Homework #4

Ben Drucker

3.3

30)

a)

$E(Y) = 0 * .6 + 1 * .25 + 2 * .1 + 3 * .05 = .6$

b)

$100 * 0^2 * .6 + 100 * 1^2 * .25 + 100 * 2^2 * .1 + 100 * 3^2 * .05 = 110$

38)

$$h(E(X)) = \frac{1}{\frac{1+2+3+4+5+6}{6}} = \frac{2}{7} = 0.\overline{285714}$$

$$E(h(X)) = \frac{1}{6}\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}\right) = .408\overline{3}$$

Gambling has a higher expected value.

3.4

46)

a)

$b(3; 8, .35) = P(X = 3) = \frac{8!}{3!(8-3)!} * .35^3(1 - .35)^{8-3} \approxeq 0.2785857790625$

b)

$b(5; 8, .6) = P(X = 5) = \frac{8!}{5!(8-5)!} * .6^5(1 - .6)^{8-5} \approxeq 0.27869184$

c)

$P(3 \leq X \leq 5) = \sum_{x=3}^{5} \frac{7!}{x!(7-x)!} * .6^x(1 - .6)^{7-x} \approxeq .7451$

d)

$P(X \geq 1) = 1 - P(X = 0) = 1 - \frac{9!}{0!(9-0)!}(.1)^0(1 - .1)^9 \approxeq .6126$

4.1)

2)

a)

$P(X < 0) = \frac{0-(-5)}{5-(-5)} = \frac{1}{2}$

b)

$P(-2.5 < X < 2.5) = \frac{2.5-(-2.5)}{5-(-5)} = \frac{1}{2}$

c)

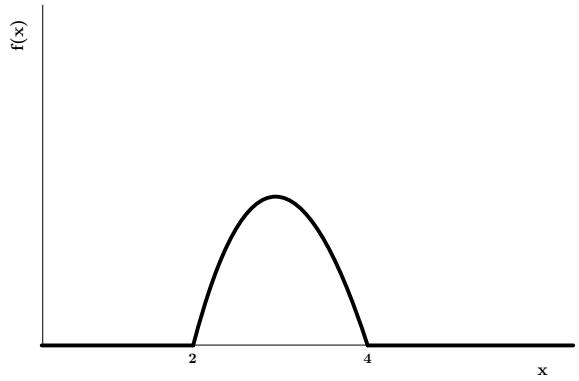
$P(-2 \leq X \leq 3) = \frac{3-(-2)}{5-(-5)} = \frac{1}{2}$

d)

$\int_k^{k+4} \frac{1}{10} dx = \frac{(k+4)-k}{10} = \frac{4}{10}$

0.1 6)

a)



b)

$$\int_2^4 k[1-(x-3)^2]dx = k\left[x - \frac{(x-3)^3}{3}\right]_2^4 = k\left(4 - \frac{1}{3} - 2 - \frac{1}{3}\right) = \frac{4}{3}k = 1 \Rightarrow k = \frac{3}{4}$$

c)

$P(X \geq 3) = \int_3^4 \frac{3}{4}[1-(x-3)^2]dx = \frac{1}{2}$

d)

$P(2.75 \geq X \geq 3.25) = \int_{2.75}^{3.25} \frac{3}{4}[1-(x-3)^2]dx \approxeq 0.367188$

e)

$P(2 \leq X < 2.5 \cup 3.5 \leq X < 4) = \int_0^{2.5} \frac{3}{4}[1-(x-3)^2]dx + \int_{3.5}^4 \frac{3}{4}[1-(x-3)^2]dx = 0.3125$

4.2)

18)

a)

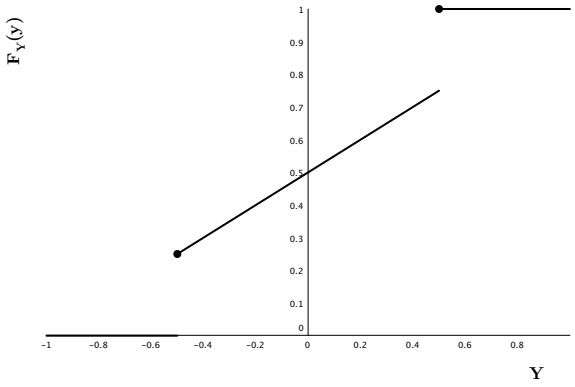
$P(Y = .5) = \frac{.5}{2} = .25$

b)

For $|X| < .5$:

$$F_Y(y) = P(Y = y) = P(Y = -.5) + P(-.5 \leq X < 1) = \frac{1}{4} + \int_{-.5}^{y} \frac{1}{2} dy = \frac{1}{4} + \frac{y + .5}{2} = \frac{y + 1}{2}$$

$$F_Y(y) = \begin{cases} \frac{1}{4} & \text{if } Y = -.5 \\ \frac{y+1}{2} & \text{if } |Y| < .5 \\ 1 & \text{if } Y = .5 \end{cases}$$

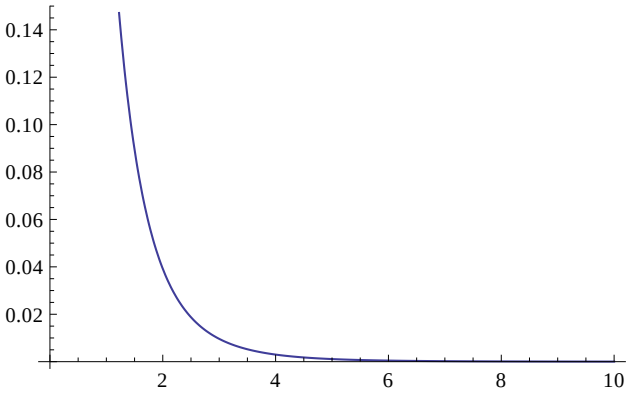


26)

a)

$\int_0^\infty k \left(1 + \frac{x}{2.5}\right)^{-7} dx = 1 \Rightarrow \frac{5}{12} k = 1 \Rightarrow k = \frac{12}{5}$

b)



c)

$E(X) = \int_0^\infty xf(x)dx = \int_0^\infty \frac{12}{5}x\left(1+\frac{x}{2.5}\right)^{-7}dx = .5$
$E(X^2) = \int_0^\infty x^2f(x)dx = \int_0^\infty \frac{12}{5}x^2\left(1+\frac{x}{2.5}\right)^{-7}dx = .625$
$SD(X) = \sqrt{E(X^2)-E(X)^2} = \sqrt{.375} \approxeq .6124$

d)

$$Y = \begin{cases} 0, & 0 \le x \le .5 \\ .8(X-.5), & .6 \le x \le 2.5 \\ 0, & \text{Elsewhere} \end{cases}$$

$$\begin{aligned} E(Y) &= E[.8(X-.5)] \\ &= \int_{.6}^{2.5}[.8(x-.5)]f(x)dx \\ &= \int_{.6}^{2.5}[.8(x-.5)]\frac{12}{5}\left(1+\frac{x}{2.5}\right)^{-7}dx \\ &\approxeq .121 \\ E(Y) &= \$121 \end{aligned}$$

4.3

32)

a)

$P(X \le 15) = .5$

b)

$P(X \le 17.5) = P(Z < \frac{17.5-15}{1.25} \approxeq .9772$

c)

$P(X \geq 10) = 1 - P(X \leq 10) = 1 - P(Z < \frac{10-15}{1.25} \approxeq 1$

d)

$P(14 \leq X \leq 18) = P(\frac{14-15}{1.25} < Z < \frac{18-15}{1.25} \approxeq .9918 - .2119 = .7799$

e)

$P(|X - 15| \leq 3) = P(12 \leq X \leq 18) = P(\frac{12-15}{1.25} < Z < \frac{18-15}{1.25}) \approxeq .9918 - .0082 = .9836$