

Resource Competition in an Ecosystem

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1. Introduction

The goal of this project was to create a model of an ecosystem where two species are competing for resources. The main characteristic of the model is the rivalry between species regarding the limited resources available in the ecosystem. Every species will have different nutritional needs as well as reproduction rates. Additionally, the model will include seasonal changes and their influence on the availability of resources. Python will be used as the modeling environment due to its simplicity of use and detailed documentation.

2. Calculating the resource mass

First, there is a need for a way to calculate the mass of nutrition, in our case grass in the model. To do that we will first use the Beer-Lambert law (1) which describes the effect of sunlight on photosynthesis and subsequently plant growth.

$$I = I_0 * e^{-K*L} \quad (1)$$

where:

- I is the light intensity (or irradiance) at a certain depth within the vegetation canopy,
- I_0 is the incident light intensity at the top of the canopy,
- K is the extinction coefficient that represents how quickly light is attenuated as it passes through the vegetation,
- L is the leaf area index (LAI), a measure of the total leaf area per unit ground area.

The next step is to calculate the grass mass (M) with consideration to the Beer-Lambert law, and we can do so by using a simplified linear relationship as follows:

$$M = \alpha * I \quad (2)$$

where:

- α represents the efficiency of converting intercepted sunlight into plant biomass.

So finally we get:

$$M = \alpha * I_0 * e^{-K*L} \quad (3)$$

where:

- M is the mass of grass,
- α is the efficiency of converting intercepted sunlight into grass biomass. It reflects the ability of the grass to utilize sunlight for photosynthesis and growth.

The formula suggests that the amount of grass mass is directly proportional to the intercepted sunlight. The last thing needed is the calculation of α in regards to the temperature to include the seasonal changes in the environment:

$$\alpha(T) = \alpha_0 * e^{-RT/E_a} \quad (4)$$

where,

- $\alpha(T)$ is the temperature-dependent efficiency,
- α_0 is the efficiency at a reference temperature,
- E_a - activation energy
- R is gas consistency,
- T is the temperature in Kelvin

With those equations, we can take into account both the light changes and the temperature differences that are due to the seasonal changes in the calculation of the resource mass available in the environment.

The model presented above however assumes that the grass mass is proportional to the light flux density within the canopy, which is not the case. The light flux density within the canopy decreases exponentially with depth so that the upper leaves receive more light than the lower ones. This would result in a non-uniform distribution of photosynthesis and biomass within the canopy. The model accounts for this by using the mean light transmission coefficient of the canopy, which is:

$$M = \alpha * I_0 * \frac{(1-e^{-K*L})}{K} \quad (5)$$

This way, the canopy photosynthesis and growth representation are represented more accurately.

3. Resource competition

In this project, a model is created where two species compete for resources without any additional inference between them. Both species have access to the resource of nutrition. The survival of the species in the ecosystem is calculated as follows:

$$S'(t) = (S^{(0)} + M - S(t))D - \left(\frac{m_1}{y_1}\right)\left(\frac{x_1(t)S(t)}{a_1 + S(t)}\right) - \left(\frac{m_2}{y_2}\right)\left(\frac{x_2(t)S(t)}{a_2 + S(t)}\right) \quad (6)$$

$$x_1'(t) = \frac{m_1 x_1(t) S(t)}{a_1 + S(t)} - D_1 x_1(t) \quad (7)$$

$$x_2'(t) = \frac{m_2 x_2(t) S(t)}{a_2 + S(t)} - D_2 x_2(t) \quad (8)$$

where:

- $S(0) > 0, x_1(0) > 0, x_2(0) > 0$
- D is the dilution rate
- $x_i(t)$ is the number of i-th species at the time
- $S(t)$ is the concentration of the nutrient at the time t
- m_i is the maximum growth ('birth') rate of the i-th species
- D_i is the death rate for the i-th species
- y_i is the yield factor for the i-th species feeding on the nutrient
- a_i is the half-saturation constant for the i-th species, which is the nutrient concentration at which the functional response of the species is half minimal

The first model presented above assumes that the number of dying is proportional to those currently alive. Another assumption with this model is that there are no significant time lags in the system and the functional response of the species obeys Holling's nonlearning curve. When analyzing the behavior of the solutions of this system of ordinary differential equations we can observe the survival of the species. The model also takes into consideration the model for grass growth mentioned in the second paragraph.

4. Advantages and Disadvantages of the Model

Advantages:

- The model is flexible and can be parameterized to represent different scenarios and experimental setups. This makes it applicable to a wide range of ecological and biological systems.

- The model provides insights into the interactions between populations and their environment. It allows us to study how changes in parameters impact the dynamics of the populations.
- The model can be used to study competition for limited resources between different species. It helps understand conditions under which multiple species can coexist or when one species outcompetes the others.

Disadvantages:

- The model makes several simplifying assumptions, such as constant parameters, homogeneous mixing, and linear Monod kinetics. These assumptions may not fully capture the complexity of real-world ecosystems.
- The model assumes perfect mixing, neglecting spatial heterogeneity within the ecosystem. In natural environments, spatial variations can have a significant impact on population dynamics.
- Determining model parameters can be challenging.
- The model focuses on nutrient dynamics but may not explicitly include other environmental factors that could influence populations in more complex ecosystems.

5. Implementation

To implement the described model, the presented formulas were used to calculate the resource mass and the number of both species. Most of the parameters in the model are constants, with the exception being the changing temperature (T), which simulates seasonal changes in the environment and the efficiency of converting intercepted sunlight into grass biomass since it is dependent on the temperature.

The parameter values were taken from existing work on the topic of resource competition. Parameters from two different papers regarding the species were considered and compared for this project (PAPERS). The values necessary to calculate resources were taken from multiple sources and adjusted to fit the possible scenario represented in the previously mentioned papers.

The model was made in Python programming language, and the Pygame library was used to present the simulation. The challenge was to integrate the equations with the

visual simulation to present results accurate to the calculations. The idea was to make the number of species and resources dependent on the result of the equation in the current time step. The visual part of the simulation will be more of a symbolic representation of the situation in the environment. In contrast, the data on the plot as well as the numbers presented during the simulation show the exact numbers that come from the theory behind the simulation. The visual simulation shows both of the species moving randomly on the board.

In the first part of the code, we have **getColour** to define the point type of the cells. There are 5 different point types in our project. These are; Dirt (represents the ground without grass), Grass, Blue agent, Red agent, and Grass2 to indicate more dense grass on the ground with a darker shade of green.

```
def getColour(self):
    if self.pointType == 0:
        return (139, 69, 19) # Dirt
    elif self.pointType == 1:
        return (0, 255, 0) # Grass
    elif self.pointType == 2:
        return (0, 0, 255) # Blue
    elif self.pointType == 3:
        return (255, 0, 0) # Red
    elif self.pointType == 4:
        return (0, 120, 0) # Grass2
```

Secondly, we have the **createBoard** function which helps us to create an initial board according to the predefined number of Red and Blue agents and the amount of grass.

```
def createBoard(self, alpha_0, R, T0, Ea, I0, K0, L0, m1, y1, x1,
a1, m2, y2, x2, a2, S0, D, t_values, dt, w):
    col = self.xSize
    row = self.ySize

    grass_count, x1, x2 = self.calculate_grass(alpha_0, R, T0,
Ea, I0, K0, L0, m1, y1, x1, a1, m2, y2, x2, a2, S0, D, t_values, dt,
w)

    grass_count = min(grass_count, col * row)
    grass_indices = random.sample(range(col * row), grass_count)
    g = 0
    for x in range(col):
        for y in range(row):
```

```

        if g < grass_count and (x * row + y) in
grass_indices:
            self.points[x][y].pointType = 1
            g += 1
        else:
            self.points[x][y].pointType = 0
self.init_species()

```

calculate_grass function includes the mathematical equations that our simulation built on. These formulas calculate the amount of grass based on the parameters of the simulation and based on the result of the formula function calls related functions if we need to add or remove grass. Also, it calculates the capacity of the ecosystem and decides the maximum number of species that can live in the ecosystem, and calls the related function in case of adding or removing species.

```

for i, t in enumerate(t_values):
    T = T0 + 0.1 * t
    K = -0.42
    L = 8.5

    alpha_t = alpha_0 * np.exp(-R * T / Ea)
    M = a * I0 * math.exp(-K * L)
    dS = (S0 + (M * math.sin(w * t))) * D - ((m1 / y1) * (x1
* self.S / (a1 + self.S))) - ((m2 / y2) * (x2 * self.S / (a2 +
self.S)))
    self.S = max(self.S + dS * dt, 0)
    grass_values.append(int(self.S))

    dx1 = (m1 * x1 * self.S / (a1 + grass_num)) - D1 * x1
    dx2 = (m2 * x2 * self.S / (a2 + grass_num)) - D2 * x2
    x1 += dx1 * dt
    x2 += dx2 * dt

    diff1 = x1 - blue_agents
    diff2 = x2 - red_agents

    if (diff1) >= 1:
        self.add_species(3, int(diff1))

    elif (diff1) <= 0:
        self.remove_species(3, int(diff1))

```

```

if (diff2) >= 1:
    self.add_species(2, int(diff2))

elif (diff2) <= 0:
    self.remove_species(2, int(diff2))

return (grass_values[-1], x1, x2)

```

move_species function detects the location of each agent on the table and decides their movements. An agent can be decided to move with 0.7 probability and not to move with 0.3 probability. They have 4 options to move. They can move a total of 1 square in each direction in each iteration.

```

if self.points[x][y].pointType in [2, 3]:

    if random.random() < 0.7:
        direction = random.randint(0, 4)
        if direction == 0:
            new_x, new_y = x + 0, y + 0

        elif direction == 1:
            new_x, new_y = x + 1, y + 0

        elif direction == 2:
            new_x, new_y = x + 0, y + 1

        elif direction == 3:
            new_x, new_y = x - 1, y + 0

        elif direction == 4:
            new_x, new_y = x + 0, y - 1

        if 0 <= new_x < self.xSize and 0 .....

            self.points[new_x][new_y].pointType =
copy.deepcopy(self.points[x][y].pointType)
            self.points[x][y].pointType = 0

```

The simulation contains two plots. The first one is **update_plot_S** for grass amount and **update_plot_pop** for the population of agents. Also, there are tables too for these numbers. We can check the result of the simulation from these places.

```

def update_plot_S(x, y):
    line_s.set_xdata(np.append(line_s.get_xdata(), x))
    line_s.set_ydata(np.append(line_s.get_ydata(), y))
    ax_s.relim()
    ax_s.autoscale_view()
    ax_s.legend()
    fig.canvas.draw()
    plt_img = np.frombuffer(fig.canvas.tostring_rgb(),
dtype=np.uint8)
    plt_img = plt_img.reshape(fig.canvas.get_width_height()[::-1] +
(3,))
    plt_img = np.fliplr(plt_img)
    plt_img = np.rot90(plt_img, 1)
    plt_surf = pygame.surfarray.make_surface(plt_img)
    screen.blit(plt_surf, (660, 0))

def update_plot_pop(x, y1, y2):
    line_x1.set_xdata(np.append(line_x1.get_xdata(), x))
    line_x1.set_ydata(np.append(line_x1.get_ydata(), y1))
    line_x2.set_xdata(np.append(line_x2.get_xdata(), x))
    line_x2.set_ydata(np.append(line_x2.get_ydata(), y2))
    ax_x.relim()
    ax_x.autoscale_view()
    ax_x.legend()

```

6. Creating the project and difficulties met along the way

Work on this project began with finding the right topic, one that could have a practical application and would be well documented in other available works, to have something to rely on while creating and validating the end result. Finally, resource competition was selected as the topic for this project.

The next step was to look through the available literature on the topic and compile the knowledge to find the best solution to use for this project. An essay was created to introduce the project and present the results of the conducted research. The more extensive knowledge of the topic allowed us to specify the details of the project and the exact type of competition that we wanted to implement. The aim was to create a model where two species would compete for a limited resource without any additional inference between them.

Finally, the next step was to implement the model. At first, the idea was to use the NetLogo environment. However, after learning more about the usage and programming language used in NetLogo a decision was made to use Python and

Pygame library instead. To start all the necessary equations were implemented and a simple plot was created to visualize the results in a set amount of timesteps. Later the simulation was made, where green spots on the board represented the resource and moving agents in two different colors represented the individuals of the two species. Lastly, one functionality was added where the resource had two levels of density (represented by darker and lighter green colors on the simulation).

The Last step of the project is this updated version of the essay as well as the presentation of the project.

The first encountered difficulty was working with NetLogo, which proved to be more complicated than working with Python, which was already familiar.

The main difficulty, however, while working on the model was figuring out how to create an accurate simulation and have the elements reliant on the equations provided in the theoretical research. The final result is still not perfect as there is no implementation that would connect an instance of an animal to an instance of the resource that is being consumed by it in a specific time moment. The results of the equations are displayed during the simulation, so the actual math-backed values are still visible but the moving agents are more symbolic of the situation, where only the number of agents (animals) in each timestep is true to the results of the equations but their movement and behavior are random. The mathematical equations do not take into account the movement of the agents and their need to physically reach the resource to be able to consume it, thus using the equations that were chosen including it would require some more work, or maybe figuring out a different method of defining the simulation, it's rules and functioning.

Another big difficulty in the project is finding the right parameter values necessary to make the simulation imitate a real-life scenario and validate it. Multiple papers were analysed in search of the parameters that would be fitting in the context of this project but only two alternative sets of parameter values were found. The parameters regarding species were more explicit in the found papers, however, the ones concerning the resources were more difficult to find, since no one paper would describe the values of all the necessary parameters in the same context. Those parameters for resources were compiled from a few different papers describing them in different environments, which can cause a disturbance in the final results of the simulation.

Other than the mentioned above there weren't many other difficulties encountered while working on this project.

7. Results

The main idea behind modeling the simulation of resource competition is considering the practical applications for this project. In this paragraph, the results of the model will be compared to the results acquired in the literature on the topic.

Below is the first example of acquired results that match the pattern of behavior of the simulation in the literature. The initial state of the model is different, but the eventual state of the model is in agreement with the results from the analyzed paper. This example is described as a case where “the supply of resource is sufficient to support B (b), there is an interval where a finite population B survives on the available resource. The decline of species B is brought about by its competitor A eating away the required food. Eventually, a higher population of A is reached because of a higher supply”.

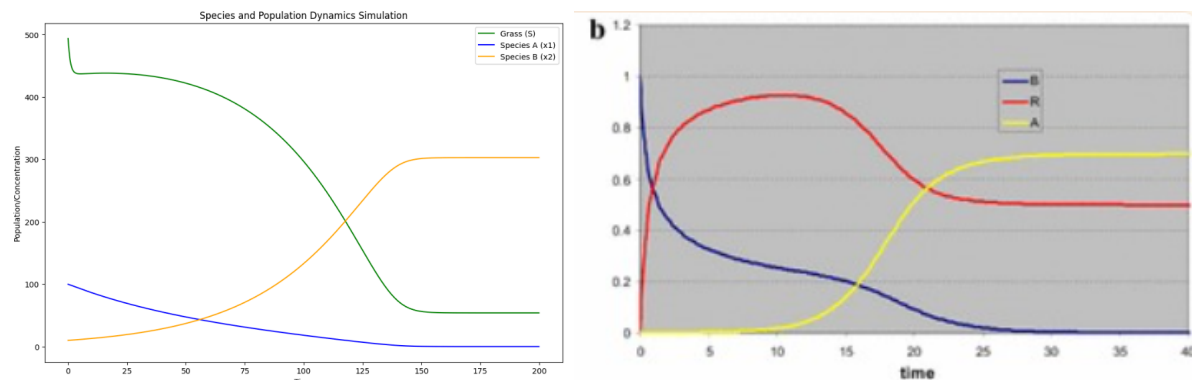


Fig. Comparison of the results. Left: our simulation, Right: source: <https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4559563/>

Another example where it was possible to acquire results that imitate an example from literature, this one is described as: “the stable resource level is below the critical level for maintenance of B (a), this consumer simply disappears, and at some later time A grows to its stationary level.”

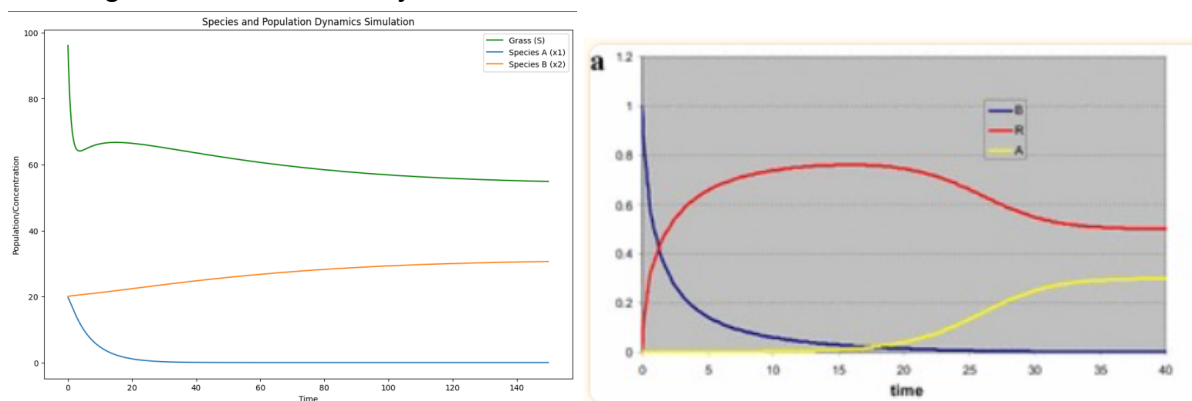


Fig. Comparison of the results. Left: our simulation, Right: source: <https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4559563/>

The next example is a modified version of the model that uses a different way of calculating temperature change (using a sin function instead of gradually changing the temperature with every step of the simulation) and shows a longer simulation process, that showcases the oscillating change in the conditions and species survival that is dependant on it. The work we compare it to is from a set of simulations implemented in NetLogo. In our simulation there is also an unexpected spike at the beginning, we are unsure why it happens.

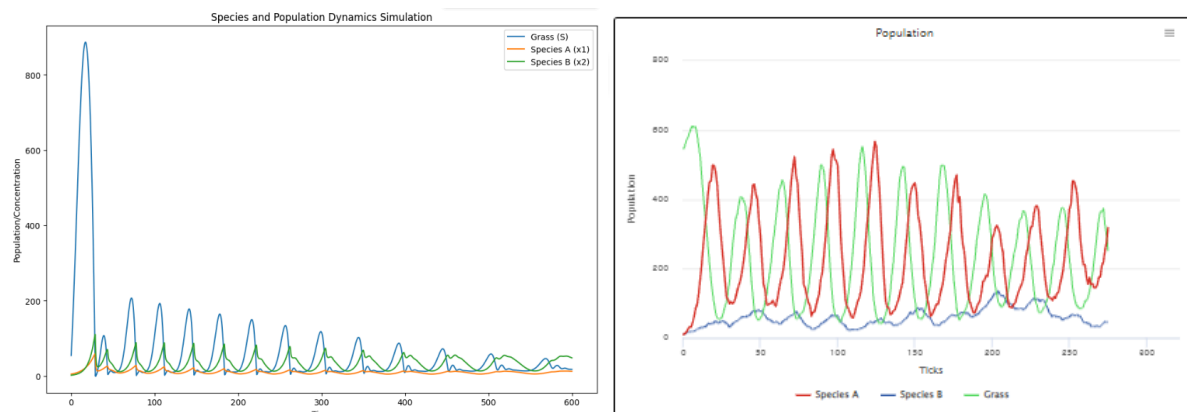


Fig. Comparison of the results. Left: our simulation, Right: source: <https://www.netlogoweb.org/launch#http://ccl.northwestern.edu/netlogo/community/Ecological%20Competition%20Model.nlogo>

Shown in the plots above is the similarity between results accomplished in papers and other works related to resource competition and ones produced by our models after adjusting the parameters. In a lot of papers, there was no specification of what parameters were used, so it made it difficult to replicate the results exactly.

Another important aspect of the simulation evaluation is its limited ability to adjust the parameters randomly. With some configurations and changes to the parameters, the simulation seems to crash, stopping after just around 100-200 time steps. Due to limited time, it was not possible to find what exactly was causing the interruption. Some experiments planned later on might be difficult to execute and it might be a challenge to find the right parameters for the simulation to show examples for some questions posed in the analysis part of this project. This shows the lack of robustness in the model, which definitely should be worked on in the future since it's crucial for the model to be robust.

8. Analysis

To analyze the practical uses of modeling further and highlight its importance the following analysis questions were posed and tested. The goal of the analysis is to showcase different use cases for such simulations. The following questions were analyzed:

- a) **Which parameters influence the speed of extinction the most? How sensitive are species populations to changes in key parameters, such as reproduction rates and consumption rates? Which species is a less fit competitor when it comes to survival?**

The balance of ecosystems is susceptible to many factors that may speed up the extinction of species. Identifying and understanding the parameters that have the most significant influence on extinction is important for devising conservation strategies. Checking which variables impact the survival of species the most allows us to gain insights into the dynamics of ecosystems and develop targeted interventions to lessen the threats to biodiversity. Conducting sensitivity analyses for different parameters like reproduction rates and consumption rates can reveal which factors have the most significant impact on species populations. This helps prioritize interventions and management strategies.

As the first step of this analysis, there needs to be a basis we can compare to. The following plot represents a simulation for a set of parameters identical for both of the species. During the experiments, the value of only one parameter for one species will be changed to observe how it will affect that species' survival and the general state of the environment.

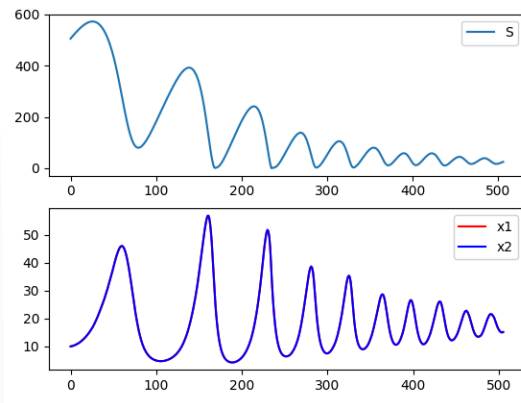


Fig. Results of the simulation for the same parameters for both species.

In the first experiment, we will change the maximum growth rate of one of the species. The values will be decreased/increased by 30% rather than a specific value, to ensure that the changes for the parameters have similar weight.

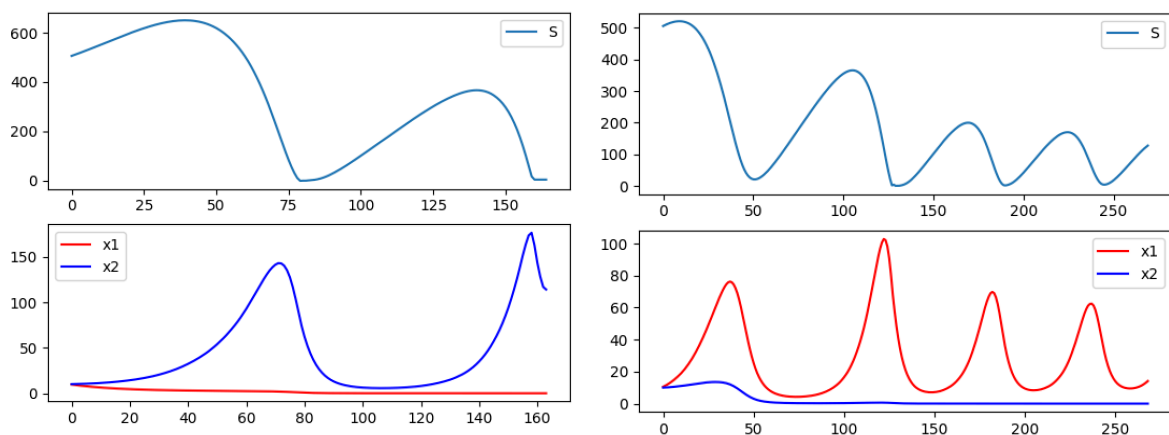


Fig. Left: maximum growth rates for x1 decreased by 30%, Right: max growth rates for x1 increased by 30%.

It is clear from the simulation results how changing growth rates influences the environment. The growth rate seems to have a big influence on how the species survive and how their resource consumption changes (we can see that by how both of the species are influenced).

The next parameter we will observe is the yield factor:

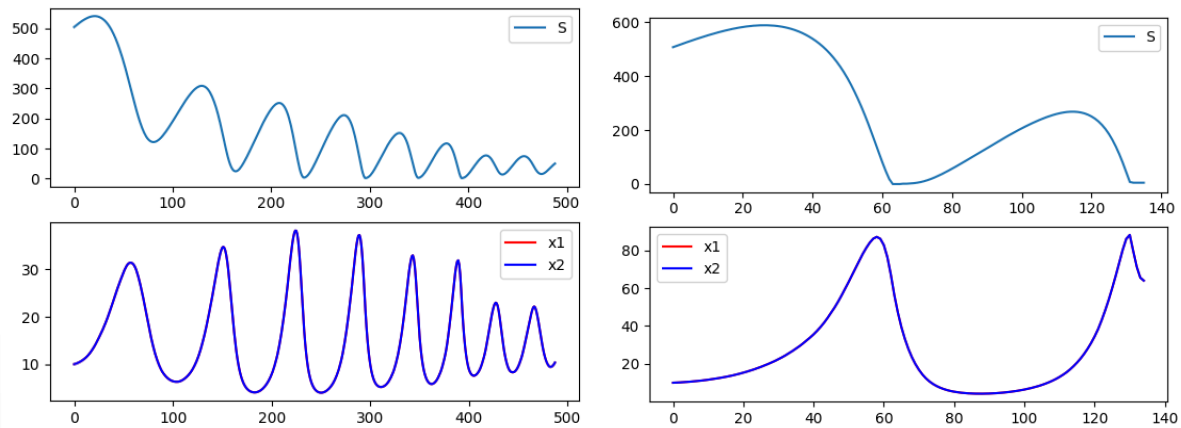


Fig. Left: yield factor for x1 decreased by 30%, Right: yield factor for x1 increased by 30%.

The results in this case show that modifying the yield factor values by 30% has little to no influence on the environment and species survival.

The next factor checked is the half-saturation constant:

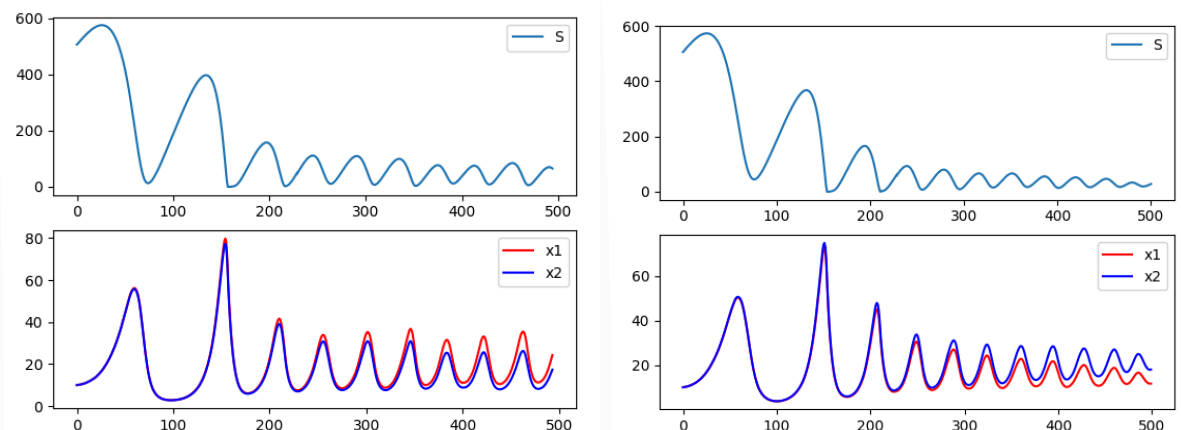


Fig. Left: half-saturation constant for x1 decreased by 30%, Right: half-saturation constant for x1 increased by 30%.

In this example we also see a slight change, however, it is more visible than with the yield factor, especially over time. Decreasing the half-saturation constant seems to increase the number of surviving species. It is possible that the percentage change was too insignificant to show true change in the result.

The last parameter of the species that we will analyze is the death rate:

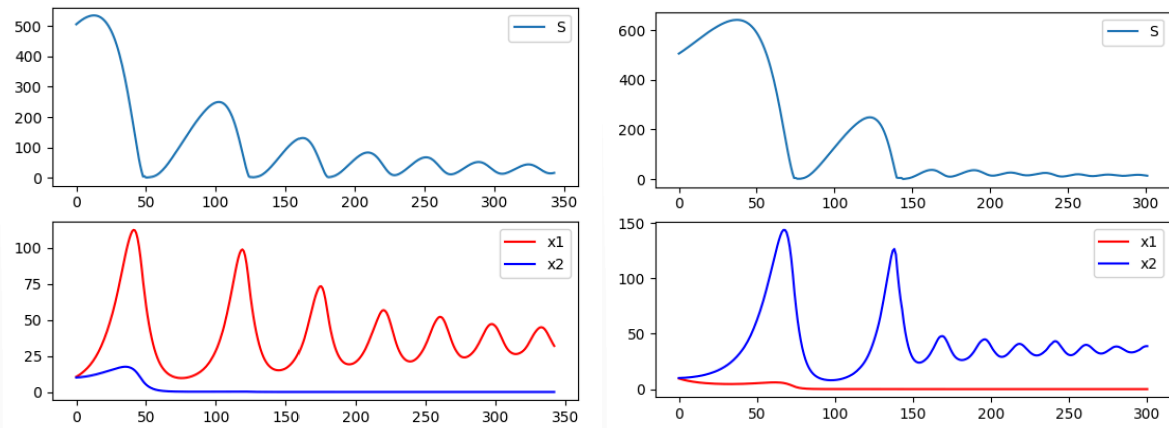


Fig. Left: the death rate for x1 decreased by 30%, Right: the death rate for x1 increased by 30%.

The results in this last example seem the most obvious since the death rate is bound to influence the species' survival the most. And we can see in this example the species with a higher death rate very quickly goes extinct.

To sum up the experiments we can see that maximum growth rates as well as death rates influence the survival of the species in the highest degree, while the yield factor and half-saturation constant are less influential when it comes to the number of surviving species.

b) How does temperature change influence resources and animal survival?

Temperature, a fundamental environmental variable, plays a big role in shaping ecosystems and influencing the availability of resources crucial for animal survival. Understanding the relationship between temperature fluctuations and resource and species dynamics is crucial for understanding the influence of climate variations on species. Exploring how temperature change impacts resource abundance and, consequently, animal survival, shows insight that is important for wildlife management and conservation practices.

The following experiments will display the results of the simulation for different values of the T parameter which represents temperature in our model (no other parameter is changed in both the simulations).

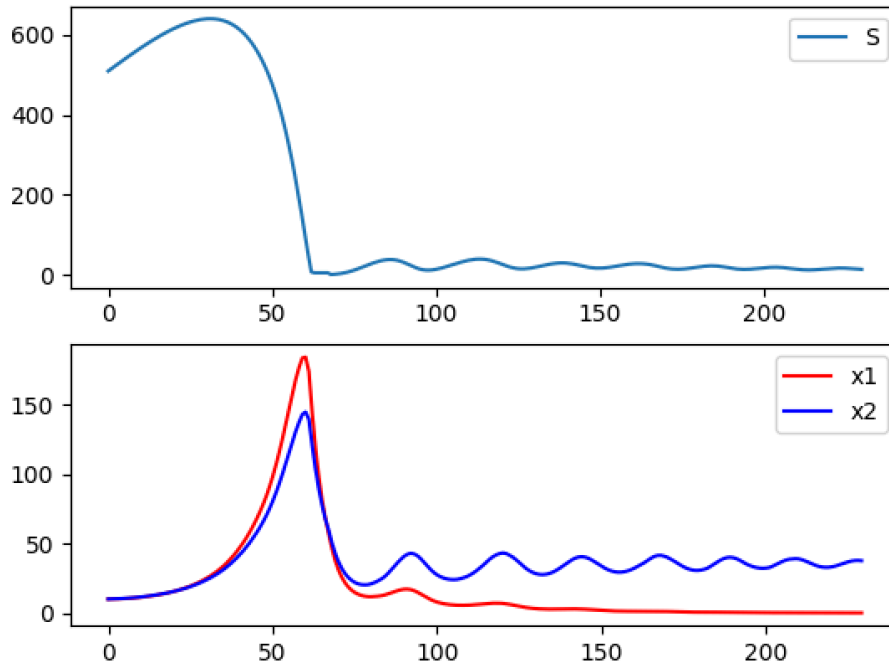


Fig. Simulation for temperature $T=300$.

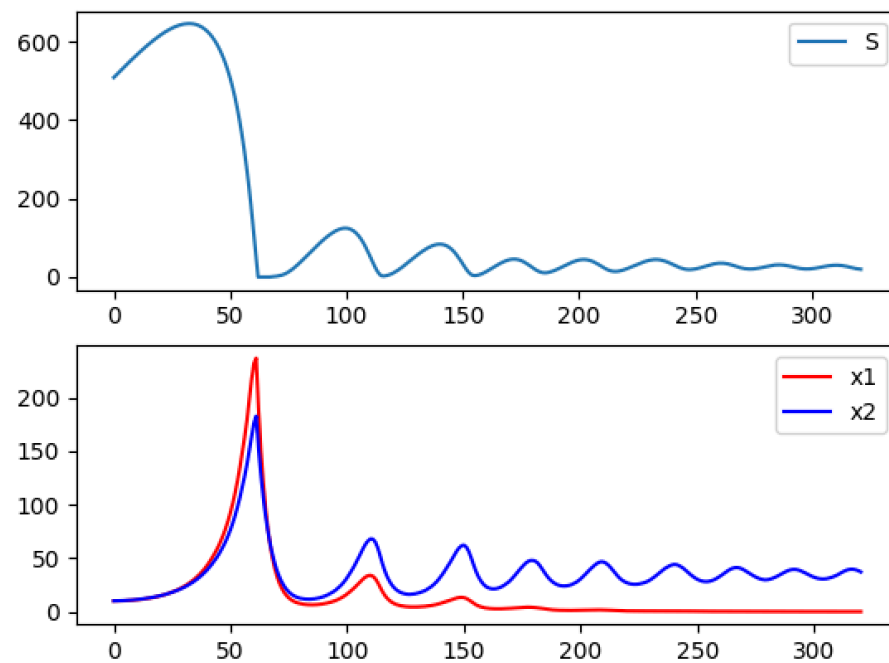


Fig. Simulation for temperature $T=280$.

In the following plots, we can see that the decreased temperature caused a decrease in the available resources. The intention was to check the results for a wider variety of temperature values but the simulation seemed to crash when the parameter was outside of the slim range, which was mentioned in the earlier paragraph, where the robustness of the model was mentioned.

c) How does the initial population size of species impact their long-term survival and resource utilization?

Investigating the consequences of different initial population sizes for different species can provide insights into the perseverance of populations over time. This can help identify critical thresholds for population viability.

The following figures represent the result of the simulation for different initial numbers of the population size (no other parameters will be changed in between the simulations, both species have different parameter values).

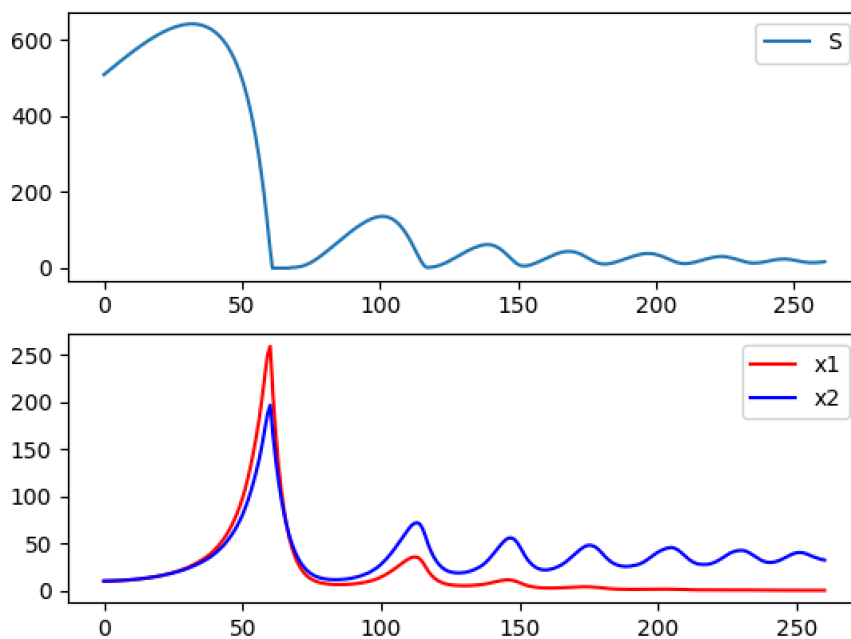


Fig. Simulation results for initial population size: $x_1=10$, $x_2=10$.

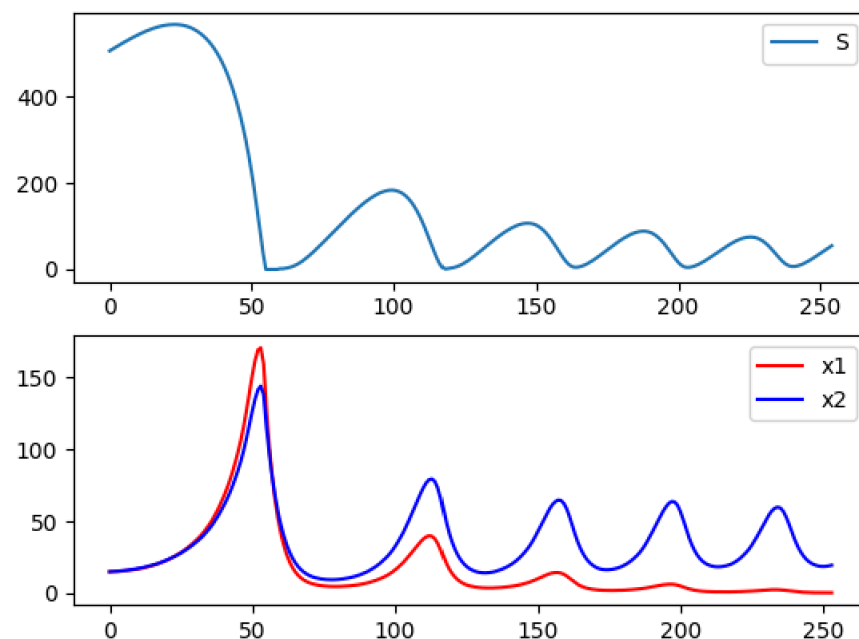


Fig. Simulation results for initial population size: $x_1=15$, $x_2=15$.

The first two plots show similar patterns regarding the survival of the species. When their number slightly prolongs the time of survival of the x2 species.

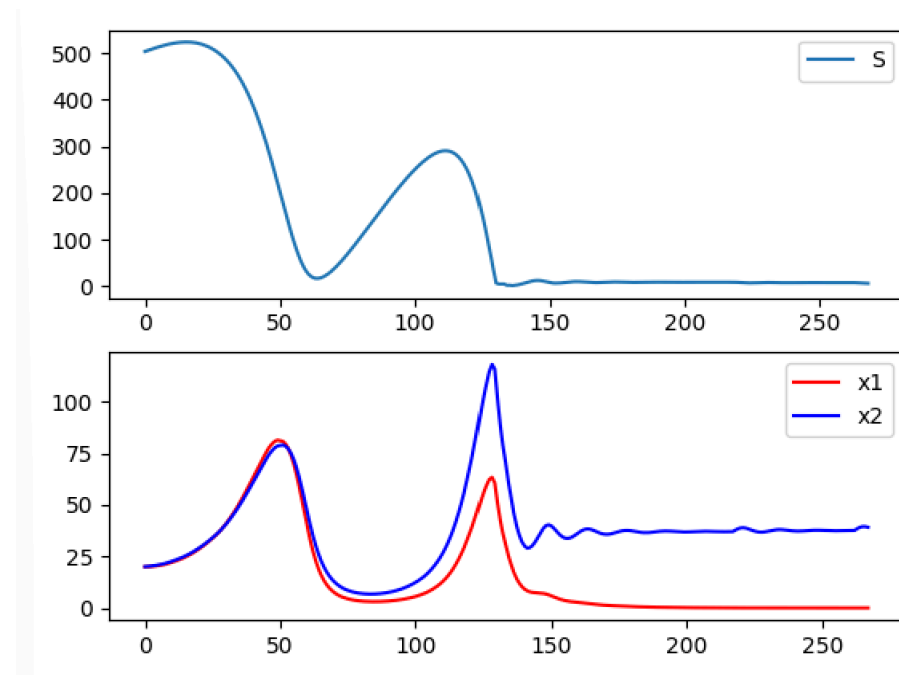


Fig. Simulation results for initial population size: $x_1=20$, $x_2=20$.

For a higher initial number of species, the pattern changes quite a lot.

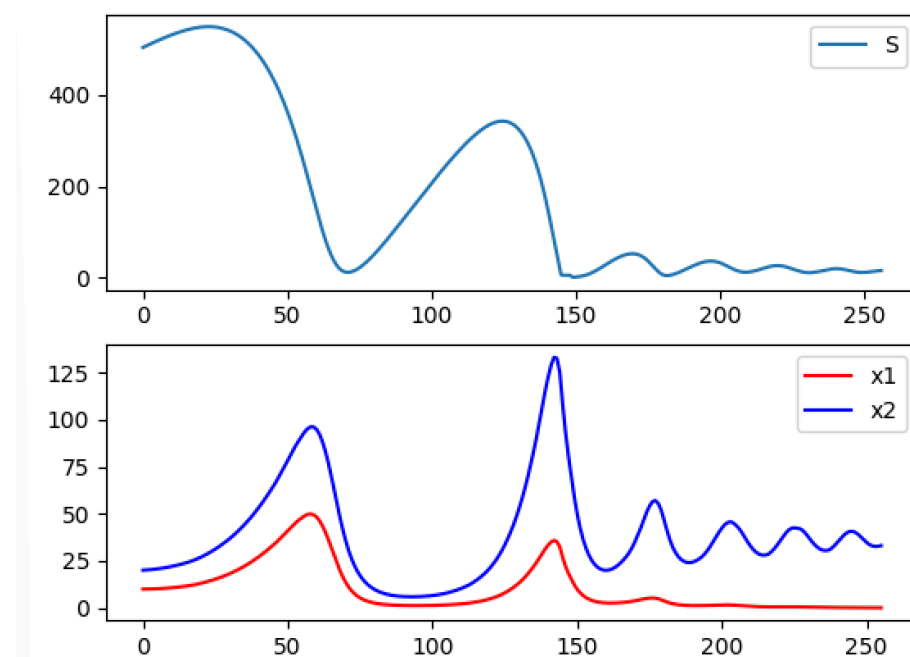


Fig. Simulation results for initial population size: $x_1=10$, $x_2=20$.

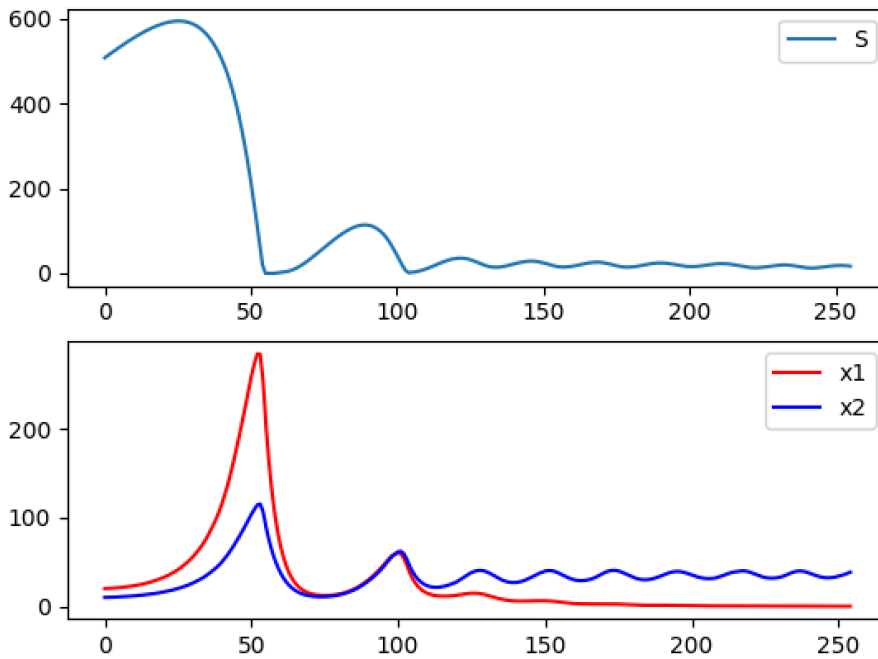


Fig. Simulation results for initial population size: $x_1=20$, $x_2=10$.

The last two plots show how the number of initial populations can prolong the survival of the species, but in the end, in all cases, the same species ends up extinct. That can suggest that the initial population of the species might not be of any help in helping prevent its extinction if the species is not fit to survive in the first place.

d) Can resource levels be artificially adjusted to prevent species extinction?

The balance between species and their environment is often threatened due to various factors, from climate change to habitat destruction. The consequences of human activities have already proven to be dire for a lot of species of both plants and animals, however, we can try to do better and analyze any capacity to bring a positive change and help the survival of endangered species instead of threatening it more. One idea of such influence would be questioning whether artificially adjusting resource levels could be a means to prevent species extinction. Can human inference guided by simulation results such as the one we are trying to accomplish in this project, offer a solution to sustain species population, without leading to side effects that could concur from intervening with the amount of resources? To try and answer that question we will conduct the following experiment.

In the simulation created for this project, inference in the initial number of resources makes the simulation unstable and it causes interruption after around 200 steps. It might suggest some error in the way the simulation works. From the experiments performed in the other experiments, it will become more apparent that the biggest influence on the survival of species is held by the parameters connected

with the species themselves like their maximum growth rate. However, in an improved version of the model, it would be beneficial to further study the influence of the fluctuating value of the initial resource in the environment.

e) What happens when resource availability fluctuates over time, and how do species adapt to such variations?

Investigation of how the fluctuation of resources can influence the environment can offer insights into the adaptability of species to changing environmental conditions. From all the conducted experiments it can be seen that the population is tied to the resource numbers. Whenever the resource fluctuates the population fluctuates as well. We can see that pattern in all the conducted experiments and all results of the simulation.

The scope of this project and the created model is limited, however, it could be extended to consider some other aspects of an environment. With such extensions, we could pose some other possible analysis questions for more complex simulations:

- Under what conditions can two species coexist?
- How do invasive species impact native ecosystems and their resource dynamics?
- What role does spatial distribution play in resource competition and species coexistence?
- How does the availability of key resources affect reproductive strategies and population dynamics?
- What impact do human activities, such as urbanization and agriculture, have on resource competition and biodiversity?
- How do environmental stressors, such as pollution and climate change, influence species interactions and resource utilization?

9. Summary

After implementing the chosen model according to the presented equations and rules the results of the simulation were compared to a real scenario to check the model's accuracy and correctness of the implementation. Analysis was conducted to answer some questions about the usefulness of creating such a model and some possible scenarios in the created environment that could help better understand the behavior and dependencies between species and the resources available in their environment. The model was evaluated and provided results analogical to those in the available papers regarding the topic of resource competition between species.

10. References

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