Gaing, Z.-L.: Particle Swarm Optimization to Solving the Economic Dispatch Considering the Generator Constraints. IEEE Tans. on Power Syst. 18(3), 1187-1195

Article *in* Power Systems, IEEE Transactions on · September 2003

DOI: 10.1109/TPWRS.2003.814889 · Source: IEEE Xplore

1 author:

Zwe-Lee Gaing
Kao Yuan University

Particle Swarm Optimization to Solving the Economic Dispatch Considering the Generator Constraints

Zwe-Lee Gaing

Abstract—**This paper proposes a particle swarm optimization (PSO) method for solving the economic dispatch (ED) problem in power systems. Many nonlinear characteristics of the generator, such as ramp rate limits, prohibited operating zone, and nonsmooth cost functions are considered using the proposed method in practical generator operation. The feasibility of the proposed method is demonstrated for three different systems, and it is compared with the GA method in terms of the solution quality and computation efficiency. The experimental results show that the proposed PSO method was indeed capable of obtaining higher quality solutions efficiently in ED problems.**

Index Terms—**Economic dispatch, genetic algorithm, particle swarm optimization, prohibited operating zone.**

I. Introduction

Economic dispatch (ED) problem is one of the fundamental issues in power system operation. In essence, it is an optimization problem and its objective is to reduce the total generation cost of units, while satisfying constraints. Previous efforts on solving ED problems have employed various mathematical programming methods and optimization techniques. These conventional methods include the lambda-iteration method, the base point and participation factors method, and the gradient method. [1], [2], [18], [19]. In these numerical methods for solution of ED problems, an essential assumption is that the incremental cost curves of the units are monotonically increasing piecewise-linear functions. Unfortunately, this assumption may render these methods infeasible because of its nonlinear characteristics in practical systems. These nonlinear characteristics of a generator include discontinuous prohibited zones, ramp rate limits, and cost functions which are not smooth or convex. Furthermore, for a large-scale mixed-generating system, the conventional method has oscillatory problem resulting in a longer solution time. A dynamic programming (DP) method for solving the ED problem with valve-point modeling had been presented by [1], [2]. However, the DP method may cause the dimensions of the ED problem to become extremely large, thus requiring enormous computational efforts.

In order to make numerical methods more convenient for solving ED problems, artificial intelligent techniques, such as the Hopfield neural networks, have been successfully employed to solve ED problems for units with piecewise quadratic fuel cost functions and prohibited zones constraint [3], [4]. However, an unsuitable sigmoidal function adopted in the Hopfield model may suffer from excessive numerical iterations, resulting in huge calculations.

In the past decade, a global optimization technique known as genetic algorithms (GA) or simulated annealing (SA), which is a form of probabilistic heuristic algorithm, has been successfully used to solve power optimization problems such as feeder reconfiguration and capacitor placement in a distribution system [1], [5]–[7]. The GA method is usually faster than the SA method because the GA has parallel search techniques, which emulate natural genetic operations. Due to its high potential for global optimization, GA has received great attention in solving ED problems. In some GA applications, many constraints including network losses, ramp rate limits, and valve-point zone were considered for the practicability of the proposed method. Among these, Walters and Sheble presented a GA model that employed units' output as the encoded parameter of chromosome to solve an ED problem for valve-point discontinuities [5]. Chen and Chang presented a GA method that used the system incremented cost as encoded parameter for solving ED problems that can take into account network losses, ramp rate limits, and valve-point zone [8]. Fung *et al.* presented an integrated parallel GA incorporating simulated annealing (SA) and tabu search (TS) techniques that employed the generator's output as the encoded parameter [9]. For an efficient GA method, Yalcinoz have used the real-coded representation scheme, arithmetic crossover, mutation, and elitism in the GA to solve more efficiently the ED problem, and it can obtain a high-quality solution with less computation time [10].

Though the GA methods have been employed successfully to solve complex optimization problems, recent research has identified some deficiencies in GA performance. This degradation in efficiency is apparent in applications with highly *epistatic* objective functions (i.e., where the parameters being optimized are highly correlated) [the crossover and mutation operations cannot ensure better fitness of offspring because chromosomes in the population have similar structures and their average fitness is high toward the end of the evolutionary process] [11], [16]. Moreover, the premature convergence of GA degrades its performance and reduces its search capability that leads to a higher probability toward obtaining a local optimum [11].

Particle swarm optimization (PSO), first introduced by Kennedy and Eberhart, is one of the modern heuristic algorithms. It was developed through simulation of a simplified social system, and has been found to be robust in solving continuous nonlinear optimization problems [13]–[17]. The PSO

Manuscript received February 12, 2003. This work was supported by the National Science Council of Taiwan, R.O.C.

The author is with the Electrical Engineering Department, Kao-Yuan Institute of Technology, Kaohsiung 821, Taiwan, R.O.C. (e-mail: zlgaing@ms39.hinet.net).

Digital Object Identifier 10.1109/TPWRS.2003.814889

technique can generate high-quality solutions within shorter calculation time and stable convergence characteristic than other stochastic methods [14]–[17]. Although the PSO seems to be sensitive to the tuning of some weights or parameters, many researches are still in progress for proving its potential in solving complex power system problems [16]. Researchers including Yoshida *et al.* have presented a PSO for reactive power and voltage control (VVC) considering voltage security assessment. The feasibility of their method is compared with the reactive tabu system (RTS) and enumeration method on practical power system, and has shown promising results [18]. Naka *et al.* have presented the use of a hybrid PSO method for solving efficiently the practical distribution state estimation problem [19].

In this paper, a PSO method for solving the ED problem in power system is proposed. The proposed method considers the nonlinear characteristics of a generator such as ramp rate limits and prohibited operating zone for actual power system operation. The feasibility of the proposed method was demonstrated for three different systems [8], [20], respectively, as compared with the real-coded GA method in the solution quality and computation efficiency.

II. Problem Description

The ED is one subproblem of the unit commitment (UC) problem. It is a nonlinear programming optimization one. Practically, while the scheduled combination units at each specific period of operation are listed, the ED planning must perform the optimal generation dispatch among the operating units to satisfy the system load demand, spinning reserve capacity, and practical operation constraints of generators that include the ramp rate limit and the prohibited operating zone [12].

A. Practical Operation Constraints of Generator

For convenience in solving the ED problem, the unit generation output is usually assumed to be adjusted smoothly and instantaneously. Practically, the operating range of all online units is restricted by their ramp rate limits for forcing the units operation continually between two adjacent specific operation periods [1], [2]. In addition, the prohibited operating zones in the input-output curve of generator are due to steam valve operation or vibration in a shaft bearing. Because it is difficult to determine the prohibited zone by actual performance testing or operating records, the best economy is achieved by avoiding operation in areas that are in actual operation. Hence, the two constraints of generator operation must be taken into account to achieve true economic operation.

1) Ramp Rate Limit: According to [3], [5], and [8], the inequality constraints due to ramp rate limits for unit generation changes are given

1) as generation increases

$$P_i - P_i^0 \le UR_i \tag{1}$$

2) as generation decreases

$$P_i - P_i^0 \le DR_i \tag{2}$$

where P_i is the current output power, and P_i^0 is the previous output power. UR_i is the upramp limit of the i-th generator (MW/time-period); and DR_i is the downramp limit of the i-th generator (MW/time period).

2) Prohibited Operating Zone: References [2], [3], and [8] have shown the input-output performance curve for a typical thermal unit with many valve points. These valve points generate many prohibited zones. In practical operation, adjusting the generation output P_i of a unit must avoid unit operation in the prohibited zones. The feasible operating zones of unit i can be described as follows:

$$\begin{aligned} P_i^{\min} &\le P_i \le P_{i,1}^l \\ P_{i,j-1}^u &\le P_i \le P_{i,j}^l,\ j = 2,3,\ldots,n_i \\ P_{i,n_i}^u &\le P_i \le P_i^{\max} \end{aligned} \tag{3}$$

where j is the number of prohibited zones of unit i.

B. Objective Function

The objective of ED is to simultaneously minimize the generation cost rate and to meet the load demand of a power system over some appropriate period while satisfying various constraints. To combine the above two constraints into a ED problem, the constrained optimization problem at specific operating interval can be modified as

$$\min F_t = \sum_{i=1}^{m} F_i(P_i) = \sum_{i=1}^{m} \alpha_i + \beta_i P_i + \gamma_i P_i^2. \tag{4}$$

Constraints

i) power balance

$$\sum_{i=1}^{m} P_i = P_D + P_L, i = l, \ldots, m. \tag{5}$$

ii) generator operation constraints

$$\max(P_i^{\min}, P_i^0 - DR_i) \le P_i \le \min(P_i^{\max}, P_i^0 + UR_i) \text{ and} \tag{6}$$

$$P_i \in \begin{cases} P_i^{\min} \le P_i \le P_{i,1}^l \\ P_{i,j-1}^u \le P_i \le P_{i,j}^l,\ j = 2,3,\ldots,n_i,\ i = l,\ldots,m. \\ P_{i,n_i}^u \le P_i \le P_i^{\max} \end{cases} \tag{7}$$

iii) line flow constraints

$$|P_{Lf,k}| \le P_{Lf,k}^{\max}, k = l, \ldots, L \tag{8}$$

where the generation cost function $F_i(P_i)$ is usually expressed as a quadratic polynomial; a_i, b_i, and c_i are the cost coefficients of the i-th generator; m is the number of generators committed to the operating system; P_i is the power output of the i-th generator; $P_{Lf,k}$ is the real power flow of line j; k is the number of transmission lines; and the total transmission network losses is a function of unit power outputs that can be represented using $\boldsymbol{B}$ coefficients

$$P_L = \sum_{i=1}^{m} \sum_{j=1}^{m} P_i B_{ij} P_j + \sum_{i=1}^{m} B_{0i} P_i + B_{00}. \tag{9}$$

III. Overview of Particle Swarm Optimization

In 1995, Kennedy and Eberhart first introduced the PSO method [13], motivated by social behavior of organisms such as fish schooling and bird flocking. PSO, as an optimization tool, provides a population-based search procedure in which individuals called particles change their positions (states) with time. In a PSO system, particles fly around in a multidimensional search space. During flight, each particle adjusts its position according to its own experience, and the experience of neighboring particles, making use of the best position encountered by itself and its neighbors. The swarm direction of a particle is defined by the set of particles neighboring the particle and its history experience.

Let x and v denote a particle coordinates (position) and its corresponding flight speed (velocity) in a search space, respectively. Therefore, the i-th particle is represented as $x_i = (x_{i1}, x_{i2}, \ldots, x_{id})$ in the ***d***-dimensional space. The best previous position of the i-th particle is recorded and represented as $pbest_i = (pbest_{i1}, pbest_{i2}, \ldots, pbest_{id})$. The index of the best particle among all the particles in the group is represented by the $gbest_d$. The rate of the velocity for particle i is represented as $v_i = (v_{i1}, v_{i2}, \ldots, v_{id})$. The modified velocity and position of each particle can be calculated using the current velocity and the distance from $pbest_{id}$ to $gbest_d$ as shown in the following formulas:

$$\begin{aligned} v_{id}^{(t+1)} =& w \cdot v_{id}^{(t)} + c_1{}^*\mathrm{rand}()^*(pbest_{id} - x_{id}^{(t)}) \\ &+ c_2{}^*\mathrm{Rand}()^*(gbest_d - x_{id}^{(t)}), \end{aligned} \tag{10}$$

$$\begin{aligned} x_{id}^{(t+1)} =& x_{id}^{(t)} + v_{id}^{(t+1)}, i = 1, 2, \ldots, n, \\ d =& 1, 2, \ldots m \end{aligned} \tag{11}$$

where

n	number of particles in a group;
m	number of members in a particle;
t	pointer of iterations (generations);
w	inertia weight factor;
c_1, c_2	acceleration constant;
$\mathrm{rand}(), \mathrm{Rand}()$	uniform random value in the range [0,1];
$v_i^{(t)}$	velocity of particle i at iteration t, $V_d^{\min} \leq v_{id}^{(t)} \leq V_d^{\max}$;
$x_i^{(t)}$	current position of particle i at iteration t.

In the above procedures, the parameter $V^{\max}$ determined the resolution, or fitness, with which regions are to be searched between the present position and the target position. If $V^{\max}$ is too high, particles might fly past good solutions. If $V^{\max}$ is too small, particles may not explore sufficiently beyond local solutions. In many experiences with PSO, $V^{\max}$ was often set at 10–20% of the dynamic range of the variable on each dimension.

The constants c_1 and c_2 represent the weighting of the stochastic acceleration terms that pull each particle toward the $pbest$ and $gbest$ positions. Low values allow particles to roam far from the target regions before being tugged back. On the other hand, high values result in abrupt movement toward, or past, target regions. Hence, the acceleration constants c_1 and c_2 were often set to be 2.0 according to past experiences.

Suitable selection of inertia weight w in (12) provides a balance between global and local explorations, thus requiring less iteration on average to find a sufficiently optimal solution. As originally developed, w often decreases linearly from about 0.9 to 0.4 during a run. In general, the inertia weight w is set according to the following equation:

$$w = w_{\max} - \frac{w_{\max} - w_{\min}}{iter_{\max}} \times iter \tag{12}$$

where $iter_{\max}$ is the maximum number of iterations (generations), and $iter$ is the current number of iterations.

IV. Development of the Proposed Method

In this paper, the process to solve a constrained ED problem using a PSO algorithm was developed to obtain efficiently a high-quality solution within practical power system operation. The PSO algorithm was utilized mainly to determine the optimal generation power of each unit that was submitted to operation at the specific period, thus minimizing the total generation cost. Before employing the PSO method to solve the ED problem, two definitions must be made as follows.

A. Representation of Individual String

For an efficient evolutionary method, the representation of chromosome strings of the problem parameter set is important. In this paper, we adopted the generation power output of each unit is as a gene, and many genes comprise an individual. Each individual within the population represents a candidate solution for solving the ED problem. For example, if there are n units that must be operated to provide power to loads, then the i-th individual Pg_i can be defined as follows:

$$Pg_i = [P_{i1}, P_{i2}, \ldots, P_{id}], i = 1, 2, \ldots, n \tag{13}$$

where n means population size, d is the number of generator, and P_{id} is the generation power output of the d-th unit at i-th individual. The dimension of a population is $n \times d$. These genes in each individual are represented as real values. The matrix representation of a population is as follows: (See the equation at the bottom of the next page).

B. Evaluation Function

We must define the evaluation function f (it is called fitness function in GA) for evaluating the fitness of each individual in the population. In order to emphasize the "best" chromosome and speed up convergence of the iteration procedure, the evaluation value is normalized into the range between 0 and 1. The evaluation function f is adopted as (14). It is the reciprocal of the generation cost function F_{cost} and power balance constraint P_{pbc} as in (4) and (5). This implies if the values of $F_{\mathrm{cost}}(Pg_i)$ and $P_{pbc}(Pg_i)$ of individual Pg_i were small, then its evaluation value would be large

$$f = \frac{1}{F_{\cos t} + P_{pbc}} \tag{14}$$

where

$$F_{\cos t} = 1 + abs\frac{\left(\sum_{i=1}^{n} F_i(P_i) - F_{\min}\right)}{(F_{\max} - F_{\min})} \tag{15}$$

$$P_{pbc} = 1 + \left(\sum_{i=1}^{n} P_i - P_D - P_L \right)^2 \tag{16}$$

$F_{\max}$ maximum generation cost among all individuals in the initial population;

$F_{\min}$ minimum generation cost among all individuals in the initial population.

In order to limit the evaluation value of each individual of the population within a feasible range, before estimating the evaluation value of an individual, the generation power output must satisfy the constraints in (6)–(8). If one individual satisfies all constraints, then it is a feasible individual and F_{cost} has a small value. Otherwise, the F_{cost} value of the individual is penalized with a very large positive constant.

C. Calculation Processes of the Proposed Method

This paper presents a quick solution to the constrained ED problem using the PSO algorithm to search for optimal or near optimal generation quantity of each unit. The search procedures of the proposed method were as shown below.

Step 1 Specify the lower and upper bound generation power of each unit, and calculate $F_{\max}$ and $F_{\min}$. Initialize randomly the individuals of the population according to the limit of each unit including individual dimensions, searching points, and velocities. These initial individuals must be feasible candidate solutions that satisfy the practical operation constraints.

Step 2 To each individual Pg of the population, employ the $\boldsymbol{B}$-coefficient loss formula to calculate the transmission loss P_L.

Step 3 Calculate the evaluation value of each individual Pg_i in the population using the evaluation function f given by **(14)**.

Step 4 Compare each individual's evaluation value with its $pbest$. The best evaluation value among the $pbests$ is denoted as $gbest$.

Step 5 Modify the member velocity v of each individual Pg_i according to **(17)**

$$\begin{aligned} v_{id}^{(t+1)} =& w \cdot v_i^{(t)} + c_1{}^*\text{rand}()^* \left(pbest_{id} - Pg_{id}^{(t)} \right) \\ &+ c_2{}^*\text{Rand}()^* \left(gbest_d - Pg_{id}^{(t)} \right) \\ i =& 1, \ldots, n. d = 1, \ldots m \end{aligned} \tag{17}$$

where n is the population size, m is the number of units, and the w value is set by **(12)**.

Step 6 If $v_{id}^{(t+1)} > V_d^{\max}$, then $v_{id}^{(t+1)} = V_d^{\max}$. If $v_{id}^{(t+1)} < V_d^{\min}$, then $v_{id}^{(t+1)} = V_d^{\min}$.

Step 7 Modify the member position of each individual Pg_i according to **(18)**

$$Pg_{id}^{(t+1)} = Pg_{id}^{(t)} + v_{id}^{(t+1)}. \tag{18}$$

$Pg_{id}^{(t+1)}$ must satisfy the constraints, namely the prohibited operating zones and ramp rate limits, described by **(6) and (7)**, respectively. If $Pg_{id}^{(t+1)}$ violates the constraints, then $Pg_{id}^{(t+1)}$ must be modified toward the near margin of the feasible solution.

Step 8 If the evaluation value of each individual is better than the previous $pbest$, the current value is set to be $pbest$. If the best $pbest$ is better than $gbest$, the value is set to be $gbest$.

Step 9 If the number of iterations reaches the maximum, then go to **Step 10**. Otherwise, go to **Step 2**.

Step 10 The individual that generates the latest $gbest$ is the optimal generation power of each unit with the minimum total generation cost.

V. Numerical Examples and Results

To verify the feasibility of the proposed PSO method, three different power systems were tested. In these examples, the ramp rate limits and prohibited zones of units were taken into account in practical application, so the proposed PSO method was compared with an elitist GA search method [10]. At each sample system, under the same evaluation function

Individual No.	P_{i1}	P_{i2}	$\ldots$	P_{id-1}	P_{id}
1	420.03	150.32	$\ldots$	75.12	45.55
2	390.28	165.39	$\ldots$	80.23	41.93
.	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
.	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
n	412.88	156.84	$\ldots$	78.11	42.78

population

and individual definition, we performed 50 trials using the two proposed methods to observe the variation during the evolutionary processes and to compare their solution quality, convergence characteristic, and computation efficiency.

A reasonable $\boldsymbol{B}$ loss coefficients matrix of power system network was employed to draw the transmission line loss and satisfy the transmission capacity constraints. The software was written in Matlab language and executed on a Pentium III 550 personal computer with 256-MB RAM.

Although the PSO method seems to be sensitive to the tuning of some weights or parameters, according to the experiences of many experiments, the following PSO and real-coded GA parameters can be used [10], [14], [18].

i) **PSO Method**
- $\text{population size} = 100$;
- $\text{generations} = 200$;
- inertia weight factor w is set by (12), where $w_{\max} = 0.9$ and $w_{\min} = 0.4$;
- the limit of change in velocity of each member in an individual was as $V_{P_d}^{\max} = 0.5P_d^{\max}$, $V_{P_d}^{\min} = -0.5P_d^{\min}$;
- acceleration constant $c_1 = 2$ and $c_2 = 2$.

ii) **GA Method**
- $\text{population size} = 100$;
- $\text{generations} = 200$;
- crossover rate $P_c = 0.8$;
- mute rate $P_m = 0.01$;
- crossover parameter $a = 0.5$.

A. Case Study

Example 1: Six-Unit System: The system contains six thermal units, 26 buses, and 46 transmission lines [18]. The load demand is 1263 MW. The characteristics of the six thermal units are given in Tables I and II. In normal operation of the system, the loss coefficients $\boldsymbol{B}$ with the 100-Mva base capacity are as follows: (See equation at the bottom of the page).

In this case, each individual Pg contains six generator power outputs, such as P_1, P_2, P_3, P_4, P_5, and P_6, which are generated randomly. The dimension of the population is equal to 6×100. Through the evolutionary process of the proposed methods, their best solutions are shown in Table III, respectively, that satisfy the system constraints, such as the ramp rate limits and prohibited zones of units. Table IV listed the statistic results that involved the generation cost, evaluation value, and average CPU time.

TABLE I
GENERATING UNIT CAPACITY AND COEFFICIENTS

Unit	P_i^{min}	P_i^{max}	α_i (\$)	β_i (\$/MW)	γ_i (\$/MW^2)
1	100	500	240	7.0	0.0070
2	50	200	200	10.0	0.0095
3	80	300	220	8.5	0.0090
4	50	150	200	11.0	0.0090
5	50	200	220	10.5	0.0080
6	50	120	190	12.0	0.0075

TABLE II
RAMP RATE LIMITS AND PROHIBITED ZONES OF GENERATING UNITS

Unit	P_i^0	URi (MW/h)	DRi (MW/h)	Prohibited zones (MW)
1	440	80	120	[210 240] [350 380]
2	170	50	90	[90 110] [140 160]
3	200	65	100	[150 170] [210 240]
4	150	50	90	[80 90] [110 120]
5	190	50	90	[90 110] [140 150]
6	110	50	90	[75 85] [100 105]

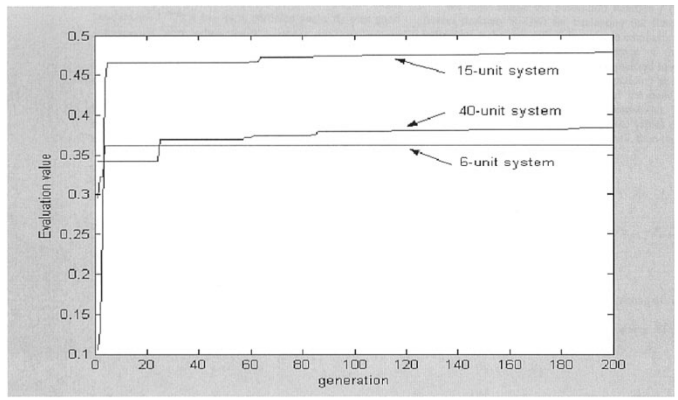

Fig. 1. Convergence property using PSO method.

Example 2: 15-Unit System: The system contains 15 thermal units whose characteristics are given in Tables V and VI [2]. The load demand of the system is 2630 MW. The $\boldsymbol{B}$ loss coefficients matrix was shown in the Appendix.

To simulate this case, each individual Pg contains 15 generator power outputs. The dimension of the population is equal to 15×100. The experimental results are shown in Tables VII and VIII, that also satisfy the system constraints.

Example 3: 40-Unit System: The system consists of 40 units in the realistic Taipower system that is a large-scale and mixed-generating system where coal-fired, oil-fired, gas-fired, diesel, and combined cycle are all present [8]. The load demand of the system is 8550 MW. Owing to the limits of space, the parameters of units and the $\boldsymbol{B}$ loss coefficients matrix cannot be listed.

$$B_{ij} = \begin{bmatrix} 0.0017 & 0.0012 & 0.0007 & -0.0001 & -0.0005 & -0.0002 \\ 0.0012 & 0.0014 & 0.0009 & 0.0001 & -0.0006 & -0.0001 \\ 0.0007 & 0.0009 & 0.0031 & 0.0000 & -0.0010 & -0.0006 \\ -0.0001 & 0.0001 & 0.0000 & 0.0024 & -0.0006 & -0.0008 \\ -0.0005 & -0.0006 & -0.0010 & -0.0006 & 0.0129 & -0.0002 \\ -0.0002 & -0.0001 & -0.0006 & -0.0008 & -0.0002 & 0.0150 \end{bmatrix},$$

$$B_{oi} = 1.0e^{-03*}[\,-0.3908 \quad -0.1297 \quad 0.7047 \quad 0.0591 \quad 0.2161 \quad -0.6635\,],$$

$$B_{oo} = 0.056.$$

TABLE III
BEST SOLUTION OF 6-UNIT SYSTEM (BEST INDIVIDUAL)

Unit power output	PSO method	GA method
$P_1(MW)$	447.4970	474.8066
$P_2(MW)$	173.3221	178.6363
$P_3(MW)$	263.4745	262.2089
$P_4(MW)$	139.0594	134.2826
$P_5(MW)$	165.4761	151.9039
$P_6(MW)$	87.1280	74.1812
Total power output(MW)	1276.01	1276.03
Ploss(MW)	12.9584	13.0217
Total generation cost ($/h)	15,450	15,459

TABLE IV
COMPARISON BETWEEN BOTH METHODS (50 TRIALS)

	Generation Cost ($)			Evaluation Value			
	Max.	Min.	Average	Max.	Min.	Standard Deviation	Average CPU time
PSO	15,492	15,450	15,454	0.3611	0.3602	0.0002	14.89
GA	15,524	15,459	15,469	0.3609	0.1602	0.0570	41.58

TABLE V
GENERATING UNIT DATA

Unit	P_i^{min}	P_i^{max}	α_i ($)	β_i ($/MW)	γ_i ($/MW^2)	URi (MW/h)	DRi (MW/h)	P_i^0
1	150	455	671	10.1	0.000299	80	120	400
2	150	455	574	10.2	0.000183	80	120	300
3	20	130	374	8.8	0.001126	130	130	105
4	20	130	374	8.8	0.001126	130	130	100
5	150	470	461	10.4	0.000205	80	120	90
6	135	460	630	10.1	0.000301	80	120	400
7	135	465	548	9.8	0.000364	80	120	350
8	60	300	227	11.2	0.000338	65	100	95
9	25	162	173	11.2	0.000807	60	100	105
10	25	160	175	10.7	0.001203	60	100	110
11	20	80	186	10.2	0.003586	80	80	60
12	20	80	230	9.9	0.005513	80	80	40
13	25	85	225	13.1	0.000371	80	80	30
14	15	55	309	12.1	0.001929	55	55	20
15	15	55	323	12.4	0.004447	55	55	20

TABLE VI
PROHIBITED ZONES OF GENERATING UNITS

Unit	Prohibited zones (MW)
2	[185 225] [305 335] [420 450]
5	[180 200] [305 335] [390 420]
6	[230 255] [365 395] [430 455]
12	[30 40] [55 65]

TABLE VII
BEST SOLUTION OF 15-UNIT SYSTEM (BEST INDIVIDUAL)

Unit power output	PSO method	GA method
$P_1(MW)$	439.1162	415.3108
$P_2(MW)$	407.9727	359.7206
$P_3(MW)$	119.6324	104.4250
$P_4(MW)$	129.9925	74.9853
$P_5(MW)$	151.0681	380.2844
$P_6(MW)$	459.9978	426.7902
$P_7(MW)$	425.5601	341.3164
$P_8(MW)$	98.5699	124.7867
$P_9(MW)$	113.4936	133.1445
$P_{10}(MW)$	101.1142	89.2567
$P_{11}(MW)$	33.9116	60.0572
$P_{12}(MW)$	79.9583	49.9998
$P_{13}(MW)$	25.0042	38.7713
$P_{14}(MW)$	41.4140	41.9425
$P_{15}(MW)$	35.6140	22.6445
Total power output(MW)	2662.4	2668.4
Ploss(MW)	32.4306	38.2782
Total generation cost ($/h)	32,858	33,113

To simulate this case, each individual Pg contains 40 generator power outputs. The dimension of the population is equal to 40×100. The experimental results and computation time are shown in Tables IX and X.

Fig. 1 shows the convergence tendency of the evaluation values of the above three sample systems using PSO method, respectively. It shows that the PSO has good convergence property, thus resulting in good evaluation value and low generation cost.

TABLE VIII
COMPARISON BETWEEN BOTH METHODS (50 TRIALS)

	Generation Cost ($)			Evaluation Value			
	Max.	Min.	Average	Max.	Min.	Standard Deviation	Average CPU time
PSO	33,331	32,858	33,039	0.4915	0.4619	0.0070	26.59
GA	33,337	33,113	33,228	0.4738	0.4089	0.0087	49.31

TABLE IX
BEST SOLUTION OF 40-UNIT TAIPOWER SYSTEM (BEST INDIVIDUAL)

	PSO method	GA method
Total power output	8637.26	8641.08
Ploss(MW)	87.24	89.76
Total generation cost ($/h)	130,380	135,070

TABLE X
COMPARISON OF BETWEEN BOTH METHODS (50 TRIALS)

	Generation Cost ($)			Evaluation Value			
	Max.	Min.	Average	Max.	Min.	Standard Deviation	Average CPU time
PSO	137,740	130,380	134,970	0.3980	0.3713	0.0077	59.45
GA	137,980	135,070	137,760	0.3690	0.3428	0.0061	81.80

B. Comparison of Two Methods

1) Solution Quality: As seen in Tables IV, VIII, and X, the PSO method can obtain lower average generation cost than the GA method, thus resulting in the higher quality solution. Moreover, through 50 trials, the PSO method yields smaller standard deviation of evaluation values in the ***Examples 1*** and ***2*** that are thermal-unit systems. Though ***Example 3***, a large-scale mixed-generating system with the variety units, generates larger standard deviation, the generation cost of each trial is still lower than the GA. Figs. 2 and 3 showed the distribution outline of the best solution of each trial, almost all generation costs obtained by the PSO method were lower, thus verifying that the PSO method has better quality of solution and convergence characteristic.

2) Dynamic Behaviors Observation: During the evolutionary processing of the two proposed methods, after each iteration, the mean value (μ) and the standard deviation (σ) of the evaluation values of all individuals in the population were recorded for observing the dynamic convergence behavior of the individuals in population. The formulas for calculating the mean value and the standard deviation of evaluation values are as follows, respectively:

$$\mu = \frac{\sum_{i=1}^{n} f(Pg_i)}{n} \tag{19}$$

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(f(Pg_i) - \mu)^2} \tag{20}$$

where $f(Pg_i)$ is the evaluation value of individual Pg_i, and n is the population size.

Fig. 4 displays the recorded data in ***Example 3***. As seen in the simulation, with the same number of iterations, though both proposed methods can obtain stable mean evaluation value (μ) under the same evaluation function and simulation conditions, the GA brings premature convergence such that the evaluation value and mean value are smaller. Conversely,

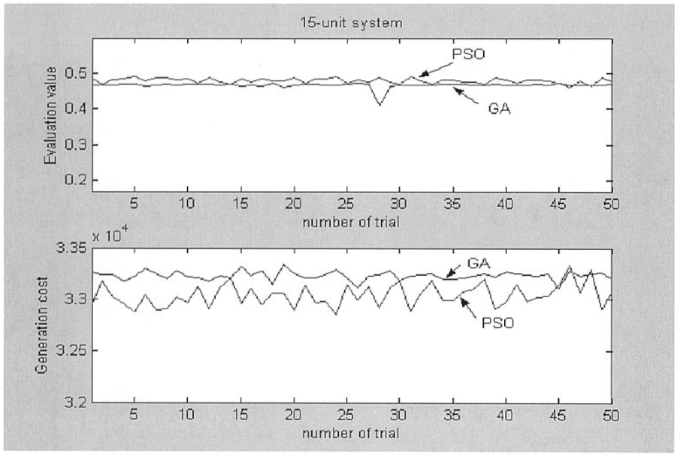

Fig. 2. Distribution of generation cost of both methods (***Example 2***, 50 trials).

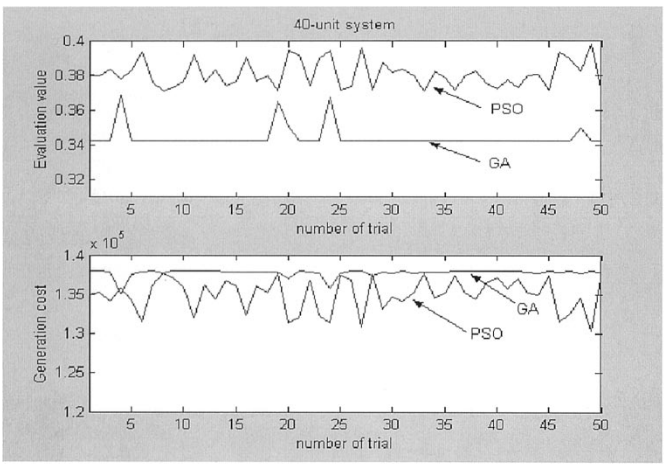

Fig. 3. Distribution of generation cost and evaluation value (***Example 3***, 50 trials).

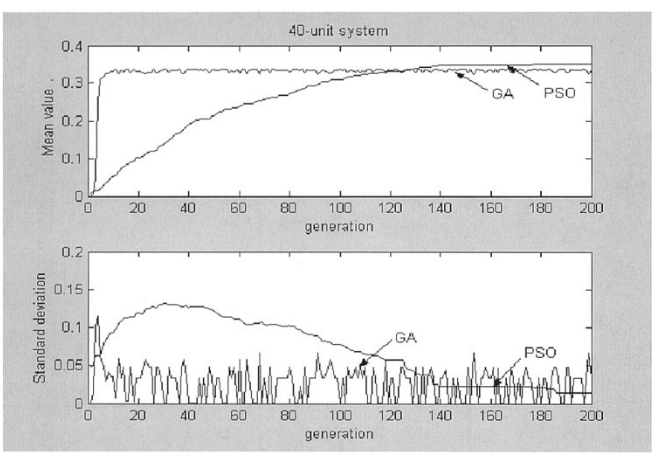

Fig. 4. Tendency of both μ and σ using the two methods in ***Example 3***.

the PSO has better evaluation value and mean value, showing that it can achieve better solution. Simultaneously, we can also find that the convergence tendency of the standard deviation (σ) of evaluation values in the PSO is much faster than the GA, because the latter presented fluctuation resulted from the mutation in the GA method. This can prove that the PSO method has better convergence efficiency in solving the power optimization problems. The results also indicate the drawbacks of the GA method mentioned in [11] and [16].

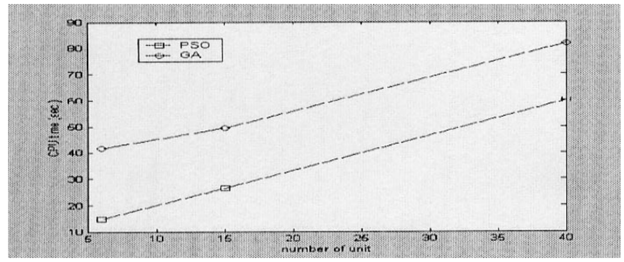

Fig. 5. Comparison of computation performance.

TABLE XI
COMPARISON OF COMPUTATION EFFICIENCY OF TWO METHODS

Example	Method	Generations (iterations)	20	50	100	150	200
6-unit	PSO	Generation cost ($)	15,473	15,467	15,466	15,462	15,458
		CPU time (sec.)	1.53	3.73	7.41	11.15	14.89
		CPU time/ per- iteration (sec.)	0.06	0.06	0.06	0.06	0.06
	GA	Generation cost ($)	15,620	15,615	15,608	15,608	15,605
		CPU time (sec.)	4.34	10.49	21.31	31.53	42.07
		CPU time/ per- iteration (sec.)	0.22	0.22	0.22	0.22	0.22
15-unit	PSO	Generation cost ($)	33,289	33,182	3,3105	33,094	33,049
		Total CPU time (sec.)	2.74	6.71	13.41	19.94	26.70
		CPU time/ per- iteration (sec.)	0.12	0.11	0.12	0.12	0.12
	GA	Generation cost ($)	33,301	33,221	33,202	33,173	33,151
		Total CPU time (sec.)	4.95	12.31	24.72	37.1	49.3
		CPU time per iteration (sec.)	0.27	0.27	0.27	0.27	0.27
40-unit	PSO	Generation cost ($)	136,581	136,279	135,377	135,181	134,980
		CPU time (sec.)	6.10	15.15	31.09	49.60	60.21
		CPU time/ per- iteration (sec.)	0.32	0.31	0.31	0.32	0.32
	GA	Generation cost ($)	137,980	137,910	137,870	137,810	137,810
		CPU time (sec.)	8.20	20.91	41.13	61.90	81.65
		CPU time/ per- iteration (sec.)	0.44	0.44	0.44	0.44	0.44

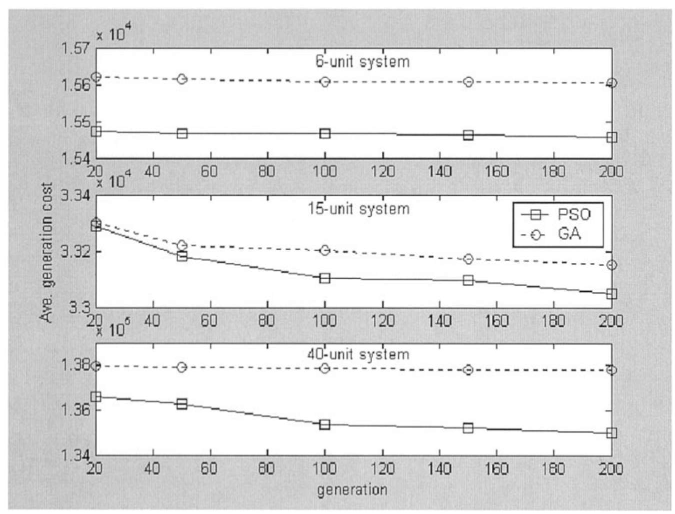

Fig. 6. Average generation cost versus generation (iteration).

3) Computation Efficiency: The comparison of computation efficiency of both methods is shown in Fig. 5. We can find that the PSO method has better computation performance. In addition, each sample system is performed 20 times at different number of iterations (generations) using the two proposed methods. Table XI lists the statistic data that include the average generation costs (average objective function values) of the best individual at different number of iterations with the average CPU time per-iteration for each sample system. Figs. 6 and 7 display the graphs of the average generation cost and average CPU time at different number of iterations, respectively.

$$B_{ij}=\begin{bmatrix}
0.0014 & 0.0012 & 0.0007 & -0.0001 & -0.0003 & -0.0001 & -0.0001 & -0.0001 & -0.0003 & 0.0005 & -0.0003 & -0.0002 & 0.0004 & 0.0003 & -0.0001 \\
0.0012 & 0.0015 & 0.0013 & 0.0000 & -0.0005 & -0.0002 & 0.0000 & 0.0001 & -0.0002 & -0.0004 & -0.0004 & -0.0000 & 0.0004 & 0.0010 & -0.0002 \\
0.0007 & 0.0013 & 0.0076 & -0.0001 & -0.0013 & -0.0009 & -0.0001 & 0.0000 & -0.0008 & -0.0012 & -0.0017 & -0.0000 & -0.0026 & 0.0111 & -0.0028 \\
-0.0001 & 0.0000 & -0.0001 & 0.0034 & -0.0007 & -0.0004 & 0.0011 & 0.0050 & 0.0029 & 0.0032 & -0.0011 & -0.0000 & 0.0001 & 0.0001 & -0.0026 \\
-0.0003 & -0.0005 & -0.0013 & -0.0007 & 0.0090 & 0.0014 & -0.0003 & -0.0012 & -0.0010 & -0.0013 & 0.0007 & -0.0002 & -0.0002 & -0.0024 & -0.0003 \\
-0.0001 & -0.0002 & -0.0009 & -0.0004 & 0.0014 & 0.0016 & -0.0000 & -0.0006 & -0.0005 & -0.0008 & 0.0011 & -0.0001 & -0.0002 & -0.0017 & 0.0003 \\
-0.0001 & 0.0000 & -0.0001 & 0.0011 & -0.0003 & -0.0000 & 0.0015 & 0.0017 & 0.0015 & 0.0009 & -0.0005 & 0.0007 & -0.0000 & -0.0002 & -0.0008 \\
-0.0001 & 0.0001 & 0.0000 & 0.0050 & -0.0012 & -0.0006 & 0.0017 & 0.0168 & 0.0082 & 0.0079 & -0.0023 & -0.0036 & 0.0001 & 0.0005 & -0.0078 \\
-0.0003 & -0.0002 & -0.0008 & 0.0029 & -0.0010 & -0.0005 & 0.0015 & 0.0082 & 0.0129 & 0.0116 & -0.0021 & -0.0025 & 0.0007 & -0.0012 & -0.0072 \\
-0.0005 & -0.0004 & -0.0012 & 0.0032 & -0.0013 & -0.0008 & 0.0009 & 0.0079 & 0.0116 & 0.0200 & -0.0027 & -0.0034 & 0.0009 & -0.0011 & -0.0088 \\
-0.0003 & -0.0004 & -0.0017 & -0.0011 & 0.0007 & 0.0011 & -0.0005 & -0.0023 & -0.0021 & -0.0027 & 0.0140 & 0.0001 & 0.0004 & -0.0038 & 0.0168 \\
-0.0002 & -0.0000 & -0.0000 & -0.0000 & -0.0002 & -0.0001 & 0.0007 & -0.0036 & -0.0025 & -0.0034 & 0.0001 & 0.0054 & -0.0001 & -0.0004 & 0.0028 \\
0.0004 & 0.0004 & -0.0026 & 0.0001 & -0.0002 & -0.0002 & -0.0000 & 0.0001 & 0.0007 & 0.0009 & 0.0004 & -0.0001 & 0.0103 & -0.0101 & 0.0028 \\
0.0003 & 0.0010 & 0.0111 & 0.0001 & -0.0024 & -0.0017 & -0.0002 & 0.0005 & -0.0012 & -0.0011 & -0.0038 & -0.0004 & -0.0101 & 0.0578 & -0.0094 \\
-0.0001 & -0.0002 & -0.0028 & -0.0026 & -0.0003 & 0.0003 & -0.0008 & -0.0078 & -0.0072 & -0.0088 & 0.0168 & 0.0028 & 0.0028 & -0.0094 & 0.1283
\end{bmatrix}$$

$$B_{0i}=\begin{bmatrix} -0.0001 & -0.0002 & 0.0028 & -0.0001 & 0.0001 & -0.0003 & -0.0002 & -0.0002 & 0.0006 & 0.0039 & -0.0017 & -0.0000 & -0.0032 & 0.0067 & -0.0064 \end{bmatrix}$$

$B_{00}=0.0055;$

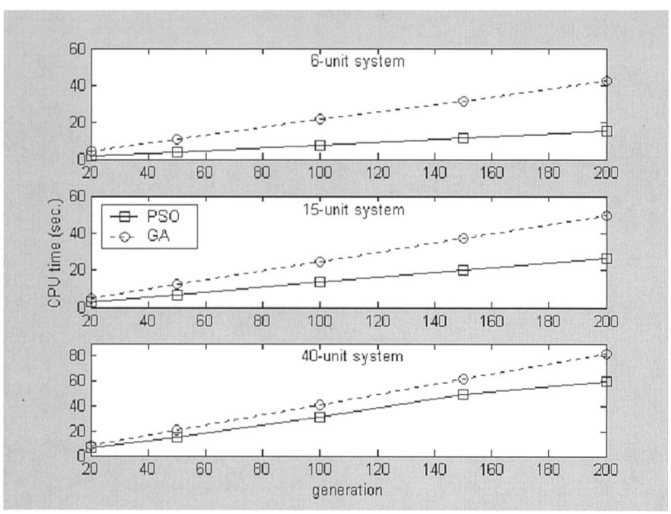

Fig. 7. Average CPU time versus generation (iteration).

As can be seen, because the PSO method does not perform the selection and crossover operations in evolutionary processes, it can save some computation time compared with the GA method, thus these data are evidence of the superior properties of the PSO method.

VI. Discussion and Conclusion

In this paper, we have successfully employed the PSO method to solve the ED problem with the generator constraints. The PSO algorithm has been demonstrated to have superior features, including high-quality solution, stable convergence characteristic, and good computation efficiency. Many nonlinear characteristics of the generator such as ramp rate limits, valve-point zones, and nonsmooth cost functions are considered for practical generator operation in the proposed method. The results show that the proposed method was indeed capable of obtaining higher quality solution efficiently in ED problems.

In addition, in order to verify it being superior to the GA method, many performance estimation schemes are performed, such as

- solution quality and convergence characteristic;
- dynamic convergence behavior of all individuals in population during the evolutionary processing;
- computation efficiency.

It is clear from the results that the proposed PSO method can avoid the shortcoming of premature convergence of GA method and can obtain higher quality solution with better computation efficiency and convergence property.

Appendix

The $\boldsymbol{B}$ loss coefficients matrix of a 15-unit system with a base capacity of 100 Mva is shown as follows: (See equation at the top of the page).

Acknowledgment

The author greatly acknowledges the technical support of Taiwan Power Company.

References

[1] A. Bakirtzis, V. Petridis, and S. Kazarlis, "Genetic algorithm solution to the economic dispatch problem," *Proc. Inst. Elect. Eng.–Gen., Transm. Dist.*, vol. 141, no. 4, pp. 377–382, July 1994.

[2] F. N. Lee and A. M. Breipohl, "Reserve constrained economic dispatch with prohibited operating zones," *IEEE Trans. Power Syst.*, vol. 8, pp. 246–254, Feb. 1993.

[3] C.-T. Su and G.-J. Chiou, "A fast-computation hopfield method to economic dispatch of power systems," *IEEE Trans. Power Syst.*, vol. 12, pp. 1759–1764, Nov. 1997.

[4] T. Yalcinoz and M. J. Short, "Neural networks approach for solving economic dispatch problem with transmission capacity constraints," *IEEE Trans. Power Syst.*, vol. 13, pp. 307–313, May 1998.

[5] D. C. Walters and G. B. Sheble, "Genetic algorithm solution of economic dispatch with valve point loading," *IEEE Trans. Power Syst.*, vol. 8, pp. 1325–1332, Aug. 1993.

[6] K. P. Wong and Y. W. Wong, "Genetic and genetic/simulated — Annealing approaches to economic dispatch," *Proc. Inst. Elect. Eng.*, pt. C, vol. 141, no. 5, pp. 507–513, Sept. 1994.
[7] G. B. Sheble and K. Brittig, "Refined genetic algorithm — Economic dispatch example," *IEEE Trans. Power Syst.*, vol. 10, pp. 117–124, Feb. 1995.
[8] P.-H. Chen and H.-C. Chang, "Large-Scale economic dispatch by genetic algorithm," *IEEE Trans. Power Syst.*, vol. 10, pp. 1919–1926, Nov. 1995.
[9] C. C. Fung, S. Y. Chow, and K. P. Wong, "Solving the economic dispatch problem with an integrated parallel genetic algorithm," in *Proc. PowerCon Int. Conf.*, vol. 3, 2000, pp. 1257–1262.
[10] T. Yalcionoz, H. Altun, and M. Uzam, "Economic dispatch solution using a genetic algorithm based on arithmetic crossover," in *Proc. IEEE Proto Power Tech. Conf.*, Proto, Portugal, Sept. 2001.
[11] D. B. Fogel, *Evolutionary Computation: Toward a New Philosophy of Machine Intelligence*, 2 ed. Piscataway, NJ: IEEE Press, 2000.
[12] K. S. Swarup and S. Yamashiro, "Unit commitment solution methodology using genetic algorithm," *IEEE Trans. Power Syst.*, vol. 17, pp. 87–91, Feb. 2002.
[13] J. Kennedy and R. Eberhart, "Particle swarm optimization," *Proc. IEEE Int. Conf. Neural Networks*, vol. IV, pp. 1942–1948, 1995.
[14] Y. Shi and R. Eberhart, "A modified particle swarm optimizer," *Proc. IEEE Int. Conf. Evol. Comput.*, pp. 69–73, May 1998.
[15] Y. Shi and R. C. Eberhart, "Empirical study of particle swarm optimization," in *Proc. Congr. Evol. Comput.*, NJ, 1999, pp. 1945–1950.
[16] R. C. Eberhart and Y. Shi, "Comparison between genetic algorithms and particle swarm optimization," *Proc. IEEE Int. Conf. Evol. Comput.*, pp. 611–616, May 1998.
[17] P. J. Angeline, "Using selection to improve particle swarm optimization," *Proc. IEEE Int. Conf. Evol. Comput.*, pp. 84–89, May 1998.
[18] H. Yoshida, K. Kawata, Y. Fukuyama, S. Takayama, and Y. Nakanishi, "A particle swarm optimization for reactive power and voltage control considering voltage security assessment," *IEEE Trans. Power Syst.*, vol. 15, pp. 1232–1239, Nov. 2000.
[19] S. Naka, T. Genji, T. Yura, and Y. Fukuyama, "Practical distribution state estimation using hybrid particle swarm optimization," *Proc. IEEE Power Eng. Soc. Winter Meeting*, vol. 2, pp. 815–820, 2001.
[20] H. Saadat, *Power System Analysis*. New York: McGraw-Hill, 1999.

Zwe-Lee Gaing received the M.S. and Ph.D. degrees from the Electrical Engineering Department at National Sun Yat-Sen University, Kaohsiung, Taiwan, R.O.C., in 1992 and 1996, respectively.

Currently, he is an Associate Professor in the Electrical Engineering Department at Kao-Yuan Institute of Technology, Kaohsiung, Taiwan, R.O.C. His research interests are artificial intelligence with application to power system operation and control.