$$(f *_w g)(x) := \int_{\mathbb{R}^1} \overline{f(y)}\, g(x+y)\, w(x+y,y)\, dy$$

$$E[\psi] = \frac{1}{N} \sum_{j=1}^{N} \| \psi *_{T_j} \psi - \mathcal{F}(g_j) \|_{L^2}^2 + R(\psi)$$

Ann. $T_j(x,y) = \overline{w_j(x)}\, w_j(y)$

$$\Rightarrow \psi *_{T_j} \psi = (\psi w_j) \star (\psi w_j)$$

$$\mathcal{F}^{-1}(\psi *_{T_j} \psi) = (2\pi)^{d/2}\, \overline{\mathcal{F}^{-1}(\psi w_j)}\, \mathcal{F}^{-1}(\psi w_j)$$

$$\Rightarrow E[\psi] = \frac{1}{N} \sum_{j=1}^{N} \| (2\pi)^{d/2} \underbrace{\overline{\mathcal{F}^{-1}(\psi w_j)}\, \mathcal{F}^{-1}(\psi w_j)}_{\varphi(\mathcal{F}^{-1}(\psi w_j))} - g_j \|_{L^2}^2 + R(\psi)$$

mit $\varphi(z) := \bar{z} z = |z|^2$

$\mathbb{C} \sim \mathbb{R}^2 \Rightarrow \varphi : \mathbb{R}^2 \to \mathbb{R}$

Struktur diskret

$\psi \in \mathbb{C}^M$, $A \in \mathbb{C}^{M \times M}$, $W_j \in \mathbb{C}^{M \times M}$ diagonal Matrix), $G_j \in \mathbb{R}^{M \times M}$

$$E[\psi] = \underbrace{\sum_{j=1}^{N} \| \varphi(A W_j \psi) - G_j \|^2}_{D(\psi)} + R(\psi)$$

$$F(\psi) := \| \varphi(B\psi) - G \|^2 \qquad B \in \mathbb{C}^{M \times M},\ G \in \mathbb{R}^M$$

$$\partial_{\operatorname{Re}\psi_i} F(\psi) = \partial_{\operatorname{Re}\psi_i} \sum_{k=1}^{M} \Big(\varphi\Big(\sum_{l=1}^{M} B_{kl} \psi_l \Big) - G_k \Big)^2 \qquad \varphi : \mathbb{R}^2 \to \mathbb{R}$$

$$= 2 \sum_{k=1}^{M} \Big(\varphi\Big(\sum_{l=1}^{M} B_{kl}\psi_l \Big) - G_k \Big) \nabla \varphi\Big(\sum_{l=1}^{M} B_{kl}\psi_l \Big) \cdot \overbrace{\partial_{\operatorname{Re}\psi_i}(B_{ki}\psi_i)}^{(\operatorname{Re} B_{ki},\ \operatorname{Im} B_{ki})}$$

$$= 2 \sum_{k=1}^{M} \varphi((B\psi)_k) - G_k \Big) \Big[\partial_1 \varphi((B\psi)_k) \cdot \operatorname{Re} B_{ki} + \partial_2 \varphi((B\psi)_k) \operatorname{Im} B_{ki}$$

$$= 2 \Big((\operatorname{Re} B^T)(\varphi(B\psi) - G) \odot \overset{= 2\operatorname{Re}}{\partial_1 \varphi} (B\psi) \Big)_i$$

$$+ 2 \Big((\operatorname{Im} B^T)(\varphi(B\psi) - G) \odot \underset{= 2\operatorname{Im}}{\partial_2 \varphi} (B\psi) \Big)_i$$

$$\Rightarrow \nabla_{\operatorname{Re}\psi} F(\psi) = 4 \operatorname{Re} B^T (\varphi(B\psi) - G) \odot \operatorname{Re}(B\psi) + 4 \operatorname{Im} B^T (\varphi(B\psi) - G) \odot \operatorname{Im}(B\psi)$$

$$= 4\,\mathrm{Re}\, B^T\Big(\underbrace{\varphi(B\psi)\odot \mathrm{Re}(B\psi)}_{\varphi_1(B\psi)} - G\odot \underbrace{\mathrm{Re}(B\psi)}_{=\frac12 \partial_1\varphi(B\psi)}\Big)$$

mit $\varphi_1(z) = |z|^2\,\mathrm{Re}\, z = \varphi(z)\,\frac12 \partial_1\varphi(z)$, $\varphi_1(a,b) = a^3 + ab^2$

$\uparrow$ nicht holomorph da
$\downarrow$ reellwertig aber nicht konstant

$$+4\,\mathrm{Im}\, B^T\Big(\underbrace{\varphi(B\psi)\odot \mathrm{Im}(B\psi)}_{\varphi_2(B\psi)} - G\odot \underbrace{\mathrm{Im}(B\psi)}_{=\frac12 \partial_2\varphi(B\psi)}\Big)$$

mit $\varphi_2(z) = |z|^2\,\mathrm{Im}\, z = \varphi(z)\,\frac12 \partial_2\varphi(z)$, $\varphi_2(a,b) = a^2 b + b^3$

$$\nabla_{\mathrm{Im}\,\psi} F(\psi) = -4\,\mathrm{Im}\, B^T(\varphi(B\psi) - G)\odot \mathrm{Re}(B\psi) + 4\,\mathrm{Re}\, B^T(\varphi(B\psi)-G)\odot \mathrm{Im}(B\psi)$$

$$= -4\,\mathrm{Im}\, B^T(\varphi_1(B\psi) - G\odot\mathrm{Re}(B\psi)) + 4\,\mathrm{Re}\, B^T(\varphi_2(B\psi) - G\odot \mathrm{Im}(B\psi))$$

$$\Big(\nabla_{\mathrm{Re}\,\psi} F(\psi), \nabla_{\mathrm{Im}\,\psi} F(\psi)\Big) = 4\,\mathrm{Re}\, B^T\Big[(\varphi_1(B\psi), \varphi_2(B\psi)) - (G\odot \mathrm{Re}(B\psi), G\odot(\mathrm{Im}(B\psi)))\Big]$$
$$+ 4\,\mathrm{Im}\, B^T\Big[(\varphi_2(B\psi), -\varphi_1(B\psi)) - (G\odot \mathrm{Im}(B\psi), -G\odot \mathrm{Re}(B\psi))\Big]$$

$$\nabla_{\mathrm{Re}\,\psi} F(\psi) + i\nabla_{\mathrm{Im}\,\psi} F(\psi) = 4\,\mathrm{Re}\, B^T\big[\varphi_1(B\psi) + i\varphi_2(B\psi) - G\odot B\psi\big]$$
$$+ 4\,\mathrm{Im}\, B^T\big[-i(\varphi_1(B\psi) + i\varphi_2(B\psi)) - G\odot(-iB\psi)\big]$$

$$= 4\,\mathrm{Re}\, B^T\big[\underbrace{\varphi_1(B\psi) + i\varphi_2(B\psi)}_{=|z|^2 z} - G\odot B\psi\big]$$
$$- i4\,\mathrm{Im}\, B^T\big[\varphi_1(B\psi) + i\varphi_2(B\psi) - G\odot B\psi\big]$$

$$= \frac12\big(\underbrace{\partial_1\varphi(z) + i\partial_2\varphi(z)}_{=:\varphi'(z)}\big)$$

$$= 2\,\overline{B}^T\big[\varphi_3(B\psi) - 2G\odot B\psi\big]$$

mit $\varphi_3(z) = 2|z|^2 z = \varphi(z)\varphi'(z)$, bzw $\varphi_3(a,b) = (2a^3 + 2ab^2, 2a^2b + 2b^3)$

$$= 2\overline{B}^T\big[(\varphi(B\psi) - G)\odot\varphi'(B\psi)\big] = \Big(2\sum_{k=1}^M \overline{B}_{ki}\big(\varphi((B\psi)_k) - G_k\big)\varphi'((B\psi)_k)\Big)_{i=1}^M$$

$$\overset{\varphi=|z|^2}{=} \Big(4\sum_{k=1}^M |(B\psi)_k|^2 - G_k \,\big|\, \underbrace{\overline{B}_{ki}\sum_{l=1}^M B_{kl}\psi_l}\Big)_{i=1}^M$$

$$= \sum_{l=1}^M \underbrace{\overline{B}_{ki} B_{kl}}_{(\overline{b}_k\otimes b_k)_{il}}\psi_l = ((\overline{b}_k\otimes b_k)\psi)_i$$

$$\psi^{k+1} = \mathrm{prox}_{\tau_k R}\big(\psi^k - \tau_k\nabla D(\psi^k)\big)$$

$$\boxed{\begin{array}{l}\varphi(a,b) = a^2 + b^2\\ \Rightarrow \partial_1\varphi = 2a \quad \partial_2\varphi = 2b\end{array}}$$

$$x,y\in\mathbb{C} \Rightarrow xy = (\mathrm{Re}\,x\,\mathrm{Re}\,y - \mathrm{Im}\,x\,\mathrm{Im}\,y) + i(\mathrm{Re}\,x\,\mathrm{Im}\,y + \mathrm{Im}\,x\,\mathrm{Re}\,y)$$

$$\Rightarrow \partial_{\mathrm{Re}\,y}(xy) = \mathrm{Re}\,x + i\,\mathrm{Im}\,x = x$$

$$\partial_{\mathrm{Im}\,y}(xy) = -\mathrm{Im}\,x + i\,\mathrm{Re}\,x = i^2\,\mathrm{Im}\,x + i\,\mathrm{Re}\,x = ix$$

$$\varphi(z) := |z| = \sqrt{\mathrm{Re}\,z^2 + \mathrm{Im}\,z^2} \Rightarrow \partial_{1/2}\varphi(z) = \frac{1}{2|z|}\,2\,\mathrm{Re}\,z \quad (\mathrm{Im}) \Rightarrow \varphi'(z) = \frac{z}{|z|} \text{ für } z\neq 0$$

$$\Rightarrow \varphi(z)\cdot\varphi'(z) = |z|\cdot\frac{z}{|z|} = z$$

$$\sum_{j=1}^{K} \|\varphi(A_j\varphi) - G_j\|_2^2 = \sum_{j=1}^{K}\sum_{i=1}^{N}\Big(\underbrace{|(A_j\varphi)_i|}_{=(A_j^{i}\,,\,\varphi)}{}^2 - G_j\Big)^2$$

A_j^{i}: i-th row of A_j

$$\varphi'(z) = \frac{z}{|z|}$$

$$\Rightarrow |\varphi'(z)-\varphi'(y)| \overset{z,y\neq 0}{=} \left|\frac{z}{|z|}-\frac{y}{|y|}\right| = \frac{\big||y|z-|z|y\big|}{|z||y|} = \frac{\big||y|z-|z|z+|z|z-|z|y\big|}{|z||y|}$$

$$\le \frac{\overbrace{\big||y|-|z|\big|}^{\text{reverse }\Delta\text{-ineq.}\ \le |y-z|}|z| + |z||z-y|}{|z||y|} = \frac{\big||y|-|z|\big|}{|y|} + \frac{|z-y|}{|y|} \le \frac{2}{|y|}|z-y|$$

$\rightarrow$ Problem bei $y=0$:

Vertauschen der Rollen von z,y liefert $|\varphi'(z)-\varphi'(y)| \le \frac{2}{|z|}|y-z|$ $\Rightarrow |\varphi'(z)-\varphi'(y)| \le \frac{2}{\max(|y|,|z|)}|z-y|$

$$|\varphi'(\varepsilon z)-\varphi'(-\varepsilon z)| = \left|\frac{\cancel{\varepsilon} z}{|\cancel{\varepsilon} z|} - \frac{-\cancel{\varepsilon} z}{|-\cancel{\varepsilon} z|}\right| = \left|\frac{2z}{|z|}\right| = 2 \ge L\,|\varepsilon z-(-\varepsilon z)|$$

für jedes L falls $\varepsilon(L)$ klein genug.

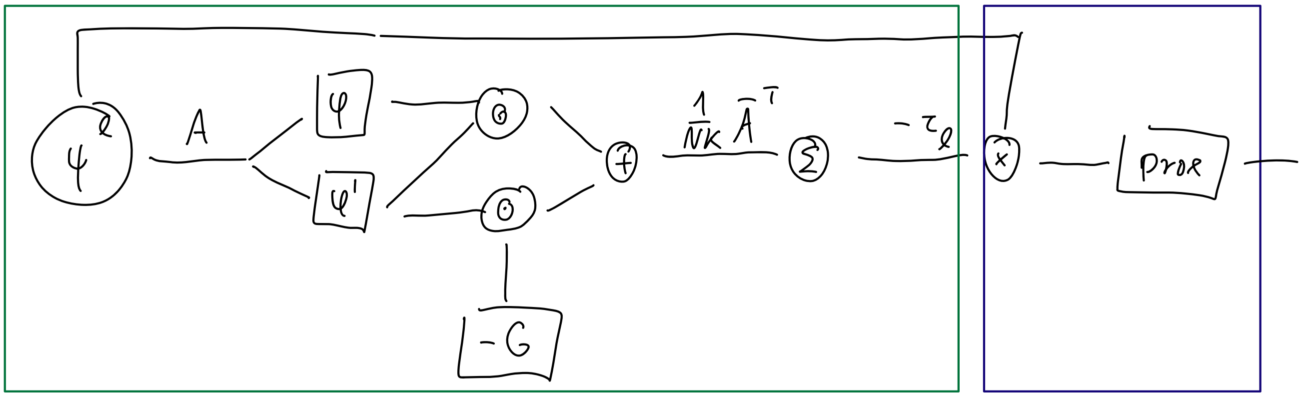
$$f_{\bar z} = \frac{1}{2}(f_x + i f_y), \quad f_z = \frac{1}{2}(f_x - i f_y)$$

f reell $\Rightarrow \overline{f_z} = f_{\bar z}$

$$\varphi(z) = \begin{cases} \frac{1}{2}|z|^2 & |z|\le\delta \\ \delta\left(|z|-\frac{1}{2}\delta\right) & \text{else} \end{cases}, \quad \varphi'(z) = \begin{cases} z & |z|\le\delta \\ \delta\frac{z}{|z|} & \text{else} \end{cases}$$

$$\Rightarrow \varphi'(z)\varphi(z) = \begin{cases} \frac{1}{2}|z|^2 z & |z|\le\delta \\ \delta^2\left(z-\frac{1}{2}\delta\frac{z}{|z|}\right) & \text{else} \end{cases},$$

$\delta = 1$ combines WF + RWF



ψ^ℓ
A
ψ
ψ'
$-G$
$\frac{1}{NK}\bar{A}^T$
Σ
$-\tau_\ell$
prox