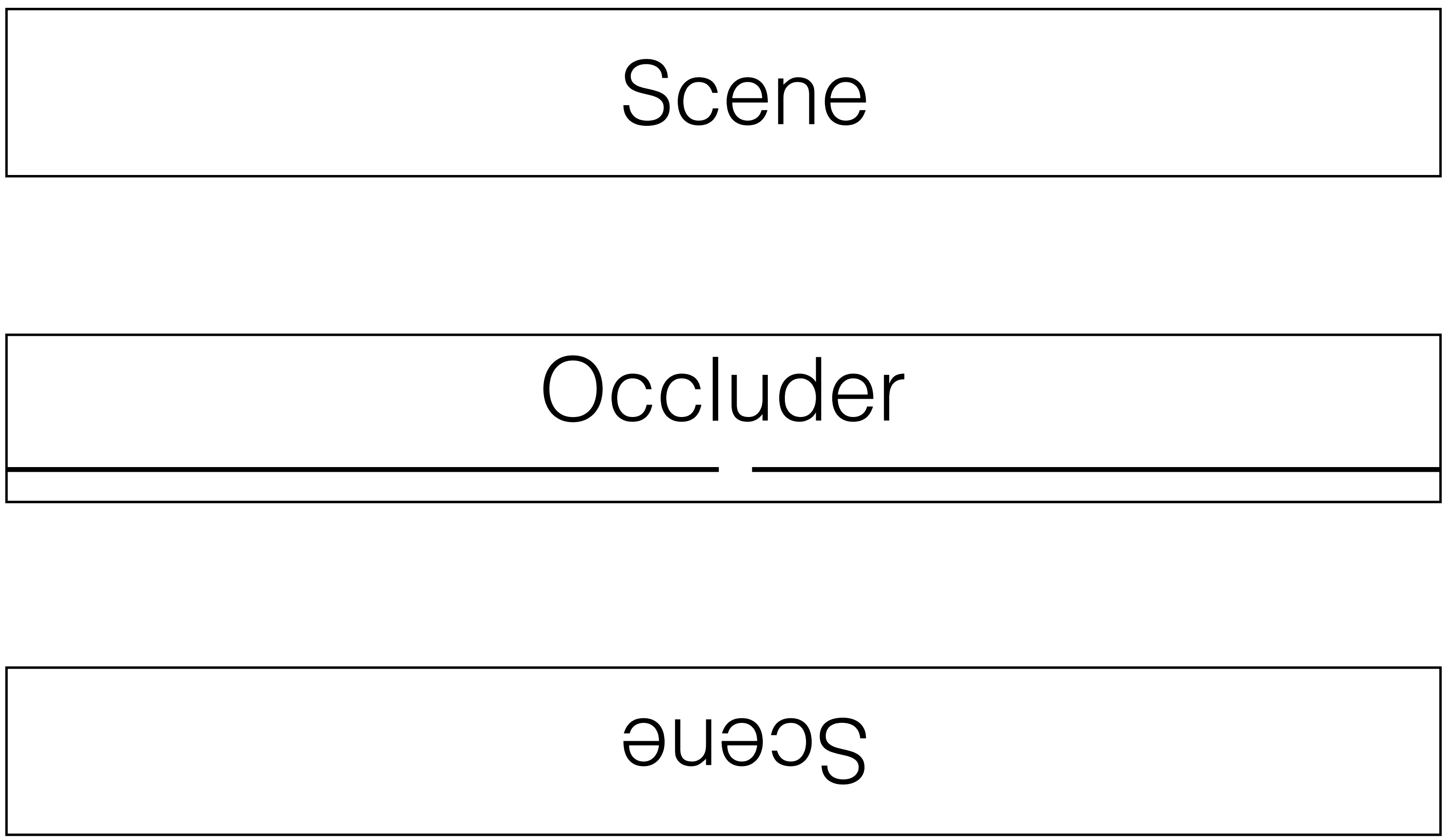
Occlusion imaging is a kind of imaging approach in which the pattern shadows cast by an occluder are used to infer a scene.

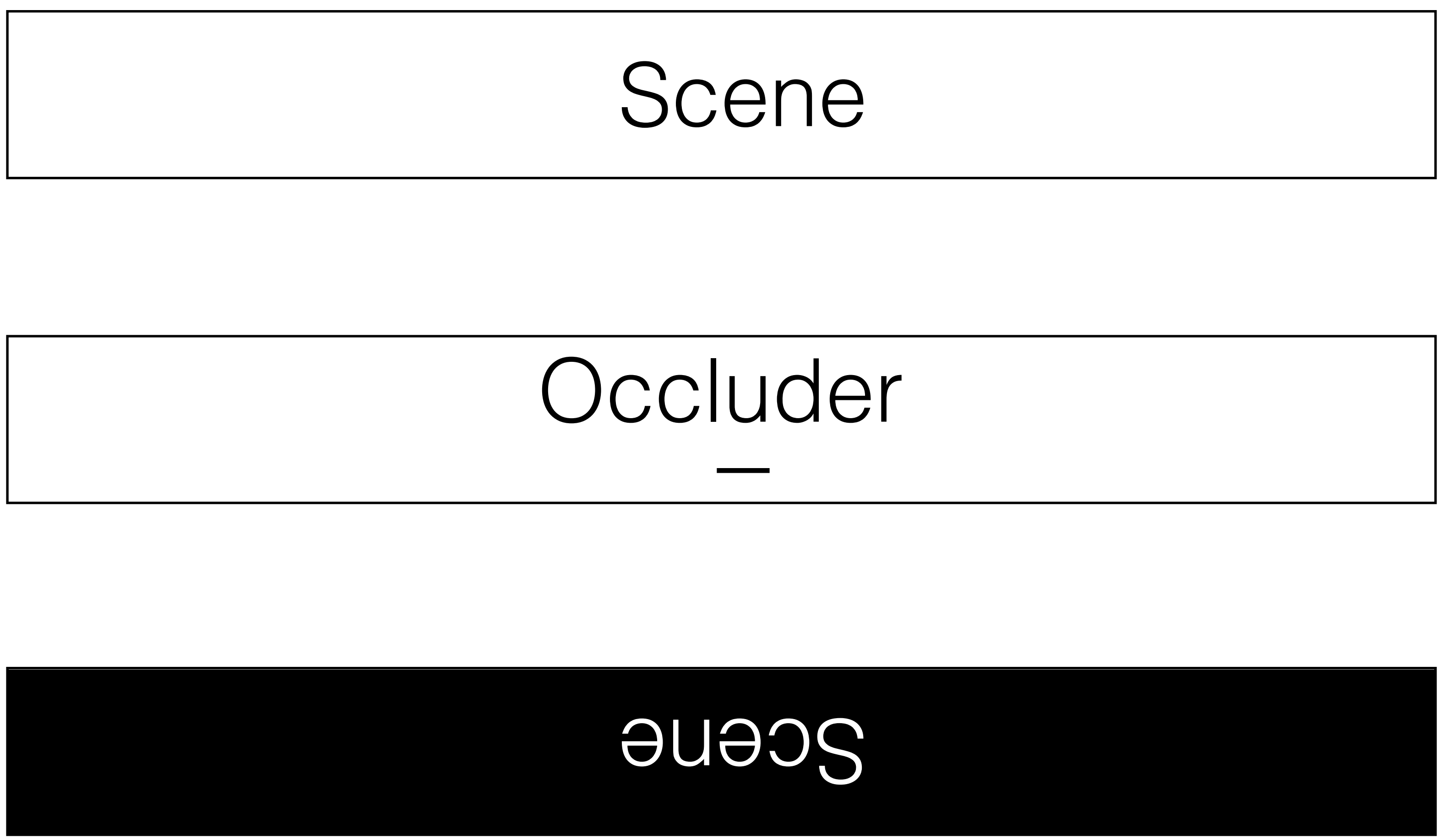


(In general, I'm going to ignore the intensity effects of distance and angle, and just take the sum of the light in the scene that is visible from each point on the wall as the intensity on the wall. This is equivalent to assuming d is large.)

There a few notable examples of occlusion imaging. One of them is the ancient and well-known idea of the pinhole camera.
https://en.wikipedia.org/wiki/Pinhole_camera



My research group is also interested in studying the reverse approach, called the “pinspeck camera.”
https://en.wikipedia.org/wiki/Pinspeck_camera



My group also is interested in the “edge camera.”

Scene

Occluder

Anti-derivative of scene

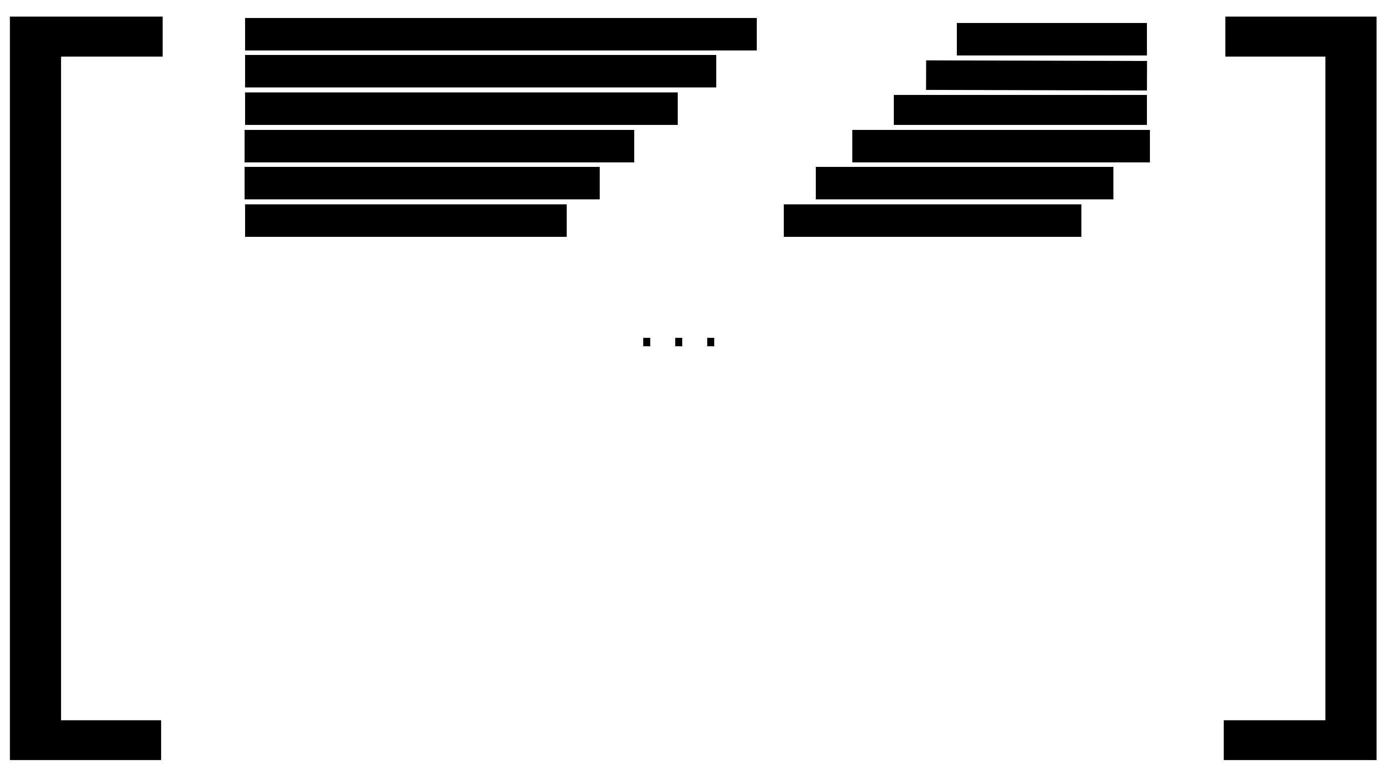
Anyway, solving this problem in the absence of noise for any old occluder isn't hard—because the image we see is just the scene convolved with the occluder, the solution is a simple deconvolution.

Nonetheless, it's plausible that some occluders yield better images, especially in the presence of noise. Is an edge occluder better than a pinhole occluder? Vice-versa?

My advisor wonders what the "best" occluder looks like. Obviously, this may depend on the specifics of the problem, signal-to-noise levels, etc… but it would be nice to have a sense of the solution.

One approach to rating an occluder's goodness could be to, for each scene in a set of orthogonal "basis scenes," find the resulting image we get.

If we arrange all these resulting image vectors into a matrix and study its properties, we can hopefully learn something about how good the occluder is.



And you've probably guessed what kind of matrix this is by now—because our view of the image is basically a "sliding window" of the occluder for the simplest set of basis scenes, the matrix is going to be circulant! (Or would be, if the occluder repeated itself once. Whatever. It's basically a circulant matrix.)

How can we use the aforementioned matrix to rate the goodness of a scene? Well, we can certainly look at the rank of the matrix to tell us the dimensionality of the space of distinguishable scenes. But that's not interesting enough. All of the occluder cameras I mentioned earlier—pinhole, pinspeck, and edge—will give you full-rank matrices here. And most random occluders will do the same.

So instead of taking the number of non-zero eigenvalues, I had the thought of using the product of the eigenvalues instead. We want the eigenvalues to not just be zero but as large as possible, since roughly speaking, the larger the eigenvalues are, the more noise we can tolerate before we starting getting a worse reconstruction.

At the same time, the product of the eigenvalues has the desirable property that any full-matrix will beat any non-full-rank matrix, etc.

So I figured this was a reasonable metric. And of course, the product of the eigenvalues of a matrix is its determinant. So that's how I got to the point of wanting to find the {0,1} circulant matrix with the largest determinant. For each row of the circulant matrix, the 1's in the row correspond to where there's no occlusion in the occluder slide, and the 0's correspond to where there is occlusion. The determinant of the matrix is how "good" that occluder is.

So I tried to find a formula for the $\{0,1\}$ circulant matrix with the maximum determinant at each size of matrix, and that's what led me to your work in Hadamard's maximal determinant problem.

Anyway, I hope that all made sense. Thanks a lot for all your advice! Please let me know if there's a hole in this approach somewhere (no pun intended) or something better I could be doing. For example, maybe the determinant isn't actually the right metric to use? (That would be a shame, since then I wouldn't be able to use all this fascinating research about the maximal determinant problem!)