Assignment 3

Adarsh Bhende (CS21BTECH11008)

***Abstract*—solution for Assignment 3 - Papoulis Example-3.6**

Papoulis Example-3.6:A coin with $P(h) = p$ is tossed n times. We maintain that the probability p(k) that heads shows k times is given by

$$\mathbf{P_n}(\mathbf{k}) = \binom{n}{k} \mathbf{p}^k \mathbf{q}^{n-k} \tag{1}$$

$$q = 1 - p \tag{2}$$

Solution: Since the probability of each elementary event equals $\mathbf{p}^k \mathbf{q}^{n-k}$
We conclude that,

P(k heads in any order)= $\binom{n}{k} \mathbf{p}^k \mathbf{q}^{n-k}$

Special Case:when If n=3 and k=2,then their are three ways of getting two heads,namely.hht,hth,thh.
Hence,
$p_3(2) = 3\mathbf{p}^2\mathbf{q}$

Success or Failure of an event A in n Independent Trials

We are given an experiment S and an event A with

$$P(A) = p \tag{3}$$

$$P(\bar{A}) = q \tag{4}$$

$$p + q = 1 \tag{5}$$

We repeat the experiment n times and the resulting product space we denote by $\mathbf{s}^n$
thus, $\mathbf{S}^n = S * ... * S$

Fundamental Theorem

$\mathbf{P_n}(\mathbf{k})$ = P(A occurs k times in any order)= $\binom{n}{k} \mathbf{p}^k \mathbf{q}^{n-k}$

Proof

The event A occurs k times in any order is a cartesian product $\mathbf{B_1} * \mathbf{B_2} .... * \mathbf{B_n}$,where k of the event $\mathbf{B_i}$ equal A and the remaining $n-k$ equal $\bar{A}$.
the probability of this event equals
P($\mathbf{B_1}$).....P($\mathbf{B_n}$)= $\mathbf{p}^k \mathbf{q}^{n-k}$
because
P($\mathbf{B}_i$) = p if $\mathbf{B_i} = A$
or
P($\mathbf{B}_i$) = q if $\mathbf{B_i} = \bar{A}$
In other words,
P(A occurs k times in a specific order)= $\mathbf{p}^k \mathbf{q}^{n-k}$
The event (A occurs k times in any order) is the union of $\binom{n}{k}$ events (A occurs i times in a specific order) and since the events are mutually exclusive,we conlude that $\mathbf{P_n}(\mathbf{k})$ is given by equation(1)
Hence proved