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Enrico Franco

Mechanical Engineering Department, Imperial College, London, UK

ABSTRACT
In this work the control of underactuated mechanical systems with dry friction on actuated and unactuated joints is investigated. A new interconnection-and-damping-assignment passivity-based-control (IDA-PBC) design is presented, which includes the adaptive estimation of the friction forces and the introduction of a nonlinear dissipative term in the closed-loop system dynamics. As a result, the traditional IDA-PBC is complemented with an additional matching condition and the control law is augmented with a new term that accounts for the Coulomb friction forces on all joints. Two adaptive control paradigms are considered for comparison purposes and stability conditions are discussed. The control design is detailed for two demonstrative examples: the disk-on-disk system; the Acrobot system. The effectiveness of the proposed design is demonstrated with numerical simulations.

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1. Introduction

Control via energy shaping has been successfully applied to underactuated mechanical systems employing two main formulations: Controlled Lagrangians (CL) as in (Bloch, Chang, Leonard, & Marsden, 2001; Bloch, Leonard, & Marsden, 2000); interconnection-and-damping-assignment passivity-based-control (IDA-PBC) as in (Ortega, Spong, Gomez-Estern, & Blankenstein, 2002) and references therein. While in its traditional formulation IDA-PBC consists of two separate terms, namely the energy-shaping and the damping-injection control, and neglects dissipative forces, recent research has suggested that relaxing these structural constraints might allow covering a wider range of applications (Crasta, Ortega, & Pillai, 2015; Donaire, Ortega, & Romero, 2016). While control via energy shaping is inherently robust to disturbances, the practical importance of friction in underactuated mechanical systems has motivated a considerable stream of research. The effect of viscous friction was studied within the CL approach by (Chang, 2010; Woolsey et al., 2004), while continuous and linear friction forces were considered in IDA-PBC by (Delgado & Kotyczka, 2014; Gómez-Estern & Van der Schaft, 2004). Furthermore, the compensation of nonlinear and discontinuous friction forces on actuated joints was investigated in (Ryalat & Laila, 2016; Sandoval, Kelly, & Santibáñez, 2011). Beyond energy-shaping control, a technical solution for dry friction compensation on both actuated and unactuated joints based on sliding-mode-control was proposed in (Martinez, Alvarez, & Orlov, 2008) considering the case of small displacements and under specific assumptions on the magnitude of the friction forces. Recently, more sophisticated IDA-PBC designs with enhanced robustness characteristics were proposed in (Haddad, Chemori, & Belghith, 2017) and in (Donaire, Romero, Ortega, Siciliano, & Crespo, 2017; Ryalat, Laila, & Torbati, 2015) but are only applicable if the disturbances are restricted to the actuated joints. In summary, control via energy shaping of underactuated mechanical systems with dry friction on both actuated and unactuated joints remains an open problem with important practical implications.

This work presents a new control design that builds upon the IDA-PBC formulation and extends it in two main ways for the purpose of friction compensation on all joints: the adaptive estimation of the friction forces; the introduction of a new nonlinear dissipative term in the closed-loop system dynamics. Differently from prior research, the purpose of the dissipative term introduced here is to produce a new matching condition, which can be expressed as a partial-differential-equation (PDE) for separable port-controlled Hamiltonian (PCH) systems or as an algebraic equation for non-separable PCH systems. Differently from previous works, no assumption is made on the magnitude of the friction forces and no prior knowledge of the friction coefficients is required. Instead, two alternative methods are employed and compared for the purpose of adaptive friction compensation: the Immersion and Invariance (I&I) formulation (Astolfi & Ortega, 2003); the time-delay-control (TDC) method (Youcef-Toumi & Ito, 1988). The effectiveness of the proposed approach is demonstrated with numerical simulations on two illustrative examples.

The rest of the paper is organised as follows: in Section 2 the IDA-PBC design and the corresponding matching conditions are briefly reviewed for completeness; Section 3 presents the new IDA-PBC designs and discusses stability conditions; Section 4 illustrates the simulation results for the disk-on-disk system and Section 5 for the Acrobot system. Section 6 contains concluding remarks.

CONTACT Enrico Franco ef1311@imperial.ac.uk Mechanical Engineering Department, Imperial College, Exhibition Road, London SW7 2AZ, UK

2. Problem formulation

An underactuated mechanical system with position $q \in \mathbb{R}^n$ and momenta $p = M\dot{q}$, subject to friction forces $Q_f(\dot{q}) \in \mathbb{R}^n$ and control input $u \in \mathbb{R}^m$ can be represented in PCH form as:

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & I^n \\ -I^n & 0 \end{bmatrix} \begin{bmatrix} \partial_q H \\ \partial_p H \end{bmatrix} + \begin{bmatrix} 0 \\ G \end{bmatrix} u - \begin{bmatrix} 0 \\ Q_f \end{bmatrix} \tag{1}$$

The input matrix is $G(q) \in \mathbb{R}^{n\times m}$, with $\text{rank}(G) = m < n$, the open-loop Hamiltonian is $H = \frac{1}{2}p^T M^{-1} p + V$, where $M(q) = M^T > 0$ is the inertia matrix and $V(q)$ is the potential energy. Finally, $\partial_q(\cdot)$ denotes the partial derivative in q, and I^n indicates the $n \times n$ identity matrix. IDA-PBC aims to achieve the following closed-loop system dynamics, where $H_d = \frac{1}{2}p^T M_d^{-1} p + V_d$ is the closed-loop Hamiltonian, with inertia matrix $M_d = M_d^T > 0$, potential energy V_d, and desired equilibrium $q^* = \text{argmin}(V_d)$:

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & M^{-1}M_d \\ -M_d M^{-1} & J_2 - GK_v G^T \end{bmatrix} \begin{bmatrix} \partial_q H_d \\ \partial_p H_d \end{bmatrix} \tag{2}$$

The term $J_2 = -J_2^T$ is a free-parameter matrix typically defined as a linear function of the momenta, while $K_v = K_v^T > 0$ is a constant gain matrix. The IDA-PBC control for the disturbance-free system (1) is defined as (Ortega et al., 2002):

$$\begin{aligned} u &= u_{es} + u_{di} \\ u_{es} &= (G^T G)^{-1} G^T (\partial_q H - M_d M^{-1} \partial_q H_d + J_2 M_d^{-1} p) \\ u_{di} &= -K_v G^T \partial_p H_d \end{aligned} \tag{3}$$

The energy-shaping control u_{es} assigns the desired closed-loop equilibrium, while the damping-injection control u_{di} achieves asymptotic stability if the output $y = G^T \partial_p H_d$ is detectable and if $Q_f = 0$ in (1). The matrix M_d and the potential energy V_d are defined by the following matching conditions, termed kinetic-energy PDE and potential-energy PDE:

$$G^\perp \left(\partial_q \left(\frac{1}{2} p^T M^{-1} p \right) - M_d M^{-1} \partial_q \times \left(\frac{1}{2} p^T M_d^{-1} p \right) + J_2 M_d^{-1} p \right) = 0 \tag{4a}$$

$$G^\perp (\partial_q V - M_d M^{-1} (\partial_q V_d)) = 0 \tag{4b}$$

The term $G^\perp$ is a full-rank left-annihilator of G defined so that $G^\perp G = 0$.

System (1) can also be expressed in Euler-Lagrange form as follows, where the open-loop Lagrangian is $L(q, \dot{q}) = \frac{1}{2}\dot{q}^T M \dot{q} - V$:

$$\frac{d}{dt} \partial_{\dot{q}} L(q, \dot{q}) - \partial_q L(q, \dot{q}) = G(q)u - Q_f(\dot{q}) \tag{5}$$

Defining the closed-loop Lagrangian $L_c(q, \dot{q}) = \frac{1}{2}\dot{q}^T M_c \dot{q} - V_c$ and the desired equilibrium $q^* = \text{argmin}(V_c)$, the closed-loop dynamics (2) can be expressed as:

$$\frac{d}{dt} \partial_{\dot{q}} L_c(q, \dot{q}) - \partial_q L_c(q, \dot{q}) = 0 \tag{6}$$

The Euler-Lagrange matching conditions corresponding to (4) are:

$$G^\perp \left(\partial_q \left(\frac{1}{2} \dot{q}^T M \dot{q} \right) - \partial_q (M\dot{q})\dot{q} - M M_c^{-1} \times \left(\partial_q \left(\frac{1}{2} \dot{q}^T M_c \dot{q} \right) - \partial_q (M_c \dot{q}) \dot{q} \right) \right) = 0 \tag{7a}$$

$$G^\perp (\partial_q V - M M_c^{-1} (\partial_q V_c)) = 0 \tag{7b}$$

Finally, (4) and (7) are equivalent if the following conditions hold: $V_c = V_d$; $M_c = M M_d^{-1} M$; $J_2 = M_d M^{-1} (\partial_q (M M_d^{-1} p)^T - \partial_q (M M_d^{-1} p)) M^{-1} M_d$ (Blankenstein, Ortega, & Van Der Schaft, 2002).

3. Main result

In this section two IDA-PBC designs for underactuated mechanical systems with Coulomb friction on all joints are outlined. The first design is intended for separable PCH systems (i.e. with constant inertia matrix) with $n = 2$ and presents an explicit solution of the relevant PDE. The second design employs a more general approach and is applicable to non-separable PCH systems (i.e. with non-constant inertia matrix) but does not provide an explicit solution of the matching conditions. In both cases, stability conditions are discussed drawing a parallel to the standard IDA-PBC. Finally, the friction forces are estimated adaptively and the adaptation laws are combined with the new IDA-PBC designs.

The following assumptions are introduced to define the control problem for system (1):

Assumption 3.1: *Coulomb friction forces are acting on all joints, $Q_f = \text{diag}[\text{sign}(\dot{q})]F_c$, where $F_c \in \mathbb{R}_+^n$ is a vector of positive, constant coefficients assumed exactly known through an appropriate adaptation law to be defined, while the elements of the diagonal matrix $\text{diag}[\text{sign}(\cdot)] \in \mathbb{R}^{n\times n}$ correspond to the standard sign function:*

$$\text{sign}(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 1 \end{cases} \tag{8}$$

Assumption 3.2: *(1) is a separable PCH system with $n = 2$ and underactuation degree one, the input matrix is $G = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, and the inertia matrix is:*

$$M = \begin{bmatrix} m_{11} & m_{12} \\ m_{12} & m_{22} \end{bmatrix}, \det(M) = \Delta > 0; \tag{9a}$$

$$M_d = \begin{bmatrix} k_1 & k_2 \\ k_2 & k_3 \end{bmatrix}, \det(M_d) = \Delta_d > 0 \tag{9b}$$

3.1 IDA-PBC design for separable systems

We consider system (1) under Assumptions 3.1–3.2, which is representative of several canonical examples of underactuated mechanical systems (e.g. disk-on-disk, inertia-wheel-pendulum). The underlying idea of the proposed approach consists in including a dissipative term $\partial_{\dot{q}} D_c$ in the closed-loop dynamics as follows:

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & M^{-1}M_d \\ -M_d M^{-1} & J_2 \end{bmatrix} \begin{bmatrix} \partial_q H_d \\ \partial_p H_d \end{bmatrix} - \begin{bmatrix} 0 \\ M_d M^{-1} \end{bmatrix} \partial_{\dot{q}} D_c \tag{10}$$

Equating (1) and (10), the resulting matching conditions can be expressed as:

$$G^{\perp}\left(\partial_q\left(\frac{1}{2}p^T M^{-1} p\right) - M_d M^{-1} \partial_q \times \left(\frac{1}{2}p^T M_d^{-1} p\right) + J_2 M_d^{-1} p\right) = 0 \tag{11a}$$

$$G^{\perp}(\partial_q V - M_d M^{-1}(\partial_q V_d)) = 0 \tag{11b}$$

$$G^{\perp}(\mathrm{diag}[\mathrm{sign}(\dot{q})]F_c - M_d M^{-1}(\partial_{\dot{q}} D_c)) = 0 \tag{11c}$$

The dissipative term $\partial_{\dot{q}} D_c$ is included in a third PDE that expresses the damping-injection in the presence of friction and is solvable analytically if Assumptions 3.1–3.2 are satisfied. Substituting M, M_d from (9), the damping-injection PDE in (11c) becomes:

$$\partial_{\dot{q}_1} D_c(m_{22}k_2 - m_{12}k_3) + \partial_{\dot{q}_2} D_c(m_{11}k_3 - m_{12}k_2) = \Delta F_{c2}\mathrm{sign}(\dot{q}_2) \tag{12}$$

Solving (12) in $\dot{q}_2$ we obtain:

$$D_c = \Delta F_{c2}|\dot{q}_2|/(m_{11}k_3 - m_{12}k_2) + \frac{k_v}{2}(\dot{q}_1 + \gamma\dot{q}_2)^2 \tag{13}$$

where $k_v > 0$ is a parameter analogous to K_v in (3) and γ is a constant term defined as $\gamma = -(m_{22}k_2 - m_{12}k_3)/(m_{11}k_3 - m_{12}k_2)$. Introducing the friction-compensation term $u^* = (G^T G)^{-1} G^T(\mathrm{diag}[\mathrm{sign}(\dot{q})]F_c - M_d M^{-1}(\partial_{\dot{q}} D_c))$, the new IDA-PBC control law for separable systems is:

$$\begin{aligned} u &= u_{es} + u^* \\ u^* &= F_{c1}\mathrm{sign}(\dot{q}_1) - F_{c2}\mathrm{sign}(\dot{q}_2)\frac{(m_{11}k_2 - m_{12}k_1)}{(m_{11}k_3 - m_{12}k_2)} \\ &\quad - k_v(\dot{q}_1 + \gamma\dot{q}_2)\frac{\Delta_d}{(m_{11}k_3 - m_{12}k_2)} \end{aligned} \tag{14}$$

where u_{es} is defined as in (3). Notably, u^* replaces u_{di} in (14) and accounts for the friction forces on all joints ($Q_f \in \mathbb{R}^n$). Nevertheless, u_{di} is included in u^* for a particular choice of k_v, as demonstrated in the following Lemma.

Lemma 3.1: *Consider system (1) under Assumption 3.1, 3.2 and control input (14) resulting in the closed-loop dynamics (10). Then the friction-compensation term u^* contains the conventional damping-injection u_{di} (3) if $k_v = K_v(m_{11}k_3 - m_{12}k_2/\Delta_d)^2$.*

Proof: Computing u_{di} for the system at hand, and recalling that $p = M\dot{q}$ we obtain:

$$u_{di} = -K_v(\dot{q}_1 + \gamma\dot{q}_2)\frac{(m_{11}k_3 - m_{12}k_2)}{\Delta_d} \tag{15}$$

Finally, if $k_v = K_v(m_{11}k_3 - m_{12}k_2/\Delta_d)^2$ the last term in u^* corresponds to (15). ■

Remark 3.1: Since the dissipative term $\partial_{\dot{q}} D_c$ is not included in the kinetic-energy PDE, it is not required to be quadratic in p. This is an important difference from previous works (Crasta et al., 2015; Donaire, Mehra, et al., 2016) where an additional term quadratic in p is introduced in the closed-loop dynamics (2) as an alternative to the free matrix J_2 and consequently appears in the kinetic-energy PDE.

Remark 3.2: If no Coulomb friction is present on the unactuated joint (e.g. $F_{c2} = 0$), then $D_c = k_v/2(\dot{q}_1 + \gamma\dot{q}_2)^2$ in (13) and can be interpreted as Rayleigh dissipation function. In this case, the Euler-Lagrange equivalent of (10) is:

$$\frac{d}{dt}\partial_{\dot{q}} L_c(q, \dot{q}) - \partial_q L_c(q, \dot{q}) + \partial_{\dot{q}} D_c(\dot{q}) = 0 \tag{16}$$

where $\partial_{\dot{q}} D_c = R(\dot{q})\dot{q}$ with $R(\dot{q}) \geq 0$ (Teo, Donaire, & Perez, 2013). If $F_{c2} \neq 0$, then $\partial_{\dot{q}} D_c$ has a nonlinear and discontinuous structure therefore the dissipation condition $\dot{q}^T \partial_{\dot{q}} D_c(\dot{q}) \geq 0$ is generally not satisfied and D_c is not a Rayleigh dissipation function. Sufficient conditions on the friction coefficient F_{c2} for a stable equilibrium in $q = q^*, p = 0$ are discussed in the following proposition, which employs a similar approach to Proposition 3.1 in (Haddad et al., 2017).

Proposition 3.1: *Consider system (1) under Assumptions 3.1–3.2 and control input (14) with k_v defined as in Lemma 3.1 resulting in the closed-loop dynamics (10). Then $\dot{H}_d \leq 0$ and q^* is a stable equilibrium point if*

$$\left|\frac{\Delta F_{c2}|\dot{q}_2|}{m_{11}k_3 - m_{12}k_2}\right| < K_v|\partial_p H_d^T G|^2.$$

Proof: The closed-loop Hamiltonian $H_d = \frac{1}{2}p^T M_d^{-1} p + V_d$ is taken as Lyapunov function candidate. Computing its time-derivative and substituting $\dot{q}, \dot{p}$ from (10) gives:

$$\dot{H}_d = \partial_q H_d^T(M^{-1}M_d\partial_p H_d) + \partial_p H_d^T(-M_d M^{-1}\partial_q H_d - M_d M^{-1}\partial_{\dot{q}} D_c) \tag{17}$$

Simplifying common terms and substituting M, M_d from (9) and D_c from (13) gives:

$$\dot{H}_d = -k_v(\dot{q}_1 + \gamma\dot{q}_2)^2 - \Delta F_{c2}|\dot{q}_2|/(m_{11}k_3 - m_{12}k_2) \tag{18}$$

Substituting $k_v = K_v(m_{11}k_3 - m_{12}k_2/\Delta_d)^2$ it follows from Lemma 3.1 that the quadratic term in (18) corresponds to: $K_v|\partial_p H_d^T G|^2$. In particular, the numerator of the second term in

(18) is positive by hypothesis, while the sign of the denominator depends on the elements of M, M_d. In general we have:

$$\dot{H}_d \leq -K_v|\partial_p H_d^T G|^2 + \Delta F_{c2} |\dot{q}_2| / |m_{11}k_3 - m_{12}k_2| \quad (19)$$

Finally,

$$\left|\frac{\Delta F_{c2}|\dot{q}_2|}{m_{11}k_3 - m_{12}k_2}\right| < K_v|\partial_p H_d^T G|^2$$

implies $\dot{H}_d \leq 0$ which concludes the proof. ■

Remark 3.3: If friction is neglected on both joints as in the standard IDA-PBC (3), the Lyapunov derivative (17) becomes after simplifying common terms:

$$\dot{H}_d = \partial_p H_d^T(-GK_vG^T\partial_q H_d - \mathrm{diag}[\mathrm{sign}(\dot{q})]F_c) \quad (20)$$

Computing the products in (20) under Assumption 3.1, 3.2 we obtain:

$$\begin{aligned}\dot{H}_d \leq\ & \frac{1}{\Delta_d}|(m_{11}k_3 - m_{12}k_2)F_{c1}|\dot{q}_1| \\ & + (m_{12}k_3 - m_{22}k_2)F_{c1}\dot{q}_1\mathrm{sign}(\dot{q}_2)| \\ & + \frac{1}{\Delta_d}|(m_{11}k_1 - m_{11}k_2)F_{c2}\dot{q}_1\mathrm{sign}(\dot{q}_2) \\ & + (m_{22}k_1 - m_{12}k_2)F_{c2}|\dot{q}_2|| - K_v|\partial_p H_d^T G|^2 \quad (21)\end{aligned}$$

Compared with (19), the Lyapunov derivative (21) contains additional terms that depend on the velocities $\dot{q}_1, \dot{q}_2$ and on their cross product. In practice, the friction forces can degrade the performance of the closed-loop system (see Section 4–5). If only the Coulomb friction on the actuated joint is compensated employing the alternative control

$$u^{*'} = F_{c1}\mathrm{sign}(\dot{q}_1) - k_v(\dot{q}_1 + \gamma\dot{q}_2)\frac{\Delta_d}{(m_{11}k_3 - m_{12}k_2)}$$

we can simplify (21) as:

$$\begin{aligned}\dot{H}_d \leq -K_v|\partial_p H_d^T G|^2 + & \frac{1}{\Delta_d}|(m_{11}k_1 - m_{11}k_2)F_{c2}\dot{q}_1\mathrm{sign}(\dot{q}_2) \\ & + (m_{22}k_1 - m_{12}k_2)F_{c2}|\dot{q}_2|| \quad (22)\end{aligned}$$

Comparing (22) with (21) and (19) confirms that, although friction compensation on the actuated joint is clearly beneficial, the new IDA-PBC design (14) includes and goes beyond that strategy. While (19),(22) are unaffected by F_{c1}, asymptotic stability cannot in general be concluded in the presence of Coulomb friction on the unactuated joints, which is in agreement with (Gómez-Estern & Van der Schaft, 2004).

3.2 IDA-PBC design for non-separable systems

In this section we consider system (1) under Assumption 3.1, while Assumption 3.2 is removed, and a generalisation of the result in Section 3.1 is presented. To this end we observe that D_c in (13) consists of a part quadratic in $\dot{q}$ and of a nonlinear part proportional to the friction forces that we indicate with $\mathcal{F}$. Similarly, u^* (14) can be separated into a damping-injection term and a friction-compensation term. We therefore propose to rewrite the closed-loop dynamics (10) as follows:

$$\begin{bmatrix}\dot{q}\\ \dot{p}\end{bmatrix} = \begin{bmatrix}0 & M^{-1}M_d\\ -M_dM^{-1} & J_2 - GK_vG^T\end{bmatrix}\begin{bmatrix}\partial_q H_d\\ \partial_p H_d\end{bmatrix} - \begin{bmatrix}0\\ M_dM^{-1}\end{bmatrix}\mathcal{F} \quad (23)$$

The damping-injection PDE (12) becomes in this case:

$$G^{\perp}(\mathrm{diag}[\mathrm{sign}(\dot{q})]F_c - M_dM^{-1}\mathcal{F}) = 0 \quad (24)$$

Notably, (24) is an algebraic equation and can be solved analytically even if G, M_d, M are not constant. The new control law becomes then:

$$\begin{aligned} u &= u_{es} + u_{di} + u^* \\ u^* &= (G^TG)^{-1}G^T(\mathrm{diag}[\mathrm{sign}(\dot{q})]F_c - M_dM^{-1}\mathcal{F}) \quad (25)\end{aligned}$$

Differently from (14), the conventional damping-injection term is included in (25) alongside the new friction-compensation term u^*. Stability conditions are discussed in the following proposition.

Proposition 3.2: *Consider system (1) under Assumption 3.1 and control input (25) resulting in the closed-loop dynamics (23). Then $\dot{H}_d \leq 0$ and q^* is a stable equilibrium point if $|\partial_p H_d^T M_dM^{-1}\mathcal{F}| < \lambda_{min}\{K_v\}|\partial_p H_d^T G|^2$, with $\lambda_{min}\{K_v\}$ the minimum eigenvalue of K_v.*

Proof: Taking the closed-loop Hamiltonian H_d as Lyapunov function candidate, computing its time-derivative, substituting $\dot{q}, \dot{p}$ from (23) and simplifying common terms we obtain:

$$\dot{H}_d = -\partial_p H_d^T GK_vG^T\partial_p H_d - \partial_p H_d^T M_dM^{-1}\mathcal{F} \quad (26)$$

Without making any assumption on the sign of $\mathcal{F}$, we can write:

$$\dot{H}_d \leq -\lambda_{\min}\{K_v\}|\partial_p H_d^T G|^2 + |\partial_p H_d^T M_dM^{-1}\mathcal{F}| \quad (27)$$

In particular, $|\partial_p H_d^T M_dM^{-1}\mathcal{F}| < \lambda_{\min}\{K_v\}|\partial_p H_d^T G|^2$ implies $\dot{H}_d \leq 0$ which concludes the proof. ■

Remark 3.4: If system (1) satisfies Assumption 3.2 then (24) becomes:

$$\mathcal{F}_1(m_{22}k_2 - m_{12}k_3) + \mathcal{F}_2(m_{11}k_3 - m_{12}k_2) = \Delta F_2\mathrm{sign}(\dot{q}_2) \quad (28)$$

Solving (28) in $\mathcal{F}_2$ with $\mathcal{F}_1 = 0$ we obtain:

$$\mathcal{F}_2 = \Delta F_2\mathrm{sign}(\dot{q}_2)/(m_{11}k_3 - m_{12}k_2) \quad (29)$$

Substituting (29) in (25) confirms that the latter corresponds to (14) for an appropriate value of k_v (see Lemma 3.1). Notably, compared to the standard IDA-PBC, (25) only requires solving the additional algebraic equation (24), which is particularly beneficial considering the ongoing research efforts that aim to obviate the solution of PDE in IDA-PBC (Donaire, Mehra, et al., 2016; Nunna, Sassano, & Astolfi, 2015).

3.3 Adaptive IDA-PBC with I&I

In this section the constant Coulomb friction coefficients $F_c \in \mathbb{R}^n_+$ are estimated adaptively with the I&I formulation (Astolfi, Karagiannis, & Ortega, 2007; Astolfi & Ortega, 2003) and integrated within the proposed IDA-PBC designs (14),(25). To this end, the vector of estimation errors is defined as follows:

$$z = \tilde{F}_c - F_c = \hat{F}_c + \beta(\dot{q}) - F_c \tag{30}$$

where $z, \beta, F_c \in \mathbb{R}^n$. The adaptation law is chosen as follows, with the tuning parameter $\alpha > 0$:

$$\begin{aligned}\dot{\hat{F}}_c &= \alpha \mathrm{diag}[\mathrm{sign}(\dot{q})]M^{-1}(-\partial_q H + Gu - \mathrm{diag}[\mathrm{sign}(\dot{q})] \\ &\quad \times (\hat{F}_c + \beta(\dot{q})) - \dot{M}\dot{q}) \\ \beta &= -\alpha \mathrm{diag}[\mathrm{sign}(\dot{q})]\dot{q}\end{aligned} \tag{31}$$

Proposition 3.3: *Consider system (1) under Assumption 3.1 and adaptation law (31). The estimation errors z are bounded and the terms* diag[sign(q̇)]z *converge to zero asymptotically for* $\alpha > 0$.

Proof: A Lyapunov function candidate is chosen as $W = \frac{1}{2}z^T z$. Computing the time-derivative of (30) and substituting $\dot{p} = M\ddot{q} + \dot{M}\dot{q}$ from (1), we obtain:

$$\begin{aligned}\dot{z} &= \dot{\hat{F}}_c + \partial_{\dot{q}}(\beta)M^{-1}(-\partial_q H + Gu - \mathrm{diag}[\mathrm{sign}(\dot{q})] \\ &\quad \times (\tilde{F}_c - z) - \dot{M}\dot{q})\end{aligned} \tag{32}$$

Computing the time-derivative of W and substituting (31), (32) gives:

$$\begin{aligned}\dot{W} &= z^T \partial_{\dot{q}}(\beta) M^{-1} \mathrm{diag}[\mathrm{sign}(\dot{q})]z \\ &= -\alpha(z^T \mathrm{diag}[\mathrm{sign}(\dot{q})]M^{-1}\mathrm{diag}[\mathrm{sign}(\dot{q})]z) \le 0\end{aligned} \tag{33}$$

Consequently z is bounded and $\mathrm{diag}[\mathrm{sign}(\dot{q})]z$ converges to zero asymptotically.

Combining the IDA-PBC design (25) and the adaptation law (31), the complete I&I-IDA-PBC for non-separable systems becomes:

$$\begin{aligned}u &= u_{es} + u_{di} + u^* \\ u^* &= (G^T G)^{-1} G^T (\mathrm{diag}[\mathrm{sign}(\dot{q})](\hat{F}_c + \beta(\dot{q})) - M_d M^{-1}\mathcal{F}) \\ \dot{\hat{F}}_c &= \alpha\ \mathrm{diag}[\mathrm{sign}(\dot{q})]M^{-1}(-\partial_q H + Gu - \mathrm{diag}[\mathrm{sign}(\dot{q})] \\ &\quad \times (\hat{F}_c + \beta(\dot{q})) - \dot{M}\dot{q}) \\ \beta(\dot{q}) &= -\alpha \mathrm{diag}[\mathrm{sign}(\dot{q})]\dot{q}\end{aligned} \tag{34}$$

In particular, $\mathcal{F}$ is computed from (24) with the estimates $\tilde{F}_c$ replacing the actual friction coefficients, while $\alpha > 0$ is the only additional parameter to the standard IDA-PBC design. Similarly, combining the IDA-PBC design (14) and the adaptation law (31), gives the I&I-IDA-PBC for separable systems (see Section 4). ■

Proposition 3.4: *Consider system (1) under Assumption 3.1 and control input (34) resulting in the closed-loop dynamics (23). Then* $\dot{H}_d \le 0$ *and* q^* *is a locally stable equilibrium point if:* $|\partial_p H_d^T M_d M^{-1}\mathcal{F}| < \lambda_{min}\{K_v\}|\partial_p H_d^T G|^2$*;* $\alpha > 0$*; the output* $y = G^T \partial_p H_d$ *is detectable.*

Proof: Computing $\mathcal{F}$ from (24) with the estimates $\tilde{F}_c$ and substituting in (26) gives:

$$\begin{aligned}\dot{H}_d &= -\partial_p H_d^T G K_v G^T \partial_p H_d - \partial_p H_d^T (M_d M^{-1}\mathcal{F} \\ &\quad + \mathrm{diag}[\mathrm{sign}(\dot{q})]z)\end{aligned} \tag{35}$$

Local stability of the equilibrium q^* is concluded employing the following argument (Khalil, 1996): q^* is a stable equilibrium if $|\partial_p H_d^T M_d M^{-1}\mathcal{F}| < \lambda_{min}\{K_v\}|\partial_p H_d^T G|^2$ in case $\mathrm{diag}[\mathrm{sign}(\dot{q})]z = 0$, while asymptotic stability is concluded if the output $y = G^T \partial_p H_d$ is detectable (see Proposition 3.2); $\mathrm{diag}[\mathrm{sign}(\dot{q})]z$ is bounded and converges to zero asymptotically (see Proposition 3.3). ■

Corollary 3.1: *Consider system (1) under Assumptions 3.1–3.2 and control input (14) with* k_v *defined as in Lemma 3.1 and* $\tilde{F}_{c1}, \tilde{F}_{c2}$ *estimated according to (31) resulting in the closed-loop dynamics (10). Then* $\dot{H}_d \le 0$ *and* q^* *is a locally stable equilibrium point if:*

$$\left|\frac{\Delta F_{c2}|\dot{q}_2|}{m_{11}k_3 - m_{12}k_2}\right| < K_v|\partial_p H_d^T G|^2;$$

$\alpha > 0$*; the output* $y = G^T \partial_p H_d$ *is detectable.*

The proof is a particular case of Proposition 3.4 and a direct consequence of Proposition 3.1 and Proposition 3.3 hence it is omitted for brevity.

Remark 3.5: While the I&I-IDA-PBC (34) is specifically intended for Coulomb friction, different friction models can be considered with appropriate changes to the adaptation law (31). For illustrative purposes we consider the simultaneous presence of Coulomb Friction and stiction resulting in: $Q_f = \mathrm{diag}[\mathrm{sign}(\dot{q})]F_c + \mathrm{diag}[\delta(\dot{q})]F_s$, where $F_s \in \mathbb{R}^n$ is the vector of stiction coefficients and the diagonal matrix $\mathrm{diag}[\delta(\dot{q})] \in \mathbb{R}^{n\times n}$ has elements

$$\delta(\dot{q}) = \begin{cases} 0 & \dot{q} \neq 0 \\ 1 & \dot{q} = 0 \end{cases}.$$

Since stiction forces are null if $\dot{q} \neq 0$, the adaptation law (31) remains unchanged. The second part of the adaptation law for $\dot{q} = 0$ is: $\dot{\hat{F}}_s = \alpha M^{-1}(-\partial_q H + Gu - \hat{F}_s - \dot{M}\dot{q})$. Finally, the estimated friction forces become in this case: $\hat{Q}_f = \mathrm{diag}[\mathrm{sign}(\dot{q})](\hat{F}_c - \alpha \mathrm{diag}[\mathrm{sign}(\dot{q})]\dot{q}) + \mathrm{diag}[\delta(\dot{q})]\hat{F}_s$.

3.4 Adaptive IDA-PBC with TDC

For comparison purposes, in this section the friction forces Q_f are estimated with the TDC method (Youcef-Toumi & Ito, 1988) which is particularly relevant in view of the implementation on digital microcontrollers. TDC employs previous values of control input and system states to estimate a lumped disturbance resulting from external forces and modelling errors. The main advantages of TDC are robustness and simple implementation. As a drawback, asymptotic convergence of the estimation errors to zero cannot be concluded. Denoting with $\tau > 0$ an arbitrarily small delay, which in a digital implementation might correspond to the sampling interval, and with $\xi \in \mathbb{R}^n_+$ an array of arbitrarily small positive constants, TDC assumes a bounded variation of the lumped disturbance Q_f so that at every instant $t > \tau$, $|Q_f(t) - Q_f(t-\tau)| < \xi$. Consequently, Q_f is estimated from the open-loop dynamics (1) at the previous instant:

$$\tilde{Q}_f \cong Q_f(t-\tau) = -\partial_q H(t-\tau) + Gu(t-\tau) - \dot{p}(t-\tau) \tag{36}$$

Typically, the time-derivative of the momenta $\dot{p}$ are not directly measurable but can be computed as:

$$\dot{p}(t-\tau) = \frac{p(t) - p(t-\tau)}{\tau}.$$

This introduces a further error in the estimate (36), which is however accounted for within ξ in this work. Differently from (31), TDC does not rely on the structure of Q_f and is therefore applicable to generic disturbances. The delay τ is the tuning parameter in TDC (36) and is limited in practice by the maximum sampling frequency available. As a general rule, a smaller delay τ is required for a plant with faster dynamics (Youcef-Toumi & Ito, 1988).

Combining the IDA-PBC design (25) and the TDC estimation (36), the TDC-IDA-PBC for non-separable systems becomes:

$$\begin{aligned} u &= u_{es} + u_{di} + u^* \\ u^* &= (G^TG)^{-1}G^T(-\partial_q H(t-\tau) + Gu(t-\tau) \\ &\quad - \dot{p}(t-\tau) - M_dM^{-1}\mathcal{F}) \end{aligned} \tag{37}$$

In particular, $\mathcal{F}$ is computed from (24) with the estimates $\tilde{Q}_f$ replacing the friction forces. Similarly, combining the IDA-PBC design (14) and the adaptation law (36), gives the TDC-IDA-PBC for separable systems (see Section 4).

Proposition 3.5: *Consider system (1) under Assumption 3.1 and control input (37) resulting in the closed-loop dynamics (23). Define the arbitrarily small delay τ and the constants $\xi \in \mathbb{R}^n_+$ so that $|Q_f - \tilde{Q}_f| < \xi$ at any instant. Then $\dot{H}_d \le 0$ and q^* is a stable equilibrium point if $|\partial_p H_d^T(M_dM^{-1}\mathcal{F} + \xi)| < \lambda_{min}\{K_v\}|\partial_p H_d^T G|^2$.*

Proof: Since by hypothesis $|Q_f - \tilde{Q}_f| < \xi$, substituting $Q_f = \tilde{Q}_f + \xi$ in (1) and equating to (23) gives:

$$\dot{p} = -M_dM^{-1}\partial_p H_d - GK_vG^T\partial_p H_d - M_dM^{-1}\mathcal{F} - \xi \tag{38}$$

Substituting (38) in the Lyapunov derivative (26) gives:

$$\dot{H}_d = -\partial_p H_d^T GK_vG^T\partial_p H_d - \partial_p H_d^T(M_dM^{-1}\mathcal{F} + \xi) \tag{39}$$

Finally, if $|\partial_p H_d^T(M_dM^{-1}\mathcal{F}+\xi)| < \lambda_{min}\{K_v\}|\partial_p H_d^T G|^2$ we have:

$$\dot{H}_d \le -\lambda_{min}\{K_v\}|\partial_p H_d^T G|^2 + |\partial_p H_d^T(M_dM^{-1}\mathcal{F} + \xi)| \le 0 \tag{40}$$

which concludes the proof. ■

Corollary 3.2: *Consider system (1) under Assumptions 3.1–3.2 and control input (14) with k_v defined as in Lemma 3.1 and $\tilde{F}_{c1}, \tilde{F}_{c2}$ estimated according to (36) so that $|\tilde{F}_{c1} - F_{c1}| < \xi_1, |\tilde{F}_{c2} - F_{c2}| < \xi_2$, where $\xi_1, \xi_2 \in \mathbb{R}_+$ are arbitrarily small constants. Then q^* is a stable equilibrium point if*

$$\left|\frac{\Delta F_{c2}|\dot{q}_2|}{m_{11}k_3 - m_{12}k_2}\right| + |\partial_p H_d^T \xi| < K_v|\partial_p H_d^T G|^2.$$

The proof is a particular case of Proposition 3.5 and is omitted for brevity.

Remark 3.6: Differently from the I&I adaptation (31), the TDC estimate (36) introduces an error in the Lyapunov derivative (39) which further restricts the stability sufficient-condition (40). Conversely, owing to the robustness of TDC, Assumption 3.1 can be relaxed for control (37) and other forces in addition to Coulomb friction can be accounted for. While TDC is particularly suitable for discrete-time systems, it was originally formulated for the continuous-time domain and is employed in this fashion within the TDC-IDA-PBC. Although the extension of (37) to discrete-time systems is possible employing a discrete-time version of IDA-PBC as in (Franco, 2018; Laila & Astolfi, 2006; Sümer & Yalçin, 2011), this aspect is beyond the scope of the present work.

4. Disk-on-disk system

4.1 Control design

The disk-on-disk system consists of one unactuated disks (disk-2) that rolls without slipping on an actuated disk (disk-1). The equations of motion are (Donaire et al., 2017):

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{12} & m_{22} \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ -\gamma_2 \sin(q_2) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} u - \begin{bmatrix} F_{c1}\text{sign}(\dot{q}_1) \\ F_{c2}\text{sign}(\dot{q}_2) \end{bmatrix} \tag{41}$$

The inertia matrix M is constant with elements: $m_{11} = r_1^2(m_1 + m_2)$; $m_{12} = -m_2r_1(r_1 + r_2)$; $m_{22} = 2m_2(r_1 + r_2)^2$. The parameters m_1, m_2, r_1, r_2 are the mass and radius of the actuated and unactuated disks respectively and g is the gravity constant. The angles q_1 of the actuated disk and q_2 of the unactuated disk are measured from the vertical, with $q_1, q_2 \in (-\pi; \pi)$.The open-loop potential energy is $V = \gamma_2\cos(q_2)$, with $\gamma_2 = m_2g(r_1 + r_2)$. The control aim is to stabilise the disks in the upright position (i.e. $q_1, q_2 = 0$). Since system (41) satisfies Assumptions 3.1–3.2 the IDA-PBC design for separable systems (14) is employed in conjunction with the adaptation laws (31) and (36) according to Corollary 3.1,3.2.

The inertia matrix M, M_d are defined as (9) and the matching conditions (11) become:

$$\begin{bmatrix} 0 & 1 \end{bmatrix}\left(\begin{bmatrix} 0 \\ -\gamma_2 sin(q_2) \end{bmatrix} - \begin{bmatrix} k_1 & k_2 \\ k_2 & k_3 \end{bmatrix} \times \begin{bmatrix} m_{22} & -m_{12} \\ -m_{12} & m_{11} \end{bmatrix}\begin{bmatrix} \partial_{q_1} V_d \\ \partial_{q_2} V_d \end{bmatrix}\frac{1}{\Delta}\right) = 0 \tag{42a}$$

$$\begin{bmatrix} 0 & 1 \end{bmatrix}\left(\begin{bmatrix} \tilde{F}_{c1}\text{sign}(\dot{q}_1) \\ \tilde{F}_{c2}\text{sign}(\dot{q}_2) \end{bmatrix} - \begin{bmatrix} k_1 & k_2 \\ k_2 & k_3 \end{bmatrix} \times \begin{bmatrix} m_{22} & -m_{12} \\ -m_{12} & m_{11} \end{bmatrix}\begin{bmatrix} \partial_{\dot{q}_1} D_c \\ \partial_{\dot{q}_2} D_c \end{bmatrix}\frac{1}{\Delta}\right) = 0 \tag{42b}$$

The closed-loop potential-energy is:

$$V_d = \gamma_2 \cos(q_2)\frac{\Delta}{(m_{11}k_3 - m_{12}k_2)} + \frac{k_p}{2}(q_1 + \gamma q_2)^2 \tag{43}$$

V_d has a minimum in $q_1, q_2 = 0$ if $(m_{11}k_3 - m_{12}k_2) < 0, k_p > 0$, where γ is defined as

$$\gamma = -\frac{(m_{22}k_2 - m_{12}k_3)}{(m_{11}k_3 - m_{12}k_2)}.$$

The dissipative term D_c is defined as in (13) with F_{c1}, F_{c2} replaced by their estimates $\tilde{F}_{c1}, \tilde{F}_{c2}$ therefore control (14) becomes:

$$\begin{aligned}
u &= u_{es} + u^* \\
u_{es} &= \gamma_2 \sin(q_2)\frac{(m_{11}k_2 - m_{12}k_1)}{(m_{11}k_3 - m_{12}k_2)} \\
&\quad - k_p \frac{\Delta_d}{(m_{11}k_3 - m_{12}k_2)}(q_1 + \gamma q_2) \\
u^* &= \tilde{F}_{c1}\mathrm{sign}(\dot{q}_1) - \frac{(m_{11}k_2 - m_{12}k_1)}{(m_{11}k_3 - m_{12}k_2)}\tilde{F}_{c2}\mathrm{sign}(\dot{q}_2) \\
&\quad - k_v \frac{\Delta_d}{(m_{11}k_3 - m_{12}k_2)}(\dot{q}_1 + \gamma \dot{q}_2)
\end{aligned} \tag{44}$$

The terms $k_p, k_v > 0$ are tuning parameters and u_{es} corresponds to (Donaire et al., 2017). The coefficients of the energy-shaping control u_{es} and of the friction-compensation u^* show a clear symmetry, which results from the similar structure of potential-energy PDE (42a) and damping-injection PDE (42b). Expressing the I&I adaptation law (31) for system (41) gives:

$$\begin{aligned}
\tilde{F}_{c1} &= (\hat{F}_{c1} - \alpha \Delta |\dot{q}_1|) \\
\tilde{F}_{c2} &= (\hat{F}_{c2} - \alpha \Delta |\dot{q}_2|) \\
\dot{\hat{F}}_{c1} &= \alpha \mathrm{sign}(\dot{q}_1)((u - \tilde{F}_{c1}\mathrm{sign}(\dot{q}_1))m_{22} \\
&\quad - (\gamma_2 \sin(q_2) - \tilde{F}_{c2}\mathrm{sign}(\dot{q}_2))m_{12}) \\
\dot{\hat{F}}_{c1} &= \alpha \mathrm{sign}(\dot{q}_2)((\gamma_2 \sin(q_2) - \tilde{F}_{c2}\mathrm{sign}(\dot{q}_2))m_{11} \\
&\quad - (u - \tilde{F}_{c1}\mathrm{sign}(\dot{q}_1))m_{12})
\end{aligned} \tag{45}$$

Conversely, the TDC estimation (36) becomes in this case:

$$\begin{aligned}
\tilde{F}_{c1}\mathrm{sign}(\dot{q}_1) &= u(t-\tau) - (m_{11}\ddot{q}_1(t-\tau) + m_{12}\ddot{q}_2(t-\tau)) \\
\tilde{F}_{c2}\mathrm{sign}(\dot{q}_2) &= \gamma_2 \sin(q_2)(t-\tau) - (m_{12}\ddot{q}_1(t-\tau) \\
&\quad + m_{22}\ddot{q}_2(t-\tau))
\end{aligned} \tag{46}$$

4.2 Simulations

Simulations were conducted with initial conditions $q_1 = 0; q_2 = 0.175; p_1 = p_2 = 0$ employing the following parameters as in (Donaire et al., 2017): $m_1 = 0.235; m_2 = 0.0216; r_1 = 0.15; r_2 = 0.075; g = 9.81; k_1 = 0.4; k_2 = -0.03; \quad k_3 = 0.003$. The tuning parameters of (44) are: $k_p = 0.00005; k_v = 0.00018; \alpha = 1000; \tau = 0.0001$. The Coulomb friction coefficients are chosen as $F_{c1} = 0.01; F_{c2} = 0.0001$ considering that disk-1 is larger and heavier compared to disk-2. Notably, the technical solution presented in (Martinez et al., 2008) would not be applicable in this case since it assumes higher friction on the unactuated joint than on the actuated joint.

Figure 1 depicts q_1, q_2 with the standard IDA-PBC (i.e. $\tilde{F}_{c1} = \tilde{F}_{c2} = 0$) and in case friction compensation is only performed on the actuated joint (i.e. $\tilde{F}_{c2} = 0$). Neglecting Coulomb friction altogether results in large errors on q_1 and slow oscillations on q_2. Friction compensation on the actuated disk clearly reduces the position errors on disk-1 although it is not sufficient to reach the desired equilibrium. It is worth highlighting how in this case a relatively small Coulomb friction on the unactuated joint ($F_{c2} \ll F_{c1}$) has large adverse effects on the system performance. Figure 2 depicts q_1, q_2 with the I&I-IDA-PBC (44)–(45) and the corresponding disturbance estimates. Although small oscillations are still present on q_1, q_2, the error on q_1 is greatly reduced, while the I&I estimates (45) converges to the correct values of F_{c1}, F_{c2}. Finally, Figure 3 refers to the TDC-IDA-PBC (44)–(46): with the chosen parameters the system response is the same as with the I&I adaptation law (45), however the estimated Coulomb friction coefficients appear less smooth compared to the I&I adaptation. This is a known shortcoming of the TDC approach, which differently from I&I does not ensure the convergence of the estimation errors to zero. Nevertheless, since the delay τ is small compared to the system dynamics, the latter acts as a low-pass filter and the large spikes in the TDC estimates are not transmitted to the disk positions.

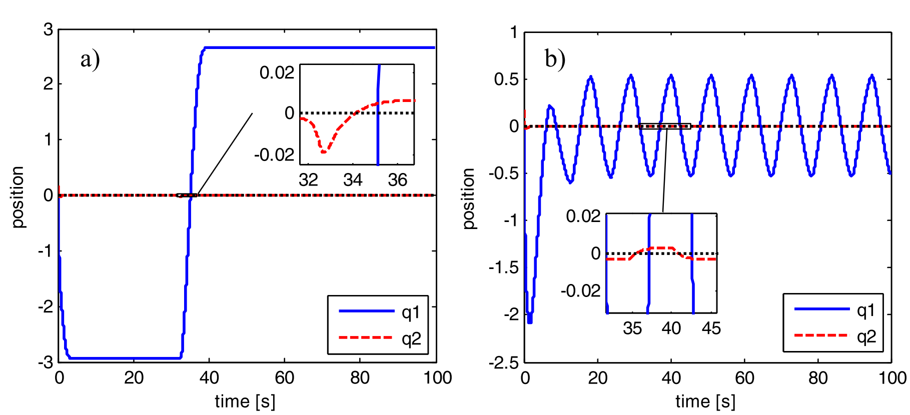

Figure 1. Disk-on-disk system: (a) position with standard IDA-PBC ($\tilde{F}_{c1} = \tilde{F}_{c2} = 0$); (b) position with IDA-PBC and friction compensation on q_1 only (i.e. $\tilde{F}_{c2} = 0$).

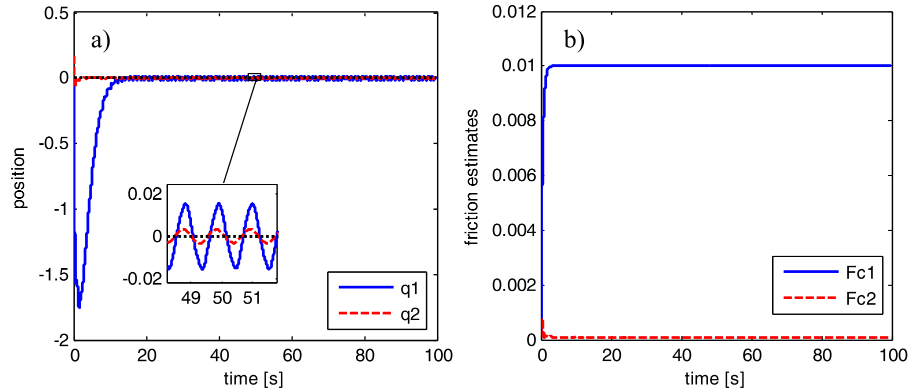

Figure 2. Disk-on-disk system: (a) position with I&I-IDA-PBC (44)–(45); (b) estimates of the Coulomb friction coefficients $\tilde{F}_{c1}, \tilde{F}_{c2}$ with I&I (45).

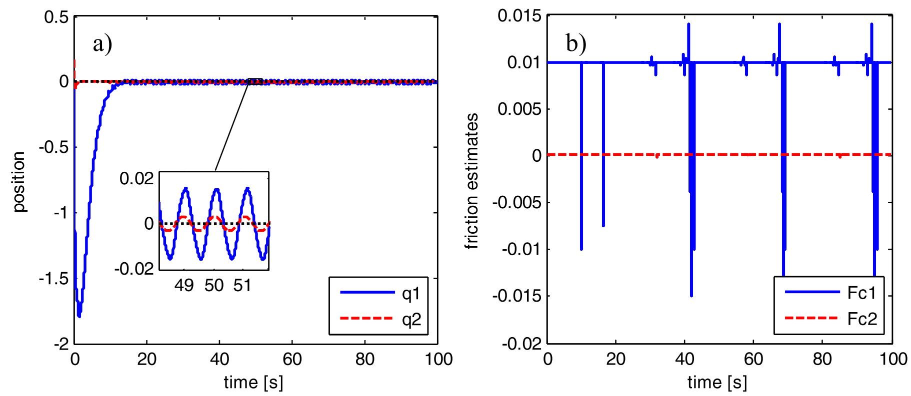

Figure 3. Disk-on-disk system: (a) position with TDC-IDA-PBC (44)–(46); (b) estimates of the Coulomb friction coefficients $\tilde{F}_{c1}, \tilde{F}_{c2}$ with TDC (46).

5. Acrobot System

5.1 Control design

The Acrobot system consists of an articulated pendulum with a single actuator at the elbow joint (q_2) and an unactuated shoulder joint (q_1). The equations of motion given in matrix form are:

$$\dot{p} = -\partial_q\left(\frac{1}{2}p^T M^{-1} p + V\right) + Gu - \begin{bmatrix} F_{c1}\mathrm{sign}(\dot{q}_1) \\ F_{c2}\mathrm{sign}(\dot{q}_2) \end{bmatrix} \tag{47}$$

where the potential energy is $V = g(c_4\cos(q_1) + c_5\cos(q_1 + q_2))$, the input matrix is $G^T = \begin{bmatrix} 0 & 1 \end{bmatrix}$ and the left-hand annihilator is $G^{\perp} = \begin{bmatrix} 1 & 0 \end{bmatrix}$. The open-loop inertia matrix is:

$$M = \begin{bmatrix} c_1 + c_2 + 2c_3\cos(q_2) & c_2 + c_3\cos(q_2) \\ c_2 + c_3\cos(q_2) & c_2 \end{bmatrix},$$

with determinant $\Delta > 0$. The terms c_1, c_2, c_3, c_4, c_5 are constant system parameters, while g is the gravity constant. The control aim consists in stabilising the upright position ($q_1 = q_2 = 0$). In this case, the IDA-PBC design for non-separable systems (25) is employed in conjunction with the adaptation laws (31) and (36) according to Proposition 3.4, 3.5. The damping-injection matching condition (24) with F_{c1}, F_{c2} replaced by their estimates $\tilde{F}_{c1}, \tilde{F}_{c2}$ is:

$$\begin{aligned} &\mathcal{F}_1 + \mathcal{F}_2 \frac{((c_1 + c_2 + 2c_3\cos(q_2))k_2 - (c_2 + c_3\cos(q_2))k_1)}{(c_2k_1 - (c_2 + c_3\cos(q_2))k_2)} \\ &\quad = \tilde{F}_{c1}\frac{\mathrm{sign}(\dot{q}_1)\Delta}{(c_2k_1 - (c_2 + c_3\cos(q_2))k_2)} \end{aligned} \tag{48}$$

Solving (48) with $\mathcal{F}_2 = 0$ gives:

$$\mathcal{F}_1 = \tilde{F}_{c1}\frac{\mathrm{sign}(\dot{q}_1)\Delta}{(c_2k_1 - (c_2 + c_3\cos(q_2))k_2)} \tag{49}$$

The IDA-PBC (Mahindrakar, Astolfi, Ortega, & Viola, 2006) is employed as baseline (see Appendix) and is complemented with the friction-compensation u^* (25):

$$\begin{aligned} u &= u_{es} + u_{di} + u^* \\ u^* &= \tilde{F}_{c2}\mathrm{sign}(\dot{q}_2) - \frac{(c_2k_2 - k_3c_2 - k_3c_3\cos(q_2))}{(c_2k_1 - k_2c_2 - k_2c_3\cos(q_2))}\tilde{F}_{c1}\mathrm{sign}(\dot{q}_1) \end{aligned} \tag{50}$$

The I&I adaptation law (31) becomes in this case:

$$\begin{aligned} \tilde{F}_{c1} &= (\hat{F}_{c1} - \alpha|\dot{q}_1|) \\ \tilde{F}_{c2} &= (\hat{F}_{c2} - \alpha|\dot{q}_2|) \end{aligned}$$

$$\begin{bmatrix}\dot{\hat{F}}_{c1}\\ \dot{\hat{F}}_{c2}\end{bmatrix} = \alpha \begin{bmatrix}\mathrm{sign}(\dot q_1) & 0\\ 0 & \mathrm{sign}(\dot q_2)\end{bmatrix}\left[Gu - \partial_q\left(\frac{1}{2}p^T M^{-1}p + V\right) - \begin{bmatrix}\tilde F_{c1}\mathrm{sign}(\dot q_1)\\ \tilde F_{c2}\mathrm{sign}(\dot q_2)\end{bmatrix} - \dot M M^{-1} p\right] \tag{51}$$

where

$$\dot M = \begin{bmatrix}-2c_3\sin(q_2)\dot q_2 & -c_3\sin(q_2)\dot q_2\\ -c_3\sin(q_2)\dot q_2 & 0\end{bmatrix}.$$

The TDC estimation (36) is defined as follows, where the delay is indicated with the subscript for brevity:

$$\begin{bmatrix}\tilde F_{c1}\mathrm{sign}(\dot q_1)\\ \tilde F_{c2}\mathrm{sign}(\dot q_2)\end{bmatrix} = -\partial_q\left(\frac{1}{2}p^T_{t-\tau}(M^{-1}_{t-\tau})p_{t-\tau} + V_{t-\tau}\right) + Gu_{t-\tau} - \dot p_{t-\tau} \tag{52}$$

5.2 Simulations

Simulations were conducted with initial position $q_1 = \pi; q_2 = 0$ employing the following parameters: $c_1 = 2.3333; c_2 = 5.3333;$

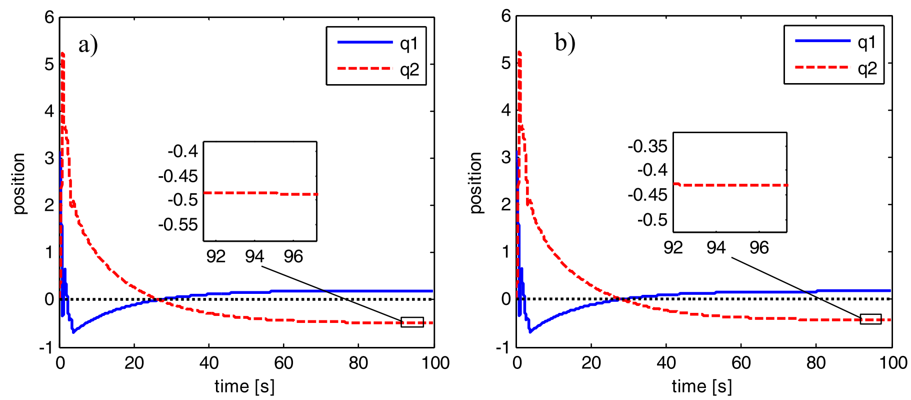

Figure 4. Acrobot system: (a) joint position with baseline IDA-PBC ($\tilde F_{c1} = \tilde F_{c1} = 0$); (b) joint position with IDA-PBC and friction compensation on q_2 only (i.e. $\tilde F_{c1} = 0$ in (50)).

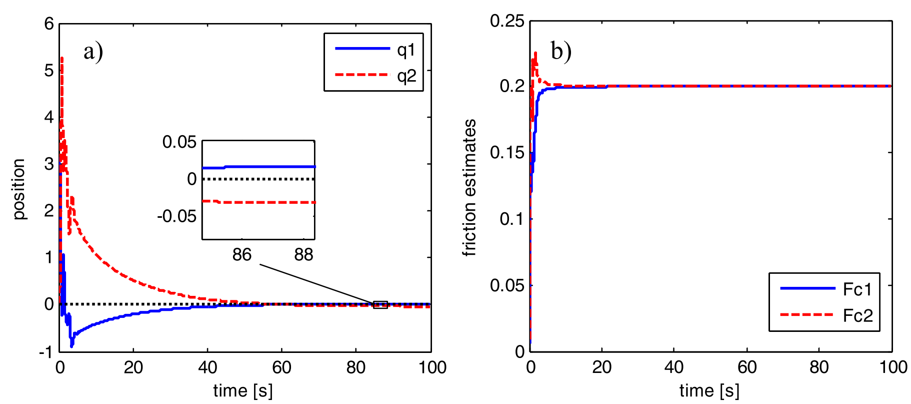

Figure 5. Acrobot system: (a) joint position with I&I-IDA-PBC (50)–(51); (b) estimates of the Coulomb friction coefficients $\tilde F_{c1}, \tilde F_{c2}$ with I&I (51).

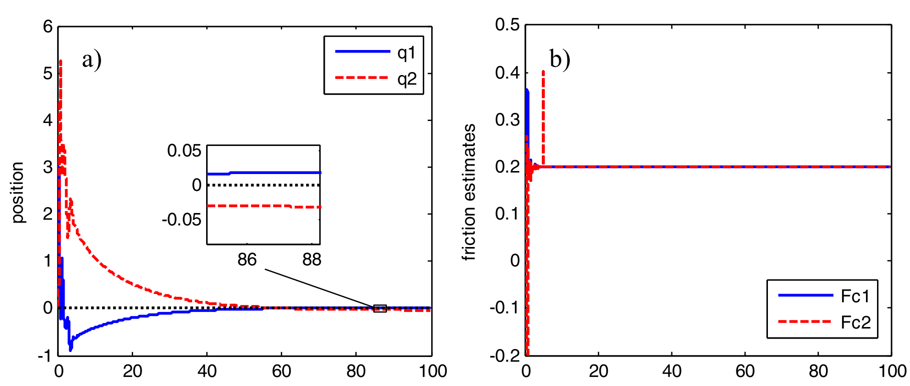

Figure 6. Acrobot system: (a) joint position with TDC-IDA-PBC (50)–(52); (b) estimates of the Coulomb friction coefficients $\tilde F_{c1}, \tilde F_{c2}$ with TDC (52).

$c_3 = 2; c_4 = 3; c_5 = 2; g = 9.81; k_1 = 0.3386; k_2 = 1; \quad k_3 = 5.9073$. The parameters of the IDA-PBC controller (50) are: $\mu = -0.6019; k_0 = -350; k_u = 10; K_v = 30; \alpha = 2; \quad \tau = 0.0001$. Considering that both joints are similar, the Coulomb friction coefficients in (47) are set equal: $F_{c1} = 0.2; F_{c2} = 0.2$.

Figure 4 depicts q_1, q_2 for the baseline IDA-PBC without friction compensation, and in case friction compensation is only performed on the actuated joint (i.e. $\tilde{F}_{c1} = 0$). In the former case the final position is $q_1 = 0.198; q_2 = -0.489$, while in the latter case it is $q_1 = 0.176; q_2 = -0.431$ confirming the limitations of friction compensation on the actuated joint alone. Figure 5 depicts the joint positions for the I&I-IDA-PBC (50)–(51) and the corresponding disturbance estimates. In this case, q_1, q_2 converge to a narrow band around zero, settling at $q_1 = 0.018; q_2 = -0.036$. Additionally, the I&I estimates (51) converge to the actual values of F_{c1}, F_{c2}. Figure 6 shows the joint position with TDC-IDA-PBC (50)–(52) and the corresponding estimates of the friction coefficients, which are comparable to I&I-IDA-PBC (50)–(51).

6. Conclusions

This paper presented a new friction compensation strategy for underactuated mechanical systems with Coulomb friction on actuated and unactuated joints. Two IDA-PBC designs were proposed considering the case of separable PCH systems and non-separable PCH systems, while sufficient-conditions for stability were discussed. Additionally, two adaptive paradigms were employed for comparison purposes. Simulations on two illustrative examples highlighted the effectiveness of the proposed approach, which goes beyond the traditional friction compensation on actuated joints. Future work will attempt to relax Assumption 3.1 considering different types of disturbances, while the results will be applied to systems with higher degree of underactuation and validated with experiments.

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ORCID

Enrico Franco http://orcid.org/0000-0001-9991-7377

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Appendix

Baseline IDA-PBC for the Acrobot system (Mahindrakar et al., 2006). The closed-loop inertia matrix is

$$M_d = \begin{bmatrix} k_1 & k_2 \\ k_2 & k_3 \end{bmatrix},$$

where $k_1 > 0$, $\Delta_d = k_1 k_3 - k_2^2 > 0$ and $\partial_q V_d$ is defined as follows, with the tuning parameters k_0, k_u, μ:

$$\begin{aligned} \partial_{q_1} V_d &= -k_0 \sin(q_1 - \mu q_2) - b_1 \sin(q_1) - b_2 \sin(q_1 + q_2) + \\ &\quad - b_3 \sin(q_1 + 2q_2) - b_4 \sin(q_1 - q_2) + k_u(q_1 - \mu q_2) \\ \partial_{q_2} V_d &= k_0 \mu \sin(q_1 - \mu q_2) - b_2 \sin(q_1 + q_2) - 2b_3 \sin(q_1 + 2q_2) \\ &\quad + b_4 \sin(q_1 - q_2) - k_u(q_1 - \mu q_2) \end{aligned} \tag{A1}$$

The coefficients b_1, b_2, b_3, b_4 are respectively:

$$\begin{aligned} b_1 &= \frac{g}{2k_2}\left(c_3 c_4 \pm 2c_4\sqrt{c_1 c_2}\right); b_2 = \frac{g\mu}{2k_2(\mu+1)}\left(c_3 c_4 \pm 2c_5\sqrt{c_1 c_2}\right) \\ b_3 &= \frac{g\mu c_3 c_5}{2k_2(\mu+2)}; b_4 = \frac{g\mu c_3 c_4}{2k_2(\mu-1)} \end{aligned} \tag{A2}$$

The energy-shaping control u_{es} and damping-injection control u_{di} are:

$$\begin{aligned} u_{es} &= \frac{1}{2}\partial_{q_2}(p^T M^{-1} p) + \partial_{q_2} V - \begin{bmatrix} k_2 & k_3 \end{bmatrix} M^{-1} \partial_q V_d \\ u_{di} &= \frac{K_v}{\Delta_d}(k_2 p_1 - k_1 p_2) \end{aligned} \tag{A3}$$