NO MERCY

JEE ADVANCED PRACTICE SHEET

CHAPTER

BASIC MATHEMATICS

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SECTION-A

JEE MAINS

ACIDIC QUESITONS

QUESTION 1.

Solve the equations $\frac{\sqrt{x^2+1}+\sqrt{x^2-1}}{\sqrt{x^2+1}-\sqrt{x^2-1}} = \frac{2x^2-1}{2}$

QUESTION 2.

The largest integral value of x satisfying $\sqrt{18^x-5} \le \sqrt{2(18^x+12)} - \sqrt{18^x+5}$ is

(A) 0

(B) 1

(C) 2

(D) No integral value of x possible

QUESTION 3.

Let $y = \sqrt{\log_2 3 \cdot \log_2 12 \cdot \log_2 48 \cdot \log_2 192 + 16} - \log_2 12 \cdot \log_2 48 + 10$. Find $y \in N$.

QUESTION 4.

Sum of all the solutions of the equation

$\log_6(x^2-1) - \log_6\sqrt{(x-6)^2} = \log_6(x+1)^2$ is $a+\sqrt{b}, (a, b \in N)$. Then $a+b$ is equal to

QUESTION 5.

If x, y are integral solutions of $2x^2 - 3xy - 2y^2 = 7$, then value of $|x + y|$ is

(A) 2

(B) 4

(C) 6

(D) 2 or 4 or 6

QUESTION 6.

If $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b}$, then $a^a \cdot b^b \cdot c^c =$

(A) 3

(B) 1

(C) 4

(D) 2

QUESTION 7.

The value of $\log_2 \cdot \log_3 \ldots \log_{100} 100^{99^{98^{\cdot^{\cdot^{\cdot^{2^1}}}}}}$ is

(A) 0

(B) 1

(C) 2

(D) 100!

QUESTION 8.

Exhaustive set of values of x satisfying $\log_{|x|}\left(x^2 + x + 1\right) \geq 0$ is

(A) $(-1, 0)$

(B) $(-\infty, -1) \cup (1, \infty)$

(C) $(-\infty, \infty) - \{-1, 0, 1\}$

(D) $(-\infty, -1) \cup (-1, 0) \cup (1, \infty)$

QUESTION 9.

The minimum value of $f(x) = |x - 1| + |x - 2| + |x - 3|$ is equal to

(A) 1

(B) 2

(C) 3

(D) 0

QUESTION 10.

Number of real value of x satisfying the equation $\log_2\left(2x^2 + \sqrt{2}\right) = \frac{\sqrt{x^2+1}}{x^2+2}$ is

(A) 0

(B) 1

(C) 2

(D) 3

QUESTION 11.

The value of $x + y + z$ satisfying the system of equations

$\log_2 x + \log_4 y + \log_4 z = 2$

$\log_3 y + \log_9 z + \log_9 x = 2$

$\log_4 z + \log_{16} x + \log_{16} y = 2$ is

(A) $\frac{175}{12}$

(B) $\frac{349}{24}$

(C) $\frac{353}{24}$

(D) $\frac{112}{3}$

QUESTION 12.

Number of non-negative integral values of x satisfying the inequality $\frac{2}{x^2-x+1} - \frac{1}{x+1} - \frac{2x-1}{x^3+1} \geq 0$ is

(A) 0

(B) 1

(C) 2

(D) 3

QUESTION 13.

Suppose $x, y, z > 0$ and different than one and $\ln x + \ln y + \ln z = 0$.

If $\frac{1}{e^k} = x^{\frac{1}{\ln y}+\frac{1}{\ln z}} \cdot y^{\frac{1}{\ln z}+\frac{1}{\ln x}} \cdot z^{\frac{1}{\ln x}+\frac{1}{\ln y}}$. The k =

QUESTION 14.

If $x = \sqrt[3]{7+5\sqrt{2}} - \frac{1}{\sqrt[3]{7+5\sqrt{2}}}$, then the value of $x^3 + 3x - 14$ is equal to

QUESTION 15.

If $\sqrt{\log_4\{\log_3\{\log_2(x^2-2x+a)\}\}}$ is defined $\forall\, x \in R$, then the set of values of 'a' is

(A) $[9, \infty)$

(B) $[10, \infty)$

(C) $[15, \infty)$

(D) $[2, \infty)$

QUESTION 16.

The solution set of inequality $\frac{(3^x - 4^x)\cdot\ln(x+2)}{x^2-3x-4} \leq 0$ is

(A) $(-\infty, 0] \cup (4, \infty)$

(B) $(-2, 0] \cup (4, \infty)$

(C) $(-1, 0] \cup (4, \infty)$

(D) $(-2, -1) \cup (-1, 0] \cup (4, \infty)$

QUESTION 17.

Number of positive solution which satisfy the equation
$\log_2 x \cdot \log_4 x \cdot \log_6 x = \log_2 x \cdot \log_4 x + \log_2 x \cdot \log_6 x + \log_4 x \cdot \log_6 x$?

(A) 0

(B) 1

(C) 2

(D) infinite

QUESTION 18.

The set of values of x satisfying simultaneously the inequalities
$\frac{\sqrt{(x-8)(2-x)}}{\log_{0.3}\left(\frac{10}{7}(\log_2 5-1)\right)} \geq 0$ and $2^{x-3} - 31 > 0$ is

(A) singleton set

(B) an empty set

(C) an infinite set

(D) a set consisting of exactly two elements

QUESTION 19.

Match the following columns:

	Column-I		Column-II
A.	If $a = 3(\sqrt{8+2\sqrt{7}} - \sqrt{8-2\sqrt{7}})$, $b = \sqrt{(42)(30)+36}$ then the value of $\log_a b$ is equal to	p.	-1
B.	If $a = \sqrt{4+2\sqrt{3}} - \sqrt{4-2\sqrt{3}}$, $b = \sqrt{(42)(30)+36}$ then the value of $\log_a b$ is equal to	q.	1
C.	If $a = \sqrt{3+2\sqrt{2}}, b = \sqrt{3-2\sqrt{2}}$ then the value of $\log_a b$ is equal to	r.	2
D.	If $a = \sqrt{7+\sqrt{7^2-1}}, b = \sqrt{7-\sqrt{7^2-1}}$, then the value of $\log_a b$ is equal to	s.	$2+2\log_2 3$

(A) A → s, B → p, C → q, D → p

(B) A → r, B → p, C → r, D → p

(C) A → r, B → s, C → p, D → p

(D) A → p, B → q, C → p, D → r

QUESTION 20. [JEE Main-2023]

The number of integral solutions x of $\log_{\left(x+\frac{7}{2}\right)}\left(\frac{x-7}{2x-3}\right)^2 \geq 0$ is

(A) 6

(B) 8

(C) 5

(D) 7

QUESTION 21. [JEE Main-2021]

The sum of all roots of the equation
$x + 1 - 2\log_2(3+2^x) + 2\log_4(10-2^{-x}) = 0$, is

(A) $\log_2 12$

(B) $\log_2 13$

(C) $\log_2 11$

(D) $\log_2 14$

SECTION-B

JEE ADVANCED

CHALLENGER QUESITONS

CRITICAL THINKING QUESITONS

QUESTION 22.

Prove that : $2^{\left(\sqrt{\log_a \sqrt[4]{ab}+\log_b \sqrt[4]{ab}}-\sqrt{\log_a \sqrt[4]{\frac{b}{a}}+\log_b \sqrt[4]{\frac{a}{b}}}\right)\cdot\sqrt{\log_a b}} = \begin{cases} 2 & \text{if } b \geq a > 1 \\ 2^{\log_a b} & \text{if } 1 < b < a \end{cases}$

QUESTION 23.

The roots of the equation $|x| = 49^{\left(\frac{1}{2}+\log_{\frac{1}{7}}27+\log_{343}81\right)}$ **include**

(A) One positive number greater than 1 only

(B) Two real number

(C) Two irrational number

(D) One negative rational number

QUESTION 24.

Solution set of the inequality $(\log_2 x)^4 - \left(\log_{1/2}\frac{x^3}{8}\right)^2 + 9\log_2\left(\frac{32}{x^2}\right) < 4\left(\log_{1/2} x\right)^2$ **is** $(a,b)\cup(c,d)$ **then the correct statement is**

(A) $a = 2b$ and $d = 2c$

(B) $b = 2a$ and $d = 2c$

(C) $\log_c d = \log_b a$

(D) there are 4 integers in (c,d)

QUESTION 25.

Match the following columns:

	Column-I	Column-II	
A.	If $p = \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}$ then $\log_{(5+2\sqrt{6})} p$ is	p.	0
B.	If $r = \frac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ then $\log_{9+2\sqrt{15}}(1/r)$ is	q.	2
C.	If $t = \frac{3+\sqrt{6}}{5\sqrt{3}-2\sqrt{12}-\sqrt{32}+\sqrt{50}}$ then $\log_{\sqrt{3}} t^2$ is	r.	-1
D.	If $k = \frac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} + \frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}$ then $\log_e(k+1)$ is	s.	1

(A) A → t, B → s, C → r, D → q

(B) A → s, B → r, C → q, D → p

(C) A → r, B → p, C → r, D → s

(D) A → t, B → q, C → s, D → r

QUESTION 26.

Find the number of integral solution of the equation

$\log_{\sqrt{x}}(x+|x-2|) = \log_x(5x-6+5|x-2|)$.

QUESTION 27.

If the solution set of $(0.3)^{\log_{\frac{1}{3}}\log_2\frac{3x+6}{x^2+2}} > 1$ **is** $\left(\frac{-1}{\alpha}, \alpha\right)$**, then** α **=**

QUESTION 28.

Suppose n **be an integer greater than 1. Let** $a_n = \frac{1}{\log_n 2002}$.

Suppose $b = a_2 + a_3 + a_4 + a_5$ **and** $c = a_{10} + a_{11} + a_{12} + a_{13} + a_{14}$**. Then find the value of** $(c-b)$.

QUESTION 29.

The complete solution set of the inequality $\frac{1}{\log_4\frac{x+1}{x+2}} < \frac{1}{\log_4(x+3)}$**, is** $(-a, \infty)$**, then determine** 'a'.

QUESTION 30.

Consider the system of equations

$\log_{10}(2000xy) - \log_{10}x \cdot \log_{10}y = 4$

$\log_{10}(2yz) - \log_{10}y \cdot \log_{10}z = 1$

and $\log_{10}(zx) - \log_{10}z \cdot \log_{10}x = 0$**,** $x, y, z \leq 90$**, then** $x+y+z$ **=**

QUESTION 31.

If x **and** y **satisfying both the equations** $\log_3 x + \log_2 y = 2; 3^x - 2^y = 23$

then sum of all the values of x **and** y **is**

QUESTION 32.

The solution set of $\log_3(\sqrt{x}+|\sqrt{x}-1|)=\log_9(4\sqrt{x}-3+4|\sqrt{x}-1|)$ is $[\alpha,\beta]\cup\{\gamma\}$, then $\gamma^{\alpha+\beta}$ =

CRITICAL THINKING QUESTION (CTQ) 33.

If $\log_4(x+2y)+\log_4(x-2y)=1$, then the minimum value of $|x|-|y|$ is

CRITICAL THINKING QUESTION (CTQ) 34.

Find sum of all possible natural numbers 'n' for which $\frac{5n^2-7n+84}{n}$ is divisible by 5.

CRITICAL THINKING QUESTION (CTQ) 35.

The value of $\left[2008+\log_{\left(\frac{6561}{256}\right)}\left(\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}}\ldots\ldots}}\right)\right]$ is

(where [•] is G.I.F.)

CHALLENGER QUESTION 36.

If the sum of all real numbers x and y such that the following system of inequalities holds:

$$\left\{\begin{array}{c}4^{-x}+27^{-y}=\frac{5}{6}\\ \log_{27}y-\log_4 x\geq\frac{1}{6}\\ 27^{y}-4^{x}\leq 1\end{array}\right\}$$ is k, then find $6k$.

CRITICAL THINKING QUESTION (CTQ) 37.

Solve the inequality $\log_2\left(x^{12}+3x^{10}+5x^8+3x^6+1\right)<1+\log_2\left(x^4+1\right)$

(A) $x\in\left(-\sqrt{\frac{-1+\sqrt{6}}{2}},\sqrt{\frac{-1+\sqrt{5}}{2}}\right)$

(B) $x\in\left(-\sqrt{\frac{-1+\sqrt{5}}{2}},\sqrt{\frac{-1+\sqrt{5}}{2}}\right)$

(C) $x\in\left(-\sqrt{\frac{-1+\sqrt{6}}{2}},\sqrt{\frac{-1+\sqrt{6}}{2}}\right)$

(D) $x\in\left(-\sqrt{\frac{-1+\sqrt{7}}{2}},\sqrt{\frac{-1+\sqrt{7}}{2}}\right)$

CRITICAL THINKING QUESTION (CTQ) 38.

For what values of a, the inequality (for x)
$\log_{\frac{1}{a}}\left(\sqrt{x^2+ax+5}+1\right)\cdot\log_5\left(x^2+ax+6\right)+\log_a 3\geq 0$ has exactly one solution?

QUESTION [IIT-JEE-1980] 39.

If $x>y>0$, then show that the expression
$\left(\sqrt{2}\left(2x+\sqrt{x^2-y^2}\right)\left(\sqrt{x-\sqrt{x^2-y^2}}\right)\right)$ can be simplified to
$\sqrt{(x+y)^3}-\sqrt{(x-y)^3}$

CRITICAL THINKING QUESTION (CTQ) 40.

If $\frac{(x-3)^{\frac{-|x|}{x}}\sqrt{(x-4)^2}(\pi-x)}{\sqrt{-x}\left(-x^2+x-1\right)(|x|-9)}<0$, then number of integers x satisfying the inequality is

QUESTION 41.

Solve the equation : $\left(x^2+x-57\right)^{3x^2+3}=\left(x^2+x-57\right)^{10x}$

QUESTION [IIT-JEE-1997/JEE Mains-2019] 42.

Find the set of all solutions of the equation
$2^{|y|}-|2^{y-1}-1|=2^{y-1}+1$

Answer Key

Section-A

1.	No Solution	13. 3	
2.	(D)	14. 0	
3.	6	15. (A)	
4.	21	16. (D)	
5.	(B)	17. (C)	
6.	(B)	18. (A)	
7.	(B)	19. (C)	
8.	(D)	20. (A)	
9.	(B)	21. (C)	
10.	(B)		
11.	(C)		
12.	(D)		

Section-B

22.	Prove
23.	(B, D)
24.	(B, C)
25.	(B)
26.	1
27.	2
28.	1
29.	1
30.	7
31.	5
32.	4
33.	$\sqrt{3}$
34.	63
35.	2007
36.	5
37.	(B)
38.	2
39.	Prove
40.	8
41.	$x=\left\{\frac{-1\pm\sqrt{233}}{2},\frac{-1\pm\sqrt{299}}{2},7,3,\frac{1}{3}\right\}$
42.	$y\in[1,\infty)\cup\{-1\}$