Assignment 2

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***Abstract*—solution for Assignment 2 - ICSE Class 12 Maths 2019 Q.18(a)**

Q18(a) Problem: Draw a sketch and find the area bounded by the curve $x^2 = y$ and $x + y = 2$
Solution: The given parabola $x^2 - y = 0$ can written in vector form as

$$\mathbf{x}^T\mathbf{a}\mathbf{x} + \mathbf{b}^T\mathbf{x} + \mathbf{c} = 0 \tag{1}$$

with the parameters,

$$\mathbf{a} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \mathbf{c} = 0 \tag{2}$$

The line $x + y = 2$ can be written as

$$\mathbf{x} = \mathbf{p} + \lambda\mathbf{m} \tag{3}$$

where $\mathbf{p}$ is a point and $\mathbf{m}$ is the direction vector of the line
Choosing $\mathbf{p}$ as $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, we get:

$$\mathbf{x} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{4}$$

The intersection of this line with the parabola is given by

$$\mathbf{x_i} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \lambda_i \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{5}$$

where λ_i is given by,

$$\lambda_i = \frac{1}{\mathbf{m}^T\mathbf{a}\mathbf{m}} \left(-\mathbf{m}^T\left(\mathbf{a}\mathbf{p} + \mathbf{b}\right) \pm \sqrt{\left[\mathbf{m}^T\left(\mathbf{a}\mathbf{p} + \mathbf{b}\right)\right]^2 - \left(\mathbf{p}^T\mathbf{a}\mathbf{p} + 2\mathbf{u}^T\mathbf{p} + c\right)\left(\mathbf{m}^T\mathbf{a}\mathbf{m}\right)}\right) \tag{6}$$

Substituting the values in above equation, we get

$$\lambda_i = 0, -3 \tag{7}$$

Using these values of λ, the intersection points are,

$$\begin{pmatrix} -2 \\ 4 \end{pmatrix} \text{and} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{8}$$

Hence, the required area is equal to,

$$= \int_{-2}^{1} (2 - x)dx - \int_{c} x^2 dx \tag{9}$$

$$= \left|2x - \frac{x^2}{2}\right|_{-2}^{1} - \left|\frac{x^3}{3}\right|_{-2}^{1} \tag{10}$$

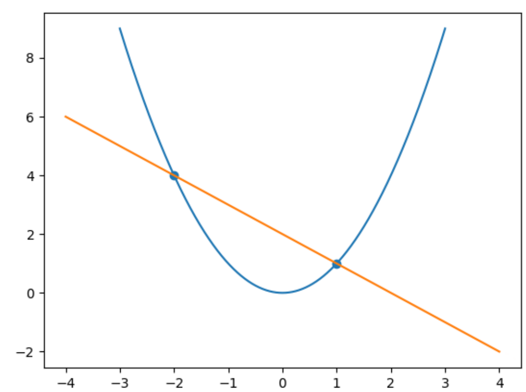
$$= 2 - \frac{1}{2} + 4 + \frac{4}{2} - \frac{1}{3} - \frac{8}{3} \tag{11}$$

$$= \frac{12 - 3 + 24 + 12 - 2 - 16}{6} \tag{12}$$

$$= \frac{9}{2} sq.units \tag{13}$$

$$= 4.5 sq.units \tag{14}$$

I. Rough Sketch



Answer: Area bounded is $\frac{9}{2}$ sq.units i.e, 4.5 sq. units