Statistical Analysis

JOHNS HOPKINS
CAREY BUSINESS SCHOOL

Statistical Process Control

1. What is Quality?
2. History of the Quality Movement
3. Total Quality Management
4. Control Charts
   - x bar and R charts
   - p charts and c charts
5. Process Capability

What is Quality?

- Perfection
- Consistency
- Waste elimination
- Speed of delivery
- Provide a good and usable product
- Compliance with policies and procedures
- Doing it right the first time
- Delighting or pleasing customers
- Total customer service and satisfaction

Quality from Manufacturing Point of View

- Quality
  - Fitness for use
  - Extent to which customer expectations are met
- Types of quality
  - Quality of design
  - Quality of conformance
  - Quality of performance

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Quality from Customers’ Perspective

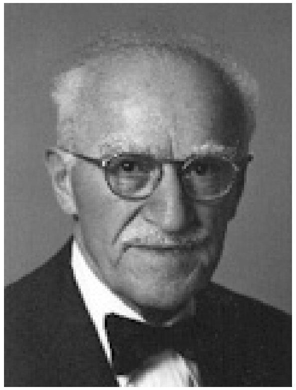
- *Operation*- Does the product do what it is designed to do?
- *Reliability and Durability*- this reflects the probability of a products’ failing or deteriorating
- *Conformance*- the degree to which the product meets specifications
- *Serviceability*- speed and accuracy of repair
- *Appearance*- perceived quality (subjective) the look, the touch, and the feel of the product
- *Perceived Quality*- brand name, or image of the product

History of the Quality Movement

1924	Statistical Quality Control/Control Charts, Shewart/Bell Labs
Late 20’s	Statistical Acceptance Sampling, Bell Labs
1946	American Society for Quality Control created
1950	W. Edwards Deming introduces statistical quality control in Japan
1951	Deming Prize established in Japan
1980’s	Total Quality Management (TQM)
1988	Malcolm Baldrige National Quality Awards established in the U.S.
1990’s	ISO 9000, international quality standards adopted

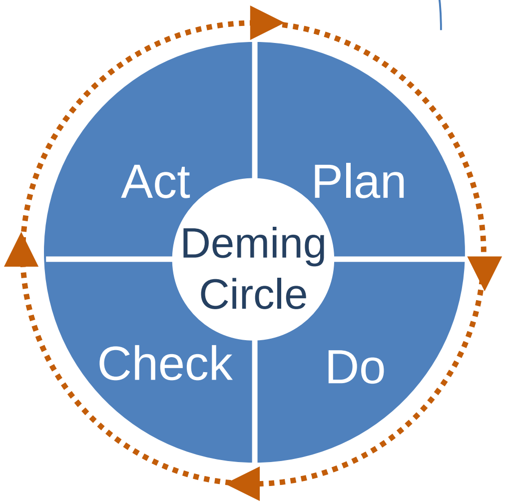
Influential Figures of Quality Control

Joseph Juran
1904 – 2008



W. Edwards Deming
1900-1993

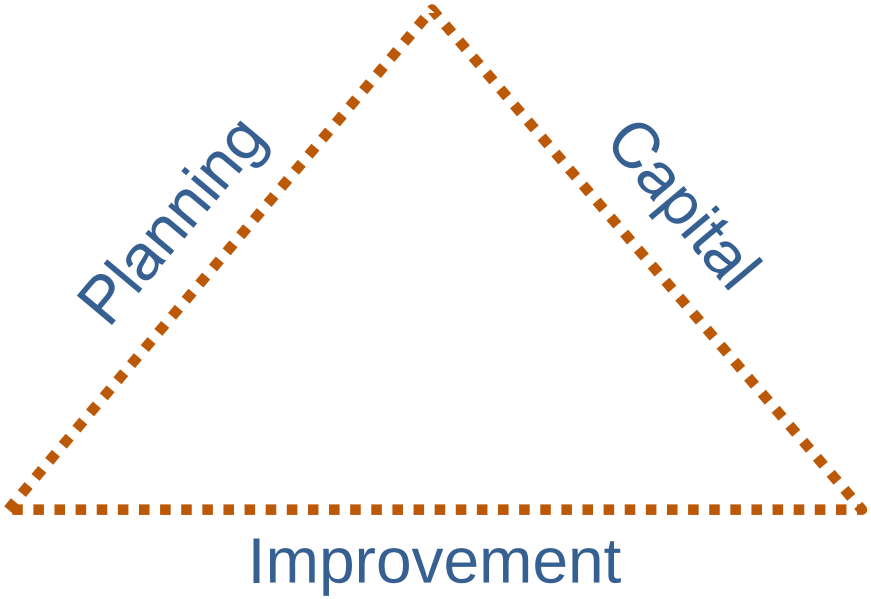
Kaoru Ishikawa
1915 – 1989



Joseph Juran - Quality Trilogy

Movement toward proactive prevention, process oriented, etc.

- Planning
- Capital
- Improvement



W. Edwards Deming's 14 Points

1. Create constancy of purpose toward improvement of product and service with a plan to become competitive, stay in business, and provide jobs
2. Adopt a new philosophy
3. Cease dependence on mass inspection
4. End the practice of awarding business on the basis of price tag
5. Improve constantly and forever the system of production and service to improve quality and productivity
6. Institute training
7. Institute leadership
8. Drive out fear, so that everyone may work more effectively for the company

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W. Edwards Deming's 14 Points *continued*

9. Break down organizational barriers
10. Eliminate slogans, exhortations and arbitrary numerical goals and targets for the workforce which urge the workers to achieve new levels of productivity and quality without providing methods
11. Eliminate work standards and numerical quotas
12. Remove barriers that rob employees of their pride of workmanship
13. Institute vigorous program of education and self-improvement
14. Take action to accomplish the transformation

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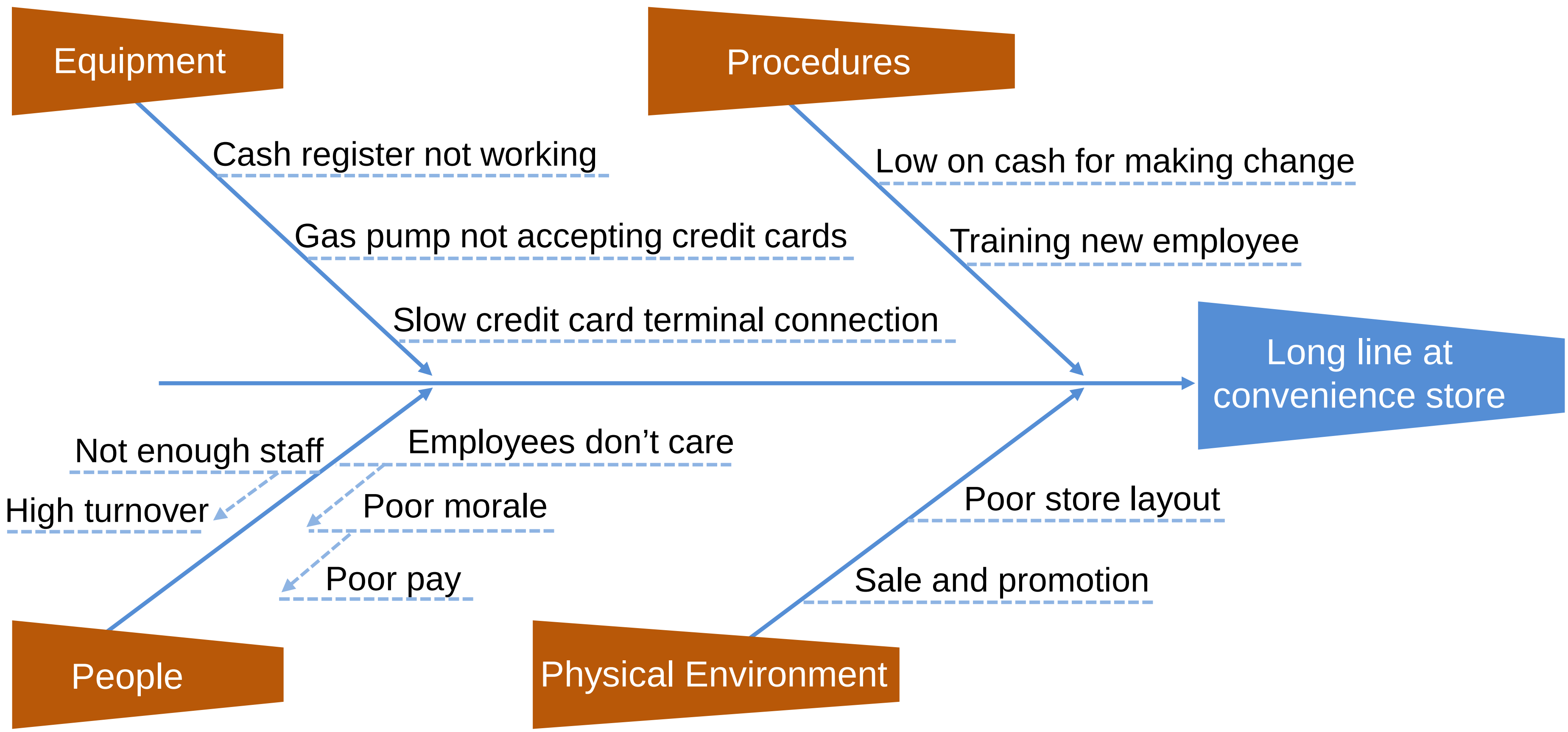
Ishikawa’s Cause-and-Effect Concentration Diagram

A cause-and-effect diagram for “long line at convenience store”

Also known as fishbone chart



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ISO 9000

- Series of international quality standards
- Establishes structures and processes for quality control systems at every step of the production process – design, raw materials, in-process monitoring, and so on
- Imposes quality discipline
- Broad acceptance internationally

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Malcolm Baldrige National Quality Awards

- First presented in 1988
- Presented by the U.S. Commerce Department
- Named for late Malcolm Baldrige
- Established to
  a) Promote quality awareness
  b) Recognize quality achievements by US companies
  c) Publicize successful quality strategies
- Past winners include Motorola, Federal Express, 3M, Ritz-Carlton.

Total Quality Management (TQM)

TQM is comprised of

- Statistical Process Control
- Shewhart Charts
- Control Charts
- Six Sigma
  - Define
  - Measure
  - Analyze
  - Improve
  - Control

Natural variation is normal random variation versus a cause or reason to make an adjustment to the process.

Quality Control (QC)

Historical inspection approach

- Inspection of output
- Action on output
- Scrap, rework, downgrade (expensive!)

Statistical Process Control (SPC)

- Monitor and study process variation
- Goal: Continuous process improvement

Process Variability

Back to statistics again

Assignable variation: caused by material, tools, worker related problems.

Common variation (random variation): caused by type of process, equipment etc.

If we eliminate assignable variation, we will bring the process within control. The process itself may still produce the variation due to common causes.

Process Variability *continued*

Products from any process have variability and a dimension of the product we are interested in may have certain probability distribution with mean μ and standard deviation σ.

We estimate these parameters (μ and σ) using random samples. From samples we can calculate sample mean ($\bar{x}$) and standard deviation (s).

Common and Assignable Causes

1. When a process is influenced only by common cause variation, the process will be in statistical control.

2. When a process is influenced by one or more assignable causes, the process will not be in statistical control; such as an overused pattern, worn-out or broken part, defective materials, change in operator.

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Sampling a Process and Rational Subgrouping

Must decide which process variables to study

- Best to study a *quantitative variable* (meaning we employ *measurement data*)

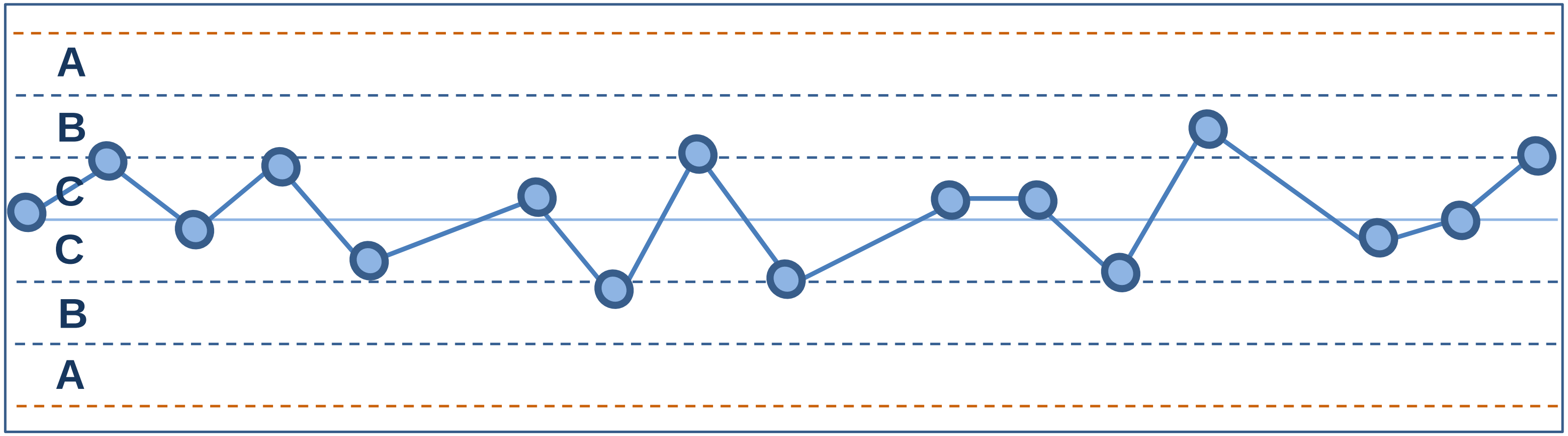
We will take a series of samples over time

- Usually called subgroups
- Usually of size two to six
- Usually observed over a short period of time

Want to observe often enough to detect important process changes. Control charts are used to audit the processes.

Control Charts

- A control chart employs a center line, upper control limit and lower control limit
- The center line represents average performance
- The upper and lower control limits are established so that when in control almost all plot points will be between the limits



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Variables and Attributes

Control charts for variables – things that we can measure

$\bar{x}$-charts — are in continuous units, i.e. diameter, time, length, height, or weight

R-charts — measures of range

Control charts for attributes – things that we count

p-charts — p stands for proportions, i.e., proportion that pass or fail, material faults, percentage of phone calls not answered within 5 rings, percentage of dissatisfied customers, etc.

c-charts — c stands for things that we can count, i.e., number of defects, complaints, data entry errors, etc.

Control Chart *example*

1 We have collected 20 samples of size 5 each

n \ m	1	2	3	4	…	17	18	19	20
1	30	55	19	36	…	27	29	30	30
2	30	51	25	36	…	27	32	34	31
3	34	47	21	37	…	26	34	34	29
4	33	33	44	35	…	29	35	35	29
5	32	48	28	38	…	27	34	36	29

2 Now we calculate sample mean for 20 samples

x bar	31.8	46.8	27.4	36.4	…	27.2	32.8	33.8	29.6

3 Now we calculate sample range for 20 samples

Range	4	22	25	3	…	3	6	6	2

4 Now we calculate the grand mean and average range

$$\overline{\overline{X}} = (31.8 + 46.8 + ... + 33.8 + 29.6) / 20 = 32.17$$

$$\overline{R} = (4 + 22 + ... + 6 + 2) / 20 = 9.00$$

20 Samples

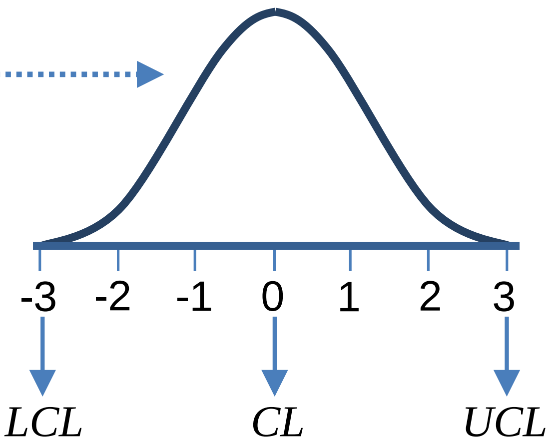
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	**30**	**55**	**19**	36	36	34	32	42	**29**	29	36	20	33	27	27	36	27	**29**	**30**	30
2	30	51	25	36	35	**38**	31	**40**	**24**	**28**	35	**22**	37	24	29	**23**	27	32	34	**31**
3	**34**	47	21	37	**36**	36	**39**	42	26	**34**	**39**	21	**26**	**22**	**26**	**45**	**26**	34	34	**29**
4	33	**33**	**44**	**35**	**31**	**29**	37	**46**	25	33	32	**20**	29	25	28	35	**29**	**35**	35	29
5	32	48	28	**38**	36	36	**24**	44	29	32	**26**	21	**38**	**31**	**31**	31	27	34	**36**	29
Mean	31.8	46.8	27.4	36.4	34.8	34.6	32.6	42.8	26.6	31.2	33.6	20.8	32.6	25.8	28.2	34.0	27.2	32.8	33.8	29.6
R	4	22	25	3	5	9	15	6	5	6	13	2	12	9	5	22	3	6	6	2

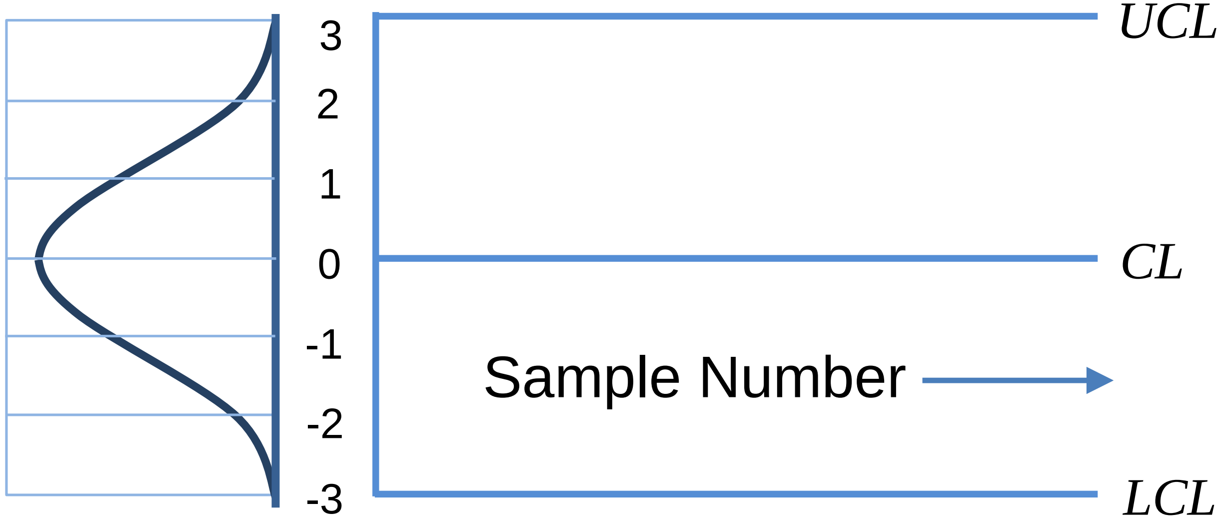
$\bar{x}$ Chart

We start with m samples of size n each. We are dealing with the distribution of the average $\bar{x}$, (normally distributed).

Most observations should be within 3 std. dev. from the mean. So we set up control limits at these points.

We turn around this figure by 90˚ before plotting:





Pattern Analysis

- An observation beyond the control limits indicates the presence of an assignable cause
- Other types of patterns also indicate the presence of an assignable cause
- These patterns are more easily described in terms of control chart zones of A, B, and C.

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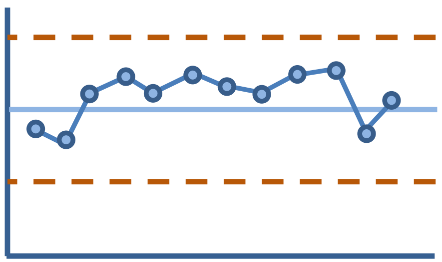
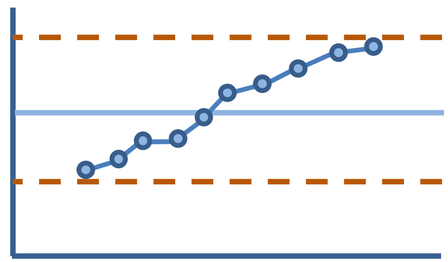
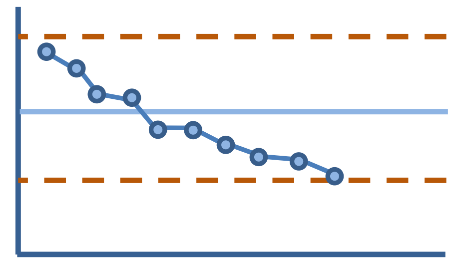
Out of Control Patterns



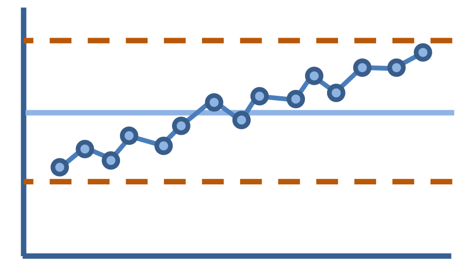
(a) A run on one side of the center line



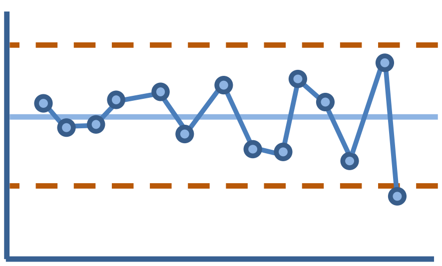
(b) A run up



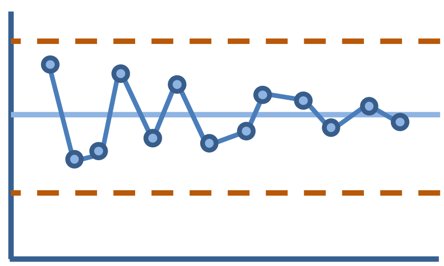
(c) A run down



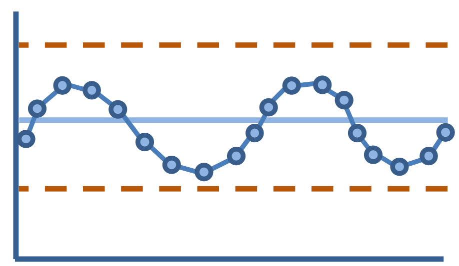
(d) A trend (here, increasing)



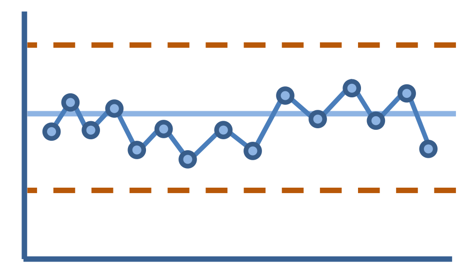
(e) Fanning out



(f) Funneling In



(g) A cycle



(h) An alternating pattern

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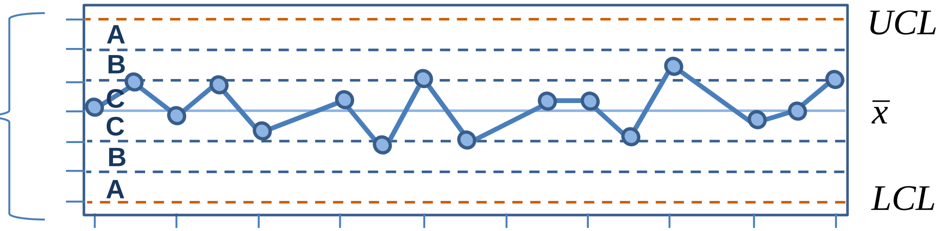
In Control Patterns



(a) A control chart with A, B, and C zones

(b) Calculating zone boundaries for $\bar{x}$ and R charts in the hole location case

	Zone Boundaries	$\bar{x}$ Chart	*R* Chart
(.135%)			
	Upper Control Limit:	$\bar{\bar{x}} + A_2\bar{R} = 3.0396$	$D_4\bar{R} = .1427$
Zone A (2.145%)			
	Upper A-B Boundary:	$\bar{\bar{x}} + \frac{2}{3}(A_2\bar{R}) = 3.0266$	$\bar{R} + \frac{2}{3}(D_4\bar{R} - \bar{R}) = .1176$
Zone B (13.59%)			
	Upper B-C Boundary:	$\bar{\bar{x}} + \frac{1}{3}(A_2\bar{R}) = 3.0136$	$\bar{R} + \frac{1}{3}(D_4\bar{R} - \bar{R}) = .0926$
Zone C (34.13%)			
	Center Line:	$\bar{\bar{x}} = 3.0006$	$\bar{R} = .0675$
Zone C (34.13%)			
	Lower A-B Boundary:*	$\bar{\bar{x}} - \frac{1}{3}(A_2\bar{R}) = 2.9876$	$\bar{R} - \frac{1}{3}(D_4\bar{R} - \bar{R}) = .0424$
Zone B (13.59%)			
	Lower B-C Boundary:*	$\bar{\bar{x}} - \frac{2}{3}(A_2\bar{R}) = 2.9746$	$\bar{R} - \frac{2}{3}(D_4\bar{R} - \bar{R}) = .0174$
Zone A (2.145%)			
	Lower Control Limit:*	$\bar{\bar{x}} - A_2\bar{R} = 2.9617$	$D_4\bar{R} = \text{does not exist}$
(.135%)			

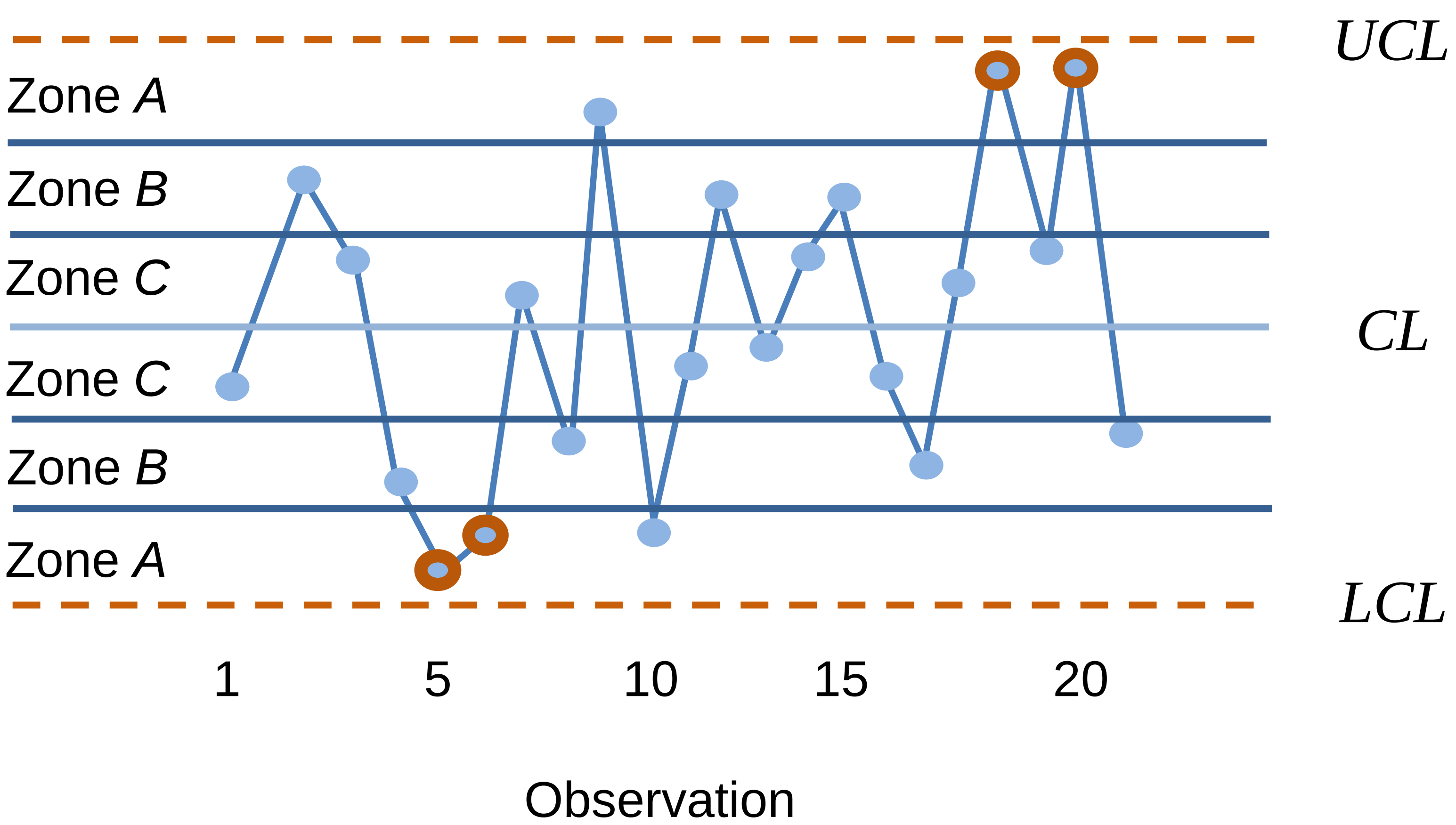
Pattern Analysis for $\bar{x}$ and R Charts

Any of the following patterns is evidence of the likely presence of an assignable cause of variation

- One point beyond zone A (three standard deviation limits)
- Two of three consecutive points in zone A (the two standard deviation warning limits, or beyond) on one side of the center line
- Four of five consecutive points in zone B (the one standard deviation limits, or beyond) on one side of the center line
- A run of eight consecutive points (runs up, down or on the same side of center line)
- Any nonrandom pattern – trend, fanning out, cycle or alternating pattern

Otherwise, the process is in statistical control

Two of Three in A or Beyond



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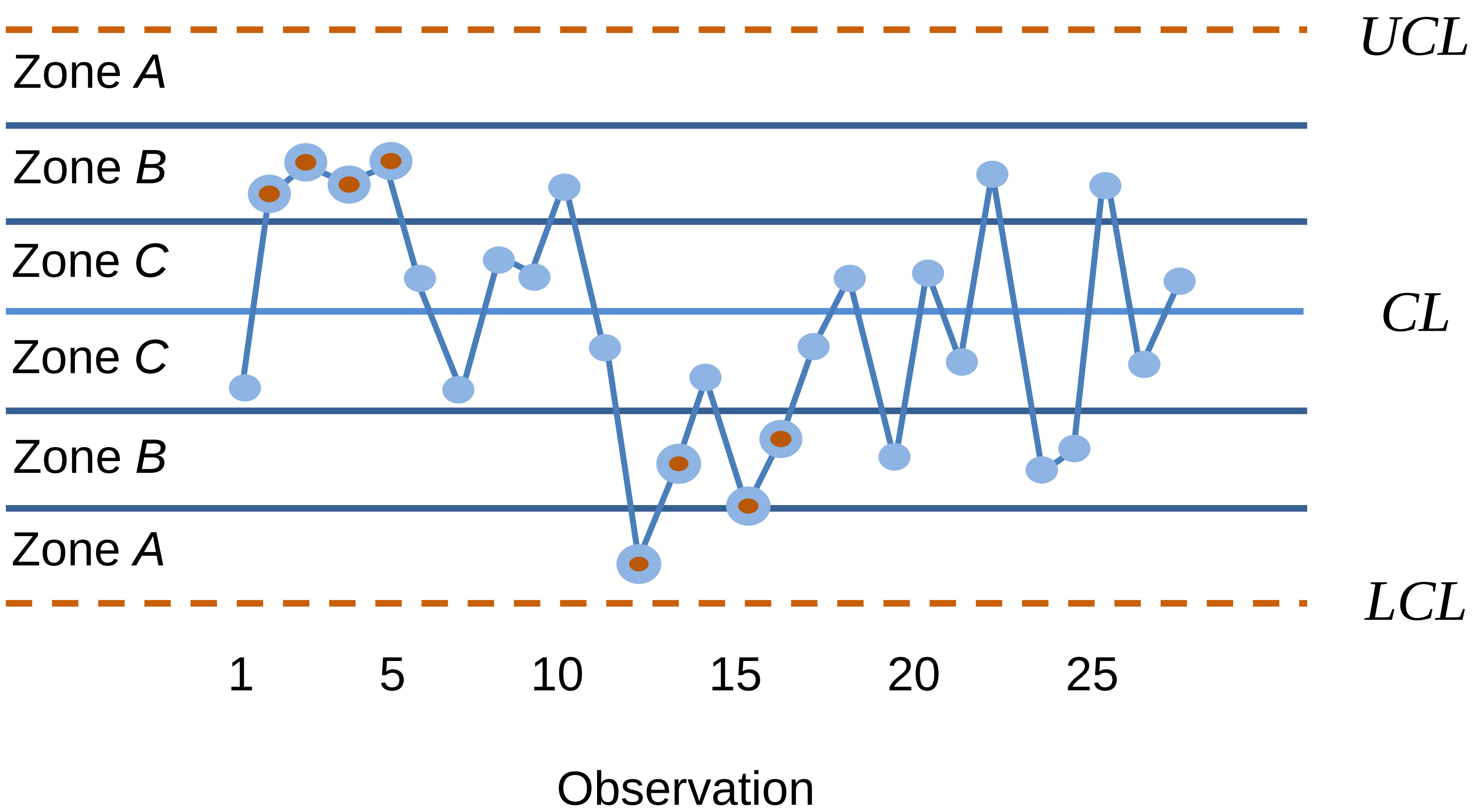
Four of Five in B or Beyond



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$\bar{x}$ Chart

Limits for this chart are given by

$$CL = \mu_{\bar{x}} \approx \bar{\bar{x}}$$

$$UCL = \mu_{\bar{x}} + 3\sigma_{\bar{x}} \approx \bar{\bar{x}} + A_2\bar{R}$$

$$LCL = \mu_{\bar{x}} - 3\sigma_{\bar{x}} \approx \bar{\bar{x}} - A_2\bar{R}$$

Use A_2 values from Table of Control Chart Constants

Sample Size, n	Mean Factor, A_2	Upper Range, D_4	Lower Range, D_3
2	1.880	3.268	0

Control Charts: Stage 1

Stage 1: to establish the control limits

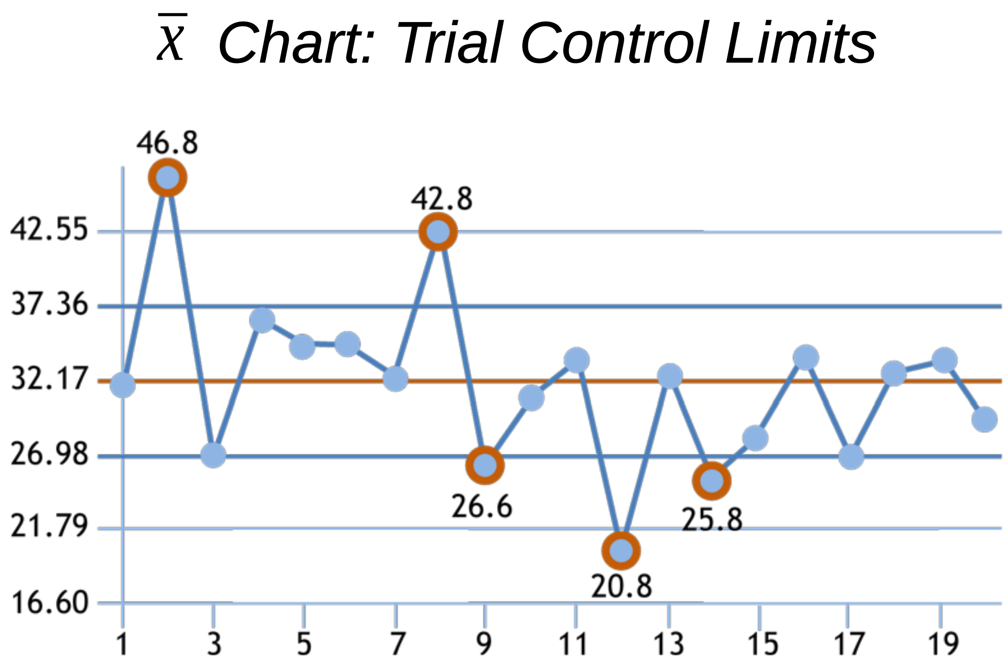
Sample #	1	2	3	4	5	6	7	8	9	10
Mean	31.8	**46.8**	27.4	36.4	34.8	34.6	32.6	**42.8**	**26.6**	31.2
Range	4	22	25	3	5	9	15	6	5	6
Sample #	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Mean	33.6	**20.8**	32.6	**25.8**	28.2	34	27.2	32.8	33.8	29.6
Range	13	2	12	9	5	22	3	6	6	2

$\overline{\overline{x}} = 32.17$ $\quad \overline{R} = 9.00$ $\quad For\ n = 5,\ A_2 = 0.577$ $\quad CL = 32.17 = \overline{\overline{x}}$

$UCL = 32.17 + 0.577(9) = 37.36$ *Which points outside* UCL*?*

$LCL = 32.17 - 0.577(9) = 26.98$ *Which points outside* LCL*?*

$\overline{x}$ Chart : Trial Control Limits



- Points 2 (*46.8*), 8 (*42.8*), 9 (*26.6*), 12 (*20.8*), 14 (*25.8*) are outside limits.
- Find assignable cause and eliminate these points.
- Find new control limits.

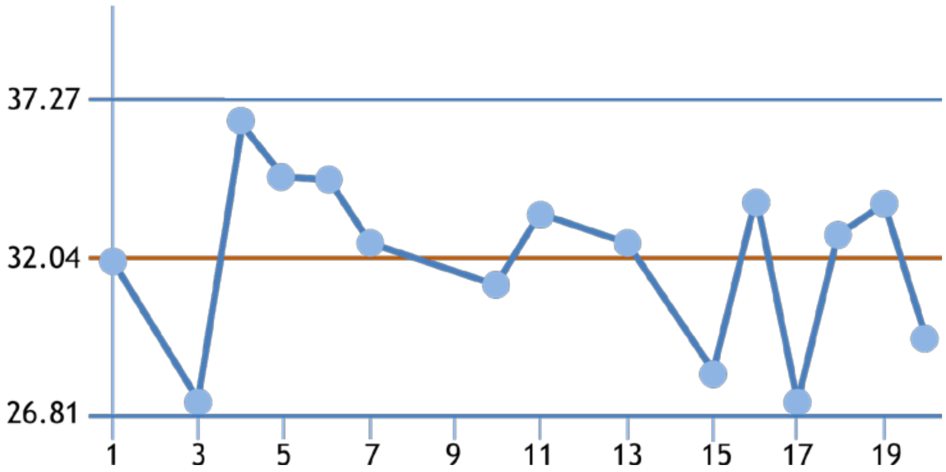
Revision 1, $m = 15$

$$\bar{\bar{x}} = 32.04 \qquad \bar{R} = 9.07 \qquad CL = 32.04 = \bar{\bar{x}}$$

$$UCL = 32.04 + 0.577(9.07) = 37.27$$

$$LCL = 32.04 - 0.577(9.07) = 26.81$$

Revised $\bar{x}$ Chart



All points are within limits. The process is now in control and the limits become stable.

R Chart

We have m samples of size n each and plot sample range values. So we are dealing with the distribution of R.
Distribution of R is not normal.

We need control limits:	LCL	CL	UCL
Estimates for limits are:	$D_3\overline{R}$	$\overline{R}$	$D_4\overline{R}$

Use D3 and D4 values from Table of Control Chart Constants

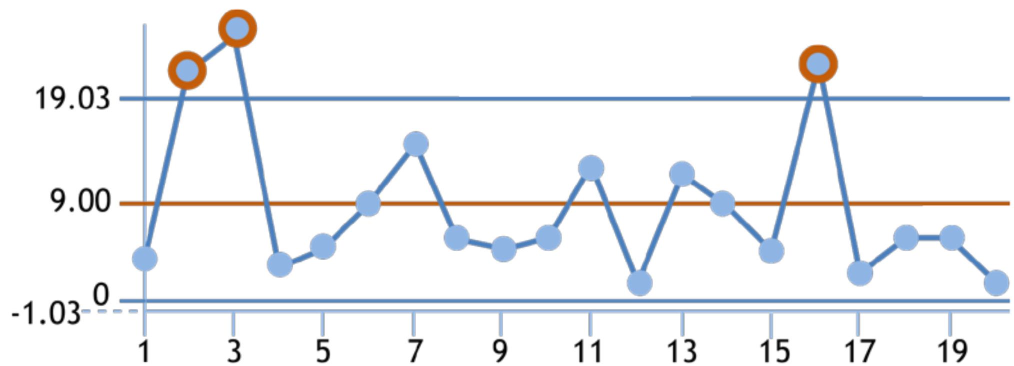
Sample Size, n	Mean Factor, A_2	Upper Range, D_4	Lower Range, D_3
2	1.880	3.268	0

R Chart

$$\bar{\bar{x}} = 32.17 \qquad \bar{R} = 9.00 \qquad n = 5,\ D_3 = 0.0,\ D_4 = 2.115$$

$$CL = 9.00,\ UCL = 2.114(9.00) = 19.03,\ LCL = 0.00(9.00) = 0.00$$

R Chart: Trial Control Limits

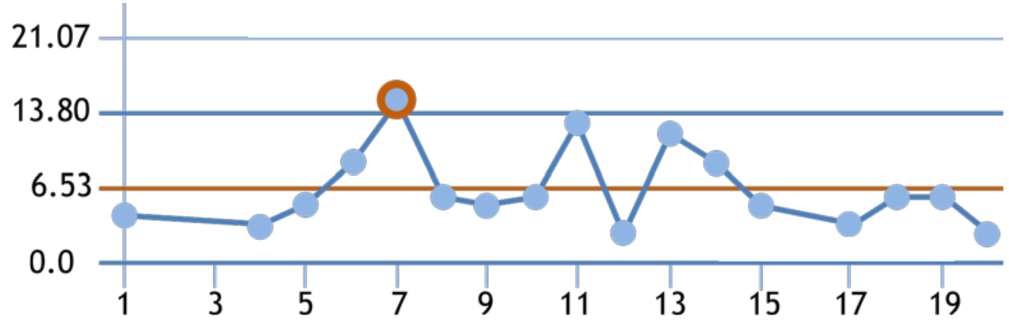


- Points 2, 3, 16 are outside limits.
- Find assignable cause and eliminate these points.
- Find new control limits.

R Chart *revision 1 ($m = 17$)*

$$\bar{R} = 6.53 \qquad CL = 6.53, UCL = 13.80, LCL = 0$$

R Chart: Revised Control Limits

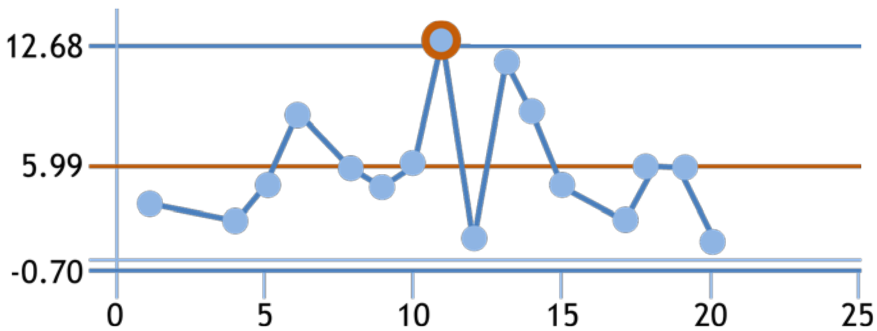


- Point 7 is outside limits
- Perform revision 2

R Chart *revision 2 ($m = 16$)*

$$\overline{R} = 6.00 \qquad CL = 5.99, UCL = 12.68, LCL = 0$$

R Chart: Control Limits (revision 2)

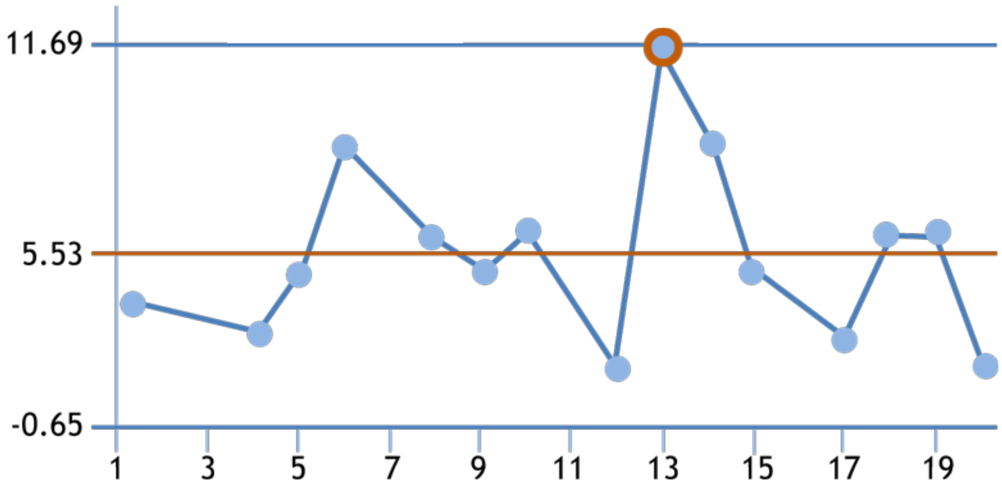


- Point 11 is outside limits
- Perform revision 3

R Chart *revision 3 ($m = 15$)*

$$\overline{R} = 5.53 \qquad CL = 5.53, UCL = 11.7, LCL = 0$$

R Chart: Control Limits (revision 3)

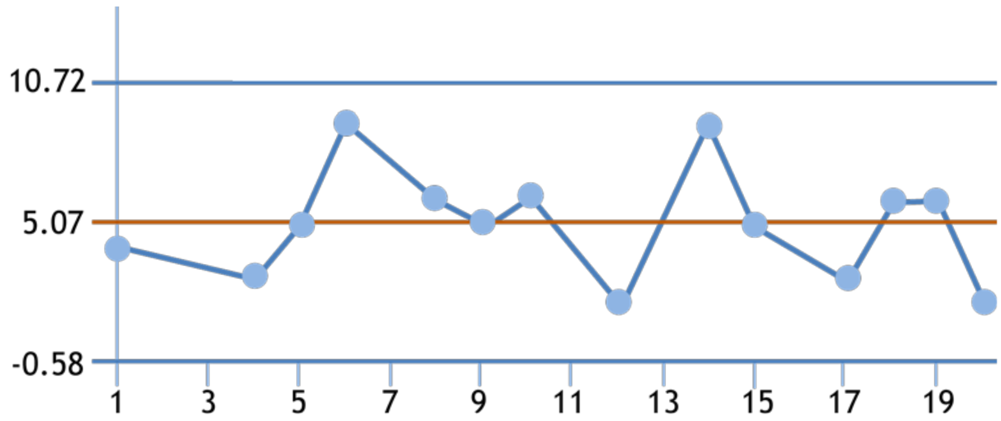


- Point 13 is outside limits.
- Perform revision 4

R Chart *revision 4 ($m=14$)*

$$\overline{R}=5.07 \qquad CL=5.07, UCL=10.73, LCL=0$$

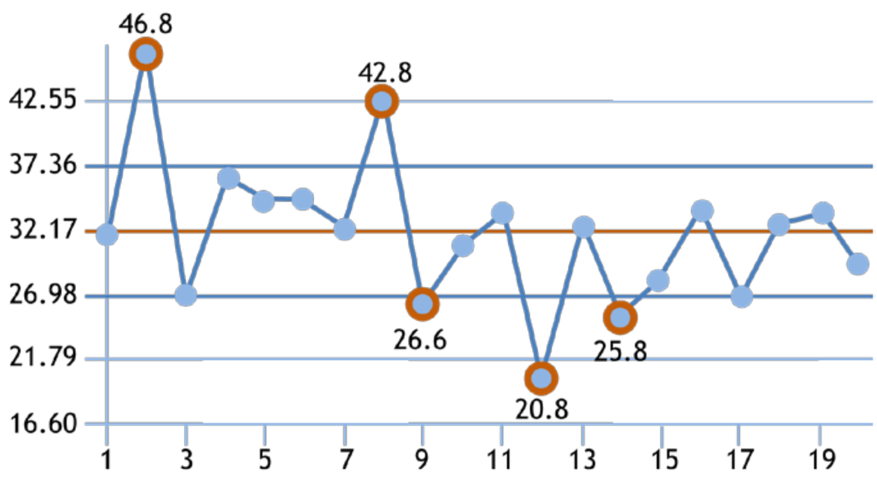
R Chart: Control Limits (revision 4)



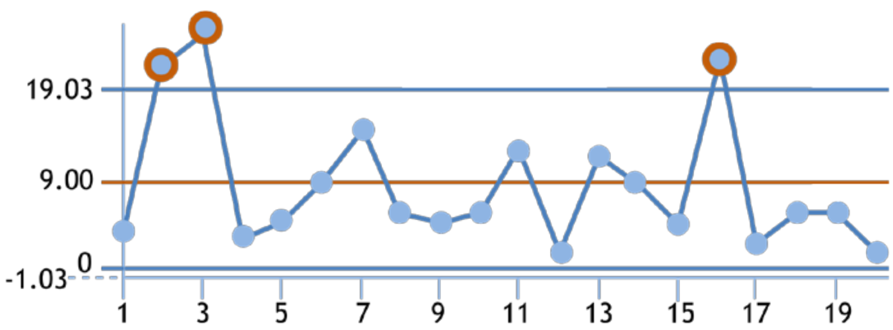
All points are in control now!

Using $\bar{x}$ and R Charts Simultaneously

$\bar{x}$ Chart: Trial Control Limits



R Chart: Trial Control Limits



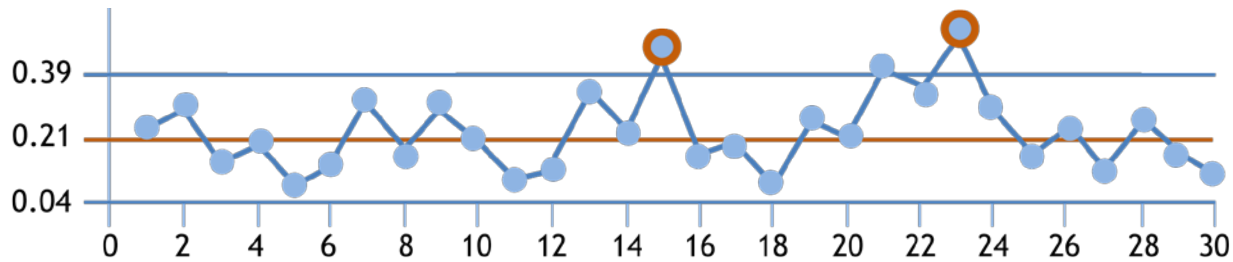
	UCL	LCL	Out of Control	UCL	LCL	Out of Control	Eliminate
Trial	37.36	26.98	2,8,9,12,14	19.04	0	2,3,16	2,3,8,9,12,14,16
Rev. 1	36.20	28.30	4,15,17	14.48	0	7	4,7,15,17
Rev. 2	36.79	28.72	None	14.81	0	None	None

Control Charts: Stage 2

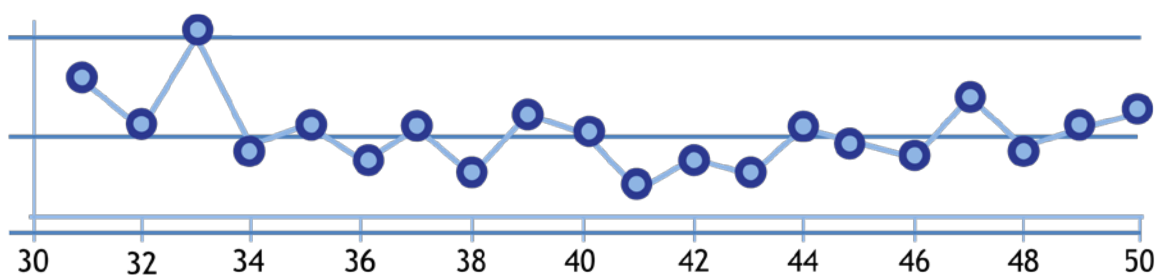
Stage 2: To maintain control limits

- Keep on taking new samples as per established procedure and plot new points.
- If process goes out of control, take corrective action (but do not recalculate control limits).
- Revise limits whenever major changes occur.

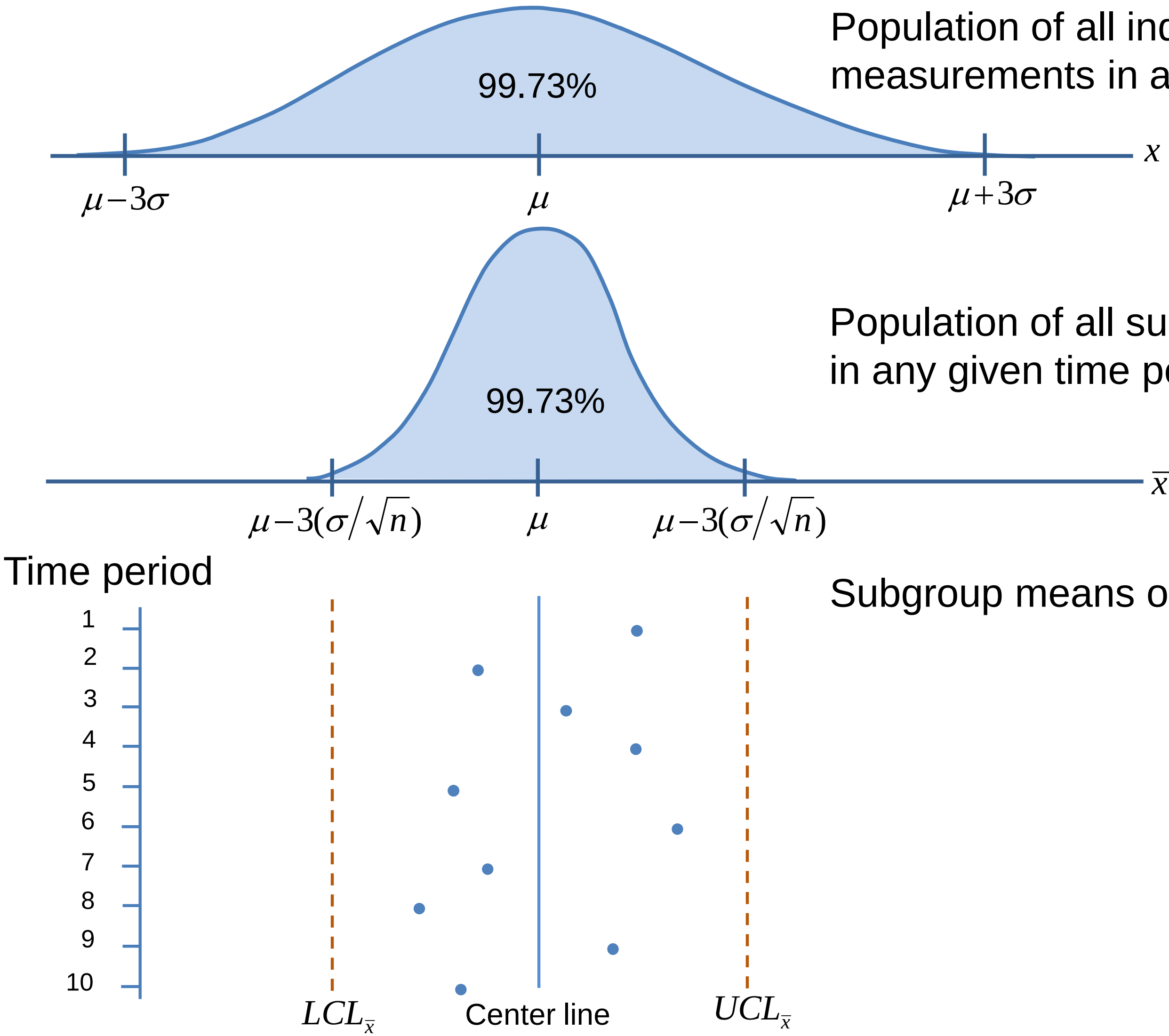
Trial Control Limits



Stable Control Limits



$\overline{x}$ Chart



Population of all individual process measurements in any given time period

Population of all subgroup means in any given time period

Subgroup means observed over time

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$\bar{x}$ and R Chart: Control Limits

$LCL_{\bar{x}}$	$UCL_{\bar{x}}$
$\mu - 3(\sigma/\sqrt{n})$	$\mu + 3(\sigma/\sqrt{n})$
$\bar{\bar{x}} - 3\left(\frac{\bar{R}/d_2}{\sqrt{n}}\right)$	$\bar{\bar{x}} + 3\left(\frac{\bar{R}/d_2}{\sqrt{n}}\right)$
$\bar{\bar{x}} - \left(\frac{3}{d_2\sqrt{n}}\right)\bar{R}$	$\bar{\bar{x}} + \left(\frac{3}{d_2\sqrt{n}}\right)\bar{R}$
$\bar{\bar{x}} - A_2\bar{R}$	$\bar{\bar{x}} + A_2\bar{R}$

Control Limits for $\bar{x}$ Charts

Estimate μ by $\bar{\bar{x}}$, σ by $\bar{R}/d_2$

$$A_2 = \frac{3}{d_2\sqrt{n}}$$

Center line is $\bar{\bar{x}}$

LCL_R	UCL_R
$D_3\bar{R}$	$D_4\bar{R}$

Control Limits for R Charts

Center line is $\bar{R}$

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Prevention Using Control Charts

1. Reduce common cause variation in order to create leeway between the natural tolerance limits and the specification limits
2. Use control charts to establish statistical control and to monitor the process
3. When the control charts give out-of-control signals, take immediate action on the process to reestablish control before out-of-specification product is produced

Variables and Attributes

Control charts for variables – things that we can measure

$\bar{x}$-charts	are in continuous units, i.e. diameter, time, length, height, or weight
R-charts	measures of range

Control charts for attributes – things that we count

p-charts	p stands for proportions, i.e., proportion that pass or fail, material faults, percentage of phone calls not answered within 5 rings, percentage of dissatisfied customers, etc.
c-charts	c stands for things that we can count, i.e., number of defects, complaints, data entry errors, etc.

Control Limits for Attributes

Control Limits and Center Line for a p Chart

LCL_p $\quad$ UCL_p

$$p-3\sqrt{\frac{p(1-p)}{n}} \qquad p+3\sqrt{\frac{p(1-p)}{n}}$$

$$Estimate\ \pi\ by\ \overline{p}=\frac{\text{total nonconforming, all subgroups}}{\text{total inspected, all subgroups}}$$

$$\overline{p}-3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} \qquad \overline{p}+3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} \qquad Center\ line\ is\ \overline{p}$$

Control Limits and Center Line for a c Chart

$$LCL_c=\overline{c}-3\sqrt{\overline{c}} \qquad UCL_c=\overline{c}+3\sqrt{\overline{c}} \qquad Center\ line\ is\ \overline{c}$$

p Control Chart

A company that makes slacks controls its production process by periodically taking a sample of 100 slacks from the production line. Each pair of slacks is inspected for defective features. Control limits are developed using three standard deviations from the mean as the limit. During the last 16 samples taken, the proportion of defective items per sample was recorded as follows:

.01	.02	.01	.03	.02	.01	.00	.02
.00	.01	.03	.02	.03	.02	.01	.00

Step 1 Determine the mean proportion defective, the UCL, and the LCL.

Step 2 Draw a control chart and plot each of the measurements on it.

Step 3 Does it appear that the process for making slacks is in control?

p Control Chart *continued*

Step 1

The mean proportion defective (center line) is

$$CL = \frac{\begin{pmatrix} .01+.02+.01+.03+.02+.01+.00+.02+ \\ .00+.01+.03+.02+.03+02+.01+.00 \end{pmatrix}}{16}$$

$$= .015$$

$$UCL = .015 + 3\sqrt{\frac{.015(.985)}{100}}$$

$$= .015 + .0365$$

$$= .0515$$

$$LCL = .015 - 3\sqrt{\frac{.015(.985)}{100}}$$

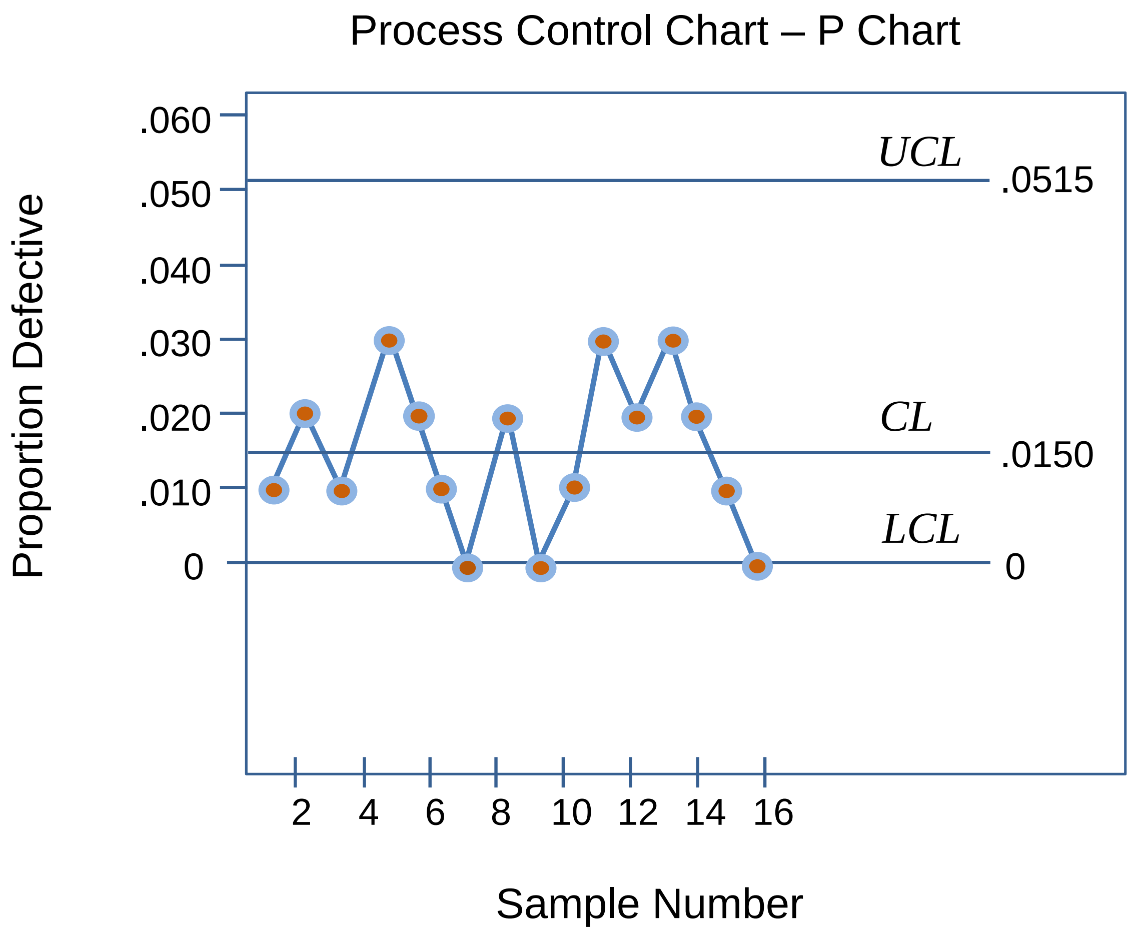
$$= .015 - .0365$$

$$= -.0215, \textit{ which is negative}$$

$$\textit{Therfore, the } LCL = 0$$

p Control Chart *continued*

Step 2



Step 3

All the points are within the control limits. We can conclude that the process is in control.

c Chart *example*

The Hearth Home Inn receives several complaints per day about it's staff. Over a nine day period (where days are the units of measure), the owner received the following numbers of calls from irate guests: 3, 0, 8, 9, 6, 7, 4, 9, 8, for a total of 54 complaints.

To compute 99.7% control limits, we take:

$$\bar{c} = \frac{54}{9} = 6 \text{ complaints per day}$$

Thus,

$$UCL_c = \bar{c} + 3\sqrt{\bar{c}} = 6 + 3\sqrt{6} = 6 + 3(2.45) = 13.35$$

$$LCL_c = \bar{c} - 3\sqrt{\bar{c}} = 6 - 3\sqrt{6} = 6 - 3(2.45) = 0$$

(since we cannot have a negative control limit)

After the owner plotted a control chart summarizing these data and posted it prominently in the staff locker room, the number of calls received dropped to an average of three per day. Can you explain why this may have occurred?

Process Capability

Suppose we are making a product using certain process and we need the process to produce at least 99.5% good products. This is called the *target process capability*. We need to ask two questions:

1. *Is the process capable?*
If the answer is no, we need to find a better process.
If the answer is *yes*, we start using the process and periodically ask the second question.

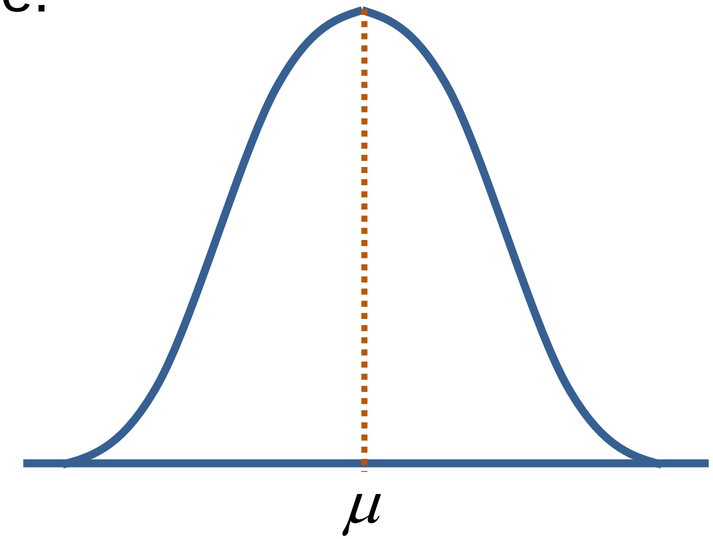
2. *How is the process doing at any given time?*
If the answer to the second question reveals capability has gone below the target value, we take some corrective action.

Answer to the first question is expressed using a measure called *"process capability ratio"* $[C_p]$.
Answer to the second question is expressed using a measure called *"process capability index"* $[C_{pk}]$.

Process Capability Analysis

The methodology used for finding $[C_p]$ and $[C_{pk}]$ is called process capability analysis. Here we make certain assumptions:

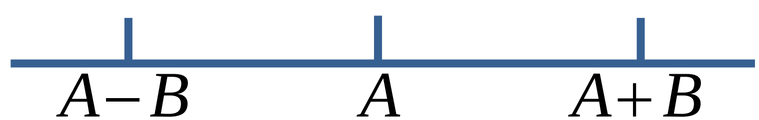
- We are measuring only one critical dimension X.
- X is normally distributed.
- Process is stable.



Process Capability Analysis *continued*

The specification for this dimension is $A \pm B$ where

- "$A+B$" is the *Upper Tolerance Limit* $[UTL]$
- "A" is the nominal value and
- "$A-B$" is the *Lower Tolerance Limit* $[LTL]$
- If the dimension of the component produced falls between LTL and UTL, we accept the component. If it falls outside, we reject.



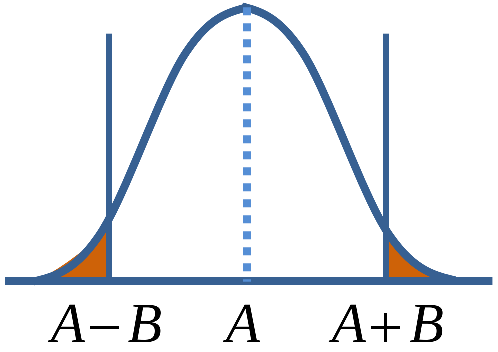
When we do the analysis, we collect random samples to estimate mean and std. deviation and to ensure that the process is stable.

Process Capability Ratio (C_P)

The *formula* for the *process capability ratio* is given by

$$C_P = (UTL - LTL)/(6\sigma)$$

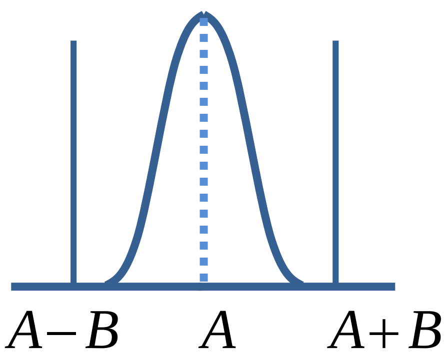
- Here are the specifications $A \pm B$
- Now we will superimpose normal distribution on the specifications.



It is assumed that the process mean is at the nominal value "A".
The *orange area* shows *probability of rejection*.

If standard deviation (σ) of the *process becomes smaller*

- C_P becomes *higher*
- Probability of *rejection* becomes *smaller*.



What is the Meaning of C_p?

Here is a table of probabilities for different values of C_p

C_p	Probability of	
	acceptance	*rejection*
1/3	0.68269	0.31731
0.5	0.86637	0.13363
2/3	0.95450	0.04550
1.0	0.99730	0.00270
1.584	0.999998	0.000002
2.0	1.000000	0.000000

For many products, $C_p = 1.0$ may be acceptable.

However, for many other products, this means rejection of 0.0027 or 2700 parts per million (ppm). This may not be acceptable.

Companies strive for $C_p = 1.584$ (rejection of 2 ppm).

The ideal is $C_p = 2$ with rejection of about 2 parts per billion. For $C_p = 2$, UTL and LTL are at a distance of 6σ from the nominal value.

How is Process Capability Used?

First we select target process capability (based on customer requirement / our own standard / industry standard).

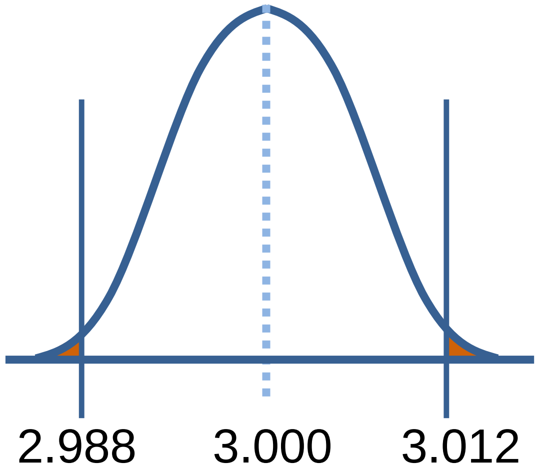
Using standard deviation estimate, we determine process capability ratio (C_p). If this exceeds target value, we use the process and periodically calculate C_{pk}. If C_{pk} falls below target value, we need some simple adjustments (sharpen the tool, adjust the setting, etc).

If C_p value is less than the target value, we won't be able to use the process; we need a long term solution. Either or both approaches below will work:

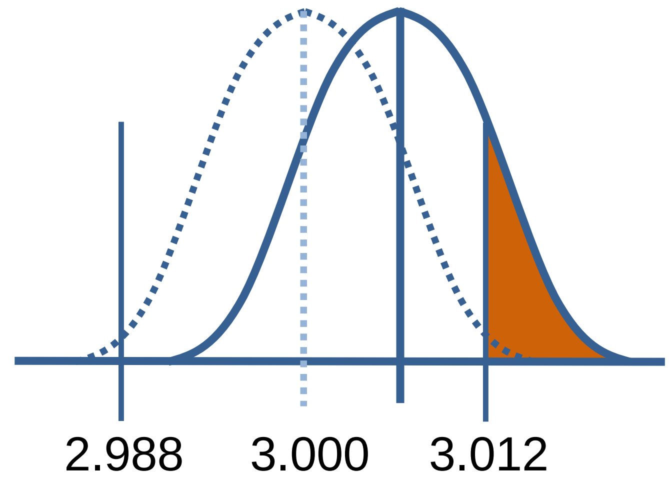
- Use better process, train operators, use better quality raw material. Sometimes tolerance limits can be relaxed,
- Reduce variability (std. deviation).

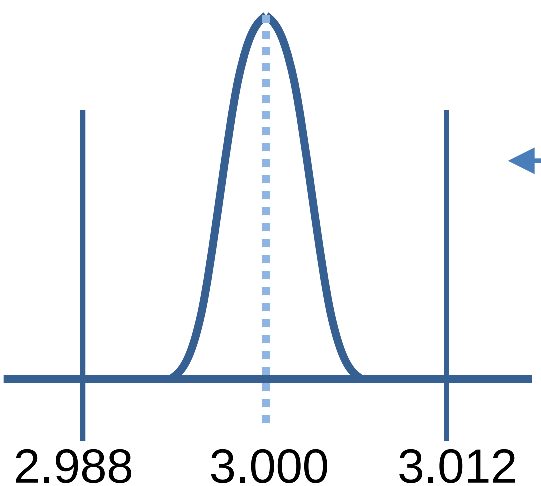
Effect of Reduced Variability

When σ is large, you produce high % defectives.

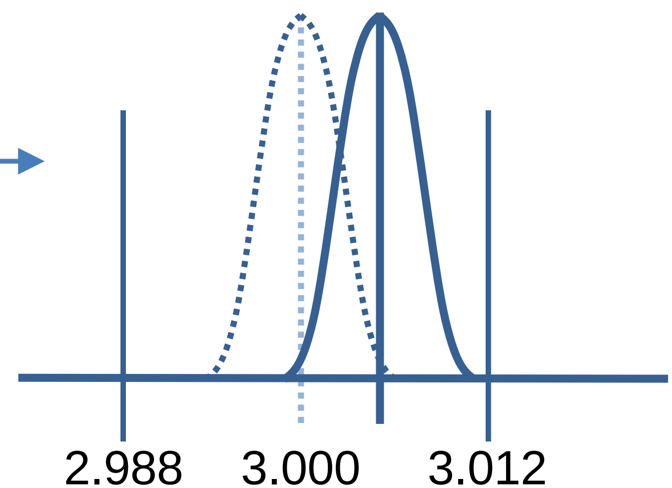


When μ is shifted, you produce even more % defectives.





See the effect of small σ even when μ shifts.



Comparison of a Process with Specifications: Capability Studies

Natural tolerance limits for a normally distributed process in statistical control will contain about 99.73 percent of the process observations and is given by

$$\left[\bar{\bar{x}} \pm 3\left(\frac{\bar{R}}{d_2}\right)\right] = \left[\bar{\bar{x}} - 3\left(\frac{\bar{R}}{d_2}\right),\ \bar{\bar{x}} + 3\left(\frac{\bar{R}}{d_2}\right)\right]$$

If the natural tolerance limits are inside the process specification limits, we say that the process is **capable** of meeting specifications

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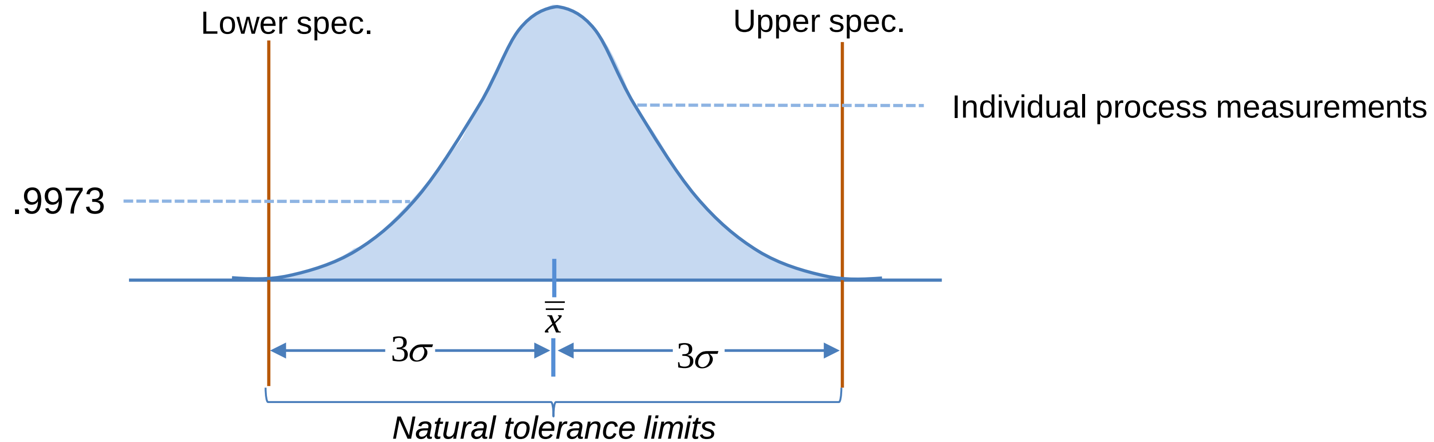
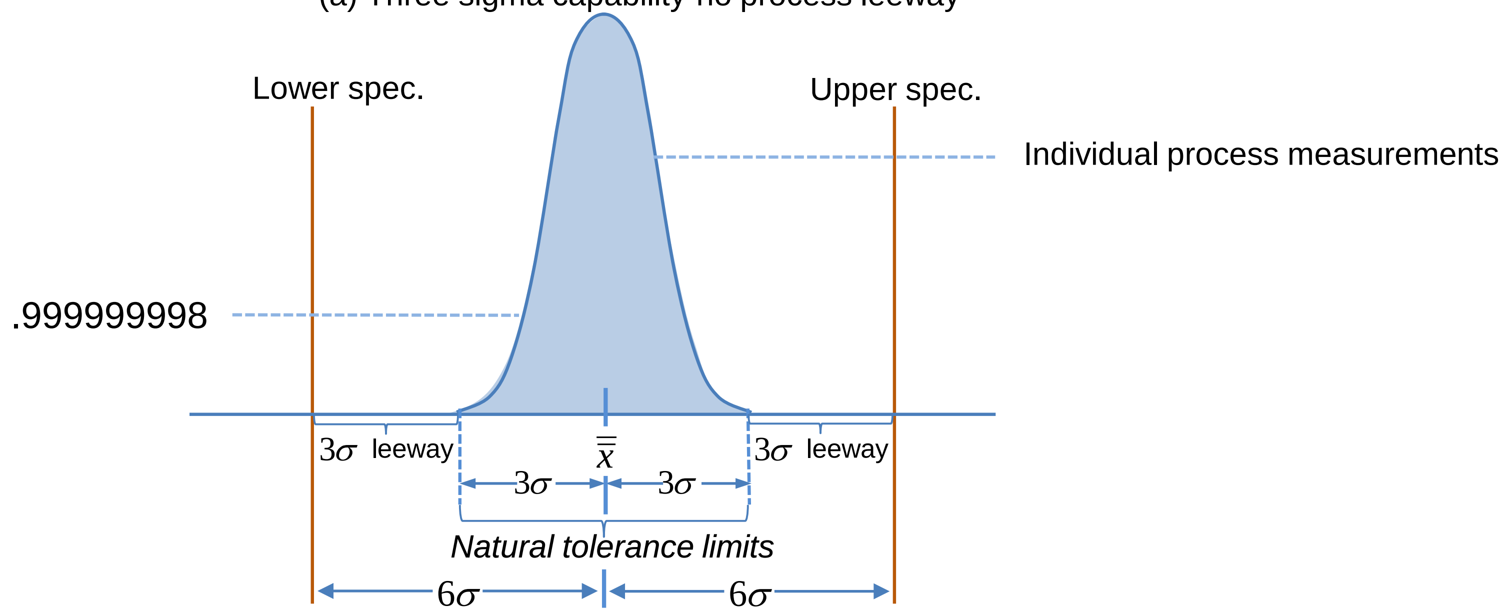
Sigma Level Capability and Process Leeway



(a) Three sigma capability-no process leeway



(b) Six sigma capability-three standard deviations of process leeway

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