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Economic dispatch using hybrid grey wolf optimizer

T. Jayabarathi [a], T. Raghunathan [a,*], B.R. Adarsh [a], Ponnuthurai Nagaratnam Suganthan [b]

[a] School of Electrical Engineering, VIT University, Vellore 632014, India
[b] EEE Department, Nanyang Technological University, Singapore

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ABSTRACT

This paper presents the application of one of the latest swarm intelligence algorithms, the grey wolf optimizer, for solving economic dispatch problems that are nonlinear, non-convex and discontinuous in nature, with numerous equality and inequality constraints. Grey wolf optimizer is a new metaheuristic algorithm that is loosely based on the behavior of the grey wolves. The optimizer has been hybridized to include crossover and mutation for better performance. Four economic dispatch problems (6, 15, 40, and 80 generators), with prohibited operating zones, valve point loading effect and ramp rate limit constraints have been solved, with and without transmission losses. The losses are calculated using *B*-coefficients. The results obtained are compared with those reported using other methods in the literature. The comparisons show that the hybrid grey wolf optimizer used in this paper either matches or outperforms the other methods.

1. Introduction

Economic dispatch (ED) in power systems is an important, real-world optimization problem that has minimization of generation cost as its objective. Given the importance of ED, solving the ED problem has been attempted from the early 1970's [1]. These early attempts employed classical, gradient based techniques such as lambda-iteration method, gradient method and dynamic programming [2]. However, gradient based methods require that the function to be optimized be differentiable, continuous, and convex, to successfully locate the global optimum. The ED problem has to satisfy a number of constraints including the presence of prohibited operating zones (POZ), valve point loading effect, and ramp rate constraints, which make the problem a non-convex and discontinuous one. These complicating factors gave rise to modifications to the gradient based methods that have continued up to the present. Some of these modifications are a branch and bound method applied to a quadratic programming approach [3], an improved lambda-iteration method using a two stage approach [4], and the use of the concept of decline rate, instead of incremental cost [5].

These complicating factors also led to a huge interest in gradient free, evolutionary computation (EC) or metaheuristic methods of optimization being employed to solve the ED problem. Some of these methods are the evolutionary programming (EP), PSO [9], firefly algorithm, biogeography-based optimization [12], teaching-learning algorithm [13], bee swarm optimization [14], cuckoo search algorithm [15], random drift particle swarm optimization [16], honey bee mating optimization [17], and chaotic bat algorithm [18]. [6]. contains the application of EP to the basic ED problem [7], the application of EP to the ED problem with multiple fuel options, and [8] the application of EP to all the variants of the basic ED problem [10]. has the application of the firefly algorithm to the basic ED problem, and [11] the application of the same firefly algorithm to the reserve constrained ED problem.

Given the criticism that metaheuristic methods are computationally intensive, a hybrid of both gradient based search and gradient free search approach too has been tried [19]. contains a hybrid of the metaheuristic cross-entropy and the gradient based sequential quadratic programming (SQP) [20], has a hybrid of the metaheuristic harmony search and the modified subgradient methods, and [21] has the metaheuristic ant swarm optimization

* Corresponding author. Tel.: +91 9489969616.
E-mail addresses: raghunathan.t@vit.ac.in (T. Raghunathan), nathraghu@yahoo.com (P.N. Suganthan).

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hybridized with SQP.

Gradient free, 'hard computing' methods – distinct from metaheuristic, or soft computing methods – have also been proposed [22]. contains a fully decentralized approach [23], a game-theoretic formulation [24], a distribution auction-based algorithm, and [25] a distribution consensus-based algorithm, for solving the ED problem.

Metaheuristic methods in turn fall into several broad categories like evolutionary algorithms (EA's), swarm intelligence and immune algorithms. Another kind of hybridization to improve the performance of a method is to hybridize the method from one category with operators from another method from a different category. Examples of this approach are the hybrid differential evolution with biogeography-based optimization [26], krill herd [27], hybrid harmony search [28], and PSOGSA [29].

This paper presents one such hybrid algorithm to solve the ED problem. The grey wolf optimizer (GWO), a swarm intelligence algorithm, is hybridized by incorporating the operators of mutation and crossover from EAs, and is referred to as the hybrid GWO (HGWO) hereafter. The main contributions of this paper are improving the performance of GWO and applying it to the economic dispatch (ED) problem. Another contribution of the paper is the use of a self-adaptive penalty approach, to deal with constraints, thereby eliminating ad hoc ways of dealing with constraints. The results obtained are either comparable with or outperform those obtained by other methods in the literature.

This paper is organized as follows: Section II explains the problem formulation, Section III summarizes the basic grey wolf optimizer, Section IV develops the hybrid grey wolf optimizer (HGWO) used in this paper, Section V outlines the constraint handling method adopted in this paper to the ED problem, Section VI applies the HGWO to solve the ED problem, Section VII contains the results and discussion, and the final Section VIII concludes the paper.

2. Problem formulation

The objective function of the ED problem is to minimize the fuel cost of thermal power plants for a given load demand while subject to various constraints.

2.1. Objective function

The cost or objective function of the ED problem is the quadratic fuel cost equation of the thermal generating units, and is given by

$$\min_{P\in R^{N_g}} \quad F=\sum_{j=1}^{N_g} F_j(P_j)=\sum_{j=1}^{N_g}\left(a_j+b_jP_j+c_jP_j^2\right) \tag{1}$$

where N_g is the total number of generating units or generators, $F_j(P_j)$ is the fuel cost in \$/hr, P_j is the power generated in MW, and a_j, b_j and c_j are cost coefficients of jth generator.

Practical generators are subject to valve point loading effect that introduces ripples into the cost function [2]. The objective function when the valve point effect is taken into account becomes

$$\begin{aligned}\min_{P\in R^{N_g}} \quad F&=\sum_{j=1}^{N_g} F_j(P_j)\\ &=\sum_{j=1}^{N_g}\left(a_j+b_jP_j+c_jP_j^2\right) \ +\left| e_j\sin\left(f_j\left(P_j^{\min}-P_j\right)\right)\right|\end{aligned} \tag{2}$$

where e_j and f_j are constants of the valve-point effect of the jth generator.

2.2. Optimization constraints

The equality and inequality constraints for the ED problem are the real power balance criterion, and real power generation limits, given by

$$\sum_{j=1}^{N_g} P_j = P_D + P_L \tag{3}$$

$$P_j^{\min} \le P_j \le P_j^{\max} \tag{4}$$

where P_D is the total power demand, $P_j^{\min}$ and $P_j^{\ \max}$ are the minimum and maximum power generation limits of the jth generator, and P_L represents the line losses given by

$$P_L=\sum_{j=1}^{N_g}\sum_{i=1}^{N_g} P_jB_{ji}P_i \ + \sum_{j=1}^{N_g} B_{0j}P_j + B_{00} \tag{5}$$

P_j and P_i are the real power injection at jth and ith buses, respectively. B_{00}, B_{0j}, B_{ji} are the loss coefficients which can be assumed to be constant under normal operating conditions.

2.3. Practical operating constraints of generators

1) Prohibited operating zones (POZ)

The prohibited zones are due to steam valve operation or vibration in shaft bearing. The feasible operating zones of jth generator can be described as follows

$$P_j\in\begin{cases} P_j^{\min} \le P_j \le P_{j,1}^{\,l} \\ P_{j,\ k-1}^{\,u} \le P_j \le P_{j,k}^{\,l}, \quad k=2,3,\dots n_j, \ \ j=1,2,\dots N_g \\ P_{j,\ n_j}^{\,u} \le P_j \le P_j^{\max} \end{cases} \tag{6}$$

where n_j is the number of prohibited zones of jth generator. $P_{j,k}^{\,l}$, $P_{j,k}^{\,u}$ are the lower and upper power output of the kth prohibited zone of the jth generator, respectively.

2) Ramp Rate Limits

The physical limitations of starting up and shutting down of generators impose ramp rate limits, which are modeled as follows. The increase in generation is limited by

$$P_j - P_j^0 \le \ UR_j \tag{7}$$

Similarly, the decrease is limited by

$$P_j^0 - P_j \le \ DR_j \tag{8}$$

where P_j^0 is the previous output power, UR_j and DR_j are the up-ramp limit and the down-ramp limit respectively, of the jth generator.

Combining (7) and (8) with (4) results in the change of the effective operating or generation limits to

$$\underline{P_j} \ \le P_j \ \le \ \overline{P_j} \tag{9}$$

where

$$\underline{P_j} = \max\left(P_j^{\min},\ P_j^0 - DR_j\right) \tag{10}$$

$$\overline{P_j} = \min\left(P_j^{\max},\ P_j^0 + UR_j\right) \tag{11}$$

Combining this with (6), the ED problem can be formulated as

$$\begin{aligned}
&\min_{P\in R^{N_g}} F = \sum_{j=1}^{N_g} F_j(P_j) = \sum_{j=1}^{N_g}\left(a_j + b_jP_j + c_jP_j^2\right) + \left|e_j \sin\left(f_j\left(P_j^{\min} - P_j\right)\right)\right| \\
&\text{s.t.} \sum_{j=1}^{N_g} P_j = P_D + P_L \\
&\underline{P_j} \le P_j \le P_{j,1}^l \\
&P_{j,k-1}^u \le P_j \le P_{j,k}^l,\ k = 2,3,\ldots n_j, j = 1,2,\ldots N_g \\
&P_{j,n_j}^u \le P_j \le \overline{P_j}
\end{aligned} \tag{12}$$

3. Grey wolf optimizer

Like any other swarm intelligence (SI) algorithm, the grey wolf optimizer (GWO) is based on loose mimicry of the collective behavior of a group of individual agents. The individual agent here is the grey wolf (Canis Lupus), an apex predator at the top of the food chain. Grey wolves live in packs, and have a very strict social dominant hierarchy as shown in Fig. 1 [30]. The alpha wolves are at the top, and the omega wolves are the bottom of the pyramid of decreasing dominance from the top to the bottom. The beta wolves as the second layer from the top and the delta wolves as the penultimate layer complete the pyramid of social hierarchy of a pack of grey wolves.

Mathematical modeling of a pack of grey wolves also involves a second feature, which is the group hunting behavior. This comprises three phases [30]: (a) tracking, chasing and approaching the prey (b) pursuing, encircling and harassing the prey until it stops moving, and (c) attacking the prey.

Mathematical modeling of the social hierarchy for solving any optimization problem involves classification of the fittest or best solution as the alpha (α), the second and third best solutions as beta (β) and delta (δ) respectively. All other solutions are classified as omega (ω).

The hunting behavior is loosely modeled by the following two operators [30]:

3.1. Encircling prey

The distance between any wolf and the prey is given by

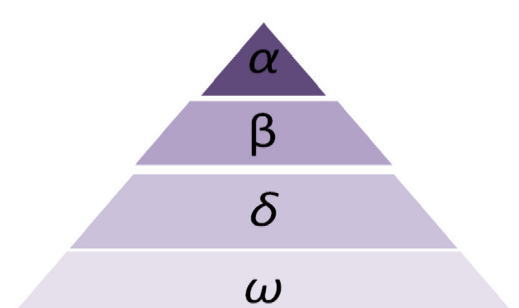

Fig. 1. Social hierarchy of a pack of grey wolves [30].

$$\vec{D} = \left|\vec{C} \otimes \vec{X}^p(t) - \vec{X}(t)\right| \tag{13}$$

$$\vec{C} = 2\ \vec{r}_1 \tag{14}$$

where $\vec{X}^p$ is the position vector of the prey, $\vec{X}$ the position vector of a wolf, and t indicates the iteration number. $\vec{r}_1$ is a vector of random numbers in the range [0, 1], of the same dimensions as $\vec{X}^p$ and $\vec{X}$. The $\otimes$ between $\vec{C}$ and $\vec{X}^p$ means corresponding component wise multiplication.

3.2. Hunting

Hunting involves moving closer to the prey using the information obtained in encircling, given by (13) and (14). This is given by

$$\vec{X}(t+1) = \vec{X}^p(t) - \vec{A} \otimes \vec{D} \tag{15}$$

$$\vec{A} = a\,(2\ \vec{r}_2 - 1) \tag{16}$$

where a is linearly decreased from 2 to 0 over the course of iterations, and $\vec{r}_2$ a vector of random numbers in the range [0, 1], and of the same dimensions as $\vec{X}^p$, $\vec{X}$ and $\vec{D}$. The $\otimes$ between $\vec{A}$ and $\vec{D}$ means corresponding component wise multiplication, as in (13).

The position of the prey $\vec{X}^p$, or the optimum solution being searched for in the solution landscape being unknown, the metaphor of social hierarchy of wolves is used. It is assumed that the α, β and δ wolves have the best knowledge of the prey, and hence, their positions are used for updating the positions of all the other (omega) wolves. Using these three best solutions in the decreasing order of their fitness, the distances between any wolf $\vec{X}$ and these three best wolves are given by

$$\vec{D}_\alpha = \left|\vec{C}_1 \otimes \vec{X}^\alpha - \vec{X}\right|, \vec{D}_\beta = \left|\vec{C}_2 \otimes \vec{X}^\beta - \vec{X}\right|, \vec{D}_\delta = \left|\vec{C}_3 \otimes \vec{X}^\delta - \vec{X}\right| \tag{17}$$

These distances can be used to obtain the new position of the wolf $\vec{X}(t+1)$ using the following equations.

$$\vec{X}_1 = \vec{X}^\alpha - \vec{A}_1 \otimes \vec{D}_\alpha, \vec{X}_2 = \vec{X}^\beta - \vec{A}_2 \otimes \vec{D}_\beta, \vec{X}_3 = \vec{X}^\delta - \vec{A}_3 \otimes \vec{D}_\delta \tag{18}$$

$$\vec{X}(t+1) = \frac{\vec{X}_1 + \vec{X}_2 + \vec{X}_3}{3} \tag{19}$$

Applying the two operators of encircling and hunting repeatedly, the prey or the best solution is located.

4. Hybrid grey wolf optimizer

The GWO as proposed by Ref. [30] has been summarized in Section III [30]. has also reported the performance of the algorithm on a suite of twenty nine unimodal and multimodal benchmark functions, and concluded that the GWO produces comparable results against other metaheuristic algorithms such as the PSO, GSA, DE, EP and ES (expansions of these abbreviations are contained in Table 1). However [30], has also expressed the need for adding more operators to improve the exploratory capability (page 50), further suggesting that these may be EA based, such as mutation (page 51). It may be noted that none of the functions in Ref. [30] was discontinuous. The ED problem is inherently discontinuous due to

Table 1
Table of abbreviations.

Optimization technique	Abbreviation
Backtracking Search Algorithm	BSA
Bacterial Foraging Optimization	BF
Biogeography-Based Optimization	BBO
Chaotic Particle Swarm Optimization	CPSO
Chaotic Particle Swarm Optimization with the Sequential Quadratic Programming	CPSO-SQP
Civilized Swarm Optimization	CSO
Cross-Entropy Method And Sequential Quadratic Programming	CE-SQP
Cuckoo Search Algorithm	CSA
Differential Evolution	DE
Differential Evolution with Biogeography-Based Optimization	DE/BBO
Differential Harmony Search Algorithm	DHS
Dimensional Steepest Decline Method	DSD
Distributed Auction-Based Algorithm	AA (Dist.)
Distributed Lambda-Consensus	λ-Consensus
Distributed Sobol Particle Swarm Optimization and Tabu Search Algorithm	DSPSO–TSA
Enhanced Gradient-Based Simplified Swarm Optimization	EGSSOA
Evolutionary Programming	EP
Evolutionary Strategy	ES
Fast lambda-Iteration Method	F λ-I
Fuzzy Adaptive Particle Swarm Optimization	FAPSO
Genetic Algorithm	GA
Genetic Algorithm – Ant Colony Optimization (special class)	GA-API
Genetic Algorithm – Binary	GA Binary
Gravitational Search Algorithm	GSA
Hybrid Chemical Reaction Optimization with Differential Evolution	HCRO-DE
Hybrid Grey Wolf Optimizer	HGWO
Hybrid Harmony Search with Arithmetic Crossover	ACHS
Improved Differential Evolution	IDE
Iteration Particle Swarm Optimization	IPSO
Krill herd Algorithm	KHA
Mixed Integer Nonlinear and Non-convex Programming	MINLP
Mixed-Integer Quadratically Constrained Quadratic Programming	MIQCQP
Modified Artificial Bee Colony Algorithm	MABC
Multiple Tabu Search	MTS
New Adaptive Particle Swarm Optimization	NAPSO
New Particle Swarm Optimization with Local Random Search	NPSO-LRS
Oppositional Real Coded Chemical Reaction Optimization	ORCCRO
Parallel Particle Swarm Optimization with Modified Stochastic Acceleration Factors	PSO-MSAF
Particle Swarm Optimization	PSO
Particle Swarm Optimization with the Sequential Quadratic Programming Technique	PSO-SQP
Passive Congregation-based Particle Swarm Optimization	PC-PSO
Quantum Particle Swarm Optimization	QPSO
Random Drift Particle Swarm Optimization	RDPSO
Real Coded Chemical Reaction Optimization	RCCRO
Self-Organizing Hierarchical Particle Swarm Optimization	SOH-PSO
Shuffled Differential Evolution	SDE
Simple Particle Swarm Optimization	SPSO
Simulated Annealing	SA
Simulated Annealing like Particle Swarm Optimization	SA-PSO
Society-Civilization Algorithm	SCA
Spatial Adaptive Play	SAP
Species-based Quantum Particle Swarm Optimization	SQPSO
Tabu Search	TS
θ-Particle Swarm Optimization	θ – PSO

the presence of POZs, described in Section II·C.1.

The ED problem is also multimodal, as explained in detail in Ref. [19]. An effective mechanism to effectively explore and exploit a multimodal solution space is to use the DE-type of mutation and crossover operators. It may be noted that the DE-type mutation gives DE one of its main assets, termed as *contour matching* [31]. Contour matching refers to the self-adaptive property of the solution population to automatically explore the most promising regions of the solution space, once they are detected.

The authors of this paper have found that the addition of a DE-type mutation and uniform crossover improves the performance of the GWO algorithm, when applied to the ED problem. Hence, the work reported in this paper incorporates these operators to the GWO, thereby hybridizing it to produce the HGWO. These operators are explained next.

4.1. Mutation

The classical DE mutation variant termed as DE/best/1 [32] is used in this paper. This is given by

$$X^i = X^{\mathrm{gBest}} + W(X^p - X^q), \qquad i = 1, 2, \ldots, N_w, \; i \neq \mathrm{gBest} \tag{20}$$

where
$p, \; q \in [1, 2, \ldots, N_w]$, randomly chosen, $p \neq q \neq i \neq \mathrm{gBest}$,
W is the weighting or scaling factor, $\in [0.4, \; 1]$ [32]. In this paper, we use a high value of 1 for W, which means that this mutation leans towards exploration, rather than exploitation. This choice is justifiable on the ground that, in a multimodal search space, more exploration than exploitation is needed to locate all the

optima quickly. N_w is the total number of wolves in the herd or set of trial solution vectors, and X^{gBest} is the global best wolf in the whole of the iterative process so far up to the current iteration. The best wolf in the pack in any given iteration is compared with this global best wolf. If the best wolf is better than the global best wolf, it becomes the new global best wolf.

The condition $i \neq$ gBest ensures elitism. That is, the global best wolf X^{gBest} is preserved as it is and is not mutated.

4.2. Crossover

Uniform or binomial crossover [32] is used. After crossover, the jth component of the ith wolf is given by

$$x_j^i = \begin{cases} x_j^r & \text{if } \; rand_j^{\,i} < C_r \\ x_j^i & \text{else} \end{cases} \qquad j = 1, 2, \ldots, D, \; i = 1, 2, \ldots, N_w \tag{21}$$

where
$r \in [1, 2, \ldots, N_W]$, chosen randomly, $r \neq i rand_j^{\,i} \in [0, 1]$ is randomly generated $\forall\ j, i$.

In this paper, crossover probability or crossover rate C_r is not a constant, static value, as in classical DE. A dynamic C_r as defined in Ref. [33] is used:

$$C_r = 0.2 \times \widehat{F}^{\,i,\,\text{best}} \tag{22}$$

$$\widehat{F}^{\,i,\,\text{best}} = \frac{F^{\,i} - F^{\text{best}}}{F^{\text{worst}} - F^{\text{best}}} \qquad i = 1, 2, \ldots, N_W \tag{23}$$

F^{best} and F^{worst} are the best and the worst fitness values in the wolf pack in the current iteration, $F^{\,i}$ is the fitness or objective function value of the ith wolf.

Equations (23) and (22) mean that C_r is 0 for the best wolf and 0.2 for the worst wolf in the pack of the current generation or iteration. $C_r = 0$ ensures elitism. That is, the best wolf or solution is preserved and does not face crossover. It also ensures that the probability of crossover of any solution is directly proportional to its relative fitness.

4.3. Constraint handling in ED problems

The ED problem as defined by (12) comprises numerous inequality constraints, which make the solution space disjointed. However, these constraints are simpler to handle; the trial solution values could be fixed at the limits that are violated, during the solution process. A more difficult task is to satisfy the power balance constraint. For, this being an equality constraint, is always active, and must be always satisfied. Solutions that satisfy all the inequality constraints could still be infeasible due to their not satisfying this equality constraint. A binary classification of such solutions into feasible and infeasible ones on the basis of violation of this equality constraint, and discarding such infeasible solutions is however a poor remedy. For, it is easily probable that there could be no feasible solutions at all, since it is rare that solutions that satisfy all the inequality constraints would automatically satisfy an equality constraint. The simplest approach of handling such infeasible solutions is to impose a penalty that is proportional to the violation. This approach comes with the disadvantages of the static penalty value being problem dependent, and user chosen. To eliminate these disadvantages, a superior approach that makes the penalty value self-adaptive [34],[35], is used in this paper. The cost or fitness or objective function $F(X)$ is modified to

$$\overline{F}(X) = d(X) + p(X) \tag{24}$$

where the distance value $d(X)$ is given by

$$d(X) = \begin{cases} \upsilon(X), \; \text{if } r_f = 0 \\ \sqrt{F''(X)^2 + \upsilon(X)^2}, \; \text{otherwise} \end{cases} \tag{25}$$

The violation value $\upsilon(X)$ is given by

$$\upsilon(X) = \frac{\sum_{n=1}^{N_e} w_n \times h_n(X)}{\sum_{n=1}^{N_e} w_n} \tag{26}$$

where $h_n(X) = 0, \; n = 1, 2, \ldots, N_e$ are the N_e number of equality constraints, $w_n = 1/h_{\max_n}$ is a weight parameter, and $h_{\max_n}$ is the maximum violation of the constraint h_n obtained so far. r_f and $F''(X)$ are defined as

$$r_f = \frac{\text{no of feasible solutions}}{\text{population size}} \tag{27}$$

$$F''(X) = \frac{F(X) - F_{\min}}{F_{\max} - F_{\min}} \tag{28}$$

$F_{\min}$ and $F_{\max}$ are the minimum and maximum values of the cost function $F(X)$ in the current population.

The penalty value $p(X)$ is given by

$$p(X) = \left(1 - r_f\right) M(X) + r_f N(X) \tag{29}$$

$$M(X) = \begin{cases} 0, & \text{if} r_f = 0 \\ \upsilon(X), & \text{otherwise} \end{cases} \tag{30}$$

$$N(X) = \begin{cases} 0, & \text{if} r_f = 0 \\ F''(X), & \text{otherwise} \end{cases} \tag{31}$$

4.4. Implementation of HGWO to ED problem

Step 0: In the initialization step, generate randomly, only feasible solutions or wolves, as in Ref. [6]. Doing so, the population comprises N_W number of feasible wolves or solutions, each comprising N_g number of generators:

$$\begin{bmatrix} P_1^1 & P_2^1 & \cdots & P_j^1 & \cdots & P_{N_g}^1 \\ P_1^2 & P_2^2 & \cdots & P_j^2 & \cdots & P_{N_g}^2 \\ \vdots & & \ddots & & & \vdots \\ P_1^i & P_2^i & \cdots & P_j^i & \cdots & P_{N_g}^i \\ \vdots & & & & \ddots & \vdots \\ P_1^{N_w} & P_2^{N_w} & \cdots & P_j^{N_w} & \cdots & P_{N_g}^{N_w} \end{bmatrix} = \begin{bmatrix} P^1 \\ P^2 \\ \vdots \\ P^i \\ \vdots \\ P^{N_w} \end{bmatrix}, \; j = 1, 2, \ldots, N_g, \; i = 1, 2, \ldots, N_W \tag{32}$$

Step 1: Evaluate the fitness values of all the wolves using (2). Identify the alpha, beta and delta wolves P^{α}, P^{β}, P^{δ} and the global best solution P^{gBest}. In the very first iteration, $P^{\text{gBest}} = P^{\alpha}$.

Step 2: Apply the operator of encircling (13) to compute

$$\vec{D}_{\alpha} = \left|\vec{C}_1 \otimes P^{\alpha} - P^{i}(t)\right|, \vec{D}_{\beta} = \left|\vec{C}_2 \otimes P^{\beta} - P^{i}(t)\right|, \vec{D}_{\delta} = \left|\vec{C}_3 \otimes P^{\delta} - P^{i}(t)\right| \tag{33}$$

Apply the operator of hunting (15), to compute $\hat{P}^1$, $\hat{P}^2$ and $\hat{P}^3$. Using (33), we get

$$\hat{P}^1 = P^{\alpha} - \vec{A}_1 \otimes \vec{D}_{\alpha}, \hat{P}^2 = P^{\beta} - \vec{A}_2 \otimes \vec{D}_{\beta}, \hat{P}^3 = P^{\delta} - \vec{A}_3 \otimes \vec{D}_{\delta} \tag{34}$$

Compute the population of the next generation, $P(t+1)$:

$$P^{i}(t+1) = \frac{\hat{P}^1 + \hat{P}^2 + \hat{P}^3}{3}, \quad i = 1, 2, \ldots, N_w \tag{35}$$

Step 3: Apply the operators of mutation (20) and crossover (21), with $P_j^{\,i}$ replacing x_j^i.
Step 4: Check if effective generation limits and POZ limits are violated. Fix the generation at the limit that is violated. This takes care of the inequality constraints. After this is done, violation of the power balance equality constraint (3) is dealt with by using the self-adaptive penalty approach outlined in Section V. It may be noted that this is the only equality constraint in the ED problem, and (3) is modified to

$$h(P) = \sum_{j=1}^{N_g} P_j - P_D - P_L = 0 \tag{36}$$

Consequently, $\nu(P) = h(P)$.

Step 5:: Is the stopping criterion satisfied? If yes, STOP. Else, repeat steps 1 to 5 until the stopping criterion is satisfied.

Table 2
Optimal generation and cost obtained by HGWO for test system 1 (6 generators with POZs, ramp rate limits and loss).

Unit	$P_j^{\min}$	$P_j^{\max}$	POZ	Generation
1	100.00	500.00	[210, 240]; [350, 380]	446.6069
2	50.00	200.00	[90, 110]; [140, 160]	172.5618
3	80.00	300.00	[150, 170]; [210, 240]	265.4896
4	50.00	150.00	[80, 90]; [110, 120]	137.0542
5	50.00	200.00	[90, 110]; [140, 150]	166.7302
6	50.00	120.00	[75, 85]; [100, 105]	87.0212
Cost ($/hr) 15,442				
Transmission loss (MW) 12.4639				

5. Results and discussion

To test the effectiveness of the proposed HGWO algorithm, four different test systems of varying computational difficulty levels have been solved using HGWO. The results obtained are compared with some of the optimization techniques listed in Table 1, and contained in [3–54].

The number of wolves N_w used is 30. For fair comparison, 50 independent trial runs are made and the results of the best and mean fuel costs are tabulated for each test system. The stopping criterion used is the maximum number of iterations, which is 300 for the first three test systems in this paper, and 600 for the last, 80 generator test system. The programs are implemented in Matlab® on a personal computer with 3.3 GHz processor and 4 GB RAM, running on Windows 7.

5.1. Test system 1

This is a small system comprising six generators meeting a load demand of 1263 MW, and includes transmission loss, POZs and ramp rate limits. Though it is a small system, the presence of transmission loss, POZs and ramp rate limits makes it a practical, realistic problem. The system data are taken from [54],[55]. Table 2 presents the optimal generations and cost obtained by HGWO for the test system. The optimal cost and the corresponding transmission loss obtained by HGWO are 15,442 $/hr and 12.4639 MW, respectively. It may be noted that the generations (Column 5)

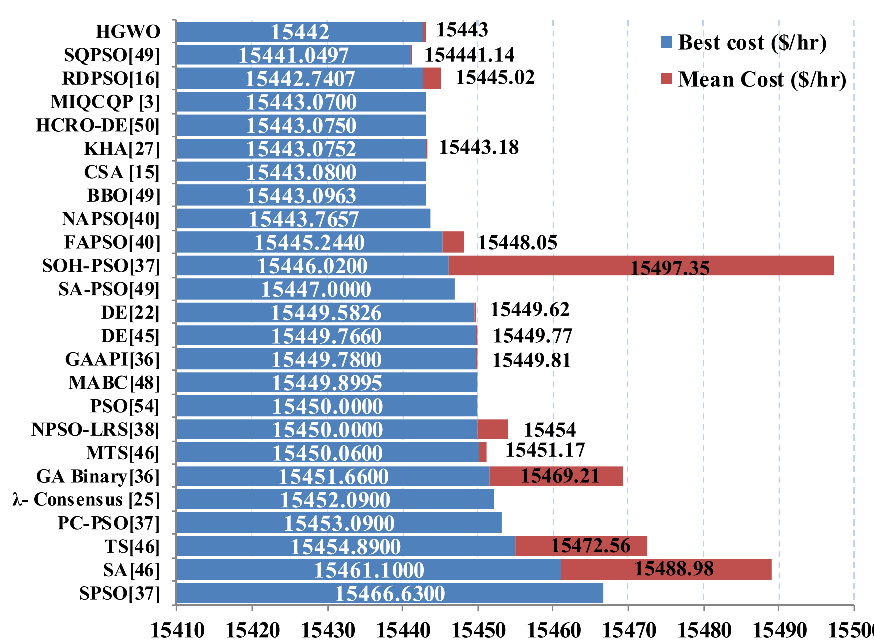

Fig. 2. Comparison of fuel costs (in $/hr) for Test System 1.

Table 3
Optimal generation and cost obtained by HGWO for test system 2 (15 generators with POZs, loss, without and with ramp rate limits).

Unit	Case 1 (without ramp rate limits)		Case 2 (with ramp rate limits)		POZ	Generation	
	P_j^{min}	P_j^{max}	$\underline{P_j}$	$\overline{P_j}$		Case 1	Case 2
1	150	455	280	455	–	455.0000	455.0000
2	150	455	180	380	[185,225]; [305,335]; [420,450]	455.0000	380.0000
3	20	130	20	130	–	130.0000	130.0000
4	20	130	20	130	–	130.0000	130.0000
5	150	470	150	170	[180,200]; [305,335]; [390,420]	230.5226	170.0000
6	135	460	280	460	[230,255]; [365,395]; [430,455]	460.0000	460.0000
7	135	465	230	430	–	465.0000	430.0000
8	60	300	60	160	–	60.0000	60.0001
9	25	162	25	162	–	25.0000	50.0350
10	25	160	25	160	–	35.4926	160.0000
11	20	80	20	80	–	74.9908	80.0000
12	20	80	20	80	[30,40]; [55,65]	79.9989	80.0000
13	25	85	25	85	–	25.0000	25.0000
14	15	55	15	55	–	15.0000	31.9997
15	15	55	15	55	–	15.0000	15.0000
Transmission loss (MW)						26.0049	27.0348
Cost ($/hr)						32,540	32,679

satisfy the generation limit constraints (Columns 2 and 3) and do not fall in the POZ (Column 4).

Fig. 2 shows the comparison of the statistical results of different algorithms such as the KHA, SA, TS, DE, and PSO that have been reported recently. It is seen that, the mean and the best cost obtained by the HGWO are the least of all, with the exception of the SQPSO. The mean and best cost in SQPSO are lower than for the HGWO since the power balance constraint is not satisfied in SQPSO, as mentioned in the last column of Table 4, page 318 of [49]. The total generation therein is less than the demand and loss by 0.1499 MW for SQPSO.

Fig. 3 shows the graphs of the convergence of the solutions with iterations for the basic GWO and the HGWO, for a typical run. It is seen that, after about half the total number of iterations, the HGWO begins to outperform the GWO, and continues to do so until the stopping criterion of 300 iterations is reached. This outperformance is both in terms of the rate of convergence and getting a lower final value of the generation cost.

The computational cost of the hybridization of the GWO to produce the HGWO can be measured in two ways. The first is in terms of the number of operators. The GWO uses the two operators of encircling and hunting. Hybridization entails the addition of the two operators of mutation and crossover. The second way of measuring the computational cost is in terms of the average execution time per run for the HGWO and GWO. This average execution time per run, obtained from fifty trial runs, for the HGWO is 1.145 s against 0.580 s for the GWO, for the Test System 1.

5.2. Test system 2

This test system consists of 15 generators meeting a load demand of 2630 MW, and includes transmission loss, POZ and ramp rate limits. As in the previous Test System 1, the presence of transmission loss and POZs makes it a practical, realistic problem. The system data are taken from [54],[55]. To further reflect the reality, this problem has been solved for the two cases of without and with ramp rate limits. Table 3 presents the optimal generations and the costs obtained. The optimal cost and the corresponding

Table 4
Optimal generation and cost obtained by HGWO for test system 3, case 1 (40generators with POZs, valve point effect, without loss).

Unit	P_j^{min}	P_j^{max}	POZ	Generation	Unit	P_j^{min}	P_j^{max}	POZ	Generation
1	36.0	114.0	–	110.8010	21	254.0	550.0	–	523.2794
2	36.0	114.0	–	110.8000	22	254.0	550.0	–	523.2794
3	60.0	120.0	–	97.4000	23	254.0	550.0	–	523.2794
4	80.0	190.0	–	179.7333	24	254.0	550.0	–	523.2794
5	47.0	97.0	–	87.7998	25	254.0	550.0	–	523.2795
6	68.0	140.0	–	140.0000	26	254.0	550.0	–	523.2794
7	11.0	300.0	–	259.5998	27	10.0	150.0	–	10.0000
8	13.0	300.0	–	284.5996	28	10.0	150.0	–	10.0000
9	13.0	300.0	–	284.5998	29	10.0	150.0	–	10.0000
10	13.0	300.0	[130,150]; [200,230]; [270,290]	130.0000	30	47.0	97.0	–	87.7999
11	94.0	375.0	[100,140]; [230,280]; [300,350]	94.0000	31	60.0	190.0	–	190.0000
12	94.0	375.0	[100,140]; [230,280]; [300,350]	94.0000	32	60.0	190.0	–	190.0000
13	12.0	500.0	[150,200]; [250,300]; [400,450]	214.7598	33	60.0	190.0	–	190.0000
14	12.0	500.0	[200,250]; [300,350]; [450,490]	394.2793	34	90.0	200.0	–	164.7998
15	12.0	500.0	–	394.2793	35	90.0	200.0	–	194.3955
16	12.0	500.0	–	394.2795	36	90.0	200.0	–	200.0000
17	22.0	500.0	–	489.2796	37	25.0	110.0	–	110.0000
18	22.0	500.0	–	489.2794	38	25.0	110.0	–	110.0000
19	24.0	550.0	–	511.2794	39	25.0	110.0	–	110.0000
20	242.0	550.0	–	511.2793	40	242.0	550.0	–	511.2794
Cost ($/hr) 121,412									

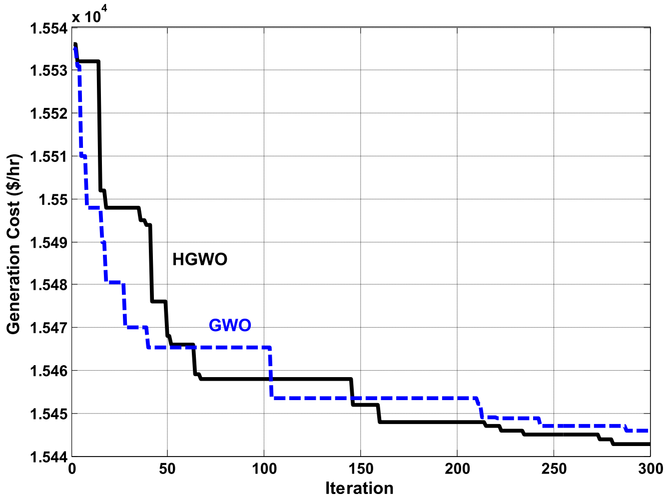

Fig. 3. Typical convergence characteristic of the GWO and HGWO for the Test System 1.

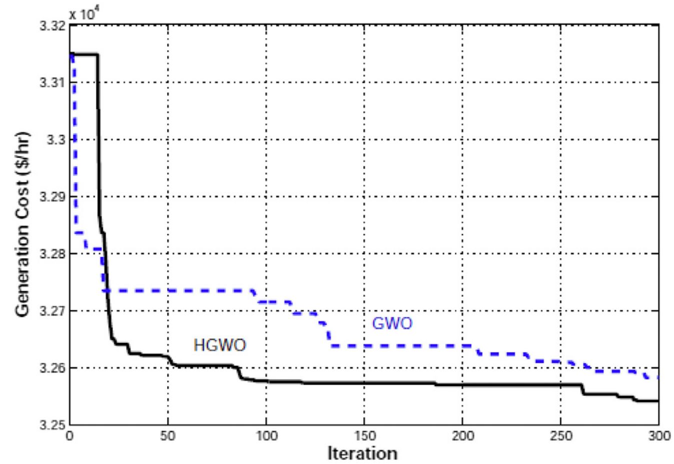

Fig. 5. Typical convergence characteristic of the GWO and HGWO for the Test System 2.

transmission loss obtained by HGWO are 32,540 $/hr and 26.0049 MW respectively, for Case 1 (without ramp rate limit). It may be noted that the generations (Column 7) satisfy the generation limit constraints (Columns 2 and 3) and do not fall in the POZ (Column 6). Fig. 4 shows the comparison of the statistical results of the HGWO and other algorithms that have been reported recently. It is seen that, the best cost obtained by the HGWO is the least of all methods. The mean cost too is also the least with the sole exception of the EGSSOA [39]. The average execution time per run, obtained from fifty trial runs, for the HGWO is 1.292 s against 0.698 s for the GWO, for the Test System 2.

Fig. 5 shows the graphs of the convergence of the solutions with iterations for the basic GWO and the HGWO, for a typical run, for Test System 2. It is seen that the HGWO outperforms the GWO both in terms of the rate of convergence and getting a lower final value of the generation cost.

In Case 2 (with ramp rate limit), the minimum and maximum generation limits are replaced by the effective generation limits $\underline{P_j}$ and $\overline{P_j}$ as given by (10) and (11). For this case, the optimal cost and the corresponding transmission loss obtained by HGWO are 32,679 $/hr and 27.0348 MW, respectively (Table 3). As in Case 1, the inequality constraints of $\underline{P_j}$, $\overline{P_j}$ and POZs are satisfied. Fig. 6 shows the comparison of the statistical results of the HGWO and other algorithms that have been reported recently. It is seen that, the best and mean cost obtained by the HGWO are the least of all methods.

5.3. Test system 3

This test system consists of 40 generators, includes transmission losses, POZs and valve point effect. The demand of this system is 10,500 MW. As in the previous Test Systems, the presence of transmission loss and POZs makes it a practical, realistic problem. The valve point effect adds a further element of reality to the problem. The fuel costs and power generation limits data are taken from [56]. As this reference does not contain POZ data, the POZ data is from Ref. [40]. Even though all practical power systems have transmission loss, this problem also considers the case without loss, for a fair comparison with existing results. Accordingly, two cases

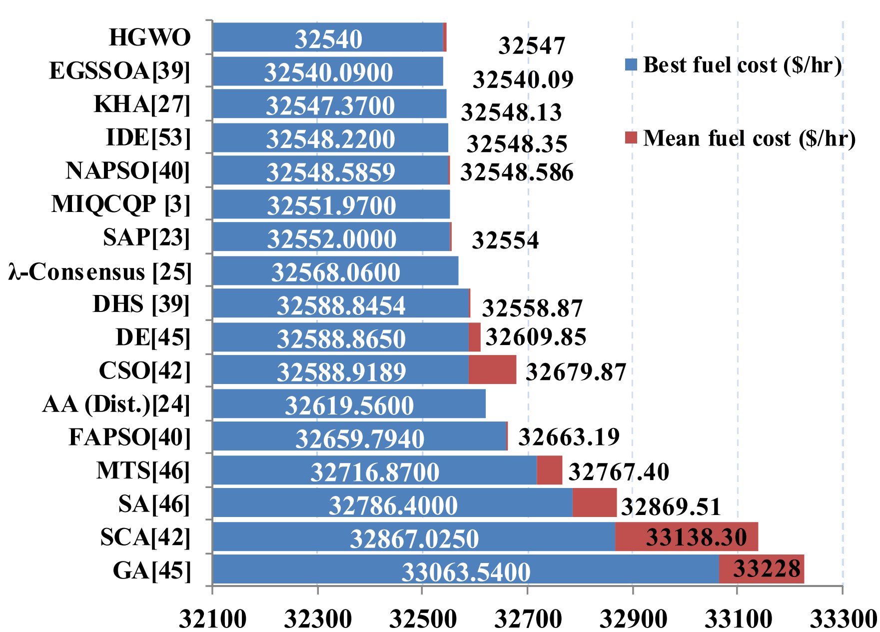

Fig. 4. Comparison of fuel costs (in $/hr) for Test System 2, Case 1 (without ramp rate limits).

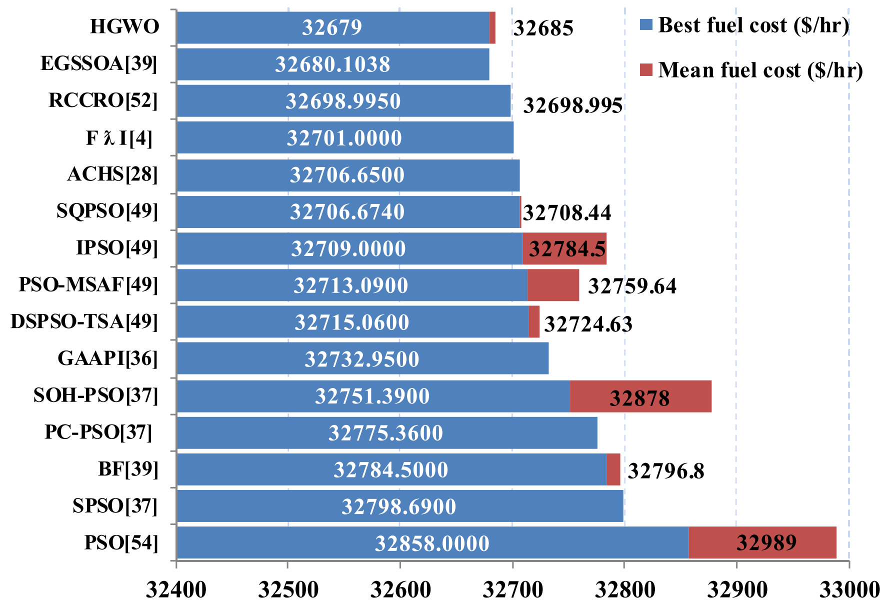

Fig. 6. Comparison of fuel costs (in $/hr) for Test System 2, Case 2 (with ramp rate limits).

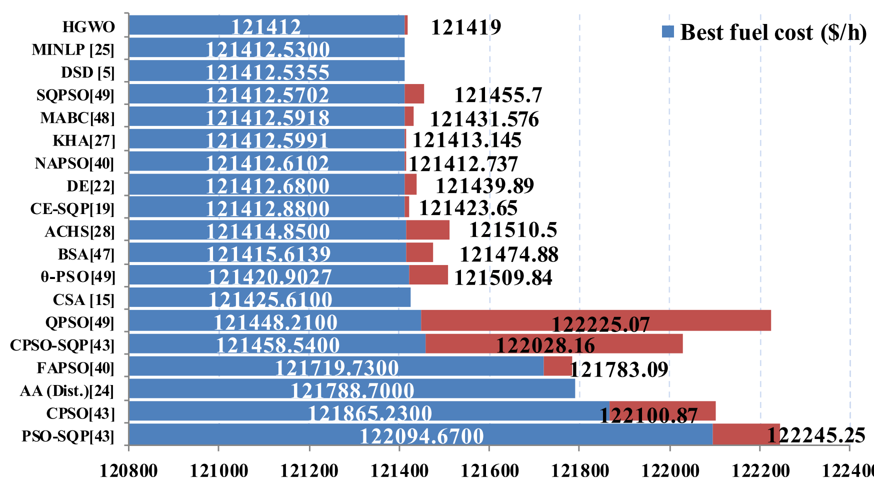

Fig. 7. Comparison of fuel costs (in $/hr) for Test System 3, Case 1 (without loss).

have been considered: without and with transmission losses. Table 4 shows the optimal generations and fuel cost obtained by HGWO, for Case 1 (without loss). The optimal cost obtained is 121,412 $/hr. As for the previous test systems, none of the constraints shown in the table are violated. Fig. 7 shows the comparison of the statistical results of the HGWO and other recent algorithms applied to this problem. It is seen that, the best and mean cost obtained by the HGWO are the least of all results, with the exception of DSD [5] and MINLP [21]. The average execution time per run, obtained from fifty trial runs, for the HGWO is 1.223 s against 0.656 s for the GWO, for this case.

Fig. 8 shows the graphs of the convergence of the solutions with iterations for the basic GWO and the HGWO, for a typical run, for Test System 3. It is seen that the HGWO outperforms the GWO both in terms of the rate of convergence and getting a lower final value of the generation cost.

Case 2: [56] does not consider transmission losses. To make the problem more realistic, transmission losses too have been considered for the work in this paper. The *B*-coefficients data is from Ref. [36]. Table 5 shows the optimal generation and fuel cost

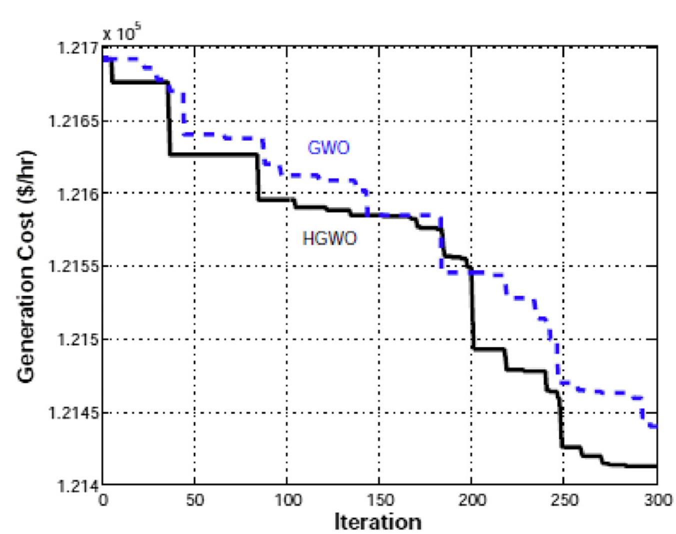

Fig. 8. Typical convergence characteristic of the GWO and HGWO for the Test System 3.

Table 5
Optimal generation and cost obtained by HGWO for test system 3, case 2 (40 generators without POZs, with valve point effect, and loss).

Unit	P_j^{min}	P_j^{max}	Generation	Unit	P_j^{min}	P_j^{max}	Generation
1	36.0	114.0	113.9796	21	254.0	550.0	
2	36.0	114.0	113.9998	22	254.0	550.0	
3	60.0	120.0	119.9993	23	254.0	550.0	523.3009
4	80.0	190.0	190.0000	24	254.0	550.0	523.2692
5	47.0	97.0	88.5401	25	254.0	550.0	524.0356
6	68.0	140.0	105.4806	26	254.0	550.0	523.4889
7	110.0	300.0	299.9951	27	10.0	150.0	10.1234
8	135.0	300.0	300.0000	28	10.0	150.0	10.1698
9	135.0	300.0	299.9923	29	10.0	150.0	10.0145
10	130.0	300.0	280.5061	30	47.0	97.0	87.7622
11	94.0	375.0	243.6484	31	60.0	190.0	189.9976
12	94.0	375.0	168.7975	32	60.0	190.0	189.9989
13	125.0	500.0	484.1454	33	60.0	190.0	189.9978
14	125.0	500.0	484.0388	34	90.0	200.0	199.9996
15	125.0	500.0	484.4070	35	90.0	200.0	164.9274
16	125.0	500.0	484.9438	36	90.0	200.0	164.8150
17	220.0	500.0	489.6908	37	25.0	110.0	109.9999
18	220.0	500.0	489.2950	38	25.0	110.0	109.9960
19	242.0	550.0	511.3919	39	25.0	110.0	109.9908
20	242.0	550.0	511.3770	40	242.0	550.0	511.9850
Cost (S/h) 136,681							
Transmission loss (MW) 978.9673							

obtained HGWO, for Case 2, which does not contain POZs.

The optimal cost and the corresponding transmission loss obtained by HGWO are 136,681 $/hr and 978.9673 MW respectively (Table 5). As in Case 1, generation limit constraints are satisfied. Fig. 9 shows the comparison of the statistical results of the HGWO and other recently reported algorithms. It is seen that, the best and mean cost obtained by the HGWO are the least of all methods, in this case. The best cost for HGWO is lower than the next higher reported figure of 136,855.19 $/hr by 0.127%. The average execution time per run, obtained from fifty trial runs, for the HGWO is 1.461 s against 1.062 s for the GWO, for this case.

5.4. Test system 4

In order to study the performance of the HGWO algorithm on high dimensional practical problems, a large system with 80 generators is considered next. The duplicate of Test System 3, Case 1 is added to Test System 3, Case 1 to form this Test System 4. This system is considered with POZs and valve point effect. Although practical systems have transmission loss, it is not considered here, for a fair comparison against the other results in the literature. The optimal generations for this system were found to differ from those in Test System 3, Case 1, only in the second decimal place onwards. The optimal cost obtained is 242,825 $/hr, and this differs from the double of that for Test System 3, Case 1, only by 0.07 $/hr.

Fig. 10 shows the comparison of the statistical results of the HGWO and other algorithms. It is seen that, the best cost obtained by the HGWO is comparable with that in Refs. [23], but lower than all other results.

Fig. 11 shows the graphs of the convergence of the solutions with iterations for the basic GWO and HGWO, for a typical run. As for all the previous test systems, the HGWO outperforms the GWO in terms of the rate of convergence, and getting a lower generation cost, for this system too. This outperformance of the HGWO over GWO is clearly evident here, for this high dimensional problem. Right from the beginning, the HGWO achieves a lower cost than the

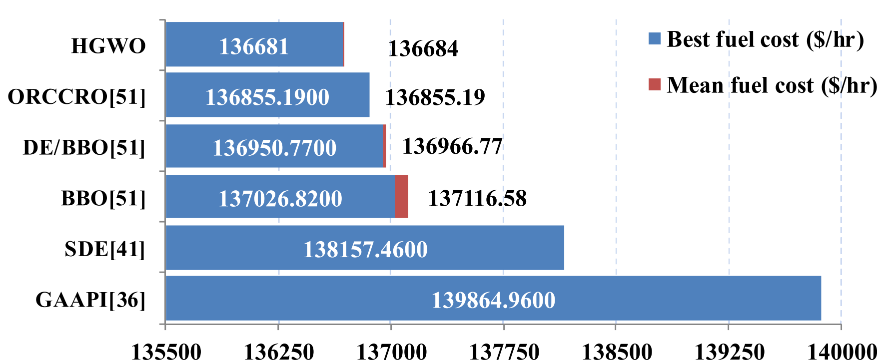

Fig. 9. Comparison of fuel costs (in $/hr) for Test System 3, Case 2 (with loss).

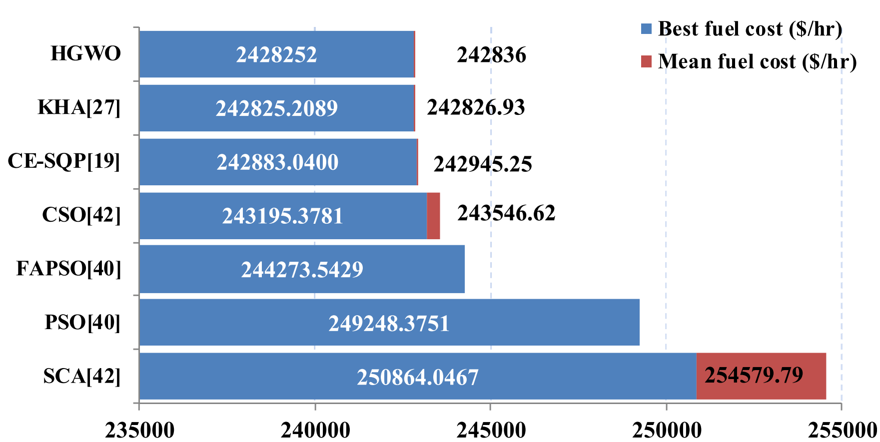

Fig. 10. Comparison of fuel costs (in $/hr) for Test System 4 (only case, without loss).

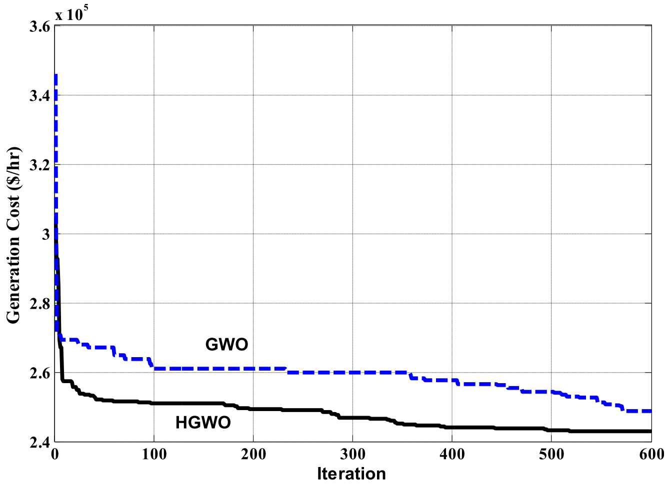

Fig. 11. Typical convergence characteristic of the GWO and HGWO for the Test System 4.

GWO, and continuously improves on it, to reach a lower final cost than the GWO. The average execution time per run, obtained from fifty trial runs, for the HGWO is 2.813 s against 1.285 s for the GWO, for the Test System 4.

6. Conclusion

This paper presented the application of the HGWO algorithm for solving the economic dispatch problem that is nonlinear, non-convex and discontinuous in nature, with numerous equality and inequality constraints. The basic GWO was hybridized with the addition of mutation and crossover operators for enhanced performance. This enhancement in performance is in terms of the rate of convergence and a lower final cost, as demonstrated by the comparison of the performances of the GWO and HGWO for the four Test Systems considered in this paper. One measure of the complexity of a problem is the dimensionality of the problem. By this measure, the superiority of the HGWO over the GWO becomes apparent as the complexity of the problem increases.

One of the issues in handling constraints in complex real life optimization problems such as the economic dispatch problem is that the user has to come up with ways of handling the constraints, which is usually by trial and error. This makes the approach ad hoc, user and problem dependent. A self-adaptive penalty approach that is problem independent and does not require the user to choose the penalty value has been successfully demonstrated in this paper.

Four economic dispatch problems (6, 15, 40, and 80 generators), with prohibited operating zones, valve point effect and ramp rate limits were solved with and without transmission losses by the HGWO proposed in this paper, and compared with other methods in the literature. The results show that the costs obtained by HGWO are either comparable, or lower than those reported by other methods (Test System 3, Case 2). The HGWO is thus as good a performer as the other methods, if not a better performer.

Future work in the area of power system optimization can explore the application of the HGWO to other problems in the area.

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