

Sample Problems for the Final Exam

NOTE: This is not the format of the final – the final exam will have two parts. Part One will consist of 4 required questions and Part II will consist of 2 questions and you will have to select one of them.

1. Suppose that the following production function represents a firm's ability to manufacture pencils: $f(L,K) = 3^{1/2}K^{1/4}L^{1/4}$.
- Show that this production function exhibits decreasing returns to scale.
 - Assume that the wage rate is equal to 9 and the rental rate on capital is equal to 4. In addition assume that the firm has a fixed cost of production equal to 16. Find the compensated factor demands for labor and capital. Find the cost of the cost minimizing input bundle (ie find the variable cost curve) and the total cost curve.
 - Find the marginal cost function and the average cost function. Graph the marginal cost function and the average cost function in a single diagram. Show that the optimal size of the firm is 2. What is the minimum value of the average cost curve?
 - If this firm is a price taker then what is the firm's supply curve?
 - Suppose that there is a second firm in the pencil business, which has an identical total cost function and that these two firms are the only two firms in the industry then what is the industry supply curve?
 - If the demand for pencils is given by the inverse market demand curve $P = 100 - Q$ then what is the market clearing price and quantity for the pencil market?
 - Do you expect that there will be entry or exit in the long run? What will be the long run price of pencils? How many units will be traded in the long run?
2. The bolt industry currently consists of 20 producers, all of whom operate with identical total cost functions given by $TC(Q) = 16 + Q^2$. Quantity is measured in tons of bolts.

- What is the marginal cost curve associated with this total cost curve? What is the average cost curve? What is the optimal size of the firm? What is the minimum value of the average cost curve? Illustrate your answer in a diagram.
- What is the individual firm's supply curve? What is the market supply curve?

Suppose that demand for bolts is given by $Q^D = 132 - P$. Price is measured in dollars per ton.

- What will be the market clearing price and quantity traded in the short-run? How many units will each firm supply to the market? Will firms be earning normal, super-normal or sub-normal profits? Briefly explain your answer without calculating the firm's level of profits.
- What is the long-run price of bolts? Do you predict that there will be entry or exit from this industry?
- What is the long-run quantity traded in this market? How many units will each firm produce in the long-run? How many firms will there be in the long-run?

3. The current price of a pound of peanuts is \$1. At that price consumers are purchasing 900 million pounds of peanuts. If we measure peanuts in units of 100 million pounds then

the current demand is 9. Finally assume that the estimated own price elasticity of demand at (9,1) is $(-4/9)$ and that the own price elasticity of supply is $(4/3)$. The government would like to increase the income of peanut farmers. It is considering adopting one of the two following programs to increase farm income:

1) Deficiency Payment Program: In this program the government promises peanut farmers that they will receive \$1.10 per pound of peanuts regardless of the price in the market. Specifically farmers sell their peanuts in the market and the government makes up the difference between the price that the farmers receive per pound sold in the market and \$1.10 per pound. Hence the cost to the government of this program is equal to the product of the quantity sold and the difference in prices.

2) Peanut Purchase Program: The government pledges to purchase peanuts in the market until the price of peanuts rises to \$1.10 per pound. Hence the total cost to the government is the product of the number of pounds of peanuts purchased and \$1.10.

- a) Show that under both programs peanut farmers supply the same amount of peanuts. Conclude that peanut farmers are indifferent between the two programs (that is peanut farmers receive exactly the same amount of money under the two programs).
- b) Evaluate the cost to the government of each of these programs. Which one is cheaper for the government?
- c) Evaluate the total **social** cost of each of these programs. Specifically evaluate the costs and benefits of each of these programs to consumers and farmers. Then add these costs and benefits to the government's expenditures to determine the social cost of the each program. Which one has the lowest social cost?

4. The demand and supply of soybeans in the United States is given by the following: $Q_{US}^D = 218 - 4P$ and $Q_{US}^S = -3 + .25P$ where quantity is measured in millions of tons and price is measured in dollars per ton.

- a) What is the market-clearing price and quantity of soybeans in the US? Illustrate your answer in a diagram.

In an effort to raise the income of farmers the government would like to raise the price of soybeans. One way to raise the price of soybeans would be to restrict the supply of soybeans. In order to decrease the supply of soybeans the government forces farmers to cut back on their production of soybeans to only 8 million tons.

- b) In a new diagram illustrate (no numbers) the impact of the quota on the price charged in the market and the quantity traded. At this price is there excess supply or excess demand?
- c) In your diagram for (b) illustrate the changes in producer surplus caused by the quota. What are the two changes in producer surplus? Which of these changes increase the income of farmers? Which of these changes decreases the income of the farmers?
- d) What is the new price and quantity traded for soybeans after the quota is imposed?
- e) What is the dollar value of the transfer from consumers to farmers? What is the deadweight loss of farmers? What is the net effect of the quota on the income of farmers? Was the governments program successful at raising the income of farmers?

Instead of imposing a quota on the farmers the government decides to levy a tax on the consumers of soybeans and to distribute all of the revenues from that tax to the farmers.

- f) Suppose that the tax is \$8.5 per ton of soybeans. Illustrate the impact of this tax in a supply and demand curve diagram. Will this tax plan raise the income of farmers more than the quota? Briefly explain your answer.

5. Sheila and Bruce are going to the beach. Sheila is bringing 12 ounces of soda (x) and 10 bags of chips (y). Bruce is bringing 8 ounces of soda and 8 bags of chips. Sheila's and Bruce's preferences over soda and chips can be represented by the utility functions:

$$U^S(x^S, y^S) = \ln x^S + 3 \ln y^S \text{ and } U^B(x^B, y^B) = \ln x^B + \ln y^B.$$

- a) What is Sheila's MRS at her endowment? What is Bruce's MRS at his endowment? Is the endowment point Pareto efficient? Illustrate in the Edgeworth box diagram the endowment point and a pair of indifference curves through the endowment point.

Instead of trading directly with each other Sheila and Bruce trade with a market. Sheila and Bruce are both price takers on both the soda and chip markets. Each can buy and sell a good at the same price. Let P_x be the price of an ounce of soda and P_y be the price of a bag of chips.

- b) Write the two equations that define the best bundle for Sheila given that the prices are P_x and P_y . Do the same for Bruce.
c) Using your answer for part (b) find Sheila's best bundle when $P_x = 5$ and $P_y = 6$. Is Sheila a net demander or a net supplier of chips (y)? Find Bruce's best bundle at the same prices. Show that when $P_x = 5$ and $P_y = 6$ neither market clears. Which market has excess supply? Illustrate the budget line associated with these prices in your diagram for part (a). Indicate the best bundles for Sheila and Bruce on this budget line.
d) Using your answer for part (b) find Sheila's best bundle when $P_x = 1$ and $P_y = 2$. Show that when $P_x = 1$ and $P_y = 2$ both markets clear. Illustrate the budget line associated with the general equilibrium prices in a new Edgeworth box diagram. Indicate the best bundles on the budget line and the endowment point.

6. Sheila and Bruce are on a hike. Sheila has brought with her 8 bottles of water (x) and 5 granola bars (y). Bruce has brought 4 bottles of water and 8 granola bars. Sheila and Bruce have preferences that can be represented by the following utility functions:

$$U^S(x^S, y^S) = \ln x^S + \ln y^S \text{ and } U^B(x^B, y^B) = \ln x^B + 2 \ln y^B.$$

- a) Given their preferences is the endowment point a Pareto efficient allocation of water and granola bars?
b) Given their marginal rates of substitution at the endowment point who values water more highly? What will be the pattern of trade associated with every mutually beneficial trade? Find one trade that will make both Sheila and Bruce strictly better off.
c) Illustrate in an Edgeworth box diagram the endowment point, a pair of indifference curves passing through the endowment point and the trade that makes both of them better off (from part (b)).
d) An allocation is envy free if Bruce prefers his bundle to Sheila's bundle and Sheila prefers her bundle to his. Show (algebraically) that the endowment point is an envy free allocation. Illustrate graphically in your Edgeworth box diagram that the endowment is envy free. If you cannot illustrate it in the diagram that you have already drawn then sketch a new

Edgeworth box below.

7. The country of Utopia is a closed economy so it does not trade with the world economy. HD Chemicals is the only producer of Dexinitrite, an important additive to fertilizers, in Utopia. The demand for Dexinitrite in Utopia is given by the demand curve: $Q = 180 - 2P$ where P is the price per ton of Dexinitrite and Q is measured in tons. The cost of producing Dexinitrite is given by $C = .25Q^2 + 400$.

- a) Given that HD Chemicals is a monopolist what price will it charge for Dexinitrite? What quantity will it sell? What will be its profits? Illustrate your answer in a diagram.

Utopia decides to open some of its markets to trade. To protect HD Chemicals the market for Dexinitrite will not be opened to trade. However, HD Chemicals will be allowed to sell Dexinitrite on the world market. The world market for Dexinitrite is a perfectly competitive market and the current world price of Dexinitrite is \$50 per ton. HD Chemicals can sell as much Dexinitrite as it would wish to the world market at the price of \$50 per ton and can choose any price that it would wish in its home market in Utopia (so it can price discriminate).

- b) Given that HD Chemicals can price discriminate between its domestic and foreign customers then what price will it charge to its home customers? How much Dexinitrite will it sell at home? How much Dexinitrite will it export? Are domestic customers better or worse off now that HD Chemicals can export Dexinitrite? What are the profits of HD Chemicals? Is HD Chemicals better off now that it can export?
- c) Illustrate your answer to (b) in a diagram similar to the one in (a) that is a diagram showing the domestic demand for Dexinitrite and the marginal cost of production. Indicate the domestic sales of HD Chemicals and the foreign sales of HD Chemicals.
- d) If the world price of Dexinitrite falls to only \$44 per ton then will HD Chemicals change the domestic price of Dexinitrite? How much will it sell at home? How much will it export at this price? Illustrate this answer in the diagram in part (c).
- e) What is the lowest world price of Dexinitrite at which HD Chemicals will choose to export Dexinitrite? Is there a world price such that at that world price HD Chemicals will supply no Dexinitrite to its Utopia market?
- f) Illustrate the export supply curve of HD Chemicals.

8. Suppose that the total cost curve for a typical firm in the olive oil industry is $TC(Q) = .05Q^2 + 15Q$. Quantity (Q) is measured in millions of gallons per year.

- a) What is the supply curve of a typical firm in the olive oil industry? If there are 10 firms in the industry each of which has the typical cost curve then what is the industry supply curve of olive oil?

Demand per year for olive oil is $Q^D = 600 - 100P$. Quantity (Q) is measured in millions of gallons per year and price (P) is measured in dollars per gallon.

- b) If the market is perfectly competitive then what will be the market clearing price and quantity traded in the olive oil market? Illustrate your answer in a market supply and demand diagram. How many gallons will each firm supply? Illustrate your answer in a diagram of individual firm supply curve and individual inverse firm demand curve.
- c) If the olive oil producers formed a cartel and decided to choose a price and quantity that would maximize industry profits then what price and quantity will the cartel choose? Illustrate your answer in your diagram from part (b).

9. Suppose that there are two firms, upstream and downstream, such that the upstream firm is the sole supplier of an input, K , to the downstream firm and the downstream firm is the only buyer of the input from the upstream firm. The downstream firm sells its output in a perfectly competitive market at price $p = 16$. The downstream firm also acts as a price taker on the input market for K . The upstream firm on the other hand acts as a monopolist in the market for K . Suppose that the downstream firm's production function is given by $x = K^{1/2}$ and it has no fixed costs of production. Suppose that the upstream firm's cost function is given by $C(K) = K$.

- a) Find the downstream firm's demand curve for the input K as a function of the price of K . (Hint you may first find the compensated demand for K and the cost function for the downstream firm. Using this cost function find the profit-maximizing level of output. Finally substitute the level of output into the compensated factor demand).
- b) Given the downstream firm's demand curve find the quantity of K that maximizes the upstream firm's profits. Find the price associated with this quantity of K .
- c) Find the profits of each firm.
- d) If the downstream firm had the opportunity to purchase the upstream firm at a price 16, would it do so?