

# GATE EE Q.17

EE23BTECH11203 - Adarsh A\*

**Question :** A continuous-time system that is initially at rest is described by,

$$\frac{dy(t)}{dt} + 3y(t) = 2x(t)$$

where  $x(t)$  is the input voltage and  $y(t)$  is the output voltage.

The impulse response of the system is?

(A)  $3e^{-2t}$

(B)  $\frac{1}{3}e^{-2t}u(t)$

(C)  $2e^{-3t}u(t)$

(D)  $2e^{-3t}$

(GATE EE 2023)

**Solution:**

Parameter	Value	Description
$x(t)$	-	Input voltage
$y(t)$	-	Output voltage
$h(t)$	$\frac{y(t)}{x(t)}$	Impulse response
$X(s)$	-	Input voltage in s-domain
$Y(s)$	-	Output voltage in s-domain
$H(s)$	$\frac{Y(s)}{X(s)}$	Impulse response in s-domain

Input Table

Given equation is,

$$\frac{dy(t)}{dt} + 3y(t) = 2x(t) \quad (1)$$

Applying Laplace transform,

$$x(t) \xrightarrow{\mathcal{L}} X(s) \quad (2)$$

$$y(t) \xrightarrow{\mathcal{L}} Y(s) \quad (3)$$

From the differentiation property,

$$\frac{dy(t)}{dt} \xleftrightarrow{\mathcal{L}} sY(s) \quad (4)$$

The equation becomes,

$$sY(s) + 3Y(s) = 2X(s) \quad (5)$$

$$Y(s)(s + 3) = 2X(s) \quad (6)$$

$$H(s) = \frac{Y(s)}{X(s)} \quad (7)$$

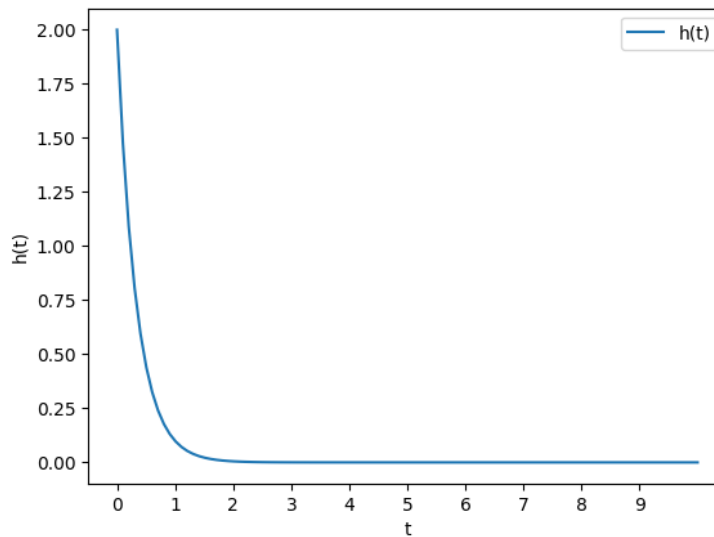
$$H(s) = \frac{2}{s + 3} \quad (8)$$

$$\frac{1}{s + a} \xleftrightarrow{\mathcal{L}} e^{-at}u(t) \quad (9)$$

Using these results,

$$h(t) = 2e^{-3t}u(t) \quad (10)$$

Plot of  $h(t)$  vs  $t$  :



(a) Plot of  $h(t)$  vs  $t$