Tutorial 2

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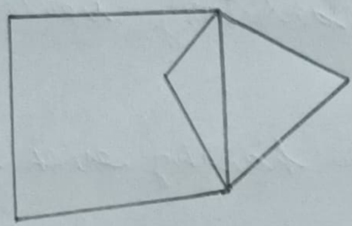
S4CSE
23

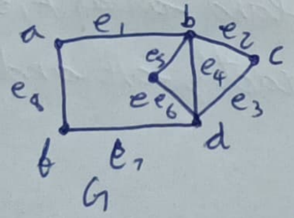
1) Consider the graph G given below



Define Euler graph. Is G an Euler? If yes, write an Euler line from G?

A). Euler line is a closed walk which contains all the edges of a graph.

- A graph is said to be Euler if it contains Euler line.



- Here all the vertices in G has even degree, hence it is an Euler graph.

- Euler line:
  $c\, e_3\, d\, e_7\, f\, e_8\, a\, e_1\, b\, e_5\, e\, e_6\, d\, e_4\, b\, e_2\, c$

2) For a Eulerian graph G, prove the following properties: i) The degree of each vertex of G is even. ii) G is an edge - disjoint union of Cycles.

A). Let G be the Eulerian graph,

1) ∵ G is a Euler graph, ∴ it contains an Euler line.

· If we consider a vertex v in this closed walk, when the walk meets v it goes through two new edges incident on v one for entering and one for exit

. This is true for any arbitary vertex and also for terminal vertex.

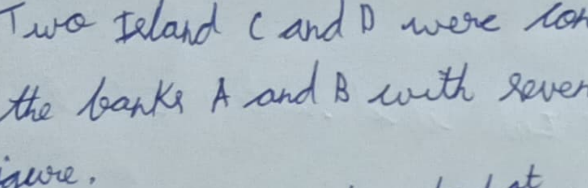
∴ All the vertices are having even degree.

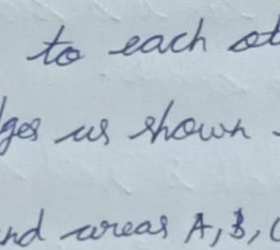
3) Discuss the Konigsberg Bridge problem. Is there any solution to the problem? Justify your answer

A). Two Island C and D were connected to each other and to the banks A and B with seven bridges as shown in the figure.

· The problem was to start at any land areas A, B, C, D. walk over each of the seven bridges exactly one and return to the starting point.

· Euler represented this problem by means of a graph, vertices represent the land areas and the edges represents the bridges.

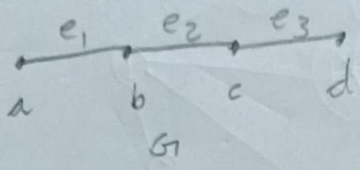




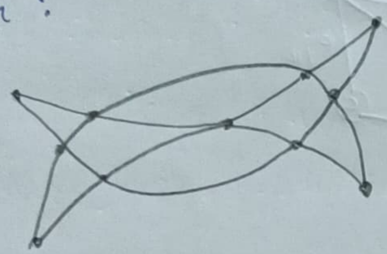
· The solution doesnot exists as we cannot form an Euler line, ∵ all the vertices are not having even degrees.

4) Draw a graph that has a Hamiltonian path but doesnot have a Hamiltonian circuit?

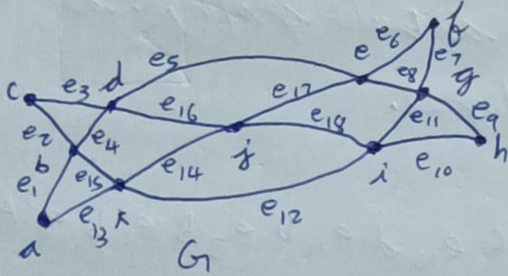
A)



5) Define Euler graph. Check whether the graph is an euler graph or not. If yes, give the Euler line and justify your answer?



A)

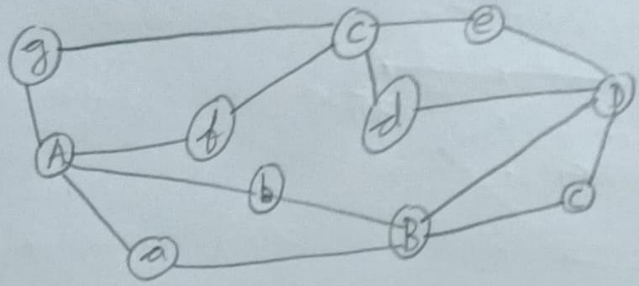


· Yes, the graph G is Euler, ∵ all the vertices are having even degree.

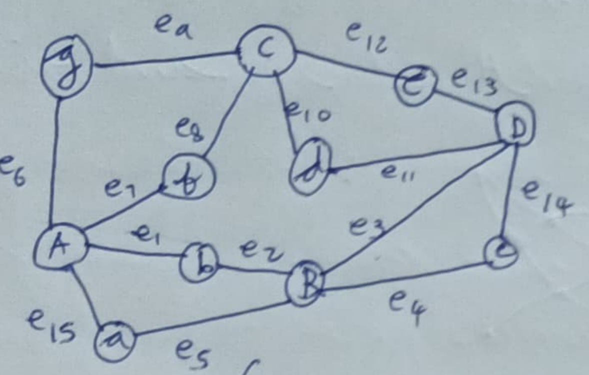
· Euler line:-

$j\ e_{17}\ e\ e_8\ f\ e_7\ g\ e_{11}\ i\ e_{12}\ k\ e_{15}\ b\ e_2\ c\ e_3\ d\ e_{16}\ j\ e_{18}\ i$
$e_{10}\ h\ e_9\ g\ e_8\ e\ e_5\ d\ e_4\ b\ e_1\ a\ e_{13}\ k\ e_{14}\ j$

6) Check whether the given graph is an Euler graph and if yes, give the Euler line. Justify your answer?



A)

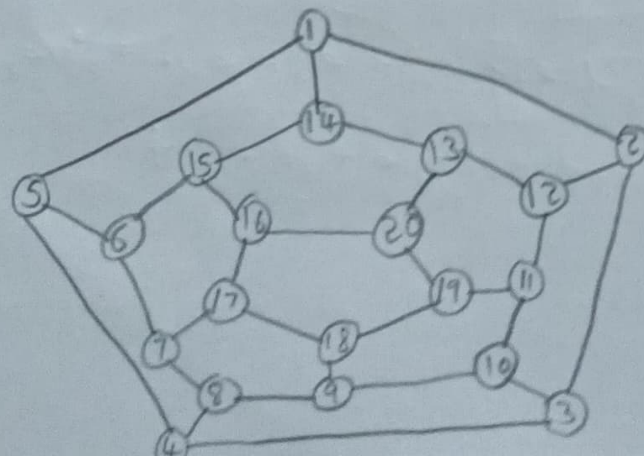


G

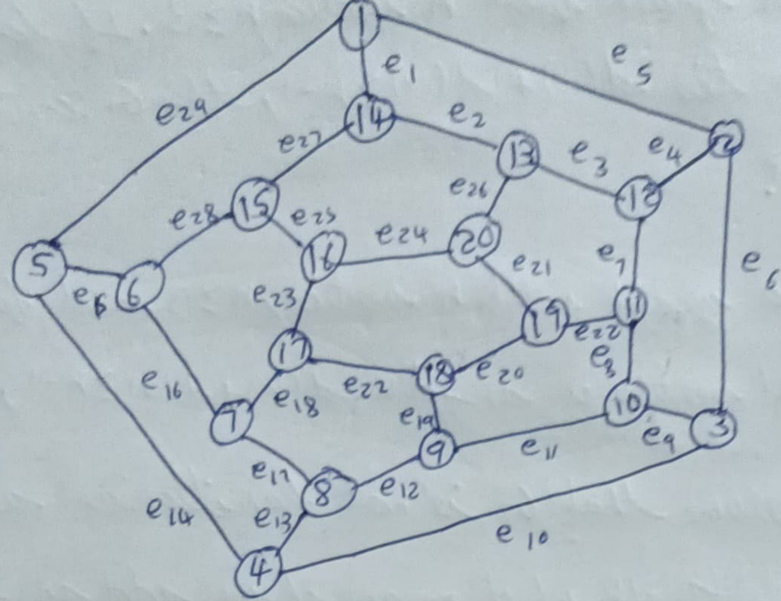
- The Given Graph G is an Euler graph, ∵ all the vertices are having even degree.
- Euler line:

$A\ e_7\ f\ e_8\ C\ e_{10}\ d\ e_{11}\ D\ e_3\ B\ e_2\ b\ e_1\ A\ e_6\ g\ e_9\ C$
$e_{12}\ e\ e_{13}\ D\ e_{14}\ c\ e_4\ B\ e_5\ a\ e_{15}\ A$

7) Give the Hamiltonian circuit of the following graph?



A)



. Hamiltonian Circuit :-

$1\, e_5\, 2\, e_6\, 3\, e_9\, 10\, e_{11}\, 9\, e_{12}\, 8\, e_{13}\, 4\, e_{14}\, 5\, e_{15}\, 6\, e_{16}\, 7\, e_{18}\, 17\, e_{22}\, 18$

$e_{20}\, 19\, e_{22}\, 11\, e_7\, 12\, e_3\, 13\, e_{26}\, 20\, e_{24}\, 16\, e_{25}\, 15\, e_{27}\, 14\, e_1\, 1$

8) Let G be a graph with exactly two connected components, both being Euler. What is the minimum number of edges that need to be added to G to obtain an Euler graph?

A). Let G be the graph, C_1 and C_2 be the two connected Eulerian components.

. Let the vertices $v_1 \in C_1$, $v_2 \in C_2$, $v_3 \in C_2$ and $v_2 v_3 \notin E(G)$ then creating the edges $v_1 v_2$, $v_1 v_3$, $v_2 v_3$ will get G to be Euler as all the vertices are having even degree.

∴ Minimum no; of edges that need to be added = $\underline{3}$

9) Let G be a graph with n vertices, $n \geq 3$. If for all non adjacent vertices $d(u) + d(v) \geq n-1$, then G contains an Hamiltonian path?

A). Let G be a graph with n vertices, $n \geq 3$ and for all non adjacent vertices u and v $d(u)+d(v) \geq n-1$,

. Let us assume that G is not hamiltonian and we assume G is a maximal graph, which means if we add an edge to G it becomes hamiltonian.

. Let u and v be two non-adjacent vertices in G,

. Let $d(u) = \underline{k}$, $\Rightarrow d(v) \geq n-k-1 \rightarrow ①$

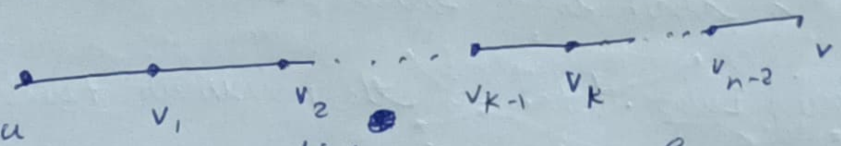


fig 1

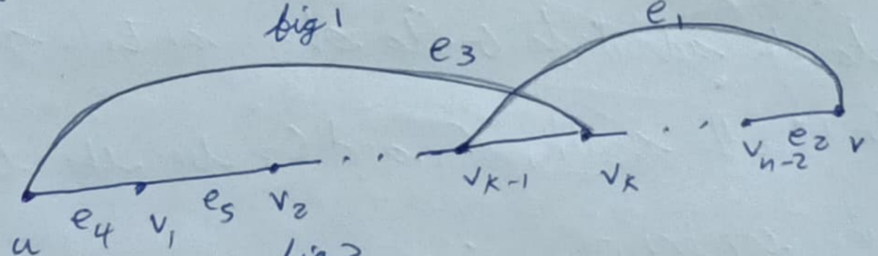


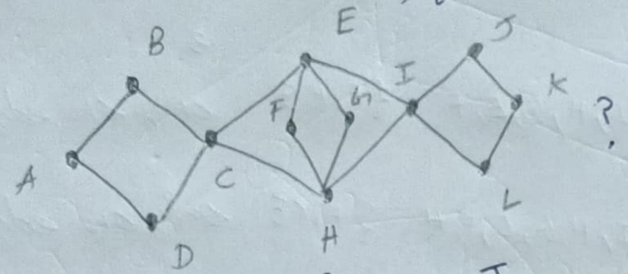
fig 2

. If u is adjacent to v_k then v cannot be adjacent to v_{k-1}, because if v_{k-1} is adjace to v_k then we can form a hamiltonian circuit $v_{k-1}\, e_1\, v\, e_2\, v_{n-2} \ldots v_k\, e_3\, u\, e_4\, v_1\, e_5\, v_2 \ldots v_{k-1}$ [from fig2]

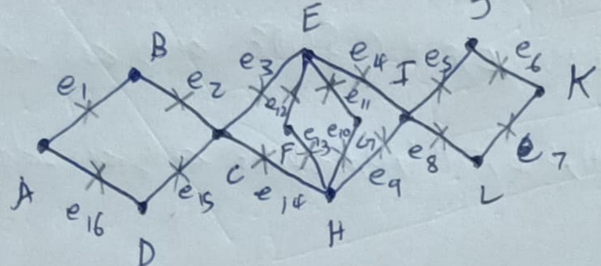
$\therefore$ if v is not adjacent to atleast k of $n-1$ vertices.

$\therefore d(v) \leq n-1-k$, but $d(v) \geq n-k-1$ [from ①] which is a contradiction, hence G is Hamiltonian, $\therefore G$ contains a Hamiltonian path.

10) Using Fleury's algorithm, find Euler line in the graph



A)



Current Path	Next Edge	Reasoning
π: A	{A, B}	No edge in A is a bridge. Choose any one
π: A, B	{B, C}	Only one edge remains from B
π: A, B, C	{C, E}	{C, D} is a bridge choose {C, E} or {C, H}
π: A, B, C, E	{E, I}	No edge in E is a bridge choose any one
π: A, B, C, E, I	{I, J}	No edge from I is a bridge choose any one
π: A, B, C, E, I, J	{J, K}	Only one edge from J remains
π: A, B, C, E, I, J, K	{K, L}	only on edge from K remains
π: A, B, C, E, I, J, K, L	{L, I}	only one edge from L remains
π: A, B, C, E, I, J, K, L, I	{I, H}	only one edge from I remains
π: A, B, C, E, I, J, K, L, I, H	{H, G}	~~No edge~~ {H, C} is a bridge, ∴ choose {H, F} or {H, G}
π: A, B, C, E, I, J, K, L, I, H, G	{G, E}	Only one edge from G remains

π: A, B, C, D, E, I, J, K, L, I, H, G, E   {E, F}   only one edge from E remai

π: A, B, C, D, E, I, J, K, L, I, H, G, E, F   {F, H}   only one edge from F remains

π: A, B, C, D, E, I, J, K, L, I, H, G, E, F, H   {H, C}   only one edge from H remain

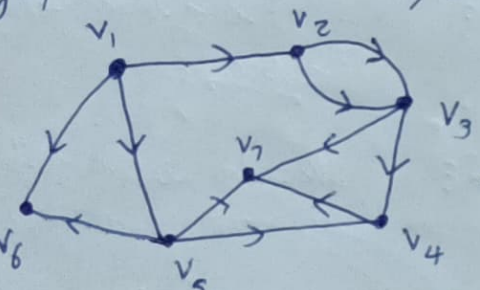
π: A, B, C, D, E, I, J, K, L, I, H, G, E, F, H, C   {C, D}   only one edge from C rema

π: A, B, C, D, E, I, J, K, L, I, H, G, E, F, H, C, D, A   {D, A}   only one edge from D remains

· Euler line:

$A\, e_1\, B\, e_2\, C\, e_3\, D\, e_4\, I\, e_5\, J\, e_6\, K\, e_7\, L\, e_8\, I\, e_9\, H\, e_{10}\, G\, e_{11}\, E\, e_{12}\, F\, e_{13}\, H$
$e_{14}\, C\, e_{15}\, D\, e_{16}\, A$

11) Find the in-degrees and out-degrees of the vertices of the digraphe shown below. Also, verify the handshaking dilemma?



A) $d^+(v_6) = 0$, $d^+(v_1) = 3$, $d^+(v_2) = 2$, $d^+(v_3) = 2$

$d^-(v_6) = 2$, $d^-(v_1) = 0$, $d^-(v_2) = 1$, $d^-(v_3) = 2$

, $d^+(v_4) = 1$, $d^+(v_5) = 3$, $d^+(v_6) = 0$, $d^+(v_7) = 0$

$d^-(v_4) = 2$ $d^-(v_5) = 1$ $d^-(v_6) = 2$ $d^-(v_7) = 3$

Sum of Indegrees $= d^-(v_1) + d^-(v_2) + d^-(v_3) + d^-(v_4) + d^-(v_5)$
$+ d^-(v_6) + d^-(v_7) = 0 + 1 + 2 + 2 + 1 + 2 + 3 = \underline{11}$

Sum of outdegrees $= d^+(v_1) + d^+(v_2) + d^+(v_3) + d^+(v_4) + d^+(v_5) + d^+(v$
$+ d^+(v_7) = 3 + 2 + 2 + 1 + 3 + 0 + 0$
$= \underline{11}$

∴ Sum of Indegrees = Sum of Outdegrees , hence verified the Handshaking dilemma.

2) ii) . G is Euler graph :-

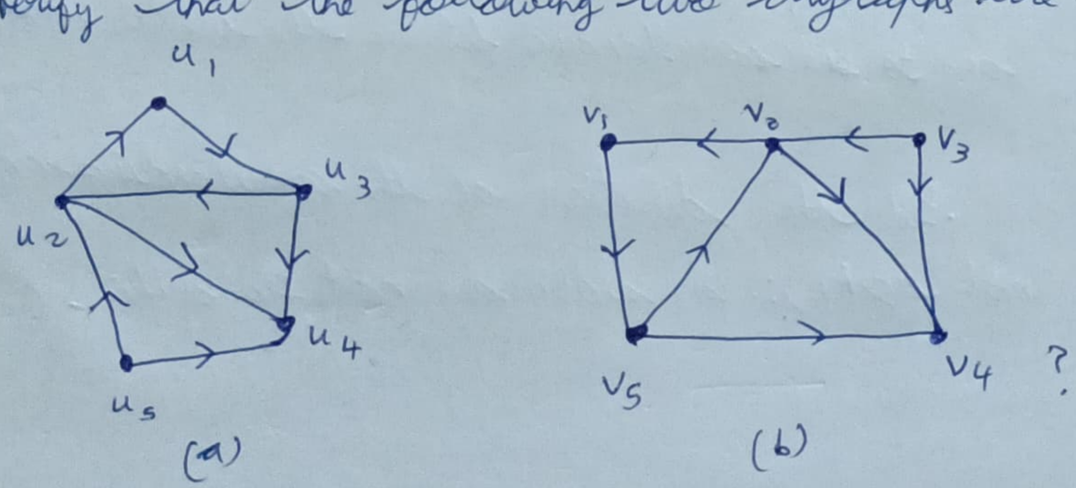
∴ All the vertices are having even degree.

. Let us consider a vertex v_1, ∵ it is having even degree it is adjacent to vertex v_2, ∴ v_2 is also having even degre there must be atleast one edge between v_2 and v_3 proceeding like this we end up at v_1, thus forming a circuit C. Remove C from G.

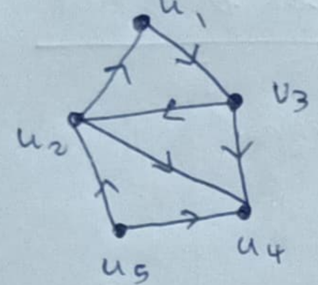
∴ After removal all the vertices in G are of even degree. (not necessarily connected). Again form a circuit C', remove it in the same manner as we removed C. Repeat until no edges are left.

∴ G is an edge-disjoint union of cycles.

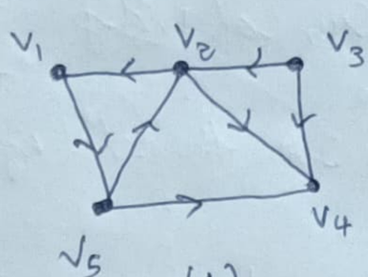
12) Verify that the following two digraphs are isomorphic



d)

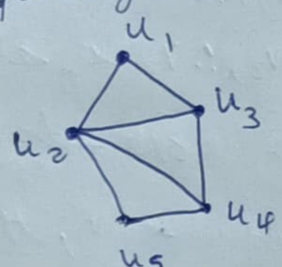


(a)

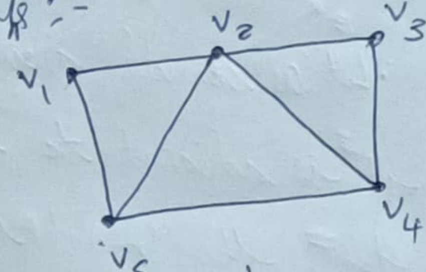


(b)

· Corresponding Undirected graphs :-



G_1'



G_2'

· Vertex correspondence :-

$u_2 \to v_2$

$u_3 \to v_4$

$u_4 \to v_5$

$u_5 \to v_1$

$u_1 \to v_3$

· Edge correspondence :-

$u_2u_3 \to v_2v_4$, $u_2u_5 \to v_2v_1$

$u_2u_4 \to v_2v_5$, $u_2u_1 \to v_2v_3$

$u_5u_4 \to v_1v_5$, $u_1u_3 \to v_3v_4$

∴ G_1' and G_2' are isomorphic ∵ there is one to one correspondence between the vertices and edges.

· But the directions of the corresponding edges are not agree, ∴ (a) and (b) are not Isomorphic.

13) Prove that a complete symmetric diagraph of n vertices contains $n(n-1)$ edges and a complete assymetric digraph of n vertices contains $n(n-1)/2$?

A). Let us assume a complete Undirected graph G with n vertices,

∴ we know that maximum number of edges

$$= \frac{n(n-1)}{2}$$

- Replace all the unidirectional edge with directional edges, ∵ we know that assymmetric digraph have atmost one directed edge between a pair of vertices.

Hence no: of edges in a complete assymetric digraph =

$$\frac{n(n-1)}{2}$$

- We know that diagraphs in which for every edge (a,b) there is also an edge (b,a), ∴ replace all the edges in the Undirected graph G with two directed edges.

Hence the no: of edges in a compalete symetric digraph

$$= 2 * \frac{(n*(n-1))}{2} = n(n-1)$$