Energy injection for mechanical systems through the method of Virtual Nonholonomic Constraints

by

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Abstract

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Acknowledgements

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Contents

List of Symbols

Symbol	Definition
$\mathbb{R}^n$	Real numbers in n dimensions
q_u	Unactuated coordinates
q_a	Actuated coordinates
p_u	Conjugate of momentum to q_u
p_a	Conjugate of momentum to q_a
A	Some really long description to see how this works when we write long text inside the table.

TEST CITATIONS:
[1] [2] [3] [4] [5] [6] [7] [8] [9] [10] [11] [12] [13] [14] [15] [16] [17] [18] [19] [20] [21] [22]

Chapter 1

Introduction

1.1 Literature Review

1.2 Statement of Contributions

1.3 Outline of the Thesis

Chapter 2

Development of Virtual Nonholonomic Constraints

2.1 Mechanical Systems

2.1.1 Lagrangian Formulation of Mechanical Systems

2.1.2 Hamiltonian Formulation of Mechanical Systems

2.1.3 Simply Actuated Hamiltonian Systems

2.2 Virtual Nonholonomic Constraints

Chapter 3

Application of VNHCS: The Variable Length Pendulum

3.1 Motivation

3.2 The VLP Constraint

Theorem 1. *For the variable-length pendulum, define* $\theta := \operatorname{arctan}_2(p, q)$*. A VNHC of the form* $l = l(\theta)$ *injects energy if there exists* $l_{avg} \in \mathbb{R}_{>0}$ *such that*

$$\left(l(\theta) - l_{avg}\right) sin(2\theta) \leq 0 \;\forall \theta \in \mathbb{S}^1$$

with the property that the inequality is strict for almost every θ*.*

Proof. Choose, as a candidate anti-Lyapunov function, the energy for the average-length pendulum

$$E_{avg}(q, p) := \frac{1}{2}\frac{p^2}{ml_{avg}^2} + mgl_{avg}(1 - \cos(q))$$

which is non-negative and has derivative

$$\dot{E}_{avg} = \frac{-g\sin(q)p\left(l(\theta)^3 - l_{avg}^3\right)}{l_{avg}^2 l(\theta)^2}$$

We will show that E_{avg} is increasing.

Observe that $\operatorname{sgn}\left(\sin(q)p\right) = \operatorname{sgn}\left(\sin(2\theta)\right)$ and, by Lemma **TODO: REF LEMMA**, $\operatorname{sgn}\left(l(\theta)^3 - l_{avg}^3\right) = \operatorname{sgn}\left(l(\theta) - l_{avg}\right)$.

Then the derivative of E_{avg} is almost always positive, since

$$\begin{aligned}\operatorname{sgn}\left(\dot{E}_{avg}\right) &= \operatorname{sgn}\left(-\sin(q)p\left(l(\theta)^3 - l_{avg}^3\right)\right)\\ &= -\operatorname{sgn}\left(\sin(2\theta)\left(l(\theta) - l_{avg}\right)\right)\\ &\geq 0 \text{ (by assumption)}\end{aligned}$$

Hence, E_{avg} is an anti-Lyapunov function with positive derivative, so the variable-length pendulum is gaining energy. □

3.3 Simulation Results

Chapter 4

Application of VNHCs: The Acrobot

4.1 Motivation

4.2 Previous Approaches

4.3 The Acrobot Constraint

4.3.1 Proving the Acrobot Gains Energy

4.4 Experimental Results

Chapter 5

Conclusion

5.1 Limitations of this Work

5.2 Future Research

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