

# NCERT 9.1 Q.14

EE23BTECH11203 - Adarsh A\*

**Question :** The Fibonacci sequence is defined by  $1 = a_1 = a_2$  and

$$a_n = a_{n-1} + a_{n-2}, n > 2$$

Find  $\frac{a_{n+1}}{a_n}$ , for  $n = 1, 2, 3, 4, 5$

**Answer :**

Parameter	Value	Description
$n$	$\geq 0$	Non negative Integer
$x(n)$	$x(n) = x(n-1) + x(n-2) + u(-n)$	$(n+1)^{th}$ term
$y(n)$	$\frac{x(n+1)}{x(n)}$	Required function
$x(0)$	1	1 <sup>st</sup> term
$x(1)$	1	2 <sup>nd</sup> term

Input Table

Here,  $a_1 = 1, a_2 = 1$

$$a_n = a_{n-1} + a_{n-2}, n > 2 \quad (1)$$

Applying  $z$  transform,

$$X(z) = z^{-1}X(z) + z^{-2}X(z) + z^{-0} \quad (2)$$

$$= \frac{1}{1 - z^{-1} - z^{-2}} \quad (3)$$

$$= \frac{1}{(1 - \alpha z^{-1})(1 - \beta z^{-1})}, |z| > |\alpha| \quad (4)$$

$$\text{Where, } \alpha = \frac{1 + \sqrt{5}}{2} \text{ and } \beta = \frac{1 - \sqrt{5}}{2}$$

Using partial fractions,

$$X(z) = \frac{\alpha}{(\alpha - \beta)(1 - \alpha z^{-1})} - \frac{\beta}{(\alpha - \beta)(1 - \beta z^{-1})} \quad (5)$$

$$a^n u(n) \xleftrightarrow{z} \frac{1}{1 - \alpha z^{-1}} \quad |z| > |\alpha|$$

Substituting this result,

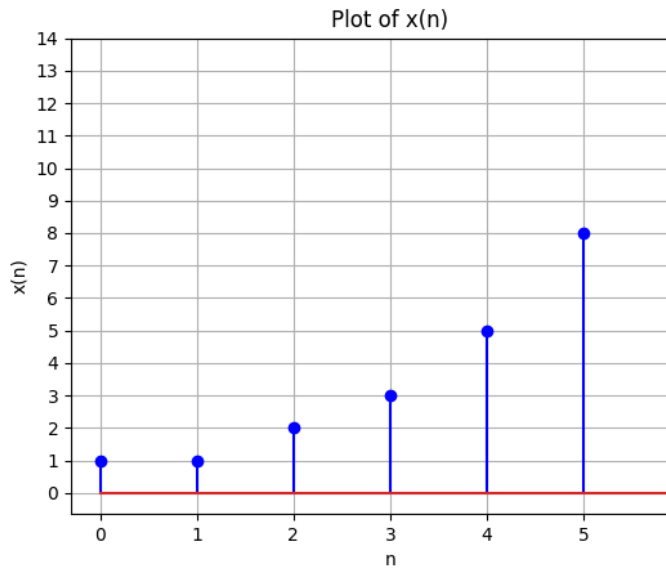
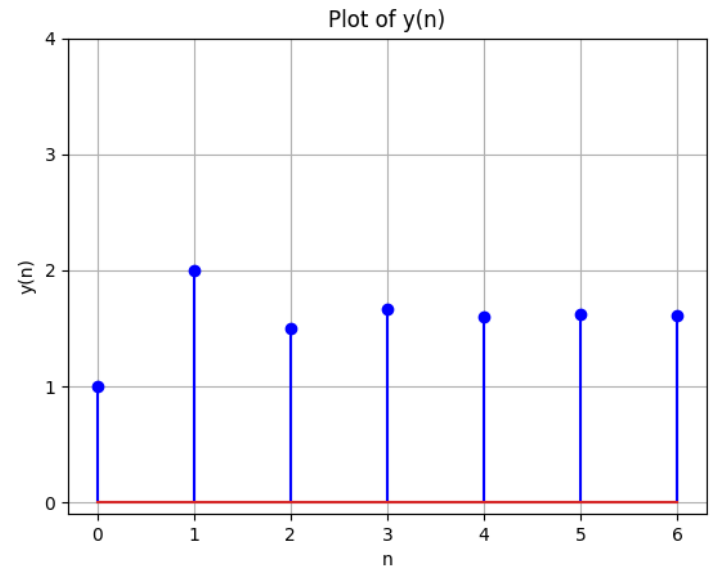
$$x(n) = \frac{\alpha}{(\alpha - \beta)} (\alpha^n u(n)) - \frac{\beta}{(\alpha - \beta)} (\beta^n u(n)) \quad (6)$$

$$x(n) = \frac{\alpha^{n+1} - \beta^{n+1}}{\alpha - \beta} u(n) \quad (7)$$

$$x(n) = \frac{(1 + \sqrt{5})^{n+1} - (1 - \sqrt{5})^{n+1}}{2^{n+1} \sqrt{5}} u(n) \quad (8)$$

$$y(n) = \frac{x(n+1)}{x(n)} \quad (9)$$

$$y(n) = \frac{1}{2} \left[ \frac{(1 + \sqrt{5})^{n+2} - (1 - \sqrt{5})^{n+2}}{(1 + \sqrt{5})^{n+1} - (1 - \sqrt{5})^{n+1}} \right] \quad (10)$$

(a) Plot of  $x(n)$  vs  $n$ (b) Plot of  $y(n)$  vs  $n$