1 Multiple Choice

1. Which of these are resistant measures of center and/or spread?

   (a) median
   (b) standard deviation
   (c) range
   (d) IQR
   (e) a) and (d)
   (f) a), (b), and (d)

   Ans: (e). Median and range are resistant. Standard deviation and range are not.

2. Which of the following are sufficient for completely describing a distribution

   (a) pmf or pdf
   (b) cdf
   (c) family and parameters
   (d) mean and variance
   (e) a), (b), or (c)
   (f) any of the above

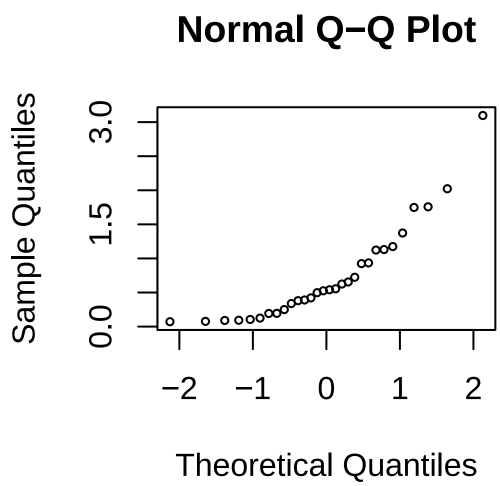
   Ans: (e). Mean and variance are not sufficient. Different distributions can have the same mean and variance; for example, consider Binomial$(3, .05)$ and $N(1.5, \sqrt{0.75})$.

3. Consider the following Normal Q-Q plot. The sample comes from what type of distribution?

   (a) normal distribution
   (b) heavy-tailed distribution
   (c) light-tailed distribution
   (d) right-skewed distribution
   (e) left-skewed distribution

   Ans: (d). High sample values are bigger than the theoretical quantiles and low sample values are also bigger than their corresponding quantiles. So this is a right-skewed distribution.

4. Suppose I have a sample $x_1, \ldots, x_n$ with sample standard deviation 4. Suppose $y_i = -2x_i$ for $i = 1, \ldots, n$. What is the standard deviation of $y_1, \ldots, y_n$?



(a) 4
(b) -4
(c) 8
(d) -8
(e) 16

Ans: (c). $\sigma_Y = 2\sigma_X = 8$**.**

5. Suppose someone guesses on all the previous questions. What distribution would his or her number of correct answers follow?

(a) discrete uniform
(b) hypergeometric
(c) binomial
(d) geometric
(e) none of the above

Ans: (e). Since some questions have 6 choices and some 5, we do not have constant p**.**

6. Suppose $X \sim N(1, 2)$ and $Y \sim N(0, 1)$ are independent random variables. What is the probability that $\min(X, Y) < -1$?
(Hint: How would you write this in set notation?)

(a) 0.025

(b) 0.29

(c) 0.32

(d) 0.53

(e) 0.63

Ans: (b). $P(\min(X, Y) < -1) = P(X < -1$ **or** $Y < -1)$**. This is the union of two events, so**

$$P(\min(X, Y) < -1) = F_X(-1) + F_Y(-1) - F_X(-1)F_Y(-1)$$

Apply the 68-95-99.7 rule to get $F_X(-1) = F_Y(-1) = (1 - 0.68)/2 = 0.16$ **and**

$$P(\min(X, Y) < -1) = 0.32 - 0.16^2 = 0.32 - 0.0256 \approx 0.29$$

2 Word problems

1. A factory procures parts from 3 different suppliers S_1, S_2, and S_3. Half of the parts come from S_1, a third from S_2, and the remaining proportion from S_3. Parts from S_1 are defective with probability 0.1, parts from S_2 are defective with probability 0.2, and parts from A_3 are defective with probability 0.3.

   (a) Draw a tree diagram for this problem.

   (b) What is the total probability that a part will be defective?
   Ans: Apply the Law of total probability.
   $$\frac{1}{2}\cdot\frac{1}{10}+\frac{1}{3}\cdot\frac{1}{5}+\frac{1}{6}\cdot\frac{3}{10}=\frac{1}{6}$$

   (c) If an item is defective, what is the conditional probability that it came from the supplier S_1?
   Ans: (1/20)/(1/6) = 6/20 = 3/10.

   (d) Our factory's product will work as long as at least 8 of its 10 parts are in working order. What is the probability that one of our products is broken due to defective parts?
   Ans: Let $X \sim$ Bin$(100, 1/6)$ be the number of broken parts. We need
   $$1 - P(X \leq 2) = 1 - \sum_{k=0}^{2}\binom{10}{k}\left(\frac{1}{6}\right)^{k}\cdot\left(\frac{5}{6}\right)^{10-k} \approx 0.22$$

   (e) We introduce some redundancy in order to improve the reliability of our product. Now it will continue working as long as at least 60 of its 100 parts are not defective. What is the probability that one of our products is broken due to defective parts?
   (Hint: We can use a binomial approximation here.)
   Ans: Use a normal approximation with mean $\mu = 100/6$ and standard deviation $\sigma = \sqrt{500/36}$. The probability that one of our products is broken is approximately $1 - \Phi((40.5-\mu)/\sigma) = 1 - \Phi(6.40) \approx 0$.

   (f) Our improved product passes quality control only if a batch of 250 products has no more than 5 broken products. What is the probability that a batch passes quality control?
   (Hint: We can use another binomial approximation here.)
   Ans: This problem was supposed to be done with a Poisson approximation. The expected number of broken products is $\lambda = 250\cdot p$, where p is your answer from e. Then
   $$P(X \leq 5) = \sum_{x=0}^{5}\frac{\lambda^{x}e^{-\lambda}}{x!}$$

2. Let X be a continuous random variable with probability density function

$$f_X(x) = \begin{cases} k/x^4 & x > 1 \\ 0 & \text{otherwise} \end{cases}.$$

(a) Find the value of k.

Ans: Since

$$1 = \int_{-\infty}^{\infty} f_X(x)dx = \int_1^{\infty} \frac{k}{x^4} dx = \left[-\frac{k}{3x^3} \right]_1^{\infty} = \frac{k}{3},$$

$k = 3$**.**

(b) What is the cumulative density function $F_X(x)$? Remember to define $F_X(x)$ for all values of $x \in (-\infty, \infty)$.

Ans: For $x < 1$**,** $F_X(x) = 0$**. Else,**

$$F_X(x) = \int_{-\infty}^{x} f_X(t)dt = \int_1^{x} \frac{3}{t^4} dt = \left[-\frac{1}{t^3} \right]_1^{x} = 1 - \frac{1}{x^3}$$

(c) What is $E[X]$?

Ans:

$$E[X] = \int_{-\infty}^{\infty} x f_X(x)dx = \int_1^{\infty} \frac{3}{x^3} dx = \frac{3}{2}$$

(d) What is $\mathrm{Var}(X)$?

Ans:

$$E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)dx = \int_1^{\infty} \frac{3}{x^2} dx = 3$$

So Var$(X) = E[X^2] - (E[X])^2 = 3 - 9/4 = 3/4$**.**