

See discussions, stats, and author profiles for this publication at: <https://www.researchgate.net/publication/260523554>

Fast –Iteration Method for Economic Dispatch With Prohibited Operating Zones

Article *in* Power Systems, IEEE Transactions on · March 2014

DOI: 10.1109/TPWRS.2013.2287995

CITATIONS

116

READS

1,604

4 authors, including:



Junpeng Zhan

Alfred University

31 PUBLICATIONS 1,319 CITATIONS

SEE PROFILE

Fast λ -Iteration Method for Economic Dispatch With Prohibited Operating Zones

J. P. Zhan, Q. H. Wu, *Fellow, IEEE*, C. X. Guo, *Member, IEEE*, and X. X. Zhou, *Fellow, IEEE*

Abstract—This letter presents a new method, the fast λ -iteration (F λ I) method, to solve the economic dispatch (ED) problem considering the prohibited operating zones (POZs) and ramp rate limits of generation units. Necessary conditions for the optimal solution of the ED problem are presented and proved. The efficiency of the method has been verified on a 15-unit system and a Korea 140-unit system.

Index Terms—Economic dispatch, fast λ -iteration, necessary condition, optimal solution, prohibited operating zone.

I. INTRODUCTION

THE prohibited operating zone (POZ) exists in a thermal or hydro generating unit due to the physical limits of power plant components, e.g., vibrations in a shaft bearing are amplified in a certain operating region. The POZs make the solution space of the economic dispatch (ED) problem discontinuous, and consequently most of mathematical programming methods fail to tackle this problem. This letter presents a fast λ -iteration (F λ I) method to solve the problem.

Considering the constraints of the generation limits, the power load balance, the ramp rate limits, and the POZs, the ED model can be formulated as

$$\text{Min : } F_I(\mathbf{X}) = \sum_{g \in I} f_g(P_g) = \sum_{g \in I} (a_g + b_g P_g + c_g P_g^2) \quad (1)$$

$$\text{s.t. } P_g^{\min} \leq P_g \leq P_g^{\max}, \sum_{g \in I} P_g - P_D - P_L = 0 \quad (2)$$

$$P_g - P_g^0 \leq UR_g, \quad P_g^0 - P_g \leq DR_g \quad (3)$$

$$P_g^{\min} \leq P_g \leq P_g^{1L} \text{ or } P_g^{1R} \leq P_g \leq P_g^{2L}$$

$$\text{or } \dots \text{ or } P_g^{Q_g R} \leq P_g \leq P_g^{\max} \quad (4)$$

where $F_I(\mathbf{X})$ denotes the cost objective; N the number of generation units; $I = \{1, 2, \dots, N\}$; \mathbf{X} a solution (P_1, P_2, \dots, P_N) ; subscript g Unit g ; P_g the power output; a_g , b_g , and c_g the coefficients of fuel cost function; P_g^{\min} and P_g^{\max} the lower and upper power output limits, respectively; P_D the load demand; P_g^0 the initial power output; UR_g and DR_g the ramp up and down rate limits, respectively; Q_g the number of

POZs in Unit g ; P_g^{qL} and P_g^{qR} ($q = 1, 2, \dots, Q_g$) the lower and upper boundaries of the q st POZ of Unit g , respectively. P_L denotes the transmission line loss, which can be calculated based on the Kron's loss formula as follows [1]:

$$P_L = \sum_{i=1}^N \sum_{j=1}^N P_i B_{ij} P_j + \sum_{i=1}^N B_{i0} P_i + B_{00} \quad (5)$$

where B_{ij} , B_{i0} , and B_{00} are B-coefficients.

II. NECESSARY CONDITIONS AND THE F λ I METHOD

In the following, two necessary conditions for a solution to be an optimal solution of (1)–(5) where $P_L = 0$ are given and proved:

- 1) If $\mathbf{X}^0 = (P_1^0, P_2^0, \dots, P_N^0)$ is an optimal solution of the original model (1)–(5), then any sub-solution $\mathbf{X}^1 = (P_{i_1}^0, P_{i_2}^0, \dots, P_{i_k}^0)$ where $\{i_1, i_2, \dots, i_k\} = I_1 \subsetneq I$ should be an optimal solution of (1)–(5) where I is replaced by I_1 .
- 2) In an optimal solution of the original model, each unit's output should have the equal incremental cost value or be located at a generation boundary or be located at a POZ boundary.

Proof 1): Proof by contradiction, suppose that \mathbf{X}^1 is not optimal. Then there exists a sub-solution \mathbf{X}^2 such that $F_{I_1}(\mathbf{X}^2) < F_{I_1}(\mathbf{X}^1) \Rightarrow F_{I_1}(\mathbf{X}^2) + F_{I-I_1}(\mathbf{X}^0) < F_{I_1}(\mathbf{X}^1) + F_{I-I_1}(\mathbf{X}^0) = F_I(\mathbf{X}^0)$. Thus there exists another solution having a smaller objective value than \mathbf{X}^0 . There is a contradiction. \square

Proof 2): The proof consists of two parts. **Part 1:** According to 1) and the equal incremental cost criterion [1], each unit's output in an optimal solution of the ED problem having no POZ should have the equal incremental cost value or be located at a generation boundary.

Part 2: The original model (1)–(5) can be decomposed into $\sum_{g=1}^N (Q_g + 1)$ sub-models by restricting the power output of each unit to only one continuous operating zone. A sub-model can be expressed as (1)–(3), (5) plus only one of the $(Q_g + 1)$ constraints in (4). Then in each sub-model, each unit has only one continuous operating zone and has no POZ. An example of both the original model and the sub-model is plotted in sub-figures (a) and (b) of Fig. 1, respectively.

According to Part 1 of Proof 2), for an optimal solution of the sub-model, each unit's output should have the equal incremental cost value or be located at a generation boundary. Considering that the generation boundaries of the sub-models are the generation boundaries or the POZ boundaries of the original model and that the optimal solution of the original model must be the optimal solution of one of the sub-models, we can come to the conclusion stated in 2). \square

According to the necessary conditions given above, if a unit's output is located at a POZ, then it should be modified to a boundary of the POZ. The question is that which is its best output, the upper or lower boundary? For the convenience of illustration, the first derivative cost functions of 3 units are plotted in Fig. 1, where the coordinates of the 4 points marked are:

Manuscript received April 24, 2013; revised July 12, 2013; accepted August 13, 2013. Date of publication November 05, 2013; date of current version February 14, 2014. This work was supported in part by the China Scholarship Council and in part by Guangdong Innovative Research Team Program, China. Paper no. PESL-00066-2013.

J. P. Zhan is with the College of Electrical Engineering, Zhejiang University, Hangzhou 310027, China. He is also with the Department of Electrical Engineering and Electronics, University of Liverpool, Liverpool L69 3GJ, U.K.

Q. H. Wu is with the South China University of Technology, Guangzhou 510641, China. He is also with Department of Electrical Engineering and Electronics, University of Liverpool, Liverpool L69 3GJ, U.K.

C. X. Guo is with the College of Electrical Engineering, Zhejiang University, Hangzhou 310027, China.

X. X. Zhou is with China Electric Power Research Institute, State Grid Corporation of China, China.

Color versions of one or more of the figures in this paper are available online at <http://ieeexplore.ieee.org>.

Digital Object Identifier 10.1109/TPWRS.2013.2287995

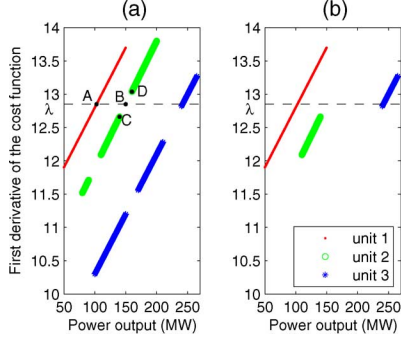


Fig. 1. First derivative of the cost function with respect to the power output for each of the 3 units. (a) Original model. (b) Sub-model.

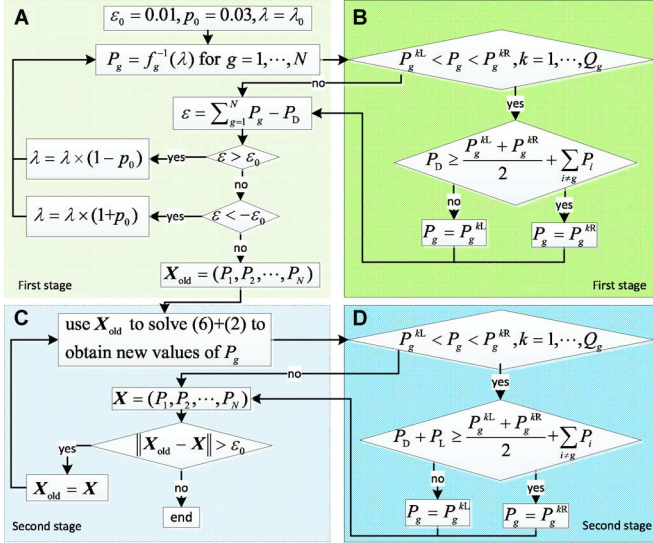


Fig. 2. Flowchart of the F λ I method.

$A(P_{10}, b_1 + 2c_1 P_{10})$, $B(P_{20}, b_2 + 2c_2 P_{20})$, $C(P_{21}, b_2 + 2c_2 P_{21})$, and $D(P_{22}, b_2 + 2c_2 P_{22})$. Points A and B have the same first derivative cost values, i.e., $b_1 + 2c_1 P_{10} = b_2 + 2c_2 P_{20}$, and B is the middle of C and D, i.e., $2P_{20} = P_{21} + P_{22}$. In order to decide which of the upper and the lower boundaries of the POZ is better, two solutions of a 2-unit ED model are compared, i.e., comparing the total cost values of the 2 solutions $(P_D - P_{21}, P_{21})$ and $(P_D - P_{22}, P_{22})$: $f_1(P_D - P_{21}) + f_2(P_{21}) - [f_1(P_D - P_{22}) + f_2(P_{22})] = a_1 + b_1(P_D - P_{21}) + c_1(P_D - P_{21})^2 + a_2 + b_2 P_{21} + c_2 P_{21}^2 - [a_1 + b_1(P_D - P_{22}) + c_1(P_D - P_{22})^2 + a_2 + b_2 P_{22} + c_2 P_{22}^2] = (P_{21} - P_{22})[-b_1 - 2c_1 P_D + c_1(P_{21} + P_{22}) + b_2 + c_2(P_{21} + P_{22})] = (P_{21} - P_{22})[-b_1 - c_1(2P_D - P_{21} - P_{22}) + b_1 + 2c_1 P_{10}] = (P_{21} - P_{22})2c_1(P_{20} + P_{10} - P_D)$. Thus, if $P_D > P_{20} + P_{10}$ ($P_D < P_{20} + P_{10}$), then it is better for Unit 2 to be located at the upper boundary D (the lower boundary C); if $P_D = P_{20} + P_{10}$, then either boundary can consist of the optimal solution.

Now we can give the details of the F λ I that consists of two stages. In the first stage, a λ iteration is used to find the optimal solution of (1)–(5) where $P_L = 0$. The flowchart of the first stage is given in Blocks A and B in Fig. 2. Block A is similar to the λ -iteration method which can refer to [1, Ch. 3.3]. In the second stage, the model (1)–(5) considering P_L is solved based on the solution obtained in the first stage. The necessary condition of an optimal solution can be expressed as [1]

$$\frac{df_g}{dP_g} - \lambda \left(1 - \frac{\partial P_L}{\partial P_g}\right) = b_g + 2c_g P_g - \lambda \left(1 - \frac{\partial P_L}{\partial P_g}\right) = 0. \quad (6)$$

The collection, (6) plus the load balance constraint in (2), is known as the coordination equations. The coordination equa-

TABLE I
RESULTS OBTAINED BY DIFFERENT METHODS ON THE 15-UNIT SYSTEM

Method	$N_{\text{Func Eval}}$	Time (s)	Cost (\$/h)
F λ I	26	0.015	32701
FMINCON [4]	385	0.09	32882
PSO [2]	2×10^4	—	33020
AIS [5]	2×10^3	—	32854

TABLE II
RESULTS OBTAINED BY 2 METHODS ON THE KOREA 140-UNIT SYSTEM

$N_{\text{unit, POZ}}$	Method	$N_{\text{Func Eval}}$	Time (s)	Cost (\$/h)
4	F λ I	29	0.0221	1.65567×10^6
	FMINCON [4]	3525	1.391	1.6560×10^6
15	F λ I	26	0.0209	1.65569×10^6
	FMINCON [4]	3313	2.852	1.6763×10^6
30	F λ I	26	0.0237	1.65773×10^6
	FMINCON [4]	3348	4.878	1.7076×10^6

tions can be solved using an iterative procedure as given in Blocks C and D in Fig. 2. Block C is similar to the iterative procedure detailed in [1, Ch. 3.2]. Blocks B and D in Fig. 2 are used to deal with the POZs.

III. SIMULATION RESULTS AND ANALYSIS

The results of the ED problem on a 15-unit system [2] and a Korea 140-unit system [3] obtained by the F λ I and other methods are given in Tables I and II, respectively. The initial solution of the FMINCON [4] is set to be $(P_g^{\min} + 1/N \times (P_D - \sum_{g \in I} P_g^{\min}) + r)$ where r is a random number and its results in the tables are the average values of 1000 independent runs. The number of function evaluated (denoted as $N_{\text{Func Eval}}$) and the time consumption are adopted to evaluate the computational load. It can be seen that the F λ I can obtain the best results in terms of both precision and computational load and that the computational load of the F λ I method is very low and it increases slightly as the unit number increases from 15 to 140.

The complexity of the ED problem will increase as the number of units having POZs (denoted as $N_{\text{unit, POZ}}$) increases. To verify the efficiency of the F λ I, we increase the $N_{\text{unit, POZ}}$ from 4 to 15 and 30 with the results tabulated in Table II. We can see that the precision and the computational load of the F λ I are not sensitive to the $N_{\text{unit, POZ}}$, and its superiority to the FMINCON becomes more obvious as the $N_{\text{unit, POZ}}$ increases.

IV. CONCLUSION

Based on the necessary conditions for the optimal solution, the F λ I method has been developed to solve the ED problem considering POZs. Numerical simulation results have shown that the method performs very well in terms of both the precision and the computational load on both the 15-unit system and the Korea 140-unit system.

REFERENCES

- [1] A. J. Wood and B. F. Wollenberg, *Power Generation, Operation and Control*. New York, NY, USA: Wiley, 1984.
- [2] Z.-L. Gaing, "Closure to discussion of 'particle swarm optimization to solving the economic dispatch considering the generator constraints'," *IEEE Trans. Power Syst.*, vol. 19, no. 4, pp. 2122–2123, Nov. 2004.
- [3] J.-B. Park, Y.-W. Jeong, J.-R. Shin, and K. Lee, "An improved particle swarm optimization for nonconvex economic dispatch problems," *IEEE Trans. Power Syst.*, vol. 25, no. 1, pp. 156–166, Feb. 2010.
- [4] X. Liu, "On compact formulation of constraints induced by disjoint prohibited-zones," *IEEE Trans. Power Syst.*, vol. 25, no. 4, pp. 2004–2005, Nov. 2010.
- [5] B. K. Panigrahi, S. R. Yadav, S. Agrawal, and M. K. Tiwari, "A clonal algorithm to solve economic load dispatch," *Elect. Power Syst. Res.*, vol. 77, no. 10, pp. 1381–1389, 2007.