

# JEE ADVANCED PRACTICE SHEET

CHAPTER

# **BASIC MATHEMATICS**

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# SECTION-A

**JEE MAINS** 

**ACIDIC QUESITONS** 

# QUESTION 1.

Solve the equations  $\frac{\sqrt{x^2+1}+\sqrt{x^2-1}}{\sqrt{x^2+1}-\sqrt{x^2-1}} = \frac{2x^2-1}{2}$ 



#### QUESTION 2.

The largest integral value of x satisfying

$$\sqrt{18^x - 5} \le \sqrt{2(18^x + 12)} - \sqrt{18^x + 5}$$
 is





 $\bigcirc$  No integral value of x possible

#### QUESTION 3.

Let  $y = \sqrt{\log_2 3 \cdot \log_2 12 \cdot \log_2 48 \cdot \log_2 192 + 16} - \log_2 12 \cdot \log_2 48 + 10$ . Find  $y \in N$ .



#### QUESTION 4.

Sum of all the solutions of the equation

 $\log_6\!\left(x^2-1\right)-\log_6\!\sqrt{(x-6)^2}=\log_6(x+1)^2$  is  $a+\sqrt{b}, (a,b\in N).$  Then a+b is equal to



If x, y are integral solutions of  $2x^2 - 3xy - 2y^2 = 7$ , then value of |x + y| is

- (A) 2
- (B)
- (c)
- D 2 or 4 or 6

- If  $\frac{\log a}{b-c}=\frac{\log b}{c-a}=\frac{\log c}{a-b}$  , then  $a^a\cdot b^b\cdot c^c=$
- A 3
- (B)
- (c)
- (D) 2

## QUESTION 7.

The value of  $\log_2 \cdot \log_3 .... \log_{100} 100^{99^{98}}$  is

- (A) (
- (B)
- (c)
- D 100!



#### QUESTION 8.

Exhaustive set of values of x satisfying  $\log_{|x|}\!\left(x^2+x+1\right)\geq 0$  is

- (-1, 0)
- B (-∞, -1) ∪ (1, ∞)
- (c) (-∞, ∞) {-1, 0, 1}
- $(-\infty, -1) \cup (-1, 0) \cup (1, \infty)$

#### QUESTION 9.

The minimum value of f(x) = |x-1| + |x-2| + |x-3| is equal to

- (A) 1
- (B) 2
- (c) :
- D 0



#### QUESTION 10.

Number of real value of x satisfying the equation  $\log_2(2x^2+\sqrt{2})=\frac{\sqrt{x^2+1}}{x^2+2}$  is

- A 0
- B 1
- (c) :
- D 3

The value of x + y + z satisfying the system of equations

$$\log_2 x + \log_4 y + \log_4 z = 2$$

$$\log_3 y + \log_9 z + \log_9 x = 2$$

$$\log_4 z + \log_{16} x + \log_{16} y = 2$$
 is

- $\frac{175}{12}$
- $\frac{349}{24}$
- $\frac{353}{24}$
- $\frac{112}{3}$

Number of non-negative integral values of  $\boldsymbol{x}$  satisfying the inequality

$$\frac{2}{x^2 - x + 1} - \frac{1}{x + 1} - \frac{2x - 1}{x^3 + 1} \ge 0 \text{ is}$$

- (A) 0
- (B)
- **(c)**
- (D) 3

# QUESTION 13.

Suppose x, y, z > 0 and different than one and  $\ln x + \ln y + \ln z = 0$ .

If 
$$\frac{1}{e^k} = x^{\frac{1}{\ln y} + \frac{1}{\ln z}} \cdot y^{\frac{1}{\ln z} + \frac{1}{\ln x}} \cdot z^{\frac{1}{\ln x} + \frac{1}{\ln y}}$$
 . The  $k =$ 



### QUESTION 14.

If  $x = \sqrt[3]{7+5\sqrt{2}} - \frac{1}{\sqrt[3]{7+5\sqrt{2}}}$ , then the value of  $x^3+3x-14$  is equal to

#### QUESTION 15.

If  $\sqrt{\log_4\{\log_3\{\log_2(x^2-2x+a)\}\}}$  is defined  $\forall x \in R$ , then the set of values of 'a' is

- (A) [9,∞)
- B [10, ∞)
- (c) [15, ∞)
- D [2, ∞)



# QUESTION 16.

The solution set of inequality  $\frac{(3^x-4^x)\cdot \ln(x+2)}{x^2-3x-4} \leq 0$  is

- B (-2, 0] ∪ (4, ∞)
- C (-1,0]∪(4,∞)
- D (-2, -1) ∪ (-1, 0] ∪ (4, ∞)

# Number of positive solution which satisfy the equation

 $\log_2 x \cdot \log_4 x \cdot \log_6 x = \log_2 x \cdot \log_4 x + \log_2 x \cdot \log_6 x + \log_4 x \cdot \log_6 x$ 

- A
- B
- (c) 2
- D infinite



## QUESTION 18.

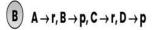
# The set of values of $\boldsymbol{x}$ satisfying simultaneously the inequalities

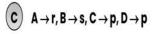
$$\frac{\sqrt{(x-8)(2-x)}}{\log_{0.0}(\frac{10}{7}(\log_{2}5-1))} \geq 0$$
 and  $2^{x-3}-31>0$  is

- (A) singleton set
- B an empty set
- c an infinite set
- D a set consisting of exactly two elements



# 

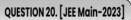




#### QUESTION 19.

# Match the following columns:

Column-I		Column-II	
A.	If $a=3(\sqrt{8+2\sqrt{7}}-\sqrt{8-2\sqrt{7}})$ , $b=\sqrt{(42)(30)+36}$ then the value of $\log_a b$ is equal to	p.	4
В.	If $a=\sqrt{4+2\sqrt{3}}-\sqrt{4-2\sqrt{3}}$ , $b=\sqrt{(42)(30)+36}$ then the value of $\log_a b$ is equal to	q.	1
C.	If $a=\sqrt{3+2\sqrt{2}}$ , $b=\sqrt{3-2\sqrt{2}}$ then the value of $\log_a b$ is equal to	r.	2
D.	If $a=\sqrt{7+\sqrt{7^2-1}}$ , $b=\sqrt{7-\sqrt{7^2-1}}$ , then the value of $\log_a b$ is equal to	S.	2 + 2log <sub>2</sub> 3



The number of integral solutions x of  $\log_{\left(x+\frac{7}{2}\right)}\left(\frac{x-7}{2x-3}\right)^2 \geq 0$  is

- (A) 6
- (B)
- (C) 5
- (D) 7



# QUESTION 21. [JEE Main-2021]

The sum of all roots of the equation

$$x + 1 - 2\log_2(3 + 2^x) + 2\log_4(10 - 2^{-x}) = 0$$
, is

- (A) log<sub>2</sub> 12
- B log<sub>2</sub> 13
- C log<sub>2</sub> 11
- D log<sub>2</sub> 14



PW



#### QUESTION 22.

 $\left(\sqrt{\log_a \sqrt[4]{ab} + \log_b \sqrt[4]{ab}} - \sqrt{\log_a \sqrt[4]{\frac{b}{a}} + \log_b \sqrt[4]{\frac{a}{b}}}\right)\sqrt{\log_a b} = \begin{bmatrix} 2 & \text{if } b \ge a > 1 \\ 2^{\log_a b} & \text{if } 1 < b < a \end{bmatrix}$ 

# SECTION-B

JEE ADVANCED

**CHALLENGER QUESITONS** 

**CRITICAL THINKING QUESITONS** 

#### QUESTION 23.

The roots of the equation  $|x|=49^{\left(\frac{1}{2}+\log_1 27+\log_{343} 81\right)}$  include

- One positive number greater than 1 only
- B Two real number
- Two irrational number
- One negative rational number

#### QUESTION 25.

# Match the following columns:

Column-I		Column-II	
A.	If $p = \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}$ then $\log_{(5+2\sqrt{6})} p$ is	p.	0
В.	If $r=rac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ then $\log_{9+2\sqrt{15}}(1/r)$ is	q.	2
C.	If $t = \frac{3+\sqrt{6}}{5\sqrt{3}-2\sqrt{12}-\sqrt{32}+\sqrt{50}}$ then $\log_{\sqrt{3}} t^2$ is	r.	-1
D.	If $k = \frac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} + \frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}$ then $\log_e(k+1)$ is	s.	1



# QUESTION 24.

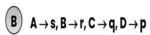


Solution set of the inequality  $(\log_2 x)^4 - \left(\log_{1/2} \frac{x^3}{8}\right)^2 + 9\log_2\left(\frac{32}{x^2}\right) < 4\left(\log_{1/2} x\right)^2$ is  $(a,b) \cup (c,d)$  then the correct statement is

- a = 2b and d = 2c
- b = 2a and d = 2c
- $\log_c d = \log_b a$
- there are 4 integers in (c, d)



 $A \rightarrow t, B \rightarrow s, C \rightarrow r, D \rightarrow q$ 



- $\bigcirc$  A  $\rightarrow$  r, B  $\rightarrow$  p, C  $\rightarrow$  r, D  $\rightarrow$  s
- $oxed{D}$  A  $\rightarrow$  t, B  $\rightarrow$  q, C  $\rightarrow$  s, D  $\rightarrow$  r



## **6.**

 $\log_{\sqrt{x}}(x+|x-2|) = \log_x(5x-6+5|x-2|).$ 

Find the number of integral solution of the equation



QUESTION 27.

If the solution set of  $(0.3)^{\frac{\log_2\log_2\frac{3x+6}{x^2+2}}} > 1$  is  $(\frac{-1}{\alpha}, \alpha)$ , then  $\alpha$  =

#### QUESTION 28.

Suppose n be an integer greater than 1. Let  $a_n=\frac{1}{\log_n 2002}$ . Suppose  $b=a_2+a_3+a_4+a_5$  and  $c=a_{10}+a_{11}+a_{12}+a_{13}+a_{14}$ . Then find the value of (c-b).



# QUESTION 29.

The complete solution set of the inequality  $\frac{1}{\log_4\frac{x+1}{x+2}} < \frac{1}{\log_4(x+3)}$ , is  $(-a, \infty)$ , then determine 'a'.

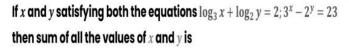
#### QUESTION 30.

# Consider the system of equations

$$\begin{array}{l} \log_{10}(2000xy) - \log_{10}x \cdot \log_{10}y = 4 \\ \log_{10}(2yz) - \log_{10}y \cdot \log_{10}z = 1 \\ \text{and } \log_{10}(zx) - \log_{10}z \cdot \log_{10}x = 0 \text{, } x,y,z \leq 90 \text{, then } x+y+z = 0 \end{array}$$



## QUESTION 31.







P<sub>W</sub>

The solution set of  $\log_3(\sqrt{x}+|\sqrt{x}-1|)=\log_9(4\sqrt{x}-3+4|\sqrt{x}-1|)$  is  $[\alpha,\beta]\cup\{\gamma\}$ , then  $\gamma^{\alpha+\beta}=$ 

If  $\log_4(x+2y) + \log_4(x-2y) = 1$ , then the minimum value of |x| - |y| is

## CRITICAL THINKING QUESTION (CTQ) 34.



CRITICAL THINKING QUESTION (CTQ) 35.



Find sum of all possible natural numbers 'n' for which  $\frac{5n^2-7n+84}{n}$  is divisible by 5.

The value of 
$$\left[2008 + \log_{\left(\frac{6561}{256}\right)} \left(\frac{1}{3\sqrt{2}} \sqrt{4 - \frac{1}{3\sqrt{2}}} \sqrt{4 - \frac{1}{3\sqrt{2}}} \sqrt{4 - \frac{1}{3\sqrt{2}}} \dots \right)\right]$$
 is (where  $[\bullet]$  is G.I.F.)

#### **CHALLENGER QUESTION 36.**



CRITICAL THINKING QUESTION (CTQ) 37.



If the sum of all real numbers  $\boldsymbol{x}$  and  $\boldsymbol{y}$  such that the following system of inequalities holds:

$$\begin{cases} 4^{-x} + 27^{-y} = \frac{5}{6} \\ \log_{27} y - \log_4 x \ge \frac{1}{6} \\ 27^y - 4^x \le 1 \end{cases}$$
 is  $k$ , then find  $6k$ .

Solve the inequality  $\log_2(x^{12} + 3x^{10} + 5x^8 + 3x^6 + 1) < 1 + \log_2(x^4 + 1)$ 

## CRITICAL THINKING QUESTION (CTQ) 38.

# For what values of a, the inequality (for x)

 $\log_{\frac{1}{2}}(\sqrt{x^2+ax+5}+1)\cdot\log_{5}(x^2+ax+6)+\log_{a}3\geq 0$  has exactly one solution?



## QUESTION [IIT-JEE-1980] 39.

If x > y > 0, then show that the expression

$$\left( \sqrt{2} \left( 2x + \sqrt{x^2 - y^2} \right) \left( \sqrt{x - \sqrt{x^2 - y^2}} \right) \right)$$
 can be simplified to 
$$\sqrt{(x + y)^3} - \sqrt{(x - y)^3}$$

## CRITICAL THINKING QUESTION (CTQ) 40.

If  $\frac{(x-3)^{\frac{-|x|}{x}}\sqrt{(x-4)^2(\pi-x)}}{\sqrt{-x}(-x^2+x-1)(|x|-9)}<0$ , then number of integers x satisfying the inequality is



## QUESTION 41.

Solve the equation:  $(x^2 + x - 57)^{3x^2+3} = (x^2 + x - 57)^{10x}$ 

## QUESTION [IIT-JEE-1997/JEE Mains-2019] 42.

# Find the set of all solutions of the equation

$$2^{|y|} - |2^{y-1} - 1| = 2^{y-1} + 1$$



# **Answer Key**

# Section-B

- Section-A No Solution 13. 3 (D) 3. 6
- 22. Prove
  - 34. 63 35. 2007

- 14. 0 15. (A)
- 23. (B, D) 24. (B, C)
  - 36. 5

- 16. (D)
- 25. (B)
- 37. (B)

- (B) (B)
- 17. (C) 18. (A)
- 26. 1 27. 2
- 38. 2 39. Prove

- 7. (B)
- 19. (C)
- 28. 1
- 40.8

- 8. (D) 9. (B)
- 20. (A) 21. (C)
- 29. 1 30. 7
- **41.**  $x = \left\{ \frac{-1 \pm \sqrt{233}}{2}, \frac{-1 \pm \sqrt{299}}{2}, 7, 3, \frac{1}{3} \right\}$ **42.**  $y \in [1, \infty) \cup \{-1\}$

- 10. (B)
- 11. (C)
- 31. 5 32. 4

- 12. (D)
- **33.**  $\sqrt{3}$

