Lecture 21: Heaps and PQs

10/14/2020

The Priority Queue Interface

The Priority Queue Interface

```
/** (Min) Priority Queue: Allowing tracking and removal of the
  * smallest item in a priority queue */
public interface MinQP<Item> {
    // Adds the item to the priority queue
    public void add(Item x);

    // Returns the smallest item in the priority queue
    public Item getSmallest();

    // Removes the smallest item from the priority queue
    public Item removeSmallest();

    // Returns the size of the priority queue
    public int size();
}
```

- Useful if you want to keep track of the "smallest", "largest", "best" etc. seen so far

Usage example: Unharmonious Text

- Imagine that you're part of the US Happiness Enhancement team
  - Your job: Monitor text messages of the citizens to make sure they are not having any unharmonious conversations
  - Prepare a report of M messages that seem most unharmonious
- Naive approach: Create a list of all messages sent for the entire day. Sort it using your comparator. Return the M messages that are largest

Naive Implementation: Store and Sort

- Potentially uses a huge amount of memory Theta(N), where N is number of texts
  - Goal: Do this in Theta(M) memory using a MinPQ
  - `MinPQ<String> unharmoniousTexts = new HeapMinPQ<Transaction>(cmptr);`

Better Implementation: Track the M Best

- Can track top M transactions using only M memory. API for MinPQ also makes code very simple (don't need to do explicit comparisons)

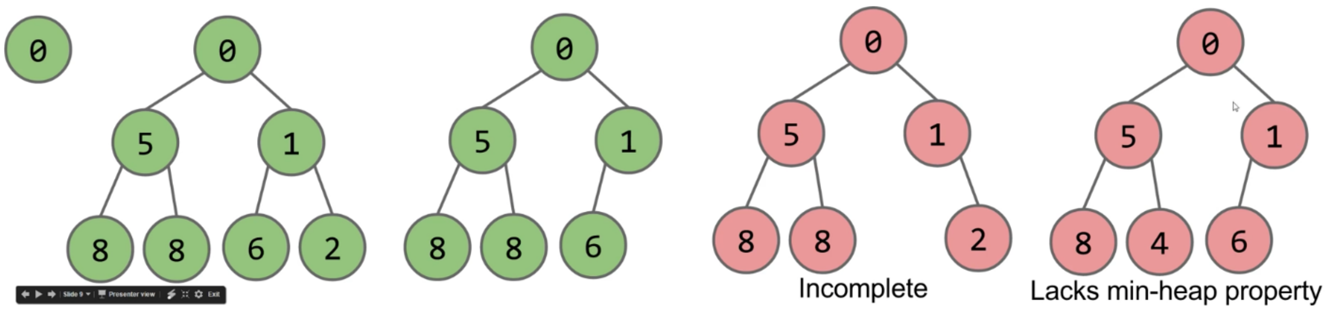
How Would we Implement a MinPQ?

- Some possibilities:
  - Ordered Array
    - add: Theta(N)
    - getSmallest: Theta(1)
    - removeSmallest: Theta(N)
  - Bushy BST: Maintaining bushiness is annoying. Handling duplicate priorities is awkward
    - add: Theta(log N)
    - getSmallest: Theta(log N)
    - removeSmallest: Theta(log N)
  - HashTable: No good! Items go into random places

Heaps

Introducing the Heap

- BSTs would work, but need to be kept bushy and duplicates are awkward
- Binary min-heap: Binary tree that is **complete** and obeys **min-heap property**
  - Min-heap: Every node is less than or equal to both of its children
  - Complete: Missing items only at the bottom level (if any), all nodes are as far left as possible



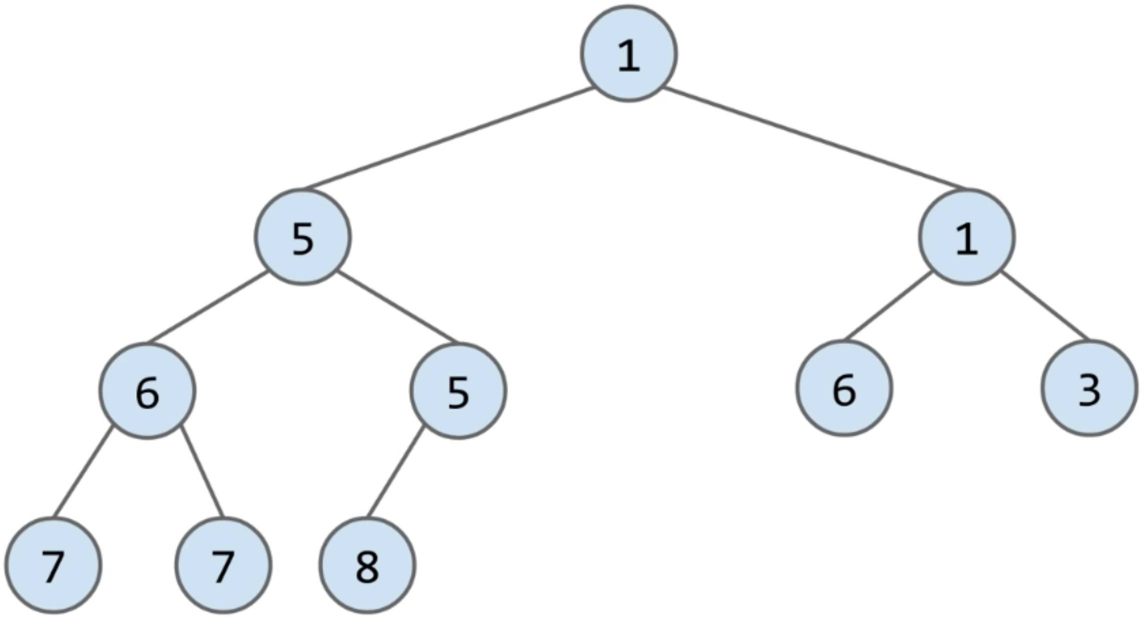
What Good are Heaps?

- Heaps lend themselves very naturally to implementation of a priority queue
- Questions:
  - How would you support `getSmallest()`
    - Return the root

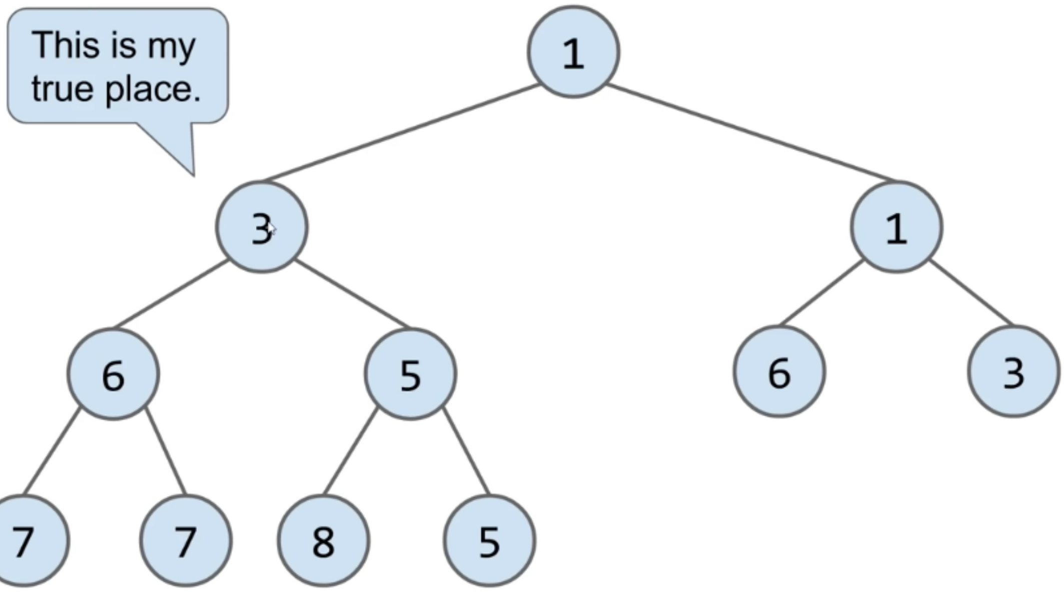
How Do We Add to a Heap?

- Challenge: Come up with an algorithm for `add(x)`

- How would we insert 3?



  - Add to end of heap temporarily
  - Swim up to the hierarchy to rightful place



- Delete min
  - Swap the last item in the heap into the root
  - Then sink your way down the hierarchy, yielding to the most "qualified" items

Heap Operations Summary

- Given a heap, how do we implement PQ operations?
  - `getSmallest()` - return the item in the root node
  - `add(x)` - place the new employee in the last position, and promote as high as possible
  - `removeSmallest()` - assassinate the president (of the company), promote the rightmost person in the company to president. Then demote repeatedly, always taking the "better"successor

Tree Representations

How do we represent a tree in Java?

- Approach 1a, 1b, and 1c: Create mapping from node to children

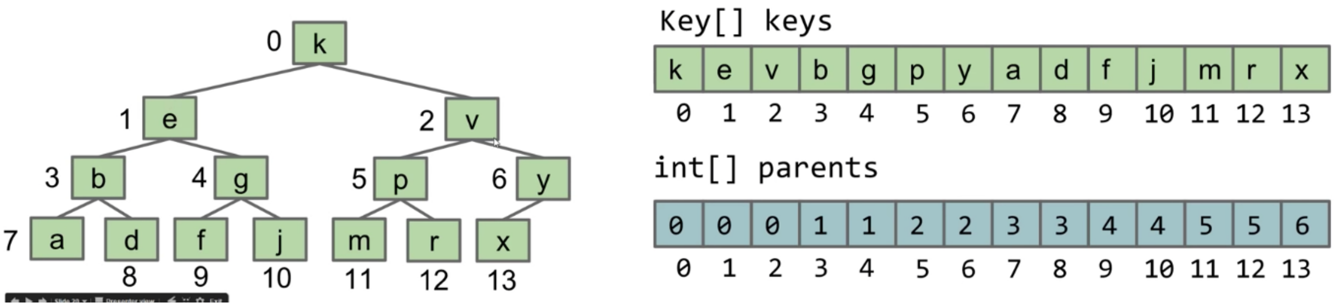
```
public class Tree1A<Key> {
    Key k;
    Tree1A left;
    Tree1A middle;
    Tree1A right;
}
```

```
public class Tree1B<Key> {
    Key k;
    Tree1B[] children;
    ...
}
```

```
// Sibling tree
public class Tree1C<Key> {
    // Nodes at the same level point to each other's siblings
    Key k;
    Tree1C favoredChild;
    Tree1C sibling;
}
```

- Approach 2: Store keys in an array. Store parentIDs in an array
    - Similar to what we did with disjointSets

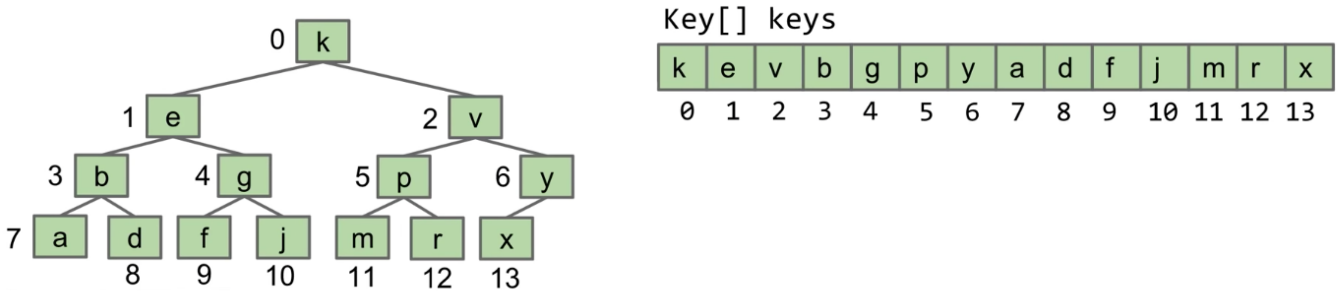
```
public class Tree2<Key> {
    Key[] keys;
    int[] parents;
    ...
}
```



- Approach 3: Store keys in an array. Don't store structure anywhere

    - To interpret array: Simply assume tree is complete
    - Obviously only works for "complete" trees

```
public class Tree3<Key> {
    Key[] keys;
}
```



A Deep Look at Approach 3

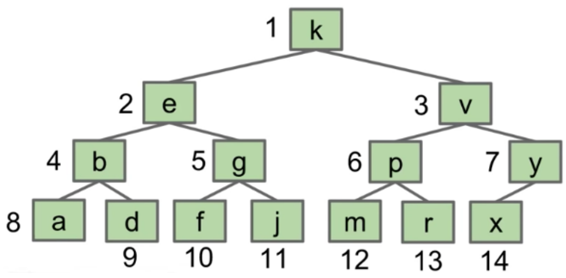
- Write the `parent(k)` method for approach 3

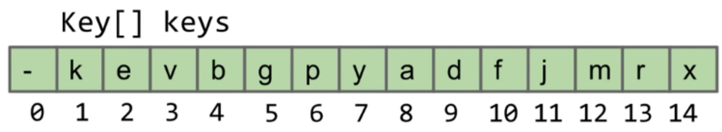
```
public void swim(int k) {
    if (keys[parent(k)] > keys[k]) {
        swap(k, parent(k));
        swim(parent(k));
    }
}
```

```
public int parent(int k) {
    if (k == 0) {
        return 0;
    }
    return (k - 1) / 2;
}
```

Approach 3B (book implementation): Leaving One Empty Spot in the Front

- Approach 3b: Store keys in an array. Offset everything by 1 spot
    - Same as 3, but leave spot 0 empty
    - Makes computation of children/parents "nicer"
        - leftChild(k) = k * 2
        - rightChild(k) = k * 2 + 1
        - parent(k) = k / 2





In next week's lab, you'll implement a MinPQ.

Heap Implementation of a Priority Queue

- Heap
  - add: Theta(log N)
  - getSmallest: Theta(1)
  - removeSmallest: Theta(log N)
- Notes:
  - Why "priority queue"? Can think of position in tree as its "priority"
  - Heap is log N time AMORTIZED (some resizes, but no big deal)
  - BST can have constant getSmallest if you keep a pointer to smallest
  - Heaps handle duplicate priorities much more naturally than BSTs
  - Array based heaps take less memory (very roughly about 1/3) the memory of representing a tree with approach 1a)

Some Implementation Questions

- How does a PQ know how to determine which item in a PQ is larger?
  - What could we change so that there is a default comparison?
- What constructors are needed to allow for different orderings?

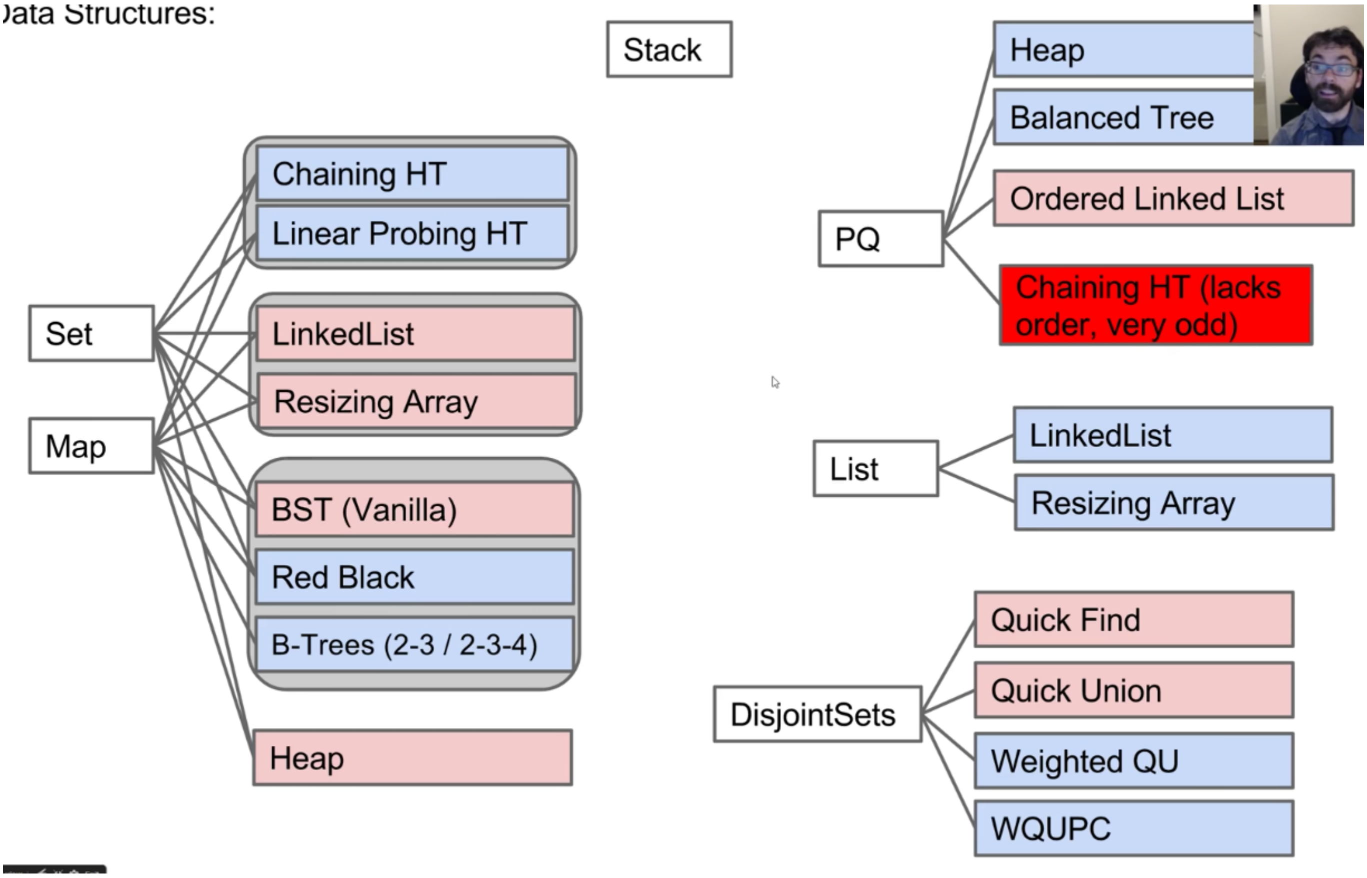
Data Structures Summary

The Search Problem

- Given a stream of data, retrieve information of interest
  - Examples:
    - Website users post to personal page. Serve content only to friends
    - Given logs for thousands of weather stations, display weather map for specified date and time

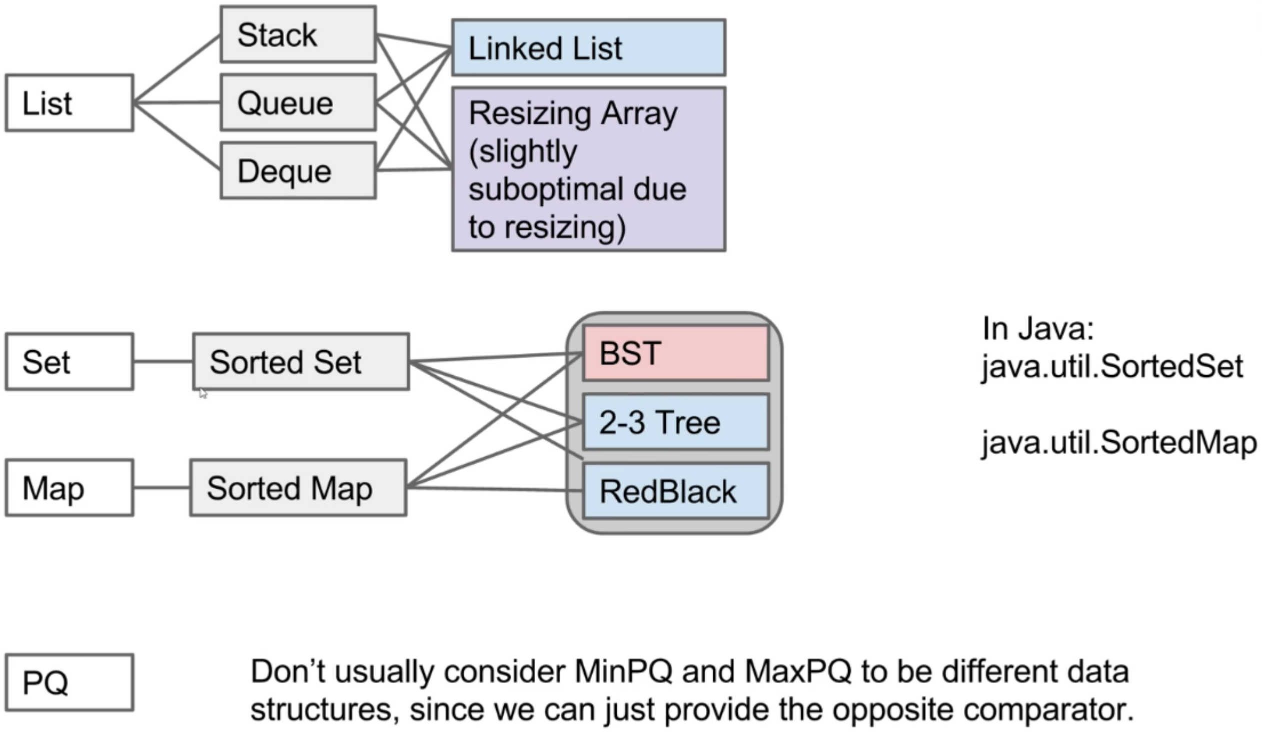
Search Data Structures (The particularly abstract ones)

Name	Storage Operation(s)	Primary Retrieval Operation	Retrieve By:
List	`add(key)` `insert(key, index)`	`get(index)`	index
Map	`put(key, value)`	`get(key)`	key identity
Set	`add(key)`	`containsKey(key)`	key identity
PQ	`add(key)`	`getSmallest()`	key order (a.k.a. key size)
Disjoint Sets	`connect(int1, int2)`	`isConnected(int1, int2)`	two int values



- Abstraction often happens in layers!
  - PQ -> Heap Ordered Tree -> Tree -> {Approach 1A, 1B, 1C, 2, 3, 3B}
  - External Chaining HT -> Array of Buckets -> Bucket -> {ArrayList, Resizing Array, LinkedList, BST (requires comparable items)}

- Specialized searching data structures:



Data Structures

- Data Structure: A particular way of organizing data
  - We've covered many of the most fundamental abstract data types, their common implementations, and the tradeoffs thereof