MATHEMATICS

Paper & Solution

Code: 30/1/1

Time: 3 Hrs. Max. Marks: 80

General Instuctions :

1. All questions are compulsory.

*2. The question paper consists of **25** questions divided into three sections —**A, B** and **C**. Section - **A** contains **7** questions of **2** marks each, Section - **B** is of **12** questions of **3** marks each and Section - **C** is of **6** questions of **5** marks each.*

3. There is no overall choice. However, an internal choice has been provided in two questions of two marks each, two questions of three marks each and two questions of five marks each.

4. In question on construction, the drawing should be neat and exactly as per the given measurements.

5. Use of calculators is not permitted. However, you may ask for Mathematical tables.

SECTION – A

1. If x + k is the GCD of $x^2 - 2x - 15$ and $x^3 + 27$, find the value of k.

Solution:

(x+k) is G.C.D of $x^2-2x-15$ and x^3+27

$\therefore$ x=-k will satisfy both expressions

$\Rightarrow$ $(-k)^2-2(-k)-15=0$

$K^2+2k-15=0$

K=-5, 3-...........(1)

Also $(-k)^3+27=0$

K=3........(2)

From (1) and (2) k=3

2. Solve for x and y :

$$x+\frac{6}{y}=6$$

$$3x-\frac{8}{y}=5$$

OR

$$\frac{x+1}{2}+\frac{y-1}{3}=8$$

$$\frac{x-1}{3}+\frac{y+1}{2}=9$$

Solution:

$$x+\frac{6}{y}=6 \quad (1)$$

$$3x-\frac{8}{y}=5 \quad (2)$$

Multiply (1) with 3 and then subtract (1) from (2)

$$3x-\frac{8}{y}=5$$

$$3x+\frac{18}{-y}=18$$

$$\frac{-26}{y}=-13$$

$$y=2$$

$$x=3$$

OR

$$\frac{x+1}{2}+\frac{y-1}{3}=8 \quad \Rightarrow \frac{x}{2}+\frac{y}{3}=8-\frac{1}{2}+\frac{1}{3} \quad \Rightarrow \frac{x}{2}+\frac{y}{3}=\frac{47}{6}.......(1)$$

$$\frac{x-1}{3}+\frac{y+1}{2}=9 \quad \Rightarrow \frac{x}{3}+\frac{y}{2}=9+\frac{1}{3}-\frac{1}{2} \quad \Rightarrow \frac{x}{3}+\frac{y}{2}=\frac{53}{6}.......(2)$$

$$\text{solving}: 2\times(1)-3\times(2) \quad x+\frac{2}{3}y=\frac{47}{3}$$

$$x+\frac{3}{2}y=\frac{53}{2}$$

$$-\quad - \qquad -$$

$$-\frac{5}{6}y=\frac{-65}{6}$$

$$y=13, x=7$$

3. Find the sum of first 25 terms of an A.P. whose n^{th} term is 1 - 4n.

Solution:

Let's assume first first term = a

Common difference =d

Given T_n = 1 - 4n

a+(n-1)d = 1 - 4n

(a-d) + nd = 1 - 4n

On comparing the like terms

$\Rightarrow$ d=-4, a-d=1

a = 5

sum of first 25 terms

$$=\frac{25}{2}\left[2a+(24)d\right]$$
$$=\frac{25}{2}[10-96]$$
$$=-1075$$

4. P and Q are points on sides CA and CB respectively of ΔABC, right angled at C.
Prove that
$AQ^2 + BP^2 = AB^2 + PQ^2$

OR

In Fig. 1, DE || AB and FE || DB.
Prove that $DC^2 = CF.AC$

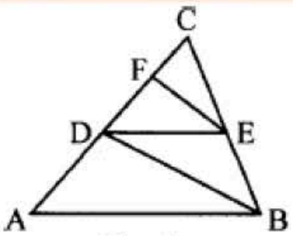


Fig. 1

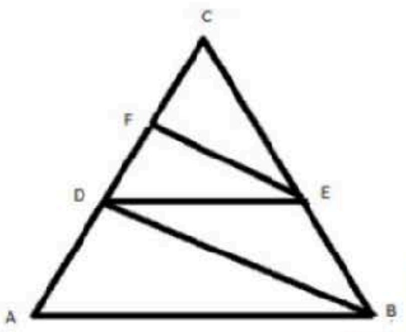
Solution:
We will be using Pythagoras Theorem in order to solve this problem.

So,

$$\begin{aligned}AQ^2+BP^2 &= \left(AC^2+CQ^2\right)+\left(BC^2+PC^2\right)\\ &= \left(AC^2+BC^2\right)+\left(CQ^2+PC^2\right) \qquad \left[AC^2+BC^2=AC^2, CQ^2+PC^2=PQ^2\right]\\ &= AB^2+PQ^2\end{aligned}$$

Hence Proved.

OR



$In\ \Delta CAB, since\ DE\ II\ AB,$

$$\frac{CD}{DA}=\frac{CE}{EB}\quad [By\ BPT]\quad ....(1)$$

$In\ \Delta CDB, since\ FE\ II\ DB,$

$$\frac{CF}{FD}=\frac{CE}{EB}\quad [By\ BPT]\quad ....(2)$$

$From\ (1)\ and\ (2), we\ get$

$$\frac{CD}{DA}=\frac{CF}{FD}$$

$$\Rightarrow \frac{DA}{CD}=\frac{FD}{CF}$$

$$\Rightarrow \frac{DA}{CD}+1=\frac{FD}{CF}+1$$

$$\Rightarrow \frac{AD+CD}{CD}=\frac{FD+CF}{CF}$$

$$\Rightarrow \frac{AC}{CD}=\frac{CD}{CF}$$

$$\Rightarrow CD^2 = AC\times CF$$

5. The mean of the following frequency distribution is 62.8. Find the missing frequency *x*.

Class	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	5	8	*x*	12	7	8

Solution:

class	frequency (f)	class mark (x_i)	$x_i f_i$
$0-20$	5	10	50
$20-40$	8	30	240
$40-60$	x	50	$50x$
$60-80$	12	70	840
$80-100$	7	90	630
$100-120$	8	110	880
	$\Sigma fi = 40 + x$		$\Sigma x_i f_i = 2640 + 50x$

$$Mean = \frac{\Sigma x_i f_i}{\Sigma f_i}$$

$$62.8 = \frac{2640+50x}{40+x}$$

$$2512+62.8x = 2640+50x$$

$$12.8x = 128$$

$$\boxed{x = 10}$$

6. Cards marked with numbers 3, 4, 5, ..., 50 are placed in a box and mixed thoroughly. One card is drawn at random from the box. Find the probability that number on the drawn card is

(i) divisible by 7.

(ii) a number which is a perfect square.

Solution:

Total possible outcomes = 48

(i) Total outcomes which are divisible by 7=7, 14, 21, 28, 35, 42, 49

Probability=$\frac{7}{48}$

(ii) Outcomes which are perfect squares = 4, 9, 16, 25, 36, 49

Probability=$\frac{6}{48} = \frac{1}{8}$

7. A washing machine is available for Rs. 13,500 cash or Rs. 6,500 as cash down payment followed by three monthly instalments of Rs. 2,500 each. Find the rate of interest charged under instalment plan.

Solution:

A washing machine is available for Rs. 13,500 cash or Rs. 6,500 as cash down payment followed by three monthly installments of Rs.2,500 each. Find the rate of interest charged under installment plan.

In Solving this question

Cash price of washing machine= Rs.135000

Cash down payment = Rs.6500

Balance due = Rs (13500-6500)=rs.7000

No. of equal installments =3

Amount of each installment = rs.2500

Amount paid in installment = Rs. 7500

Therefore, interest paid in installment scheme = rs (7500-7000)= rs.500

Principal for the 1st month = Rs.7000

Principal for the 2nd month = Rs.4500

Principal for the 3rd month = Rs.2000

Total = Rs.13500

Let the rate of interest be r % per annum

I=p*r*1/1200

Then, I = 13500*r*1/100*12

$\Rightarrow$500=13500*r*1/100*12

$\Rightarrow$r = 500*12/135

$\Rightarrow$r = 44.4%

SECTION – B

Questions number 8 to 19 carry 3 marks each.

8. Solve the following system of equations graphically :

2x + 3y = 8 ; x + 4y = 9

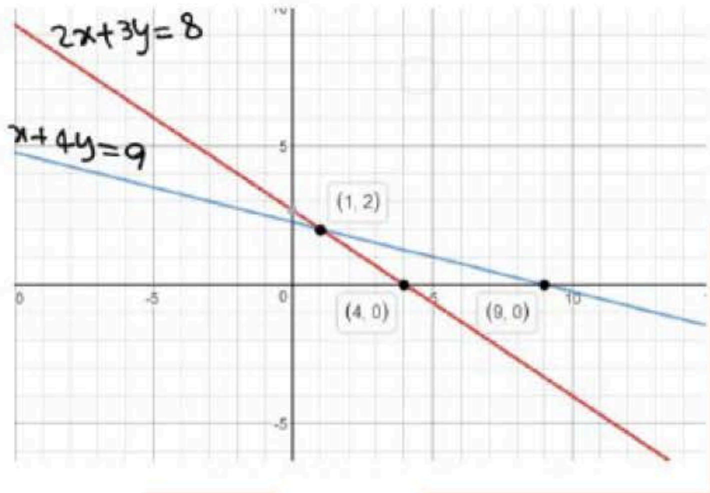
Solution:

$$2x+3y=8 \Rightarrow y=\frac{8-2x}{3}$$

$$x+4y=9 \Rightarrow y=\frac{9-x}{4}$$

X	1	4
$y=\frac{8-2x}{3}$	2	0

X	1	5	9
$y=\frac{9-x}{4}$	2	4	0



Solution for the system of equations x = 1, y = 2

9. Simplify :

$$\frac{x}{x-y}-\frac{y}{x+y}-\frac{2xy}{x^2-y^2}$$

Solution:

$$\frac{x}{x-y}-\frac{y}{x+y}-\frac{2xy}{x^2-y^2}$$

We know that $x^2-y^2=(x-y)(x+y)$

$\therefore$ take L.C.M

$$\Rightarrow \frac{x(x+y)-y(x-y)-2xy}{x^2-y^2}$$

$$\Rightarrow \frac{x^2+xy-xy+y^2-2xy}{x^2-y^2}$$

$$=\frac{(x-y)^2}{(x-y)(x+y)}=\frac{x-y}{x+y}$$

10. Which term of the A.P. 3, 15, 27, 39,……. will be 132 more than its 54th term ?

Solution:

3, 15, 27, 39, ……….

Lets say T_n will be 132 more than 54th term

$T_n=3+(n-1)(15-3)=3+(n-1)12$

$T_{54}=3+(53)(12)$

$\Rightarrow T_n=132+T_{54}$

$3+(n-1)12=132+3+(53)12$

$(n-54)12=132$

$n=11+54$

$n=65$

11. In Fig. 2, TA is a tangent to the circle from a point T and TBC is a secant to the circle. If AD is the bisector of $\angle$ CAB, prove that ADT is isosceles.

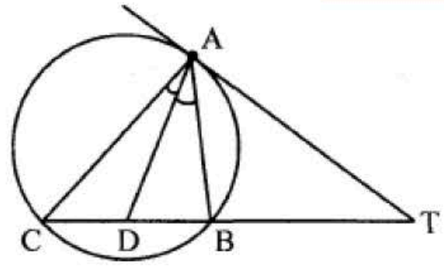
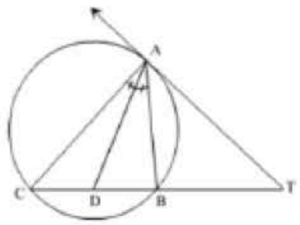


Fig. 2

OR

In ABC, AD BC and AD^2 = BD.DC. Prove that BAC is a right angle.

Solution:



Given: TA is tangent of the circle, TBC is the secant AD is the bisector of $\angle CAB$

To prove: ΔADT is an isosceles triangle

Proof: In order to prove this, we will use alternate segment theorem

$\Rightarrow \angle TAB = \angle BCA$ ....(1)

AD is bisector of $\angle BAC \Rightarrow \angle BAD = \angle DAC$ ....(2)

Now, $\angle TAD = \angle TAB + \angle BAD$

$= \angle BCA + \angle DAC$ (using (1) and (2))

$= \angle DCA + \angle DAC$

$= 180^\circ - \angle CDA$ [

$\because$

In $\Delta CAD, \angle CAD + \angle DCA = 180^\circ$]

$\Rightarrow TD = TA$ (sides opposite to equal angles are equal)

Hence, ΔADT is an isosceles Δ

OR

Given In triangle ABC, AD is perpendicular to BC & AD: = BD.DC

To Prove : BAC = 90 degree

Proof : in right triangles ADB & ADC we have $AB^2= AD^2 +BD^2$ ……….(1) & $AC^2 = AD^2 + DC^2$…. (2)

from 1 & 2 , $AB^2 + AC^2= 2AD^2.BD^2 + DC^2$

$= 2BD . CD + BD^2 + CD^2$ { AC^2 BD. CD(GVEN)}

$= (BD + CD)^2 BC^2$

Thus in triangle ABC we have, $AB^2 + AC^2 = BC^2$

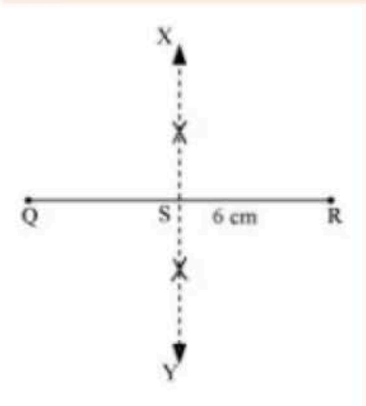
hence triangle ABC is a right triangle right angled at A

so BAC = 90 degree (proved)

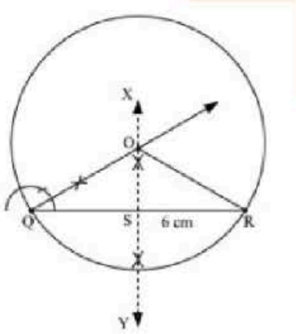
12. Draw a ΔPQR with base QR = 6 cm, vertical angle P = 60° and median through P to the base is of length 4.5 cm.

Solution:

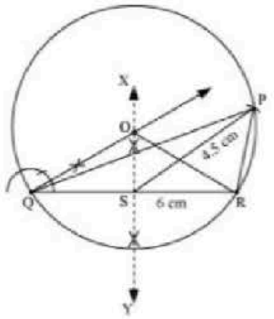
Draw QR= 6c. Draw its perpendicular bisector XY which intersects QR at S



(ii) At Q, draw an angel of measure 30°, intersecting XY at point O. Taking O as the centre and radius OQ= OR, draw a circle.



(iii) From S, draw an arc of radius 4.5 cm, intersecting the circle at point P, Join PQ, PR, and PS



It can be observed that ∠ OQR = 30 [Being radii]

∴ ∠ QOR = 180 - (∠ OQR + ∠ ORQ) = 180 - 60 = 120

We know that the angel subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle

∴ ∠ QOR = 2∠ QPR

$$\angle QPR = \frac{1}{2}\angle QOR = \frac{1}{2}\times 180^{\circ} = 60^{\circ}$$

Thus, ΔPQR is the required triangle, where QR = 6 cm, $\angle P = 60^{\circ}$, and the median PS = 4.5 cm

13. A toy is in the form of a cone mounted on a hemisphere of common base radius 7 cm. The total height of the toy is 31 cm. Find the total surface area of the toy.

$\left(\text{Use } \pi = \frac{22}{7}\right)$

Solution:

height of toy = 31 cm

radius = 7cm

therefore, height of the cone = 31-7=24 cm

slant height , l

$l^2 = r^2 + h^2$

=7*7 + 24*24 =625

therefore l = root of 625 = 25cm

Area of toy = area of cone+ area of hemisphere

= pi r l + 2pi r^2

pi r (l+ 2r) = 22/7*7 (25+ 14)

22 * 39 = 858 cm^2

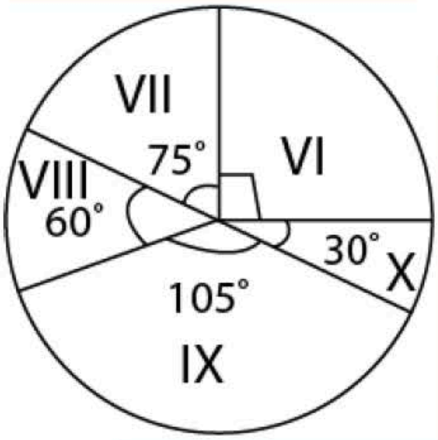
14. The enrolment of a secondary school in different classes is given below :

Class	VI	VII	VIII	IX	X
Enrolment	600	500	400	700	200

Draw a pie chart to represent the above data.

Solution:

Class	Eurollment	fraction	central angle
VI	600	$\frac{1}{4}$	$\frac{1}{4} \times 360^\circ = 90^\circ$
VII	500	$\frac{5}{24}$	$\frac{5}{24} \times 360^\circ = 75^\circ$
VIII	400	$\frac{1}{6}$	$\frac{1}{6} \times 360^\circ = 60^\circ$
IX	700	$\frac{7}{24}$	$\frac{7}{24} \times 360^\circ = 105^\circ$
X	200	$\frac{1}{12}$	$\frac{1}{12} \times 360^\circ = 30^\circ$
Total	2400		



Steps:

1) Draw a circle of any radius and mark its centre

2) Draw any radius

3) Take up the first component (90) place protractor on the radius and draw a new radius marks 90 with it

4) From the new radius draw the next component's central angle

5) Repeat the same for all the components.

15. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball from the bag is thrice that of a red ball, find the number of blue balls in the bag.

Solution:

Let the no. Of blue balls be x

No. Of red balls = 5

Total no of balls = no. Of blue balls + no. Of red balls + x+5

Probability of getting blue ball = x / x+ 5

Probability of getting red ball = 5/ x +5

According to the question,

X/x+5=2*5/x + 5

Cancelling x + 5 from both terms as they are common

X: 10

So, there are 10 blue balls in the bag

16. Prove that :

$$\frac{\cos A}{1-\tan A}+\frac{\sin A}{1-\cot A}=\sin A+\cos A$$

OR

Evaluate without using trigonometric tables :

$$\frac{3\cos 55^\circ}{7\sin 35^\circ}-\frac{4(\cos 70^\circ.\text{cosec } 20^\circ)}{7(\tan 5^\circ.\tan 25^\circ.\tan 45^\circ.\tan 65^\circ.\tan 85^\circ)}$$

Solution:

$$\frac{\cos A}{1-\tan A}+\frac{\sin A}{1-\cot A}$$

$$=\frac{\cos A}{1-\frac{\sin A}{\cos A}}+\frac{\sin A}{1-\frac{\cos A}{\sin A}}$$

$$=\frac{\cos^2 A}{\cos A-\sin A}+\frac{\sin^2 A}{\sin A-\cos A}$$

$$=\frac{\cos^2 A-\sin^2 A}{\cos A-\sin A}$$

$$=\frac{(\cos A-\sin A)(\cos A+\sin A)}{\cos A-\sin A}$$

$$=\cos A+\sin A$$

OR

$$\frac{3\cos 55^\circ}{7\sin 35^\circ} - \frac{4(\cos 70^\circ \operatorname{cosec} 20^\circ)}{7(\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ)}$$

$$\tan 5^\circ = \cot 85^\circ \Rightarrow \tan 5^\circ \tan 85^\circ = 1$$

$$\tan 25^\circ = \cot 65^\circ \Rightarrow \tan 25^\circ \tan 65^\circ = 1$$

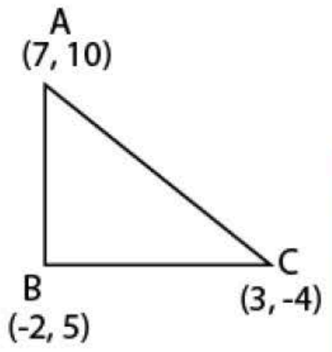
$$\cos 70^\circ.\operatorname{cosec} 20^\circ \Rightarrow \cos 70^\circ \sec 70^\circ = 1$$

$$\therefore \frac{3\cos 55^\circ}{7\sin 55^\circ} - \frac{4}{7}$$

$$\Rightarrow -\frac{1}{7}$$

17. Show that the points (7, 10), (-2, 5) and (3, -4) are the vertices of an isosceles right triangle.

Solution:



Construction: join AB, BC and CA.

Proof: The distance between AB.

$$|AB| = \sqrt{(-2-7)^2 + (5-10)^2}$$

$$= \sqrt{81+25}$$

$$= \sqrt{106}$$

The distance between BC.

$$|BC| = \sqrt{(3+2)^2 + (-4-5)^2}$$

$$= \sqrt{25+81}$$

$$= \sqrt{106}$$

And, the distance between CA,

$$|CA| = \sqrt{(7-3)^2 + (10+4)^2}$$

$$= \sqrt{16+196}$$

$$= \sqrt{212}$$

$$\Rightarrow AB^2 = 106, BC^2 = 106, CA^2 = 212$$

Hence,

$$AB^2 + BC^2 = 106 + 106 = 212 = CA^2$$
$$\Rightarrow \Delta ABC$$

Is right angles and it is right angled at B.

Also,

$|AB| = \sqrt{106} = |BC| \Rightarrow \Delta ABC$ is Isosceles

18. In what ratio does the line *x* - y - 2 = 0 divides the line segment joining (3, - 1) and (8, 9) ?
Solution:
let t line x-y^2=0 divid t line segment in t ratio k :1 at point C

so coordinates of C are

{(8k + 3 / k+1), (9k – 1 / k+1)}

C lies on x -y -2 = 0

so subs coordinates of x, y in tis eq.

(8k +3/k+1)-(9k-t / k+1)-2 =0

8k +3 - 9k + 1 -2k-2 / k+1 =0

-3k +2/ k+1 =0

-3k+2=0

$\Rightarrow$ k=2/3

i.e. 2:3

19. A man borrows money from a finance company and has to pay it back in two equal half-yearly instalments of Rs. 7,396 each. If the interest is charged by the finance company at the rate of 15% per annum, compounded semi-annually, find the principal and the total interest paid.
Solution:
Amount of each instalment = Rs 7396

Number of instalments = 2

Amount paid in instalments = 2$\times$ Rs 7396 = Rs 14792

Rate of interest = $\frac{15}{2}$% half yearly

Principal for the first installment is given by

$$P_1 = Rs\left[7396 + \left(1 + \frac{15}{200}\right)\right] = Rs\left(7396 \times \frac{200}{215}\right) = Rs\,6880$$

Principal for the second installment is given by

$$P_2 = Rs\left[7396 + \left(1 + \frac{15}{200}\right)^2\right] = Rs\left[7396 \times \frac{200}{215} \times \frac{200}{215}\right] = Rs\,6400$$

Therefore toal principal = $P_1 + P_2$ = Rs 6880 + Rs 6400 = Rs 13280

Total interest paid = Rs 14792 – Rs 13280 = Rs 1512

SECTION – C

Questions number 20 to 25 carry 5 marks each.

20. If a line is drawn parallel to one side of a triangle, to intersect the other two sides in distinct points, prove that the other two sides are divided in the same ratio.

Using the above, prove the following :

In Fig. 3, DE || BC and BD = CE. Prove that ABC is an isosceles triangle.

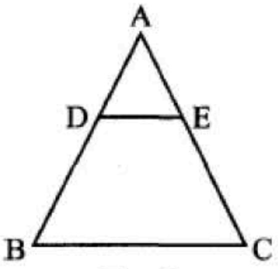


Fig. 3

Solution:

Construction: Join BE, and CD and draw a per pendicular EM at AB, and DN at AC.

$$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2} \times EM \times AD}{\frac{1}{2} \times BD \times EM}$$

$$\Rightarrow \frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{AD}{BD} \qquad \text{.............(1)}$$

Similarly

$$\frac{ar(\Delta ADE)}{ar(\Delta CDE)} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times EC \times DN}$$

$$\Rightarrow \frac{ar(\Delta ADE)}{ar(\Delta CDE)} = \frac{AE}{EC} \qquad \text{............(2)}$$

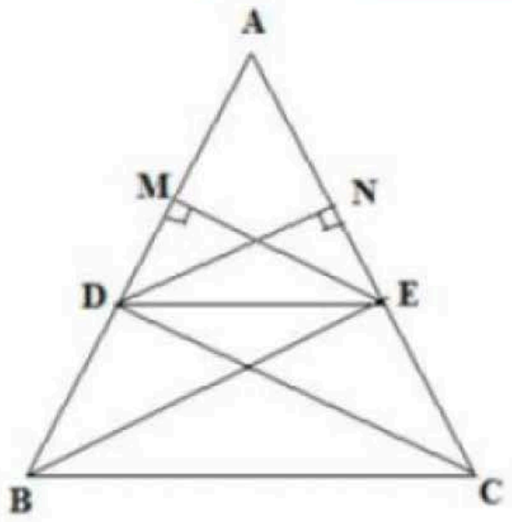
But $ar(\Delta BDE) = ar(\Delta CDE)$ triangle on same base DE and between the same parallels DE and BC)

Thus, equation (2) becomes,

$$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{AE}{EC} \quad \text{............(3)}$$

From equation (2) and (3), we have

$$\frac{AD}{BD} = \frac{AE}{EC}$$



If DE || BC

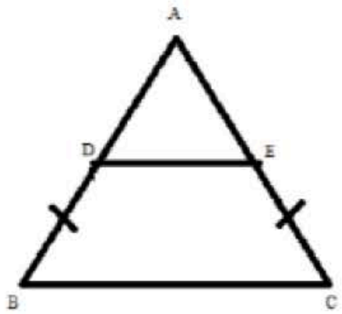
From the fig.

$\frac{AD}{BD} = \frac{AE}{CB}$ (by basic proportionality theorem)

Adding 1 on both sides

$$\Rightarrow \frac{AD}{BD} + 1 = \frac{AE}{CE} + 1$$
$$\Rightarrow \frac{AD + BD}{BD} = \frac{AE + CE}{CE}$$
$$\Rightarrow \frac{AB}{BD} = \frac{AC}{CE}$$

As from the given data



BD = CE

So

$$\frac{AB}{BD}=\frac{AC}{BD}$$
$$\Rightarrow AB=AC$$

As two sides of the triangle are equal.

$\therefore$ ABC is a osceles triangle.

21. Prove that the sum of either pair of opposite angles of a cyclic quadrilateral is 180°. Using the above, find *x* and y in Fig. 4.

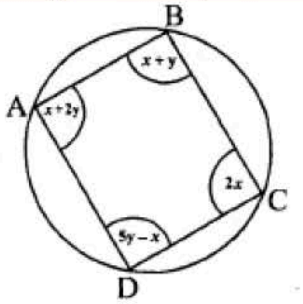
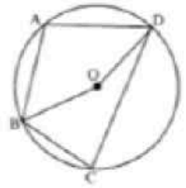


Fig. 4

Solution:

Consider a cyclic quadrilateral ABCD inscribed a circle with centre at 0

In order to prove this theorem join OB and OD



Now, $\angle BAD=\frac{1}{2}$reflex $\angle BOD$

And $\angle BCD = \frac{1}{2}\angle BOD$

Now $\angle BAD + \angle BCD = \frac{1}{2}\text{reflex}\ \angle BOD + \frac{1}{2}\angle BOD$

$= \frac{1}{2}\left[\angle BOD + \text{reflex}\angle BOD\right]$

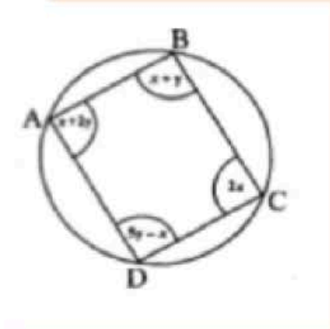
$= \frac{1}{2} \times 360^\circ$

$\Rightarrow \angle BAD + \angle BCD = 180..........(1)$

Similarly, by joining OA and OC it can be proved that

$\angle ABC + \angle ADC = 180...........(2)$

Equation (1) and equation (2) shows that the sum of opposite angles of a cyclic quadrilateral is 180 degree.



Sum of opposite angles in a cyclic quadrilateral = 180

$x + y + 5y - x = 180^\circ$

$y = 30^\circ$

Also $x + 2y + 3x = 180 \Rightarrow x = 30^\circ$

22. The difference of two numbers is 5 and the difference of their reciprocals is $\frac{1}{10}$. Find the numbers.

OR

By increasing the list price of a book by Rs. 10 a person can buy 10 less books for Rs. 1,200. Find the original list price of the book.

Solution:

Lets say the two numbers are x, y (y > x)

Given $y - x = 5 \Rightarrow y = 5 + x$ .....(1)

$$\frac{1}{x}-\frac{1}{y}=\frac{1}{10}$$
$$\frac{5}{xy}=\frac{1}{10}$$
$$xy=50$$

$\therefore x(5+x)=50$ [From (1)]

$$x^2+5x-50=0$$
$$(x+10)(x-5)=0$$
$$x=-10, 5$$

$$y=-5, 10$$

$\therefore$ Numbers are -10, 5 or 5, 10

OR

Let the list price of book be Rs x

Number of books bought R. 1200

$$=\frac{1200}{x}$$

When the list price of the book is Increased by Rs 10. then

New list in tee of book - Rs (x + 10)

Number of books bought for Rs 1200

$$=\frac{1200}{x+10}$$

Gives what the list price of the book is increase by Rs 10, then the person can buy 10 leas books.

$$\therefore \frac{1200}{x}-\frac{1200}{x+10}=10$$
$$\Rightarrow \frac{1200(x+10)-1200x}{x(x+10)}=10$$
$$\Rightarrow 1200x+12000-1200x=10x^2+100x$$
$$\Rightarrow 10x^2+100x-12000=0$$
$$\Rightarrow x^2+10x-1200=0$$
$$\Rightarrow x^2+40x-30x-1200=0$$
$$\Rightarrow x(x+40)-30(x+40)=0$$

$$\Rightarrow (x-30)(x+40)=0$$
$$\Rightarrow x=30 \text{ or } x=-40$$

$\Rightarrow x = 30$ (Price of the book cannot be negative)

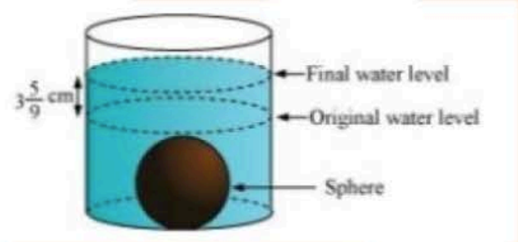
Thus, the list price of the book is Rs. 30.

23. A sphere, of diameter 12 cm, is dropped in a right circular cylindrical vessel, partly filled with water. If the sphere is completely submerged in water, the water level in the cylindrical vessel rises by $3\frac{5}{9}$ cm. Find the diameter of the cylindrical vessel.

OR

A solid right circular cone of diameter 14 cm and height 8 cm is melted to form a hollow sphere. If the external diameter of the sphere is 10 cm, find the internal diameter of the sphere.

Solution:



Let the radius of the cylindrical vessel be R

Diameter of the sphere = 12 cm

$\therefore$ Radius of the sphere = r= 6 cm

Volume of the sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3}\times\pi\times(6\ cm)^3 = 288\pi\ cm^3$

Rise in the water level $= h = 3\frac{5}{9}cm = \frac{32}{9}cm$

Increase in the volume of water in the cylindrical vessel $= \pi R^2 h = \left(\pi R^2 \times \frac{32}{9}\right)cm^3$

But,

Increase in the volume of water = Volume of the sphere

$\Rightarrow \pi R^2 \times \frac{32}{9} = 288\pi$

$\Rightarrow R^2 = \frac{288\times 9}{32} = 81$

$\Rightarrow R = \sqrt{81} = 9$

Thus, diameter of the cylindrical vessel = 2R cm=2×9 cm= 18 cm

OR

Diameter of the cone = 14 cm

∴ Radius of the cone = r = 7 cm

Height of the cone = h = 8 cm

External diameter of the hollow sphere = 10 cm

∴ External radius of the hollow sphere = r_1 = 5 cm

Let the internal radius of the hollow sphere be r_2 cm

Since the solid cone is melted to form a hollow sphere, we have:

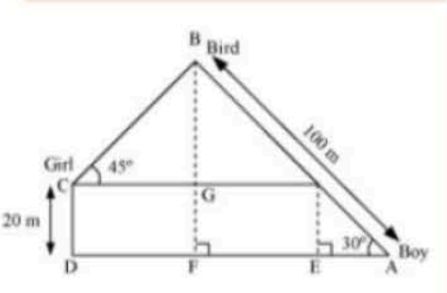
Volume of the solid cone = volume of the hollow sphere

$$\Rightarrow \frac{1}{3}\pi r^2 h = \frac{4}{3}\pi\left(r_1^3 - r_2^3\right)$$
$$\Rightarrow (7)^2 \times 8 = 4(5^3 - r_2^3)$$
$$\Rightarrow 4r_2^3 = 500 - 392 = 108$$
$$\Rightarrow r_2^3 = 27$$
$$\Rightarrow r_2 = 3$$

Thus, internal diameter of the sphere = $2r_2$ cm = (2×3) cm = 6 cm

24. A boy standing on a horizontal plane finds a bird flying at a distance of 100 m from him at an elevation of 30°. A girl standing on the roof of 20 metre high building, finds the angle of elevation of the same bird to be 45°. Both the boy and the girl are on opposite sides of the bird. Find the distance of bird from the girl.

Solution:



Here, A, B, and C are the positions of the boy, bird, and the girl respectively

In right ΔBFA:

$$\frac{FB}{AB} = \sin 30^\circ = \frac{1}{2}$$
$$\Rightarrow \frac{FB}{100\ cm} = \frac{1}{2}$$
$$\Rightarrow FB = 50\ cm$$

Now, BG = FB – GF = FB – CD = (50-20) cm = 30 cm

In right ΔBGC:

$$\frac{BG}{BC} = \sin 45^\circ$$
$$\Rightarrow \frac{30\ cm}{BC} = \frac{1}{\sqrt{2}}$$
$$\Rightarrow BC = 30\sqrt{2}\ cm$$

Thus, the distance of the bird from the girl is $30\sqrt{2}\ cm$

25. Ms. Shahnaz earns Rs. 35,000 per month (excluding HRA). She donates Rs. 30,000 to Prime Minister Relief Fund (100% exemption) and Rs. 40,000 to a Charitable Hospital (50% exemption). She contributes Rs. 5,000 per month to Provident Fund and Rs. 25,000 per annum towards LIC premium. She purchases NSC worth Rs. 20,000. She pays Rs. 2,300 per month towards income tax for 11 month. Find the amount of income tax she has to pay in 12^{th} month of the year.

Use the following to calculate income tax :

(a) **Saving :**	100% exemption for permissible savings upto Rs. 1,00,000
(b) **Rates of income tax for ladies**	
Slab	**Income tax**
(i) Upto Rs. 1,35,000	No tax
(ii) From Rs. 1,35,001 to Rs. 1,50,000	10% of taxable income exceeding Rs. 1,35,000
(iii) From Rs. 1,50,001 to Rs. 2,50,000	Rs. 1,500 + 20% of the amount excedding Rs. 1,50,000
(iv) From Rs. 2,50,001 and above	Rs. 21,500 + 30% of the amount exceeding Rs. 2,50,000
(c) **Education Cess :**	2% of Income tax payable

Solution:

Gross income of Ms.Shahnaz in one year = 12 $\times$ Rs 35000 = 420000

Donation

Donation to the Prime Minister Relief Fund = Rs 30000

Donation to a charitable hospital = Rs 40000

=Rs 30000 + Rs 20000

=Rs 50000

Savings

PF = 12 $\times$ Rs 5000 = Rs 60000

LIC = **Rs** 25000

NSC = Rs 20000

Total savings = Rs 105000

Savings is subjected to a maximum of Rs 100000

So, taxable income = Rs (420000 – 50000 - 100000)= Rs 270000

Now, income tax = Rs 21500 + 30% of (Rs 270000 – Rs 250000)

$$= Rs 21500 + \frac{30}{100} \times \left(Rs\ 20000 \right)$$

= Rs 21500 + Rs 6000

= Rs 27500

Educational cess = 2% of income tax $= \frac{2}{100} \times Rs\ 27500 = Rs\ 550$

Total tax to be paid = Rs 27500 + Rs 550 = Rs 28050

Income tax paid for 11 months = Rs 2300

Total tax to be paid = Rs 27500 + Rs 550 = Rs 28050

Income tax paid for 11 months = Rs 2300 $\times$11 = Rs 25300

Therefore, income tax to be paid in the 12th month of the year = Rs 28050 – Rs 25300 = Rs 2750