Homework #5

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4.2

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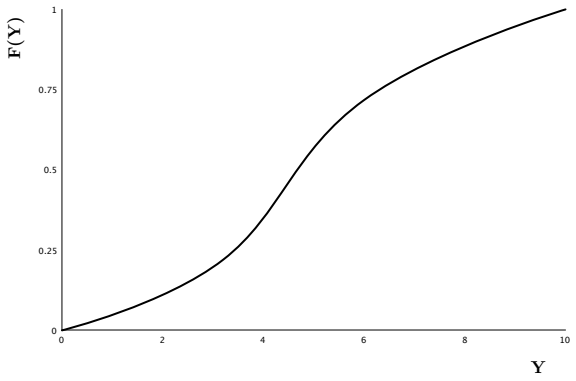
a)

$$F(Y) = \begin{cases} 0 \le y \le 5 & \Rightarrow \int_0^y \frac{y}{25} dy = \frac{y^2}{50} \\ 5 \le y \le 10 & \Rightarrow \int_0^y f(y)dy = \int_0^5 f(y)dy + \int_5^y f(y)dy \\ & = \frac{1}{2} + \int_0^y \left[\frac{2}{5} - \frac{y}{25}\right] dy = \frac{2y}{5} - \frac{y^2}{50} - 1 \end{cases}$$



b)

$0 < p \le .5 \Rightarrow p = F(y_p) = \frac{y_p^2}{50} \rightarrow y_p = \sqrt{50p}$
$.5 < p \le 1 \Rightarrow p = \frac{2y_p}{5} - \frac{y_p}{50} - 1 \rightarrow y_p = 10 - 5\sqrt{2 - 2p}$

c)

$E(Y) = \int_0^5 y\frac{y}{25}dy + \int_5^{10} y\left(\frac{2}{5} - \frac{y}{25}\right) dy = 5$
$V(Y) = \left(\int_0^5 \frac{y^3}{25}\, dy + \int_5^{10} y^2\left(\frac{2}{5} - \frac{y}{25}\right)\, dy\right) - 5^2 = \frac{25}{6}$
For a single bus, the values are simply halved. So: $E(X) = 2.5, V(X) = \frac{25}{12}$

4.3

40

a)

$P(X \leq 40) = P\left(Z \leq \frac{40-43}{4.5}\right) \approxeq 0.2546$
$P(X > 60) = 1 - P(Z < \frac{60-43}{4.5}) \approxeq 1 - 0.999...$

b)

$P(Z < z) = .75 \rightarrow z = .67 \rightarrow .67 = \frac{x-43}{4.5} \Rightarrow x = 46.015$

46)

a)

$$P(67 \leq X \leq 75) = P\left(\frac{67-70}{3} < Z < \frac{75-70}{3}\right) \approxeq .953 - .159 = .794$$

b)

$Z_{.05/2} = Z_{.025} = 1.96; 1.96 * 3 = 5.88.$

c)

$E(RV) = .794 * 10 = 7.94$

d)

$P(X \leq 73.84) = 0.89973$
$P(p = 0.9, n = 10, x = 9) = .387$
$P(p = 0.9, n = 10, x = 10) = 0.349$
$p = 1 - 0.387 - .349 = .264$

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a)

$p(1.72) - p(.55) = .2485$
$p(.55) - p(0)) + (p(1.72) - p(0))$