Name : Adarsha Kanel
UCID: 30049820
CPSC 449
Tutorial T03

Assignment 3
Theory component

Question 2

Prove, by structural induction for a polynomial p

degree p $\geq$ degree (d p)

1. Definitions

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data Polynomial = PConst Integer |
PVar |
PAdd Polynomial Polynomial |
PMul Polynomial Polynomial

degree :: Polynomial -> Integer
degree (PConst n)   =    0                              (degree.1)
degree PVar         =    1                              (degree.2)
degree (PAdd p1 p2) =    max (degree p1) (degree p2)    (degree.3)
degree (PMul p1 p2) =    (degree p1) + (degree p2)      (degree.4)

d :: Polynomial -> Polynomial
d (PConst n)   =   PConst 0                             (d.1)
d PVar         =   PConst 1                             (d.2)
d (PAdd p1 p2) =   PAdd (d p1) (d p2)                   (d.3)
d (PMul p1 p2) =   PAdd (PMul p1 (d p2)) (PMul (d p1) p2)   (d.4)
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2. Proof goals

We want to prove the two goals of the induction proof:

Since, polynomial p can be either a PVar or (PConst Integer) in base case, for base case, we have to prove:

a. degree (PConst Integer) $\geq$degree (d PConst Integer) (BASE.1)
b. degree (PVar) $\geq$degree (d PVar) (BASE.2)

Since, polynomial p can be either a (PAdd p1 p2) or (PMul p1 p2) in the induction step, we have to prove

a. degree (PAdd p1 p2)$\geq$ degree(d (PAdd p1 p2)) (IND.1)
b. degree (PMul p1 p2)$\geq$ degree(d (PMul p1 p2)) (IND.2)

On the assumption that:

a. degree (p1) $\geq$degree (d (p1)) (HYP.1)
b. degree (p2) $\geq$degree (d p2)) (HYP.2)

3. Proving the Base Case

Base case 1

degree (PConst Integer) $\geq$ degree (d PConst Integer) BASE.1

Left-hand side(LHS)

degree (PConst Integer)
= 0 by (degree.1)

Right-hand side (RHS)

degree (d PConst Integer) = degree
= degree (PConst 0) by (d.1)
= 0 by (degree.1)

So, LHS = RHS, so degree (PConst Integer) $\geq$ degree (d PConst Integer) is satisfied

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Base case 2
degree (PVar) $\geq$ degree (d PVar) BASE.2

Left-hand side(LHS)

degree (PVar)
= 1 by (degree.2)

Right-hand side (RHS)

degree (d PVar)
= degree (PConst 1) by (d.2)
= 0 by (degree.1)

So, LHS > RHS, so degree (PVar) $\geq$ degree (d PVar) is satisfied

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So, both base cases have been proven so LHS $\geq$ RHS for the base case

4. Proving the induction step

a. degree (PAdd p1 p2) $\geq$ degree(d (PAdd p1 p2)) IND.1

Left-hand side(LHS)
degree (PAdd p1 p2)
= max (degree p1) (degree p2) by (degree.3)
$\geq$ max(degree (d p1)) (degree p2) by (HPY.1)

$\geq$ max (degree (d p1)) (degree (d p2)) by (HYP.2)

Right-hand side (RHS) will be
degree(d (PAdd p1 p2))
= degree (PAdd (d p1) (d p2)) by (d.3)
= max (degree (d p1)) (degree (d p2)) by (degree.3)

So, LHS $\geq$ RHS when p = (PAdd p1 p2)

B. degree (PMul p1 p2) $\geq$ degree(d (PMul p1 p2)) IND.2

Left-hand side(LHS)
degree (PMul p1 p2)
= (degree p1) + (degree p2) by (degree.4)

Right-hand side(RHS)
degree(d (PMul p1 p2))
= degree (PAdd (PMul p1 (d p2)) (PMul (d p1) p2)) by (d.4)
= max (degree (PMul p1 (d p2))) (degree (PMul (d p1) p2)) by (degree.3)
= max (degree (p1) + degree (d p2)) ((degree (d p1)) + (degree p2)) by(degree.4)

So, max will chose the biggest degree between (degree p1) + (degree (d p2)) and (degree (d p1)) + (degree p2)

So, when (degree p1)+(degree (d p2))) $\geq$((degree (d p1)) + (degree p2)), we get

= max (degree p1) + (degree (d p2)) (degree (d p1)) + (degree p2)
= (degree p1) + (degree (d p2)) by max function
$\leq$(degree p1) + (degree p2) by (HYP.2)

So, when ((degree (d p1)) + (degree p2))$\geq$((degree p1) + (degree (d p2))),we get

= max ((degree p1) + (degree (d p2))) ((degree (d p1)) + (degree p2))
= (degree (d p1)) + (degree p2) by max function
$\leq$(degree p1) + (degree p2) by(HYP.1)

So, LHS $\geq$ RHS when p = (PMul p1 p2)

LHS $\geq$ RHS

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