Antoni Zygmund

TRIGONOMETRIC SERIES
THIRD EDITION

LaTeX 2_ε version by Ali Darijani

January 29, 2023

Darij's Publishing House

Foreward

Surely, Antoni Zygmund's *Trigonometric Series* has been, and continues to be, one of the most influential books in the history of mathematical analysis.Therefore, the current printing, which ensures the future availability of this work to the mathematical public is an event of major importance. Its tremendous longevity is a testimony to its depth and clarity. Generations of mathematicians from Hardy and Littlewood to recent classes of graduate students specializing in analysis have viewed *Trigonometric Series* with enormous admiration and have profited greatly from reading it. In light of the importance of Antoni Zygmund as a mathematician and of the impact of *Trigonometric Series*, it is only fitting that a brief discussion of his life and mathematics accompany the present volume, and this is what I have attempted to give here[1]. I can only hope that it provides at least a small glimpse into the story of this masterpiece and of the man who produced it.

Antoni Zygmund was born on December 26, 1900 in Warsaw, Poland. His parents had received relatively little education, and were of modest means, so his background was far less privileged than that of the vast majority of his colleagues. Zygmund attended school through the middle of high school in Warsaw. When World War I broke out, his family was evacuated to Poltava in the Ukraine, where he continued his studies. When the war ended in 1918, his family returned to Warsaw, where he completed pre collegiate work, and entered Warsaw University. Zygmund's main interest throughout his childhood was astronomy, but at Warsaw University at that time there were not sufficient courses offered in that subject to make it realistic as a specialization, and so Zygmund turned instead toward another of his interests, mathematics.

There were a number of excelent mathematicians and teachers who profoundly influenced Zygmund during this period. The greatest impact came from Aleksander Rajchman and Stanislaw Saks. Rajchman was a junior faculty member who was an expert on Riemann's theory of trigonometric series, and Saks a felow student who was a few years his senior. From Rajchman, he learned much of the Riemann theory,

[1] I have been fortunate to have a number of excelent references to consult regarding the life of Antoni Zygmund. The reader interested in aditional material may consult the references in the bibliography to this Foreword.

and his doctoral thesis in 1923 was on this subject. Zygmund became an active co-laborator with both Rajchman and Saks, publishing a number of important articles with each of them. In adition, Saks and Zygmund produced *Analytic Functions*, one of the classic texts on complex analysis.

One year prior to his PhD, Zygmund was appointed to an instructorship at the Warsaw Polytechnical School, and, in 1926, he was appointed Privatdozent at the University of Warsaw. He was awarded a Rockefeller fellowship, which he used to travel to England for the academic year of 1929-30 and visit G.H. Hardy at Cambridge for the first half of the year, and J.E. Litlewod at Oxford for the second half. This experience had a tremendous impact on the young Zygmund. Not only did he work with two of the greatest analysts of the time, but while in England, he also met another young mathematician, R.E.A.C. Paley, a student of Littlewod, with whom he had an extended and very fruitful collaboration. When he returned to Poland in 1930, Zygmund moved to Wilno where he took a chair in mathematics at the Stefan Batory University. It was here that Zygmund's talent and quiet charismasa as a teacher of advanced mathematics began to have a very major impact on the field. In the fall of 1930, Zygmund met a new student at the University, Jozef Marcinkiewicz. Marcinkiewicz was recognized, even when he was a student, as being tremendously talented, with the potential to become a serious mathematician. In the following year, which was only the second at Stefan Batory for both teacher and student, Zygmund decided to offer a course on trigonometric series preceded by lectures on Lebesgue integration. Marcinkiewicz attended this course, and thus began his association with Zygmund. It took just three years for Marcinkiewicz to obtain his masters degree, with a thesis that contained the highly non-trivial result that it is possible for a continuous periodic function to have interpolating polynomials corresponding to equidistant nodal points diverging almost everywhere. This result was elaborated to form his PhD thesis in 1935, and in 1937 Marcinkicwicz became Dozent in Wilno. In the period from 1935 to 1939, a collaboration between Marcinkiewicz and Zygmund developed that was incredibly successful. Though of relatively short duration, their work opened a number of new directions, and in a sense set the stage for the theory of singular integrals which would be Antoni Zygmund's greatest contribution.

The years in which Zygmund was a young profesor in Wilno, though very productive mathematically, were not easy ones. This was due in large part to Zygmund's courageous opposition to the bigotry which was all to common around him, and which was supported by the higher authorities. An example of this was his strong dislike of anti-Semitic policies with in his university. At one time,for instance, student organizations, somewhat analogous to modern day fraternities, were sufficiently influential to mandate that all Jewish students must sit on the left side of each classroom during lectures. For Zygmund, this was completely unacceptable and in response, he decided to move his classes from the larger hals to small mathematics department seminar rooms where there were only long tables in a central arrangement, and hence no seats at the left or right of the room. Another illustration of Zygmund's sensitivity to issues of social justice had to do with is university's requirement that all student associations have faculty members as their academic sponsors. Zygmund regularly sponsored associations which were not in favor with the Polish government. These

unpopular moves on Zygmund's part did not go unnoticed, and in the year 1931, as part of the political purges of the universities by the government, Zygmund was dismissed from his professorship. This immediately brought extremely strong reaction from some of the most distinguished mathematicians in Europe.From Lebesgue in France, and from Hardy and Littlewod in England came formal written protests which resulted in Zygmund's reinstatement as professor. It is therefore an important aspect of Zygmund's life that, in a very real sense, he was a crusader for human rights well before this was fashionable.

Among the many remarkable contributions of the Wilno period is the writing of the first version of this book, published in Warsaw under the title *Trigonometrical Series*. This was Zygmund's first book, and it was published as volume xivof the series Monografie Matematyczne. This is the same series in which the celebrated book *Theorie des Operations Lineaires* by S. Banach appears as volume .The tremendous success of *Trigonometrical Series* led to its expansion and revision in to a second edition, published in 1959 by Cambridge University Press, and then to no fewer than six reprinted versions after that.

The time in Wilno which featured the rapid achievement of success came to a sudden end in September 1939 as World War 2 erupted. At that time, both Zygmund and Marcinkicwicz were mobilized as reserve oficers in the Polish army, and, as a result of the temporary "friendship" between Germany and Russia, Poland was partitioned. The Soviets were given control of much of the country, including the part containing Wilno, and they preceded to round up and execute many of the Polish officer corps in the Katyn Forest massacre. Most likely, this is how Marcinkiewicz perished. Almost by a miracle, Zygmund was able to return to his family and escape with them to the United States, but his loss was absolutely devastating. His principal collaborators up to that time besides Marcinkicwicz had been Saks, Rajchmanand, Paley. Both Saks and Rajchman were murdered by the Nazis, and Paley had died in a tragic accident in 1933. These loses were not just mathematical. Zygmund had been extremely close to each of them, and so the war period must surely have been one of the most difficult of his life.

By 1939, Zygmund had an international reputation, and many friends all over the mathematical world. It was due to the efforts of some of these friends, such as Jacob Tamarkin, Jerzy Neyman and Norbert Wiener, that Zygmund was able to emigrate to the United States in 1940. During the time immediately prior to the United States entering in to the war, there were very few jobs available to mathematicians. Nevertheless, after teaching for a semester at MIT, Zygmund was offered and accepted a position at Mount Holyoke College in central Massachusetts. A few years later, other offers followed. In 1945, Zygmund became a professor at the University of Pennsylvania, and then, in 1947, he was offered a professorship at the University of Chicago.

University of Chicago, January 29, 2023 *Robert A. Fefferman*

Contents

Part I
Part Title

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