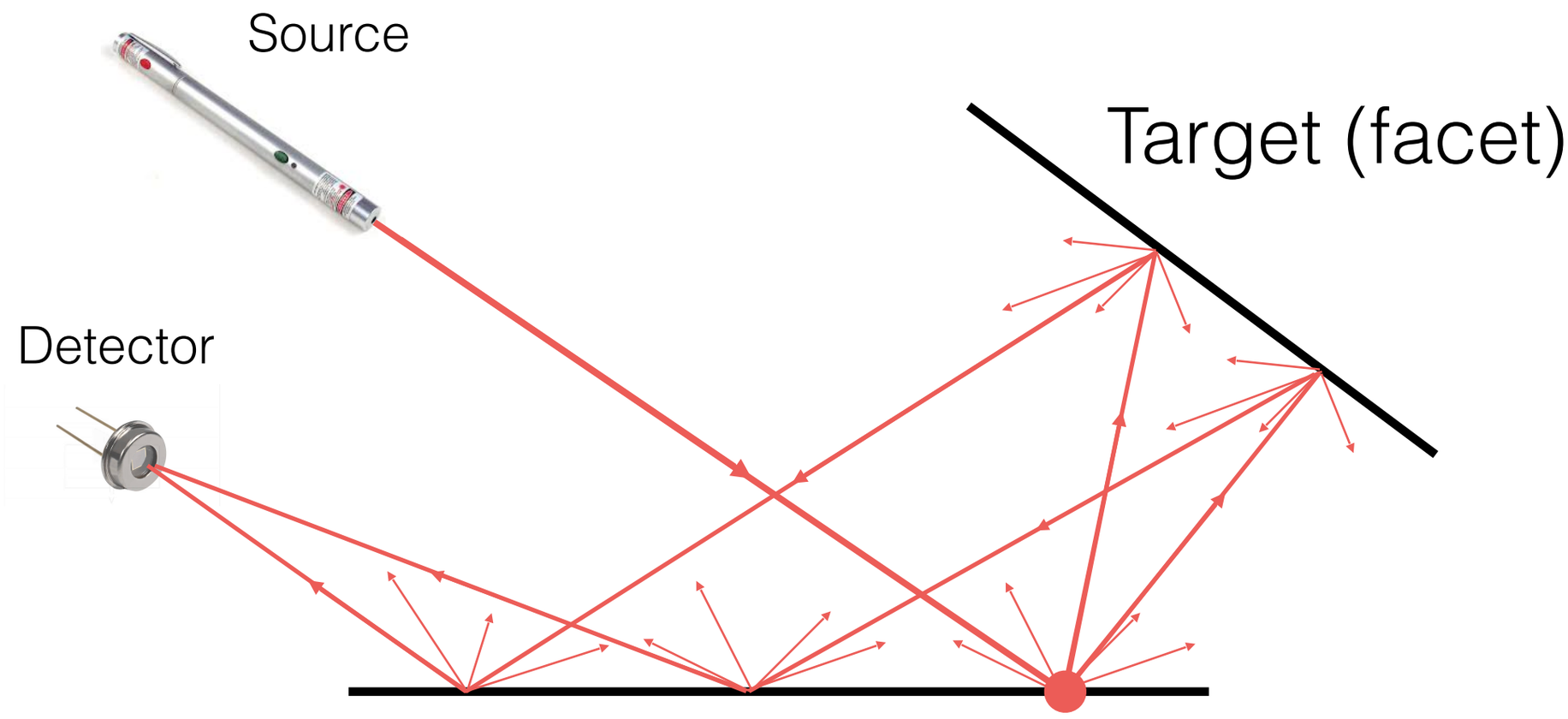
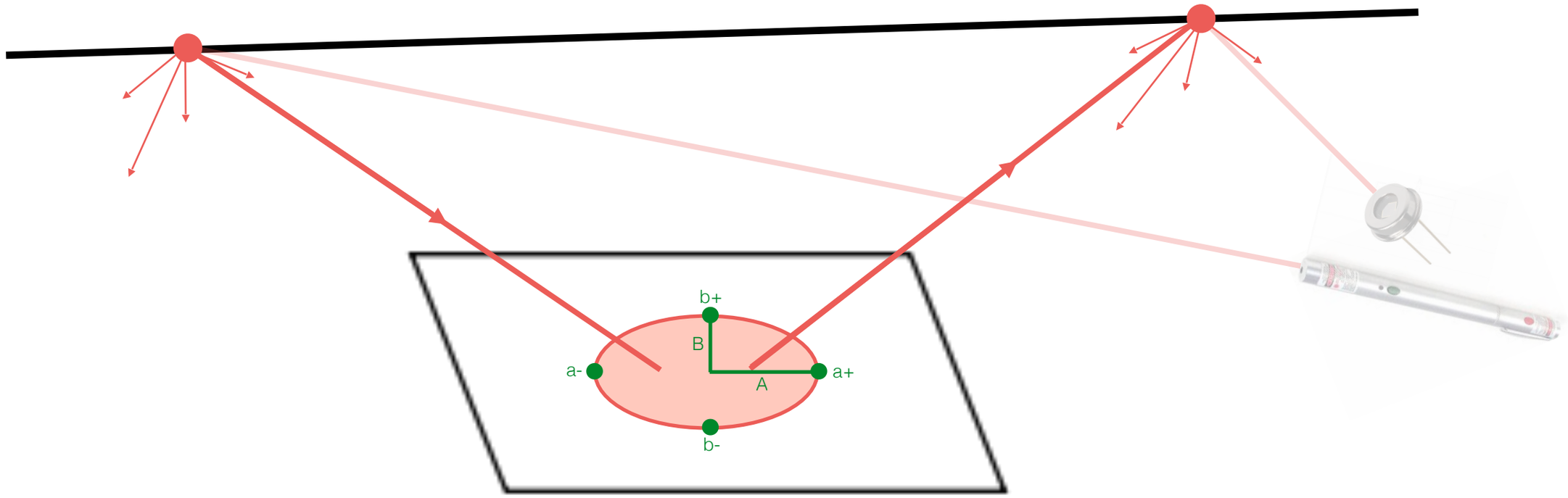
Scenario: Collimated source, unfocussed detector



Can we locate the target from a temporal sequence of observations at a single detector location and fixed source location?

We already have a forward model for the three-bounce problem with a directional source, a focused detector, and a single bounce on a locally linear (i.e. smooth) target. The solution comes from a solution to the *one*-bounce problem with a divergent source and an unfocused detector.



Target

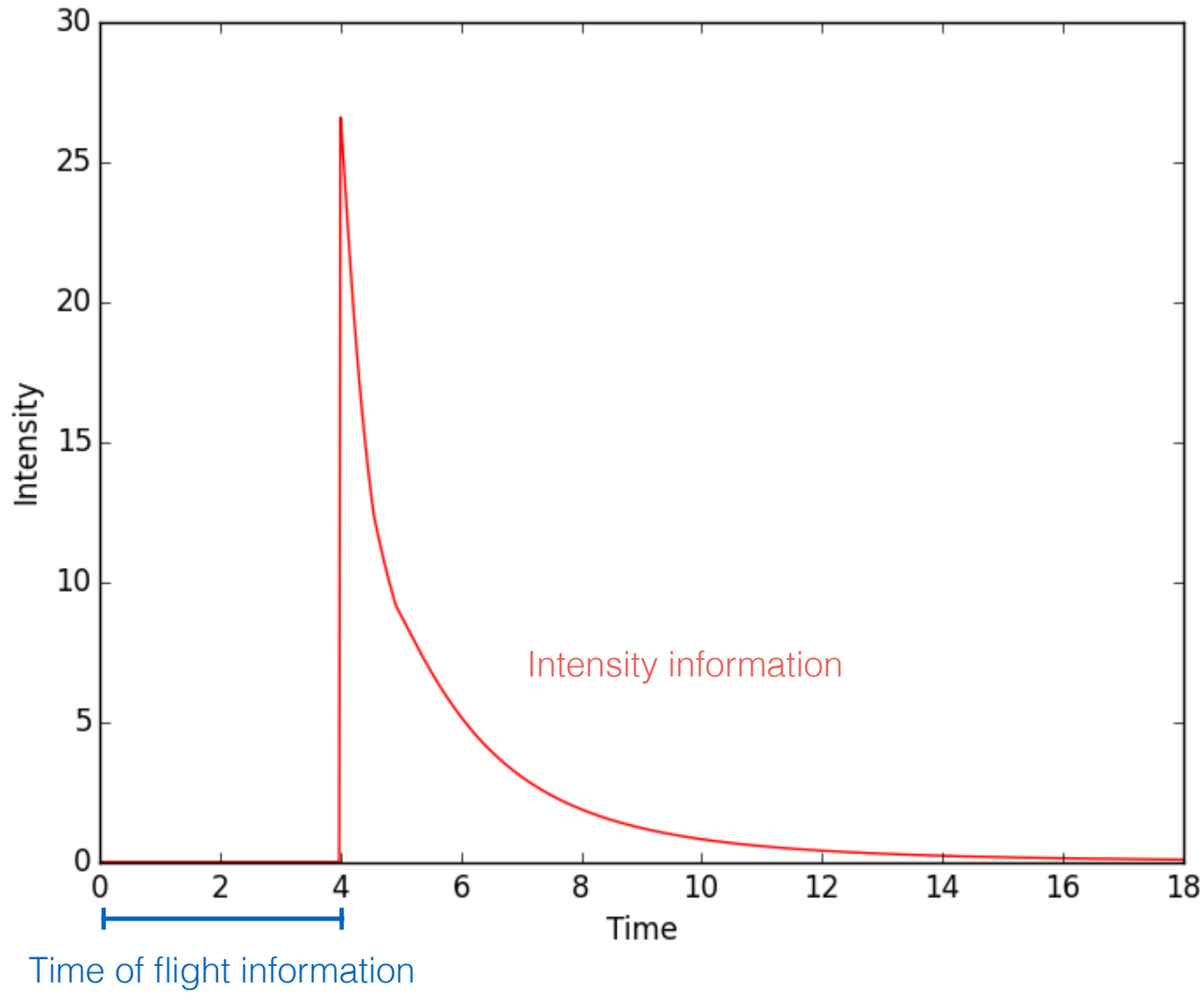
$$N_p(t) = \frac{d}{dt} N_c(t) \qquad I(t) = N_p(t) \cdot I_p(t)$$

$$N_c(t) = \frac{\pi A(t) B(t)}{(\Delta x)^2}$$

$$\hat{I}_p(t) = \frac{1}{4}\left(I_{a+}(t) + I_{b+}(t) + I_{a-}(t) + I_{b-}(t)\right)$$

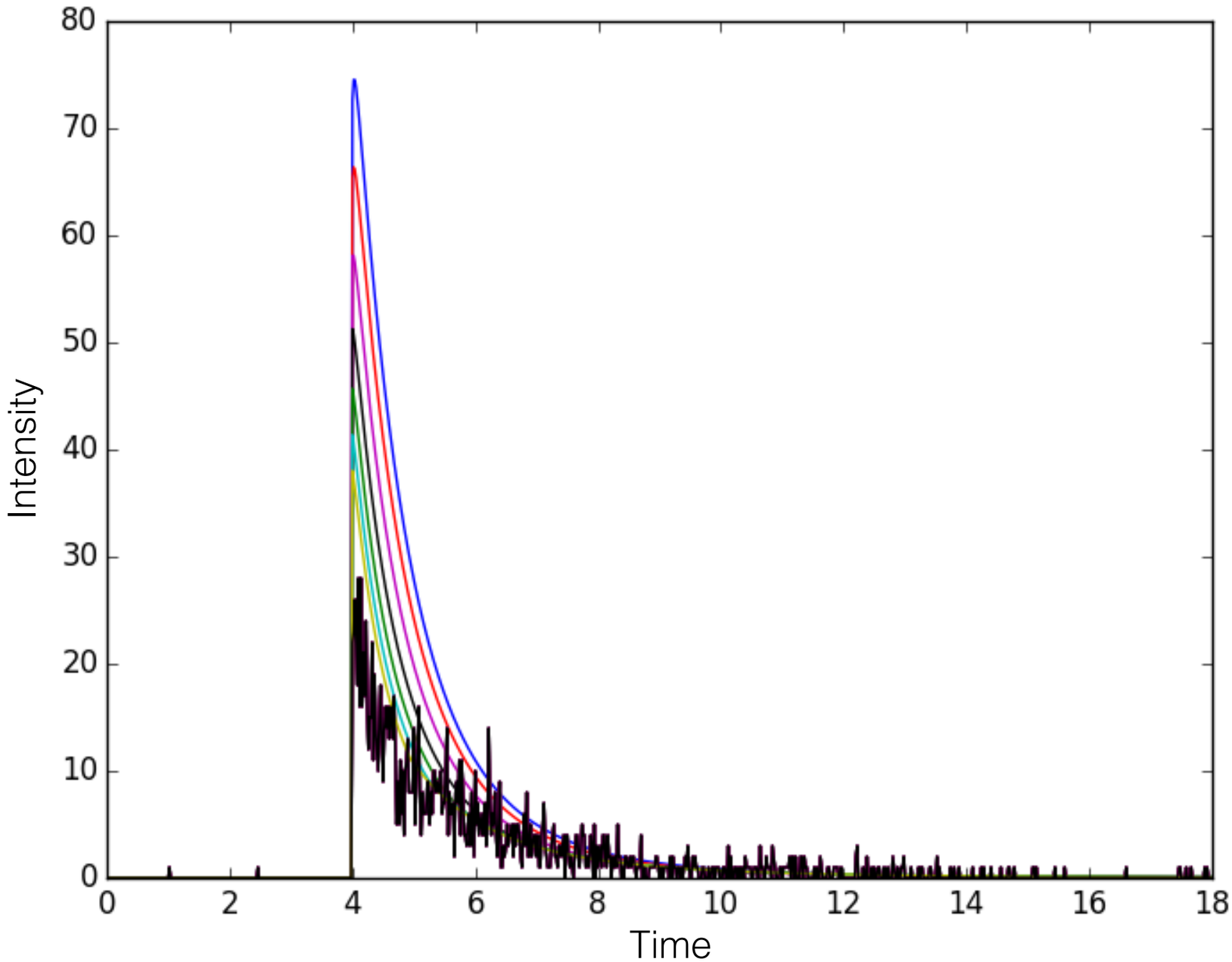
$$I_p^{p_w} = I_p^{p_s \to p_w} \cdot I_p^{p_w \to p_d}$$

Our forward model allows us to infer a temporal response from a candidate target location:



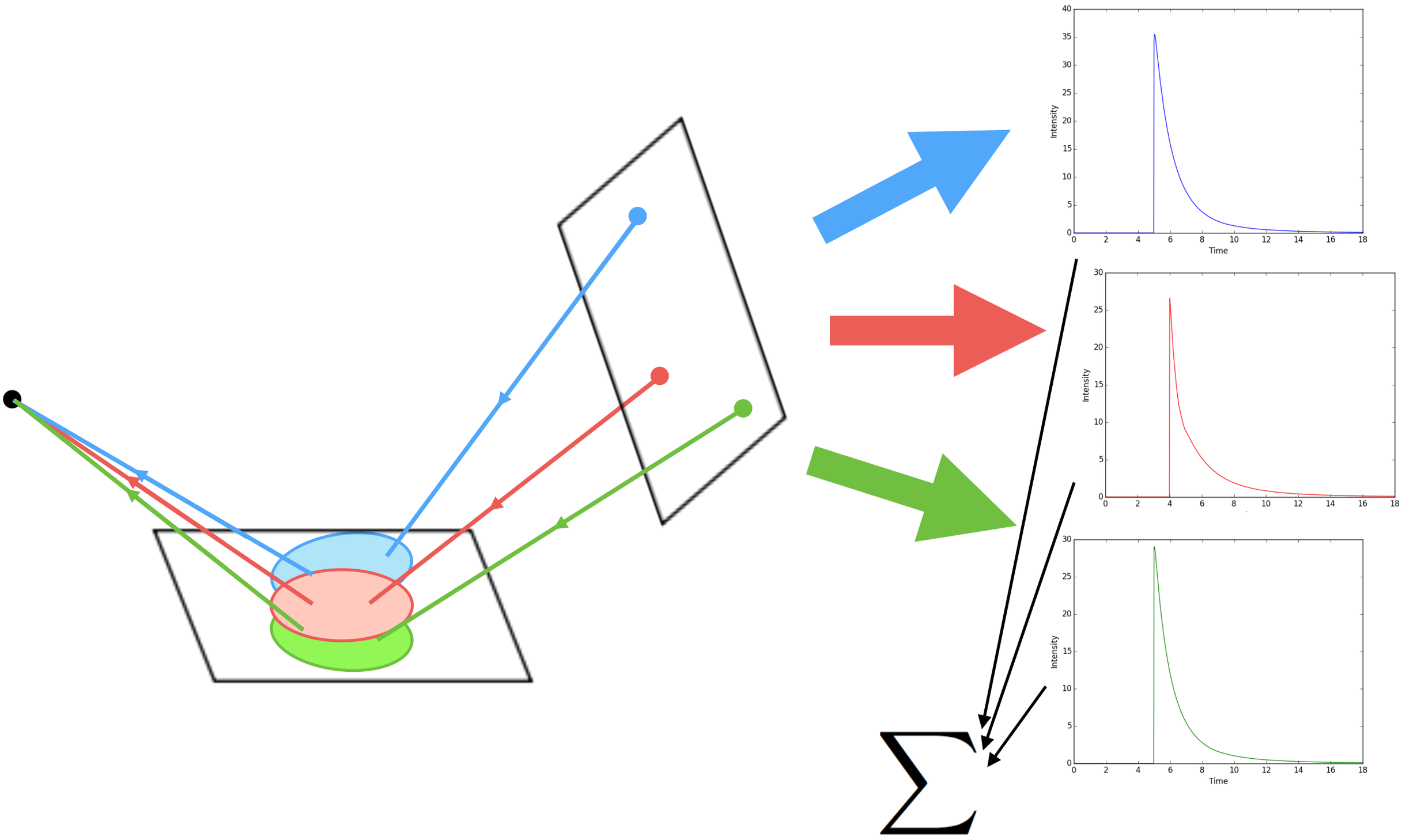
Having the full temporal response allows us to make use of *both* time-of-flight and intensity information—giving us an edge over methods that only use time-of-flight info!

Given a set of observations (black) we can perform a search over target locations that maximizes the likelihood of the observation given the target's location.

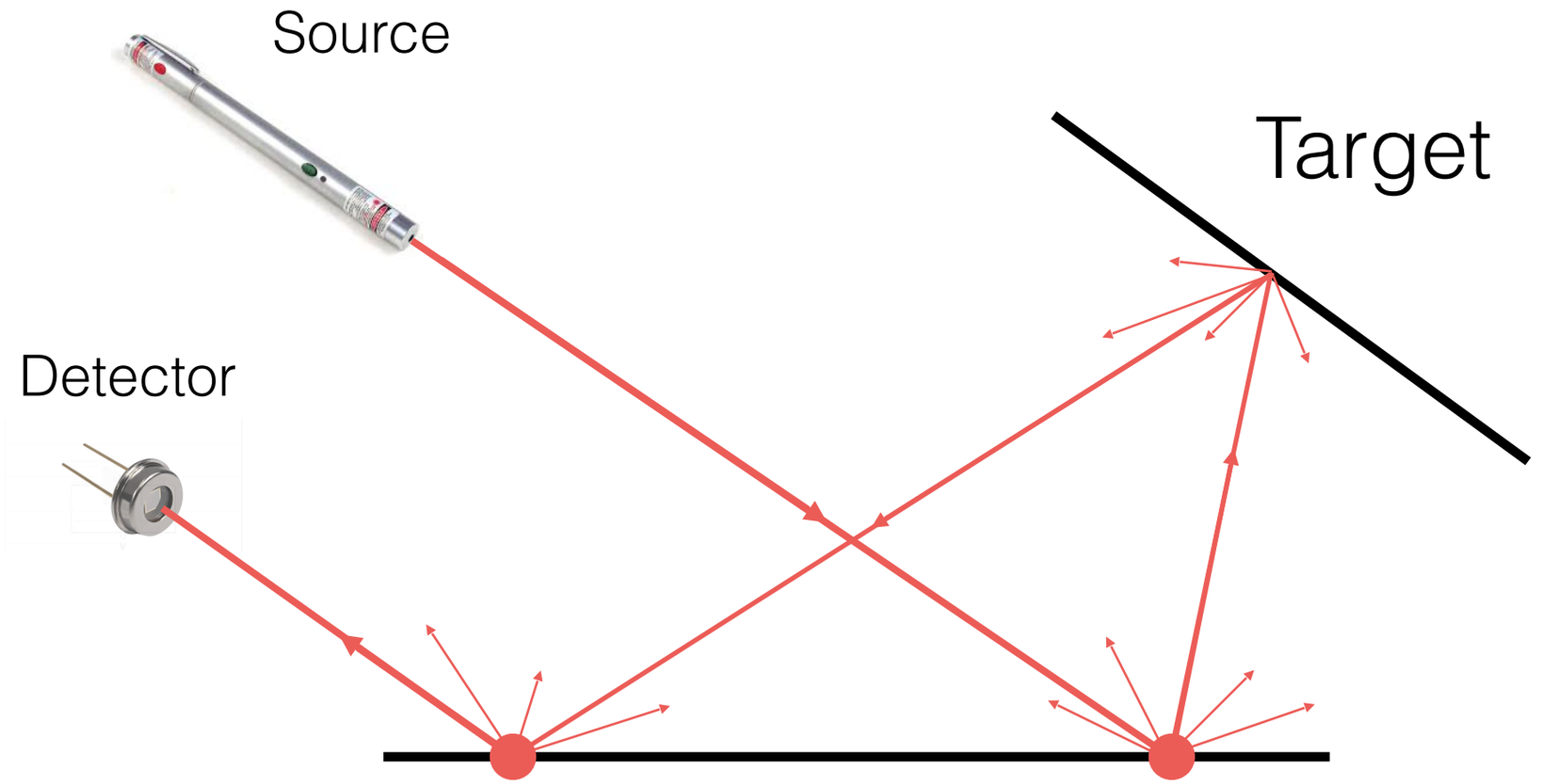


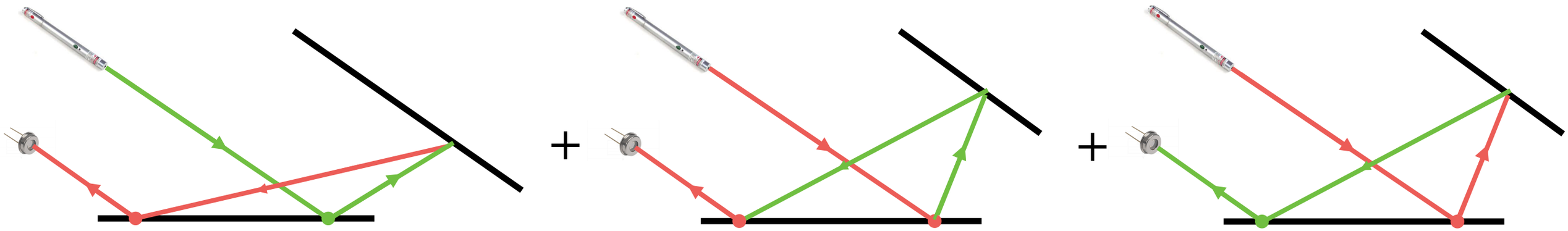
The intensity functions implied by our forward model at increasingly plausible target locations are in the other colors.

We can use that forward model to perform a sum over possible bounce points on the target wall, to get a forward model for the two-unfocussed-bounce case. Among other benefits, this lets us gather more light, letting us work at lower SNR.



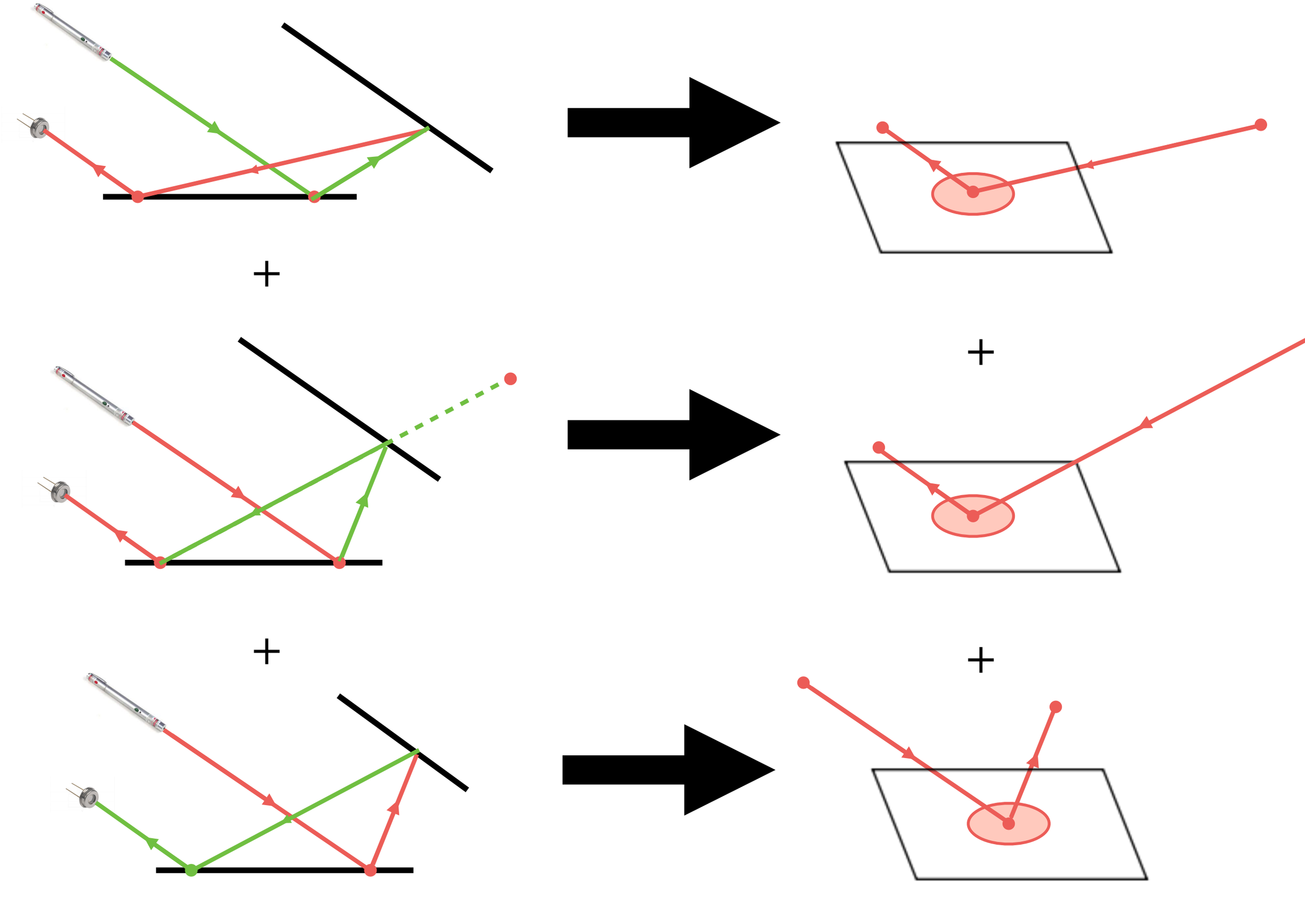
Also, the reflecting wall and target wall may have a specular component. This lets us separate the problem into a series of two-point problems.





We can approximate the overall partially-specular response as the sum of the responses if two all but two of the faces are specular, since those paths will dominate the overall response assuming a significant (>1%) specular component. Specular legs are shown above in green.

Each partially-specular sub-component can be approximated as a two-point problem, which we can add together to get a fast and accurate approximation of the true temporal response.



Source	Detector	Specularity	Can the target's location be recovered?
Collimated	Focused	None	Yes
Divergent	Focused	None	Yes, but it's hard
Collimated	Unfocused	None	Yes, but it's hard
Divergent	Unfocused	None	Maybe
Collimated	Focused	Partial	Yes
Divergent	Focused	Partial	Yes
Collimated	Unfocused	Partial	Yes
Divergent	Unfocused	Partial	Maybe
Collimated	Focused	Mirror-like	No, unless you're really lucky
Divergent	Focused	Mirror-like	Maybe
Collimated	Unfocused	Mirror-like	Maybe
Divergent	Unfocused	Mirrorlike	Maybe