

Calculus Review Questions.

1. Find  $dy/dx$  given  $x^2y - xy^2 + x = 0$
2. Given  $x^3y + xy^3 = 2$  find  $dy/dx$  at the point  $(1,1)$ .
3. Find the equation of the tangent and normal lines to  $y = x^3 - 2x^2 + 4$  at the point  $(2,4)$ .
4. Show in the space below that  $f(x) = 4x^3 + x - 3 = 0$  has exactly one real solution (you only need to show the existence of one real solution – you do not have to find the value).  
Hint: sketch a graph of the function and look at the slope. Use a fact about the slope to demonstrate the result.
- 5 a) Use the fact that  $y = e^{\ln y}$  to rewrite  $2^x$  in exponential form.  
b) Use your answer to (a) to differentiate  $2^x$ .
- 6 a) Use the chain rule to write  $dz/dt$  where  $z = f(x(t), y(t))$   
b) Use the expression from part (a) to find  $dz/dt$  given  $z = \ln(x^2 + y^2)$ ;  $x(t) = e^{-t}$  and  $y(t) = e^t$
7. Find the relative extrema (max and min) for the function  $f(x) = -7x^2 + 126x - 23$ .
  - a) Write the First Order Condition
  - b) Write the solution(s) to the First Order Condition
  - c) Use the second order conditions to classify the critical values as relative maxima, relative minima or neither.
8. Solve each of the following equations for  $x$  in terms of  $y$ :
  - a)  $\ln x = \ln 3 + \ln y$
  - b)  $\ln x = 3y$
9. Find the derivatives of
  - a)  $e^{3x+1}$
  - b)  $x^2e^x$
  - c)  $\ln(x-10)$
10. For each of the following find  $\partial f/\partial x$  and  $\partial f/\partial y$ 
  - a)  $f(x,y) = x^2 + 3xy + y^2$
  - b)  $f(x,y) = \ln x + 2 \ln y$
  - c)  $f(x,y) = e^{xy}$
11. Solve the following constrained optimization problem:  
 $\max \ln x + 2 \ln y$  subject to  $x + y = 60$