Assignment Report: Bayesian Inference on Power Plant Pumps

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1 Problem Description

We aimed to model the relationship between the number of failures and the running time of power plant pumps using a Gamma-Poisson Hierarchical model.

2 Implementation Details

Implemented in Python using PyMC3 library for Bayesian inference. MCMC and Variational Inference (VI) techniques used for posterior distribution estimation.

3 Model and Inference

3.1 Model

Gamma-Poisson Hierarchical model.

3.2 Priors

Exponential prior for alpha, Gamma prior for beta, and Gamma priors for rate parameters.

3.3 Likelihood

Poisson distribution for observed failures.

3.4 Inference Techniques ,Samplers and Parameters

MCMC and VI.

3.4.1 MCMC

The MCMC technique utilized the No-U-Turn Sampler (NUTS) sampler, a variant of Hamiltonian Monte Carlo (HMC) implemented in PyMC3. The sampler parameters were as follows:

- **Sampler:** No-U-Turn Sampler (NUTS)
- **Sampling Parameters:**
  - `draws`: 1000
  - `tune`: 1000
  - `cores`: 1
- **Convergence Diagnostics:** All $\hat{R}$ values were 1.0, indicating good convergence.
- **Trace Summary:** Mean, Standard Deviation, HDI, Effective Sample Size (ESS), and $\hat{R}$ for each parameter.

3.4.2 VI

The VI technique employed Automatic Differentiation Variational Inference (ADVI) implemented in PyMC3. The sampler parameters were set to default values:

- **Sampler:** Automatic Differentiation Variational Inference (ADVI)
- **Sampling Parameters:**
  - `method`: 'advi'
  - Default parameters
- **Variational Objective:** Lower bound used by VI in PyMC3.
- **Trace Summary:** Mean, Standard Deviation, HDI, Effective Sample Size (ESS) for each parameter.

3.4.3 Comparison

- **Sampler Type:** MCMC used a Hamiltonian Monte Carlo (HMC) variant (NUTS), while VI employed Automatic Differentiation Variational Inference (ADVI).
- **Parameter Estimation:** MCMC provided more accurate estimates with narrower HDIs and higher ESS values compared to VI.
- **Convergence:** MCMC showed good convergence with all $\hat{R}$ values at 1.0, while VI does not provide this diagnostic.

Considering the sampler type, parameter estimation accuracy, and convergence diagnostics, MCMC with the NUTS sampler appears to be more suitable for our model.

4 Results

4.1 MCMC Results

- Alpha: Mean = 0.713, SD = 0.275, HDI(3%) = 0.264, HDI(97%) = 1.179
- Beta: Mean = 0.943, SD = 0.538, HDI(3%) = 0.141, HDI(97%) = 1.918
- Lambda for each pump, with corresponding statistics

Pump	**Mean**	**SD**	**HDI(3%)**	**HDI(97%)**
1	0.060	0.026	0.019	0.110
2	0.100	0.075	0.001	0.238
3	0.090	0.037	0.027	0.156
4	0.117	0.031	0.061	0.173
5	0.606	0.304	0.136	1.186
6	0.610	0.140	0.355	0.877
7	0.906	0.737	0.008	2.279
8	0.892	0.723	0.009	2.174
9	1.606	0.783	0.369	3.004
10	1.979	0.433	1.180	2.798

Table 1: Summary statistics for each pump

- Mean Absolute Error (MAE): 0.489

4.2 VI Results

- Alpha: Mean = 0.750, SD = 0.238, HDI(3%) = 0.362, HDI(97%) = 1.187
- Beta: Mean = 0.487, SD = 0.271, HDI(3%) = 0.125, HDI(97%) = 0.989
- Lambda for each pump, with corresponding statistics

Pump	Mean	SD	HDI(3%)	HDI(97%)
1	0.436	0.160	0.188	0.733
2	0.457	0.202	0.155	0.822
3	0.438	0.162	0.169	0.720
4	0.436	0.157	0.183	0.722
5	0.792	0.407	0.224	1.567
6	0.658	0.184	0.322	0.979
7	1.454	1.078	0.175	3.412
8	1.564	1.328	0.202	3.777
9	2.103	1.173	0.412	4.324
10	2.116	0.558	1.145	3.177

Table 2: Summary statistics for each pump (VI)

4.3 Comparison of Inference Techniques

Both MCMC and VI produced similar parameter estimates. MCMC provided more detailed information on convergence and sampling.MCMC generally appears to provide more accurate and reliable estimates, as indicated by the narrower HDIs, higher ESS values, and good convergence diagnostics (R_hat = 1.0). Therefore, MCMC can be considered the preferred inference technique.

4.4 Prediction and Error Analysis

Generated posterior samples for parameters. Made predictions based on posterior distribution. Calculated MAE: 0.489.

4.5 Experimental Setup

MCMC: 1000 samples, 1000 tuning steps, single core. VI: Default settings. Runtime: MCMC sampling took 6 secs.

5 Mathematical Expression

Variational Inference (VI) in PyMC3 utilizes the Evidence Lower Bound (ELBO) as a surrogate objective function to approximate the posterior distribution. The ELBO is defined as follows:

$$ELBO = E_q[\log p(\mathbf{X}, \mathbf{Z})] - E_q[\log q(\mathbf{Z})]$$

where:

- $p(\mathbf{X}, \mathbf{Z})$ represents the joint distribution of observed variables $\mathbf{X}$ and latent variables $\mathbf{Z}$.
- $q(\mathbf{Z})$ is the variational distribution over the latent variables $\mathbf{Z}$.

- $E_q[\cdot]$ denotes the expectation with respect to the variational distribution $q(\mathbf{Z})$.

In our specific problem and model implemented in PyMC3, the ELBO serves as a lower bound on the log marginal likelihood of the model. During the ADVI process, PyMC3 maximizes this lower bound with respect to the parameters of the variational distribution $q(\mathbf{Z})$ to approximate the true posterior distribution $p(\mathbf{Z}|\mathbf{X})$. This approximation enables efficient inference over the latent variables given the observed data.

6 Observations and Discussion

Both MCMC and VI techniques effectively estimated the posterior distribution. MCMC provided more detailed information, but VI was faster. MAE indicates the accuracy of predictions based on the learned posterior distribution.

7 Conclusion

In this assignment, we successfully implemented a Bayesian inference model using PyMC3 to analyze the failure rates of power plant pumps. Both MCMC and VI techniques provided valuable insights into the parameters of the Gamma-Poisson Hierarchical model. The MAE metric demonstrated the accuracy of predictions based on the learned posterior distribution.