UNIT I

CHAPTER 2

Mathematical Induction

Syllabus

Proof by Mathematical Induction and Strong Mathematical Induction.

2.1 MATHEMATICAL INDUCTION

- It is a method of proving mathematical theorems, statements or formulae.
- In mathematical induction, induction is about **"Inductive Reasoning"**.
- It is deductive in nature.
- In mathematical induction, we first prove that the first proposition is true called as **"base of Induction"**.
- Next, we prove that if k^{th} proposition is true then $(K)^{th}$ proposition is also true known as the **"Inductive step"**.
- There are many real time examples which will show the use of mathematical induction.
  (i) From Fig. 2.1.1 given below, it is clear that, if first the dominoes falls then all the dominoes will fall.
  (ii) If the first teeth of zip is zipped successfully then we can successfully zip a proper zipper.
- For software developer, mathematical induction play important role in algorithm verification.
- It is used to generalized the results from any theorems or formulae's.
- There are two principles of mathematical induction.
  (a) First principle of mathematical induction
  (b) Second principle of mathematical induction.

Steps involved in mathematical induction

▶ **Step I : Basis of induction**

Here, check, validity or correctness of given statement say S(n) is true for the smallest integral value of n = 1 or 2 or 3 ...

▶ **Step II : Induction step**

In this step, assume given statement S(n) is true for n = k where k denotes any value of n, then it is also true that S(n) is true for n = k + 1

▶ **Step III : Conclusion**

The statement is true for all integral values of n equal to or greater than that for which it was verified in step – I.

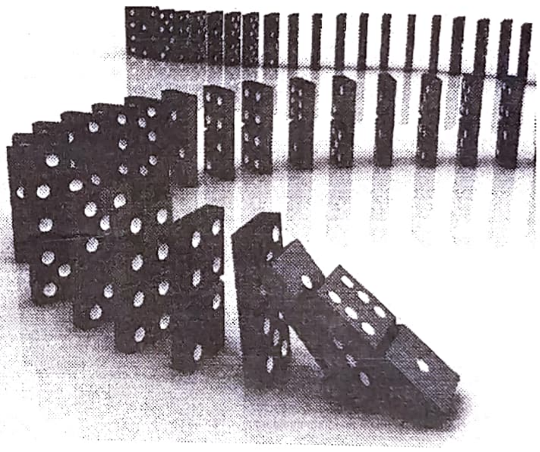

Fig. 2.1.1

2.1.1 Solved Examples

UEx. 2.1.1 **(SPPU - Q. 1(a), Dec.18, 4 Marks)**

By using mathematical induction show that :

$1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$ for all natural number values of n.

Soln. :

Let $S(n) \; : \; 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$ ...(1)

▶ **Step I : Basis of induction**

Check S (n) is true for n = 1

$$\text{LHS} = 1, \quad \text{RHS} = \frac{1(1+1)}{2} = \frac{2}{2} = 1$$

$\therefore$ LHS = RHS

Hence, S(n) is true for n = 1

▶ **Step II : Induction step**

Assume S(n) is true for n = k

Thus we get

$$S(k) \; : \; 1 + 2 + 3 + \ldots + k = \frac{k(k+1)}{2} \qquad \ldots(2)$$

Now, check S(n) is true for n = k + 1 from Equation (1),

$$S(k+1) : 1 + 2 + 3 + \ldots + k + k + 1 = \frac{(k+1)(k+2)}{2}$$

Now, solve the LHS of above statement

$$S(k+1) \;:\; \underbrace{1+2+3+\ldots+k}+(k+1)$$

$$= \frac{k(k+1)}{2} + k + 1 \quad \because \text{From Equation (2)}$$

$$= \frac{k(k+1)+2(k+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2} = \text{RHS of } S(k+1)$$

► **Step III**

Conclusion : $S(n) : 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$ is true using mathematical induction.

Ex. 2.1.2 : Prove by the mathematical induction

$$S(n) = 1 + 3 + 6 + \ldots + \frac{n(n+1)}{2} = \frac{n(n+1)(n+2)}{6}$$

☑ **Soln. :**

► **Step I : Basis of induction**

$$S(n) = 1 + 3 + 6 + \ldots + \frac{n(n+1)}{2} = \frac{n(n+1)(n+2)}{6} \quad \ldots(1)$$

For n = 1, put n = 1 in LHS and RHS of S(n)

$$S(1) : \quad \text{LHS} = \frac{1(1+1)}{2} = \frac{2}{2} = 1,$$

$$\text{RHS} = \frac{1(1+1)(1+2)}{6} = \frac{(2)(3)}{6} = \frac{6}{6} = 1$$

$$\text{LHS} = \text{RHS}$$

Hence, S(n) is true for n = 1

► **Step II : Induction step**

Assume, S(n) is true for n = k

So,

$$S(k) : 1 + 3 + 6 + \ldots + \frac{k(k+1)}{2} = \frac{k(k+1)(k+2)}{6} \quad \ldots(2)$$

Now, check S(n) is true for n = k + 1

So, put n = k + 1 in Equation (1) then,

$$S(k+1) : 1 + 3 + 6 + \ldots + \frac{k(k+1)}{2} + \frac{(k+1)(k+2)}{2}$$

$$= \frac{(k+1)(k+2)(k+3)}{6}$$

Lets start from L.H.S. of S (k + 1)

$$S(k+1) : 1 + 3 + 6 + \ldots + \frac{k(k+1)}{2} + \frac{(k+1)(k+2)}{2}$$

$$= \frac{k(k+1)(k+2)}{6} + \frac{(k+1)(k+2)}{2} \quad \because \text{From Equation (2)}$$

$$= \frac{2[k(k+1)(k+2)] + 6[(k+1)(k+2)]}{12}$$

$$= \frac{[k(k+1)(k+2)] + 3[(k+1)(k+2)]}{6}$$

$$= \frac{(k+1)(k+2)(k+3)}{6} = \text{R.H.S of } S(k+1)$$

► **Step III : Conclusion**

This show that, if S(n) is true for n = k, then it is also S(n) is true for n = k + 1. Thus, mathematical induction is true for every value of n.

Ex. 2.1.3 : Prove by mathematical induction

$$S(n) : 1 \cdot 2 + 2 \cdot 2^2 + \ldots + n \cdot 2^n = (n-1) \cdot 2^{n+1} + 2$$

☑ **Soln. :**

Given : $S(n) : 1 \cdot 2 + 2 \cdot 2^2 + \ldots + n \cdot 2^n = (n-1) \cdot 2^{n+1} + 2$ ...(1)

► **Step I : Basis of induction**

Check, S(n) is true for n = 1

Put, n = 1, $\quad \text{L.H.S} = 1 \cdot 2^1 = 2$

$\text{R.H.S} = (1 \cdot 1) \cdot 2^{1+1} + 2 = 0 \times 2^2 + 2 = 2$

Here, L.H.S. = R.H.S, Hence, S(n) is true for n = 1.

► **Step II : Induction step**

Assume, S(n) is true for n = k

Put, n = k in S(n)

$$S(k) : 1 \cdot 2 + 2 \cdot 2^2 + \ldots + k \cdot 2^k = (k-1) \cdot 2^{k+1} + 2 \quad \ldots(2)$$

Now, check S(n) is true for n = k + 1

$$S(k+1) : 1 \cdot 2 + 2 \cdot 2^2 + \ldots + k \cdot 2^k + (k+1) \cdot 2^{k+1} = (k+1-1) \cdot 2^{(k+1+1)} + 2$$

$$S(k+1) : 1 \cdot 2 + 2 \cdot 2^2 + \ldots + k \cdot 2^k + (k+1) \cdot 2^{k+1} = k \cdot 2^{(k+2)} + 2$$

Taking L.H.S. of above equation,

$$S(k+1) : 1 \cdot 2 + 2 \cdot 2^2 + \ldots + k \cdot 2^k + (k+1) \cdot 2^{k+1} = (k-1)\,2^{k+1} + 2 + (k+1) \cdot 2^{k+1}$$

$\because$ From Equation (2)

Taking 2^{k+1} common

$$= 2^{k+1}\,[(k-\cancel{1}) + (k+\cancel{1})] + 2$$

$$= 2^{k+1}\,[2k] + 2$$

$$= 2^k \cdot 2^1 \cdot 2^1 \cdot k^1 + 2 \Rightarrow (2)^{k+1+1} \cdot k + 2$$

$$= 2^{k+1+1}(k+1-1) + 2$$

$$\Rightarrow (k+1-1) \cdot 2^{k+1+1} + 2$$

$$\Rightarrow (k) \cdot 2^{k+2} + 2 = \text{R.H.S}$$

► **Step III : Conclusion**

This shows that S(n) is true for n = k then it is also true for n = k + 1.

Thus, by using mathematical induction, S(n) is true for every integral value of n.

UEx. 2.1.4 (SPPU - Q. 1(a), May 14, 4 Marks)

With the help of mathematical induction prove that,

$1^2 + 3^2 + 5^2 + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

Soln. :

Let,

$$S(n) : 1^2 + 3^2 + 5^2 + \ldots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3} \quad \ldots(1)$$

Step I : Basis of induction

Check S (n) is true for n = 1

$$\text{So, LHS} = (2 \cdot 1 - 1)^2 = (2-1)^2 = 1^2 = 1$$

$$\text{RHS} = 1\frac{(2-1)(2+1)}{3} = \frac{(1)(3)}{3} = \frac{3}{3} = 1,$$

$$\text{LHS} = \text{RHS}$$

Hence, S (n) is true for n = 1

Step II : Induction step

Assume S (n) is true for n = k

$$S(k) : 1^2 + 3^2 + 5^2 + \ldots + (2k-1)^2 = \frac{k(2k-1)(2k+1)}{3} \quad \ldots(2)$$

Now, check, S (n) is true for n = k + 1

$$S(k+1) : 1^2 + 3^2 + 5^2 + \ldots + (2k-1)^2 + (2(k+1)-1) = \frac{(k+1)(2(k+1)-1)(2(k+1)+1)}{3}$$

Solve LHS of above statement

$$\Rightarrow \frac{k(2k-1)(2k+1)}{3} + (2k+1)^2 \quad \because \text{from Equation (2)}$$

$$\Rightarrow \frac{2k+1}{3}(2k^2 - k + 3(2k+1))$$

$$\Rightarrow \frac{2k+1}{3}(2k^2 + 5k + 3)$$

$$\Rightarrow \frac{2k+1}{3}((2k+3)(k+1))$$

$$\Rightarrow \frac{(2k+1)(2k+3)(k+1)}{3} \Rightarrow \frac{(k+1)(2k+1)(2k+3)}{3}$$

$$\Rightarrow \frac{(k+1)(2(k+1)-1)\cdot(2(k+1)+1)}{3} \text{ Required RHS}$$

Step III :

Conclusion : Hence S(n) is true by mathematical induction.

Ex. 2.1.5 : Show that,

$$\frac{1^2}{1\cdot 3} + \frac{2^2}{3\cdot 5} + \ldots + \frac{n^2}{(2n-1)(2n+1)} = \frac{n(n+1)}{2(2n+1)}$$

using mathematical induction.

Soln. :

$$S(n) : \frac{1^2}{1\cdot 3} + \frac{2^2}{3\cdot 5} + \ldots + \frac{n^2}{(2n-1)(2n+1)} = \frac{n(n+1)}{2(2n+1)} \quad \ldots(1)$$

check, S(n) is true for n = 1

Put, n = 1 in Equation (1)

$$S(1) : \text{L.H.S} = \frac{1^2}{(2\times 1-1)(2\times 1+1)} = \frac{1}{(1)(3)} = \frac{1}{3}$$

$$\text{R.H.S} = \frac{1(1+1)}{2(2\times 1+1)} = \frac{\cancel{2}}{\cancel{6}} = \frac{1}{3} \quad \text{L.H.S.} = \text{R.H.S.}$$

Hence, S(n) is true for n = 1.

Step II : Induction step

Assume, S(n) is true for n = k.

$$S(k) : \frac{1^2}{1\cdot 3} + \frac{2^2}{3\cdot 5} + \ldots + \frac{k^2}{(2k-1)(2k+1)} = \frac{k(k+1)}{2(2k+1)} \quad \ldots(2)$$

Now, check S(n) is true for n = k + 1

$$S(k+1) : \frac{1^2}{1\cdot 3} + \frac{2^2}{3\cdot 5} + \ldots + \frac{k^2}{(2k-1)(2k+1)} + \frac{(k+1)^2}{[2(k+1)-1][2(k+1)+1]} = \frac{(k+1)(k+1+1)}{2(2(k+1)+1)}$$

$$S(k+1) : \frac{1^2}{1\cdot 3} + \frac{2^2}{3\cdot 5} + \ldots + \frac{k^2}{(2k-1)(2k+1)} + \frac{(k+1)^2}{(2k+1)(2k+3)} = \frac{(k+1)(k+2)}{2(2k+3)}$$

$$S(k+1) : \frac{k(k+1)}{2(2k+1)} + \frac{(k+1)^2}{(2k+1)(2k+3)} = \frac{(k+1)(k+2)}{2(2k+3)}$$

...From Equation (2)

Take L.H.S. of S (k + 1), then solve,

$$S(k+1) : \frac{k(k+1)}{2(2k+1)} + \frac{(k+1)^2}{(2k+1)(2k+3)}$$

$$= \frac{[k(k+1)]\cdot[(2k+1)(2k+3)] + [2(2k+1)(k+1)^2]}{[2(2k+1)]\cdot[(2k+1)\cdot(2k+3)]}$$

Taking $\frac{(k+1)}{(2k+1)}$ common

$$= \frac{k+1}{2k+1}\left[\frac{k(2k+3) + 2(k+1)}{2(2k+3)}\right]$$

$$= \frac{k+1}{2k+1}\left[\frac{2k^2 + 5k + 2}{2(2k+3)}\right]$$

$$= \frac{k+1}{2k+1}\left[\frac{2k^2 + 4k + k + 2}{2(2k+3)}\right]$$

$$= \frac{k+1}{2k+1}\left[\frac{2k(k+2) + 1(k+2)}{2(2k+3)}\right]$$

$$= \frac{k+1}{\cancel{2k+1}}\left[\frac{\cancel{(2k+1)}(k+2)}{2(2k+3)}\right]$$

$$= \frac{(k+1)(k+2)}{2(2k+3)} = \text{R.H.S. of } S(k+1)$$

▶ **Step III : Conclusion**

This shows that, S(n) is true for n = k then it is also true for n = k + 1.

Thus, by using mathematical induction S(n) is true for every integral value of n.

UEx. 2.1.6 (SPPU - Q. 1(a), Dec. 15, 4 Marks)

Use mathematical induction to show that $n^2 - 4n^2$ is divisible by 3 for all $n \geq 2$.

☑ **Soln. :**

Let $S(n) : n^2 - 4n^2$ is divisible by 3 for all $n \geq 2$ ...(1)

▶ **Step I : Basis of Induction**

Check S (n) is true for n = 2

$S(2) : 2^2 - 4(2)^2 \Rightarrow 4 - 4 \cdot 4 = 4 - 16 = -12$ is divisible by 3 so, S (n) is true for n = 2

▶ **Step II : Induction step**

Assume, S (n) is true for n = k

$S(k) \;:\; k^2 - 4k^2$ is divisible by 3 ...(2)

Check S (k + 1) is true for n = k + 1

$S(k+1) \;:\; (k+1)^2 - 4(k+1)^2$

$\Rightarrow (k^2 + 2k + 1) - 4(k^2 + 2k + 1)$

$\Rightarrow (k^2 + 2k + 1) - 4k^2 - 8k - 4$

$\Rightarrow (k^2 - 4k^2) - (6k + 3)$

$\Rightarrow (k^2 - 4k^2) - 3(k + 1)$

Here, $k^2 - 4k^2$ is divisible by 3 from Equation (2) and 3(k + 1) is divisible by 3.

▶ **Step III : Conclusion**

$S(n) : n^2 - 4n^2$ is divisible by 3 for all $n \geq 2$ by mathematical induction .

UEx. 2.1.7 (SPPU - Q. 1(a), May 16, 4 Marks)

Use mathematical induction to show that

$1^2 - 2^2 + 3^2 - 4^2 + \ldots + (-1)^{n-1} n^2 = (-1)^{n-1} \cdot \frac{n(n+1)}{2}$

☑ **Soln. :**

Let, S (n) :

$$1^2 - 2^2 + 3^2 - 4^2 + \ldots + (-1)^{n-1} \cdot n^2 = (-1)^{n-1} \cdot \frac{n(n+1)}{2} \qquad \ldots(1)$$

▶ **Step I : Basis of Induction**

Check S (n) is true for n = 1

$S(1) : \text{LHS} = (-1)^0 \cdot 1^2 = 1 \cdot 1 = 1$

$\text{RHS} = (-1)^0 \cdot \frac{1(1+1)}{2} = 1 \cdot \frac{2}{2} = \frac{2}{2} = 1$

Hence, S (n) is true for n = 1

▶ **Step II : Induction step**

Assume, S (n) is true for n = k

$$S(k) : 1^2 - 2^2 + 3^2 - 4^2 + \ldots + (-1)^{k-1} \cdot k^2 = (-1)^{k-1} \cdot \frac{k(k+1)}{2} \qquad \ldots(2)$$

Now, Check S (n) is true for n = k + 1

$$S(k+1) : \; 1^2 - 2^2 + 3^2 - 4^2 + \ldots + (-1)^{k-1} \cdot k^2 + (-1)^{k+1-1} \cdot (k+1)^2 = \frac{(-1)^k \cdot (k+1)(k+2)}{2}$$

Solve LHS of above statement

$$\Rightarrow (-1)^{k-1} \cdot \frac{k(k+1)}{2} + (-1)^k \cdot (k+1)^2$$

...From Equation (2)

$$\Rightarrow (-1)^k (k+1)\left[\frac{-k}{2} + (k+1)\right]$$

$$\Rightarrow (-1)^k \cdot (k+1)\left[\frac{-k + 2k + 2}{2}\right]$$

$$\Rightarrow \frac{(-1)^k \cdot (k+1) \cdot (k+2)}{2} = \text{Required RHS}$$

▶ **Step III : Conclusion**

Hence, $S(n) : \; 1^2 - 2^2 + 3^2 - 4^2 + \ldots + (-1)^{n-1} \cdot n^2 = (-1)^{n-1} \cdot \frac{n(n+1)}{2}$ is true by mathematical induction

UEx. 2.1.8 (SPPU - Q. 2(b), May 15, 4 Marks)

Prove by mathematical induction that for : n >= 1 :

$1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \ldots\ldots + n \cdot n! = (n+1)! - 1$

☑ **Soln. :**

Let,

$S(n) : 1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \ldots + n \cdot n! = (n+1)! - 1$ ...(1)

▶ **Step I : Basis of induction**

Check S(n) is true for n = 1

So, LHS $= 1 \cdot 1! = 1$,

RHS $= (1+1)! - 1 = 2! - 1 = 2 - 1 = 1$

LHS = RHS

Hence, S(n) is true for n = 1

▶ **Step II : Induction step**

Assume S(n) is true for n = k

$S(k) : \; 1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \ldots + k \cdot k! = (k+1)! - 1$ ...(2)

Now, check S(n) is true for n = k + 1

$S(k+1) : \; 1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \ldots + k \cdot k! + (k+1)(k+1)! = (k+2)! - 1$

Solve above statement by taking LHS

$\Rightarrow \underbrace{1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \ldots + k \cdot k!} + (k+1)(k+1)!$

$\Rightarrow (k+1)! - 1 + (k+1)(k+1)!$

$\Rightarrow (k+2)(k+1)! - 1$

$\Rightarrow (k+2)! - 1$

$\Rightarrow$ Required RHS

▶ **Step III : Conclusion**

S(n) : $1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \ldots + n \cdot n! = (n+1)! - 1$ is true by mathematical induction

UEx. 2.1.9 (SPPU - Q. 1(a), Dec. 13, 4 Marks)

With the help of mathematical induction prove that $8^n - 3^n$ is multiple of 5, for $n \geq 1$.

☑ **Soln. :**

Let, S (n) : $8^n - 3^n$ is multiple of 5 for $n \geq 1$

▶ **Step I : Induction Basis**

Check S (n) is true for n = 1

S (1) : $8^1 - 3^1 = 8 - 3 = 5$. Here, 5 is multiple by 5

So, S (n) is true for n = 1

▶ **Step II : Induction step**

Assume, S (n) is true for n = k that is $8^k - 3^k$ is multiple by 5

Now, check S (k + 1) is true for n = k + 1

$$
\begin{aligned}
S(k+1) : 8^{(k+1)} - 3^{(k+1)} &= 8^k \cdot 8^1 - 3^k \cdot 3^1 \\
&= 8^k (5+3) - 3^k \cdot 3 \\
&= 8^k \cdot 5 + (8^k \cdot 3 - 3^k \cdot 3) \\
&= 8^k \cdot 5 + 3(8^k - 3^k) \\
&= 5 \cdot 8^k + 3 \cdot 5^k \\
&= 5 \cdot 8^k + 5^k \cdot 3
\end{aligned}
$$

Here, $5 \cdot 8^k$ is multiple by 5 and $5^k \cdot 3$ is multiple by 5

Therefore, $8^{k+1} - 3^{k+1}$ is multiple of 5

▶ **Step III**

Conclusion : S (n) : $8^n - 3^n$ is multiple of 5 for $n \geq 1$

UEx. 2.1.10 (SPPU - Q. 1(a), May 18, 4 Marks)

Prove : $1^3 + 2^3 + 3^3 + \ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$

☑ **Soln. :**

Let, $S(n) : 1^3 + 2^3 + 3^3 + \ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$ ...(1)

▶ **Step I : Basis of Induction**

Check S (n) is true for n = 1

So, LHS = $1^3 = 1$

RHS = $\left(\frac{1(1+1)}{2}\right)^2 = \left(\frac{2}{2}\right)^2 = \frac{4}{4} = 1$

LHS = RHS, Hence, S (n) is true for n = 1

▶ **Step II : Induction step**

Assume, S (n) is true for n = k

$S(k) : 1^3 + 2^3 + 3^3 + \ldots + k^3 = \left[\frac{k(k+1)}{2}\right]^2$ ...(2)

Now, prove S (n) is true for n = k + 1

$S(k+1) : 1^3 + 2^3 + 3^3 + \ldots + k^3 + (k+1)^3 = \left[\frac{(k+1)(k+2)}{2}\right]^2$

Solve LHS to get required RHS

$S(k+1) : \underbrace{1^3 + 2^3 + 3^3 + \ldots + k^3} + (k+1)^3$

$\Rightarrow \left[\frac{k(k+1)}{2}\right]^2 + (k+1)^3$ $\quad\because$ from Equation (2)

$\Rightarrow (k+1)^2 \left[\frac{k^2}{4} + (k+1)\right]$

$\Rightarrow (k+1)^2 \left[\frac{k^2 + 4k + 4}{4}\right] \Rightarrow (k+1)^2 \left[\frac{(k+2)^2}{4}\right]$

$\Rightarrow \left[\frac{(k+1)(k+2)}{2}\right]^2$ $\Rightarrow$ Required RHS

▶ **Step III : Conclusion**

So given $S(n) : 1^3 + 2^3 + 3^3 + \ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$ is true by mathematical induction.

UEx. 2.1.11 (SPPU - Q. 1(a), May 19, 3 Marks)

Show that : $7^{2n} + (2^{3n-3})(3^{n-1})$ is divisible by 25 for all natural number n.

☑ **Soln. :**

Let, $S(n) : 7^{2n} + (2)^{3n-3} \cdot (3)^{n-1}$ is divisible by 25 ...(1)

▶ **Step I : Basis of Induction**

Check S (n) is true for n = 1

$S(1) : 7^2 + 2^{3-3} \cdot 3^0 \Rightarrow 49 + 1 \cdot 1 \Rightarrow 50$ divisible by 25

So, S (n) is true for n = 1

▶ **Step II : Induction step**

Assume, S (n) is true for n = k

$S(k) : 7^{2k} + 2^{3k-3} \cdot 3^{k-1}$ is divisible by 25 ...(2)

Now, check S (k) is true for n = k + 1

$S(k+1) = 7^{2(k+1)} + 2^{3(k+1)-3} \cdot 3^{(k+1)-1}$

$\Rightarrow 7^{2k+2} + 2^{3k} \cdot 3^k$

$\Rightarrow 7^2 (7^{2k}) + 2^{3k} \cdot 3^k$

$\Rightarrow 7^2 (7^{2k} + 2^{3k-3} \cdot 3^{k-1} - 2^{3k-3} \cdot 3^{k-1}) + 2^{3k} \cdot 3^k$

$\Rightarrow 49 \underbrace{(7^{2k} + 2^{3k-3} \cdot 3^{k-1})} - 49(2^{3k-3} \cdot 3^{k-1}) + 2^{3k} \cdot 3^k$

$\Rightarrow 49 \cdot 25C - 49 (2^{3k-3} \cdot 3^{k-1}) + 2^{3k} \cdot 3^k$

$\because$ From Equation (2)

$\Rightarrow 49 \cdot 25C - 49 (2^{3k-3} \cdot 3^{k-1}) + 3^1 \cdot 8^1 2^{3k-3} \cdot 3^{k-1}$

$\Rightarrow 49 \cdot 25C - 49 (2^{3k-3} \cdot 3^{k-1}) + 24 \cdot 2^{3k-3} \cdot 3^{k-1}$

$\Rightarrow 49 \cdot 25C - 25 (2^{3k-3} \cdot 3^{k-1})$

Here, $49 \cdot 25C \leftarrow$ divisible by 25

$25 (2^{3k-3} \cdot 3^{k-1}) \leftarrow$ divisible by 25

▶ **Step III :**

Conclusion : S (n) : $7^{2n} + 2^{3n-3} \cdot (3)^{n-1}$ is divisible by 25 by mathematical induction.

UEx. 2.1.12

(SPPU - Q. 1(b), May 17, Q. 1(a), Dec. 18, 4 Marks)

By using mathematical induction show that :

$1 + 2 + 3 + \ldots + n = \dfrac{n(n+1)}{2}$ for all natural number values of n.

☑ **Soln. :**

Let $S(n) : 1 + 2 + 3 + \ldots + n = \dfrac{n(n+1)}{2}$ ...(1)

▶ **Step I : Basis of induction**

Check S (n) is true for n = 1

$$\text{LHS} = 1, \quad \text{RHS} = \frac{1(1+1)}{2} = \frac{2}{2} = 1$$

$\therefore$ LHS = RHS

Hence, S(n) is true for n = 1

▶ **Step II : Induction step**

Assume S(n) is true for n = k

Thus we get

$$S(k) : 1 + 2 + 3 + \ldots + k = \frac{k(k+1)}{2} \quad \ldots(2)$$

Now, check S(n) is true for n = k + 1 from Equation (1),

$$S(k+1) : 1 + 2 + 3 + \ldots + k + (k+1) = \frac{(k+1)(k+2)}{2}$$

Now, solve the LHS of above statement

$$S(k+1) : \underbrace{1 + 2 + 3 + \ldots + k}+ (k+1)$$

$$= \frac{k(k+1)}{2} + k + 1 \quad \because \text{From Equation (2)}$$

$$= \frac{k(k+1) + 2(k+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2} = \text{RHS of } S(k+1)$$

▶ **Step III :**

Conclusion : $S(n) : 1 + 2 + 3 + \ldots + n = \dfrac{n(n+1)}{2}$ is true using mathematical induction.

UEx. 2.1.13 (SPPU - Q. 1(a), Dec. 16, 3 Marks)

Prove by mathematical induction :

$$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1} \text{ for } n \geq 1$$

☑ **Soln. :**

Let,

$$S(n) : \frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1} \quad \ldots(1)$$

▶ **Step I : Basis of Induction**

Check S (n) is true for n = 1

$$S(1) : \text{LHS} = \frac{1}{(2(1)-1)(2(1)+1)} = \frac{1}{3}$$

$$\text{RHS} = \frac{1}{2(1)+1} = \frac{1}{3} \quad \text{LHS} = \text{RHS}$$

Hence S (n) is true for n = 1

▶ **Step II : Induction step**

Assume S(n) is true for n = k

$$S(k) : \frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{(2k-1)(2k+1)} = \frac{k}{2k+1} \quad \ldots(2)$$

Now, have to prove S (n) is true for n = k + 1.

$$S(k+1) : \frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{(2k-1)(2k+1)} + \frac{1}{[2(k+1)-1][2(k+1)+1]} = \frac{k+1}{[2(k+1)+1]}$$

Solve LHS of above statement

$$\Rightarrow \frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{(2k-1)(2k+1)} + \frac{1}{[2(k+1)-1][2(k+1)+1]}$$

$$\Rightarrow \frac{k}{2k+1} + \frac{1}{[2(k+1)-1][2(k+1)+1]}$$

$\because$ from Equation (2)

$$\Rightarrow \frac{1}{2k+1}\left[k + \frac{1}{2k+3}\right]$$

As $\because 2(k+1) - 1 = (2k+1)$

$\because 2(k+1) + 1 = 2k + 3$

$$\Rightarrow \frac{1}{2k+1}\left[\frac{k(2k+3)+1}{2k+3}\right] \Rightarrow \frac{1}{2k+1}\left[\frac{2k^2+k+1}{2k+3}\right]$$

$$\Rightarrow \frac{1}{2k+1}\left[\frac{k(2k+1)+1}{2k+3}\right]$$

$$\Rightarrow \frac{1}{\cancel{2k+1}}\left[\frac{(\cancel{2k+1})(k+1)}{2k+3}\right] \Rightarrow \left[\frac{k+1}{2k+3}\right]$$

$$\Rightarrow \frac{k+1}{2(k+1)+1} \Rightarrow \text{Required RHS}$$

▶ **Step III :**

Conclusion : Hence S(n) is true by mathematical induction.

Ex. 2.1.14 : Prove by mathematical induction, for $n \geq 0$

$$S(n) : 1 + a + a^2 + ... + a^n = \frac{1 - a^{n+1}}{1 - a}$$

☑ **Soln. :**

Given : $S(n) : 1 + a + a^2 + \ldots + a^n = \frac{1 - a^{n+1}}{1 - a}$ ...(1)

▶ **Step I : Basis of induction**

Check, S(n) is true for n = 0 or 1

For $n = 0$, L.H.S. $= a^0 = 1$

R.H.S. $= \frac{1-a}{1-a} = 1$ } L.H.S. = R.H.S.

For n = 1, L.H.S. $= 1 + a$

R.H.S. $= \frac{1-a^2}{1-a} = \frac{(1\cancel{-a})(1+a)}{(1\cancel{-a})}$

$= 1 + a$ $\because$ L.H.S = R.H.S

Here, S(n) is true for n = 0, 1 that is S(0), S(1) are true.

▶ **Step II : Induction step**

Assume, S(n) is true for n = k (some value of n)

So,

$$S(k) : 1 + a + a^2 + \ldots + a^k = \frac{1 - a^{k+1}}{1 - a} \quad \text{...(2)}$$

check, S(n) is true for n = k + 1,

$$S(k+1) : 1 + a + a^2 + \ldots + a^k + a^{k+1} = \frac{1 - a^{k+1+1}}{1 - a}$$

$$S(k+1) : 1 + a + a^2 + \ldots + a^k + a^{k+1} = \frac{1 - a^{k+2}}{1 - a}$$

Solve above equation by taking L.H.S

$S(k+1) : 1 + a + a^2 + \ldots + a^k + a^{k+1}$

$\Rightarrow \frac{1 - a^{k+1}}{1 - a} + a^{k+1}$ ...from Equation (2)

$\Rightarrow \frac{1 - a^{k+1} + (1 - a) \cdot a^{k+1}}{1 - a}$

$\Rightarrow \frac{(1 - a^{k+1}) + a^{k+1} - a^1 \cdot a^{k+1}}{1 - a}$

$\Rightarrow \frac{1 - \cancel{a^{k+1}} + \cancel{a^{k+1}} - a^{k+1+1}}{1 - a}$

$\Rightarrow \frac{1 - a^{k+2}}{1 - a} = \text{R.H.S}$

▶ **Step III : Conclusion**

This show that, S(n) is true for n = k then it is also true for n = k + 1.

Thus, by using mathematical induction, S(n) is true for every integral value of n.

Ex. 2.1.15 : Prove that for any positive integer n, the number $n^5 - n$ is divisible by 5.

☑ **Soln. :**

$S(n) : n^5 - n$ is divisible by 5

▶ **Step I : Basis of induction**

Check, S(n) is true n = 1

So, put n = 1 in S(n)

$S(1) : 0^5 - 0 = 0$ is divisible by 5

because, 0 is divisible by every number

So, S(n) is true for n = 1

▶ **Step II : Induction step**

Assume S(n)is true for n = k

$S(k) : k^5 - k$ is divisible by 5

Now, check S(n) is true for n = k + 1

$S(k+1) : (k+1)^5 - (k+1)$

$\Rightarrow (k^5 + 5_{c_1} k^4 + 5_{c_2} k^3 + 5_{c_3} k^2 + 5_{c_4} k + 5_{c_5}) - (k+1)$

$\Rightarrow k^5 + 5k^4 + 10k^3 + 10k^2 + 5k + 1 - k - 1$

$\because$ C = combination

$$n_{c_r} = \frac{n!}{(n-r)! \times r!}$$

$\Rightarrow (k^5 - k) + 5 [k^4 + 2k^3 + 2k^2 + k]$

Here, $(k^5 - k)$ is divisible by 5. (Assumed when n = k)

$5 [k^4 + 2k^3 + 2k^2 + k]$ is also divisible by 5.

Hence, $(k+1)^5 - (k+1)$ is divisible by 5.

▶ **Step III : Conclusion**

This show that S(n) is true for n = k then it is also true for n = k + 1.

Thus, by using mathematical induction S(n) is true for every integral value of n.

Ex. 2.1.16 : Prove that $\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \ldots + \frac{n}{2^n} = 2 - \frac{n+2}{2^n}$, for $n \geq 1$ by using mathematical induction

☑ **Soln. :**

$$S(n) : \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \ldots + \frac{n}{2^n} = 2 - \frac{n+2}{2^n} \quad \text{...(1)}$$

▶ **Step I : Basis of induction**

Check S(n) is true for n = 1

Put, $n = 1$ in Equation (1)

$$S(1) : \frac{1}{2} = \text{L.H.S},$$

$$\text{R.H.S} = 2 - \frac{1+2}{2^1} = 2 - \frac{3}{2} = \frac{4-3}{2} = \frac{1}{2}$$

Here, L.H.S = R.H.S

Hence S(1) is true.

▶ **Step II : Induction step**

Assume, S(n) is true for n = k

$$S(k) : \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \ldots + \frac{k}{2^k} = 2 - \frac{k+2}{2^k} \qquad \ldots(2)$$

Now check S(n) is true for n = k + 1

$$S(k+1) : \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \ldots + \frac{k}{2^k} + \frac{k+1}{2^{k+1}} = 2 - \frac{k+3}{2^{k+1}}$$

Solve above Equation, by taking L.H.S.

$$S(k+1) : \underbrace{\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \ldots + \frac{k}{2^k}} + \frac{k+1}{2^{k+1}}$$

$$S(k+1) : 2 - \frac{k+2}{2^k} + \frac{k+1}{2^{k+1}}$$

$$\Rightarrow 2 - \left[\frac{k+2}{2^k} - \frac{k+1}{2^{k+1}}\right]$$

$$\Rightarrow 2 - \left[\frac{2^{k+1}(k+2) - 2^k(k+1)}{2^k \cdot 2^{k+1}}\right]$$

$$\Rightarrow 2 - \left[\frac{2^k \cdot 2^1 \cdot k + 2^k \cdot 2^1 \cdot 2^1 - 2^k \cdot k^1 - 2^k}{2^k \cdot 2^{k+1}}\right]$$

$$\Rightarrow 2 - \left[\frac{\cancel{2^k}(2k + 4 - k - 1)}{\cancel{2^k} \cdot 2^{k+1}}\right]$$

$$\Rightarrow 2 - \left(\frac{k+3}{2^{k+1}}\right) \Rightarrow \text{R.H.S.}$$

▶ **Step III :**

Conclusion

This, show that S(n) is true for n = k then it is also true for n = k + 1.

Thus, by using mathematical induction, S(n) is true for every integral value of n.

UEx. 2.1.17 **SPPU - Dec. 12**

Show that, $1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}$.

☑ **Soln. :**

$$S(n) : 1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4} \qquad \ldots(1)$$

▶ **Step I : Basis of induction**

Check, S(n) is true for n = 1

Therefore, put n = 1 in S(n)

$$\text{L.H.S} \Rightarrow 1^3 = 1$$

$$\text{R.H.S} \Rightarrow \frac{1^2(1+1)^2}{4} = \frac{1 \times 4}{4} = 4 \qquad \text{here, L.H.S = R.H.S}$$

Hence, S(n) is true for n = 1

▶ **Step II : Induction step**

Assume, S(n) is true for n = k

$$\text{So, } S(k) : 1^3 + 2^3 + 3^3 + \ldots + k^3 = \frac{k^2(k+1)^2}{4} \qquad \ldots(2)$$

Now, check S(n) is true for n = k + 1.

$$S(k+1) : 1^3 + 2^3 + 3^3 + \ldots + k^3 + (k+1)^3 = \frac{(k+1)^2(k+1+1)^2}{4}$$

$$S(k+1) : 1^3 + 2^3 + 3^3 + \ldots + k^3 + (k+1)^3 = \frac{(k+1)^2(k+2)^2}{4}$$

Solve L.H.S of S(k + 1)

$$S(k+1) : 1^3 + 2^3 + 3^3 + \ldots + k^3 + (k+1)^3$$

$$\Rightarrow \frac{k^2(k+1)^2}{4} + (k+1)^3 \Rightarrow (k+1)^2 \cdot \left[\frac{k^2}{4} + (k+1)\right]$$

$$\Rightarrow (k+1)^2\left[\frac{k^2 + 4k + 4}{4}\right] \Rightarrow \frac{(k+1)^2(k+1)^2}{4} \Rightarrow \text{R.H.S}$$

▶ **Step III : Conclusion**

This show that S(n) is true for n = k then it is also true for n = k + 1. Thus, by using mathematical induction S(n) is true for every integral value of n.

Ex. 2.1.18 : Let n be a positive integer. Show that any $2^n \times 2^n$ chessboard with one square removed can be covered by L-shape pieces, where each piece covers three squares at a time.

☑ **Soln. :**

Let, S(n) be the proposition that any $2^n \times 2^n$ chess board with one square removed can be covered by L-shape pieces.

▶ **Step I : Basis of induction**

Check, S(n) is true for n = 1

S(1) : Implies that any 2 × 2 chessboard with one square removed can be covered using L-shaped pieces. P(1) is true, as shown in Fig. P. 2.1.18.

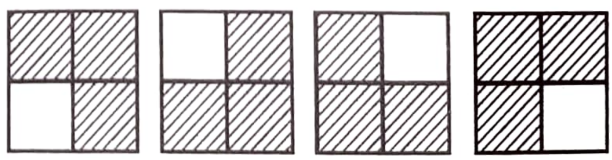

Fig. P. 2.1.18

▶ **Step II : Induction step**

Assume S(n) is true for n = k that is any $2^k \times 2^k$ chessboard with one square removed can be covered using

L-shaped pieces.

Now, we have to check that S(n) is true for n = k + 1 that is S(k + 1) is true. For this consider a $2^{k+1} \times 2^{k+1}$ chessboard with one square removed. Divide the chessboard into four equal halves of size $2^k \times 2^k$ as shown in Fig. P. 2.1.18(a).

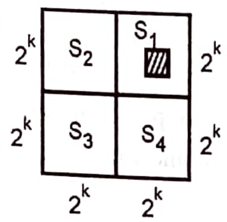

Fig. P. 2.1.18(a)

The square which has been removed, would have been removed from one of the four chessboard say S_1. Then by induction hypothesis, S_1 can be covered using L-shaped pieces. Now, from each of the remaining chessboards, remove that particular piece or tile, lying at the centre of the large chessboards.

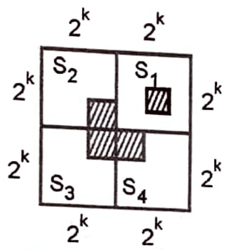

Fig. P. 2.1.18(b)

Then, by induction hypothesis each of these $2^k \times 2^k$ chessboards with a piece or tile removed can be covered by the L-shaped pieces. Also the three tiles removed from the centre can be covered by one L-shaped piece. Hence, the chessboards of $2^{k+1} \times 2^{k+1}$ can be covered by L-shaped pieces.

...Hence Proved.

Ex. 2.1.19 : Using mathematical induction show that

$$S(n) : \frac{1}{1\cdot3} + \frac{1}{3\cdot5} + \frac{1}{5\cdot7} + \ldots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

☑ **Soln. :**

Given :

$$S(n) : \frac{1}{1\cdot3} + \frac{1}{3\cdot5} + \frac{1}{5\cdot7} + \ldots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1} \quad \ldots(1)$$

▶ **Step I : Basis of induction**

Check S(n) is true for n = 1

$$\text{L.H.S} \Rightarrow \frac{1}{(2-1)(2+1)} = \frac{1}{(1)(3)} = \frac{1}{3}$$

$$\text{R.H.S} \Rightarrow \frac{1}{2+1} = \frac{1}{3}$$

Here, L.H.S = R.H.S

S(n) is true for n = 1

▶ **Step II : Induction step**

Assume, S(n) is true for n = k

$$S(k) : \frac{1}{1\cdot3} + \frac{1}{3\cdot5} + \frac{1}{5\cdot7} + \ldots + \frac{1}{(2k-1)(2k+1)} = \frac{k}{2k+1} \quad \ldots(2)$$

Now, check S(n) is true for n = k + 1

$$S(k+1) : \frac{1}{1\cdot3} + \frac{1}{3\cdot5} + \frac{1}{5\cdot7} + \ldots + \frac{1}{(2k-1)(2k+1)} + \frac{1}{[2(k+1)-1][2(k+1)+1]} = \frac{k+1}{2(k+1)+1} = \frac{k+1}{2k+3}$$

Solve L.H.S of above equation

$$S(k+1) : \underbrace{\frac{1}{1\cdot3} + \frac{1}{3\cdot5} + \frac{1}{5\cdot7} + \ldots + \frac{1}{(2k-1)(2k+1)}} + \frac{1}{[2(k+1)-1][2(k+1)+1]}$$

$$\Rightarrow \frac{k}{2k+1} + \frac{1}{(2k+1)(2k+3)}$$

$$\Rightarrow \frac{k(2k+1)(2k+3) + (2k+1)}{(2k+1)(2k+1)(2k+3)}$$

$$\Rightarrow \frac{\cancel{(2k+1)}[k\cdot(2k+3)+1]}{\cancel{(2k+1)}(2k+1)(2k+3)}$$

$$\Rightarrow \frac{2k^2+3k+1}{(2k+1)(2k+3)} \Rightarrow \frac{\cancel{(2k+1)}(k+1)}{\cancel{(2k+1)}(2k+3)}$$

$$\Rightarrow \frac{k+1}{2k+3} \Rightarrow \text{R.H.S}$$

▶ **Step III : Conclusion**

This show that S(n) is true for n = k then it is also true for n = k + 1.

Thus, by using mathematical induction S(n) is true for every integral value of n.

Ex. 2.1.20 : Show that for any positive integers n, $(11)^{n+2} + (12)^{2n+1}$ is divisible by 133.

☑ **Soln. :**

S(n) : $(11)^{n+2} + (12)^{2n+1}$ is divisible by 133.

▶ **Step I : Basis of induction**

For n = 0, $(11)^2 + (12)^1 = 121 + 12 = 133$

Which is divisible by 133

For n = 1, $(11)^3 + (12)^3 = 1331 + 1728$

$= \mathbf{3059}$

Which is divisible by 133 as $\frac{3059}{133} = 23$

So, S(n) is true for n = 0 and 1

▶ **Step II : Induction step**

Assume, S(n) is true for n = k

$S(k) : (11)^{k+2} + (12)^{2k+1}$ is divisible by 133

now, check S(n) is true for n = k + 1

$S(k+1) : \quad (11)^{(k+1)+2} + (12)^{2(k+1)+1}$

$\Rightarrow \quad (11^{k+2}) \cdot 11^1 + 12^{2k+1} \cdot 12^2$

$\Rightarrow \quad (11) \cdot (11^{k+2}) + 144.12^{2k+1}$

$\Rightarrow \quad 11.11^{k+2} + (133 + 11) \cdot 12^{2k+1}$

$\Rightarrow \quad 11.11^{k+2} + 11.12^{2k+1} + 133.12^{2k+1}$

$\Rightarrow \quad 11\,(11^{k+2} + 12^{2k+1}) + 133.12^{2k+1}$

Here, $11^{k+2} + 12^{2k+1} \rightarrow$ divisible by 133

(Assumed in n = k)

$133.12^{2k+1} \rightarrow$ divisible by 133

▶ **Step III : Conclusion**

$S(n) : (11)^{n+2} + (12)^{2n+1}$ is divisible by 133.

UEx. 2.1.21 (SPPU - Q. 2(b), Dec. 14, 3 Marks)

Use mathematical induction to show that :

$n \cdot (n^2 - 1)$ is divisible by 24 where n is any odd positive number.

☑ **Soln. :**

Given : $n \cdot (n^2 - 1)$ is divisible by 24 where n is any odd positive number.

$\Rightarrow \quad (n^3 - n)$ is divisible by 24

$\Rightarrow \quad S(n) : (n^3 - n)$

▶ **Step I : Basis of induction**

Check s(n) is true for n = 1

$S(1) \; : \; 1^3 - 1 = 0$, Here 0 is divisible by 24.

$S(3) \; : \; 3^3 - 3^3 = 24$, 24 is divisible by 24.

▶ **Step II : Induction step**

Assume, S(n) is true for n = k.

$S(k) \; : \; k^3 - k$ is divisible by 24 ...(1)

Now check, S(n) is true for n = k + 1

$S(k+1) \; : (k+1)^3 - (k+1)$

$\Rightarrow \quad k^3 + 3k^2 + 3k + 1 - k - 1$

$\Rightarrow \quad k^3 + 3k^2 + 2k$

$\Rightarrow \quad (k^3 + k) + 3k^2 + 3k + 2k + k$

$\Rightarrow \quad (k^3 - k) + 3k^2 + 3k$

$: \quad \underbrace{24 \cdot C_1} + \underbrace{3k\,(k+1)}$

k(k + 1) is multiple of 8, for k ≥ 3.

From Equation (1)

$$= 24 \cdot C_1 + 3 \cdot (8 \cdot C_2)$$
$$= 24 \cdot C_1 + 24 \cdot C_2$$
$$= 24 \cdot (C_1 + C_2)$$
$$= 24 \cdot C_3$$

So, S(k + 1) is true because $24 \cdot C_3$ is divisible by 24.

Hence,

▶ **Step III :**

Conclusion

$S(n) : n(n^2 - 1) = n^3 - n$ is divisible by 24 where n is any odd positive integer.

UEx. 2.1.22 (SPPU - Q. 1(b), Dec. 19, 3 Marks)

Show that for natural no. n :

$1^3 + 2^3 + 3^3 + \ldots + n^3 = (1 + 2 + \ldots + n)^2$

☑ **Soln. :**

▶ **Step I : Basis of Induction**

Let, $S(n): 1^3 + 2^3 + 3^3 + \ldots + n^3 = (1 + 2 + \ldots + n)^2$ ...(1)

Check S (n) is true for small value of n i.e. n = 1

LHS $= 1^3 = 1$, RHS $= 1^2 = 1$

Here, LHS = RHS So, S (n) is true for n = 1

▶ **Step II : Induction step**

Assume, S (n) is true for n = k

So, $S(K) : 1^3 + 2^3 + 3^3 + \ldots + k^3 = (1 + 2 + \ldots + k)^2$ ...(2)

Now, we have to show that S (n) is true for n = k + 1

So, put n $= k+1$ in S(n) (3)

then

$$S(K+1) : 1^3 + 2^3 + 3^3 + \ldots + K^3 + (K+1)^3$$

$$= (1+2+\ldots+(K+1))^2$$

Now, lets start from LHS of S (K + 1)

$$S(K+1) = \underbrace{1^3 + 2^3 + 3^3 + \ldots + K^3}+ (K+1)^3$$

$$= (1+2+\ldots+K)^2 + (K+1)^3 \rightarrow \text{by Equation (2)}$$

$$= \frac{K^2 (K+1)^2}{4} + (K+1)^3$$

$$\because \quad (1+2+\ldots+n^2) = \frac{n^2 (n+1)^2}{4}$$

$$= (K+1)^2 \left[\frac{K^2 + 4(K+1)}{4}\right]$$

$$= (K+1)^2 \left[\frac{K^2 + 4K + 4}{4}\right] = \frac{(K+1)^2 (K+2)^2}{4}$$

$$= (1+2+3+\ldots+(K+1))^2$$

= RHS, S (n) is true for $n = K + 1$

▶ **Step III :**

Conclusion

$S(n) : 1^3 + 2^3 + 3^3 + \ldots. + n^3 = (1+2+\ldots+n)^2$ is true by mathematical induction.

Chapter Ends ...

□□□