



BASELIUS COLLEGE KOTTAYAM

DEPARTMENT OF PHYSICS

PROJECT REPORT

**CALCULATING HUBBLE CONSTANT USING
PANTHEON TYPE Ia SUPERNOVAE
DATASET**

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Calculating Hubble Constant Using Pantheon Type Ia Supernovae Dataset

A Project Report Submitted for the Partial Fulfillment of the
requirements for the degree of Master of Science

in

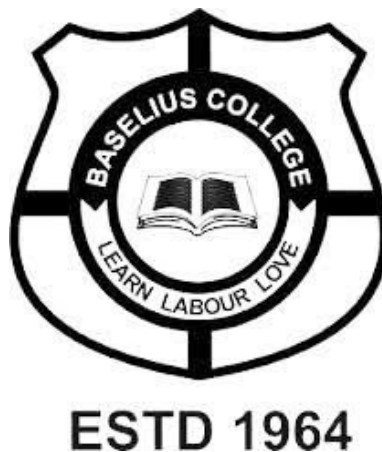
Physics

by

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Under the guidance of

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CERTIFICATE

This is to certify that we have examined the project entitled “**Calculating Hubble Constant Using Pantheon Type Ia Supernovae Dataset**”, submitted by **Adarsh T Saji** (Roll Number: *200011012171*), a postgraduate student of **Department of Physics** in partial fulfillment for the award of degree of **Masters of Science** with specialization in **Physics**. We hereby accord our approval of it as a study carried out and presented in a manner required for its acceptance in partial fulfillment for the post graduate degree for which it has been submitted. The project has fulfilled all the requirements as per the regulations of the institute and has reached the standard needed for submission.

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DECLARATION

I, **Adarsh T Saji**, hereby declare that the project entitled "**Calculating Hubble Constant Using Pantheon Type Ia Supernovae Dataset**" submitted in partial fulfillment of the requirements for the award of the degree of Master of Science from Mahatma Gandhi University Kottayam, is a bonafide record of the work carried out by myself under the guidance and supervision of Dr. Moncy V John Visiting Professor, School of Pure and Applied Physics, Mahatma Gandhi University, Kottayam

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ABSTRACT

The Pantheon Type Ia supernovae catalogue Scolnic et.al(2018) data set which contains the data of 1048 supernovae acts as the base for this dissertation. Calculation of the Hubble constant using this data is our primary aim. We also look into the existing discrepancy in the values of H_0 which is called as Hubble tension. We also examine more closely at the many models that have been developed in cosmology. In this analysis we classify the models using the deceleration parameter (q_0) [measure of the cosmic acceleration of the expansion of space in a Friedman-Lemaitre-Robertson-Walker universe]. Using the distance modulus equation for each model we calculate the H_0 value. The likelihood of occurrence of an obtained H_0 value is then measured using the Bayesian theorem. Error estimation of the values are also done.

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Chapter 1

Introduction

Cosmology is a branch of astronomy that deals with the origin, structure, and space-time relationships of the universe as a mathematically driven science, cosmological physics is usually thought to be extremely precise. But it has its downsides. One being the difficulty to evaluate Hubble's constant precisely. The Hubble constant plays a crucial part in cosmology as it describes how fast the universe is expanding, which in turn helps in calculating the universe's age and history. Modern scientific cosmology is usually considered to have begun in 1917 with Albert Einstein's publication of his final modification of general relativity in the paper "Cosmological Considerations of the General Theory of Relativity". General theory of relativity prompted all the major cosmogonists to explore its possible astronomical ramifications, thereby enhancing the ability to explore distant bodies. On 1929, Edwin Hubble calculated this constant from star measurements. Hubble used his telescope experience to make measurements of Cepheid variables. He used the work of fellow astronomer Henrietta Leavitt to predict the brightness of these stars, which enabled him to calculate their distance from earth. These measurements confirmed that the universe extends far beyond the milky way. This is regarded as the most significant cosmological milestone. But the problem is, even after 93 years of its introduction, scientists are still puzzled about its exact value. The problem is that having two different figures for the Hubble constant measured from different perspectives would simply invalidate the cosmological model. Cosmologists using the plank satellite to study the cosmic microwave background and have arrives at a high precision value of of the expansion rate which is called as the Hubble constant. Another set of astronomers observing stars and galaxies have also obtained a value of H_0 however the two values disagree. This discrepancy is called as Hubble tension.

1.1 Hubble Tension

Cosmology is a branch of astronomy that studies the origin, structure, and space-time relationships of the universe. Because it is a mathematically driven science, cosmological physics is widely regarded as extremely precise. However, astronomers have reached a fundamental stumbling block in their understanding of the universe: they cannot agree on how fast it is expanding. This is now the hottest and most mysterious research topic in cosmology.

Scientists first realized the universe was expanding in the 1920s, when the US astronomer Edwin Hubble found that the greater the distance between two galaxies, the faster they are moving apart. Hubble constant, a constant of proportionality in the relation between the velocities of remote galaxies and their distances. It expresses the rate of expansion of the universe. It remains one of the most important scientific discoveries ever made. But, even if the universe was expanding at an increasing rate, one key question remained: what is the precise rate of this expansion? Just how quickly is the cosmos flying apart? To be more specific, what exactly is the value of the Hubble constant? It is a highly valuable and highly sought-after value because it will reveal much about the origin, age, evolution, and, ultimately, fate of the universe. But even after 93 years of its introduction, scientists are still puzzled about its exact value.

The problem is that having two different figures for the Hubble constant measured from different perspectives. The first method is to use astronomical measurements to examine nearby objects and determine how fast they are moving. This is a local approach.

The other method for determining the Hubble constant has involved astronomers studying the rippling pattern of light known as the cosmic microwave background, which were formed just after the universe's big bang 13.8 billion years ago. These findings demonstrate how the early universe's expansion would have most likely resulted in an expansion that astronomers can now measure.

Until recently, these two approaches produced estimates that appeared to be consistent with each other, despite the fact that both measurements had significant uncertainties. Everyone believes that as the two values are tested with increasing precision, the differences between them will disappear. Unfortunately for astronomers hoping for a quick fix, this has not occurred. In reality, the inverse has occurred. The disparity has grown more pronounced. This disparity, known as the Hubble tension, has been growing for years, as study after study of both the early and late universe yields ever more precise results, leaving scientists on both sides worried and perplexed. After all, either faction could be measuring the universe incorrectly. However, the tension could be a true reflection of reality, necessitating exotic new physics and a dramatic revision of our understanding of cosmic evolution. The lower estimate of the Hubble constant has gotten a little lower over the years,

while the higher estimate has gotten even higher. Today, those who use cosmic background data to calculate the Hubble constant get a value of 67.4 plus or minus 0.5. By contrast the local approach gives a figure of 73.5 plus or minus 1.4.

The dissimilarity may not sound great but it is significant. This isn't just a case of two experiments disagreeing. We're measuring something entirely different. One is a measurement of how quickly the universe is expanding as we see it today. The other is a prediction based on early universe physics and measurements of how fast it should be expanding. If these values do not agree, it is very likely that we are missing a factor in the cosmological model that connects the two eras. In short, something appears to be missing from our understanding of the universe, and the Hubble constant has become the focus of a heated debate over the nature of this intangible influence.

Changing the Hubble constant from 67.4 to 73.5 implies that it must have been flying apart faster than previously assumed, implying that it is younger than the currently accepted age of 13.8 billion years. In fact, it would shorten the time to 12.7 billion years. And this does cause issues. The universe contains some very old stars with estimated ages of around 12 billion years. After all, stars take a long time to form.

There have been over 300 proposals for solutions to the cosmology crisis to date. Some argue for more physics in the CMB era. Some claim that dark energy did something strange in the recent past. Some fundamentally alter physics, interfering with our observations of supernovae. However, no single proposal can account for the wealth of cosmological evidence, and there is no consensus on a solution. The fact of concern has been that discrepancy has been increasing with the number of studies being conducted on both the early and the late universe. So we wouldn't be able to say what the age of the universe was until we had put our physics right.

1.2 Cosmological Models

Let's first discuss some of the major cosmological models. A cosmological model describes the universe's largest-scale structures and dynamics and allows researchers to investigate fundamental questions about the universe's origin, structure, evolution, and ultimate fate.

1.2.1 Einstein's Static Model

Einstein used three assumptions that were outside the scope of his equations to derive his 1917 cosmological model. The first assumption was that the universe is homogeneous and isotropic in general (i.e., the same everywhere on average at any instant in time). The second assumption was that the universe was homogeneous and isotropic, with a closed spatial geometry. Einstein's third assumption was that

the universe as a whole is static, meaning that its large-scale properties do not change over time. This assumption was made prior to Hubble's observational discovery of the universe's expansion. As a result, it is known as the Einstein Static Model. It should be noted that this model is unrealistic in light of Hubble's law.

1.2.2 de-Sitter's Model

In 1917, the Dutch astronomer Willem de-Sitter realised that by removing all matter, he could obtain a static cosmological model that differed from Einstein's. Since there is no matter to move, the solution remains stationary. If some test particles are reintroduced into the model, the cosmological term would push them apart. Astronomers began to wonder if this effect could not explain the spiral galaxies' recession.

1.2.3 Friedmann-Lemaître Model

Friedmann and Georges Lemaître independently discovered realistic solutions to Einstein's equations in 1922 and 1927, respectively. These evolutionary models correspond to cosmologies based on the Big Bang. Friedmann and Lemaître adopted Einstein's spatial homogeneity and isotropy assumption. They rejected his assumption of time independence, however, and considered both positively curved ("closed") and negatively curved ("open") universes. The difference between Friedmann's and Lemaître's approaches is that the former set the cosmological constant to zero, whereas the latter allowed for it to have a nonzero value.

The geometry of space in Friedmann's closed models is similar to Einstein's original model; however, there is a curvature to both time and space. Unlike Einstein's model, in which time runs eternally at each spatial point on an uninterrupted horizontal line that extends infinitely into the past and future, time in Friedmann's version of a closed universe has a beginning and an end when material expands from or is recompressed to infinite densities. These are known as the "big bang" and "big squeeze" instants, respectively.

Final expression of luminosity distance D in closed model is given by,

$$D = \frac{c}{H_0} \frac{1}{q_0^2} [q_0 z + (q_0 - 1)(\sqrt{1 + 2zq_0} - 1)] \quad (1.1)$$

where q_0 is the deceleration parameter. It is a dimensionless measure of the cosmic expansion of space in Friedmann-Lemaître-Robertson-Walker universe. The formula was first derived by Mattig in 1958.

The spatial and temporal behaviour of open Friedmann models differs from that of closed models. The total volume of space and the number of galaxies contained in an open universe are infinite. The three-dimensional spatial geometry is one of uniform negative curvature, which means that if circles are drawn with very long

lengths of string, the circumference to length ratio is greater than 2π . The universe's temporal history begins again with infinite density expansion from a big bang, but this time the expansion continues indefinitely, and the average density of matter and radiation in the universe would eventually become negligibly small. In such a model, time has a beginning but no end.

The calculation in this case is similar to that for the closed model, with the difference that the trigonometric functions are replaced by hyperbolic ones.

Final expression for luminosity distance are same for closed and open model.

From eq.(1.1) for $q_0 = 1$ we get,

$$D = \frac{c}{H_0} z \quad (1.2)$$

And for $q_0 = 0$ we get,

$$D = \frac{c}{H_0} z \left(1 + \frac{z}{2}\right) \quad (1.3)$$

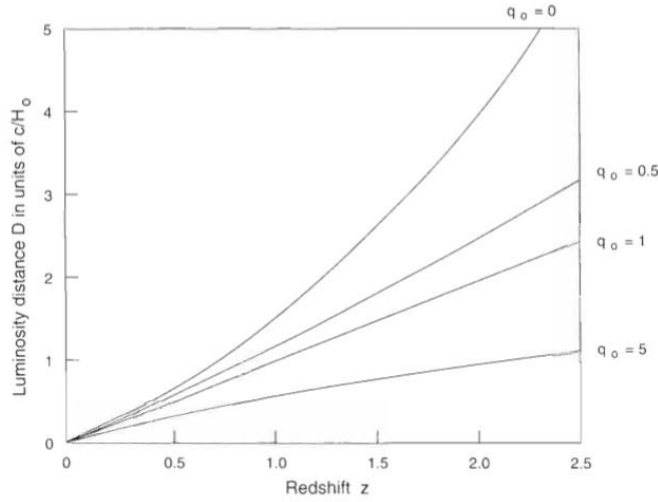


Figure 1.1: The luminosity distance expressed as a function of the redshift for various q_0 values.

Source : Introduction to Cosmology by Jayant V.Narlikar

Plot $D(q_0, z)$ as a function of z for various parametric value of q_0 . All curves start off with linear Hubble law for small redshift, but then spread out, with only the curve for $q_0 = 1$ staying linear all the way.^[1]

1.2.4 Einstein-de Sitter Model

In 1932, Einstein and de Sitter proposed that the cosmological constant be set to zero, and they developed a homogeneous and isotropic model that provides the separating case between the closed and open Friedmann models; that is, Einstein and de Sitter assumed that the universe's spatial curvature is neither positive nor

negative, but rather zero. The Einstein-de Sitter universe's spatial geometry is Euclidean (infinite total volume), but space-time is not globally flat (i.e., not exactly the space-time of special relativity). Time begins with a big bang again, and the galaxies recede indefinitely, but the recession rate (Hubble's "constant") asymptotically approaches zero as time advances to infinity. The final equation for luminosity distance of Einstein-de Sitter Model is

$$D = \frac{2c}{H_0} [(1+z) - (1+z)^{\frac{1}{2}}] \quad (1.4)$$

If we put $q_0 = 0.5$ in eq.(1.1) it is easy to see that the result eq.(1.4) for the Einstein-de Sitter Model also follows from the same formula.^[1]

1.2.5 Standard Model / Λ CDM Model

The Λ CDM model is a depiction of the big bang cosmological model, which states that the universe has three primary components. A cosmological constant symbolised by Lambda, which is related with dark energy, cold dark matter, and finally ordinary matter. It is now considered to be the standard model of big bang cosmology. This is because it provides reasonably good account for the properties of the cosmos like existence and the structure of the cosmic microwave background, large scale distribution of galaxies and also the accelerated expansion of the universe. The Λ CDM model incorporates metric space expansion which is extensively established through red shift of spectral absorption or emission lines in light from distant galaxies. The letter Λ represents the cosmological constant that is associated with dark energy in empty space which is used to explain the accelerating expansion of the space against the influence of gravity. Dark matter is postulated in order to account for gravitational effects observed in very large scale structures that cannot be accounted for by the amount of observed matter. Cold dark matter is currently hypothesized to be non-baryonic, cold, dissipation less and collision less. The model uses the Friedmann–Lemaître–Robertson–Walker metric, the Friedmann equations and the cosmological equations of state to describe the observable universe from right after the inflationary epoch to present and future. Currently, the concordance model (currently accepted or a model of the universe that assumes a minimum number of parameters) is the Λ CDM model (which includes cold dark matter and a cosmological constant). According to this model, the Universe is 13.7 billion years old and composed of 4% baryonic matter, 23% dark matter, and 73% dark energy. The Hubble constant for this model is 71 km/s/Mpc.

1.2.6 Eternal Coasting Model

The eternal coasting model^[2] is a model that is very similar to the Friedman Robertson-Walker model which assumes the presence of dark energy along with

with matter and radiation, the only difference being is that eternal coasting models constrained by the equation of state $\rho + 3p = 0$, where ρ is the energy density. This model predicted that the value $H_0 t_0$ was unity and the data that we are currently obtaining is in agreement with this. The luminosity distance in this model is given by

$$D = \frac{c}{H_0}(1+z) \ln(1+z) \quad (1.5)$$

1.3 Motive of the project

The largest dataset of SN 1a samples, that is available to the public is known as the pantheon data[3]. It contains 1048 supernovae on the redshift range $0 < z < 2.3$. Pantheon+ data, the largest dataset of SN 1a samples, has not yet been published. Our motivation was to calculate the value of the Hubble constant from these Type Ia supernovae data for various available cosmological models. In addition, we will use this data to try to understand the concept of the Hubble tension. After obtaining the values we try to find the best suitable model to explain the big bang theory with help of the Bayesian theorem. Each model has a prescribed equation for obtaining the value for distance modulus rearranging this equation we obtain the value for calculating the value of H_0 . With values of zCMB from the pantheon data we calculate the Hubble constant. This calculation is done by selecting certain ranges for z for example, $z < 0.05$, $z < 0.15$ etc. Now to find the most suitable values in each of these models with the help of Bayesian theorem. This is done by acquiring the value for $\exp(-\chi^2/2)$. In order to do this we have to find the calculated value for zCMB, With which χ can be known. We then plot a graph with H_0 and $\exp(-\chi^2/2)$ from which we can obtain the likelihood of the parameter which in this case is the likelihood of H_0 .

Chapter 2

Hubble Constant

The Hubble Constant is one of the most significant numbers in cosmology because it is needed to estimate the size and age of the universe. This number indicates the rate at which the universe is expanding, since "The Big Bang". In this section we discuss about Hubble's law, Method of calculating Hubble Constant and Hubble tension.

2.0.1 Hubble's Law

In 1929 , one of the most astonishing discovery about the motion of galaxies was published by Edwin Hubble. That discovery would demolish many of the previously held beliefs. Using the observations of distant galaxies, he showed that the universe is expanding. Not only that, the light from distant galaxies is systematically increased in wavelength, the fractional increase being proportional to the distance D of the galaxy from us. Thus if λ is the wavelength of light sent out by the galaxy, and $\lambda + \Delta\lambda = \lambda_0$ the light received, then

$$z = \frac{\Delta\lambda}{\lambda} \propto D \quad (2.1)$$

The quantity z is called the redshift. Hubble interpreted this as a Doppler effect (As an object moves away from us, the light waves emitted by the objects are stretched out, which makes them have lower pitch and moves towards end of the electromagnetic spectrum, where light has a longer wavelength. This is called redshift) and attributed a velocity of recession $v = cz$ to the source galaxy.

Hubble observation (1.1) can be written as

$$cz = H_0 D \quad (2.2)$$

And also be written as

$$v = H_0 D \quad (2.3)$$

Where H_0 is called Hubble constant. The Hubble constant is most frequently quoted

in km/s/Mpc. Though the Hubble constant H_0 is roughly constant in the velocity-distance space at any given moment in time. Hubble found that $H_0 \approx 530 \text{ km/s/Mpc}$

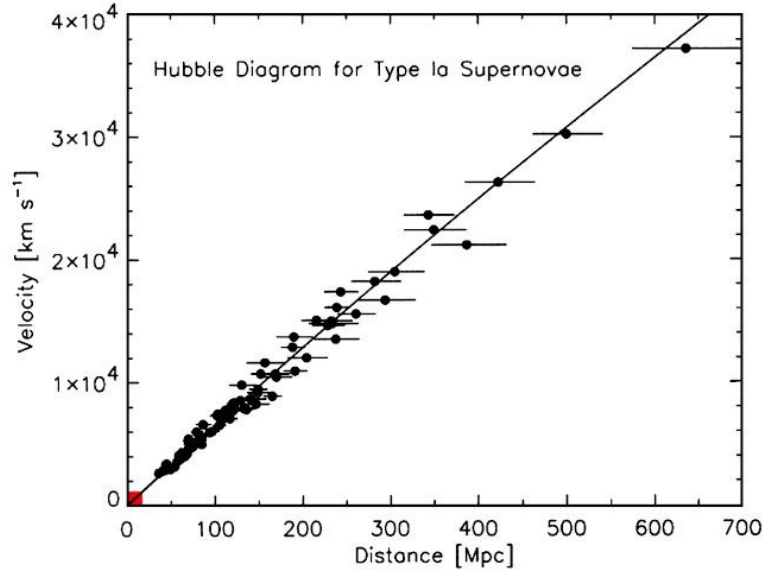


Figure 2.1: The Hubble diagram for type Ia supernovae. Source : PNAS

The Hubble constant H_0 has units of inverse time; the Hubble time t_H is simply defined as the inverse of Hubble constant.

$$\text{i.e., } t_H = \frac{1}{H_0} = \frac{1}{530 \text{ km/s/Mpc}} = 1.8 \times 10^9 \text{ years}$$

In these units $H_0^{-1} \approx 1.8 \times 10^9$ years as found by Hubble.

The Hubble time is the age it would have had if the expansion had been linear, and it is different from the real age of the universe because the expansion is not linear.

Although Hubble originally obtained $H_0 \approx 530 \text{ km/s/Mpc}$, the present estimate of H_0 is much lower. We believed to lie in the range of $50 \leq H_0 \leq 100$. Hubble Space Telescope and some ground based telescopes have narrowed this range down to around 55 to 75.

2.0.2 Hubble Constant

The Hubble constant describes how fast the universe is expanding in different distances from a particular point in space. It was first calculated in the 1920s by Edwin Hubble, he made the observation that cloud-like celestial objects were actually distant galaxies that were present outside our own galaxy. Using the research done by Henrietta Leavitt on Cepheid, Hubble derived the Cepheid's distance which lead to the formulation of the Hubble's law that was explained in the previous section.

Hubble constant acts as a constant of proportionality in the Hubble law. It is denoted as H_0 . Hubble's original value for H_0 was 500 km per Mpc in cosmological units. But modern techniques have helped in refining the initial measurement. How much was Hubble's value off by, still is a topic of debate. After the discovery that the universe is not only expanding but also accelerating in its expansion it became necessary to modify the existing model with new data, including the addition of "dark energy"- a force that pushes everything apart in the universe. After this discovery they tried to pin down the value of the Hubble constant using two methods.

1. Using the data of Cepheid variables and other astrophysical sources, which gave the value of H_0 to be 73 km/s/Mpc
2. Using the information of European Space Agency's plank satellite where H_0 was found to be 67 km/s/Mpc.

The two values obtained even though are different they are extremely precise without any overlapping between their error bars. If any one of these methods are considered to be wrong it can lead to a domino effect which can affect all the advancements that we have made in the field of cosmology. This continuous to be the biggest topic of debate in cosmology. New methods such as using the LIGO to obtain new set of data or gravitational lensing- which occurs when extremely massive object wraps and bends the space time like magnifying glass- could clear the discrepancy. one measurement is about how fast the universe is expanding as of today, while the other measurement is a prediction that is based on the physics of the early universe, this discrepancy shows us that we are missing a factor that connects these two eras in our cosmological model. This difference also affects the calculation of age of the universe, cutting of more than a billion years of existence in one case.

The introduction of the concept of dark energy can help us to validate the discrepancy in the value of H_0 . One idea contains the use of a sub atomic particle that travels close to the velocity of light, these entities are called as dark radiations, these particles can affect the speed of expansion of the universe. Another idea is the presence of a special intense dark- energy produced after the big bang which expanded the universe faster than astronomers had previously anticipated. But at

the moment no one knows when and where the final answer for the Hubble constant lies.

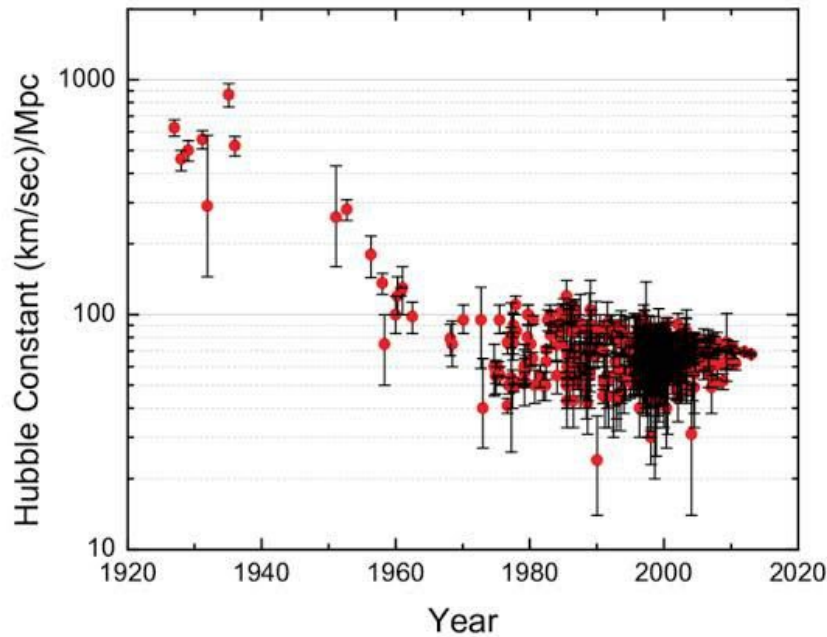


Figure 2.2: Historical evolution of Hubble constant measurements. Source: uploaded by Boris Pritychenko (<https://www.researchgate.net/profile/Boris-Pritychenko>)

2.0.3 Method of calculating Hubble Constant

Considerable progress has been made in determining the Hubble constant over the past two decades. Even though the accuracy of these methods is a topic up for debate. In this section we briefly discuss the different methods present, the major focus being high precision distance determination methods like Cepheid, tip of the red giant, maser galaxies, surface brightness fluctuations, the Tully-Fisher relation and Type 1a supernovae which is the method that is being used in this dissertation to determine the Hubble constant.

A Cepheid is a variable star having a regular cycle of brightness with a frequency related to its luminosity, so allowing estimation of its distance from the earth. This method provided the initial step on the cosmic distance ladder. Hubble used the observational data for Cepheid in galaxies to deduce a law, which stated that the more remote the galaxy is from the Milky Way, the faster its recession is. The period luminosity(pl) relation for Cepheid is a relation linking the luminosity of pulsating variable stars with their pulsation period. This pl relation provided a powerful tool that astronomers use to measure the distance to nearby galaxies and hence calculate

the Hubble's constant.

Tip of the Red Giant Branch (TRGB) is a distance indicator used in astronomy. it uses the luminosity of the brightest red giant branch stars in a galaxy as reference point to measure the distance to that galaxy. TRGB offers an alternative to Cepheid as it provides an accurate ($\sim 5\%$) distance to galaxies within 10 Mpc with a single orbit of Hubble space telescope time.

The next method is by using maser galaxies. An astrophysical maser is a naturally occurring source of stimulated spectral line emission, typically in the microwave portion of the electromagnetic spectrum. In this technique we utilize the mapping of 22.2 GHz water maser sources in the accretion disks of massive black holes located in spiral galaxies with active galactic nuclei. In the simplest version of this technique a rotation curve is measured along the major axis of the disk; proper motions are measured on the near side of the disk minor axis, and a comparison of the angular velocities in the latter measurement with the absolute velocities in km s^{-1} in the former measurements yields the distance.

For distance to elliptical galaxies and early-type spiral bulge populations we use the Surface Brightness Fluctuation method(SBF).Both TRGB and SFB use the properties of the red giant branch luminosity function to estimate distances. here pixel to pixel variance is measured, the variance in a pixel is taken to be a function of distance simply because the total number of discrete sources contributing to any given pixel increases with the square of the distance.The major difference between TRGB and SBF method is that TRGB completely relies on the brightest red giant stars while SBF method uses a luminosity weighed integral.

The Tully-Fisher relation at present is one of the most widely applied methods for distance measurements, providing distances to thousands of galaxies both in the general field and in groups and clusters. The relation can be understood in terms of of the virial relation applied to rotationally supported disk galaxies, under the assumption of a constant mass to light ratio. However, a detailed self-consistent physical picture that reproduces the Tully-Fisher relation and the role of dark matter in producing almost universal spiral galaxy rotation curves still remain a challenge.

Type Ia supernovae, unlike Cepheids, are bright enough to be seen from relatively greater distances. Astronomers compare the luminosity and apparent brightness of distant supernovae to determine the distance to which the universe's expansion can be seen. They compare those distance measurements to how light from supernovae is stretched to longer wavelengths due to space expansion. They use these two values to calculate the Hubble constant.

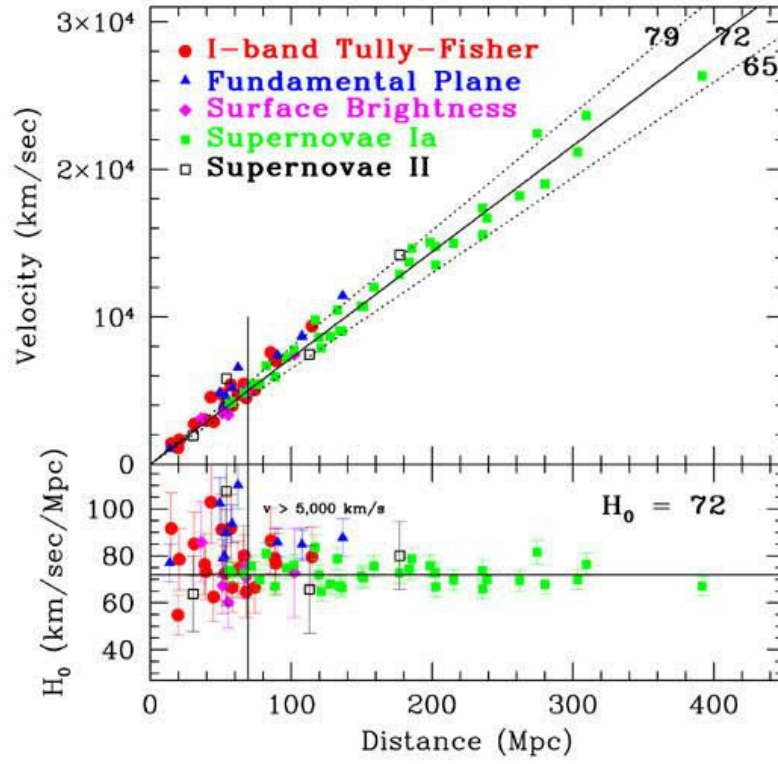


Figure 2.3: Hubble diagram using different methods. Source : Measuring and Understanding the Universe - W. Freedman

Chapter 3

Calculating Hubble Constant using SN Ia data

Using the data obtained from the Pan-STARRS1 (PS1) Medium Deep Survey, Scolnic et.al.2018 [3] deduced the optical light curves, redshifts and classification for 365 spectroscopically confirmed Type Ia supernovae. Improvements were done on PS1 SN photometry, astrometry, and calibrations which help in reducing the systematic uncertainties in the PS1 SN Ia distances. By combining the subset of 279 PS1 SNe Ia ($0.03 < z < 0.68$) with useful distance estimates of SNe Ia from the Sloan Digital Sky, SNLS and Hubble Space Telescope samples thus forming the largest combined sample of SNe Ia, with a total of 1048 SNe Ia within the range of $0.01 < z < 2.3$ hence forming the what we call as the "Pantheon Sample" (fig.3.1)

In this section value of H_0 is calculated by varying the deceleration parameter ($q_0 = 0, q_0 = \frac{1}{2}, q_0 = 1$). Then we exclusively select low redshift region and calculate H_0 . and finally H_0 for the entire dataset (1048 supernovae).

The sample set contains Target ID, zCMB, zHEL (redshifts in two different reference frames, DZ(Error in redshifts), MB (Apparent Magnitude), DMB (Error in MB). Let's discuss these terms in detail.

3.0.1 Redshift

Redshift is an extremely important phenomenon in cosmology and astronomy. When electromagnetic radiation are emitted or reflected from an object, it is shifted toward the less energetic (higher wavelength) end of the spectrum. Using this phenomenon allows for the first steps toward understanding features of our galaxy and even the universe as a whole. Our data set contains two types of redshifts: Zhel and ZCMB. ZHEL is the redshift in the Sun's reference frame. ZCMB is the redshift in CMB(Cosmic Microwave Background) frame.

3.0.2 Apparent Magnitude

Apparent magnitude can be defined as the measure of the brightness of a star or any other astronomical entity observed from Earth. An object's apparent magnitude depends upon various factors like intrinsic luminosity, its distance from the earth, and scattering of the radiation due to the presence of interstellar dust along the line of sight of the observer. The distinction between luminosity and apparent brightness is that luminosity is an intrinsic property of the star, with a fixed value, while apparent magnitude varies with the distance of the observer. In this analysis, we denote apparent brightness with 'm'. We obtained the apparent brightness of about 1048 supernovas from Scolnic et al. (2018), using which we calculated the values of the Hubble constant. In the data sheet it is denoted as MB.

3.0.3 Absolute Magnitude

Absolute magnitude(M) is the measure of the luminosity of a celestial object on an inverse logarithmic astronomical magnitude scale. If an object is viewed from a distance of exactly 10 parsecs, its absolute magnitude equals its apparent magnitude, without extinction of its light due to the presence of interstellar dust. Absolute magnitude can be specified for different wavelength regions. As the luminosity of an object increases, the numeric value of its apparent brightness decreases. During this analysis we have taken the value of M to be -19.3 . This assumption is based on the fact that we only consider type 1a supernovae. The value of M changes with the type entity we measure. Type 1a supernovae occur when a white dwarf accumulates too much mass to resist the force of gravity. This always occurs when the mass of the star reaches the Chandrasekhar limit. All type Ia supernovae have approximately the same absolute magnitude, for this very reason they are commonly used as standard candles to determine the distance to a galaxy once the stretch factor is accounted for.

Target ID (sortable)	ZCMB (sortable)	ZHEL (sortable)	DZ (sortable)	MB ($\mu - M_B$) see Eqn. 3 in Scolnic et al. 2018 (sortable)	DMB (error in MB) (sortable)
Type filter...	Type filter...	Type filter...	Type filter...	Type filter...	
03D1au	0.50309	0.50309	0.0	22.93445	0.12605
03D1aw	0.58073	0.58073	0.0	23.52355	0.1372
03D1ax	0.4948	0.4948	0.0	22.8802	0.11765
03D1bp	0.34593	0.34593	0.0	22.11525	0.111
03D1co	0.67767	0.67767	0.0	24.0377	0.2056
03D1ew	0.8665	0.8665	0.0	24.34685	0.17385
03D1fc	0.33094	0.33094	0.0	21.7829	0.10685
03D1fq	0.79857	0.79857	0.0	24.3605	0.17435
03D3aw	0.44956	0.44956	0.0	22.78895	0.14135
03D3ay	0.37144	0.37144	0.0	22.28785	0.1245
03D3ba	0.29172	0.29172	0.0	21.47215	0.12535
03D3bl	0.35582	0.35582	0.0	22.05915	0.12645
03D3cd	0.46127	0.46127	0.0	22.62945	0.13775
03D4ag	0.2836	0.2836	0.0	21.40915	0.1028
03D4at	0.63222	0.63222	0.0	23.66065	0.20445
03D4au	0.4664	0.4664	0.0	23.21635	0.1587
03D4cj	0.26862	0.26862	0.0	21.1762	0.0993
03D4cx	0.94688	0.94688	0.0	24.41155	0.1642
03D4cy	0.92491	0.92491	0.0	24.7416	0.1923
03D4cz	0.69315	0.69315	0.0	23.79645	0.24445
03D4dh	0.62522	0.62522	0.0	23.46015	0.1303
03D4di	0.89693	0.89693	0.0	24.40275	0.16645
03D4dy	0.60825	0.60825	0.0	23.3767	0.1266
03D4fd	0.78904	0.78904	0.0	24.2188	0.11585
03D4gf	0.57828	0.57828	0.0	23.2373	0.13895
03D4gg	0.59027	0.59027	0.0	23.3354	0.1666
04D1aj	0.71963	0.71963	0.0	23.82525	0.2057
04D1dc	0.21075	0.21075	0.0	20.64495	0.10525
04D1de	0.7666	0.7666	0.0	24.24705	0.1593
04D1ff	0.85852	0.85852	0.0	24.1237	0.1478
04D1hd	0.36791	0.36791	0.0	22.24455	0.1019
04D1hx	0.55875	0.55875	0.0	23.1648	0.1335
04D1hy	0.84852	0.84852	0.0	24.33175	0.14855
04D1iv	0.9964	0.9964	0.0	24.69965	0.1408
04D1jd	0.7766	0.7766	0.0	24.0942	0.17835
04D1jg	0.58294	0.58294	0.0	23.34385	0.1258
04D1kj	0.58375	0.58375	0.0	23.2686	0.09915
04D1ks	0.79656	0.79656	0.0	23.85445	0.1588
04D1oh	0.58873	0.58873	0.0	23.30085	0.13005
04D1ow	0.91348	0.91348	0.0	24.37895	0.13735
04D1pc	0.76859	0.76859	0.0	24.2078	0.16885
04D1pd	0.94846	0.94846	0.0	24.66675	0.15105
04D1pg	0.5138	0.5138	0.0	23.12745	0.1329
04D1pp	0.73362	0.73362	0.0	23.8481	0.15575
04D1pu	0.63771	0.63771	0.0	23.37	0.2329
04D1qd	0.7656	0.7656	0.0	24.10805	0.1549
04D1rh	0.43487	0.43487	0.0	22.5623	0.1145
04D1rx	0.98341	0.98341	0.0	24.828	0.153
04D1sa	0.58375	0.58375	0.0	23.46355	0.14025
04D1sl	0.70064	0.70064	0.0	23.7492	0.1738

Figure 3.1: Scolnic et al. 2018 Supernovae Catalog

3.0.4 Distance Modulus

The distance modulus can be easily defined as the difference between the apparent magnitude(m) and the absolute magnitude(M). The equation that we use in this analysis to obtain distance modulus is given as

$$\mu = m - M = 5\log(D) - 5 \quad (3.1)$$

where D is the distance in pc.

Another equation that we can use is the modified form of the Tripp equation which is given as

$$\mu = m - M + \alpha x1 - \beta c + \Delta m + \Delta n \quad (3.2)$$

where α is the coefficient of relation between luminosity and stretch, β is the coefficient of relation between luminosity and colour, Δm is the mass correction based on the host galaxy of the SN and Δn is a distance correction based on predicted biases from stimulation.

3.1 Calculation of H_0

3.1.1 $q_0 = 1$ (Hubble's Law)

From Hubble's law eq.(2.2) we get,

$$H_0 = zc/D \quad (3.3)$$

The value of D can be obtained from (3.1),

$$\mu = m - M = 5\log(D) - 5 \quad (3.1)$$

Rearranging,

$$D = \frac{1}{\log \frac{\mu+5}{5}} \quad (3.4)$$

Since low redshift will give more accurate value of H_0 due to reduced extinction of radiation we initially choose the range for z to be less than 0.05. There are 157 supernovae between this range.

The average value of H_0 with in this range is 70.7156 km/s/Mpc similarly calculating the average value of H_0 by changing the range from z less than 0.05(157 supernovae), less than 0.1(211 supernovae), less than 0.15(299 supernovae) we obtained 70.5494, 68.7576 respectively.

Now we calculate H_0 for complete dataset (high redshift region) thereby obtaining H_0 to be 60.3170 km/s/Mpc.

	A	B	C	D	E	H	K	L	M
1	Full Screen	ZCMB (sortable)	MB (μ - MB) see Eq. 3 in Scolnic et al. 2018 (sortable)	DMB (error in MB) (sortable)	m-M		D in Mpc	ZCMB*c	H0
240		0.08859	18.64345	0.10605	37.94345		387.873402584	26577	68.51978
245		0.094	18.889	0.10285	38.189		434.310170886	28200	64.93055
266		0.06641	17.9881	0.1134	37.2881		286.826979833	19923	69.45999
271	6558	0.05715	17.65695	0.1072	36.95695		246.257802674	17145	69.62216
282	12779	0.07883	18.5397	0.12205	37.8397		369.777089658	23649	63.95475
283	12781	0.08328	18.7438	0.11605	38.0438		406.218782331	24984	61.5038
287	12898	0.08286	18.5179	0.1055	37.8179		366.083369396	24858	67.90257
289	12950	0.08162	18.511	0.10425	37.811		364.921961009	24486	67.09928
350	17240	0.0718	18.0121	0.1254	37.3121		290.014692268	21540	74.2721
356	17784	0.03743	16.7708	0.1166	36.0708		163.741965801	11229	68.57741
364	18241	0.0939	18.94955	0.14945	38.24955		446.591034314	28170	63.07784
415	21502	0.08798	18.6796	0.1066	37.9796		394.384647047	26394	66.92451
416	722	0.08534	18.62285	0.1057	37.92285		384.211181714	25602	66.63523
418	774	0.09278	18.8576	0.109	38.1576		428.075132721	27834	65.0213
433	3592	0.08575	18.608	0.10035	37.908		381.592649119	25725	67.41482
457	7876	0.07523	18.2988	0.10185	37.5988		330.948182215	22569	68.19497
461	10028	0.06481	17.87655	0.1109	37.17655		272.464546982	19443	71.35974
463	10805	0.04437	16.92415	0.11995	36.22415		175.723562794	13311	75.74966
527	17186	0.07877	18.7576	0.11205	38.0576		408.808578008	23631	57.80456
531	17258	0.08833	18.4809	0.11615	37.7809		359.898469681	26499	73.6291
560	19968	0.05573	17.56165	0.10915	36.86165		235.68394554	16719	70.93822
573	2001ah	0.05948	17.71775	0.12625	37.01775		253.25031837	17844	70.45993
574	2001az	0.04093	17.0528	0.11665	36.3528		186.448974924	12279	65.85716
575	2001da	0.01705	14.85645	0.1832	34.15645		67.8094153619	5115	75.432
576	2001en	0.01531	14.59755	0.1613	33.89755		60.1880121542	4593	76.31088
577	2001fe	0.01472	14.6158	0.14905	33.9158		60.6959900694	4416	72.75604
578	2001gb	0.02673	15.97105	0.16165	35.27105		113.294805976	8019	70.77994
579	2001G	0.01732	14.8718	0.1587	34.1718		68.2904540079	5196	76.08677
580	2001ic	0.04437	16.72085	0.19805	36.02085		160.018428132	13311	83.18417
581	2001V	0.01567	14.4307	0.15405	33.7307		55.736539353	4701	84.34323
582	2002bf	0.02477	15.6593	0.1718	34.9593		98.143151588	7431	75.71593
583	2002bz	0.038	16.7664	0.13185	36.0664		163.410515487	11400	69.76295
584	2002ck	0.0299	16.2413	0.123	35.5413		128.309850807	8970	69.9089
585	2002cr	0.01012	13.90745	0.19825	33.20745		43.8016025363	3036	69.31253
586	2002de	0.0258	16.1648	0.1376	35.4648		123.86824944	7740	62.48575
587	2002dp	0.01038	14.0496	0.2048	33.3496		46.7648989225	3114	66.5884
588	2002eu	0.03791	16.5851	0.2001	35.8851		150.321118993	11373	75.65803
589	2002G	0.03507	16.45115	0.16785	35.75115		141.328581496	10521	74.44354
590	2002ha	0.01268	14.4486	0.171	33.7486		56.1978887188	3804	67.68938
591	2002he	0.02525	15.82815	0.13295	35.12815		106.079142398	7575	71.40895
592	2002kf	0.01908	15.3534	0.14115	34.6534		85.2471758147	5724	67.14592
593	2003ae	0.03489	16.36835	0.1667	35.66835		136.041057746	10467	76.94001
594	2003ch	0.0295	16.3438	0.12735	35.6438		134.511680961	8850	65.79354
595	2003cq	0.03416	16.57565	0.1563	35.87565		149.668360042	10248	68.47139
596	2003fa	0.04067	16.7938	0.1177	36.0938		165.409379047	17201	73.76247

Figure 3.2: Calculating H_0 from scolnic et.al 2018 supernovae catalog in the range of $z < 0.05$

3.1.2 $q_0 = 0.5$ (Einstein-de Sitter Model)

From Einstein de-Sitter model we have the equation for luminosity distance eq.(1.4) Rearranging,

$$H_0 = \frac{2c}{D}[(1+z) - (1+z)^{\frac{1}{2}}] \quad (3.5)$$

where D can be obtained from eq.(3.4)

Initially taking the value of z to be less than 0.05, the value of H_0 can be obtained as 71.8407. Next we calculate H_0 similar to the previous section, by considering the entire dataset, we obtain H_0 as 63.8739 km/s/Mpc.

3.1.3 $q_0 = 0$

Rearranging eq.(1.3) we get,

$$H_0 = \frac{c}{D}z(1 + \frac{z}{2}) \quad (3.6)$$

For range $z < 0.05$, we can obtain H_0 as 72.3320. similarly for complete dataset the value of H_0 is found to be 69.067 km/s/Mpc.

3.1.4 Eternal coasting model

By rearranging the equation of luminosity distance of eternal coasting model eq.(1.5) we get,

$$H_0 = \frac{c}{D}(1+z) \ln(1+z) \quad (3.7)$$

For low redshift range ($z < 0.05$), H_0 is 72.3222 km/s/MPc . For complete dataset the value of H_0 is 67.9062 km/s/Mpc.

Chapter 4

Bayesian Parameter Estimation

4.1 χ^2 (Chi square) Test

A chi-square (χ^2) statistic is a test that compares a model to actual observed data. Given the size of the sample and the number of variables in the relationship, the chi-square statistic compares the size of any discrepancies between the expected and actual results. χ^2 can be used to determine whether two variables are related or independent.

$$\chi^2 = \left(\frac{\text{Calculated} - \text{Observed}}{\text{Error}} \right)^2 \quad (4.1)$$

The calculated and observed values of apparent magnitude(m) has been taken during the evaluation of χ^2 value. Eq.(4.1) becomes,

$$\chi^2 = \left(\frac{m_c - m_o}{DMB} \right)^2 \quad (4.2)$$

m_c can be deduced from the formula,

$$m_c = (5 \log D - 5) + M \quad (4.3)$$

where absolute magnitude(M) is taken to be -19.3 . Also for the evaluation of χ^2 here we required $D_{\text{calculated}}$ which in turn is obtained by using an expected value of H_0 within the equations for D which varies depending upon the models used (Ref.eqns.(1.2),(1.3),(1.4)).

Using the above equations value of χ^2 is procured. [4]

4.2 Bayesian theory

The Bayesian theory can be used to assess the probability of a hypothesis. The Bayes theorem can be derived with the help of two fundamental axioms. Let A, B, C represent prepositions and \bar{A} represents the denial of A. AB is used to denote some

propositions that is true only if ‘A and B’ are true. Similarly the symbol $A + B$ denotes a proposition that is true if ‘either A or B; is true.

Axiom 1(sum rule) - $P(A | C) + P(\bar{A} | C) = 1$

Axiom 2(product rule) - $P(AB | C) = P(A | BC)P(B | C)$

Here, the vertical bar ‘|’ is the conditional symbol, indicating what propositions are assumed for the assignment of the probability. As a result, while estimating probabilities, we will aim to explicitly include any relevant background information.

We know that AB and BA are identical from the definition of joint occurrence of A and B, and simplifying we get

$$P(A | BC) = P(A | C) \frac{P(B | AC)}{P(B | C)}$$

This is known as the Bayes theorem. Given a data D with some background information I, we can calculate the probability of the hypothesis H by substituting $A = H, B = D$ and $C = I$, hence modifying the theorem as

$$(H | DI) = P(H | I)P(D | HI)P(D | I)$$

the RHS of the above equation is called the posterior probability for H.

The first term $P(H | I)$ is what that gives the probability of the hypothesis. This is assigned before the analysis of the hypothesis hence the name prior probability. It needs to be acknowledged that there is some unpredictability in giving this value, as prior knowledge may vary or even be lacking.

The second term in the numerator $P(D | HI)$, is the probability for the data with the assumption that both the hypothesis and the information are true.

The quantity on the denominator $P(D | I)$, is the probability to get the data irrespective of the hypothesis.

The Bayesian theorem helps in calculating the probability of hypothesis, this method differs from the frequentist approach where probability is measured for random variable. However, we should observe that the axioms that lead to Bayes’ theorem are relatively arbitrary. Another flaw in utilising Bayes’ theory is the issue of allocating prior probability. An important feature of the Bayes theorem is that we can adjust the assessment of a hypothesis when new data regarding the hypothesis is acquired. The Bayesian approach is a more general one as a variety of hypothesis are assessed with their posterior probability. This evaluation requires the prior possibility assignment of the hypothesis and the the probability of the data.

4.2.1 Bayesian parameter estimation

The theory or parameterized model is made up of a set of exclusive hypotheses that are labelled by certain parameters. For some value of each of these parameters, the model is considered to be true. Because the truth of this model is known, it is included in information I in the Bayes' theorem. Continuous or discrete parameters might be used. In this scenario, the hypothesis we wish to evaluate is the set of values for those parameters in the theory. The probability evaluation is done using the theorem that is given as

$$P(\theta | DI) = P(\theta | I) \frac{P(D | \theta I)}{P(D | I)}$$

Where the terms in the numerator are the prior probability and the likelihood function. The probability $P(D | \theta I)$ is the probability for the data, given the truth of the value of the parameter θ . It is called the the likelihood for the value of the parameter θ and can be calculated as $\exp(-\chi^2/2)$.

$$\text{Likelihood of parameter} = \exp(-\chi^2/2) \quad (4.4)$$

The first term on the RHS of the above equation $P(\theta | I)$ is the probability for the value of the parameter θ . Before analysing the data. Hence it is called as the prior probability for θ . We note that the term in the denominator, $P(D | I)$ is independent of θ and that it serves only as a normalisation constant.[\[5\]](#)

4.3 Estimation of Likelihood

Here the likelihood is calculated using the equation(4.4), with the values obtained for $\exp(-\chi^2/2)$ we plot a graph by taking the Hubble constant value between the range 40 to 100 km/s/Mpc.

4.3.1 $q_0 = 1$ (Linear Hubble Law)

Initially we set the limit for z to be less than 0.05 . With the procedures prescribed in the previous section we deduce the values for χ^2 and $\exp(-\chi^2/2)$. With the obtained values for $\exp(-\chi^2/2)$ we plot a graph with the previously defined range for H_0 .

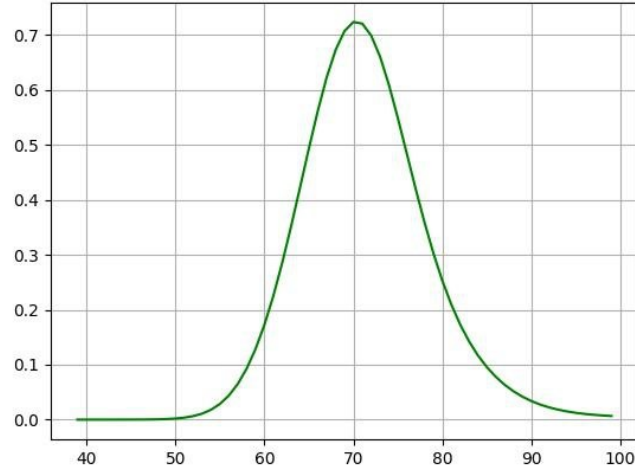


Figure 4.1: Plot of H_0 versus $\exp(-\chi^2/2)$ in the range $z < 0.05$

From the graph it is visible that the curve we have obtained is nearly a gaussian distribution, whose peak gives us the likelihood. Therefore, from the above curve likelihood is 71.

Next we select the range to be $z < 0.1$

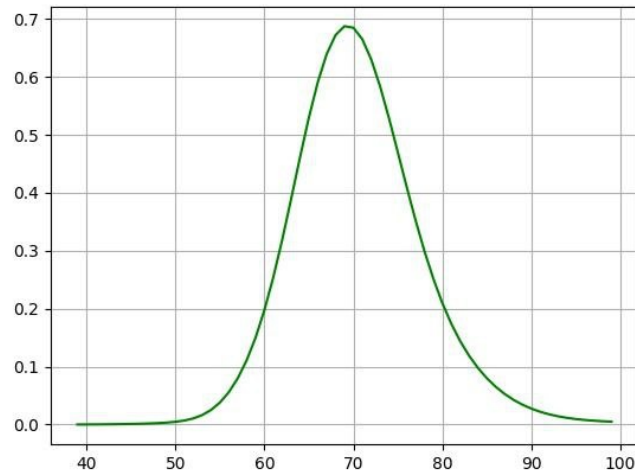


Figure 4.2: Plot of H_0 versus $\exp(-\chi^2/2)$ in the range $z < 0.1$

For this range likelihood is 70 km/s/Mpc.

Similarly we change the range to $z < 0.15$ and plotting,

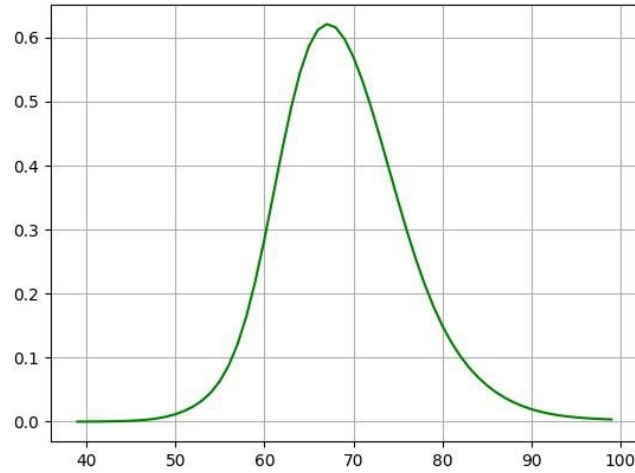


Figure 4.3: Plot of H_0 versus $\exp(-\chi^2/2)$ in the range $z < 0.15$

And for $z < 0.15$, likelihood is 68 km/s/Mpc.

Finally we take the entire dataset

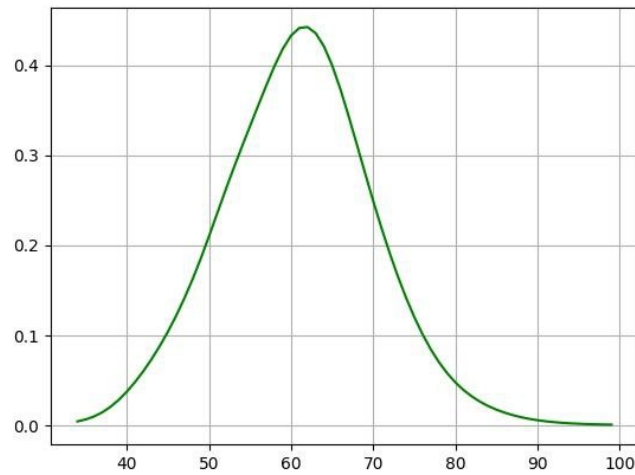


Figure 4.4: Plot of H_0 versus $\exp(-\chi^2/2)$ for the complete dataset($q_0 = 1$)

Here likelihood is 63 km/s/Mpc.

4.3.2 $q_0 = \frac{1}{2}$ (*Einstein – deSitter Model*)

Here we calculate the value of $\exp(-\chi^2/2)$ by taking the entire dataset (1048 supernovae)

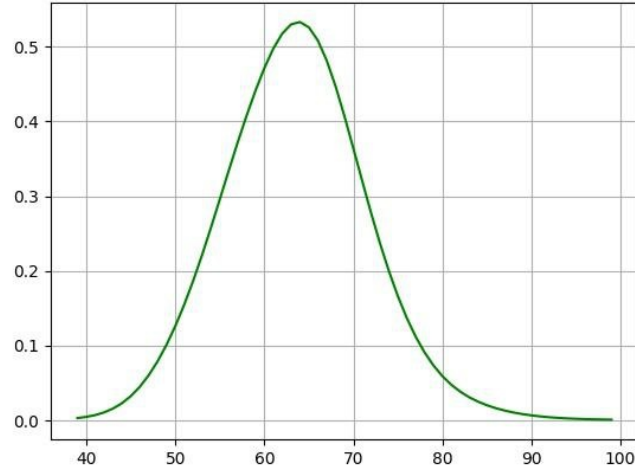


Figure 4.5: Plot of H_0 versus $\exp(-\chi^2/2)$ for the complete dataset($q_0 = \frac{1}{2}$)

From the above graph the peak is at 65 km/s/ Mpc which is the likelihood.

4.3.3 $q_0 = 0$

Similar to the previous subsection here also we take the entire dataset to attain the values of $\exp(-\chi^2/2)$

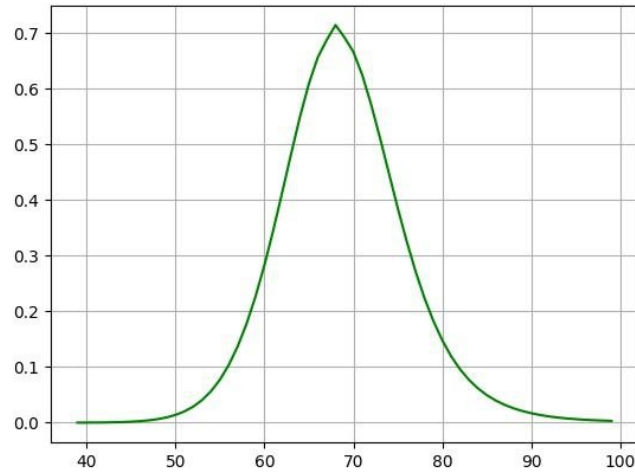


Figure 4.6: Plot of H_0 versus $\exp(-\chi^2/2)$ for the complete dataset($q_0 = 0$)

Similar to the previous graphs here the likelihood can be obtained from the peak which is 69 km/s/Mpc.

4.4 Error Estimation

If we observe all the graphs that we have obtained, it can be seen that they all are almost a gaussian distribution. The standard deviation of a gaussian distribution gives the error in our estimation. In this dissertation we calculate the sigma value directly the graph. if we observe the curve it can be seen that The distribution has equal amount of data on either side of the peak, the standard deviation quantifies the variability of the curve. The assumption that we can make about a data that follows a gaussian distribution is that the area under the curve is relative to how many standard deviation we are away from the mean. The area between plus and minus one standard deviation from the mean contains 68% of the data .Two standard deviation contains 95% of data and three standard deviation contains 99.8% of data.

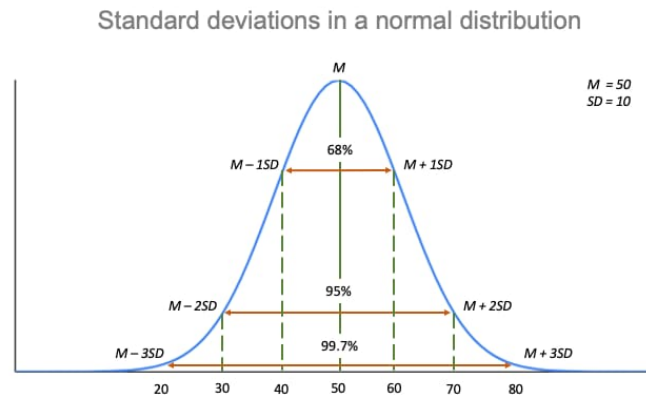


Figure 4.7: Standard deviations in a gaussian distribution.
Source: Scribbr.

Using the method provided in the above graph we can acquired the standard deviation(σ) of all the H_0 versus $\exp(-\chi^2/2)$ graphs.

- For $q_0 = 1$ within the ranges $z < 0.05$, $z < 0.1$, $z < 0.15$ and for the entire dataset the values of σ are 7.83,7.67,7.67 and 7.33 respectively.
- Similarly for $q_0 = 0$ the value of sigma for entire dataset is 7.83
- Likewise for $q_0 = 0.5$ for the entire dataset the value of sigma is 8.17

Chapter 5

Conclusion

Throughout this dissertation we have tried to acquire the value of H_0 using the supernovae data from scolnic et.al. 2018

Initially we calculate the value of H_0 in four ways using four different models. We had obtained the value of H_0 for $q_0 = 1$, $q_0 = 0.5$, $q_0 = 0$ and eternal coasting model in the low redshift region ($z < 0.05$) as 70.7156 km/s/Mpc, 71.8407 km/s/Mpc, 72.3320 km/s/Mpc and 72.3222 km/s/Mpc respectively. If we analyze these H_0 values they can found to be compatible. Hence the low redshift region can be considered to model independent, as visible in fig.(1.1).

As we increase the range of z , the values of H_0 become less compatible with each other. Due to which they are considered to be model dependent. This notion can be better understood if we look into the values of H_0 for $q_0 = 1$, $q_0 = 0.5$, $q_0 = 0$ and eternal coasting model when the entire dataset is taken into consideration. The values are $H_0 = 60.3170 \text{ km/s/Mpc}$, $H_0 = 63.8739 \text{ km/s/Mpc}$, $H_0 = 69.067 \text{ km/s/Mpc}$ and $H_0 = 67.9062 \text{ km/s/Mpc}$ respectively.

We had used Bayesian method to find the most suitable value of H_0 . This suitable value was acquired using the peak of H_0 versus $\exp(-\chi^2/2)$ curve. This peak is called as the likelihood of parameter(H_0).The curve that we had plotted using H_0 and $\exp(-\chi^2/2)$ is a normal curve (gaussian distribution). Any error that might occur can be calculated using the standard deviation, since it is a gaussian distribution.

Combining all the informations that we had stated previously the most suitable H_0 value along with it error for different models was found to be,

$$\begin{aligned} &\text{for } q_0 = 1, H_0 = 63 \pm 7.33 \text{ km/s/Mpc} \\ &\text{for } q_0 = 0.5, H_0 = 65 \pm 8.17 \text{ km/s/Mpc} \\ &\text{for } q_0 = 0, H_0 = 69 \pm 7.83 \text{ km/s/Mpc} \end{aligned}$$

Likewise the value of H_0 model independent situation $H_0 = 71 \pm 7.83 \text{ km/s/Mpc}$. The Hubble's constant is the current rate of expansion of the universe, but the rate predicted by the standard model is far slower than the rate discovered by the most accurate local observations. There are a lot of ways to evaluate the value of Hubble's constant, which creates the primary problem of its evaluation difficult. The existing dilemma is that we haven't been able to assign a single value for H_0 . In this paper, we used the local approach method to calculate the value for H_0 using the data from the scolnic et. el. 2018 data set.

A new approach of cosmologists is to resolve the difference between the predicted and observed values without jeopardising the consistency of the standard model. This can be done with the help of other cosmological phenomenons, But whether such a phenomenon exist is another topic of discussion. Certain cosmologists have now identified a certain aspect of all cosmological models that has been overlooked, which is that most dimensionless cosmic observables are substantially invariant when gravitational free-fall rates and photon-electron scattering rates are scaled uniformly. This finding made us realise that the phenomenon of cosmic background radiation can be used to bridge the gap between the predicted and observed values for the Hubble constant. This discovery aided us in reconciling cosmic background measurements, resulting in the development of a cosmological model in which a scaling transformation for the H_0 value can be performed without violating any measurements of values not protected by symmetry. The "mirror world" dark sector would result in effective scaling of gravitational free-fall speeds while keeping the precisely determined mean photon density a constant. Further modifications to this model may help us to solve the two remaining constraints: the inferred primordial deuterium and helium abundances. This finding has opened the doors to a new possibility: the existence of a parallel universe that is remarkably similar to our universe but can only be seen through the gravitational influence on our world.

Research is also being carried out to identify whether this gap is partly due to the mistakes in measurements while looking for any new missing elements to complete this model. This has grown in importance as the disparity between the values has become more evident as the quality of the data incorporated in the analysis has also increased.

At the end of the day, it is astonishing to see that the discrepancy which we had hoped in reducing has only seen growth with the number of studies being conducted on both early and late universe. This tension might be a true mirror of reality, necessitating exotic new physics and a radical reworking of our knowledge of the cosmos.

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