Homework #3

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2: Supplementary Exercises

94)

$\{T, R, E\} = \{\text{Transmitter, Relay, Receiver}\} \in N = \text{Nodes}$
R^n: relay n
$N_i, i \in \{1, 2\}$: value transmitted at node, or received in the case of E.

a)

$P(R_1^2 \cap R_1^2 \cap R_1^3 | T_1) = .8^3 = .512$

b)

$P(R_1|T_1) = 3 * (.2^2 * .8) + .8^3 = .608$

c)

$$\begin{aligned} P(T_1|R_1) &= \frac{P(T_1)P(R_1|T_1)}{P(T_0)P(R_1|T_0) + P(T_1)P(R_1|T_1)} \\ &= \frac{.7 * .8^3}{.3(1 - .8^3) + .7 * .8^3} \\ &\approxeq 0.71 \end{aligned}$$

100)

D denotes a disease carrier. T denotes a positive test.
$P(D) = .01; P(T|D) = .9; P(T|D') = .05.$

a)

$$\begin{aligned} P(TT \cup T'T') &= P(TT|D)P(D) + P(TT|D')P(D') + P(T'T'|D)P(D) + P(T'T'|D')P(D') \\ &= .9^2 * .01 + .05^2 * .99 + .1^2 * .01 + .95^2 * .99 \\ &= .90415 \end{aligned}$$

b)

$$\begin{aligned} P(D|TT) &= \frac{P(TT|D)P(D)}{P(TT)} \\ &= \frac{.9^2 * .01}{.9^2 * .01 + .05^2 * .99} \\ &\approxeq .766 \end{aligned}$$

3.1

5)

No. Roll a die until two consecutive rolls are the same. Let X equal the value of the roll. The sample space is infinitely large but X can only take on 6 possible values.

8)

- $Y = 3$: {SSS}
- $Y = 4$: {FSSS}
- $Y = 5$: {SFSSS, FFSSS}
- $Y = 6$: {SSFSSS, SFFSSS, FFFSSS, FSFSSS}
- $Y = 7$: {FFFFSSS,FFSFSSS, FSSFSSS, SSFFSSS, SFSFSSS, SFFFSSS, FSFFSSS}

3.2

14)

a)

$1 = k(1 + 2 + 3 + 4 + 5) = 15k \Rightarrow k = \frac{1}{15}$

b)

$P(y = 1 \cup y = 2 \cup y = 3) = \frac{1+2+3}{15} = \frac{2}{5}$

c)

$P(y = 2 \cup y = 3 \cup y = 4) = \frac{2+3+4}{15} = \frac{3}{5}$

d)

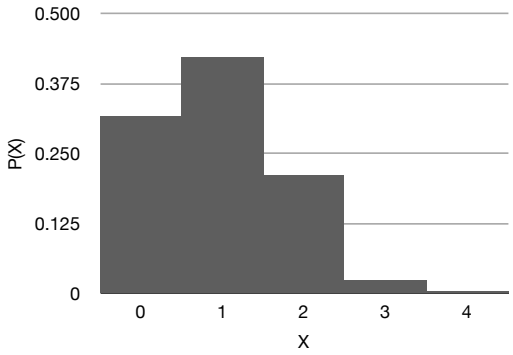
$\sum_{y=1}^{5} \frac{y^2}{50} = \frac{1+2^2+3^2+4^2+5^2}{50} = \frac{55}{50} \neq 1$
$\therefore p(y)$ is not the pmf of Y.

16)

a)

- $P(X = 0) = .75^4$
- $P(X = 1) = .75^3 * .25 * 4$
- $P(X = 2) = .75^2 * .25^2 * 6$
- $P(X = 3) = .75 * .25^3 * 2$
- $P(X = 4) = .25^4$

b)



c)

$X = 1$

d)

$P(X = 2 \cup X = 3 \cup X = 4) = .75^2 * .25^2 * 6 + .75 * .25^3 * 2 + .25^4 = \frac{61}{256}$

28)

Given $x_1 < x_2$:

$$
\begin{aligned}
F(x_2) &= P(X \leq x_2) && (1)\\
&= P(X \leq x_2 \cup x_1 < X \leq x_2) && (2)\\
&= P(X \leq x_2) + P(x_1 < X \leq x_2) && (3)\\
&= F(x_1) + P(x_1 < X \leq x_2) && (4)\\
&\geq F(x_1) && (5)
\end{aligned}
$$

$F(x_1) = F(x_2) \iff P(x_1 < X \leq x_2) = 0.$

3.3)

36)

$E(X) = -.1(1000 - 500) - -.08(5000 - 500) - .02(10000 - 500) = -600$. A premium of \$700 $(100 - -600)$ is required for an expected profit of \$100.

3.4)

60)

$E(X) = .6 * 1 + .4 * 2.5) * 25 = 40$.