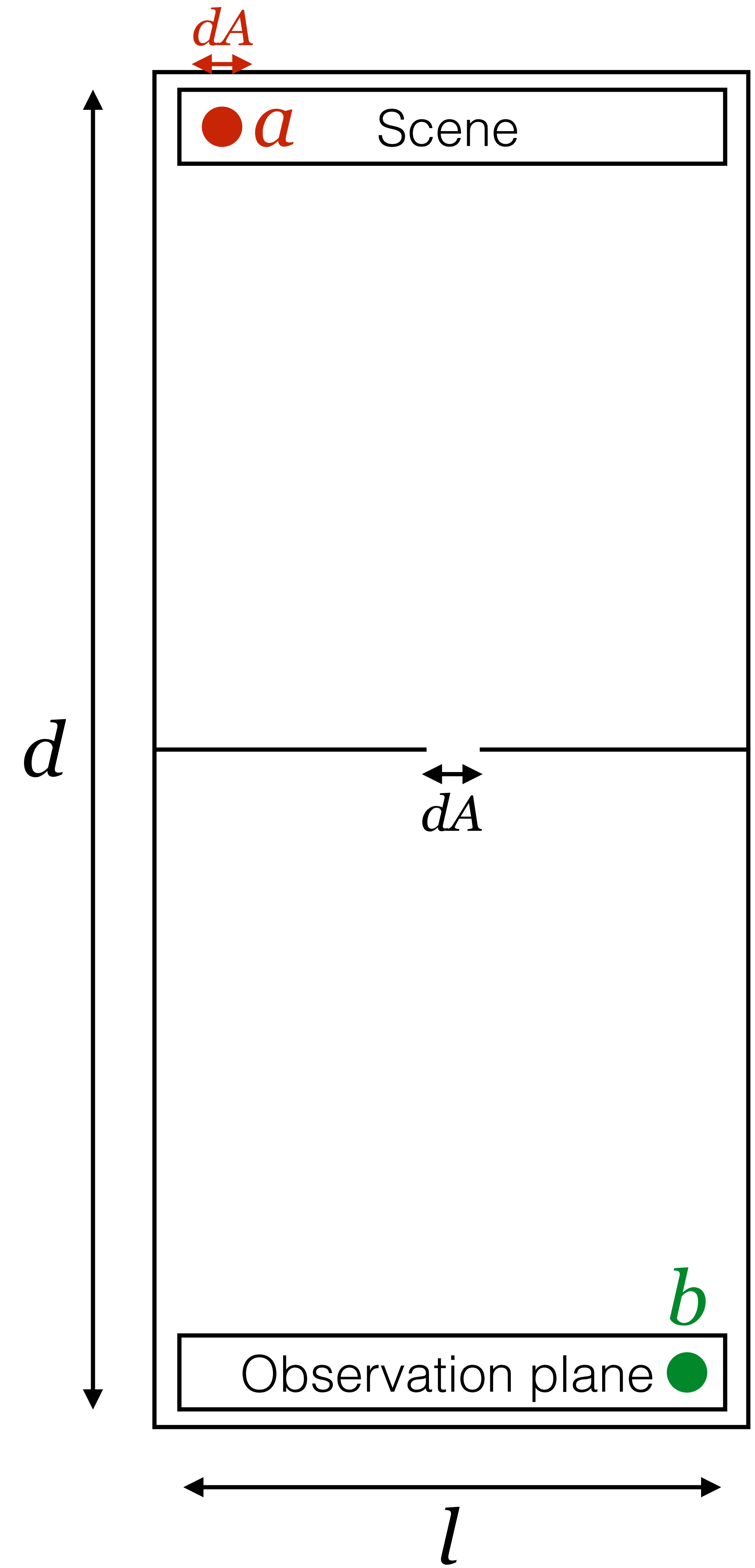
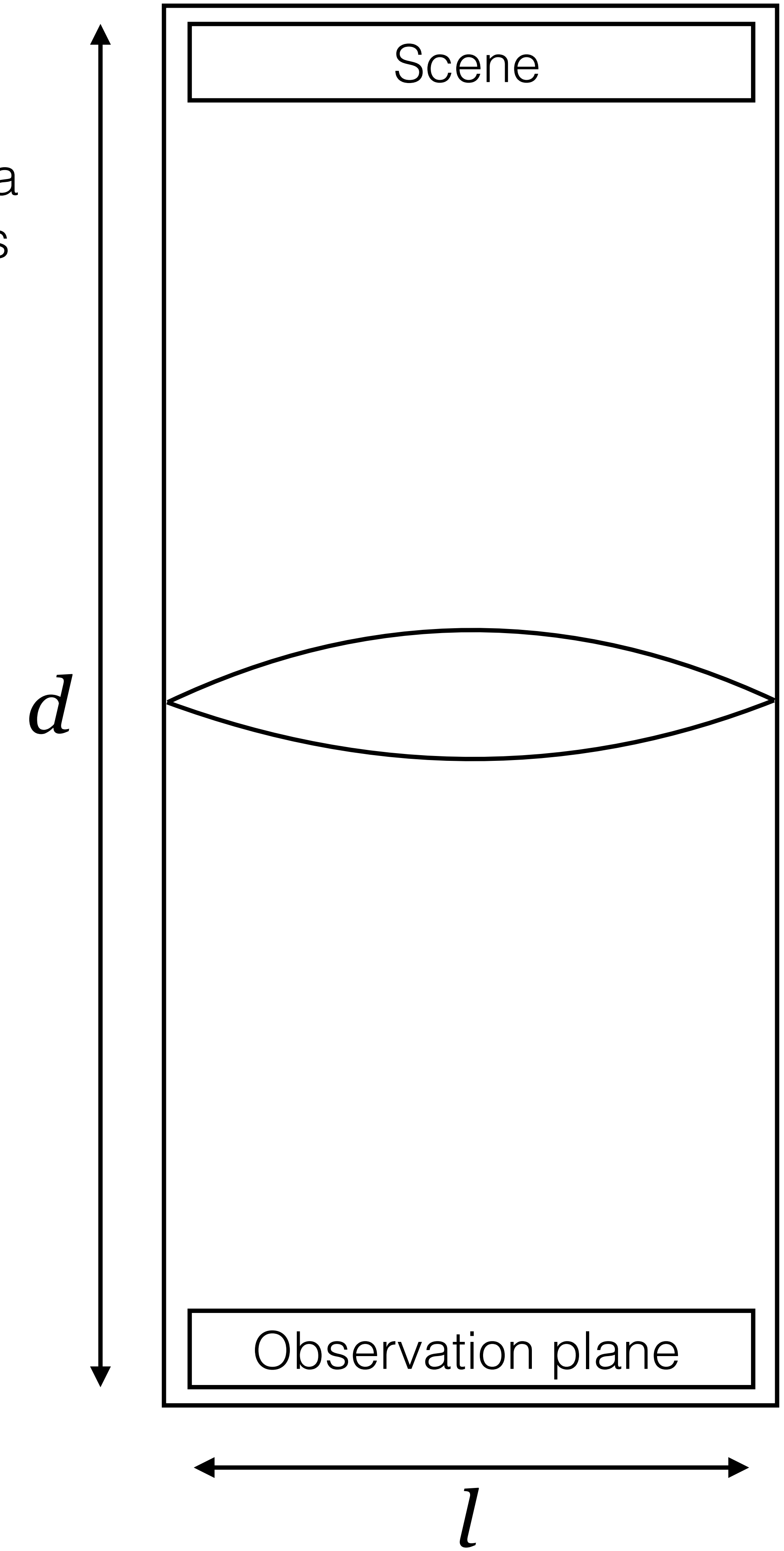
I said earlier that the transfer matrix of a pinhole camera is the identity matrix. As you're pointing out, that's inexact. In fact, if the pinhole has area $dA$, then a little patch $a$ of area $dA$ in the scene of intensity $J$ illuminates an opposite patch $b$ with intensity $J\,dA/(4\pi d^2)$ on the observation plane, because only one part in $dA/4\pi d^2$ of the light in that patch on the scene makes it through the pinhole. So in fact, the transfer matrix of a pinhole camera is $I\,dA/(4\pi d^2)$, where $I$ is the identity matrix.

(We're still making the far-field assumption here—$d >> l$—without which a pinhole camera would *definitely* not give you the identity matrix!)

(Also, I'm assuming we're working in a three-dimensional world, and the kaleidoscope on the right is a cylinder that we're seeing the cross-section of. If we were working in flatland, the numbers would change slightly, but the basic principle would remain the same.)



But what if our occluder frame were a lens, instead of a pinhole? Well, then, all the light coming from a that hits the lens now makes it to b. How much of the light coming from a hits the lens? Still assuming l >> d, the area of the lens is $\pi\, l^2/4$, so $(\pi\, l^2/4)/(4\, \pi\, (d/2)^2)$ of the light would from the patch would hit the lens. That means our transfer matrix will be $I\, l^2/4d^2$.



So before, we were assuming that $A$ was a {0,1} matrix. What we have now instead is a $\{0, q\}$ matrix, for two different values of $q$ (one for the pinhole, one for the lens). And because in both cases, $A$ is a scaled identity matrix, the expression

$$\frac{AA^T}{\sigma^2} + I$$

is going to simplify nicely. If we recall the following:

$$I(x, y) = \log(\det(\frac{AA^T}{\sigma^2} + I))$$

then we can write:

Pinhole: $I(x, y) = \log((\frac{q_p^2}{\sigma^2} + 1)^n)$ where $q_p = \frac{dA}{4\pi d^2}$

Lens: $I(x, y) = \log((\frac{q_l^2}{\sigma^2} + 1)^n)$ where $q_l = \frac{l^2}{4d^2}$

One way to think of this is that I was sort of implicitly handwaving away the factor of

$$q_p = \frac{dA}{4\pi d^2}$$

that you get in these kinds of problems, when I told you that (for example) the pinhole matrix was the identity, or the edge matrix was the upper-triangular matrix of 1's. The nice thing is, this factor is the same for all the occluder-based cameras, so it won't change the calculations of the quality of these occluders relative to each other, or relative to the bound.

But that doesn't mean it won't change the specific values for the number of bits you'll get, or the performance of these occluder-baser cameras relative to a lens-based camera. You're right; it's best to be exact about these things. I'm going to redo the calculations of the bound, with this extra factor $q_p$. As a template, I'll copy-paste a slide from the slides I sent you earlier.

We can apply this to our earlier formula to get a bound on the mutual information ***I(x, y)***. We know that the transfer matrices $A$ we get from occluders are Toeplitz {0,1} matrices. Thus, we know that

$$\frac{AA^T}{\sigma^2} + I$$

is itself a matrix with entries in between 0 and $\frac{n}{\sigma^2} + 1.$

This tells us that

$$I(x,y) = \log(\det(\frac{AA^T}{\sigma^2} + I)) \leq \log(\frac{n^{\frac{n}{2}}(\frac{n}{\sigma^2} + 1)^n}{2^{n-1}})$$

Simplifying, we have

$$I(x,y) \leq \frac{n}{2}\log n - n + 1 + n\log(\frac{n}{\sigma^2} + 1)$$

(Recall that $n$ is the side-length of $A$. In other words, it's the number of pixels of resolution on our camera.)

We’re just going to want to revise that slide as follows (revisions highlighted in red).

We can apply this to our earlier formula to get a bound on the mutual information $I(x, y)$. We know that the transfer matrices $A$ we get from occluders are Toeplitz $\{0, q_p\}$ matrices. Thus, we know that

$$\frac{AA^T}{\sigma^2} + I$$

is itself a matrix with entries in between 0 and $q_p^2 \frac{n}{\sigma^2} + 1.$

This tells us that

$$I(x, y) = \log(\det(\frac{AA^T}{\sigma^2} + I)) \leq \log(\frac{n^{\frac{n}{2}}(\frac{nq_p^2}{\sigma^2} + 1)^n}{2^{n-1}})$$

Simplifying, we have

$$I(x, y) \leq \frac{n}{2} \log n - n + 1 + n \log(\frac{nq_p^2}{\sigma^2} + 1)$$

(Recall that $n$ is the side-length of $A$. In other words, it's the number of pixels of resolution on our camera.)

One more note: this bound applies only to pure occluder-based approaches! Central to the bound is the assumption that our transfer matrix $\boldsymbol{A}$ can contain no entries greater than $\boldsymbol{q_p}$. Lens cameras violate that assumption, as you point out. So would mirrors in the occluder plane, or anything that redirects the light; anything that allows more than a $\boldsymbol{q_p}$ fraction of the light to get from point $\boldsymbol{a}$ to point $\boldsymbol{b}$.

I hope this helps to clarify things!