67/70 + 5

Homework #7

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Question 1

(a)

$f(S, B) = \frac{\min[2S, B]}{12}$ Multiplying S and B by a constant λ, we see that f exhibits constant returns to scale:

$$f(\lambda S, \lambda B) = \frac{\min[2\lambda S, \lambda B]}{12} = \frac{\lambda \min[2S, B]}{12} = \lambda f(S, B)$$

(b)

$\min[S, B]$ s.t. $f(S, B) = \overline{Q}$
$S = 6Q \quad B = 12Q$

Question 2

(a)

$$R = 250Q$$
$$MR = 250$$

(b)

$$VC = Q^2 + 200Q$$
$$MC = 2Q + 200$$

(c)

$$MC = MR \tag{1}$$
$$2Q + 200 = 250 \tag{2}$$
$$Q = 25 \tag{3}$$
$$\Pi(Q = 25) = 250 * 25 \quad (25^2 + 200 * 25) = 625 \tag{4}$$

(d)

$$P_D = 1.5 \Rightarrow \Pi = 1875$$
$$P_D = 3 \Rightarrow \Pi = -1875$$

(e)

$V_E = \frac{1}{3} * 1 + \frac{2}{3} * 0 = \frac{1}{3}$

(f)

$1875 - \frac{1}{3}n = -1875 + n - \frac{1}{3}n \Rightarrow n = 3750. \Pi = 625$

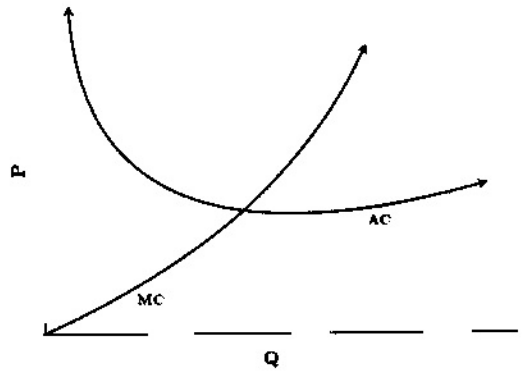
Question 3

(a)

$$\min[64x + y + 8] \text{ s.t. } 8x^{\frac{1}{3}} = \overline{Q} \Rightarrow x = \left(\frac{\overline{Q}}{8}\right)^3$$
$$VC = 64x + y = 2Q^3$$
$$TC = 64x + 8 = \frac{Q^3}{8} + 8$$

(b)

$$MC = \frac{3}{8}Q^2$$
$$AC = \frac{\frac{Q^3}{8} + 8}{Q}$$
$$MC = AC \Rightarrow Q = \sqrt[3]{32}$$



(c)

$$MRTS = \frac{\frac{4}{3}x^{-\frac{2}{3}}y^{\frac{1}{3}}}{\frac{4}{3}x^{\frac{1}{3}}y^{-\frac{2}{3}}} = \frac{y}{x} = 64 \Rightarrow y = 64x \quad (1)$$

$$\overline{Q} = 4x^{\frac{1}{3}}y^{\frac{1}{3}} = 4x^{\frac{1}{3}}(64x)^{\frac{1}{3}} \quad (2)$$

$$x = \frac{\overline{Q}^{\frac{3}{2}}}{64}, y = \overline{Q}^{\frac{3}{2}} \quad (3)$$

$$VC = 64x + y = 2Q^{\frac{3}{2}} \quad (4)$$

$$TC = VC \quad (5)$$

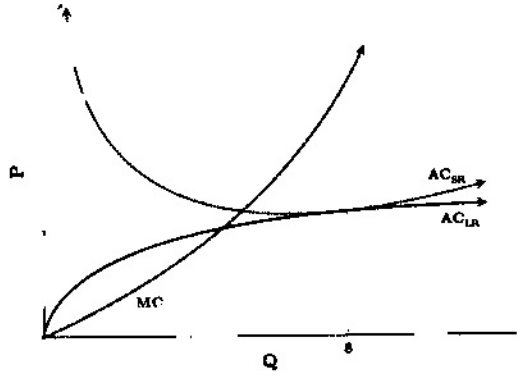
(d)

$MC = 3\sqrt{Q}$ $AC = 2\sqrt{Q}$

(e)

$8 = Q^{\frac{3}{2}} \Rightarrow Q = 4$. The long run and short run average costs must therefore be equal.

(f)



Question 4

10/10

(a)

$$MRTS = \frac{\frac{\partial Q}{\partial L}}{\frac{\partial Q}{\partial K}} = \frac{K}{L} = \frac{16}{4} = 4 \Rightarrow K = 4L \quad (1)$$

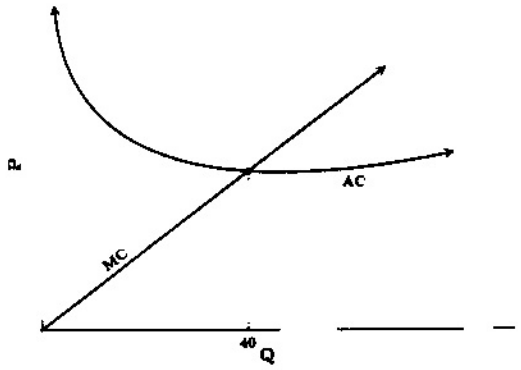
$$\overline{Q} = 20L^{\frac{1}{2}}(4L)^{\frac{1}{2}} = 20\sqrt{2L} \quad (2)$$

$$L = \frac{Q^2}{800}, K = \frac{Q^2}{200} \quad (3)$$

$$C = 16L + 4k = \frac{Q^2}{50} + \frac{Q^2}{50} = .04Q^2 \quad (4)$$

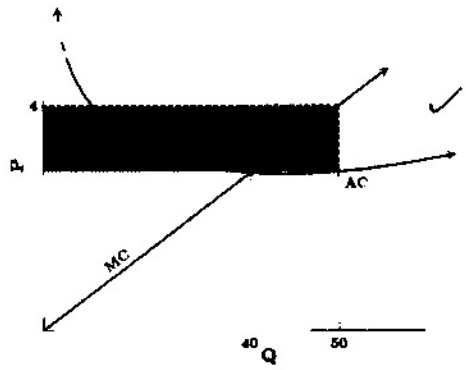
(b)

$AC = \frac{.04Q^2+64}{Q}$ $MC = .08Q$ $MC = AC \Rightarrow Q = 40$ The firm exhibits an scale economies where $Q < 40$



(c)

$Q_d(4) = 7600 \Rightarrow .08Q = 4 \Rightarrow Q = 50$ The firm earns supernormal profits.



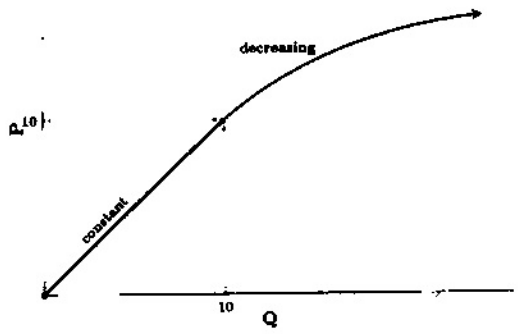
(d)

No $40 * .08 = 3.2$ is the long run market clearing price $Q_s = 40$ $Q_d = 9200 - 400 * 3.2 = 7920$ $\pi = \frac{7920}{40} = 198$

Question 5

29/30

(a)



(b)

$L = Q$ for $Q \leq 10$. $Q = 10 + \sqrt{L - 10} \Rightarrow L = (Q - 10)^2 + 10$ for $Q > 10$ (1)

$C(L) = 2L$ (2)

$VC = 102Q$ (3)

TC for $Q \leq 10 = 102Q + 230$ (4)

TC for $Q > 10 = 2Q^2 + 60Q + 450$ (5)

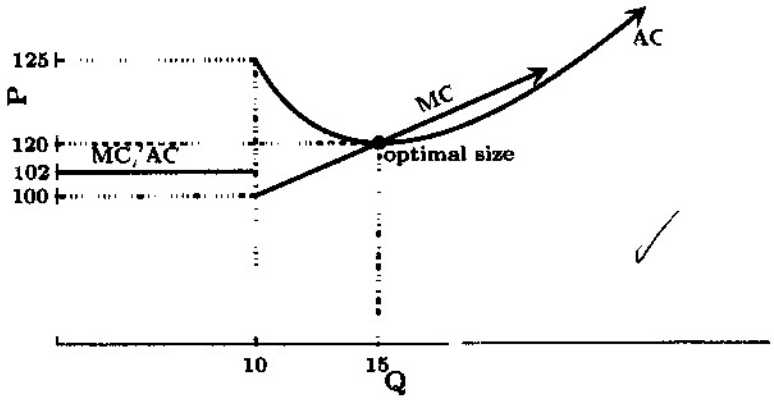
MC for $Q \leq 10 = 102$ (6)

MC for $Q > 10 = 4Q + 60$ (7)

AC for $Q \leq 10 = 102$ (8)

AC for $Q > 10 = \frac{2(Q + 15)^2}{Q}$ (9)

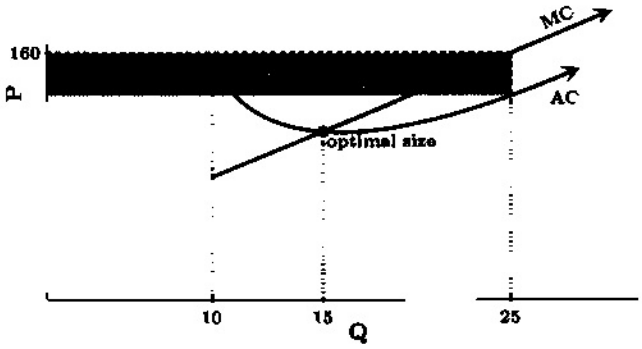
(c)



(d)

$P = 4Q_s + 60 \Rightarrow Q_s = \frac{P-60}{4}$

$Q_s(160) = 25$. The firm earns supernormal profits at this price point



(e)

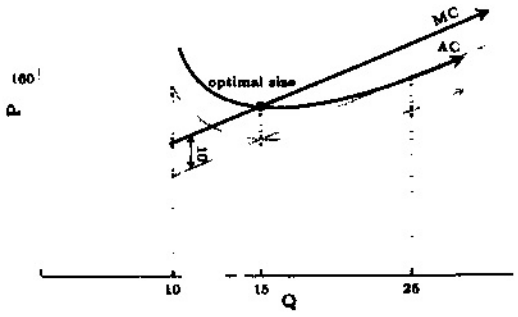
Firms will exit and the price will fall such that $Q = 15$ and in turn $P = 120$

(f)

$P = 120, Q = 270, n = \frac{270}{15} = 18$

(g)

$$VC = 2Q^2 + 70Q + 220$$
$$MC = 4Q + 70$$
$$AC = 2Q + \frac{250}{Q} + 70$$
$$MC = AC \Rightarrow Q = 15$$



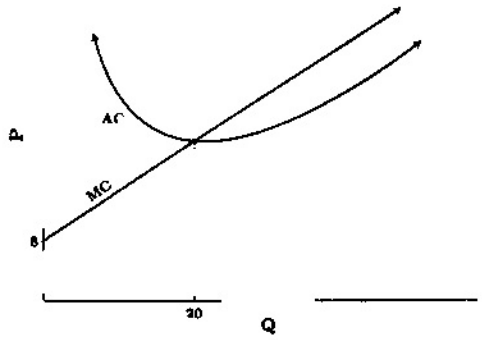
(h)

$Q = 15, P = 130$ $Q_d = 390 - 130 = 260$ $n = \frac{Q_d}{Q} = \frac{260}{15} = 17\overline{3} < 18$ Firms will exit the market

Question 6

(a)

$AC = \frac{2Q^2+5Q+80}{Q}$ $MC = 4Q + 5$ $AC = MC \Rightarrow Q = 20$

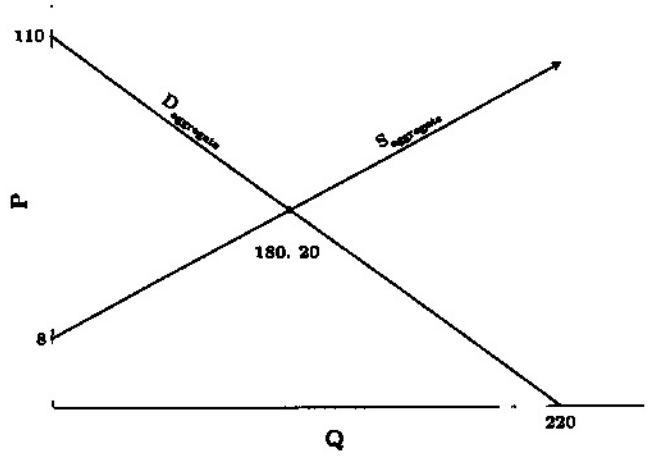


(b)

$Q_s = 2.5(P - 5)$ $Q_{s_{aggregate}} = 15(P - 5)$

(c)

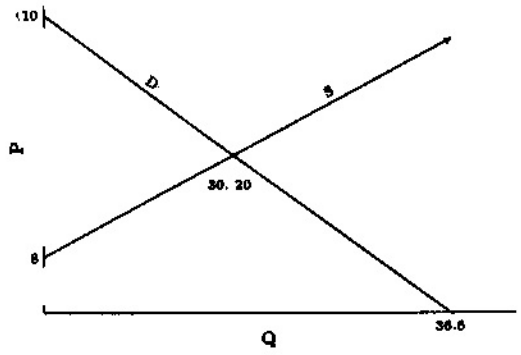
$220 - 2P = 15(P - 8) \Rightarrow P = 20$



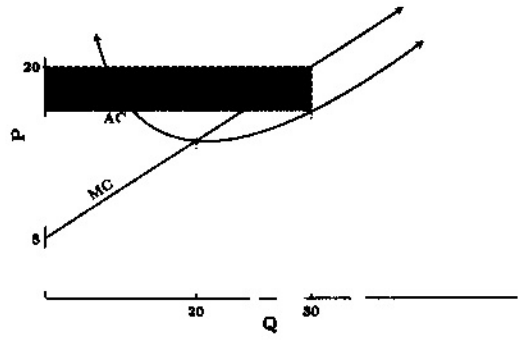
(d)

$Q_s = 15(20 - 8) = 30$ Finding elasticity

$$\eta_d = \frac{\partial Q_d}{\partial P}\frac{P}{Q_d} = -2\frac{20}{\frac{180}{6}} = -\frac{4}{3}$$



(e)

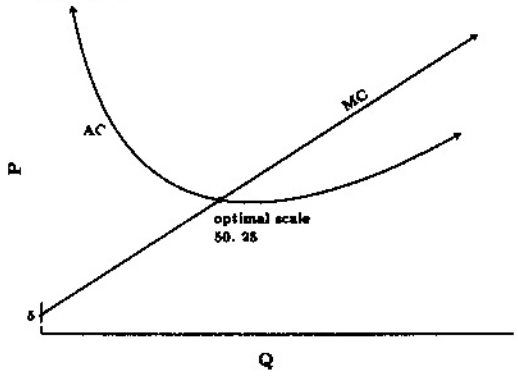


Question 7

(a)

$$AC = \frac{2Q^2 + 5Q + 500}{Q}$$
$$MC = 4Q + 5$$
$$AC = MC \Rightarrow Q = 50$$

Plugging $Q = 50$ into MC we find that the shutdown point is $P = 25$.

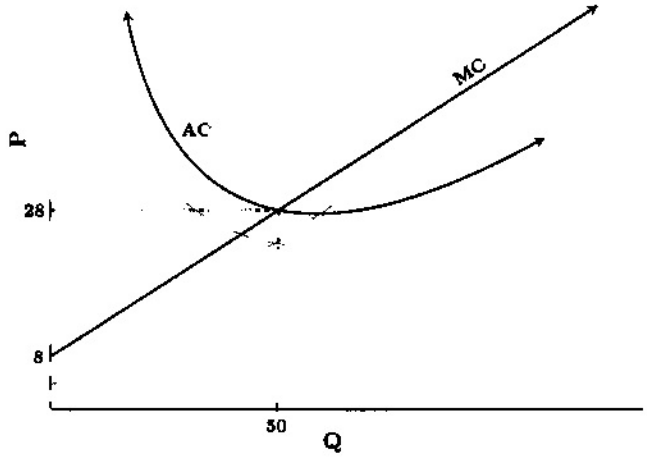


(b)

$MC(Q = 80) = 25. Q_d(25) = 300 \frac{300}{50} = 6$.

(c)

$MC = 4Q + 8. AC = \frac{2Q^2+8Q+800}{Q}$ $AC = MC \Rightarrow Q = 50$.



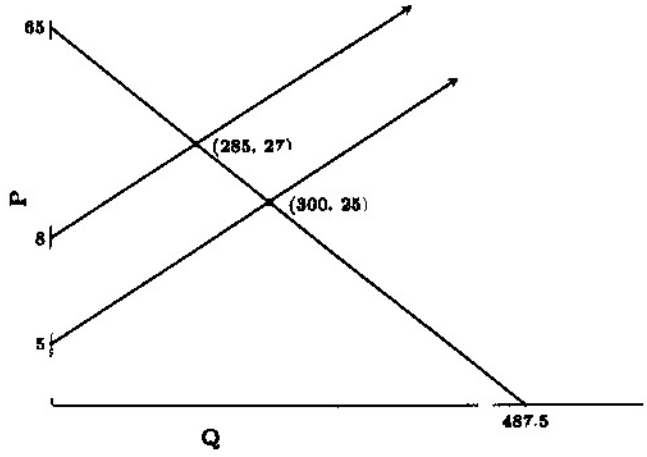
(d)

$P = 4Q + 8 \Rightarrow Q = 2.5(P - 8)$ $Q_{s_{aggregate}} = 15(P - 8)$

(e)

$15(P - 8) = 487.5 - 7.5P \Rightarrow P = 27, Q = 285$

(f)



(g)

Exit Dividing the aggregate demand (285) by the optimal quantity supplied by each firm (50), we find that $n = 5.7 < 6$ Firms would therefore exit the market

Question 4 Bonus

+5

$f(L,K) = (LK)^{1/4}$, $w = 10$, $r = 25$

$[\min 10L + 25K]$ s.t. $\bar{Q} = (LK)^{1/4}$

FOC

$10 - \frac{1}{4}\lambda L^{-3/4} K^{1/4} = 0$

$25 - \frac{1}{4}\lambda L^{1/4} K^{-3/4} = 0$

$Q = (LK)^{1/4}$

$\frac{10}{2.5} = \frac{K}{L}$, $K = 4L$

$Q = (4L^2)^{1/4} \rightarrow Q^4 \; 4L^2$

$Q^2 = 2L \rightarrow \boxed{L \; \frac{Q^2}{2}}$ ✓

$\boxed{K = 2Q^2}$ ✓

$C(Q) = 5Q^2 + 5Q^2 = 10Q^2$

$MC(Q) = 20Q$

$10 = \frac{1}{4}\lambda \left(\frac{Q^2}{2}\right)^{3/4} (2Q^2)^{1/4}$

$10 = \frac{1}{4}\lambda \left(\frac{1}{2}\right)^{-3/4} (2)^{1/4} (Q^2)^{-3/4} (Q^2)^{1/4}$

$10 = \frac{1}{2}\lambda (Q^2)^{-1/2}$

$20 = \frac{\lambda}{Q} \rightarrow \boxed{\lambda = 20Q}$ ✓

✓

The lagrange multiplier is equal to the marginal cost curve