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The Determination of Vehicle Drag Contributions from Coast-Down Tests

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THE PROBLEM OF AERODYNAMIC and rolling resistance characteristics of cars and trucks is of considerable importance to vehicle engineers as the two major contributions to external vehicle drag. Numerous techniques have been developed for their experimental determination each with particular advantages and disadvantages. For aerodynamic purposes, the methods include wind-tunnel testing of scale models (1-3)*, the testing of full-size production cars (4, 5), and coast-down testing (6-8). Wind-tunnel testing, while well established in the aircraft industry, when applied to determining the aerodynamic drag characteristics of both models and full-size cars has raised many questions of interpretation of data due to ground plane simulation problems (2, 9) and model scale effects (Reynolds number, boundary layer transition, and separation). These limitations imposed on accuracy of results, together with the inherently high costs of wind-tunnel testing are certainly severe drawbacks.

On the other hand, the coast-down technique, which avoids these problems, is affected by the limited accuracy germane to the finding of derivatives to experimentally established curves (7) and lumps together the effects of aerodynamic and other (mostly rolling) resistances. The methods for the determination of rolling resistance are primarily those of the testing of individual tires on drums or tow rigs, or the actual towing of full-size cars in a protective box (9, 10).

If these shortcomings of the coast-down technique can be overcome, its simplicity and inexpensiveness make it most attractive. The presently proposed method is based on the mathematical analysis of a simplified dynamic model which not only allows the separation of aerodynamic and rolling resistance forces, but also utilizes the closed mathematical form of the solution to eliminate the need for differentiating an experimentally determined data curve.

A representation of the coast-down process in terms of dimensionless variables also establishes a dimensionless parameter which characterizes the resistance behavior of any given vehicle. Furthermore, vehicles of considerably different design can be correlated by a single normalized coast-down function.

*Numbers in parentheses designate References at end of paper.

ABSTRACT

The problem of aerodynamic and rolling resistance characteristics of cars and trucks is of considerable importance to vehicle engineers as the two major contributions to external vehicle drag. Many testing methods have been developed including wind tunnel testing of scale models, testing of full-size production cars, and coast-down testing. The advantages and disadvantages of each method are discussed and analyzed.

ANALYSIS

In typical coast-down tests, data can be presented in the form of velocity-time curves similar to Fig. 1; it is from such information that our analysis shall now proceed.

In the absence of a propulsive force, the time rate of decrease of kinetic energy (both translational and rotating) is attributed to the power absorbed by aerodynamic drag and rolling resistance R, respectively.

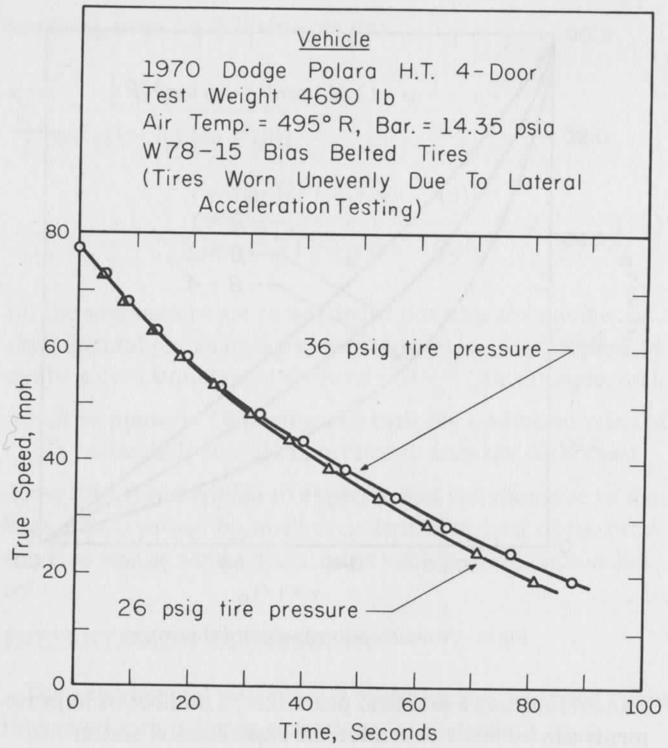

Fig. 1 - Velocity-time coast-down test

$$-\frac{d}{dt}(\text{kinetic energy}) = \frac{-v(m + \Delta m)}{g_c}\frac{dv}{dt}$$

$$= \frac{1}{2g_c}\rho v^3 AC_d + vR \qquad (1)$$

It is important to note that the mass term contains here an additive (Δ m) term to account for the effective inertia mass of the rotating components (primarily the wheels).

Without loss of generality, one may separate the variables, obtaining

$$-\frac{dv}{\dfrac{\rho AC_d v^2}{2m_{eff}} + \dfrac{Rg_c}{m_{eff}}} = dt \qquad (2)$$

where $m_{eff} = m + \Delta m$, and Δm is constant when the kinetic energy of the rotating masses can be directly related to the translational speed of the vehicle.

One notes now that recently the automotive industry has adopted the bias-belted tires as standard equipment while a few makes appear with radials. These types of tires have a markedly constant rolling resistance up to approximately 70 mph, limiting the variation to approximately 18% (Fig. 2). The standard bias-construction low-profile tire which is becoming popular has an even smaller change in this range, namely about 15%. Thus, if the coast-down tests are made for speeds below that which the major rise in rolling resistance occurs, (approximately 60-80 mph depending on the type and style of tire) (10, 11), the rolling resistance can be taken as a constant to a good approximation as will be borne out by the analysis. In addition, the assumption of a constant aerodynamic drag coefficient is reasonable due to the very large Reynolds numbers encountered. Consequently, Eq. 2 can now be integrated to yield

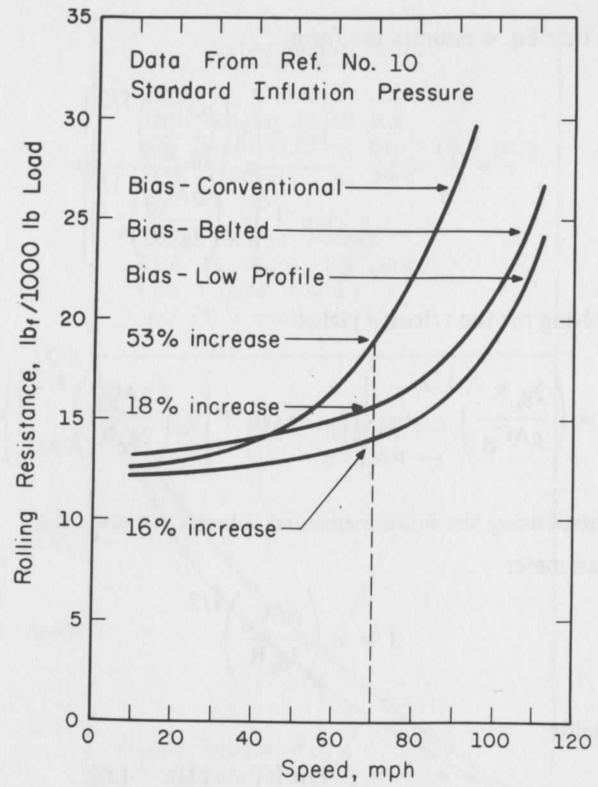

Fig. 2 - Rolling resistance of time versus speed

$$-\left[\frac{2m_{eff}^2}{\rho AC_d g_c R}\right]^{1/2} \tan^{-1}\left[v\left(\frac{\rho AC_d}{2g_c R}\right)^{1/2}\right] = t + \text{constant} \qquad (3)$$

The constant of integration can be evaluated as $v = v_o$ for $t = 0$ (v_o is the initial speed at start of coast down). Thus, one obtains

$$t = \left[\frac{2m_{eff}^2}{\rho AC_d g_c R}\right]^{1/2} \left\{\tan^{-1}\left[v_o\left(\frac{\rho AC_d}{2g_c R}\right)^{1/2}\right] - \tan^{-1}\left[v\left(\frac{\rho AC_d}{2g_c R}\right)^{1/2}\right]\right\} \qquad (4)$$

One nondimensionalizes the time variable t by the total coast-down time t_o, where $v = 0$ (which shall be subjected to further scrutiny)

$$t_o = \left[\frac{2m_{eff}^2}{\rho AC_d g_c R}\right]^{1/2} \tan^{-1}\left[v_o\left(\frac{\rho AC_d}{2g_c R}\right)^{1/2}\right] \qquad (5)$$

so that Eq. 4 assumes the form

$$\tau = \frac{t}{t_o} = 1 - \frac{\tan^{-1}\left[v\left(\frac{\rho A C_d}{2g\,R}\right)^{1/2}\right]}{\tan^{-1}\left[v_o\left(\frac{\rho A C_d}{2g_c R}\right)^{1/2}\right]} \qquad (6)$$

Solving for the velocity yields

$$v = \left(\frac{2g_c R}{\rho A C_d}\right)^{1/2} \tan\left\{(1-\tau)\tan^{-1}\left[v_o\left(\frac{\rho A C_d}{2g_c R}\right)^{1/2}\right]\right\} \qquad (7)$$

Introducing the nondimensional velocity $\nu = v/v_o$, the parameter

$$\beta = v_o\left(\frac{\rho A C_d}{2g_c R}\right)^{1/2} \qquad (8)$$

yields

$$\nu = \frac{v}{v_o} = \frac{1}{\beta}\tan\left[(1-\tau)\tan^{-1}(\beta)\right] \qquad (9)$$

Eq. 9 represents a one-parameter family, in (β), of curves as shown in Fig. 3, in the dimensionless ν-τ plane.

By comparing test data for any vehicle with the curves in Fig. 3, the appropriate β value can be obtained. With knowledge of the total coast-down time, it is now possible to write simple algebraic expressions for the rolling resistance and drag coefficient. These are, as obtained from Eqs. 5 and 8,

$$C_d = \frac{2m_{eff}\,\beta\tan^{-1}(\beta)}{v_o t_o \rho A} \qquad (10)$$

and

$$R = \text{rolling resistance} = \frac{v_o m_{eff}\tan^{-1}(\beta)}{\beta t_o g_c} \qquad (11)$$

Eqs. 9-11 allow determination of the important characteristics of the vehicle in a straightforward manner from a simple coast-down test. It is also obvious that a comparison of vehicles on the basis of their drag characteristics can be made by examining β values. Furthermore, the coast-down behavior of all cars (for that matter, all vehicles) can be represented by a single curve in the nondimensional plane by the selection of a normalized value; for example, $\beta = 1$,* through the proper choice of v_o.

*One notes that a higher value of β should be selected if it is desirable to correlate cars with relatively high aerodynamic drag contribution, or for covering coast-down tests from high initial speeds.

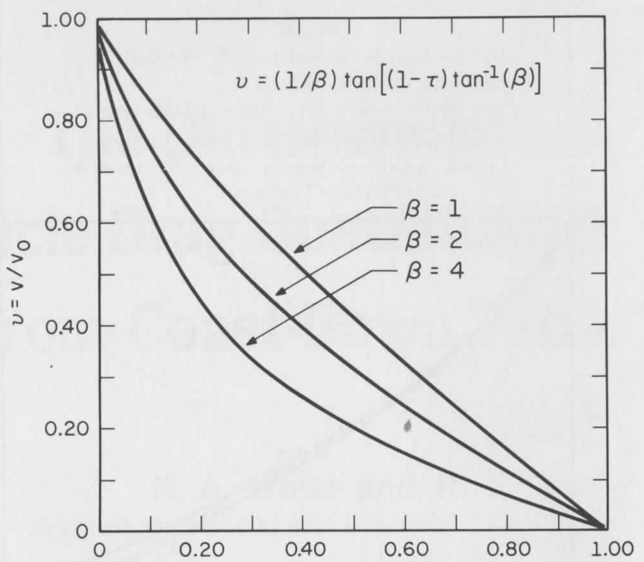

Fig. 3 - Nondimensional velocity-time plane

In establishing a practical procedure, a number of improvements can be made to reduce the experimental scatter for obtaining t_o without direct measurement, and for more accurately identifying the parameter β.

Applying the method of least squares to Eq. 9 for determining optimal values of β and t_o requires that

$$\frac{\partial}{\partial\beta}\sum\left[\nu_i - \nu(\beta,\tau_i)\right]^2 = 0 \qquad (12a)$$

$$\frac{\partial}{\partial t_o}\sum\left[\nu_i - \nu(\beta,\tau_i)\right]^2 = 0 \qquad (12b)$$

Carrying out the operations of Eq. 12 yields

$$\beta^2 - \beta\left\{\frac{\Sigma_1 + \cos^2[\tan^{-1}(\beta)]\,\Sigma_4}{\cos^2[\tan^{-1}(\beta)]\,\Sigma_3}\right\} + \frac{\Sigma_2}{\cos^2[\tan^{-1}(\beta)]\,\Sigma_3} = 0 \qquad (13)$$

where:

$$\Sigma_1 = \sum_i \nu_i \tan\left[(1-\tau_i)\tan^{-1}(\beta)\right]$$

$$\Sigma_2 = \sum_i \tan^2\left[(1-\tau_i)\tan^{-1}(\beta)\right]$$

$$\Sigma_3 = \sum_i \frac{\nu_i(1-\tau_i)}{\cos^2[(1-\tau_i)\tan^{-1}(\beta)]}$$

$$\Sigma_4 = \sum_i \frac{(1-\tau_i)\tan[(1-\tau_i)\tan^{-1}(\beta)]}{\cos^2[(1-\tau_i)\tan^{-1}(\beta)]}$$

Similarly, from Eq. 12b one obtains

$$\sum_i \frac{\nu_i\tau_i}{\cos^2[(1-\tau_i)\tan^{-1}(\beta)]} - \sum_i \frac{\tau_i\tan[(1-\tau_i)\tan^{-1}(\beta)]}{\beta\cos^2[(1-\tau_i)\tan^{-1}(\beta)]} = 0 \qquad (14)$$

All the summations are to be carried out over the number of experimental points in every coast-down test. Eqs. 13 and 14 can be solved simultaneously for β and t_o* (for example, on a digital computer). Obtaining a statistically optimized value of t_o is a particularly useful improvement since the total coast-down time is susceptible to experimental variation due to the large effects caused by small irregularities in road surface and slope, as well as wind effects as the vehicle speed approaches zero.

PRESENTATION OF RESULTS

The experimental procedure is to determine the speed versus time relationship during coast down of the vehicle from approximately 75-20 mph. After obtaining a representative set of velocity-time value pairs, Eqs. 13 and 14 can be solved simultaneously for β and t_o. With t_o being thus determined, one can now replace the original velocity-time diagram shown in Fig. 1 by a dimensionless $\nu = v/v_o$ versus $\tau = t/t_o$ plot. The analytically determined β value then establishes the theoretical solution corresponding to Eq. 9. As can be seen in Fig. 4, the agreement between experimental data points and the "least square fit" theoretical curve is most gratifying. Since no "weighting" scheme has been introduced into the least square fit approach used in Eqs. 13 and 14, care should be taken that a relatively uniform spacing of the experimental data is used, to avoid overemphasizing the importance of any portion of the test in favor of the remainder.

The apparent usefulness of obtaining an accurate (calculated) t_o value is not explicitly seen in the nondimensional (ν-τ) plot of Fig. 4. It does, however, directly effect the calculation of both aerodynamic drag coefficient and the rolling resistance as seen from Eqs. 10 and 11. To obtain the drag coefficient C_d from Eq. 10, it is necessary to determine the effective mass (m_{eff}) and the frontal area of the vehicle A, as well as the air density ρ.

The rolling resistance as calculated from Eq. 11 can be brought into the conventional form of force/1000 lbf weight by

$$F\Big|_{1000} = \frac{R g_c}{mg}\,1000 \qquad (15)$$

where m is the actual mass of the car.

*Note that the unknown t_o appears implicitly in the $\tau_i = t_i/t_o$ ratios.

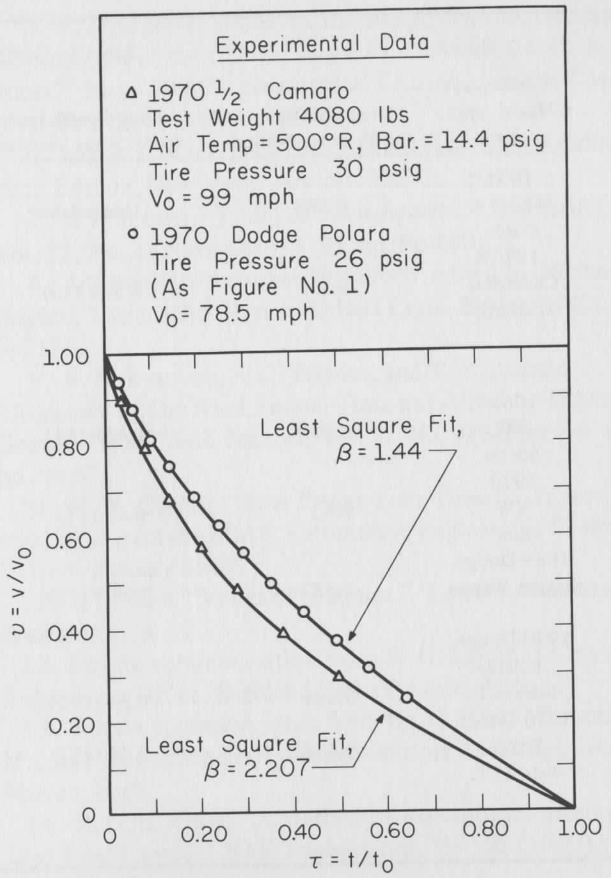

Fig. 4 - Comparison of experimental data and least square fit in dimensionless velocity-time plane

Results obtained from coast-down tests with a variety of cars with different shapes, weights, and initial speeds are shown in Table 1 together with pertinent data (where available) from other sources.

Comments have been included to expose particular conditions affecting our experimental results.

Altogether, the information resulting from the present method of analysis appears to be in reasonable agreement with most data from other sources (runs 2-5), and sufficiently accurate in isolating aerodynamic drag and rolling resistance (runs 6 and 7). Whether the somewhat larger discrepancy between our C_d values for runs 6 and 7 and that reported in (12) can be fully resolved by our comments is questionable. Yet, the accuracy of the experimental method reported in (12) cannot be fully appraised.

CONCLUSIONS

A new method for extracting aerodynamic drag coefficients and rolling resistance data from coast-down experiments is shown to yield good results.

A mathematical analysis based on an energetic model for the coast-down process leads to a closed solution in terms of two parameters which are subsequently determined by a statistical optimization procedure.

Furthermore, it follows from the analysis, that coast-down

Table 1 - Comparison of Drag Contribution

Car Make, Model, and Year	Drag Coefficient by Present Method	Drag Coefficient from Other Sources	Rolling resistance, 1000 lb load, and Tire Pressure	Test V_o mph	Test β	Comments
1931 Model A Ford	0.838	Unavailable	29.2 lb/1000 at 25 psig	50.0	1.314	Dragging mechanical brakes account for the high rolling resistance
1970½ Chevrolet Camaro	0.491	0.521 (13)	12.1 lb/1000 at 30 psig	99.0	2.207	Mechanical transmission, F-70 belted bias construction tires. Ref. 13 C_D value is from full-scale wind tunnel tests
1965 VW Beetle	0.450	0.458 (14)	12.9 lb/1000 at 26 psig	68.7	1.886	Bias construction tires on this vehicle
1970 VW Bus	0.43	0.42 (15)	17.85 lb/1000 at unknown psig	60.8	1.419	Three different types of bias construction tires on this vehicle
1969 Dodge Station Wagon	0.587	0.607 (12)	18.6 lb/1000 at 25 psig	78.5	1.564	Car equipped with studded snow tires and automatic transmission
1970 Dodge Polara 4-dr. H.T.	0.499	0.543 (12)	16.35 lb/1000 at 35 psig	77.2	1.541	Car equipped with tires worn unevenly from lateral acceleration tests. Only variable is tire pressure, showing separation of rolling resistance and aerodynamic drag
1970 Dodge Polara 4-dr. H.T.	0.498	0.543 (12)	18.7 lb/1000 at 25 psig	77.2	1.441	

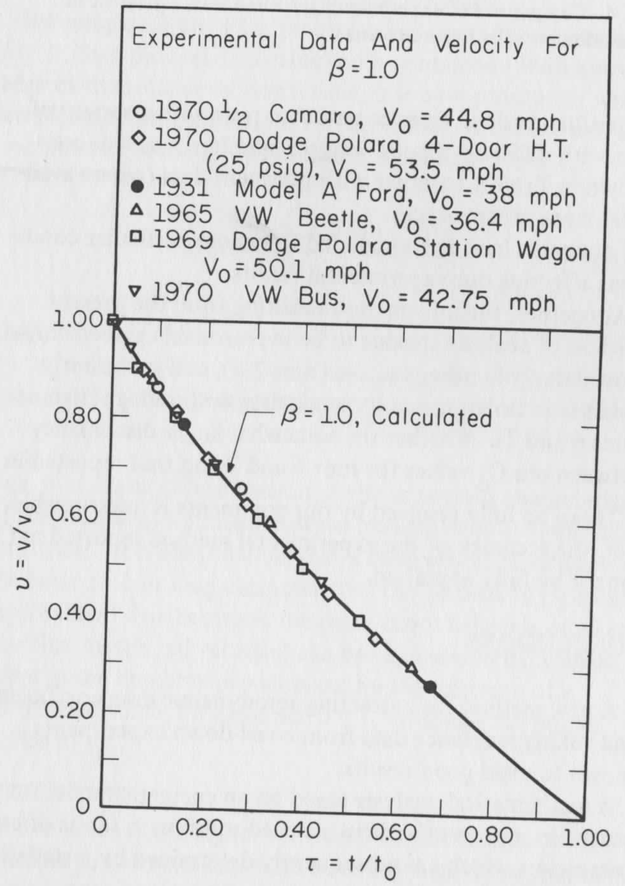

Fig. 5 - Dimensionless velocity-time plane with vehicles normalized to $\beta = 1$

behavior can be correlated by a single, nondimensionalized speed-time curve. Experiments conducted with a variety of different vehicles (but also with fans and compressors) have verified this, as shown in Fig. 5.

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NOMENCLATURE

A = frontal area of vehicle, ft^2
D = aerodynamic drag force, lbf
g = gravitational acceleration, $ft\text{-}s^{-2}$
g_c = gravitational constant, $lbm\text{-}lbf^{-1}\text{-}ft\text{-}s^{-2}$
m = mass of vehicle, lbm
m_{eff} = effective mass of vehicle, lbm
v = velocity $ft\text{-}s^{-1}$
v_o = initial velocity, $ft\text{-}s^{-1}$
V_o = initial velocity, rolling distance, mph
R = rolling resistance, lbf

t = time, s
t_o = total coast-down time from V_o to $V = 0$, s
Δm = additive mass, lbm
ρ = air density, $lbm\text{-}ft^{-3}$

Dimensionless Groups

ν = dimensionless velocity, V/V_o
τ = dimensionless time, t/t_o
C_D = aerodynamic drag coefficient, $2Dg_c/Av^2\rho$
$F\Big|_{1000}$ = dimensionless rolling resistance (Eq. 15)
β = coast-down parameter, Eq. 8

Subscripts

i = dimensionless data point pairs of ν_i and τ_i

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