

Prove he has x , s.t. $\boxed{x^2 + 4x + 7 = 0}$, mod 997, (p) without showing x while hiding x

Requirements: ① A separate verification setup ② Challenges
③ Acceptance criteria

① Setup. $V(I) = 300^I$

② Challenge. Ask prover to send $A (= 300^{x^2})$ and $B (= 300^{4x})$

③ Acceptance Criteria. Verifies that $A \cdot B \cdot 300^6 = 1$. If so, accept!

Consider $V(I) = 300^I$, 300 is a generator for prime field with $p=997$

Properties (1) $g \cdot g^{p-1} = 1$ (2) If $I = p-1$, then $V(I)$ or $300^{p-1} = 1 \pmod{997}$



$p-1$

Idea: Let $I = (x^2 + 4x + 7) - 1$, then $V(I) = 300^{x^2 + 4x + 6} \pmod{997} = 1$ ← Alternative Statement (equivalent)

Why? Simplify:

$$\underbrace{300^{x^2 \pmod{997}}}_A \cdot \underbrace{300^{4x \pmod{997}}}_B \cdot 300^6 = 1$$

$$\boxed{300^{x^2}}$$

$$= \underbrace{300 \cdot 300 \cdots 300}_{x^2 \text{ times}}$$

$$\boxed{300^{4x}}$$

$$= \underbrace{300 \cdot 300 \cdot 300 \cdots 300}_{4x \text{ times}}$$