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Homework #3
Intermediate Microeconomics
W3211 001

Question 1

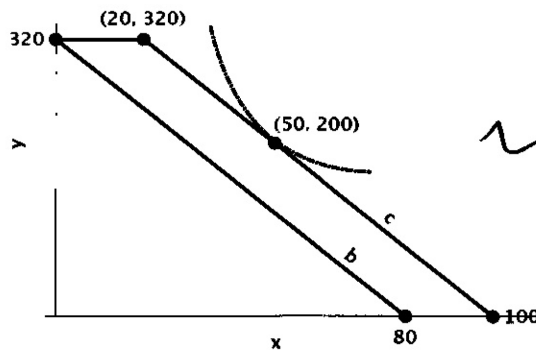
a) $xp_x + yp_y = I$

$$MRS = \frac{y}{x} = \frac{p_x}{p_y}$$

$$\mathbf{x = \frac{I}{2p_x}, y = \frac{I}{2p_y}}$$

b) Illustrated in (c)

c) $(x^*, y^*) = (50, 200)$

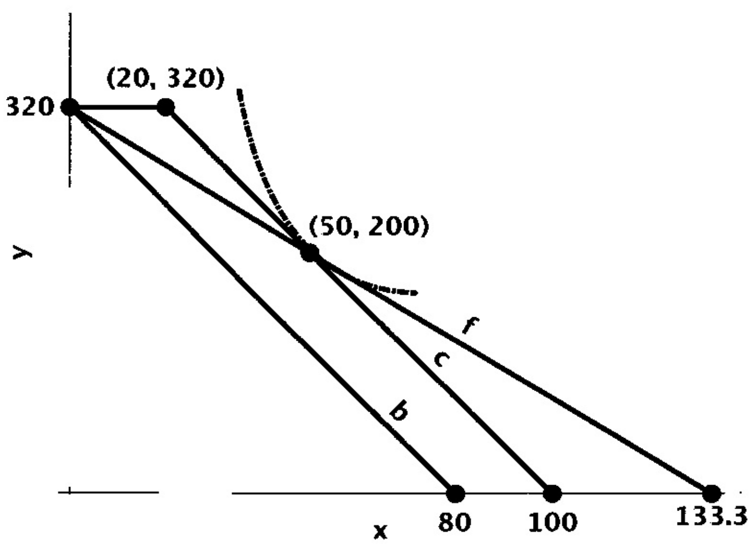


d)
$$20 \text{ gallons} * \frac{\$4}{\text{gallon}} = \$80$$

e) $320 = 50p_x + 320 * 200$

$\mathbf{p_x = \$2.40}$

f)



g) More $(x^*, y^*) = (66.\bar{6}, 160)$

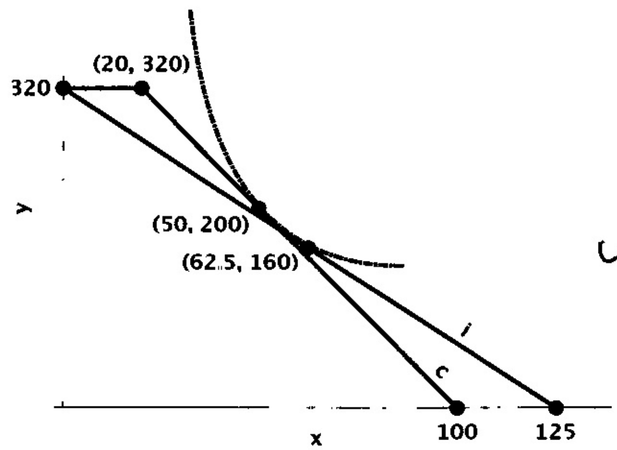
$Cost = 66\bar{6} * (4 - 2.4) = \$106.56$ which exceeds the original cost of \$80

h) $(x^*, y^*) = (62.5, 160)$

$U_c(x_c^*, y_c^*) = (50 * 200)^2 = 10^6$

$U_h(x_h^*, y_h^*) = (62.5 * 160)^2 = 10^6$

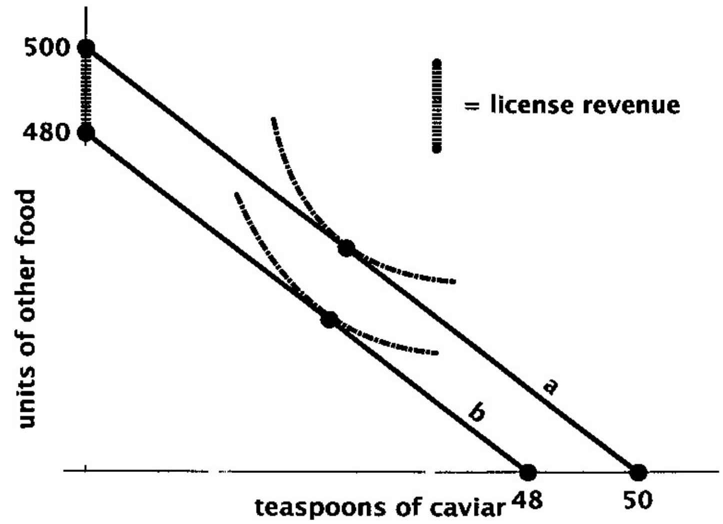
i)



j) $EV - Cost \quad \$80.$

Question 2

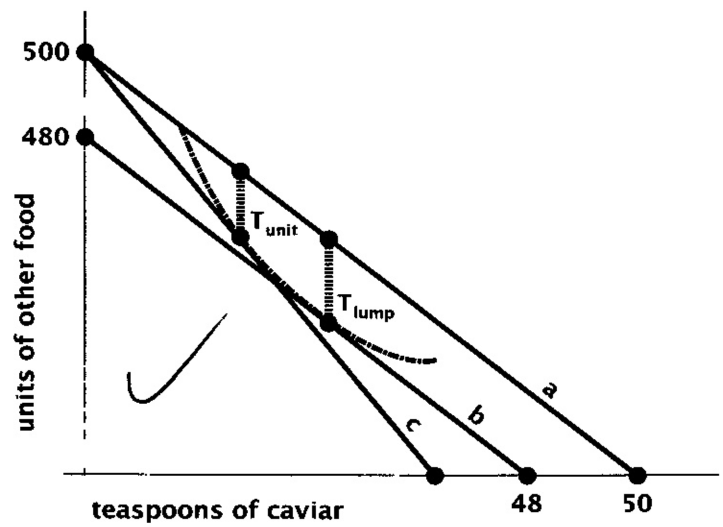
a) Opportunity cost of a teaspoon of caviar is \$10 of other goods



b)

✓

c) $T_{lump} = 20$ T_{unit} does not reach budget line b and therefore must be < 20 If a bundle were chosen below budget line b, then it would be strictly worse than the bundles on b Therefore, it is impossible for more revenue to be raised per unit than via lump sum keeping indifference constant



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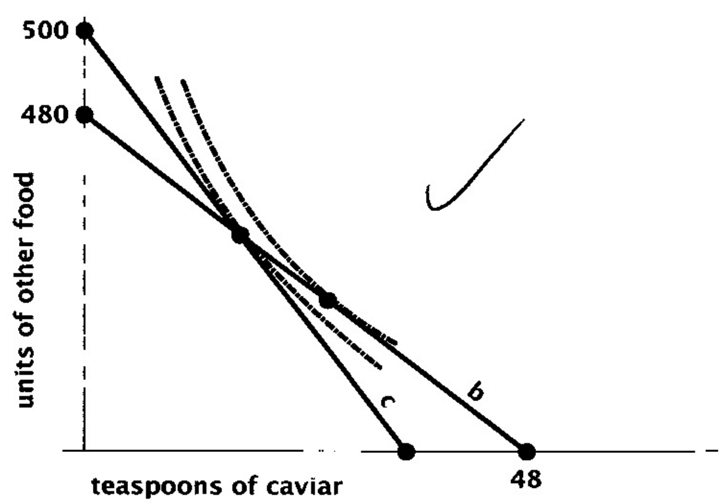
d) The best bundle for the lump sum tax *(b)* is to the right of the indifference curve that is tangent to the budget line for the unit tax *(c)* The best bundle for the lump sum is therefore strictly better than that of the unit tax



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e) 1. $p_x x + p_y y = I$

2. $\dfrac{p_x}{p_y} = \dfrac{y}{x}$

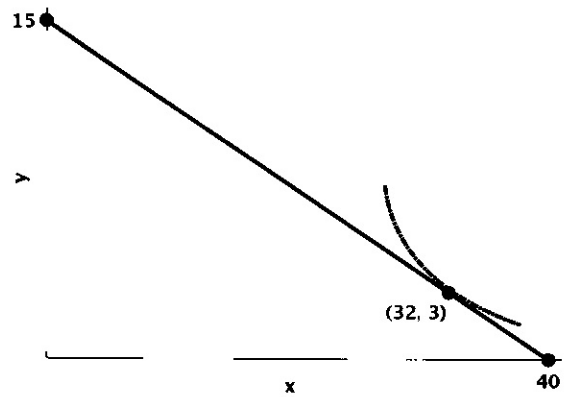
3. $tx = 20$

Question 3

a) $p_x = 3, p_y = 8, I = 120$

$$MRS = \frac{\frac{\partial u}{\partial x}}{\frac{\partial u}{\partial y}} = \frac{p_x}{p_y} \rightarrow \frac{y}{8} = \frac{3}{8} \rightarrow \mathbf{y^* = 3}$$

$$p_x x + p_y y = I \rightarrow 3x + 24 = 120 \rightarrow \mathbf{x^* = 32}$$

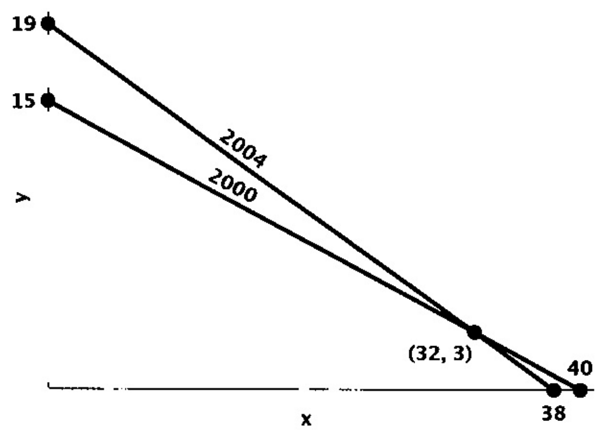


Opportunity cost of one beer is 3/8 of a movie

b) $$FWI = \frac{32*5+3*10}{32*3+3*8}(100) = \mathbf{158.\bar{3}}$$

$$i = FWI - 100 = \mathbf{58.\bar{3}\%}$$

c) $I = 1.58\bar{3} * 120 = \mathbf{190}$



d) $min_{x,y}[xp_x + yp_y = I],\ s.t.\ U(x,y) = \overline{U}$

$$U(32,3) = \overline{U} = 32 + 8\ln 3$$

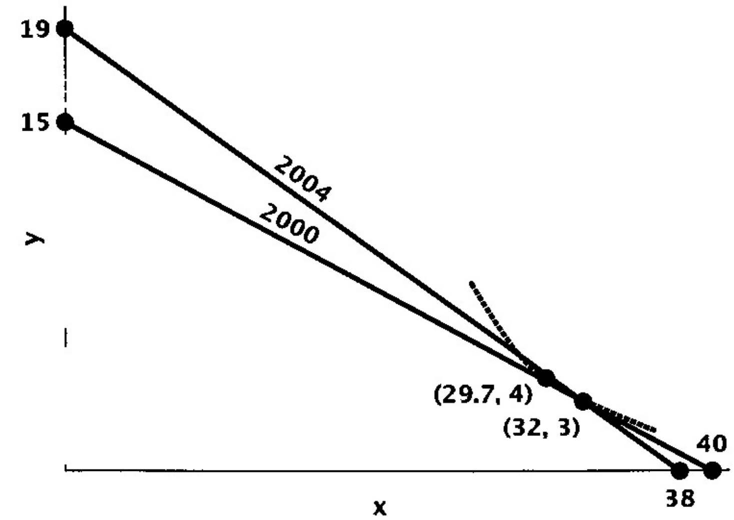
$$MRS = \frac{y}{8} = \frac{p_x}{p_y} = \frac{5}{10} \rightarrow \mathbf{y = 4}$$

$$x + 8\ln 4 = 32 + 8\ln 3 \rightarrow \mathbf{x = -8\left(\ln\frac{4}{3} - 4\right)} \approxeq \mathbf{29.7}$$

e) $C(x,y) \approxeq 5*29.7 + 10*4 = \mathbf{188.5}$

This is smaller than the \$190 her parents gave her in (c). The fixed weight index overcompensates her

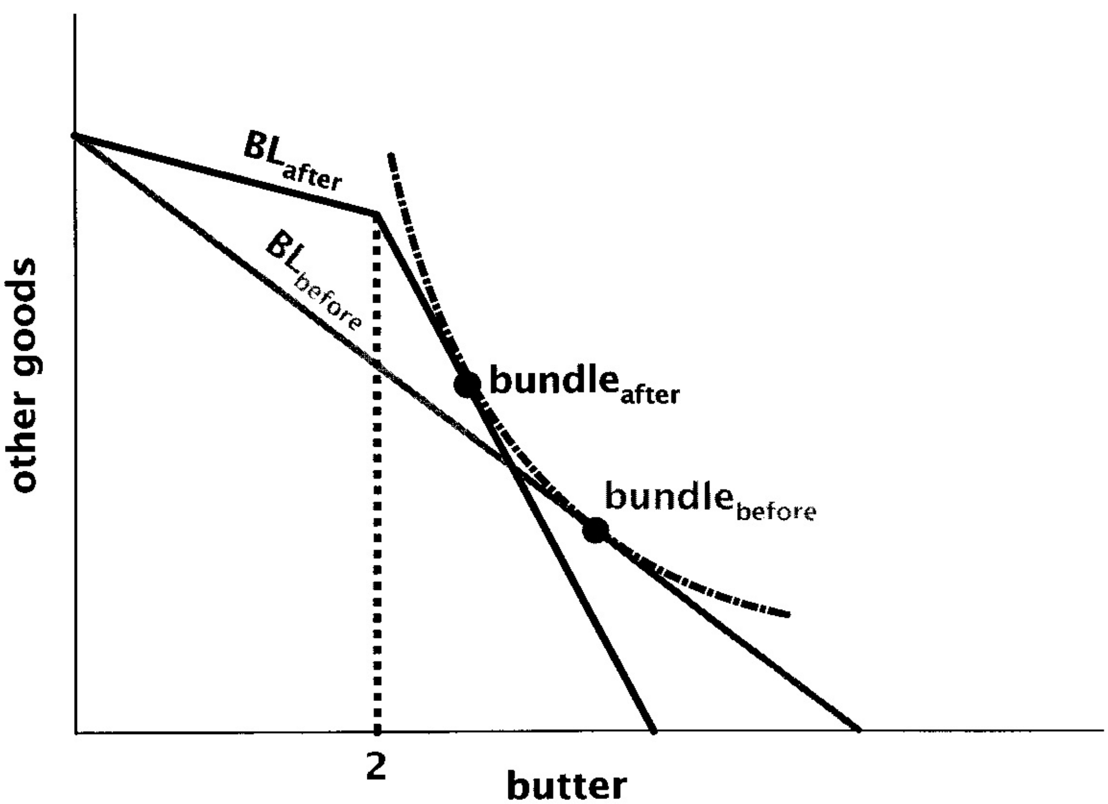
f) The indifference curve is tangent to BL_{2000} at (32,3) and intersects BL_{2004} at (29 7, 4)



Question 4

Given strictly decreasing *MRS*, the indifference curve must be tangent to BL_{after} before the intersection of the two budget lines (where subsidy exceeds tax)

Because the family is indifferent to the change, we know that the indifference curve that repsresents their best bundles is tangent to both BLafter and BLbefore The tangency to BLafter must be before the intersection, if it was after the intersection then it would be strictly worse than all bundles on the 'before' budget line Before the intersection, BL~~[illegible]~~ after is higher than BLbefore, and if the bundle is higher than the original budget line, that means the net is a subsidy, not a tax



Question 5

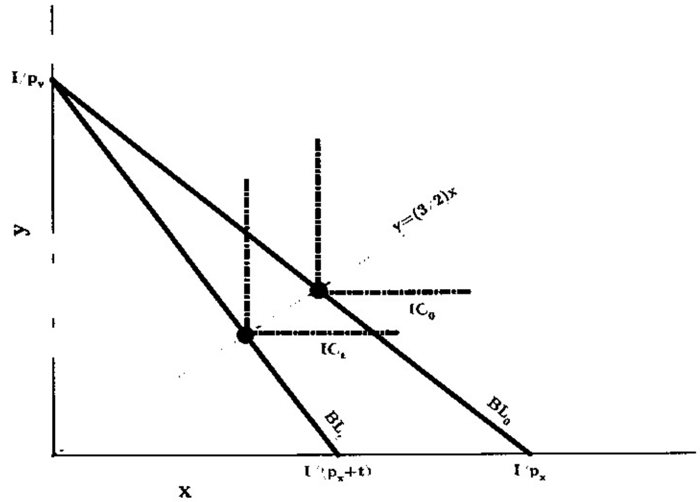
a)

$$y = \frac{3}{2}x$$

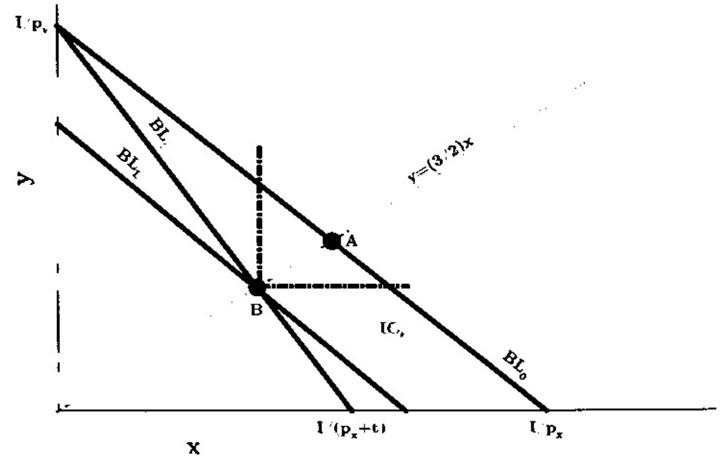
$$p_x x + p_y y = I \rightarrow x\left(p_x + \frac{3}{2}\right) = I \rightarrow \mathbf{x} = \frac{\mathbf{I}}{\mathbf{p_x} + \frac{3}{2}}$$

$$\mathbf{y} = \frac{\frac{3}{2}\mathbf{I}}{\mathbf{p_x} + \frac{3}{2}}$$

b)



c) No Bundle B is the best bundle for both the unit tax and lump sum tax scenario



d) i) $x = \dfrac{I}{p_x + t + \frac{3}{2}}$

ii) $R = tx = \dfrac{tI}{p_x + t + \frac{3}{2}}$

iii) $x = \dfrac{I - R}{p_x + \frac{3}{2}p_y} = \dfrac{I \quad \frac{tI}{p_x + \frac{3}{2}p_y + t}}{p_x + \frac{3}{2}p_y} = \dfrac{I(p_x + \frac{3}{2}p_y + t - tI}{(p_x + \frac{3}{2}p_y + t)(p_x + \frac{3}{2}p_y)} = \dfrac{I(p_x + \frac{3}{2}p_y + t - t)}{(p_x + \frac{3}{2}p_y + t)(p_x + \frac{3}{2}p_y)} = \dfrac{I(p_x + \frac{3}{2}p_y)}{(p_x + \frac{3}{2}p_y + t)(p_x + \frac{3}{2}p_y)}$

$x = \dfrac{I}{p_x + \frac{3}{2}p_y + t}$

$y = \dfrac{3}{2}x = \dfrac{\frac{3}{2}I}{p_x + \frac{3}{2}p_y + t}$

Question 6

$\nabla \frac{1}{3}\ln(x) + \frac{1}{6}\ln(y) + \frac{1}{2}\ln(z) = -\lambda * \nabla(I - x * px - y * py - z * pz)$

$\dfrac{\partial L}{\partial x} = \dfrac{1}{3x} = \lambda px$

$\dfrac{\partial L}{\partial y} = \dfrac{1}{6y\lambda py}$

$\dfrac{\partial L}{\partial z} = \dfrac{1}{2z} = \lambda pz$

$\dfrac{\partial L}{\partial \lambda} = I - xpx - ypz - zpz = 0$

$\dfrac{2y}{3x} = \dfrac{px}{py}$

$xpx - 2ypy = 0$

$\dfrac{z}{3y} = \dfrac{py}{pz}$

$3ypy - zpz = 0$

$$\begin{bmatrix} px & -2py & 0 & 0 \\ 0 & 3py & -pz & 0 \\ px & py & pz & I \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{-2py}{px} & 0 & 0 \\ 0 & 3py & -pz & 0 \\ 0 & 3py & pz & I \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{-2py}{px} & 0 & 0 \\ 0 & 1 & \frac{-pz}{3py} & 0 \\ 0 & 0 & 2pz & I \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{-2py}{px} & 0 & 0 \\ 0 & 1 & 0 & \frac{I}{6py} \\ 0 & 0 & 1 & \frac{I}{2pz} \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & \frac{I}{3px} \\ 0 & 1 & 0 & \frac{I}{6py} \\ 0 & 0 & 2pz & \frac{I}{2pz} \end{bmatrix}$$

$x = \dfrac{I}{3px}$

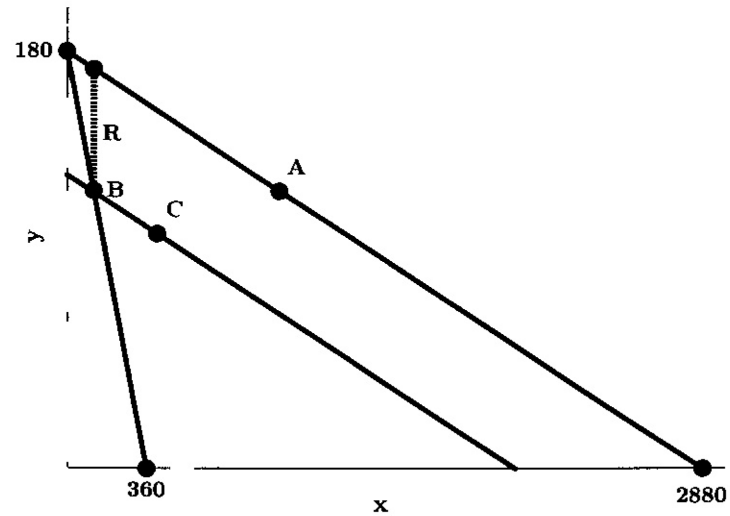
$y = \dfrac{I}{6py}$

$z = \dfrac{I}{2pz}$

Question 7

a) $x = \dfrac{2880}{3*1}, y = \dfrac{2880}{3*16}$

$\mathbf{(x^*, y^*) = (960, 120)}$



b) $x = \dfrac{2880}{3*8}$

$\mathbf{(x^*, y^*) = (120, 120)}$

$\mathbf{U(120, 120) = 1,728,000}$

c) $R = 120 * 7 = 840$

d)

e) $min[p_x x + p_y y = I, s.t. U = \overline{U} = 1,728,000$

f) $MRS = \dfrac{y}{2x} = \dfrac{p_x}{p_y}$

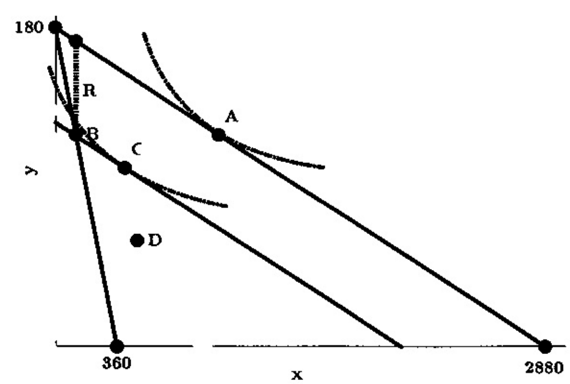
$$U = \left(\frac{2p_x x}{p_y}\right)^2 x = \frac{4x^3 p_x^2}{p_y^2}$$

$$\mathbf{x^h} = \sqrt[3]{\frac{\mathbf{p_y^2 U}}{\mathbf{4p_x^2}}}$$

$$y = \frac{2xp_x}{p_y} = \frac{2\sqrt[3]{\frac{p_y^2 U}{4p_x^2}}\, p_x}{p_y}$$

$$\mathbf{y^h} = \mathbf{2}\sqrt[3]{\frac{\mathbf{p_x U}}{\mathbf{4p_y}}}$$

g) $x^h(1, 16, 1728000) = 480, y^h(1, 16, 1728000) = 60$



h) $EV = 2880 - 1440 = 1440$

$EB = 2040 - 1440 = 600$

i) (480, 60)