A GA-API Solution for the Economic Dispatch of Generation in Power System Operation

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***Abstract*—This work proposes a novel heuristic-hybrid optimization method designed to solve the nonconvex economic dispatch problem in power systems. Due to the fast computational capabilities of the proposed algorithm, it is envisioned that it becomes an operations tool for both the generation companies and the TSO/ISO. The methodology proposed improves the overall search capability of two powerful heuristic optimization algorithms: a special class of ant colony optimization called API and a real coded genetic algorithm (RCGA). The proposed algorithm, entitled GAAPI, is a relatively simple but robust algorithm, which combines the downhill behavior of API (a key characteristic of optimization algorithms) and a good spreading in the solution space of the GA search strategy (a guarantee to avoid being trapped in local optima). The feasibility of the proposed method is first tested on a number of well-known complex test functions, as well as on four different power test systems having different sizes and complexities. The results are analyzed in terms of both quality of the solution and the computational efficiency; it is shown that the proposed GAAPI algorithm is capable of obtaining highly robust, quality solutions in a reasonable computational time, compared to a number of similar algorithms proposed in the literature.**

***Index Terms*—API, ant colony optimization, economic dispatch, genetic algorithm, global optimization, hybrid models, nonconvex optimization, power system operation, robust search.**

NOMENCLATURE

a_i, b_i, c_i	Coefficients of the fuel-cost function of unit i.
API	Ant colony algorithm for continuous domains.
$\mathbf{B}$	Transmission loss coefficients matrix.
$\mathrm{DR_i}/\mathrm{UR_i}$	Ramp-down/up rate limit of unit i (MW/h).
e_i, f_i	Coefficients for the valve-point effect of unit i.
$e(\cdot)$	Counter for the number of consecutive failures when searching inside one site.
$F_{\mathrm{total}}(\cdot)$	Total generation cost (€/h).
n	Number of dispatchable units in the system.
ns	Counter for the sites memorized by an ant.
N	Total number of sites an ant can memorize.
NL	Number of lines in the power network.
NP_i	Number of prohibited operating zones (POZ) of unit i.
popRCGA	Population size in the RCGA algorithm.
P	Predefined API constant for consecutive unsuccessful searches.
$\mathbf{P}$	Active power output vector (MW).
P_D	Total average power demand forecasted for the dispatch period (MW).
$P_i^{\max}$	Maximum active power output of unit i (MW).
$P_i^{\min}$	Minimum active power output of unit i (MW).
$P_{i,j}^{\mathrm{LB}}$	Lower boundary of POZ j of unit i (MW).
$P_{i,j}^{\mathrm{UB}}$	Upper boundary of POZ j of unit i (MW).
P_{Loss}	Total active power losses of the network (MW).
$S_l^{\max}$	Maximum thermal limit of transmission line l (MVA).
S_l	Apparent power flow through transmission line l (MVA).
SR	Total spinning reserve in the system (MW).
SR_i	Spinning reserve contribution of unit i (MW).
$\mathrm{SR}_i^{\max}$	Maximum spinning reserve contribution of unit i (MW).
$\mathrm{SR_{TOTAL}}$	Total spinning reserve required by the system (MW).
t	Dispatch period.
Ω	Set of all online units.
Θ	Set of all units with POZ restrictions.

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I. INTRODUCTION

THE economic dispatch of generation in power systems is one of the most important optimization problems for both the generating companies competing in a free electricity market and the systems operator (SO) in charge with a fair handling of transactions between electricity suppliers and their customers. The fuel cost component is still the major part of the variable cost of electricity generation, directly reflected in the electricity bills.

Economic dispatch aims at allocating the electricity load demand to the committed generating units in the most economic or profitable way, while continuously respecting the physical constraints of the power system. In a free electricity market, the (transmission/independent) system operator (TSO/ISO/SO) needs to obtain the most economic schedule of generation taking into account a number of system limitations, such as the heat rate curves, generation limits or ramping limitations of the generating units, limitations of the transmission lines, or reliability preventive parameters of the system (e.g., the power reserve). Thus, the optimization problem may be stated as a minimization problem [1], when the objective is to minimize the total cost of supplying the load (the TSO's point of view), or as a maximization problem [2], when the objective is to maximize the profit of the generating company (GENCO's point of view).

Traditional approaches to solve the ED problem use deterministic methods based on Lagrange multipliers [1], linear programming [3], quadratic programming [4], or dynamic programming (DP) [5], [6]. These algorithms, except the dynamic programming technique, require monotonically increasing incremental cost curves, or, in other words, the existence of the first and second order derivatives of the cost functions. Thus, the problem is resumed to optimizing a convex function over a convex set which is guaranteed to have a unique minimum. However, as long as the input-output characteristics of the real units are highly nonconvex because of factors such as valve-point loading, multiple fuel usage, prohibited operating zones (thermal instability points of the turbine), and nonlinear power flow equality constraints, the problem cannot be solved in a classical analytical approach anymore. Further, for a large scale system, the conventional methods have oscillatory problems resulting in a longer solution time [7]. As far as DP is concerned, this method does not impose any restrictions on the structure of the optimization system but it suffers from the "curse of dimensionality" [8], especially when the number of system variables increases.

The economic dispatch problem considering partially or entirely the above-mentioned nonlinearities and discontinuities is termed in this paper as the nonconvex economic dispatch (NCED). The complexity of the NCED problem has led to the use of heuristic methods in an attempt to reach the near global optimum solution even when including the nonconvex characteristics of the problem [9]. Methods such as genetic algorithm (GA) [10]–[12], evolutionary programming (EP) [8], simulated annealing (SA) [3], [13], neural network (NN) [14], fuzzy logic [15], Tabu search [16], and particle swarm optimization techniques (PSO) [7], [17]–[20] are only a few examples that solve the nonconvex economic dispatch problem. These methods have the advantage of not having restrictions on the generation cost function or generally on the type of the system under investigation. On the other hand, they prove to be efficient in finding near global optimal solutions within a reasonable computational time.

The main drawback in the successful solution of the above-mentioned heuristic methods may appear due to their premature convergence [20]. Thus, hybrid solutions were proposed to solve this complex optimization problem [21]. Such hybrid techniques include GA combined with SA [22], EP with sequential quadratic programming (SQP) [23], or NN methods with PSO [24].

This paper proposes a novel hybrid stochastic method to solve the nonconvex economic dispatch problem and it is an extension and improvement of [25]. The method proposed combines an ant colony approach (API) with a genetic algorithm and it is shown to provide a fast and robust solution. Four test power systems are used to validate the effectiveness and applicability of the algorithm for solving the economic dispatch problem in its different formulations. The main advantages of the optimization tool proposed are its flexibility in adding more constraints with minimum transformations in the approach, its reduced computational time, and the robustness of the solution.

The following section of the paper presents the formulation of the nonconvex economic dispatch of generation. Section III describes in detail the proposed GAAPI method used to solve the NCED problem, and gives some hints over its performance. Section IV of the paper shows the applicability of the proposed method for four different benchmark systems by a detailed analysis of the results. The last section of the paper is allocated for conclusions.

II. Economic Dispatch Formulation

The main objective of the economic dispatch of generation in power systems is to determine the output of each generating unit based on the committed generation mix for the next dispatch interval such that the total generation cost is minimized, while continuously respecting system constraints.

Modern thermal units have multiple fuel admission valves that are used to control the power output of the unit. When starting to open each steam admission valve in a turbine, a rippling effect is added to the fuel cost curve of the unit, which represents the actual effect due to the sudden influx of steam. This is called valve point effect and it is modeled by adding a sinusoidal component to the quadratic approximation of the fuel cost function [7], [12], [17]–[19]. Thus, the nonconvex economic dispatch formulation may refer, in this paper, to: 1) the nonconvex generator power output curve (nonconvexity caused by the rippling effect of the multiple steam admission valves), 2) the nonconvex set of the feasible solution set due to transmission losses, prohibited operating zones, and ramp rate limits as the constraints of the power system, and 3) both 1) and 2).

The formulation of the economic dispatch problem as a single-objective optimization problem is adopted in this paper, and two specific components of the optimization system are depicted below: the formulation of the objective function, and the formulation of the constraints to be taken into account.

A. Optimization Function

The objective function can be set up either as 1) *piecewise linear*, or 2) *convex* (*smooth*) (the typical industry format), or 3) *nonconvex* (the more realistic picture when dealing with thermal units with multiple steam admission valves). Many utilities prefer to represent their generator cost functions as single- or multiple-segment linear cost functions [1], as illustratively presented in Fig. 1.

A *convex* thermal generation function is a quadratic approximation of the incremental cost curves that could include the operation maintenance cost, and is of the form

$$F_{\text{total}}(P,t) = \sum_{i=1}^{n} a_i + b_i P_i(t) + c_i P_i^2(t). \qquad (1)$$

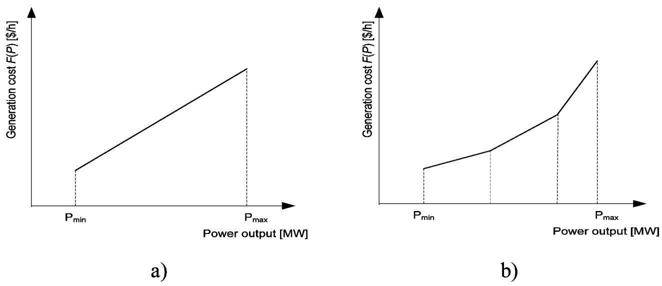

Fig. 1. Linear or piecewise linear generator cost function. (a) Single-segment representation. (b) Multiple-segment representation. $F(P)$ is the generation cost of one unit.

A *nonconvex* thermal generation function incorporates sinusoidal components that represent the valve point loading effect produced by opening the steam admission valves of the generating unit:

$$F_{\text{total}}(P,t)=\sum_{i=1}^{n} a_i+b_iP_i(t)+c_iP_i^2(t) + \left|e_i \sin\left(f_i\left(P_i^{\min}-P_i(t)\right)\right)\right| \quad (2)$$

where F_{total} is the total generation cost, the terms a_i, b_i, and c_i are the fuel-cost coefficients of unit i, and e_i, and f_i are the sinusoidal term coefficients that model the valve-point effect of unit $i; P_i(t)$ is the output power of the ith unit, and n is the number of generators in the system, all reported at the dispatch period t.

B. Constraints

1) Balance Constraint: The total electric power generation has to meet the total electric power demand and the real power losses. Hence

$$P_{\text{Loss}}(t)+P_D(t)-\sum_{i=1}^{n}P_i(t)=0 \quad (3)$$

where $P_D(t)$ is the load demand for the dispatch period $t, P_{\text{Loss}}(t)$ represents the transmission losses associated with the power flow determined for the dispatch period t, and $P_i(t)$ is the output power of unit i at the dispatch period t.

2) Transmission Constraints: The transmission power losses (P_{Loss}) can be computed through a power flow computation (DC or AC approach). However, for simplification and separability of the problem, a common practice is to approximate the total transmission losses either as a quadratic function of the power output of generating units (known as *Kron's loss formula*), or through a simplified linear formula [1]. The quadratic representation of the Kron's loss formula is adopted in this paper as follows:

$$P_{\text{Loss}}(P,t)=\sum_{i=1}^{n}\sum_{j=1}^{n}P_i(t)B_{ij}P_j(t)+\sum_{i=1}^{n}B_{i0}P_i(t)+B_{00} \quad (4)$$

The $\mathbf{B}$ matrix coefficients are assumed to be constant during the dispatch procedure. Reasonable accuracy can be expected when the actual operating conditions are close to the case at which these coefficients were computed. To determine the $\mathbf{B}$ coefficients for a new case study, a power flow program must be run in advance. In this power flow run, the security limits of the system are also taken into account. Hence

$$S_l(t)\le S_l^{\max}, \quad l=1,\dots,\text{NL} \quad (5)$$

where S_l is the apparent power flow through the transmission line, and $S_l^{\max}$ is the upper thermal limit of the apparent power admitted by the transmission line.

3) Generation Limit Constraints: For stable operation, the real power output of each generator is restricted by lower and upper limits as follows:

$$P_i^{\min}\le P_i(t)\le P_i^{\max}. \quad (6)$$

4) Ramp Rate Limits: Increasing or decreasing the output generation of each unit is restricted to an amount of power over a time interval due to the physical limitations of each unit. The generator ramp rate limits change the effective real power operating limits as follows:

$$\begin{aligned}\max\left(P_i^{\min},P_i(t-1)-\text{DR}_i\right)&\le P_i(t)\\ P_i(t)&\le\min\left(P_i^{\max},P_i(t-1)+\text{UR}_i\right)\end{aligned} \quad (7)$$

where $P_i(t-1)$ is the output power of generator i in the previous dispatch.

5) Prohibited Operating Zones (POZ): Modern generators with valve point loading have many prohibited operating zones [5]. Therefore, in practical operation, when adjusting the generation output P_i of unit i, the operation of the unit in the prohibited zones must be avoided. The feasible operating zones of unit i can be described as follows:

$$\begin{aligned}P_i^{\min}&\le P_i(t)\le P_{i,1}^{\text{LB}}\\ P_{i,j}^{\text{LB}}&\le P_i(t)\le P_{i,j}^{\text{UB}}, \quad j=2,3,\dots,\text{NP}_i\\ P_{i,\text{NP}_i}^{\text{UB}}&\le P_i(t)\le P_i^{\max}\end{aligned} \quad (8)$$

where NP_i is the number of prohibited zones of unit i.

6) Spinning Reserve Constraints: Due to security reasons in case of unexpected outage of generating units or heavily loaded transmission lines, the committed generating units are not fully loaded: 5% to 10% of the capacity of each dispatchable unit is kept available in case of emergency situations. The prohibited operating zone (POZ) constraint heavily limits the flexibility of the respective units in providing spinning reserve [11]. So, the spinning reserve constraint, which applies only to the online units which are not restricted by POZ, is stated as follows:

$$\text{SR}_i=\begin{cases}\min\left\{\left(P_i^{\max}-P_i\right),\text{SR}_i^{\max}\right\}, & \forall i\in(\Omega-\Theta)\\ 0, & \forall i\in\Theta\end{cases} \quad (9)$$

$$\text{SR}=\sum_{i=1}^{n}\text{SR}_i \quad (10)$$

$$\text{SR}\ge\text{SR}_{\text{TOTAL}} \quad (11)$$

where SR_{TOTAL} is the total spinning reserve in the system (MW), SR_i is the spinning reserve contribution of unit i (MW), $SR_i^{\max}$ is the maximum spinning reserve contribution of unit i (MW), Ω is the set of all units online, and Θ is the set of all online units with POZ.

III. GAAPI FOR THE NCED PROBLEM

This paper focuses on the solution of the NCED problem with valve point loading, ramp rate limits, and prohibited operating zones, employing a hybrid method that incorporates favorable features of two powerful optimization algorithms: RCGA and API.

It is well known that metaheuristic algorithms like GAs, EP, SA, and PSO typically work well for small dimensional, less complex problems, but fail to locate the global optima for more complicated problems [26]. They have a very good search space covering, but a weak search capability around the global solution. On the other hand, API has a good "hill climbing behavior", but does not cover the solution space very well. Therefore, by incorporating these two algorithms into one technique, it is expected to create a method that combines their good features while overcoming their disadvantages. The GA used in the proposed algorithm is a simple real coded genetic algorithm (RCGA). The API algorithm and the RCGA that form the proposed GAAPI method are described in the remainder of this section.

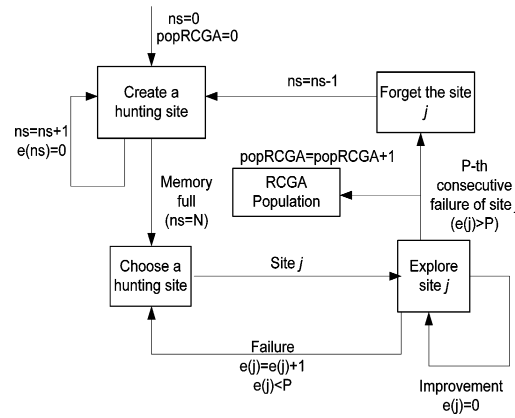

Fig. 2. Search mechanism of ants as used in the proposed method. *ns* is the counter for the sites memorized by one ant and increases from zero to N (the total number of sites an ant can memorize); $e(\cdot)$ is the counter for consecutive failures when searching one site (e.g., if site j was randomly chosen to be searched and the solution is not better than the previous one when searching the same site j, then $e(j)$ increases by one); P is the total number of consecutive search failures before one site is deleted from the memory of one ant; *popRCGA* is the counter for the population size in the RCGA algorithm.

A. Overview of the RCGA Algorithm

The main idea behind GAs is to improve a set of candidate solutions for a problem by using several genetic operators inspired from genetic evolution mechanisms observed in real life. Usually, the genetic operators used are *selection, crossover*, and *mutation*. The *selection* operator makes sure that the best member from a population survives. *Crossover* generates two new individuals (offspring) from two parent solutions, based on certain rules such as mixing them with a given probability. *Mutation* takes an individual and randomly changes a part of it with a certain probability [10].

In this paper, a real-coded genetic algorithm was adopted, considering the difficulties of binary representation when referring to a continuous search space with large dimensions. Therefore, the decision variable is represented by a real number within its lower and upper bounds.

The RCGA operators were set as follows: 1) *blend crossover* operator (BLX-α) with a probability of crossover of 0.3, and a value of α set to 0.366 [11]; 2) *uniform mutation* with a probability of mutation set to 0.35; 3) *elitism*: the best two individuals are retained with no modifications in the population of the next generation, such that the strongest genes up to this point are not lost.

A dynamic number of individuals is adopted for the population of RCGA, in a range between the total number of ants from the API algorithm and 1000. The role played by the RCGA algorithm is to give diversity in the solution generation accepting both feasible and infeasible solutions from the API algorithm. Thus, the population of the RCGA is dynamically formed from the best hunting sites in the memory of all ants (e.g., if the nest has ten ants, then the best site of each ant in the current generation is transferred to the population of the RCGA), as well as from the forgotten (erased) sites (unsuccessful sites which will be deleted from the ant's memory in the next iteration). However, because the population size cannot increase infinitely, it is bounded to 1000 individuals. When this number is exceeded (more forgotten sites than the difference between 1000 minus the number of ants), then, at first all duplicated individuals are deleted from the population (it is possible that two or more ants have the same best site or the same forgotten site, due to the overlapping search procedure of API). If the number of sites is still more than 1000, then the excess number of individuals is discarded randomly. The lower bound in the population size of the RCGA is ensured by the number of ants. Because only one generation is used with the RCGA algorithm to generate a solution, the diversity in the generation of a solution for the proposed GAAPI algorithm is ensured by this minimum number of individuals in the GA population being equal to or higher than the population of ants in the API algorithm.

B. Search Strategy of API

The API (a short for apicalis) algorithm is based on the natural behavior of pachycondyla apicalis ants described in [27]. The search mechanism of API can be summarized as shown in Fig. 2.

The following process takes place for each search agent (ant): initially, each ant checks its memory. If the number of hunting sites in its memory (*ns*) is less than the total number of sites an ant can memorize (N), then it will generate a new site in the small neighborhood of the center of the ant (the current position of the ant), save it to its memory, and use it in the next iteration as the next hunting site. Otherwise (memory of the ant is full, $\mathrm{ns} = N$), one of the sites in the memory of the ant is selected as the hunting site. The ant then performs a local random search around the neighborhood of this hunting site. If this local search is successful, the ant will repeat its exploration around the site (same site) until an unsuccessful search occurs; otherwise (if the previous exploration was unsuccessful), the ant will select an alternative site among its memorized sites. This process will be repeated until a termination criterion is reached. The termination criterion used in this phase is that the procedure will stop automatically once the number of successive unsuccessful explorations reaches a predefined value (P), or there is no improvement after a number of iterations. All the sites whose explorations reached this predefined value will go to the popula-

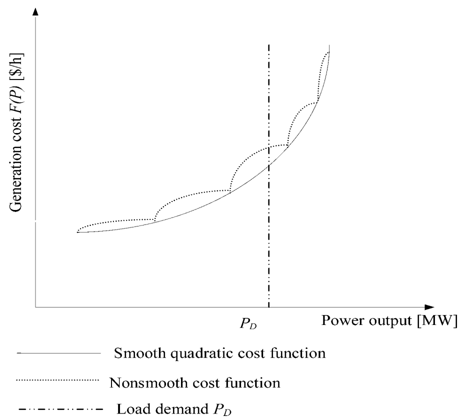

Fig. 3. Determining the first approximation of the ED solution.

tion of the RCGA, and, when the nest moves to a new position, they will be erased form the memory of the ant.

C. *GAAPI: The Proposed Solution*

The proposed hybrid GAAPI algorithm uses only one nest of ants. The first step of the proposed GAAPI algorithm is to find a starting point for the search (referred from here on as the initialization process). This starting point is given by the solution of the Lagrange multipliers method applied to the quadratic objective function (1). The reason for this choice is provided below.

If one uses a quadratic cost function or a generation function with valve point effects (Fig. 3) and ignores transmission losses and all other constraints (except the balance constraint), an approximate area containing the optimal solution may be identified. Taking the particular case presented in Fig. 3, the balance constraint is a straight line crossing the cost function at the point where the generation output equals the load demand (the geometrical view for one generator). With more constraints taken into account, this geometrical delimitation is more difficult to draw, especially when more constraints must be considered. Therefore, if at time t_0, the losses are computed using an approximate solution given for the economic dispatch problem for this time frame (t_0) and set as a constant value in the balance constraint equation, then this equation becomes linear. Then, using the quadratic approximation of the generation function, a good starting point for the next, more accurate search can be determined. This starting point is the optimal solution of the optimization system given by (1), (3), and (7).

Note that after the initialization process, each ant of the nest takes a different position according to their "experience" (e.g., some ants search/take positions closer to the nest if they are less experienced, while some others search in larger areas around the nest, up to the entire search space). The amplitude coefficient differs from one ant to another. The flowchart of the proposed GAAPI algorithm is given in Fig. 4 and the explanation of its functionality is described below.

The GAAPI algorithm uses a modified API procedure along with RCGA. The key modifications in API are summarized in points 1 and 2 below, while point 3 explains the RCGA procedure as modified to be used with API in the GAAPI algorithm.

1) Generation of New Nest: The initialization process refers only to one initial starting point (the initial position of the nest)

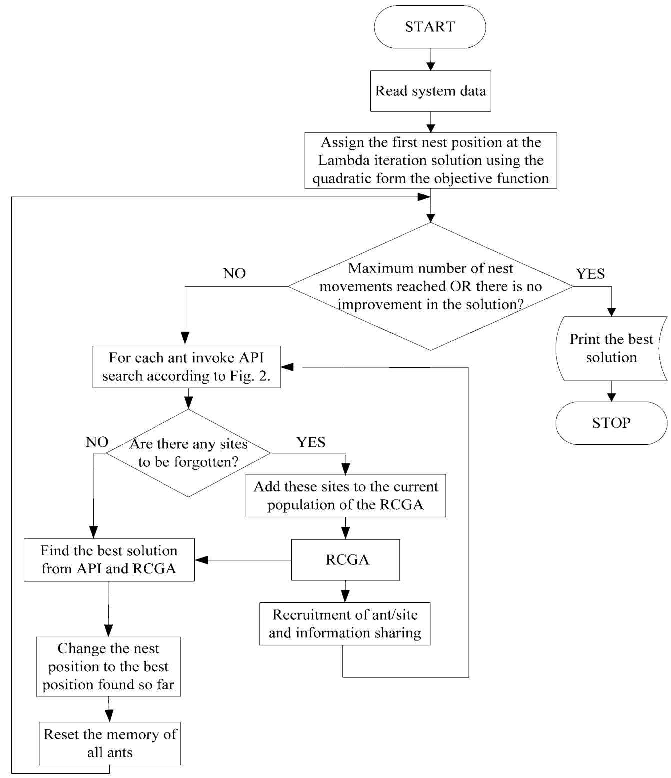

Fig. 4. GAAPI flowchart.

and this is generated with Lagrange multipliers method. After initialization with the approximate solution from the Lagrange multipliers method, the nest moves only to the best solution found in the current search cycle (between two consecutive movements of the nest). The "downhill" property is not very strong in this case, so the trapping in local minima is avoided.

2) Exploitation With API: For the sites which were explored unsuccessfully, a predefined number of consecutive iterations will be first memorized as part of the current population to be explored by the RCGA, and then erased from the memory of the ant.

3) Information Sharing With RCGA: In order to keep diversity in the solution space, information sharing is performed using a simple RCGA method. A random site is chosen in the memory of a randomly chosen ant, and it is replaced by the new RCGA solution. This can be seen as a form of communication. The RCGA procedure involves a population formed by the currently best hunting sites in the memory of all ants as well as the forgotten sites. The best solution obtained after one set of GA operations (selection, crossover, mutation) replaces the chosen site in the memory of the selected ants. This technique is applied before moving the nest to the best position so far. The RCGA contains the forgotten sites in order to keep diversity in the population.

D. *Constraint Handling in GAAPI*

There are different ways to handle constraints in a constrained optimization problem. One way is to use a penalty fitness function (optimization function) that aggregates the objective function with the constraint functions penalized [15], [17], [21].

However, the penalty parameter must be carefully chosen in order to distinguish between feasible and infeasible solutions. Sometimes this parameter tuning is a difficult task even when the problem is very well defined. Another way is to generate only feasible solutions and work only with feasible solutions during the search process. The second procedure was adopted in this paper and is briefly described below.

First, an initial solution, $P = (P_1, P_2, \ldots, P_i, \ldots, P_n)$ is generated respecting generation limits and prohibited operating zones, according to

$$P_i = P_i^{\min} + \mathrm{rand} * \left(P_i^{\max} - P_i^{\min}\right). \tag{12}$$

While the balance constraint considering losses as shown in (3) is not satisfied, a random generator is chosen as slack from the pool of n generators, and its output is fixed to meet the balance. If its limits are exceeded, then another random slack is chosen from the $(n-1)$ pool. If all the generators are checked and no one can cover the difference to meet the balance, then two generators will be chosen as slack and share the difference, and so on. When a generator is in a prohibited zone, then its output is fixed to the closest feasible bound.

IV. Numerical Results and Analysis

A. Benchmark Power Systems

In order to validate the proposed GAAPI method that solves the NCED problem, four test power systems have been used. The first test system is a 3-unit system with a valve point effect cost of generation and only the balance constraint considered. The data for this system are given in [18]. The minimum cost found for this system is 8234.07 $/h [20].

The second test system is a 6-unit power system (obtained from the IEEE 30-bus test system) and having a demand of 1263 MW [5]. The cost of generation for this system is chosen to be a smooth (quadratic) function and the nonconvexity is given by the prohibited operating zones and ramp rate limits. The reason for choosing these generator characteristics is to compare the results with other similar metaheuristic methods described in [5] and [21]. Then, the complexity of the problem is increased by using the cost of generation with valve point effect included, as in (2). The comparison for this second case covers implementations of a binary GA and a simple RCGA and compared to the best cost obtained with the GAAPI method. The data of this test system can be found in [25].

The third test system is a 15-unit system with smooth (quadratic) cost of generation, prohibited operating zones and ramp rate limits and having a demand of 2630 MW. The system data were taken from [5]. The minimum generation cost reported so far for this system is 32 751.39 $/h [17].

The fourth test system is a larger system with 40 units, a nonconvex generation function with valve point effect, and considering power losses. The load demand for this system is 10 500 MW [5], [21]. It seems that this system has not been tested by other researchers using constraints such as transmission losses. This system was chosen to demonstrate the applicability of the proposed algorithm in relatively large and complex systems. The B-loss coefficients used to compute the transmission losses of this system were derived from the B-loss coefficients of the 6-generator test system [25], by multiplication on rows and columns up to 40 units.

B. Parameter Settings

1) Parameters of API: The number of ants to perform the search is directly proportional to the dimension of the system (number of generating units). For all the test systems, the number of ants is ten times the number of generating units. Therefore, for a 3-generator test system, the number of search agents in API is $3 \times 10 = 30$ ants. The number of hunting sites which each ant can memorize is five $(N = 5)$ (as suggested in [27]). The number of consecutive search failures of each site in the memory of ants is five $(P = 5)$. The maximum number of site exploitations (searches) is directly proportional to the dimension of the system (five times the number of generating units, e.g., for a 3-generator test system, this number is $3 \times 5 = 15$).

2) Parameters of RCGA: The population size is set dynamically between the number of ants from API and 1000. The population size is a function of the number of the forgotten sites appearing during each movement of the nest. Having a variable population size of RCGA aids in increasing the probability of the generated solution being different than the API-generated solution, thus triggering the search in a region less explored (in the case of large RCGA population). In case that the API search improves the solution in an adequate pace, the role of RCGA is limited by its small population size (less diversity in the solution). The probability of crossover is 0.3 and the probability of mutation is 0.35; the factor α of the blend crossover operator is 0.366.

C. Convergence and Robustness Tests

In the power systems literature, the convergence tests in the field of economic dispatch are mainly related to the number of iterations or generations (e.g., in the case of GAs and PSO) until the solution falls below a certain threshold, and/or related to the CPU time per iteration/generation [17], [18], [20], [21], [28]. However, the number of iterations (generations) does not provide adequate information about the computational effort needed to perform this specific task in order to have the same base of comparison. The CPU time is subject to the computer infrastructure available, and therefore, it is a parameter that is difficult to be used as an evaluation criterion. Thus, the measure of the speed of convergence adopted in this paper is the mean number of (objective) function evaluations (denoted as *M-num-fun*) until the algorithm stops [26]. The mean number of function evaluations is defined as the average of the total number of function evaluations during a predefined number of independent runs of the algorithm. In other words, if the number of function evaluations in the ith independent run of the proposed algorithm is denoted as nF_i and there are a total of M runs which need to be taken into account in the evaluation process, then the mean number of function evaluations is

$$\mathrm{M{-}num{-}fun} = \frac{\sum_{i=1}^{M} \mathrm{nF}_i}{M}. \tag{13}$$

TABLE I
CHARACTERISTICS OF RASTRIGIN, ACKLEY, AND GRIENWANGK BENCHMARK FUNCTIONS

Function name/mathematical form	Description
Rastrigin $F_1 = \sum_{i=1}^{n}(x_i^2 - 10\cdot\cos(2\pi i) + 10)$	Highly multimodal function with deep local minima regularly distributed. The global minimum is at $x^* = 0$ and $f(x^*) = 0$ (n=30).
Ackley $F_2 = -20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2}\right) - \exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi i)\right) + 20 + \exp(1)$	Multimodal function with deep local minima. The global minimum of this function is $x^* = 0$ and $f(x^*) = 0$. The variables of this function are independent (n=30).
Grienwangk $F_3 = \frac{1}{4000}\sum_{i=1}^{n}x_i^2 - \prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	Multimodal function with a global minimum at $x^* = 0$ and $f(x^*) = 0$. Significant interaction between its variables due to the product term (n=30).

TABLE II
COMPARISON WITH OTHER HEURISTIC METHODS FOR THREE BENCHMARK FUNCTIONS

Function	Algorithm	M-num-fun	M-best	Std	Opt-F
Rastrigin	OGA/Q	224710	0	0	0
	M-L	305899	121.7575	7.75	
	LEA	223803	2.10E-8	3.3E-18	
	GAAPI	**24714**	**1.02E-6**	**4.58E-9**	
Ackley	OGA/Q	112421	4.4E-16	3.9E-17	
	M-L	121435	2.5993	0.094	
	LEA	105926	3.2E-16	3.0E-17	
	GAAPI	**19592**	**0.000163**	**9.1E-7**	
Grienwangk	OGA/Q	134000	0	0	
	M-L	151281	0.11894	0.0104	
	LEA	130498	6.10E-6	2.5E-17	
	GAAPI	**29647**	**2.2E-05**	**2.05E-8**	

As a means of demonstrating the convergence and robustness properties of the proposed optimization algorithm, three well-known complex, nonsmooth functions (Rastrigin, Ackley, and Grienwangk) have been used as test functions. The characteristics and the mathematical form of these functions are provided in Table I.

In this work, the robustness of the solution is characterized by two indices: 1) the standard deviation (denoted as *Std* in Table II) and 2) the average best value (denoted as *M-best* in Table II). These two indices are calculated from all optimal solutions found by the algorithm in a number of predefined independent runs (in this case, 50 runs). Note that all the algorithms in Table II terminate when the best solution cannot be improved further in 50 successive generations (nest movements) or when the optimum solution is reached.

A comparison of the proposed GAAPI algorithm with similar search techniques is given in Table II. Note that in Table II, OGA/Q denotes the orthogonal genetic algorithm with quantization [29]; M-L is the modified mean-level-set method [26]; LEA is the level-set evolution; and Latin squares algorithm [26]. *Opt-F* denotes the optimum known solution of the function to be optimized.

As it may be observed from Table II, GAAPI is a promising solution for problems which need fast, near real time convergence, and robustness of the solution as it has always the minimum number of function evaluations until the solution is close to the global known optimum. The standard deviation is also lower than most of the other algorithms used in this comparison.

TABLE III
SIX-GENERATOR TEST SYSTEM, SMOOTH COST: COMPARISON ON ROBUSTNESS

Method	**Max** (\$/h)	**Min** (\$/h)	**Average** (\$/h)
GA binary	15519.87	15451.66	15469.21
GA	15524.00	15459.00	15469.00
NPSO-LRS	15455.00	15450.0	15454.00
SOHPSO	15609.64	15446.02*	15497.35
GAAPI	15449.85	15449.78	15449.81

(*) The loss value computed with the B-Loss formula (12.95 MW) is higher than the one given by the author (12.55 MW) [20] which can lead to a higher minimum value of the cost of generation than the one reported in [20].

TABLE IV
STATISTICAL RESULTS FOR THE 15-GENERATOR TEST SYSTEM

Method	**Max** (\$/h)	**Min** (\$/h)	**Average** (\$/h)
GA	33337.00	33113.00	33228.00
SOHPSO	32945.00	32751.00	32878.00
GAAPI	32756.01	32732.95	32735.06

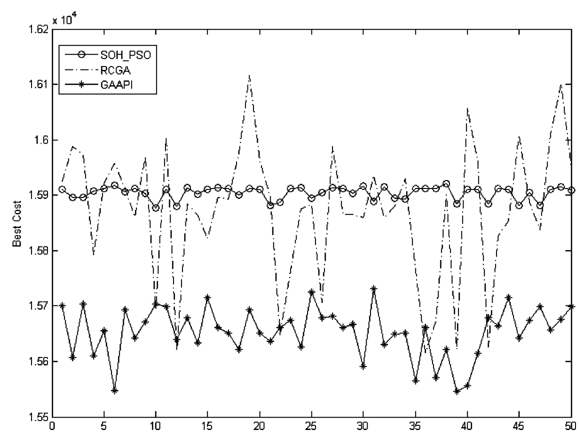

Fig. 5. Comparison on consistency of results over 50 independent runs: 6-generator test system with nonsmooth cost of generation.

Robustness tests were also carried out for two of the test power systems chosen for analysis in this paper: the second (Table III) and the third test system (Table IV). The measures used to emphasize the robustness in this case were the minimum (min), maximum (max), and the average values gathered during 50 independent runs of the algorithms. The average solution indicates the consistency of the best solution over the independent trials, always satisfying the equality and inequality constraints. Note that GAAPI gives again the best average and the best minimum cost over the independent runs of the algorithm or comparable results with other powerful heuristic methods.

In order to demonstrate the consistency of the results of the GAAPI algorithm over independent runs, a plot of the distribution of the best cost found by GAAPI for the 6-generator test system with nonconvex cost of generation is given in Fig. 5. In the same graph, the optimum cost of two other recent

TABLE V
THREE-GENERATOR TEST SYSTEM: BEST SOLUTION FOR A SMOOTH COST FUNCTION

Unit output (MW)	GA	IEP	EP	MPSO	GAAPI
P_1	300.00	300.23	300.26	300.27	300.25
P_2	400.00	400.00	400.00	400.00	399.98
P_3	150.00	149.77	149.74	149.73	149.77
Total output	850.00	850.00	850.00	850.00	850.00
Generation cost ($/h)	8237.60	8234.09	8234.07	8234.07	8234.07

TABLE VI
SIX-GENERATOR TEST SYSTEM: BEST SOLUTION FOR A SMOOTH COST FUNCTION

Unit output (MW)	LM	GA binary	RCGA	NPSO-LRS	SOH-PSO	GAAPI
P_1	447.00	456.46	474.81	446.96	447.49	447.12
P_2	173.50	168.26	178.64	173.39	173.32	173.41
P_3	264.00	258.68	262.21	262.34	263.47	264.11
P_4	138.50	132.66	134.28	139.51	139.06	138.31
P_5	166.04	170.97	151.90	164.70	165.47	166.02
P_6	87.00	89.10	74.18	89.01	87.13	87.00
Losses	13.00	13.13	13.02	12.93	12.55*	12.98
Total output	1276.00	1276.13	1276.03	1275.94	1275.55	1275.97
Generation cost ($/h)	15450.00	15451.66	15459.00	15450.0	15446.02	15449.7

(*) The loss value computed with the B-Loss formula (12.95 MW) is higher than the one given by the authors (12.55 MW) in [20].

evolutionary based algorithms (the authors' implementation of SOH-PSO [20] and RCGA [11]) is plotted for the same number of independent runs. It can be observed that GAAPI outperforms the other two functions in terms of the minimum cost of generation. Further, it is consistently giving the same result (even though SOH-PSO appears to be slightly more consistent). The proposed algorithm is clearly more consistent than the RCGA.

For the smooth 6-generator test system (Table III), it can be noticed that GAAPI gives comparable results with the NPSO-LRS, SOHPSO in terms of the minimum best solution, and better average than all other methods used in the comparison table. *GA binary* refers to the GA optimization package from MATLAB. The results used for comparison in the case of GA, NPSO-LRS, and SOHPSO were obtained from [20] and [30].

For the 15-generator test system (Table IV), it can be noticed that GAAPI gives the best results compared to the GA and SOHPSO [20] methods in terms of both minimum value and average value found in 50 independent runs.

D. Comparison of the Best Solution: Power System Benchmarks

For the first three test systems used in this work, the best solutions obtained in a predefined number of independent runs (in this work this number is 50) are compared to the corresponding values reported in the literature (Tables V–VIII). The fourth test power system, including constraints, seems to have not been used in the literature. The best solution determined using the GAAPI algorithm (in 50 independent runs) for this last test system is provided in Table IX.

TABLE VII
SIX-GENERATOR TEST SYSTEM: BEST SOLUTION FOR A NONCONVEX COST FUNCTION

Unit output (MW)	SOH-PSO	RCGA	GAAPI
P_1	419.64	495.09	499.98
P_2	188.16	150.45	199.89
P_3	198.15	223.11	225.75
P_4	150.00	149.40	124.95
P_5	200.00	147.94	150.19
P_6	120.00	109.72	74.97
Losses	12.95	12.07	13.13
Total power output	1275.95	1275.70	1276.13
Total generation cost ($/h)	15896.73	15634.70	**15607.47**

TABLE VIII
FIFTEEN-GENERATOR TEST SYSTEM: BEST SOLUTION FOR A SMOOTH COST FUNCTION

Unit output (MW)	PSO	SOH-PSO	GAAPI
P_1	455.00	455.00	454.70
P_2	380.00	380.00	380.00
P_3	130.00	130.00	130.00
P_4	129.28	130.00	129.53
P_5	164.77	170.00	170.00
P_6	460.00	459.96	460.00
P_7	424.52	430.00	429.71
P_8	60.00	117.53	75.35
P_9	25.00	77.90	34.96
P_{10}	160.00	119.54	160.00
P_{11}	80.00	54.50	79.75
P_{12}	72.62	80.00	80.00
P_{13}	25.00	25.00	34.21
P_{14}	44.83	17.86	21.14
P_{15}	49.42	15.00	21.02
Losses	30.49	32.28	30.36
Total power output	2660.44	2662.29	2660.36
Total generation cost ($/h)	32798.69	32751.39	**32732.95**

TABLE IX
FORTY-GENERATOR TEST SYSTEM: BEST SOLUTION FOR A NONCONVEX COST FUNCTION

Unit output (MW)	GAAPI	Unit output (MW)	GAAPI
P_1	114.00	P_{21}	550.00
P_2	114.00	P_{22}	550.00
P_3	120.00	P_{23}	550.00
P_4	190.00	P_{24}	550.00
P_5	97.00	P_{25}	550.00
P_6	140.00	P_{26}	550.00
P_7	300.00	P_{27}	11.44
P_8	300.00	P_{28}	11.56
P_9	300.00	P_{29}	11.42
P_{10}	205.25	P_{30}	97.00
P_{11}	226.30	P_{31}	190.00
P_{12}	204.72	P_{32}	190.00
P_{13}	346.48	P_{33}	190.00
P_{14}	434.32	P_{34}	200.00
P_{15}	431.34	P_{35}	200.00
P_{16}	440.22	P_{36}	200.00
P_{17}	500.00	P_{37}	110.00
P_{18}	500.00	P_{38}	110.00
P_{19}	550.00	P_{39}	110.00
P_{20}	550.00	P_{40}	550.00
Losses		1045.06	
Total power output		11545.06	
Total generation cost ($/h)		139864.96	

V. Conclusion

This paper presents a novel algorithm, entitled GAAPI, to solve the nonconvex economic load dispatch problem. The proposed algorithm emerges from the hybridization process of the GA and API strategies. It is designed in such a way that the various system constraints may be modeled and respected. It is also shown that starting from the solution obtained for the quadratic cost function (Lagrange multipliers method), the search space is reduced, and implicitly, the computational effort is reduced. The strategy for handling the constraints is to always generate feasible solutions and work only with these feasible solutions during the search process. Compared to the penalty method, this strategy has the advantage of not dealing with other parameter settings that complicate user ability to use the method.

The proposed algorithm is proven to always find comparable or better solutions in a number of independent trials, as compared to other methods available in the power systems literature. GAAPI has provided the global solution, both in test functions and test power systems, always satisfying the constraints. Further, through the test cases presented, its superiority in robustness is evident: it has a high probability to reach the global or quasi-global solution, especially in nonconvex formulations. GAAPI converges smoothly to the global, avoiding fast convergence that may lead to local optima.

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