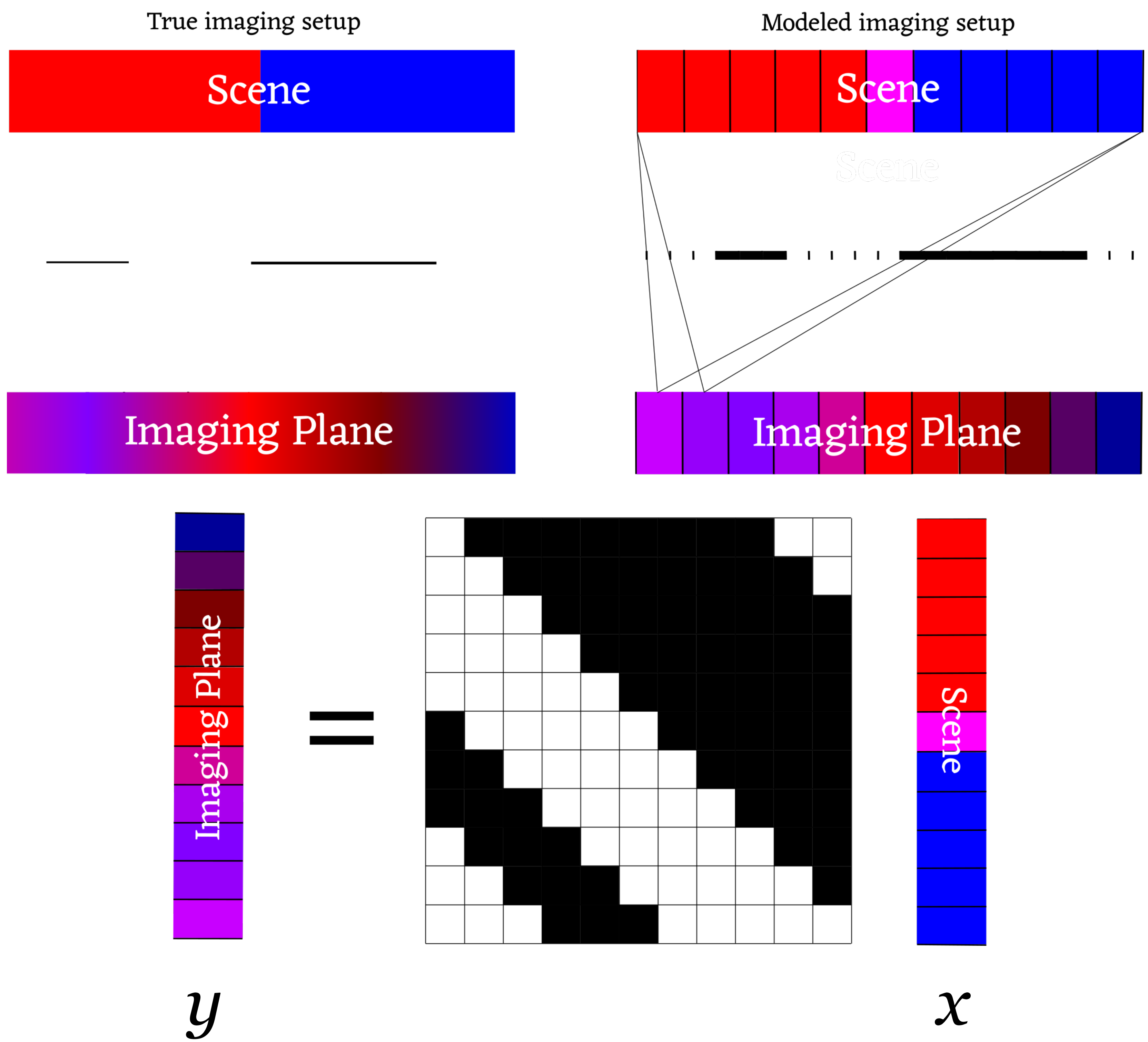
We can model a continuous scene as a discrete array of intensity values, and an occluder mask as a discrete on/off sequence. In this way, we can get a Toeplitz transfer matrix by considering the form of the occluder mask.



If we assume the net power radiated by the scene is J, and the net power of ambient ("thermal") sources is W, then we can characterize the mutual information between the scene and observation plane:

Scene covariance matrix

Transfer matrix

$$\mathcal{I} = \log\det\left(\frac{1}{n}\frac{A^T Q A}{\underbrace{pJ + W}_{\text{Overall noise variance}}} + I\right)$$

We can make progress by considering the determinant as the product of eigenvalues:

Singular values of Q

Eigenvalues of $A^T A$

$$\mathcal{I} = \sum_{i=1}^{n} \log\left(\frac{1}{pJ + W} \cdot d_i \cdot \frac{|\lambda_i(\mathbf{A})|^2}{n} + 1\right)$$

If we assume a circulant transfer matrix A, that gives us a lot of information about the form of its eigenvalues:

$$\lambda_j = c_0 + c_{n-1}\omega_j + c_{n-2}\omega_j^2 + \ldots + c_1\omega_j^{n-1}, \qquad j = 0, 1, \ldots, n-1.$$

where ω_j is the j^{th} root of unity.

The previous slide's formula for the mutual information (expressed as the sum of logs of the norm-squared of the eigenvalues) tells us that we should want to spread our power exactly evenly across the frequency spectrum, if $Q = I$ (such as with a maximum-length sequence). For general covariance matrices Q, we want:

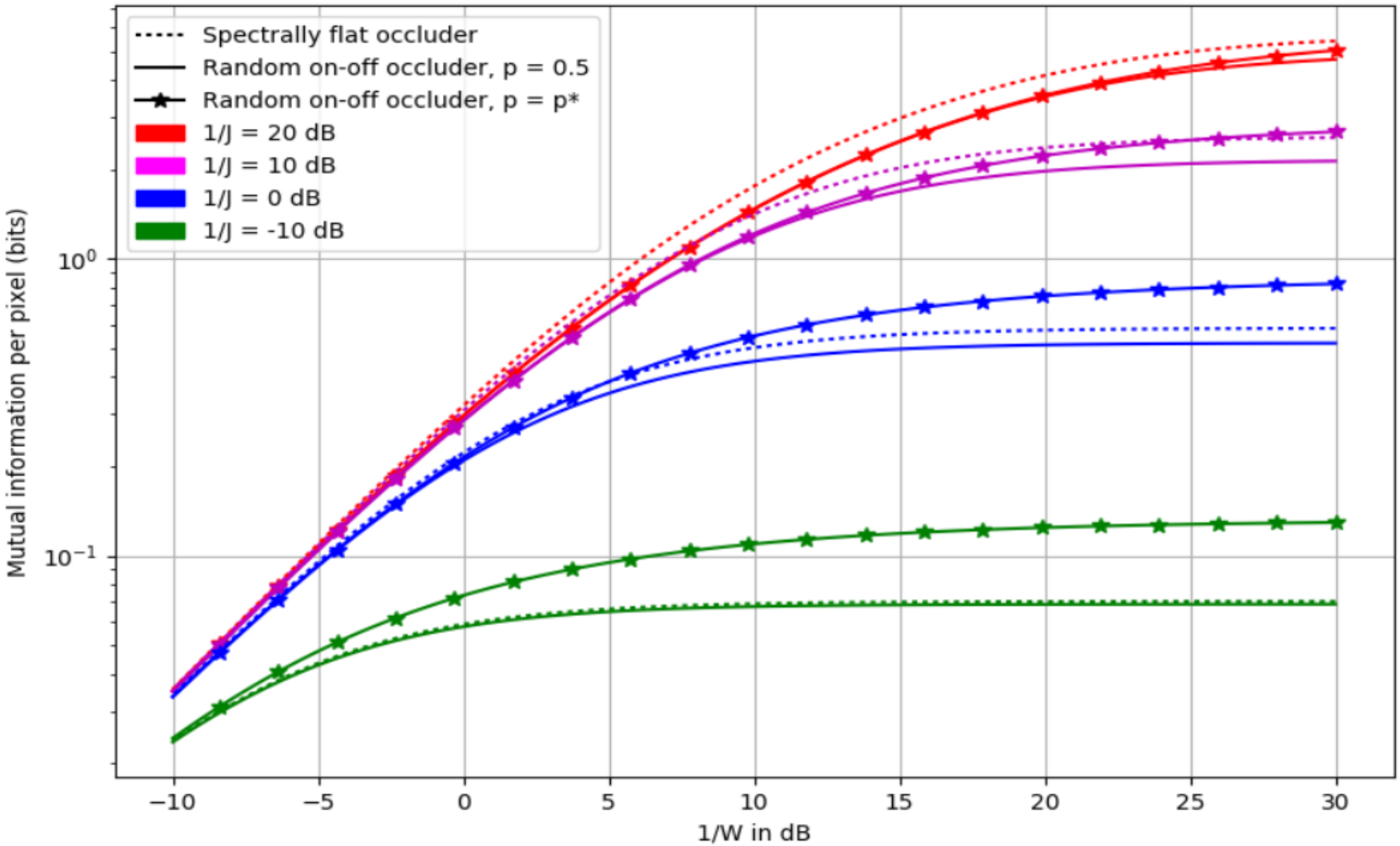
$$\lambda_2 d_2 \;=\; \lambda_3 d_3 \;=\; \ldots \;=\; \lambda_n d_n.$$

(The DC term λ_1 is equal to np regardless of the contribution of the other frequencies, and contributes negligibly to the mutual information for large enough n.)

If we instead consider a random Bernoulli mask where each part of the mask is transmissive with i.i.d. probability p, we can instead conclude from random matrix theory that:

$$\bar{\tilde{\mathcal{I}}}_p = \mathbb{E}_X\left[\log\left(\frac{p(1-p)}{W+pJ}X^2 + 1\right)\right].$$

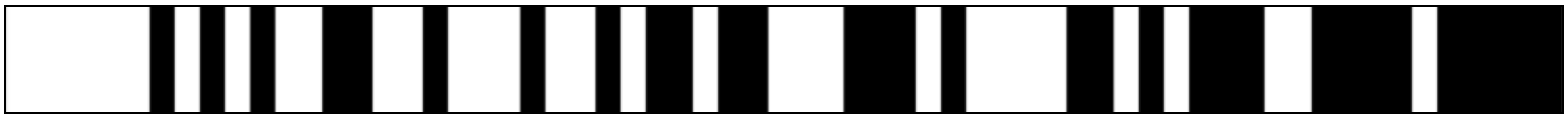
$\bar{\tilde{\mathcal{I}}}_p$ is the normalized MI per pixel, X is a random variable distributed with density $f_X(x) \;=\; |x|e^{-x^2}$. Thus in passive settings ($W >> J$) we have $p^* = 0.5$ for IID scenes.



A plot of the MI per pixel of a spectrally-flat occluder, a random on-off occluder with $p = 0.5$, and a random on-off occluder with optimally chosen $p^\star$. n has been taken to infinity, and scenes are presumed to be IID

Takeaways:

-Given an IID scene, a mask whose transmissivity is spectrally flat (i.e. conforming to a maximum length sequence or MURA code) is optimal.



An example of a spectrally-flat sequence for n *= 64.*

-Given an IID scene, a mask whose transmissivity is spectrally flat (i.e. conforming to a maximum length sequence or MURA code) is optimal in the limit of high SNR.

-If the noise is dominated by thermal (“ambient”) noise sources, such as in an outdoor NLoS setting, optimal masks will have average transmissivity 0.5, but if the noise is dominated by shot noise sources, such as in an actively-illuminated dark-room NLoS setting or a designer-camera setting, optimal masks will have lower average transmissivity.

-For correlated scenes, the power spectrum of the optimal mask will depend inversely on the average power spectrum of the expected scenes.