

## STAT W1211.001 Mid-term Examination Practice Problems

### Part I: Multiple Choice

- Which of the following is true for events  $M$  and  $N$ ?
  - $(M \cup N)' = M' \cap N'$
  - $(M \cap N)' = M' \cup N'$
  - Neither  $A$  nor  $B$  is true
  - Both  $A$  and  $B$  are true
- Which of the following statements are true?
  - The unit for the standard deviation is the same as the unit for the data values.
  - The average deviation of  $x_1, x_2, \dots, x_n$  from the mean  $\bar{x}$  may be positive or negative.
  - The average absolute deviation of  $x_1, x_2, \dots, x_n$  from the mean  $\bar{x}$  is always zero.
  - We use  $\sigma^2$  (square of the lowercase Greek letter sigma) to denote the sample variance.
- How many permutations of size 3 can be constructed from the set  $\{A, B, C, D, E\}$ ?
  - 60
  - 20
  - 15
  - 8
- If  $P(A) = .30$ ,  $P(B) = .10$  and events  $A$  and  $B$  are independent, then  $P(A' \cap B')$  is
  - .63
  - .70
  - .90
  - .40
- Which of the following statements are correct if  $y_i = x_i + 5 \forall i = 1, 2, \dots, n$ ?
  - $\bar{y} = \bar{x}$
  - $\bar{y} = \bar{x} + 5$
  - $s_y^2 = s_x^2 + 5$
  - $s_y^2 = s_x^2 + 25$
  - None of the above

6. Which of the following assumptions does not lead to the hypergeometric distribution?
- The population or set to be sampled consists of  $N$  individuals, objects, or elements (a finite population).
  - Each individual can be characterized as a success ( $S$ ) or a failure ( $F$ ), and there are  $M$  successes in the population.
  - A sample of  $n$  individuals is selected without replacement in such a way that each subset of size  $n$  is equally likely to be chosen.
  - All of the above are assumptions leading to the hypergeometric distribution.
7. The cumulative distribution function  $F(x)$  of a discrete random variable  $X$  is given by  $F(0) = .30$ ,  $F(1) = .70$ ,  $F(2) = .90$ , and  $F(3) = 1.0$ , then the value of the probability mass function  $p(x)$  at  $x = 1$  is
- .30
  - .40
  - .20
  - .80
8. If the probability density function of a continuous random variable  $X$  is

$$f(x) = \begin{cases} .5x & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

then,  $P(1 \leq x \leq 1.5)$  is

- .5625
  - .3125
  - .1250
  - .4375
9. Which of the following statements about the  $z_\alpha$  notation used in your book is correct? (Page 156, 8<sup>th</sup> edition)
- $\alpha$  of the area under the standard normal curve lies to the left of  $z_\alpha$ .
  - $1-\alpha$  of the area under the standard normal curve lies to the right of  $z_\alpha$ .
  - $z_\alpha$  is the  $100(1-\alpha)$ th percentile of the standard normal distribution
  - $z_\alpha$  is the  $100\alpha$ th percentile of the standard normal distribution

10. If  $X$  is a normally distributed random variable with a mean of 10 and standard deviation of 4, then the probability that  $X$  is between 6 and 16 is
- A. .9332
  - B. .7745
  - C. .6587
  - D. .0668
  - E. .8413
11. A continuous random variable  $X$  is uniformly distributed over the interval  $[12, 18]$ . The variance of  $X$  is
- A. 15
  - B. 6
  - C. 3
  - D. 9

*Part II: Structured Questions*

1. Let  $X$  denote the amount of space occupied by an article placed in a 1-ft<sup>3</sup> packing container. The pdf of  $X$  is

$$f(x) = \begin{cases} 90x^8(1-x) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

- a. Obtain the cdf of  $X$ .
- b. What is  $P(X \leq .5)$  [i.e.,  $F(.5)$ ]?
- c. What is  $P(.25 < X \leq .5)$ ?
- d. Compute  $E(X)$  and  $\sigma_x$ .
- e. What is the probability that  $X$  is within 1 standard deviation of its mean value?

2. Seventy percent of all vehicles examined at a certain emissions inspection station pass the inspection. Assuming that successive vehicles pass or fail independently of one another, calculate the following probabilities.
- $P(\text{all of the next three vehicles inspected pass})$
  - $P(\text{exactly one of the next three inspected passes})$
  - $P(\text{at most one of the next three vehicles inspected passes})$
  - Given that at least one of the next three vehicles passes inspection, what is the probability that all three pass?
3. a. A 10 meter by 10 meter plot of land, is divided into a grid of 100 squares. 300 seeds are scattered on this plot. Assume that each seed falls at random, so that it is equally likely to fall on any of the 100 squares. Consider the square in the upper left hand corner. What is the probability that exactly 4 seeds fall in it? What is the probability that 0 seeds fall in it?
- A certain nucleus contains  $10^6$  genes. Each gene has probability  $3.2 \times 10^{-6}$  of being mutated. What is the probability that the nucleus contains at most 2 mutated genes?
  - How many people are needed so that the probability that at least one of them has the same birthday as you is greater than 0.5?
4. Suzanne is getting ready to take an important oral examination and is concerned about the possibility of having an “on” day or an “off” day. She figures that if she has an “on” day, then each of her examiners will pass her, independently of each other, with probability 0.8, whereas if she has an “off” day, this probability will be reduced to 0.4. Suppose that Suzanne will pass the examination if a majority of the examiners pass her. If she feels that she is twice as likely to have an “off” day as she is to have an “on” day, should she request an examination with three examiners or with five examiners?