Contents

1 Wirtinger Flow

The whole thing about *Wirtinger Flow* variants started with the seminal work of Candes and Soltanolkotabi[1]. The most important improvements chronologically were done by Candes and Chen[2], Kolte and Özgür[3], and Zhang et al.[4]. For a quite extensive survey on *Wirtinger Flow* variants please refer to Liu et al.[5]. Chandra et al.[6] gathered quite number of *Phase Retrieval* methods including a couple of *Wirtinger Flow* variants in the MATLAB® problem solving environment in a uniform manner.
We quickly go over the problem formulation, difficulties, algorithms, and at the of the chapter we give some numerical experiments we are going to refer to in the subsequent chapters.

1.1 Problem Formulation

Consider the ray $\boldsymbol{x} \in \mathbb{C}^{n\times 1}$ is emitted onto the object of interest and the diffracted rays are measured as $\boldsymbol{y} \in \mathbb{R}^{m\times 1}$ and is connected to the original ray by $\boldsymbol{y} = \varphi(\boldsymbol{A}\boldsymbol{x})$, where $\boldsymbol{A} \in \mathbb{C}^{m\times n}$ and φ the usual element-wise absolute value(or the squared absolute value) from $\mathbb{C}^{m\times 1}$ to $\mathbb{R}^{m\times 1}$.
Candes and Soltanolkotabi[1] considered φ to be squared element-wise absolute value and the loss function to be quadratic. The summary for all the variants in terms of formulation is in table1.1

1.2 Difficulties

The loss function is non-convex. Two nice figures can be found in [4] that show the effect of different formulation in the curvature but at the same time the non-convexness of the presented problem. For the thorough geometric analysis of the *Phase Retrieval* problem we refer to Wright et al.[7]

Wirtinger Flow Variant	φ	loss functions
Wirtinger Flow	$\lvert\boldsymbol{z}\rvert^2$	quadratic
Truncated Wirtinger Flow	$\lvert\boldsymbol{z}\rvert^2$	quadratic
Incrementally Truncated Wirtinger Flow	$\lvert\boldsymbol{z}\rvert^2$	quadratic
Reshaped Wirtinger Flow	$\lvert\boldsymbol{z}\rvert$	quadratic
Incrementally Reshaped Flow	$\lvert\boldsymbol{z}\rvert$	quadratic

Table 1.1: φ and the loss function used in [1], [2], [3], [4]

Input: $\boldsymbol{y} = \{y_i\}_{i=1}^m$, $\{\boldsymbol{a}_i\}_{i=1}^m$;
Parameters: Lower and upper thresholds α_l, α_u for truncation in initialization, step size μ;
Initialization: Let $\boldsymbol{z}^{(0)} = \lambda_0 \tilde{\boldsymbol{z}}$, where $\lambda_0 = \frac{mn}{\sum_{i=1}^m \|\boldsymbol{a}_i\|_1} \cdot \left(\frac{1}{m}\sum_{i=1}^m y_i\right)$ and $\tilde{\boldsymbol{z}}$ is the leading eigenvector of

$$\boldsymbol{Y} \coloneqq \frac{1}{m}\sum_{i=1}^m y_i \boldsymbol{a}_i \boldsymbol{a}_i^* \mathbf{1}_{\{\alpha_l \lambda_0 < y_i < \alpha_u \lambda_0\}}. \tag{1.1}$$

Gradient loop: for $t = 0 : T-1$ do

$$\boldsymbol{z}^{(t+1)} = \boldsymbol{z}^{(t)} - \frac{\mu}{m}\sum_{i=1}^m \left(\boldsymbol{a}_i^* \boldsymbol{z}^{(t)} - y_i \cdot \frac{\boldsymbol{a}_i^* \boldsymbol{z}^{(t)}}{|\boldsymbol{a}_i^* \boldsymbol{z}^{(t)}|}\right) \boldsymbol{a}_i. \tag{1.2}$$

Output $\boldsymbol{z}^{(T)}$.
Algorithm 1: Reshaped Wirtinger Flow suggested by [**zhang2016reshaped**]

Input: $\boldsymbol{y} = \{y_i\}_{i=1}^m$, $\{\boldsymbol{a}_i\}_{i=1}^m$;
Initialization: Same as in RWF (Algorithm 1);
Parameters: Lower and upper thresholds α_l, α_u for truncation in initialization, step size μ;
Gradient loop: for $t = 0 : T-1$ do
Choose i_t uniformly at random from $\{1, 2, \ldots, m\}$, and let

$$\boldsymbol{z}^{(t+1)} = \boldsymbol{z}^{(t)} - \mu \left(\boldsymbol{a}_{i_t}^* \boldsymbol{z}^{(t)} - y_{i_t} \cdot \frac{\boldsymbol{a}_{i_t}^* \boldsymbol{z}^{(t)}}{|\boldsymbol{a}_{i_t}^* \boldsymbol{z}^{(t)}|}\right) \boldsymbol{a}_{i_t}, \tag{1.3}$$

Output $\boldsymbol{z}^{(T)}$.
Algorithm 2: Incremental Reshaped Wirtinger Flow (IRWF) suggested by [**zhang2016reshaped**]

Input: $\boldsymbol{y} = \{y_i\}_{i=1}^m$, $\{\boldsymbol{a}_i\}_{i=1}^m$;
Initialization: Same as in RWF (Algorithm 1);
Gradient loop: for $t = 0 : T-1$ do
Choose Γ_t uniformly at random from the subsets of $\{1, 2, \ldots, m\}$ with cardinality k, and let

$$\boldsymbol{z}^{(t+1)} = \boldsymbol{z}^{(t)} - \mu \cdot \boldsymbol{A}_{\Gamma_t}^* \left(\boldsymbol{A}_{\Gamma_t} \boldsymbol{z}^{(t)} - \boldsymbol{y}_{\Gamma_t} \odot \mathrm{Ph}(\boldsymbol{A}_{\Gamma_t} \boldsymbol{z}^{(t)})\right), \tag{1.4}$$

where $\boldsymbol{A}_{\Gamma_t}$ is a matrix stacking $\boldsymbol{a}_i^*$ for $i \in \Gamma_t$ as its rows, $\boldsymbol{y}_{\Gamma_t}$ is a vector stacking y_i for $i \in \Gamma_t$ as its elements, $\odot$ denotes element-wise product, and $\mathrm{Ph}(\boldsymbol{z})$ denotes a phase vector of $\boldsymbol{z}$.
Output $\boldsymbol{z}^{(T)}$.
Algorithm 3: Minibatch Incremetnal Reshaped Wirtinger Flow (minibatch IRWF) suggested by [**zhang2016reshaped**]

Bibliography

[1] E. J. Candes, X. Li, and M. Soltanolkotabi. *Phase Retrieval via Wirtinger Flow: Theory and Algorithms*. Apr. 2015.

[2] Y. Chen and E. J. Candes. *Solving Random Quadratic Systems of Equations Is Nearly as Easy as Solving Linear Systems*. 2016. arXiv: `1505.05114 [cs.IT]`.

[3] R. Kolte and A. Özgür. *Phase Retrieval via Incremental Truncated Wirtinger Flow*. 2016. arXiv: `1606.03196 [cs.IT]`.

[4] H. Zhang, Y. Zhou, Y. Liang, and Y. Chi. *Reshaped Wirtinger Flow and Incremental Algorithm for Solving Quadratic System of Equations*. 2016. arXiv: `1605.07719 [stat.ML]`.

[5] J.-W. Liu, Z.-J. Cao, J. Liu, X.-L. Luo, W.-M. Li, N. Ito, and L.-C. Guo. "Phase Retrieval via Wirtinger Flow Algorithm and Its Variants." In: *2019 International Conference on Machine Learning and Cybernetics (ICMLC)*. 2019, pp. 1–9.

[6] R. Chandra, Z. Zhong, J. Hontz, V. McCulloch, C. Studer, and T. Goldstein. *PhasePack: A Phase Retrieval Library*. 2017. arXiv: `1711.10175 [math.OC]`.

[7] J. Sun, Q. Qu, and J. Wright. "A Geometric Analysis of Phase Retrieval." In: *Foundations of Computational Mathematics* 18.5 (Aug. 2017), pp. 1131–1198.