Assignment-5

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Papoulis-Chapter-15

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Problem 15-2

Consider a marco chain $\{X_n\}$ with states $e_0, e_1, e_2, ..., e_m$ and the transition probability matrix

$$P = \begin{pmatrix} q & p & 0 & . & . & . & 0 \\ 0 & q & p & 0 & . & . & 0 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 0 & 0 & . & . & . & q & p \\ p & 0 & . & . & . & 0 & q \end{pmatrix}$$

Determine P^n, and the limiting distribution

Solution : Property involved

The following property would be involved in the problem.

Property:

The state transition probability matrix of a Markov chain gives the probabilities of transitioning from one state to another in a single time unit. It will be useful to extend this concept to longer time intervals

Solution : I

We have to note that both row sums and colunmn sums are unity in this case.Hence P represents a doubly stochastic matric here,and

$$P^n = \frac{1}{m+1}\begin{pmatrix} 1 & 1 & . & . & . & 1 & 1 \\ 1 & 1 & . & . & . & 1 & 1 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 1 & 1 & . & . & . & 1 & 1 \\ 1 & 1 & . & . & . & 1 & 1 \end{pmatrix} \tag{1}$$

$$\lim_{n\to\infty} P\{x_n = e_k\} = \frac{1}{m+1}, \qquad k = 0, 1, 2, 3, ..., m \tag{2}$$

answer

$$\lim_{n\to\infty} P\{x_n = e_k\} = \frac{1}{m+1}, \qquad k = 0, 1, 2, 3, ..., m \tag{3}$$