Lecture 18: Red Black Trees

10/7/2020

The Bad News

- 2-3 trees (and 2-3-4 trees) are a real pain to implement, and suffer from performance problems. Issues include:
  - Maintaining different node types
  - Interconversion of nodes between 2-nodes and 3-nodes
  - Walking up the tree to split nodes

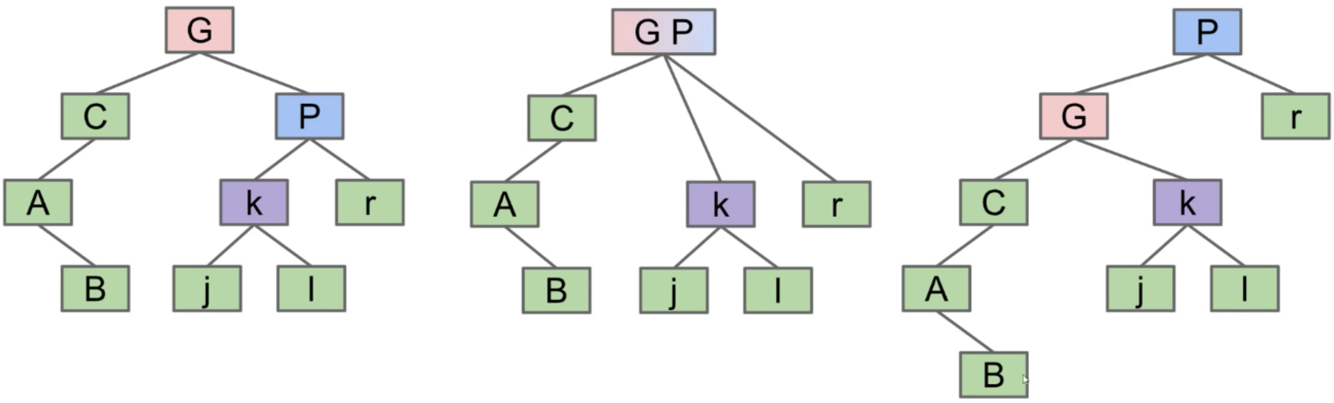
BST Structure and Tree Rotation

BSTs

- Suppose we have a BST with the numbers 1, 2, 3. Five possible BSTs
  - The specific BST you get is based on the insertion order
  - More generally, for N items, there are Catalan(N) different BSTs
- Given any BST, it is possible to move to a different configuration using "rotation"
  - In general, can move from any configuration to any other in 2n - 6 rotations

Tree Rotation Definition

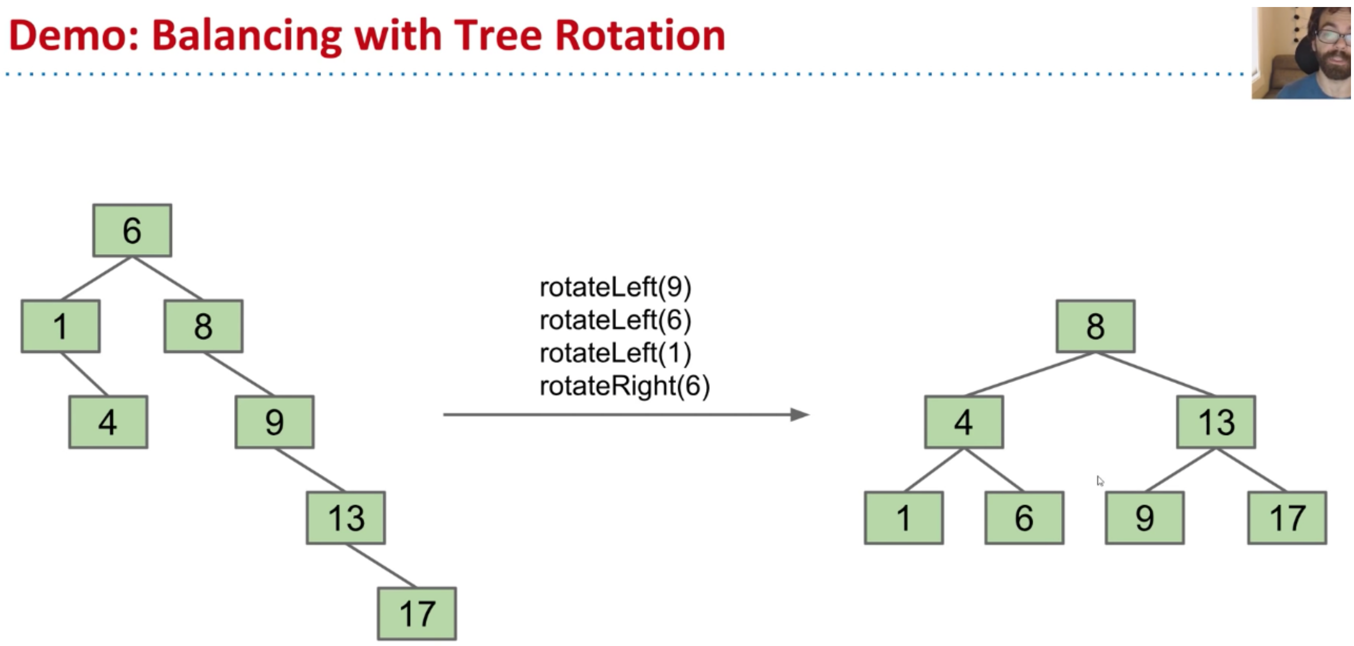
- rotateLeft(G): Let x be the right child of G. Make G the **new left child** of x
  - Preserves search tree property. No change to semantics of tree
  - Can think of as temporarily G and P, then sending G down and **left**



- rotateRight(P): Let x be the left child of P. Make P the **new right child** of x
  - Can think of as temporarily merging G and P, then sending P down and **right**

Rotation for Balance

- Rotation:
  - Can shorten (or lengthen) a tree
  - Preserves search tree property



Rotation: An Alternate Approach to Balance

- Rotation:
  - Can shorten (or lengthen) a tree
  - Preserves search tree property
- Paying O(n) to occasionally balance a tree is not ideal. In this lecture, we''l see a better way to achieve balance through rotation

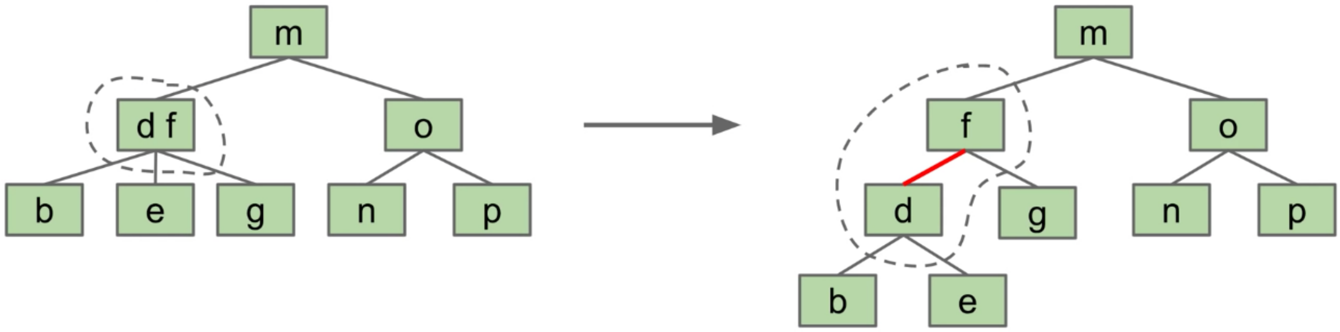
Red-Black Trees

Search Trees

- There are many types of search trees:
  - **Binary search trees**: Can balance using rotation, but we have no algorithms for doing so (yet)
  - **2-3 trees**: Balanced by construction, i.e. no rotations required
- Let's try something clever, but strange
- Our goal: Build a BST that is structurally identical to a 2-3 tree
  - Since 2-3 trees are balanced, so will our special BSTs.

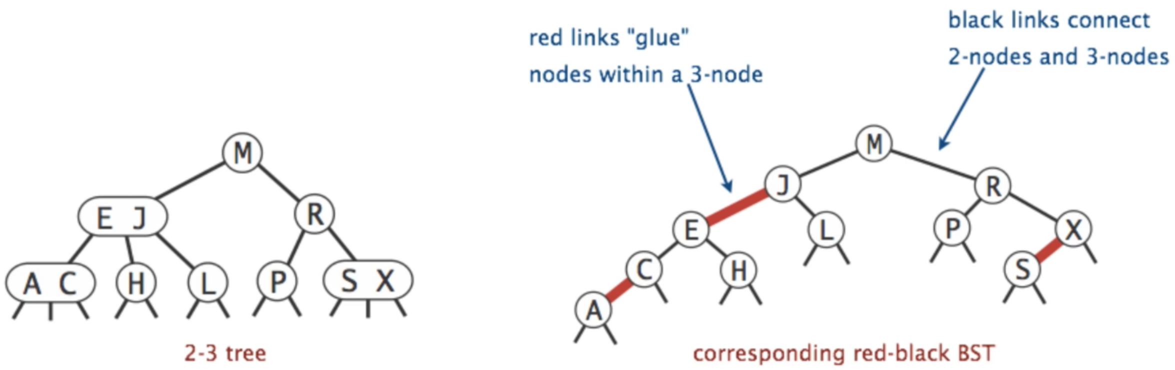
Representing a 2-3 Tree as a BST

- A 2-3 tree with only 2-nodes is trivial
  - BST is exactly the same!
- What do we do about 3-nodes?
  - Possibility 1: Create dummy "glue" nodes
    - Result is inelegant. Wasted link. Code will be ugly
  - Possibility 2: Create "glue" links with the smaller item off to the left
    - Idea is commonly used in practice



Left-Leaning Red Black Binary Search Tree (LLRB)

- A BST with left glue links that represent a 2-3 tree is often called a "Left Leaning Red Black Binary Search Tree" or LLRB
  - LLRBs are normal BSTs
  - There is a 1-1 correspondence between an LLRB and an equivalent 2-3 tree
  - The red is just a convenient fiction. Red links don't "do" anything special



Red Black Tree Properties

Left-Leaning Red Black Binary Search Tree (LLRB)

- Searching an LLRB tree for a key is easy
  - Treat it exactly like any BST

Left-Leaning Red Black Binary Search Tree (LLRB) Properties

- Some handy LLRB properties:
  - No node has two red links [otherwise it'd be analogous to a 4 node, which are disallowed in 2-3 trees]
  - Every path from root to a leaf has same number of **black links** [because 2-3 trees have the same number of links to every leaf]. LLRBs are therefore balanced
  - Logarithmic height

LLRB Construction

- Where do LLRBs come from?
  - Would not make sense to build a 2-3 tree, then convert it
  - Instead, it turns out we implement an LLRB insert as follows:
    - Insert as usual into a BST
    - Use zero or more rotations to maintain the 1-1 mapping

Maintaining 1-1 Correspondence Through Rotations

The 1-1 Mapping

- There exists a 1-1 mapping between:
  - 2-3 Tree
  - LLRB
- Implementation of an LLRB is based on maintaining this 1-1 correspondence
  - When performing LLRB operations, pretend like you're a 2-3 tree
  - Preservation of the correspondence will involve tree rotations

Design Task 1: Insertion Color

- Always use a red link when adding onto a leaf

Design Task 2: Insertion on the Right

- Right links aren't allowed, so rotateLeft
- Likewise, left links aren't allowed, so rotateRight

New Rule: Representation of Temporary 4-Nodes

- We will represent temporary 4-nodes as BST nodes with two red links
  - This state is only temporary, so temporary violation of "left leaning" is ok

Design Task 3: Double insertion on the left

- When double inserting on the left, rotate the node to the right

Design Task 4: Splitting Temporary 4-nodes

- Suppose we have the LLRB includes a temporary 4 node
  - To fix this, flip the colors of all edges touching the node
    - Note: This doesn't change the BST structure/shape

That's it!

- We've just invented the red-black BST
  - When inserting: Use a red link
  - If there is a right leaning "3-node", we have a **Left Leaning Violation**
    - Rotate left the appropriate node to fix
  - If there are two consecutive left links, we have an **Incorrect 4 Node VIolation**
    - Rotate right the appropriate node to fix
  - If there are any nodes with two red children, we have a **Temporary 4 Node**
    - Color flip the node to emulate the split operation
- Cascading operations
  - It is possible that a rotation or flip operation will cause an additional violation that needs fixing

LLRB Runtime and Implementation

LLRB Runtime

- The runtime analysis for LLRBs is simple if you trust the 2-3 tree runtime
  - LLRB tree has height O(log N)
  - Contains is trivially O(log N)
  - Insert is O(log N)
    - O(log N) to add the new node
    - O(log N) rotation and color flip operations per insert
- We will not discuss LLRB delete
  - Not too terrible really, but it's just not interesting enough to cover. See optional textbook if you're curious

LLRB Implementation

- Amazingly, turning BST into an LLRB requires only 3 clever lines of code
  - Does not include helper methods (which do not require cleverness)

Search Tree Summary

Search Tree

- In the last 3 lectures, we talked about using search trees to implement sets/maps
  - Binary search trees are simple, but they are subject to imbalance
  - 2-3 Trees (B Trees) are balanced, but painful to implement and relatively slow
  - LLRBs insertion is simple to implement (but delete is hard)
    - Works by maintaining mathematical bijection with a 2-3 trees
  - Java's TreeMap is a red-black tree (not left leaning)
    - Maintains correspondence with 2-3-4 tree (is not a 1-1 correspondence)
    - Allows glue links on either side
    - More complex implementation, but significantly faster
- There are many other types of search trees out there
  - Other self balancing trees: AVL trees, splay trees, treaps, etc.
- There are other efficient ways to implement sets and maps entirely
  - Other linked structures: Skip lists are linked lists with express lanes
  - Other ideas entirely: Hashing is the most common alternative. We'll discuss this idea in the next lecture

CSM Review

B-Trees

- B is for balanced
- Some definitions:
  - **depth** of a node is distance to the root (the root node has depth 0)
  - **height** of a tree is the depth of the lowest leaf
- Purpose of B-trees:
  - Avoids spindly trees
  - Keeps the tree with height log(n)

2-3 Trees

- Each non-leaf node can have 2 or 3 children
- Each node can be stuffed with at most 2 values
    - Once a node is overstuffed (aka 3 values), push middle value up

2-3-4 Trees

- Same idea as 2-3 trees, but now nodes can have 2, 3, or 4 children
- A node is overstuffed if it has 4 values
- Push up left-middle node

Traversals

- **Level-Order Traversals**: Nodes are visited top-to-bottom, left-to-right
- **Depth-First Traversals**: Visit deep nodes before shallow ones
- In-order Traversal: LFR - "Left, Functionality, Right"
    - In a BST, this produces a sorted list of nodes in the tree (an in-order traversal that is "depth-first search")

Rotating Nodes

- Imagine a circle around the node you rotate around and its child nodes
- "Pull" everything around in the direction you want to rotate
- Rotations do not change the in-order traversal

Left-Leaning Red Black Trees

- A **binary** search tree
- Has a 1-1 correlation with 2-3 trees
    - Values that are stuffed into one node are now connected with red links
- Invariant: all red edges lean to the left
    - Fix by rotation/color swap
- Insert nodes with a red link (in a 2-3 tree we stuff values in a leaf node)

Tree Traversals

- Pre Order
    - visit self, visit left, visit right
- Post Order
    - visit left, visit right, visit self
    - Can be done with a stack
- In Order
    - visit left, visit self, visit right
- Level Order (Breadth First Search)
    - Visit in order of tree levels
    - Iterative search!
    - Can be done with a queue