# NCERT 9.5 Q.27

EE23BTECH11203 - Adarsh A*

**Question :** A farmer buys a used tractor for Rs 12000. He pays Rs 6000 in cash and agrees to pay the balance in annual installments of Rs 500 plus 12 % interest on the unpaid amount. How much will the tractor cost him?

**Solution:**

| Parameter | Value | Description |
|---|---|---|
| $n+1$ | - | Number of years |
| $a$ | 6000 | Amount paid |
| $r$ | 6000 | Remaining Amount |
| $x(n)$ | $(1220-60n)\ u(n)$ | Amount to be paid at $(n+1)^{th}$ year |
| $y(n)$ | $\left(1220+1190n-30n^2\right)\ u(n)$ | Total amount after $(n+1)$ yrs |

Input Table

Number of years taken to pay the remaining amount,

$$n+1=\frac{r}{500} \tag{1}$$

$$n=11 \tag{2}$$

The amount to be paid by the farmer after $(n+1)$ year/s is,

$$x(n)=500+0.12\,(6000-500n) \tag{3}$$

$$x(n)=(1220-60n)\ u(n) \tag{4}$$

Some results,

$$x(n)\xleftrightarrow{\mathcal{Z}}X(z) \tag{5}$$

$$u(n)\xleftrightarrow{\mathcal{Z}}\frac{1}{1-z^{-1}}:U(z) \tag{6}$$

$$n.u(n)\xleftrightarrow{\mathcal{Z}}-z\frac{d}{dz}\frac{1}{1-z^{-1}}=\frac{z^{-1}}{\left(1-z^{-1}\right)^2} \tag{7}$$

By taking $z$ transform,

$$X(z) = \frac{1220}{1 - z^{-1}} - \frac{60z^{-1}}{(1 - z^{-1})^2} \tag{8}$$

$$X(z) = \frac{1220 - 1280z^{-1}}{(1 - z^{-1})^2} \ , \quad |z| > 1 \tag{9}$$

Convolution in time domain is multiplication in $z$ domain :

$$y(n) = x(n) * u(n) \tag{10}$$

$$Y(z) = X(z) . U(z) \tag{11}$$

$$Y(z) = \frac{1220 - 1280z^{-1}}{(1 - z^{-1})^2} . \frac{1}{1 - z^{-1}} \tag{12}$$

$$Y(z) = \frac{1220 - 1280z^{-1}}{(1 - z^{-1})^3} \ , \quad |z| > 1 \tag{13}$$

Using Partial fractions,

$$Y(z) = \frac{Az}{z - 1} + \frac{Bz}{(z - 1)^2} + \frac{Cz}{(z - 1)^3} \tag{14}$$

$$A = 1220 \tag{15}$$

$$B = 1160 \tag{16}$$

$$C = -60 \tag{17}$$

$$\frac{n(n-1)(n-2)...(n-k-2)}{k!} u(n) \xleftrightarrow{\mathcal{Z}} \frac{z}{(z-1)^k} \tag{18}$$

Using this result,

$$y(n) = 1220\, u(n) + 1160\, n\, u(n) - 60\, \frac{n(n-1)}{2}\, u(n) \tag{19}$$

$$y(n) = \left(1220 + 1190n - 30n^2\right) u(n) \tag{20}$$

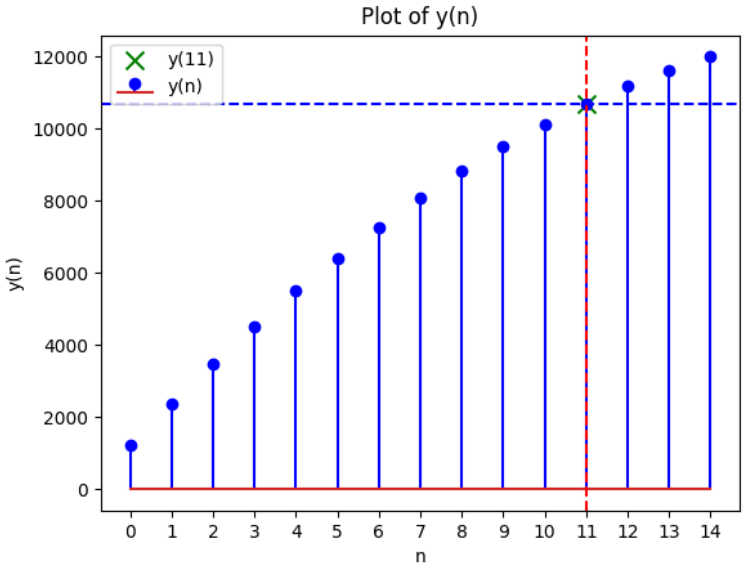
$$y(11) = 10680 \tag{21}$$

The total tractor cost to be paid,

$$= a + y(11) \tag{22}$$

$$= 16,680 \tag{23}$$

Plot of $y(n)$ $vs$ $n$ :




(a) Plot of $y(n)$ $vs$ $n$