Homework #6

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Question 1

$$\max_C [U_E] = (1 - .1)\sqrt{10,000 - rC} + .1\sqrt{10,000 - 7,500 + C(1-r)} \quad (1)$$

$$= .9\sqrt{10,000 - rC} + .1\sqrt{2,500 + C(1-r)} \quad (2)$$

$$\frac{\partial U_E}{\partial C} = -\frac{\left(\frac{1}{2}\right)(.9)(r)}{\sqrt{10,000 - rC}} + \frac{\left(\frac{1}{2}\right)(.1)(1-r)}{\sqrt{2,500 + C(1-r)}} = 0 \quad (3)$$

Solving for $C(r)$:

$$C(r) = \frac{2,500(77r^2 + 8r - 4)}{r(80r^2 - 79r - 1)}$$

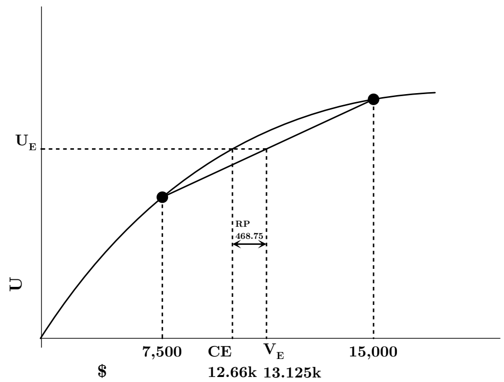
Question 2

(a)

For men, $Cost = .25 * 7,500 = 1,875$. They will purchase this policy because the cost is below their $2,343.75 maximum acceptable cost. For women, $Cost = .19 * 3,60 = 684$. They will purchase this policy because the cost is below their $745.50 maximum acceptable cost.

(b)

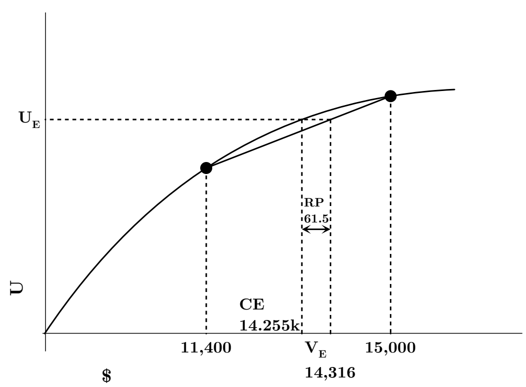
$$V_E = .75 * 15,000 + .25 * 7,500 = 13,125$$
$$CE = 15,000 - 2,343.75 = 12,656.25$$
$$RP = V_E - CE = 468.75$$



Graph is not to scale. Proportions altered for clarity.

(c)

$$
\begin{aligned}
V_E &= .81 * 15,000 + .19 * 11,400 = 14,316 \\
CE &= 15,000 - 745.5 = 14,254.5 \\
RP &= V_E - CE = 61.5
\end{aligned}
$$



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(d)

For the insurance company:

$$
\begin{aligned}
I_E &= \frac{1}{2}(-7,500 * .25 + 7,500 * r) + \frac{1}{2}(-3,600 * .19 + 3,600 * r) \\
&= 5,550r - 1279.5
\end{aligned}
$$

For women:

$$
\begin{aligned}
3,600r &\leq 745.5 \\
r &\leq 0.20708\bar{3}
\end{aligned}
$$

For men:

$$7,500r \leq 2,343.75$$
$$r \leq \frac{5}{16}$$

Treating men and women as the rate-setters and inputting their maximum acceptable values of r into the expected income function for the insurance company:

$$I_{E_{women}} \leq -140.541\bar{6}$$
$$I_{E_{men}} \leq 439.25$$

(e)

Only men will be covered. Even if the insurance company sets r such that women pay their maximum acceptable cost, I_E will be negative. The company will not offer insurance to women under any circumstances. They can only receive positive I_E when the rate is determined by what is acceptable to men. They will set $r = .25$ such that $rC = 1,875$, the cost of providing full fair insurance.

Question 3

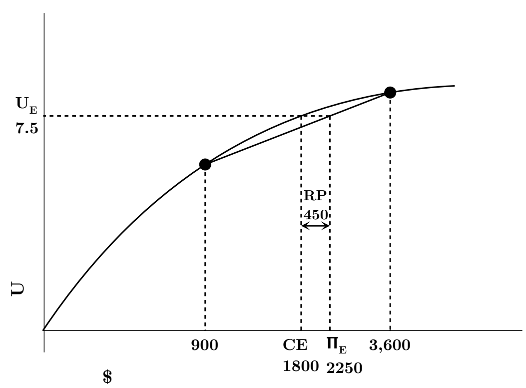
(a)

$\Pi_H = 3,600; \Pi_L = 900; \Pi_E = 2,250.$

(b)

$CE = e^{U_E} = e^{\frac{1}{2}(\ln 3600 + \ln 900)} = 1,800; RP = V_E - CE = 450$

(c)



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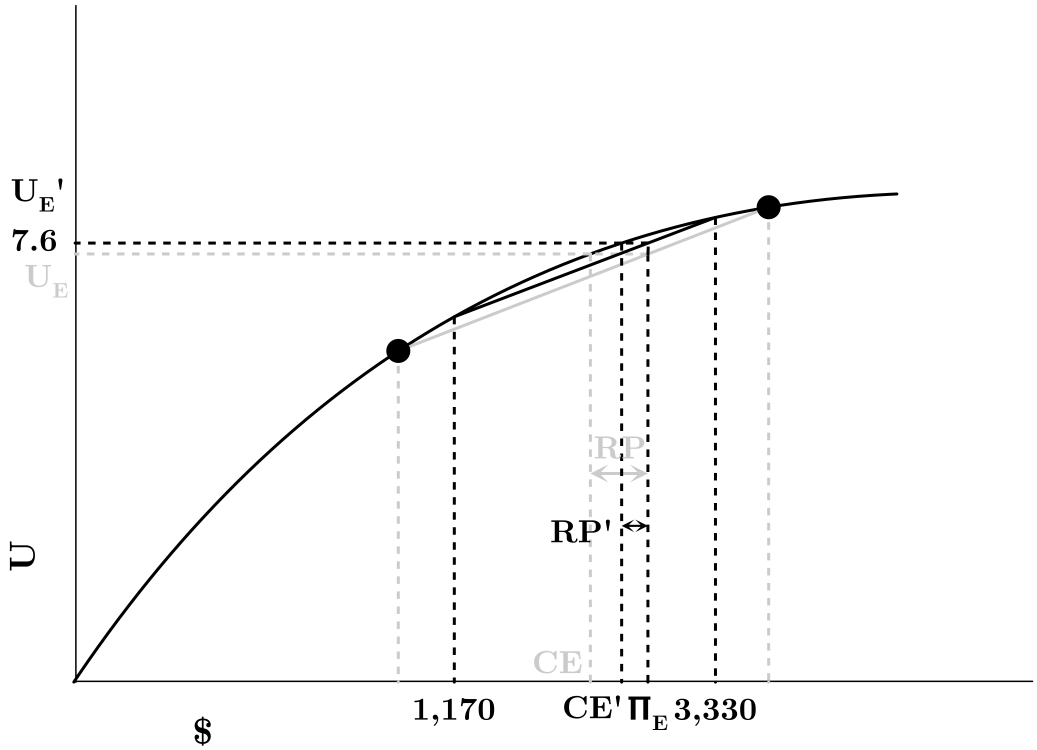
(d)

Helen is receiving insurance. Under the scheme, she transfers some of her risk to Gene, much the way a risk averse individual would to an insurance provider.

(e)

$W_E = \frac{1}{2}(.2(1900 + 4600) + 700)) = 1000; \Pi_E = 3,250.$

(f)



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$U_E = \frac{1}{2}\ln[4,600 - (350 + .2 * 4,600)] + \frac{1}{2}\ln[1900 - (350 + .2 * 1900)] = 7.591\bar{9}.$
$U_{E_{previous}} \approxeq 7.5.$
Helen's utility is greater than under the old scheme so she will be better off.

(g)

No. Helen has transferred some of her risk to Gene. He is risk averse and so the same expected income and increased risk is worse for him.

(h)

$\Pi = \frac{1}{2}[(4,600 - W_H) + (1900 - W_L)]$

(i)

$(W_L, W_H) = (100, 2800)$
This brings Helen to her certainty equivalent of 1,800.

(j)

$W_E = \frac{1}{2}(100 + 2800) = 1450$

Question 4

(a)

$W_E = .9 * 202,500 + .1 * 90,000 = 191,250; U_E = .9\sqrt{202,500} + .1\sqrt{90,000} = 435.$

(b)

$CE = U^{-1}(U_E) = 435^2 = 189,225$

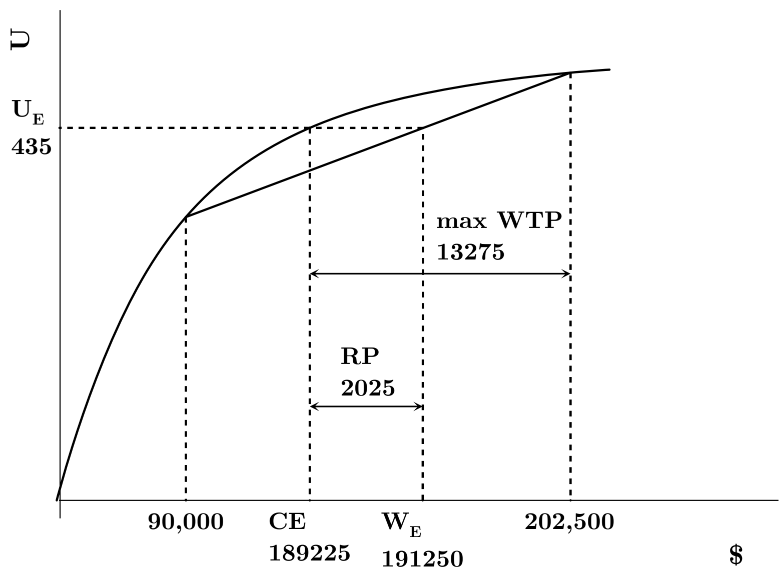
(c)

$RP = W_E - CE = 2,025$

(d)

$W_0 - CE = 202,500 - 189,225 = 13,275$

(e)



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(f)

Fire: $W = 90,000 + C - rC$
No Fire: $W = 202,500 - rC$
$U_E = .1\sqrt{90,000 + C - rC} + .9\sqrt{202,500 - rC}$

(g)

$$\frac{\partial U}{\partial C} = \frac{.05(1-r)}{\sqrt{C - rC + 90,000}} - \frac{.45r}{\sqrt{202,500 - Cr}} = 0$$

(h)

$.9(202,500 - r) + .1(90,000 + 1 - r) = 191,250$
$r = .1$

(i)

$$C(r) = \frac{202500\left(35r^2 + 2r - 1\right)}{r\left(80r^2 - 79r - 1\right)}$$
$$C(.1) = 112,500$$

(j)

$$\frac{\partial U}{\partial C} = \frac{.05(1-r)}{\sqrt{C - rC + 87,500}} - \frac{.45r}{\sqrt{200,000 - Cr}} = 0 \quad (1)$$
$$C(r) = \frac{12500\left(551r^2 + 32r - 16\right)}{r\left(80r^2 - 79r - 1\right)} \quad (2)$$
$$C(.1) = 112,500 \quad (3)$$

The amount of fair insurance she purchases is unchanged.

(k)

$$\frac{\partial U}{\partial C} = \frac{.05(1-r)}{\sqrt{C - rC + 90,000}} - \frac{.45r}{\sqrt{200,000 - Cr}} = 0 \quad (1)$$
$$C(r) = \frac{10000\left(709r^2 + 40r - 20\right)}{r\left(80r^2 - 79r - 1\right)} \quad (2)$$
$$C(.1) = 110,000 \quad (3)$$

Maria reduces the coverage she purchases.

Question 5

(a)

$P = .2(C - Cr) - .8Cr$; For $P = 0, r = .2$.

(b)

$W_E = \frac{1}{2}(120,000 + 30,000) = 75,000; U_E = \frac{1}{2}(\ln 120,000 + \ln 30,000)$

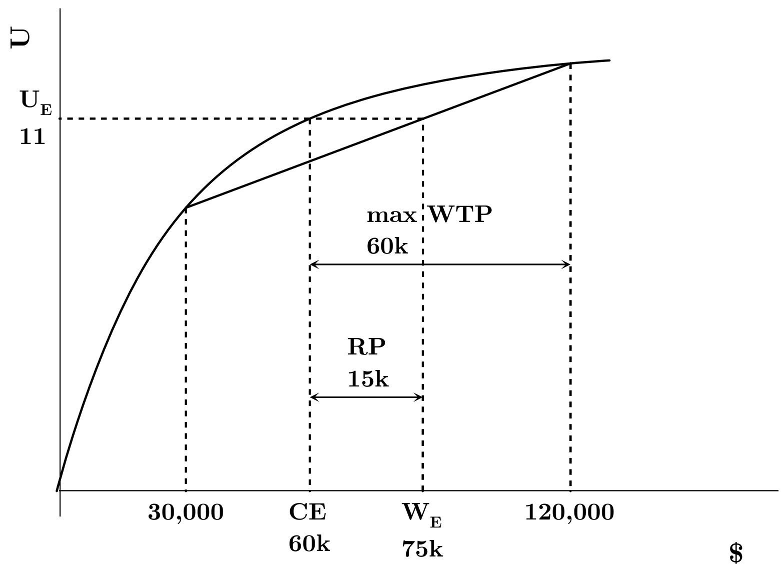
(c)

$CE = e^{\frac{1}{2}(\ln 120,000 + \ln 30,000)} = 60,000; RP = W_E - CE = 15,000$

(d)

$Cr_{max} = W_0 - CE = 60,000$

(e)



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(f)

Yes. $90,000 * .2 = 18,000 < Cr_{max} = 60,000$.

(g)

$U_E = .9\ln(120,000 - .2C) + .1\ln(30,000 + C - .2C)$

(h)

$$\frac{\partial U}{\partial C} = -\frac{.18}{120,000 - .2C} + \frac{.08}{30,000 + .8C} = 0$$

(i)

$$\left.\frac{\partial U}{\partial C}\right|_{C=90,000} = -\frac{.18}{120,000 - .2 * 90,000} + \frac{.08}{30,000 + .8 * 90,000} = -\frac{.18}{102,000} + \frac{.08}{102,000} < 0$$

(j)

$C = 26,250$

(k)

Both will purchase a policy. For Helios, $r = .1$ represents his fair price and he is risk averse. $30,000 is also more than he purchased from Lloyd's.

We know that Luna will purchase the full $30,000 from Greenwich if she does choose to buy their policy since the $r = .1$ is well below her fair price. We can find her expected utility:

$$U_E = \frac{1}{2}[\ln(120,000 - 30,000) + \ln(30,000 + 30,000 - .1 * 30,000)] \approxeq 11.18$$

Her utility under Lloyd's was:

$$U_E = \ln(120,000 - 90,000 * .2) \approxeq 11.53$$

Thus she will choose the Lloyd's policy.

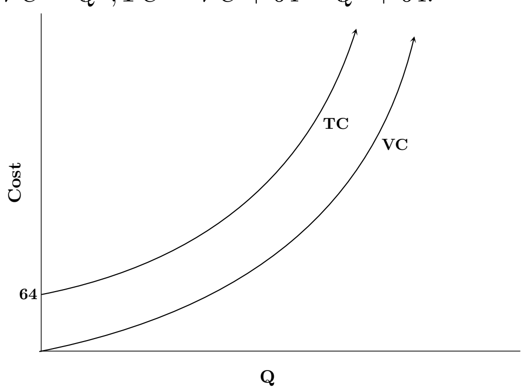
(l)

No. Previously, Lloyd's made money on low risk customers and lost money on high risk customers. Now that all low risk customers have moved to the Greenwich policy, Lloyd's will lose money by offering the policy.

Question 6

(a)

$VC = Q^2; TC = VC + 64 = Q^2 + 64.$



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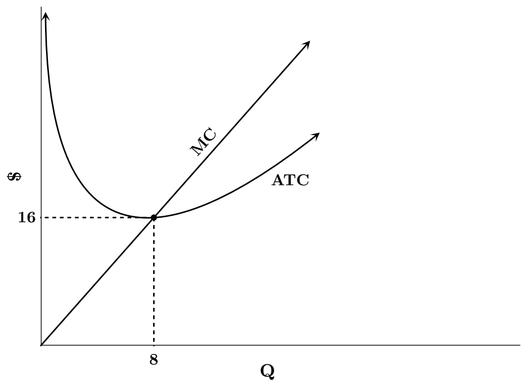
(b)

$MC = 2Q; ATC = \frac{Q^2+64}{Q}$
$\frac{\partial ATC}{\partial Q} = 1 - \frac{64}{Q^2} = 0; Q_{opt} = 8.$



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(c)

$R = 20Q; MR = 20$

(d)

$$MC = MR$$
$$2Q = 20$$
$$Q = 10$$

Question 7

a) Yes. An increase in input costs increases MC. MC now intersects MR at a smaller Q.

b) No, there is no effect on output. Although TC is changed, MC is unchanged.

c) No, there is no effect on output. Because $\Pi = 0$ at $MR = MC$, this tax does not affect the firm.

d) Yes. The slope of MC increases and so Q is reduced.

e) No, there is no effect on output. A lump-sum grant does not affect MC or MR.

f) Yes. This reduces the firms MC, increasing Q.

g) Yes. This reduces the firms MC, increasing Q.