EE5601 - HW0

B. Adarsh
cs18btech11010

(3) MLE :

a) Binomial :- $f(x) = {}^{n}C_x\, p^x (1-p)^{n-x}$

$\Rightarrow \ln L(p) = \sum_{i=1}^{n} \ln(n!) - \ln(x_i!) - \ln(n-x_i!)$

$+ x_i \ln p + (n-x_i)\ln(1-p)$

Put $\frac{d}{dp}\left[\ln(L(p))\right] = 0 \Rightarrow \frac{1}{p}\sum x_i = \frac{1}{1-p}(n - \sum x_i)$

$\Rightarrow \boxed{p_{MLE} = \frac{\sum x_i}{N}}$

b) Poisson :- $f(x) = \frac{\lambda^x e^{-\lambda}}{x!}$

$\Rightarrow \ln[L(\lambda)] = \sum_{i=1}^{N} (x_i \ln\lambda - \lambda - \ln x_i!)$

Put $\frac{d}{d\lambda}\left[\ln(L(\lambda))\right] = 0 \Rightarrow -N + \frac{\sum x_i}{\lambda} = 0$

$\Rightarrow \boxed{\lambda_{MLE} = \frac{\sum x_i}{N}}$

(c) Exponential :-

$$f(x) = \lambda e^{-\lambda x}$$

$\Rightarrow \ln[L(x)] = N(\ln \lambda) - \lambda \sum_{i=1}^{N} x_i$

$\Rightarrow$ Put $\frac{d}{d\lambda}(\ln(L(\lambda))) = 0$

$\Rightarrow \frac{N}{\lambda} - \sum_{i=1}^{N} x_i = 0$

$\Rightarrow$ So, MLE estimator is $\boxed{\frac{1}{\hat{\lambda}} = \frac{\sum_{i=1}^{N} x_i}{N}}$

(d) Gaussian :-

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$\Rightarrow \ln(L(\theta)) = \sum_{i=1}^{N}\left(\ln\left(\frac{1}{\sqrt{2\pi}\sigma}\right) - \frac{(x_i-\mu)^2}{2\sigma^2}\right)$

$\Rightarrow$ Put $\frac{d}{d\mu}[\ln(L(\theta))] = 0 \Rightarrow \sum_{i=1}^{N} x_i - \hat{\mu} = 0$

$\Rightarrow \boxed{\mu_{MLE} = \frac{\sum x_i}{N}}$

Similarly, $\frac{d}{d\sigma} \ln(L(\theta)) = 0$

$$\Rightarrow \boxed{\sigma^2_{MLE} = \frac{\sum_{i=1}^{N} (x_i - \mu)^2}{N}}$$

e) Laplacian : $f(x) = \frac{1}{2b} e^{-\left(\frac{|x-\mu|}{b}\right)} \rightarrow \underline{\theta} = \begin{bmatrix} \mu \\ b \end{bmatrix}$

$$\Rightarrow L(\theta) = \prod_{i=1}^{N} \frac{1}{2b} e^{-\left(\frac{|x-\mu|}{b}\right)}$$

Now, $\hat{\theta} = \arg\max_{\underline{\theta}} L(\theta) = \arg\max_{\underline{\theta}} \ln(L(\theta))$

$$= \arg\max_{\underline{\theta}} \left(\sum_{i=1}^{n} \ln\left(\frac{1}{2b}\right) - \frac{|x_i - \mu|}{b} \right)$$

Also, $\mu_{MLE} = \arg\max_{\mu} \sum \ln\left(\frac{1}{2b}\right) - \frac{(x_i - \mu)}{b}$

$$= \arg\min_{\mu} \sum_{i=1}^{n} \frac{|x_i - \mu|}{b}$$

This occurs when μ = median of data

$$\Rightarrow \boxed{\mu_{MLE} = \text{median of } \{x_i\}_{i=1}^{N}}$$

Now, $b_{MLE} = \arg\min_{b} \left(\sum_{i=1}^{N} \ln\left(\frac{1}{b}\right) - \frac{|x_i - \mu|}{b} \right)$

Put $\frac{\partial}{\partial b} \ln(L(\theta)) = 0 \Rightarrow -\frac{N}{b} + \frac{1}{b^2} \sum |x_i - \mu| = 0$

$$\Rightarrow \boxed{b_{MLE} = \frac{\sum_{i=1}^{N} |x_i - \mu|}{N}}$$