

# Fast Iterative Solvers

## Suggested Work for Multigrid Assignment – Part 2

### Restriction Operator

Implement a fully weighted restriction operation, as discussed in class. You will re-use this function when writing a complete multigrid solver!

For validation we approximate a square domain  $\Omega = [0, 1]^2$  using a grid as defined in the previous handout. We define a "fine mesh" with  $N = 2^n$ ; for some integer  $n \geq 1$ , and a coarse mesh using  $N^c = 2^{n-1}$ . This means that points in the coarse mesh will also be points in the fine mesh, while every other point in the fine mesh is deleted.

A function  $\mathbf{u}_{2h} = \text{RESTR}(\mathbf{u}_h; N^c)$  should be most convenient to work with, i.e. the loop ought to be over the coarse mesh nodes. Please also see the class notes, where we discussed a possible loop layout.

- Test your implementation for two grids, using  $n = 4$  and  $n = 7$  for the fine grid, respectively.
- initialize  $u_{i,j}$  on the *fine* grid such that  $u_h[i, j] = u(x_i, y_j)$  for the function  $u(x, y) = \sin(2\pi x) \sin(2\pi y)$ .
- Then transfer to the coarse grid, and measure  $\|\mathbf{e}_{2h}\|_\infty = \max_{i,j} |u_{2h}[i, j] - u(x_i, y_j)|$ . (Note that here  $(i, j)$  are *coarse* grid indices!)

### Prolongation Operator

Implement the bilinear prolongation operator, as discussed in class. You will re-use this function when writing a complete multigrid solver!

Define fine and coarse grids in the same way as for the restriction operator.

A Function  $\mathbf{u}_h = \text{PROLONG}(\mathbf{u}_{2h}; N^c)$  should be most convenient to work with, i.e. the loop ought to be over the coarse mesh nodes.

- Test your implementation for two grids, using  $n = 4$  and  $n = 7$  for the fine grid, respectively.
- initialize  $u_{i,j}$  on the *coarse* grid such that  $u_{2h}[i, j] = u(x_i, y_j)$  for the function  $u(x, y) = \sin(2\pi x) \sin(2\pi y)$ .
- Then transfer to the fine grid, and measure  $\|\mathbf{e}_h\|_\infty = \max_{i,j} |u_h[i, j] - u(x_i, y_j)|$ . (Note that here  $(i, j)$  are *fine* grid indices! )