A company that produces yogurt has plants in four locations: 1) Preston, Idaho 2) New Ulm, Minnesota 3) Carthage, Texas 4) Coalwood, West Virginia. The annual capacity of each plant to produce yogurt is 10,300, 12,400, 11,300, and 12,700 tons, respectively. To cover demand, the company wants to ship 7,700 tons to a distribution center in Modesto, California, 6,200 to South Bend, Indiana, 10,200 to Talladega, Alabama, and 8,400 to Punxsutawney, Pennsylvania. The per ton distribution costs from each plant to each distribution center are given in the table below:

Table: Shipping cost per ton, plant capacity in tons, and distribution center demand in tons

| | Distribution centers | | | | |
|---|---|---|---|---|---|
| Plants | California | Indiana | Alabama | Pennsylvania | **Capacity** |
| Idaho | $160 | $300 | $360 | $380 | **10,300** |
| Minnesota | $380 | $120 | $220 | $200 | **12,400** |
| Texas | $360 | $180 | $100 | $240 | **11,300** |
| West Virginia | $520 | $100 | $80 | $60 | **12,700** |
| **Requirements** | **7,700** | **6,200** | **10,200** | **8,400** | |

In addition to choosing how much yogurt to ship from each plant to each distribution center, the company may also choose which plants to operate. The fixed cost of operating each plant are $500,000 for the Idaho plant, $450,000 for the Minnesota plant, $380,000 for the Texas plant, and $350,000 for the West Virginia plant.

Use the GOMP to determine your objective function, decision variables, constraints, and optimal decisions. Which plants should the company operate, and how many tons of yogurt should be shipped from each plant to each distribution center if the company's goal is to minimize the combined distribution and fixed operating costs? What is the combined cost under the optimal solution?

Objective Function:

- Minimize total costs
- Total Cost = fixed costs + operating/shipping costs
  - Total Cost =
    500,000*FI +450,000*FM + 380,000*FT + 350,000*FWV +
    160*IC + 300*II + 360*IA + 380*IP +
    380*MC + 120*MI + 220*MA + 200*MP +
    360*TC + 180*TI + 100*TA + 240*TP +
    520*WVC + 100*WVI + 80*WVA + 60*WVP

Decision Variables:

- Fixed Site Cost for Idaho/Minnesota/Texas/West Virginia
  - FI, FM, FT, FWV
- Shipping Cost for Idaho - To California/Indiana/Alabama/Pennsylvania
  - IC, II, IA, IP
- Shipping Cost for Minnesota - To California/Indiana/Alabama/Pennsylvania
  - MC, MI, MA, MP
- Shipping Cost for Texas- To California/Indiana/Alabama/Pennsylvania
  - TC, TI, TA, TP
- Shipping Cost for West Virginia- To California/Indiana/Alabama/Pennsylvania
  - WVC, WVI, WVA, WVP

Constraints:

- Idaho Capacity:
  - IC + II + IA + IP <= 10,300
- Minnesota Capacity:
  - MC + MI + MA + MP <= 12,400
- Texas Capacity:
  - TC + TI + TA + TP <= 11,300
- West Virginia Capacity:
  - WVC + WVI + WVA + WVP <= 12,700
- California Demand:
  - IC + MC + TC + WVC >= 7,700
- Indiana Demand:
  - II + MI + TI + WVI >= 6,200
- Alabama Demand:
  - IA + MA + TA + WVA >= 10,200
- Pennsylvania Demand:
  - IP + MP + TP + WVP >= 8,400
- Fixed Site Costs are Binary:
  - FI, FM, FT, FWV as binary (0,1)
- Linking Constraints for Site and Usage (M = 1,000,000,000,000):
  - Idaho:
    - IC + II + IA + IP <= M*FI
  - Minnesota:
    - MC + MI + MA + MP <= M*FM
  - Texas:
    - TC + TI + TA + TP <= M*FT
  - West Virginia:
    - WVC + WVI + WVA + WVP <= M*FWV

Solution:

- Optimal Combined Cost: $4,854,000
- What Plants Should the Company Operate:
  - Idaho, Texas, and West Virginia
- Distribution Amounts:
  - Idaho:
    - California: IC - 7700
    - Indiana: II - 800
  - Texas:
    - Indiana: TI - 1100
    - Alabama: TA - 10200
  - West Virginia:
    - Indiana: WVI - 4300
    - Pennsylvania: WVP - 8400