

Homework 5

1. There are 8760 hours in a (non-) leap year. Assume that a typical upper middle income worker earns \$80 per hour and has no other source of income. The worker can use his income to purchase a consumption good priced at \$1 per unit.

- a) Illustrate the budget set of a typical upper middle income worker.

Suppose that the government levies a Social Security tax on wage income. The tax is levied only on the first \$100,000 of income. The marginal tax rate is 10% so the worker pays \$.10 on every dollar earned up to 100,000. There are no taxes levied on income earned over \$100,000.

- b) In your diagram for part (a) illustrate the effect of the tax system on the worker's budget set.

Given the wage rate and the tax system the typical worker chooses to work 40 hours per week for 50 weeks in the year for a total of 2000 hours per year. You may assume that leisure and consumption goods are not perfect complements.

- c) How much does the typical worker pay in taxes? How much does he earn after taxes? What is his MRS at his best bundle?

The government proposes to change the tax code. The government will lower the marginal tax rate to 6.25% (.0625). Simultaneously the government will tax every dollar earned (including income earned above \$100,000). In other words the worker will pay \$.0625 in taxes on every dollar earned.

- d) Show algebraically that the bundle chosen by the worker under the old tax scheme is still available under the new tax scheme. Illustrate the new budget line in your diagram in (a).
e) Will the new tax scheme encourage or discourage more work? Briefly explain your answer. Will the new tax scheme raise more money, less money or do you need more information to answer this question. Briefly explain your answer.

2. Joe is a risk-averse farmer who owns a 1000-acre farm in South Dakota. He can plant either wheat or corn or any combination of wheat and corn on his farm. At the end of the summer Joe is certain that he can sell his wheat at \$5 per bushel and his corn at \$4 per bushel. However the yield, from his planting (the amount of crops produced) will depend on the weather in the upcoming growing season. The summer season may be either dry or wet and the yields of the two crops will vary depending on the amount of rain. The yields are summarized in the following table.

	Dry Season		Wet Season	
	Bushels per acre	\$ per acre	Bushels per acre	\$ per acre
Wheat	80	\$400	100	\$500
Corn	160	\$640	110	\$440

Note that the dollars per acre is simply the product of the price per bushel and the bushels per acre.

The Farmers' Almanac predicts that the probability of a wet growing season will be .6 (60%) and the probability of the dry growing season will be .4 (40%). Joe always believes what he reads in the Farmers' Almanac so he believes these probabilities to be accurate.

- a) Let α represent the proportion of his acreage that Joe plants with wheat. If Joe would like to have a sure income (that is an income that does not vary with the seasons) then what α should he choose?
- b) Set up Joe's expected utility maximization problem as a function of α and write the first order necessary condition.
- c) Use your first order condition to show that Joe will NOT choose the α that you determined in part (a). Note that you cannot solve for α because you do not have the functional form of his utility function but you can determine that it is not the value from part (a).
- d) Is the α that you determined in part (a) too big or too small relative to the expected utility maximizing ?

3. Emmitt is a risk averse investor whose preferences over dollar wealth are given by the utility function $u(\$) = \ln(\$)$. Emmitt has the option to buy a portfolio of precious metals. If there is high inflation in the future then the portfolio will be worth \$10,000. If there is low inflation then the portfolio will be worth only \$6400. Emmitt considers high and low inflation to be equally likely.

- a) What is the expected value of the portfolio of precious metals? What is the certainty equivalent to this portfolio? What is the risk premium?
- b) Illustrate the expected utility of this portfolio in a diagram. Be sure to label the expected value, expected utility, certainty equivalent and risk premium.

Suppose that the likelihood of high inflation falls to 22.5% and that the value of the portfolio of precious metals rises to \$14,400 in the high inflation state (the value in the low inflation state remains unchanged at \$6400).

- c) Show that the expected value of the portfolio has not changed but that both the risk premium has risen? Illustrate the expected utility of the portfolio under the new circumstances in your diagram for part (b). Be sure to indicate the new certainty equivalent and the new risk premium.

4. Fred has \$W to invest. He is considering investing in a defense company, Zee Co. If Zee Co is awarded a new defense contract then the value of any investment in Zee Co will double. However, if it does not receive a new contract then the value of the investment will be cut in half. Fred believes that it is equally likely that Zee Co will receive the new contract. Finally assume that his preferences over his investment (x) can be represented by the utility function $u(x) = \ln x$.

For parts (a) - (c) Assume that Fred has put all of his money in Zee Co (so he has invested \$W in Zee Co).

- a) What is the expected value of his investment (as a function of W)? What is his expected utility of his investment?
- b) What is the certainty equivalent to his investment (as a function of W)? What is the risk premium?

- c) Illustrate his utility function and the expected utility of his investment in a diagram. Label the expected value, the certainty equivalent and the risk premium.
- d) If his only options are investing all of his money in Zee Co or keeping his money in cash (where it will have a certain value of \$W) which (if any) would he prefer?

For parts (e) - (g) below assume that Fred may put any fraction, α , of his \$W in Zee Co. The remaining fraction, $(1-\alpha)W$, will be kept in cash. Assume that the returns on money put in Zee Co and the associated probabilities are as above.

- e) Write his expected utility function as a function of α . Write the FOC associated with Fred's expected utility maximization problem.
- f) Show that at $\alpha=0$ the first derivative of expected utility is positive (so 0 does not solve the FOC). Briefly explain why this is not a surprising result.
- g) What is the value of α that maximizes his expected utility?

5. Nina has a house worth \$160,000 which represents all of her wealth. Unfortunately there is a chance that there will be a fire and that her house will suffer \$97,500 in damages. Thus leaving her a house valued at only \$62,500. The probability of the fire is .1. Finally assume that her preferences over dollar values of wealth can be represented by the utility function $u(x) = \sqrt{x}$

- a) What is her expected wealth? What is Nina's expected utility?
- b) What is the certainty equivalent to her situation?
- c) What is the risk premium associated with her situation?
- d) Illustrate her expected utility, expected wealth, certainty equivalent, and the risk premium in a diagram.

6. Farming is an inherently risky business. Two risk averse farmers, Jeb and George grow soybeans in Nebraska. A successful crop will yield \$700 in profit to each of them. An unsuccessful crop will yield nothing. They each have \$900 in the bank at the start of the growing season (so that each will have \$1600 if they are successful and \$900 if not successful).

- a) If each has a probability of a successful crop equal to .8 then what are their expected incomes?
- b) Before the growing season George suggests to Jeb that they should "pool" their risks. In particular George suggests that each of them put up \$350. If both are successful or if both of them fail then they each take back their \$350. However if only one of them should fail then he should take the entire pool of \$700. Will Jeb agree to George's proposal?

Suppose of the two of them Jeb is the more experienced farmer. Hence his chances of a successful season are higher than George's. The probability that Jeb is successful is .9 and the probability that George is successful is .8. Each of them have a utility function represented by $U(\$) = (\$)^{1/2}$.

- c) Given the new probabilities will Jeb accept George's original proposal?
- d) If George continues to put up \$350 then how much money would Jeb be just willing to put up?

7. Suppose that you own a bar near the University of Nevada at Las Vegas. Every time the basketball team wins a home game, business is brisk and you clear \$500 for the day. When it loses, business is slow and on those days you clear only \$50. Let θ be the probability that the basketball team wins. The problem is, you don't like risk; you would prefer a certain income to an uncertain one. Fortunately for you, gambling is legal in Nevada and there are many people who like to bet on sporting events. Thus you can "hedge" against low profit days by betting against the home team.

- a) Assuming that you can get fair odds (so that a bet is a fair bet) and that a bet pays off \$1 when the home team loses then how much would the losses of the bet be in the case that the home team wins? Your answer will be a function of the probability θ .
- b) Using your answer to part a how much should you bet per game so that you will be perfectly "hedged" that is how much should you bet in order that you have a certain income?
- c) If θ is .6 then how much should you bet?