EE5602 Probabilistic Graphical models

HW0

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1) Undirected graphs :-

$$p(x_1, x_2 \ldots x_n) = \frac{1}{Z} \prod_{c \in \mathcal{C}} \psi_c(x_c)$$

Where; ψ_c : Compatibility function

$\mathcal{C}$:- set of all max cliques

Z :- normalising factor

Sum product algorithm :-

Note that marginal "x_s" found from joint as:

$$p(x_1) = \sum_{x_1} \sum_{x_3} \ldots \sum_{x_n} p(x_1, x_2 \ldots x_n)$$

Consider graph $G = (V, E)$

The factored distribution is

$$p(x_1, x_2 \ldots x_n) = \frac{1}{Z} \prod_{s \in V} \psi_s(x_s) \prod_{(s,t) \in E} \psi_{st}(x_s, x_t)$$

But in a tree, with source vertex say "s", when conditioned on it, its neighbours are independent.

So exploiting that lets split the vertex and edge set as follows $V = \{s\} \cup \{\bigcup_{t \in N(s)} V_t\}$

$$\Rightarrow E = \left\{ \bigcup_{t \in N(s)} (s,t) \right\} \cup \left\{ \bigcup_{t \in N(s)} E_t \right\}$$

where; $N(s)$ represents the neighbours of 's'

$$\Rightarrow P(x_1, x_2, \dots x_n) = \frac{1}{Z} \psi_s(x_s) \left[\prod_{t \in V \setminus \{s\}} \psi_t(x_t) \right] \left[\prod_{(s,t) \in E} \psi_{s,t}(x_s, x_t) \right]$$

$$= \frac{1}{Z} \psi_s(x_s) \left[\prod_{\substack{p \in \cup V_t \\ t \in N(s)}} \psi_p(x_p) \right] \cdot \left[\prod_{t \in N(s)} \psi_{st}(x_s, x_t) \right] \left[\prod_{\substack{(s,t) \in E \\ t \notin N(s)}} \psi_{s,t}(x_s, x_t) \right]$$

Now, $$P(x_s) = \sum_{x_1} \sum_{x_2} \dots \sum_{x_{s-1}} \dots \sum_{x_n} P(x_1, x_1 \dots x_n)$$

$$= \frac{1}{Z} \left(\sum_{x_1 \dots x_{s-1}, x_{s+1} \dots x_n} \psi_s(x_s) \prod_{t \in N(s)} \psi_{s,t}(x_s, x_t) \prod_{\substack{(s,t) \in E \\ t \notin N(s)}} \psi_{s,t}(x_s, x_t) \right)$$

But for a subtree $G_T = (V_t, E_t)$ we have;

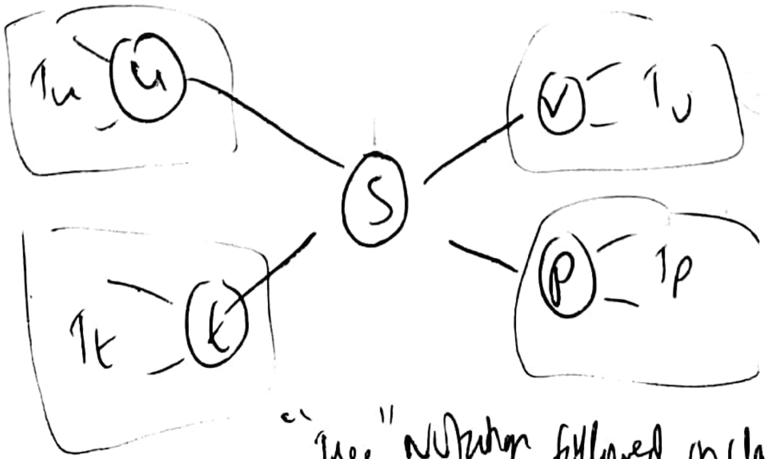
$$P(V_t; T_t) = \prod_{x \in S_t} \psi_s(x) \prod_{(s,t) \in E_t} \psi_{s,t}(x_s, x_t)$$

and $$\mu_s(x_s) = \frac{1}{Z} \psi_s(x_s) \sum_{\substack{x_1 \dots x_{s-1}, \\ x_{s+1} \dots x_n}} \prod_{t \in N(s)} P(x_{V_t}; T_t)$$

$\Rightarrow$

$\rightarrow$ $P(x_s) \propto M(x_s) = k\, \psi_s(x_s) \prod_{t \in N(s)} M_{ts}(x_s, x_t)$

where; $M_{st}(x_s, x_t) = \sum_{x \in V_t} \psi_{st}(x_s, x_t)\, p(x_{V_t}; T_t)$



"Tree" Notation followed in class