Exercise 3
3D Computer Vision

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1 Theory

Assume two fully calibrated cameras with intrinsic calibration matrices K_i and extrinsics R_i and t_i , i = 0, 1. The projections can be assumed to be free of any distortions.

1.0.1 Question 1:

Given an image point x_0 in the first view, how does this constrain the position of the corresponding point x_1 in the second image?

Solution:
As mentioned, we have a point x_0 in first view, so we can find the position of corresponding point x_1 in second image by using epipolar line and the epipolar constraint. This point (x_1) lies anywhere along an epipolar line in the second image. We can do this using the following relation:
$x_1^T F x_0 = 0$, where F is a Fundamental matrix

1.0.2 Question 2:

Assume corresponding image points $x_0 < -- > x_1$ are given. Describe in words how the 3D world point X can be computed from its projected image points

$$x_i = K_i[R_i|t_i]\begin{pmatrix} X \\ 1 \end{pmatrix}$$

Solution:
1.We have the points $x_0 <> x_1$ as mentioned in the previous question, so firstly the Fundamental matrix is derived from the above relation: $x_1^T F x_0 = 0$, where F is a Fundamental matrix.
2. Now, we can find the Rotation and Translation vectors of the camera with respect to the other static camera reference
3.Now, we find Essential Matrix, since we already have determined the rotation and translation vectors.
4. Finally, we triangulate the points and intersection of these points will give a 3-D point, if we assume that the first camera is static.

1.0.3 Question 3:

How can the epipoles be computed for the cameras? How are epipolar lines and the epipoles related?

Solution:
The epipoles can be computed using the equations $e = PC'$ and $e = P'C$. The epipoles connectes the center of the cameras.The epipolar line for a given camera passes through the epipole of the other camera.

1.0.4 Question 4:

How can the fundamental matrix be computed if no calibration is given (only the idea, no mathematical derivation)?

Solution:
Given corresponding set of points x_i, $x_i^{'}$ expanding the epipolar constrain equation: $x_1'^T F x_0$, a 3x3 matrix would give us a corresponding equation in 9 variables. Based on the number of point correspondences the system of equations, we can be easily solve it using a suitable estimation algorithm.

1.0.5 Question 5:

How can the fundamental matrix be computed if the calibration (intrinsics and pose) is given?

Solution:
If the intrinsic parameters of the camera is given, we can calculate the fundamental matrix in the following way:
1. For a point x in first image back propagate a ray with camera P.
2. Choose 2 points on the ray and project into second image with camera P'.
3. Compute the line l' through the two image points using the relation $l' = pxq$.
4. This will give the Fundamental matrix by $l' = Fx$

1.0.6 Question 6:

Name a fundamental problem of the matching technique used in the pracical part. When does it make sense to match features this way?

Solution:
We are unable to compute the structure and motion from the fundamental matrix, if we compute it using the previously mentioned epipolar constraint. This type of matching works well if the images remain unchanged.