

Cost

$$\min F = \sum (a_j + b_j P_j + c_j P_j^2) + \left| e_j \sin(f_j(P_j^{min} - P_j)) \right|$$

Subject to:

$$\sum P_j = P_D + P_L$$

Where P_L is the transmission loss given by

$$P_L = 1e - 2 (\sum \sum P_j B_{ij} P_i + \sum B_{oj} P_j + B_{oo}) \rightarrow \text{Loss eqn.1}$$

From [3], accounting for the 100 MVa Baseline in the B-coefficients

$$P_L = (1e - 4 \sum \sum P_j B_{ij} P_i + 1e - 2 \sum B_{oj} P_j + B_{oo})$$

HGWO [5] uses the loss eqn.1.

6-generator testcase

- Data is taken from [1] and [2]
- Population size=50, iterations=200

Generating unit	a_j (\$/hr)	b_j (\$/hr-MW)	c_j (\$/hr-MW ²)	P_j^{min} (MW)	POZ (MW)	P_j^{max} (MW)	Gen eqn.1	Gen eqn.2
1	240	7	0.007	100	[210, 240]; [350, 380]	500	447.0699	447.4853
2	200	10	0.0095	50	[90, 110]; [140, 160]	200	173.1795	173.2752
3	220	8.5	0.009	80	[150, 170]; [210, 240]	300	263.9154	263.5781
4	200	11	0.009	50	[80, 90]; [110, 120]	150	139.0487	139.0478
5	220	10.5	0.008	50	[90, 110]; [140, 150]	200	165.5727	165.5339
6	190	12	0.0075	50	[75, 85]; [100, 105]	120	86.6191	87.0292
Loss							12.4152	12.95951
Cost							15,442.52	15,449.76

15 generator

- Data is taken from [1] and [2]
- [2] highlights a B co-efficient which should be corrected. Most papers, use the data from the original paper [1] which leads to a lower loss value and consequently a lower cost.

- Population size=50, iterations=200

Generatin g unit	<i>aj</i> <i>(\$/hr)</i>	<i>bj</i> <i>(\$/hr-MW)</i>	<i>cj</i> <i>(\$/hr-MW2)</i>	(MW)	(MW)	Gen eqn.1	Gen eqn.2
1	0.00029900	10.10	671	280	455	455	454.9994
2	0.00018300	10.20	574	180	380	379.9999	380
3	0.00112600	8.80	374	20	130	130	129.9998
4	0.00112600	8.80	374	20	130	129.9997	130
5	0.00020500	10.40	461	150	170	169.9998	169.9999
6	0.00030100	10.10	630	280	460	459.9999	460
7	0.00036400	9.80	548	230	430	430	429.9988
8	0.00033800	11.20	227	60	160	69.6557	71.6818
9	0.00080700	11.20	173	25	162	59.9181	58.9711
10	0.00120300	10.70	175	25	160	159.9996	160
11	0.00358600	10.20	186	20	80	80	79.9998
12	0.00551300	9.90	230	20	80	80	79.9999
13	0.00037100	13.10	225	25	85	25.0001	25
14	0.00192900	12.10	309	15	55	15	15
15	0.00444700	12.40	323	15	55	15.0019	15
loss						29.5847	30.6606
cost						32692.2795	32704.3336

40 generator (no loss)

- Data is taken from [4]
- Includes valve-point effects
- Population size=50, iterations=300
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Generating unit	<i>aj</i> <i>(\$/hr)</i>	<i>bj</i> <i>(\$/hr-MW)</i>	<i>cj</i> <i>(\$/hr-MW2)</i>	<i>ej</i> <i>(\$/hr)</i>	<i>ff</i> <i>(\$/hr-MW)</i>	(MW)	(MW)	Gen
1	0.0069	6.73	94.705	100	0.084	36	114	110.7998

2	0.0069	6.73	94.705	100	0.084	36	114	110.7998
3	0.02028	7.07	309.54	100	0.084	60	120	97.3999
4	0.00942	8.18	369.03	150	0.063	80	190	179.7331
5	0.0114	5.35	148.89	120	0.077	47	97	87.7999
6	0.01142	8.05	222.33	100	0.084	68	140	140.0000
7	0.00357	8.03	287.71	200	0.042	11	300	259.5997
8	0.00492	6.99	391.98	200	0.042	13	300	284.5997
9	0.00573	6.6	455.76	200	0.042	13	300	284.5997
10	0.00605	12.9	722.82	200	0.042	13	300	130.0000
11	0.00515	12.9	635.2	200	0.042	94	375	94.0000
12	0.00569	12.8	654.69	200	0.042	94	375	94.0000
13	0.00421	12.5	913.4	300	0.035	12	500	214.7598
14	0.00752	8.84	1760.4	300	0.035	12	500	394.2794
15	0.00708	9.15	1728.3	300	0.035	12	500	394.2794
16	0.00708	9.15	1728.3	300	0.035	12	500	394.2794
17	0.00313	7.97	647.85	300	0.035	22	500	489.2794
18	0.00313	7.95	649.69	300	0.035	22	500	489.2794
19	0.00313	7.97	647.83	300	0.035	24	550	511.2794
20	0.00313	7.97	647.81	300	0.035	242	550	511.2794
21	0.00298	6.63	785.96	300	0.035	254	550	523.2794
22	0.00298	6.63	785.96	300	0.035	254	550	523.2794
23	0.00284	6.66	794.53	300	0.035	254	550	523.2794
24	0.00284	6.66	794.53	300	0.035	254	550	523.2794
25	0.00277	7.1	801.32	300	0.035	254	550	523.2794
26	0.00277	7.1	801.32	300	0.035	254	550	523.2794
27	0.52124	3.33	1055.1	120	0.077	10	150	10.0000
28	0.52124	3.33	1055.1	120	0.077	10	150	10.0000
29	0.52124	3.33	1055.1	120	0.077	10	150	10.0000
30	0.0114	5.35	148.89	120	0.077	47	97	87.7999
31	0.0016	6.43	222.92	150	0.063	60	190	190.0000
32	0.0016	6.43	222.92	150	0.063	60	190	190.0000
33	0.0016	6.43	222.92	150	0.063	60	190	190.0000
34	0.0001	8.95	107.87	200	0.042	90	200	164.7998
35	0.0001	8.62	116.58	200	0.042	90	200	194.3878

36	0.0001	8.62	116.58	200	0.042	90	200	200.0000
37	0.0161	5.88	307.45	80	0.098	25	110	110.0000
38	0.0161	5.88	307.45	80	0.098	25	110	110.0000
39	0.0161	5.88	307.45	80	0.098	25	110	110.0000
40	0.00313	7.97	647.83	300	0.035	242	550	511.2794
Cost							121,412.4219	

40 generator with loss

- Data is taken from [4]
- Includes valve-point effects
- Population size=50, iterations=300

Gener ating unit	a_j (\$/hr)	b_j (\$/hr- MW)	c_j (\$/hr- MW ²)	e_j (\$/hr)	f_j (\$/hr- MW)	(MW)	(MW)	Gen (eqn.1)	Gen (eqn.2)
1	0.0069	6.73	94.705	100	0.084	36	114	114.0000	113.9647
2	0.0069	6.73	94.705	100	0.084	36	114	113.9998	111.1133
3	0.02028	7.07	309.54	100	0.084	60	120	120.0000	119.9999
4	0.00942	8.18	369.03	150	0.063	80	190	182.8522	189.9948
5	0.0114	5.35	148.89	120	0.077	47	97	87.8070	96.8481
6	0.01142	8.05	222.33	100	0.084	68	140	105.3999	105.3869
7	0.00357	8.03	287.71	200	0.042	11	300	300.0000	299.9977
8	0.00492	6.99	391.98	200	0.042	13	300	300.0000	288.5100
9	0.00573	6.6	455.76	200	0.042	13	300	300.0000	300.0000
10	0.00605	12.9	722.82	200	0.042	13	300	279.5997	279.5868
11	0.00515	12.9	635.2	200	0.042	94	375	243.5996	243.5976

12	0.00569	12.8	654.69	200	0.042	94	375	168.7998	94.0013
13	0.00421	12.5	913.4	300	0.035	12	500	484.0392	484.0295
14	0.00752	8.84	1760.4	300	0.035	12	500	484.0392	484.0570
15	0.00708	9.15	1728.3	300	0.035	12	500	484.0392	484.0355
16	0.00708	9.15	1728.3	300	0.035	12	500	484.0392	484.0513
17	0.00313	7.97	647.85	300	0.035	22	500	489.2794	489.3240
18	0.00313	7.95	649.69	300	0.035	22	500	489.2794	489.2101
19	0.00313	7.97	647.83	300	0.035	24	550	511.2794	511.2463
20	0.00313	7.97	647.81	300	0.035	242	550	511.2796	511.2913
21	0.00298	6.63	785.96	300	0.035	254	550	523.2794	523.4741
22	0.00298	6.63	785.96	300	0.035	254	550	550.0000	523.3258
23	0.00284	6.66	794.53	300	0.035	254	550	523.2794	523.3181
24	0.00284	6.66	794.53	300	0.035	254	550	523.2794	523.3837
25	0.00277	7.1	801.32	300	0.035	254	550	523.2794	536.8175
26	0.00277	7.1	801.32	300	0.035	254	550	523.2794	523.3137
27	0.52124	3.33	1055.1	120	0.077	10	150	10.0000	10.0235
28	0.52124	3.33	1055.1	120	0.077	10	150	10.0000	10.0007
29	0.52124	3.33	1055.1	120	0.077	10	150	10.0000	10.0164
30	0.0114	5.35	148.89	120	0.077	47	97	87.7999	87.8471
31	0.0016	6.43	222.92	150	0.063	60	190	190.0000	189.9901
32	0.0016	6.43	222.92	150	0.063	60	190	190.0000	189.9284
33	0.0016	6.43	222.92	150	0.063	60	190	190.0000	189.9996
34	0.0001	8.95	107.87	200	0.042	90	200	200.0000	199.9924
35	0.0001	8.62	116.58	200	0.042	90	200	164.7998	199.9772
36	0.0001	8.62	116.58	200	0.042	90	200	164.7998	164.7575
37	0.0161	5.88	307.45	80	0.098	25	110	110.0000	109.9982
38	0.0161	5.88	307.45	80	0.098	25	110	110.0000	109.8647
39	0.0161	5.88	307.45	80	0.098	25	110	110.0000	110.0000
40	0.00313	7.97	647.83	300	0.035	242	550	511.2794	549.8566
Loss								978.4181	966.0064
Cost								136,622.6632	136,603.8729

80 generator (no loss)

- Data is obtained by repeating the 40-generator case 1 (no loss) by 2.
- Population size=50, iterations=600

Generating unit	Gen	Generating unit	Gen
1	110.7998	41	110.7998
2	110.7998	42	110.7998
3	97.3999	43	97.3999
4	179.7331	44	179.7331
5	87.7999	45	87.7999
6	140.0000	46	140.0000
7	259.5997	47	259.5997
8	284.5997	48	284.5997
9	284.5997	49	284.5997

10	130.0000	50	130.0000
11	94.0000	51	94.0000
12	94.0000	52	94.0000
13	214.7598	53	214.7598
14	394.2794	54	394.2794
15	394.2794	55	394.2794
16	394.2794	56	394.2794
17	489.2794	57	489.2794
18	489.2794	58	489.2794
19	511.2794	59	511.2794
20	511.2794	60	511.2794
21	523.2794	61	523.2794
22	523.2794	62	523.2794
23	523.2794	63	523.2794
24	523.2794	64	523.2794
25	523.2794	65	523.2794
26	523.2794	66	523.2794
27	10.0000	67	10.0000
28	10.0000	68	10.0000
29	10.0000	69	10.0000
30	87.7999	70	87.7999
31	190.0000	71	190.0000
32	190.0000	72	190.0000
33	190.0000	73	190.0000
34	164.7998	74	164.7998
35	194.3876	75	194.3980
36	200.0000	76	200.0000
37	110.0000	77	110.0000
38	110.0000	78	110.0000
39	110.0000	79	110.0000
40	511.2794	80	511.2794
Cost			242,824.9577

References:

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