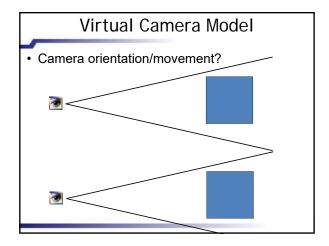
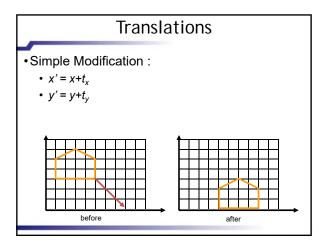
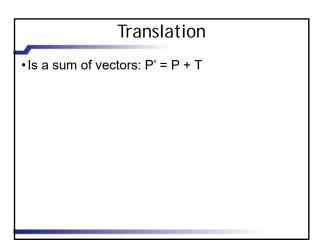


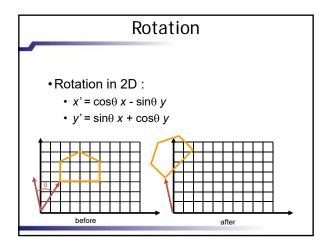
Graphics Pipeline Overview Camera Movement/Orientation Object Movement/Orientation Projection Rasterization How to perform operations in "practice"?

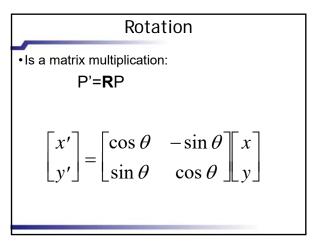


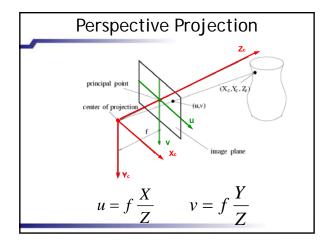
Movement and Orientation in 2D • Starting in 2D – Simpler to represent











Virtual Camera Model

Projecting a scene point with the camera:

- · Apply camera position (adding an offset)
- · Apply rotation (matrix multiplication)
- Apply projection (non-linear scaling)
 Our camera starts to become complicated and not well adapted to a hardware solution.

There has to be a better way...



What we want:

- Notation simple, concise
- Unification
 - Translation, rotation
- · Make concatenation of operations possible

And if I am allowed to dream:

- · Allow projection operation too
- Keep everything linear

Dreams can come true!

We will see...

- Graphics Pipeline:
 - Take vertices
 - Multiply with matrix
 - Fill pixels underneath triangle

Homogenous Coordinates - Definition

- Projective geometry
 - Used everywhere (Image, Vision, Robotics...)
- N-D space represented by (N+1)-D coordinates without a zero and a new equivalence relation:

Two points p, q are considered equal iff

exists a!=0 such that p*a = q

Homogenous Coordinates - Examples

- Let us focus on in the following on a 2D space (having 3D coordinates)
- · What does this equivalence relation mean?
- E.g.,

$$(1,2,1) = (2,4,2)$$

But why do we have 3 coordinates for 2D?

Two points are the same if they lie on a line through the origin (0,0,0)

Along a line -> one dimension less

Homogenous Coordinates - Affine Space

What is the relationship between the
 "standard space" and projective space?

The traditional "affine" 2D space can be
 embedded via a plane.

This affine plane
 is like a standard
 2D space
 x

Simplest embedding:
We will always set the last coordinate to 1.
To underline this special role, the last coordinate is called W.
e.g., a point is (x,y,w)

This affine plane is like a standard 2D space

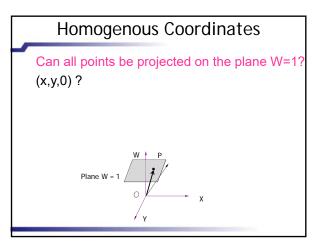
Homogenous Coordinates - Projection

How to find affine points from projective points Divide by last coordinate and keep first two:

(2x,2y,2) / 2 = (x,y,1) -> (x,y)

Does this look familiar?

Homogenous Coordinates - Embedding
 How to go from 2D to projective ?
 Imagine a mapping (x,y) -> (x,y,1) = (2x,2y,2)

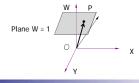


Homogenous Coordinates

Let's look at

P=(x, y, w) with $w \rightarrow 0$

P=(1,0,1)



Homogenous Coordinates

Let's look at

P=(x, y, w) with $w \rightarrow 0$

P=(1,0,0.5)=(2,0,1)

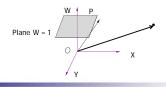


Homogenous Coordinates

Let's look at

P=(x, y, w) with $w \rightarrow 0$

P=(1,0,1/4)=(4,0,1)

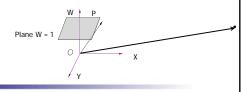


Homogenous Coordinates

Let's look at

P=(x, y, w) with $w \rightarrow 0$

P=(1,0,1/16)=(16,0,1)

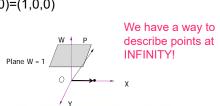


Homogenous Coordinates

Let's look at

P=(x, y, w) with $w \rightarrow 0$

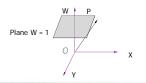
P=(1,0,0)=(1,0,0)



Points at infinity

Choosing a different plane associated with the embedding, changes infinity and much more...

We will focus on an affine space with Plane W=1! Hence, in our case Infinity always means W=0



Homogeneous Coordinates

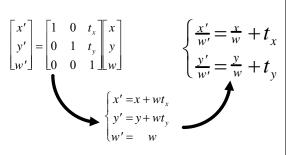
How is all this useful?

Projective geometry makes things simpler!

Overview

- · Object Movement/Orientation
- Complex Objects
- · Projection

Translations in homog. coordinates



What happens to points at infinity?

Rotation in homog. coordinates

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

$$\begin{cases} \frac{x'}{w'} = \cos \theta \frac{x}{w} - \sin \theta \frac{y}{w} \\ \frac{y'}{w'} = \sin \theta \frac{x}{w} + \cos \theta \frac{y}{w} \end{cases}$$

$$\begin{cases} x' = \cos \theta x - \sin \theta y \\ y' = \sin \theta x + \cos \theta y \\ w' = w \end{cases}$$
What happens to points at infinity?

Transformation Compositing

- All rigid transformations become linear!
 (and many others too...)
- It is possible to produce linear applications that translate and rotate

Rotation around point Q

- Rotation around point Q:
 - Translate Q to origin(T_{O}),
 - Rotate around origin (\mathbf{R}_{\odot})
 - Translate back to Q (T_{-O}).

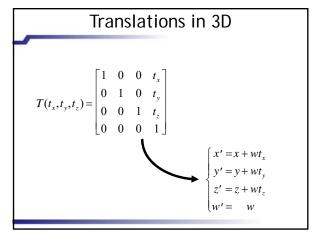
$$P' = (T_{-Q})R_{\Theta}T_{Q} P$$

And in 3 dimensions?

- •The same!
- Add a fourth coordinate, w

 $\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$

• All transformations are 4x4 matrices



Rotations in 3D

- •Rotation = axis and angle
- Rotations around x,y,z axis are simple
 - A different axis is achieved by combining matrices

Rotation about *X*-axis
$$R_{x}(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
Axis *x* not changing
$$Quick \ \text{verification}: rotation of } \pi/2$$
Should change *y* in *z*, and *z* in -*y*

$$R_{x}(\frac{\pi}{2}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation about y-axis

$$R_{y}(\theta) = \begin{bmatrix} \cos \theta & 0 & \sin \theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta & 0 & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

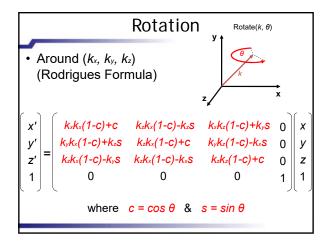
Axis y not changing

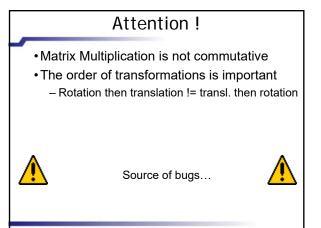
Quick verification: rotation of $\pi/2$

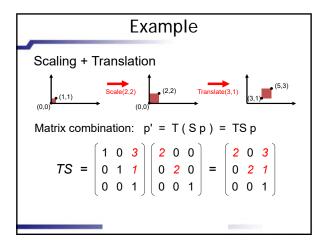
 $R_{\mathbf{y}}(\frac{\pi}{2}) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

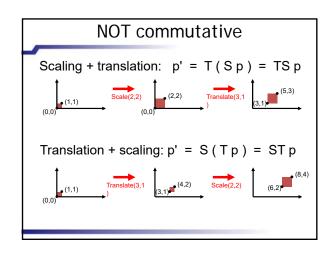
$$R_z(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
Axis z not changing

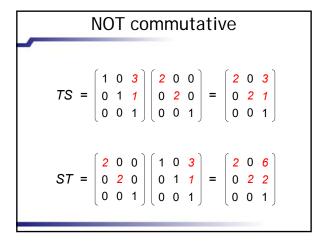
Quick verification: rotation of $\pi/2$
Should change x in y, and y in -x
$$R_z(\frac{\pi}{2}) = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$













Overview

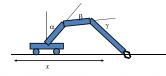
- · Object Movement/Orientation
- Complex Objects
- · Projection

Complex Objects

 Projective Geometry allows us to describe relationships by concatenating operations together.

Why concatenate operations?

- ·Often, objects are defined as combinations
 - Examples : robots, cars (wheels)...
- ·We want a natural behavior:
 - Object stays connected:
 - If you move the arm, the hand should follow



Why concatenate operations?

- Relative coordinates
 - Wheel with respect to car
 - Bolts with respect to wheel
- · Like in real life: rarely absolute coordinates!

Why concatenate operations?

- · Use relative coordinates:
 - Position of hand with respect to arm
 - Position of arm with respect to body

– ...

Why concatenate operations?

- We need absolute coordinates to draw:
 - Concatenate and apply complete matrix
 - Example: Draw an arm with hand

The arm consists of two parts:

The arm itself and a hand

Both are designed independently and placed at the origin as shown below:





Why concatenate operations?

· Drawing first the arm, then the hand, you get:



- · That does not look right!
- Instead: Produce a matrix to place the origin at the right location and then draw the object

Why concatenate operations?

- Graphics Pipeline:
 - Take vertices
 - multiply with matrix
 - Fill pixels

Why concatenate operations?

- · Concatenate and apply matrices
 - Example: Draw an arm with hand



Why concatenate operations?

- · Concatenate and apply matrices
 - Translation at position of arm (changes matrix)



Why concatenate operations?

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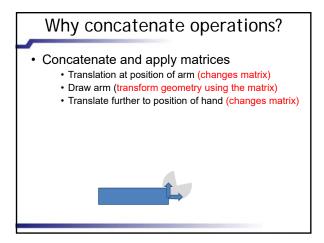


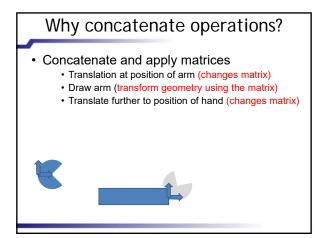
Why concatenate operations?

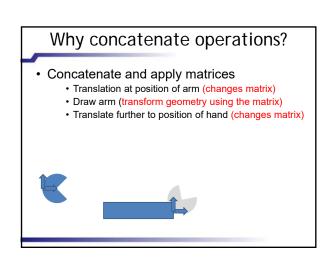
- · Concatenate and apply matrices
 - Translation at position of arm (changes matrix)
 - Draw arm (transform geometry using the matrix)

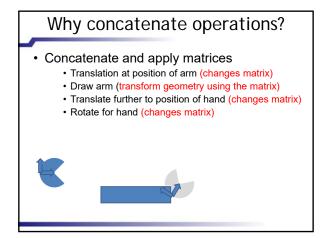


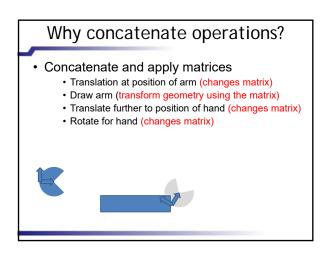
Why concatenate operations? Concatenate and apply matrices Translation at position of arm (changes matrix) Draw arm (transform geometry using the matrix) Translate further to position of hand (changes matrix)











Why concatenate operations?

- · Concatenate and apply matrices
 - Translation at position of arm (changes matrix)
 - Draw arm (transform geometry using the matrix)
 - Translate further to position of hand (changes matrix)
 - Rotate for hand (changes matrix)
 - Draw hand (transform geometry using the matrix)





Why concatenate operations?

- · Concatenate and apply matrices
 - Translation at position of arm (changes matrix)
 - Draw arm (transform geometry using the matrix)
 - Translate further to position of hand (changes matrix)
 - Rotate for hand (changes matrix)
 - · Draw hand (transform geometry using the matrix)



Why concatenate operations?

- · Concatenate and apply matrices
 - Translation at position of arm (changes matrix)
 - ROTATE (changes matrix)
 - Draw arm (transform geometry using the matrix)
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Why concatenate operations?

- · Concatenate and apply matrices
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Why concatenate operations?

- Concatenate and apply matrices
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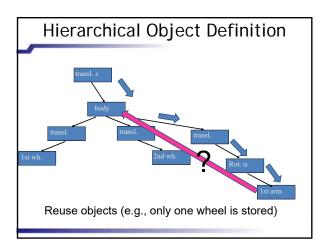
Why concatenate operations?

- · Concatenate and apply matrices
 - Translation at position of arm (changes matrix)
 - ROTATE (changes matrix)
 - Draw arm (transform geometry using the matrix)
 - Translate further to position of hand (changes matrix)
 - Rotate for hand (changes matrix)
 - Draw hand (transform geometry using the matrix)



Why concatenate operations?

- · What about even more complex objects?
- · For example:
 - A race car...
 - An airplane...



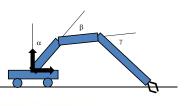
Coordonnées relatives

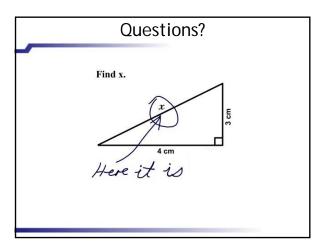
- How to go back?E.g., Body -> left arm -> Body?
- Simple Solution:
- · Keep a matrix stack!

Matrix Stack • Keep information about modifications - T (translation by x) push - draw robot body - TT² (translation to center of 1st wheel) push - draw first wheel as circle of center (0,0) - return to T: pop - TT³ (T³ translation to center of 2nd wheel) push - draw second wheel

Why concatenate operations?

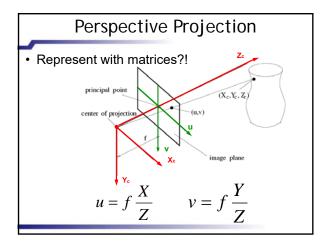
- Another simple example:
- 1. Rotation a
- 2. Translation a
- 3. Rotation b
- 4. Translation b
- 5. Rotation c
- 6. Translation c





Overview

- Object Movement/Orientation
- Complex Objects
- Projection



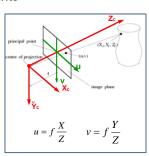
Perspective Projection

• "Homogenize" the points

$$P = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \implies \widetilde{P} = \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

• Look for M such that:

$$\widetilde{MP} = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} fX/Z \\ fY/Z \\ 1 \end{pmatrix}$$



Perspective Projection

· Hint: Think projective!

$$M\widetilde{P} = \begin{pmatrix} fX / Z \\ fY / Z \\ 1 \end{pmatrix} \Rightarrow M\widetilde{P} = \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix}$$

Perspective Projection

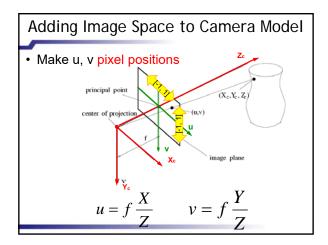
Solution:

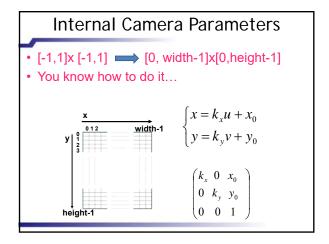
$$M = \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

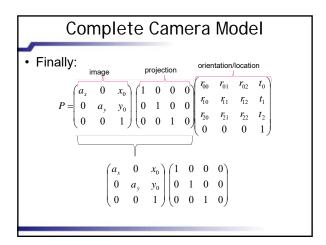
Putting things together: Camera Model

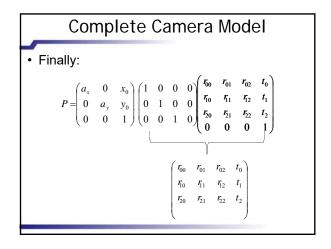
• Projection + Movement + Rotation

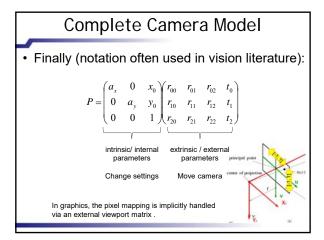
$$M = \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} r_{00} & r_{01} & r_{02} & t_0 \\ r_{10} & r_{11} & r_{12} & t_1 \\ r_{20} & r_{21} & r_{22} & t_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

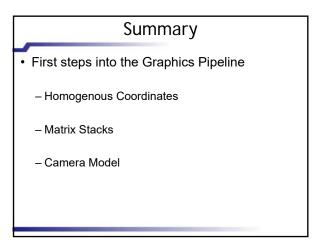












Thank you very much for your attention!

