Intermediate Micro – Homework 2 Solution

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1. A survey shows that the typical Brown undergraduate has $6,000 to spend each year on books and entertainment. That typical student spends $2000 on books and the remaining $4000 on entertainment. The Brown dean finds it outrageous that the undergraduates must spend 1/3 of their disposable income on books and so the dean sets up a program to give each undergraduate a subsidy of $500. If the income elasticity of books of the typical student is 1.5 then will the subsidy program achieve its goal of lowering the percentage of income spent on books?

SOLUTION:

Intuitively, an income elasticity of 1.5 indicates a more than proportional increase in the consumption of books than the increase in income. Hence, the percentage of income spent on books will be higher than before. The subsidy program fails to achieve the goal.

Mathematically, $Income\ Elasticity = \frac{\Delta x}{x} \cdot \frac{I}{\Delta I} = \frac{\Delta x}{2000} \cdot \frac{6000}{500} = 1.5$. Solve for Δx, we have $\Delta x = 250$. Therefore total expenditure on books is $2000 + 250 = 2250$. Total income is $6000 + 500 = 6500$. The percentage of income spent on books is $\frac{2250}{6000} = 34.6\% > \frac{2000}{6000} = 33.3\%$.

2. Frank buys hot dogs (x) and pretzels (y) at the ball park.. His preferences over these two goods can be represented by the utility function U(x,y) = $\ln$x + 3$\ln$y where x represents the number of hot dogs and y represents the number of pretzels.

a) Given his preferences find his demand functions for hot dogs (x) and pretzels (y).

SOLUTION:

Tangency: $MRS = \frac{1/x}{3/y} = \frac{P_x}{P_y}$ implies $y = \frac{3P_x}{P_y}x$.

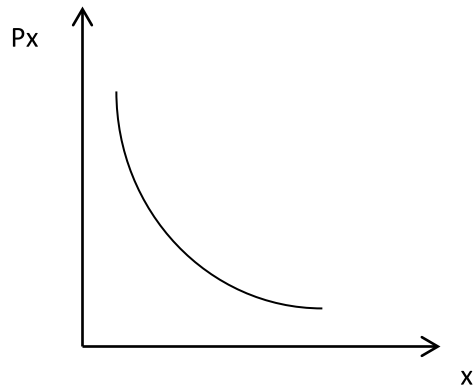
Feasibility: $P_x x + P_y y = I$.

Solving these two equations, we have $x^* = \frac{I}{4P_x}$ and $y^* = \frac{3I}{4P_y}$.

b) Suppose that the price of a pretzel is \$1 and that Frank has \$40 to spend on hot dogs and pretzels. Write Frank's demand curve for hot dogs. Illustrate his demand curve.

<u>SOLUTION</u>:

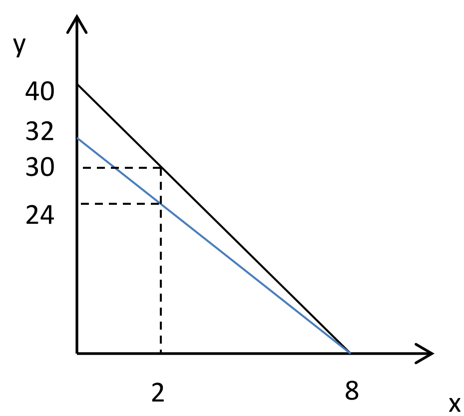
From the demand function in part a, substitute $I = 40$, we have $x^* = \frac{10}{P_x}$.



c) Suppose that the price of a hot dog is \$5 (the price of pretzels and income remain \$1 and \$40, resp.). Use your demand functions to find his best bundle. In an indifference curve diagram illustrate his best bundle at these prices.

<u>SOLUTION</u>:

By substituting into the demand functions, we have best bundle at $P_x = 5, P_y = 1, I = 40$ is $x^* = 2, y^* = 30$.



For the remainder of the question assume that the price of a pretzel rises to \$1.25 and that the price of a hot dog and his income are unchanged at P_x=\$5 and I = \$40.

d) Use your demand functions to find his new best bundle. Illustrate the new budget line and the new best bundle that you found above in your diagram for part (c). Be sure to indicate the slopes of both budget lines.

SOLUTION:

By substituting into the demand functions, we have best bundle at $P_x = 5, P_y = 1.25, I = 40$ is $x^* = 2, y^* = 24$.

Diagram see above, blue line represents the new BL. Old slope is -5, new slope is -4.

e) What is the cross price elasticity of hot dogs in this case?

SOLUTION:

Since the demand function for x is not a function of P_y, the cross elasticity is 0.

3. Bruce enjoys both donuts (x) and bagels (y) and his preferences over these two goods can be represented by the utility function U(x,y) = x + 2 ln y where x represents the number of donuts and y represents the number of bagels.

a) **Given Bruce's preferences find his demand functions for donuts (x) and bagels (y).**

SOLUTION:

Tangency: $MRS = \frac{1}{2/y} = \frac{P_x}{P_y}$ implies $y^* = \frac{2P_x}{P_y}$.

Feasibility: $P_x x + P_y y = I$.

Solving the two equations, we have: $x^* = \frac{I}{P_x} - 2, y^* = \frac{2P_x}{P_y}$.

b) **Suppose that the price of a donut is \$1 and that Bruce has \$12 to spend on donuts and bagels. Write Bruce's demand curve for bagels. Illustrate his demand curve.**

SOLUTION:

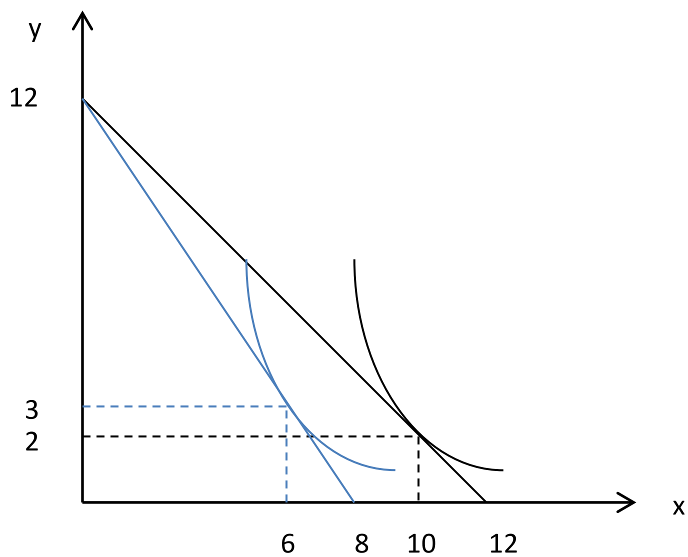
From the demand function in part a, substitute $P_x = 1$, we have the demand curve for bagels: $y^* = \frac{2}{P_y}$.

Graph is similar to that of Question 2 part b.

c) **Suppose that the price of a bagel is $1 (and the price of a donut is 1 and his income is 12). Use your demand functions to find the best bundle. In the indifference curve diagram below illustrate his best bundle at these prices.**

SOLUTION:

By substituting into the demand functions, the best bundle at $P_x = P_y = 1, I = 12$ is $x^* = 10, y^* = 2$.



d) **Suppose that the price of a donut rises to $1.5 (and the price of bagel remains $1 and his income is still $12). Use your demand functions to find his new best bundle. What is his new demand curve for bagels?**

SOLUTION:

By substituting into the demand functions, the best bundle at $P_x = 1.5, P_y = 1, I = 12$ is $x^* = 6, y^* = 3$.

New demand curve for bagels: $y^* = \frac{2P_x}{P_y} = \frac{3}{P_y}$.

e) **Illustrate the new budget line and the new best bundle that you found in part (d) in your diagram for part (c). Illustrate the new demand curve for bagels that you found in part (c) in your diagram for part (b).**

SOLUTION:

See above diagram. Demand curve diagram similar to that of Question 2 part b.

4. Greg snacks on ham (x) and cheese (y) when he watches TV. His preferences over these two goods can be represented by the utility function U(x,y) = min [x, .5y] where x represents the number of pounds of ham and y represents the number of pounds of cheese.

a) **Given his preferences find his demand functions for ham (x) and cheese (y).**

SOLUTION:

"Tangency": $x = 0.5y$ implies $y = 2x$.

Feasibility: $P_x x + P_y y = I$.

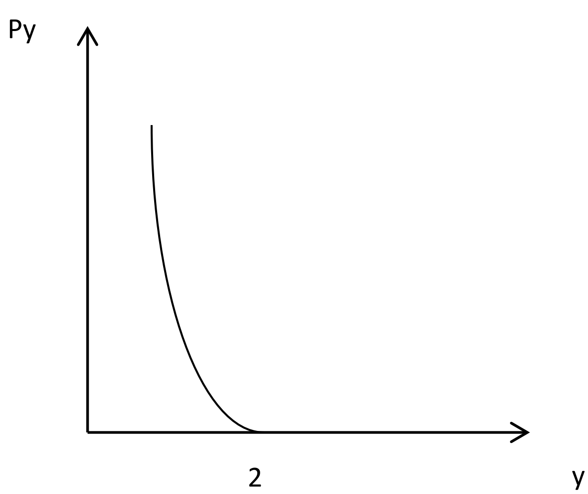
Solving these equations, we have:

$x^* = \frac{I}{P_x+2P_y}$ and $y^* = \frac{2I}{P_x+2P_y}$.

b) **Suppose that the price of a pound of ham is \$8 and that Greg has \$8 to spend on ham and cheese. Write Greg's demand curve for cheese. Illustrate his demand curve.**

SOLUTION:

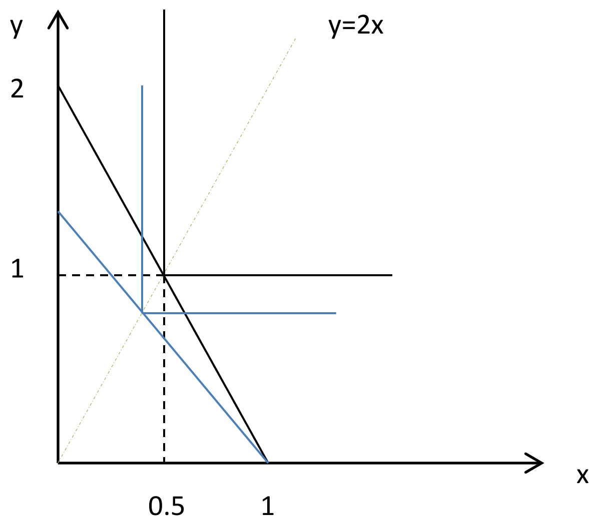
Using the demand function for cheese (y good), substitute $P_x = 8, I = 8$, we have the demand curve $y^* = \frac{8}{4+P_y}$.



c) **Suppose that the price of a pound of cheese is \$4 (the price of ham and income remain \$8 and \$8, resp.). Use your demand functions to find his best bundle. In an indifference curve diagram illustrate his best bundle at these prices.**

SOLUTION:

Substituting into the demand functions, the best bundle is $x^* = 0.5, y^* = 1$.



For the remainder of the question assume that the price of a pound of cheese rises to \$6 and that the price of ham and his income are unchanged at P_x=\$8 and I = \$8.

d) Use your demand functions to find his new best bundle.

SOLUTION:

Plug in the values, we have the new optimal bundle being $x^* = 0.4, y^* = 0.8.$

e) Illustrate the new budget line and the new best bundle that you found above in your diagram for part (c). Be sure to indicate the slopes of both budget lines.

SOLUTION:

See above diagram.

f) What is the cross price elasticity of ham for these preferences? (This question is not graded)

SOLUTION:

Evaluate at $P_x = 8, P_y = 6, I = 8, x^* = 0.4:$

$$cross\ price\ elasticity\ of\ ham = \frac{\partial x^*}{\partial P_y}\cdot\frac{P_y}{x^*} = -\frac{2I}{\left(P_x + 2P_y\right)^2}\cdot\frac{P_y}{x^*} = -0.6$$

g) What is the own price elasticity of cheese? (This question is not graded)

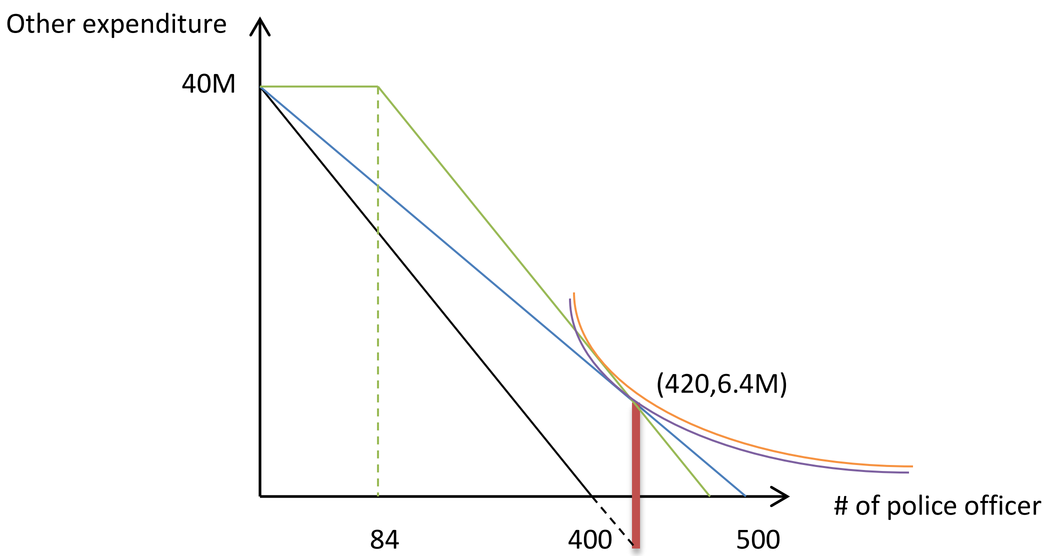
SOLUTION:

$$own\ price\ elasticity\ of\ cheese = \frac{\partial y^*}{\partial P_y} \cdot \frac{P_y}{y^*} == -\frac{4I}{(P_x + 2P_y)^2} \cdot \frac{P_y}{y^*} = -0.6$$

5. Local government officials have a total annual budget for the police department of 40 million dollars ($40,000,000). They must allocate this budget between salaries for police officers ($100,000 per officer per year including benefits) and other expenditures (equipment, vehicles, building maintenance, etc).

a) **Putting the number of officers on the x-axis and expenditure on other goods on the y-axis illustrate the budget set of the local government. What is the opportunity cost of an additional officer in terms of other expenditures?**

<u>SOLUTION</u>:



The opportunity cost of one additional police officer is $100,000 expenditure on other goods.

The federal government wants to lower the crime rate in America. One way to do so is to increase the number of police officers on the street. The federal government may assist local governments to hire more officers by subsidizing the salaries of police officers. Suppose that the federal government

offers to pay \$20,000 of the salary of every police officer in the district so that the cost to the local government of an officer falls to \$80,000.

b) **In a new diagram illustrate the effect of the subsidy plan on the budget line of the local government. Include in your diagram the original budget line. What is the new opportunity cost of police officers in terms of other expenditures?**

   SOLUTION:

   See above diagram, blue represents the new BL. New opportunity cost is \$80,000 expenditure on other goods.

c) **Suppose that under the subsidy plan the local government chooses to hire 420 police officers for the year. How much money is left over for other expenditures? What is the marginal rate of substitution of the local government at this choice (bundle)? What is the total cost to the federal government of this program? Illustrate your answer in a new diagram. In your diagram indicate the cost of the program to the federal government.**

   SOLUTION:

   Money left over = $40{,}000{,}000 - 420 \times 80{,}000 = \$6{,}400{,}000$.

   MRS at optimal bundle = opportunity cost = 80,000.

   Total cost to federal government = $420 \times 20{,}000 = 8{,}400{,}000$.

   See above diagram, red bar represents the cost.

An alternative way to increase the number of police officers is for the federal government to hire officers and provide their services for free to the local government. It costs the federal government \$100,000 per year to hire an officer (local labor contracts prohibit the government from paying lower salaries).

d) **If the federal government wants to spend the same amount on this proposed program as it did on the proposed subsidy program then how many officers can it hire for this local government? Illustrate the budget line associated with this program in a new diagram. Include in your diagram the two previous budget lines. Make sure that you indicate which**

lines have the same slope and if two lines intersect then label the point of intersection. What is the opportunity cost of officers in terms of other expenditures given this proposal?

SOLUTION:

The number of officers hired by the federal government is $8{,}400{,}000 \div 100{,}000 = 84$.

See above diagram, green represents the new BL. The intersection is (420,6.4M). The opportunity cost for the first 84 officers is zero, for the subsequent officers is $100,000.

e) **Will the local government still choose to hire a total of 420 officers (this number includes the officers supplied at no cost by the federal government)? Why or why not? Use a diagram to illustrate your answer.**

SOLUTION:

See above diagram. The local government will NOT choose to hire 420 officers under the new program because there are strictly better bundles as illustrated. The local government, under this new program, will seek to hire fewer police officers than 420. Purple IC represents the highest IC under the first program. Pink IC represents an even higher IC under the new program.

f) **Which of the two proposals (if any) will be preferred by the local government? Why? Which of the two proposals will result in a higher number of officers?**

SOLUTION:

The second proposal will be preferred by the local government, because it can achieve a higher level of utility than before (pink vs. purple IC). However, the first proposal will result in a higher number of officers (i.e. 420 officers) being hired by the local government.

Bonus : **Suppose that $x > 0$ and $y = 0$ at the solution to the consumer's choice problem. Write the first order conditions that characterize this solution. Illustrate it in an indifference curve diagram. Alternatively suppose that $x = 0$ and $y > 0$ at the solution to the consumer's problem. Write the first order conditions that characterize this solution. Use your answer to write the demand functions associated with the utility function $U(x,y) = x + 3y$.**

SOLUTION:

Consider the following general optimization problem, assume the problem is well defined (a concave objective function, all prices are positive and so is income).

$$\max_{(x,y)} U(x, y)$$

Subject to:

1. $P_x x + P_y y \leq I$.
2. $x \geq 0$.
3. $y \geq 0$.

Set up the Lagrangian as:

$$\mathcal{L} = U(x, y) + \lambda(I - P_x x - P_y y) + \mu_x x + \mu_y y$$

where λ, μ_x, μ_y are non-negative Lagrange multipliers.

Then the Kuhn-Tucker Theorem states:

1. $U_x - \lambda P_x + \mu_x = 0$
2. $U_y - \lambda P_y + \mu_y = 0$
3. $\lambda \geq 0,\ I - P_x x - P_y y \geq 0,\ \lambda(I - P_x x - P_y y) = 0$
4. $\mu_x \geq 0,\ x \geq 0,\ \mu_x x = 0$
5. $\mu_y \geq 0,\ y \geq 0,\ \mu_y y = 0$

For $x > 0, y = 0$ to be the solution to this optimization problem, we must have:

$$\mu_x = 0$$

Therefore, from equation 1, we have $U_x = \lambda P_x$. From equation 2, we have $U_y = \lambda P_y - \mu_y$.

Hence, $\frac{U_y}{U_x} = \frac{\lambda P_y - \mu_y}{\lambda P_x} = \frac{P_y}{P_x} - \frac{\mu_y}{\lambda P_x} \leq \frac{P_y}{P_x}$, equality holds when $\mu_y = 0$.

This implies that $MRS = \frac{U_x}{U_y} \geq \frac{P_x}{P_y}$.

Analogously, for $x = 0, y > 0$ to be the solution, we must have $MRS = \frac{U_x}{U_y} \leq \frac{P_x}{P_y}$.

In the case of $U(x, y) = x + 3y$, $MRS = \frac{1}{3}$. For $x > 0, y = 0$ to be the solution, we must have: $\frac{P_x}{P_y} \leq \frac{1}{3}$. In this situation, $x^* = \frac{I}{P_x}, y^* = 0$.

Similarly, when $\frac{P_x}{P_y} \geq \frac{1}{3}$, we have the solution $x^* = 0, y^* = \frac{I}{P_y}$.