# Homework #10

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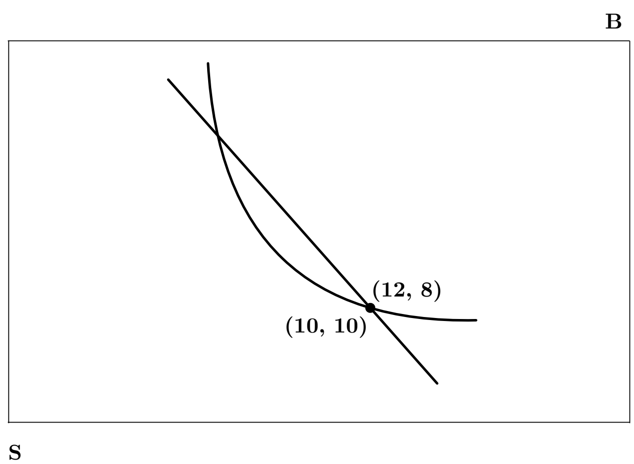
## Question 1

### (a)

$$MRS_B = \frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}} \Rightarrow MRS_B = \frac{3}{1}$$
$$MRS_S = \frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}} \Rightarrow MRS_S = \frac{y}{x}$$

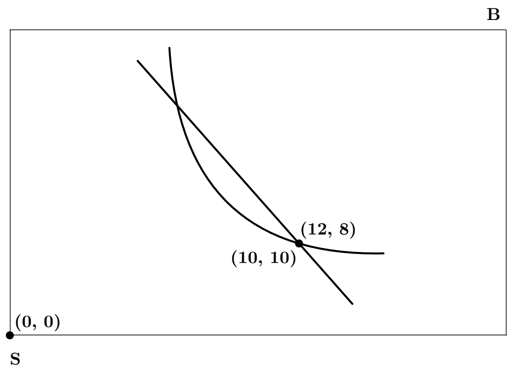



### (b)

Bruce will give up $y$ for $x$ and Sheila will give up $x$ for $y$. Sheila will be willing to trade $1 \leq x \leq 3$ for 1 $y$.

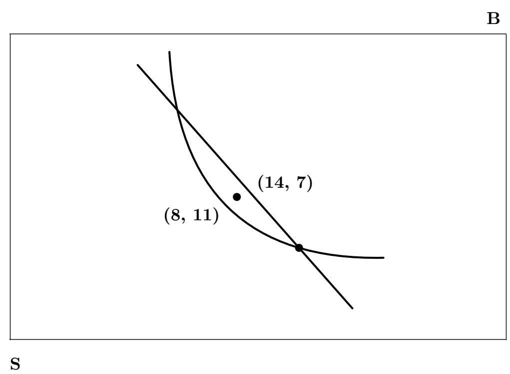
### (c)

Sheila gives up her entire allocation, making her new allocation (0,0).



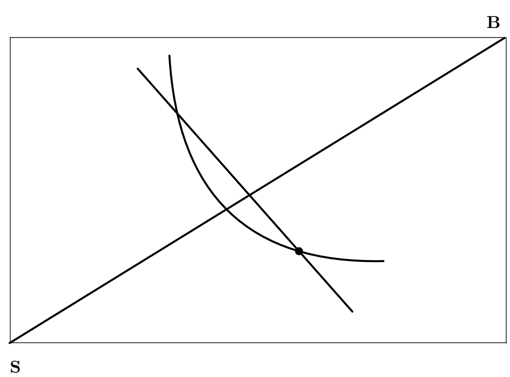

### (d)

Sheila gives up 2 $x$ for 1 $y$ from Bruce.




### (e)

$\frac{y_s}{x_s} = 3 \Rightarrow y_s = 3x_s$.



# Question 2

## (a)

Yes, since both allocations are identical.

## (b)

$MRS_S = 3, MRS_B = 1$ at these allocations. They are not Pareto efficient.

## (c)

Sheila. Bruce will trade $x$ for $y$.

## (d)

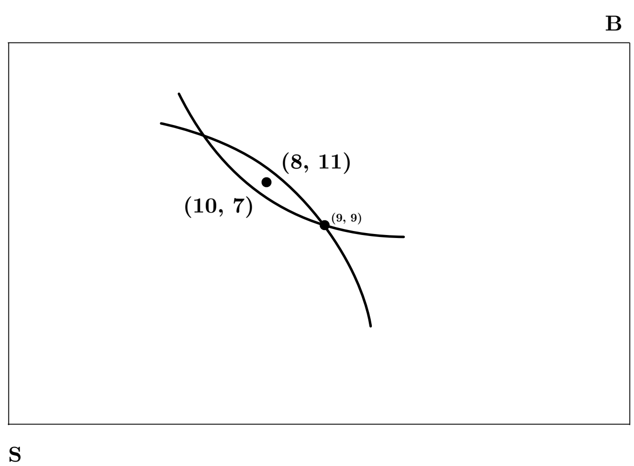
$1 \leq P \leq 3$.

## (e)

Sheila trades 2 $y$ for 1 $x$.

$$U_1^S = 3\ln 10 + \ln 8 \approxeq 8.99 > U_0^S = 4\ln 9 \approxeq 8.79$$
$$U_1^B = \ln 8 + \ln 11 \approxeq 4.48 > U_0^B = 2\ln 9 \approxeq 4.39$$

## (f)




# Question 3

## (a)

No. $MRS_S = \frac{y}{x}, MRS_B = \frac{4y}{x}$. Plugging in endowments, $MRS_S = \frac{20}{12} \neq MRS_B = \frac{4*20}{10}$, therefore the allocation is not Pareto efficient.

## (b)

Bruce. Bruce will trade $y$ for $x$. $1 \le P \le \frac{6}{5}$.

## (c)

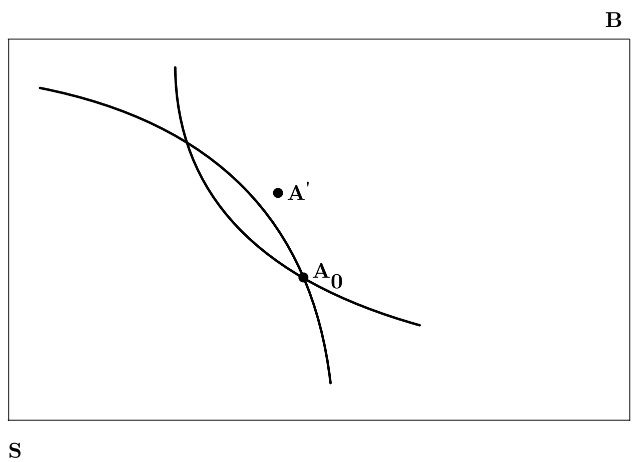
No, it does not fall within $P$.

$$U_1^S = \ln 19 + \ln 15 \approxeq 5.65 > U_0^S = \ln 20 + \ln 12 \approxeq 5.48$$
$$U_1^B = 4\ln 21 + \ln 7 \approxeq 14.12 < U_0^B = 4\ln 20 + \ln 10 \approxeq 14.29$$

## (d)

No. $\frac{3}{1} > \frac{6}{5}$ and therefore falls outside of $P$.

## (e)



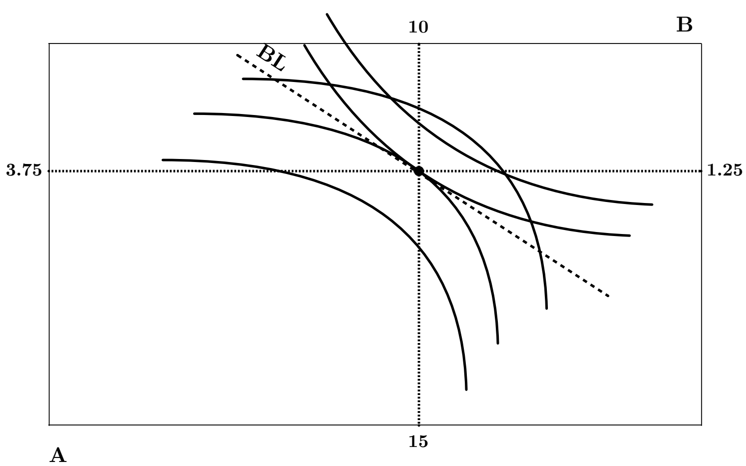
# Question 4

## (a)

Given a bundle $(x^a, y^a)$ for Melvin and $(x^b, y^b)$ for Martin and $p = \frac{p_x}{p_y}$:

$$MRS = p \Rightarrow \frac{y^a}{y^b} = p \Rightarrow y^a = px^a \tag{1}$$
$$px^a + y^a = 10p + 5 \tag{2}$$
$$2y^a = 10p + 5MRS = p \Rightarrow \frac{2y^b}{x^b} = p \Rightarrow 2y^b = px^b \tag{3}$$
$$px^b + y^b = 15p \tag{4}$$
$$3y^b = 15p \tag{5}$$
$$x^a + x^b = 25 \tag{6}$$
$$y^a + y^b = 5 \tag{7}$$
$$y^a + y^b = 5p + \frac{5}{2} + 5p = 10p + \frac{5}{2} \Rightarrow p = \frac{1}{4} \tag{8}$$
$$(y^a, y^b) = \left(\frac{15}{4}, \frac{5}{4}\right) \tag{9}$$
$$(x^a, x^b) = (15, 10) \tag{10}$$

## (b)



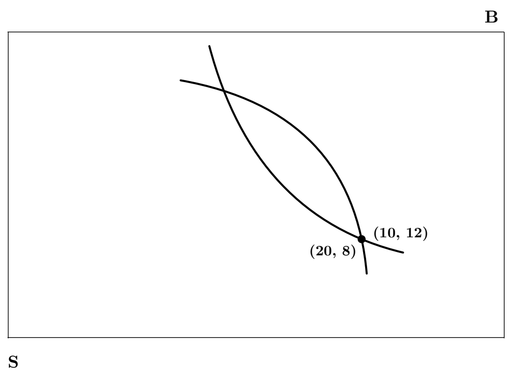

## (c)

The competitive allocation, as expected, results in no surplus or shortage.

# Question 5

## (a)

$MRS_S = \frac{54}{35}, MRS_B = \frac{36}{5}$. The endowment point is not Pareto efficient.




## (b)

For Sheila:

$$\frac{27y_s}{7x_s} = \frac{p_x}{p_y} \tag{1}$$

$$p_x x_s + p_y y_s = 20p_x + 8p_y \tag{2}$$

For Bruce:

$$\frac{6y_b}{x_b} = \frac{p_x}{p_y} \tag{3}$$

$$p_x x_b + p_y y_b = 10p_x + 12p_y \tag{4}$$

## (c)

The price ratio of 1.3 does not fall between the $MRS$ values from *(a)*.

## (d)

Sheila:

$$20 * 13 + 8 * 10 = 340 \tag{1}$$

$$12x_s + 10y_s = 340 \tag{2}$$

$$y_s = 34 - 1.3x_s \tag{3}$$

$$\frac{27y_s}{7x_s} = 1.3 \Rightarrow 9.1y_s = 27x_s \Rightarrow 27(34 - 1.3x_s) = 9.1x_s \tag{4}$$

$$x_s = 20.77, y_s = 34 - 1.3 * 20.77 = 7 \tag{5}$$

Sheila is a net supplier of chocolate.

Bruce:

$$10 * 13 + 12 * 10 = 250 \tag{1}$$

$$y_b = 25 - 1.3x_b \tag{2}$$

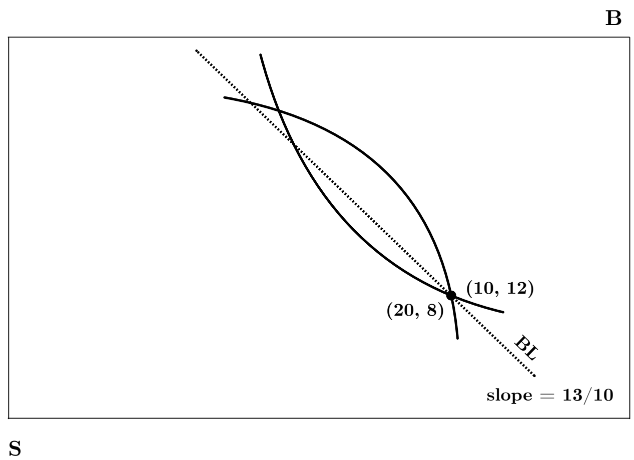
$$\frac{6y_b}{x_b} = 1.3 \Rightarrow 6y_b = 1.3x_b \Rightarrow 6(25 - 1.3x_b) = 1.3x_b \tag{3}$$

$$x_b = 16.48, y_b = 25 - 1.3 * 16.48 = 3.58 \tag{4}$$

Bruce is a net supplier of chocolate.

## (e)

There is excess supply in the market for chocolate and excess demand in the market for soda.




The slope of the budget line $BL$ is $\frac{p_x}{p_y}$.

## (f)

Sheila:

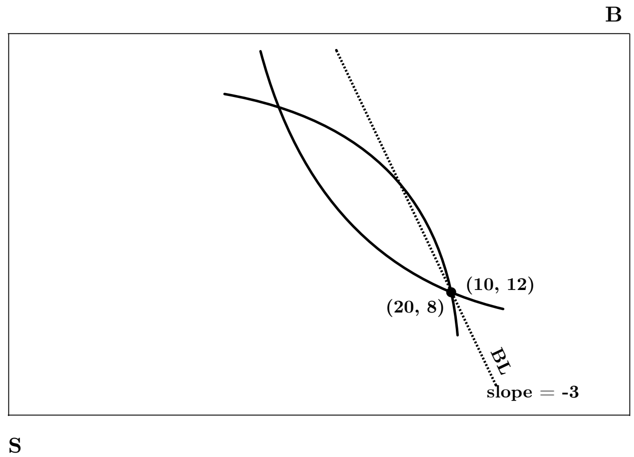
$$4x_s + y_b = 20 * 3 + 8 * 1, \frac{27y_s}{7x_s} = 3 \Rightarrow x_s = 18, y_s = 14$$

Bruce:

$$3x_b + y_b = 10 * 3 + 12 * 1, \frac{6y_b}{x_b} = 3 \Rightarrow x_b = 12, y_b = 6$$

$x_s + x_b = 30, y_s + y_b = 20$. The market clears.

### (g)




## Question 6

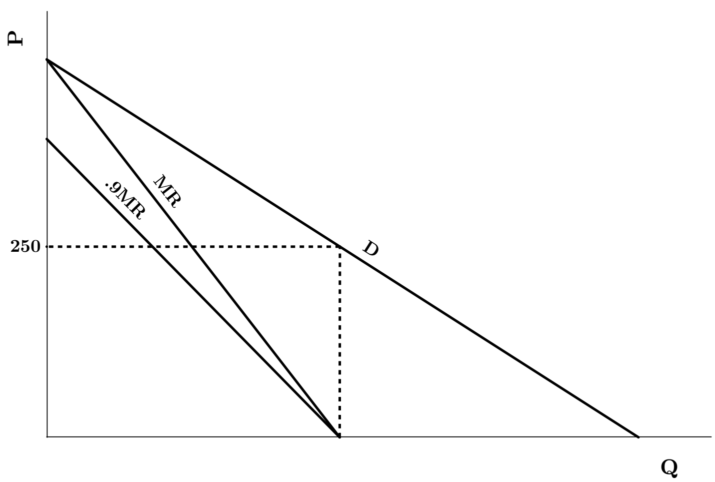
### (a)

$$MR = p\left(1 + \frac{1}{\epsilon^D}\right) \tag{1}$$

$$MR = MC = 0, p = 250 \tag{2}$$

$$250\left(1 + \frac{1}{\epsilon^D}\right) = 0 \Rightarrow \epsilon^D = -1 \tag{3}$$

### (b)




Lively bears the full tax burden.

# Question 7

## (a)

$\frac{\partial \Pi}{\partial Q} = R'(Q) - C'(Q) - t = 0.$

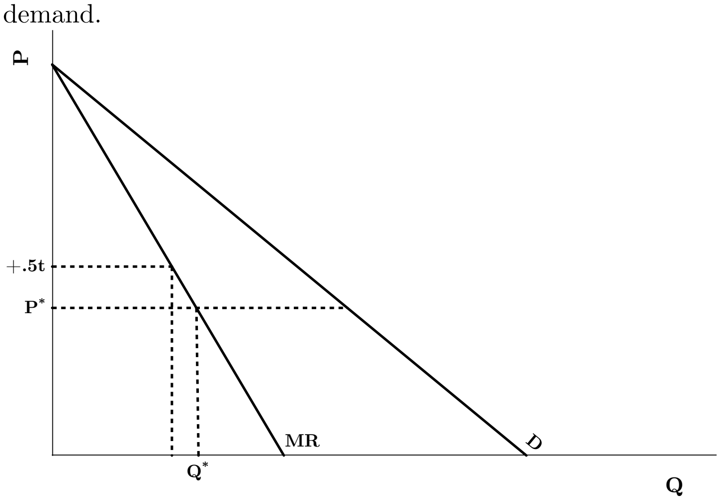
## (b)

$$\frac{d^2R}{dQ^2}\frac{dQ}{dt} - \frac{d^2C}{dQ^2}\frac{dQ}{dt} - 1 = 0$$
$$\frac{dQ}{dt} = \frac{1}{\frac{d^2R}{dQ^2} - \frac{d^2C}{dQ^2}}$$

## (c)

$$p(Q) = a - bQ \tag{1}$$
$$MR(Q) = a - 2bQ \tag{2}$$
$$a - 2bQ - c - t = 0 \tag{3}$$
$$Q = \frac{a-c-t}{2} \tag{4}$$
$$P(Q) = a - bQ = a - b\left(\frac{a-c-t}{2b}\right) = a - \frac{a-c-t}{2} = \frac{a+c+t}{2} \tag{5}$$
$$\frac{dP}{dt} = \frac{1}{2} \tag{6}$$

In the perfectly competitive case, the burden of the tax depends on the relative elasticities of supply and demand.



### (d)

$$P(Q) = Q^{\frac{1}{\epsilon}} \tag{1}$$
$$R(Q) = pQ = Q^{1+\frac{1}{\epsilon}} \tag{2}$$
$$MR = \left(1 + \frac{1}{\epsilon}\right) Q^{\frac{1}{\epsilon}} \tag{3}$$
$$MR = MC \Rightarrow \left(1 + \frac{1}{\epsilon}\right) Q^{\frac{1}{\epsilon}} \ = c + t \tag{4}$$
$$Q^{\frac{1}{\epsilon}} = \frac{c+t}{1+\frac{1}{\epsilon}} \tag{5}$$
$$Q = \left(\frac{c+t}{1+\frac{1}{\epsilon}}\right)^{\epsilon} \tag{6}$$
$$P(Q) = \frac{c+t}{1+\frac{1}{\epsilon}} \tag{7}$$
$$\frac{dP}{dt} = \frac{1}{1+\frac{1}{\epsilon}}; \epsilon < -1. \tag{8}$$
$$\frac{dP}{dt} > 1 \tag{9}$$

A tax burden exceeding 100% of $t$ was not possible in a competitive market.

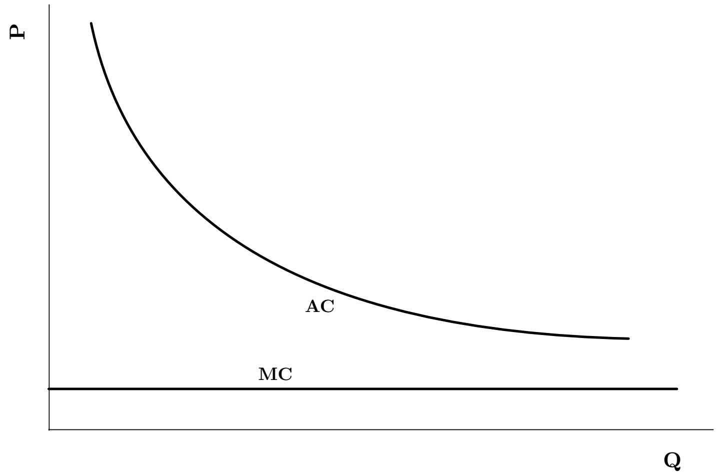
## Question 8

### (a)

$$\min[16L + 2K] \text{ s.t. } 5L^{\frac{1}{5}}K^{\frac{4}{5}} = \overline{Q}$$
$$MRTS = \frac{\frac{\partial Q}{\partial L}}{\frac{\partial Q}{\partial K}} = \frac{K}{4L} = \frac{W}{R} = 8 \Rightarrow K = 32L$$
$$5L^{\frac{1}{5}}(32L)^{\frac{4}{5}} = \overline{Q} = 80L$$
$$L = \frac{\overline{Q}}{80}, K = \frac{32\overline{Q}}{80} = \frac{2\overline{Q}}{5}$$
$$VC = 16L + 2K = \frac{16\overline{Q}}{80} + \frac{4\overline{Q}}{5} = \overline{Q}$$

### (b)

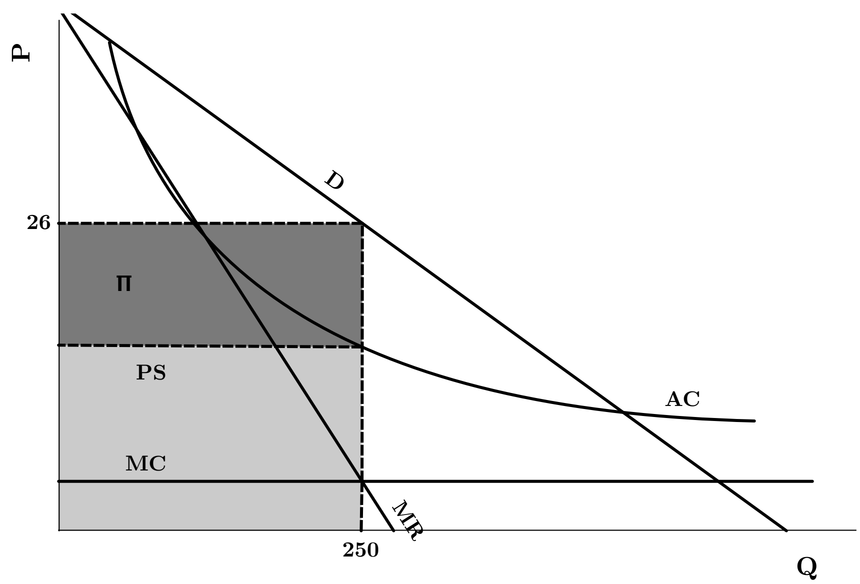
$AC = \frac{4000}{Q} + 1; MC = 1$. The cost curves exhibit scale economies for all values of $Q$.




### (c)

$$P = \frac{510 - Q}{10}$$
$$MR = 51 - \frac{Q}{5} = MC = 1$$
$$Q = 250$$
$$P = \frac{510 - 250}{10} = 26$$

### (d)




The dark shaded region is Π. The entire shaded region is $PS$.

### (e)

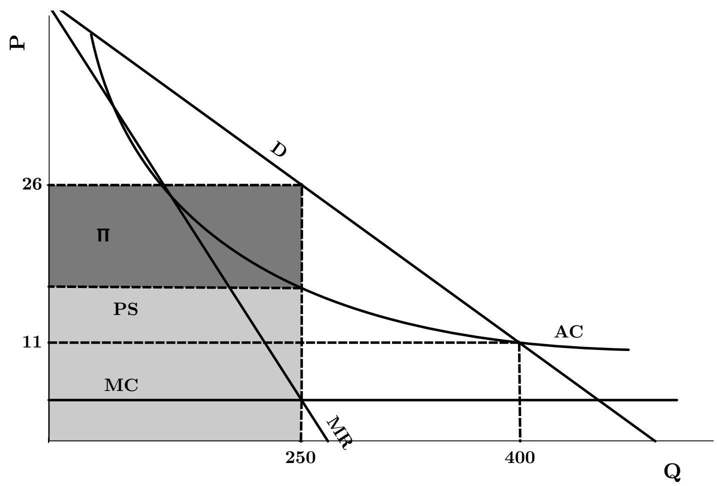
The difference between $PS$ and $\Pi$ is fixed cost.

### (f)

$$\begin{aligned} P &= AC \\ \frac{4000}{Q} + 1 &= 51 - \frac{Q}{10} \\ Q &= 100, 400 \\ P(Q = 100) = 41. P(Q = 400) &= 11. \end{aligned}$$

The regulator should set $P = 11$. At this price, $MC = AC$, so Horizon is willing to produce $Q = 400$.

### (g)




## Question 9

### (a)

$$\begin{aligned} MR = 0 &= 9.8 - .0004Q \\ Q = 24,500 &> 15,000. Q = 15,000. \\ P &= 9.8 - .0002 * 15000 = 6.8 \\ MR &= 9.8 - .0004 * 15000 = 3.8 \\ MR &= P\left(1 + \frac{1}{\epsilon^D}\right) \Rightarrow \epsilon^D = -2.27 \\ \Pi = PQ - 5Q - 25000 &= 6.8 * 15000 - 5 * 15000 - 25000 = 2,000 \end{aligned}$$

### (b)

$$
\begin{aligned}
MR = 9.8 - .0004Q = MC = 5 \\
Q = 12,000 \\
P = 9.8 - .0002 * 12000 = 7.4 \\
\Pi = 7.4 * 12000 - 5 * 12000 - 25000 = 3800
\end{aligned}
$$

### (c)

$$
\begin{aligned}
MR_1 = 9.8 - .0004Q_1 \\
MR_2 = 7 - .0002Q_2 \\
MR_1 = MR_2 = MC \Rightarrow Q_1 = 12,000, Q_2 = 10,000 \\
Q_t = 22,000 > 15,000 \\
Q_1 + Q_2 = Q_t = 15,000 \Rightarrow Q_2 = 15,000 - Q_1 \\
9.8 - .0004Q_1 = 7 - .0002 \Rightarrow Q_1 = 9667 \Rightarrow Q_2 = 5333 \\
9.8 - .0002 * 9667 = P_1 = 7.87 \\
7 - .0001 * 5333 = P_2 = 6.47 \\
\Pi = 7.87 * 5333 + 6.47 * 5333 - 5 * 15000 - 25000 = 10583.8
\end{aligned}
$$