

Homework 3

Intermediate Microeconomics

1. Each month Felix purchases household utilities. His preferences over gallons of heating oil (x) and units of other utilities (y) can be represented by the utility function:

$$U(x,y) = (xy)^2.$$

- a) Derive Felix's demand functions for heating oil and other utilities.

Suppose that the market price of heating oil is \$4 per gallon, the price of a unit of other utilities is \$1 and that Felix has \$320 to spend per month on his utilities.

- b) Illustrate Felix's budget set.

Due to the high price of heating oil the federal government gives an in-kind transfer to Felix of 20 free gallons of heating oil per month at a cost of \$80 to the government.

- c) In your diagram for part (b) illustrate Felix's new budget set. Using your demand functions from part (a) find Felix's best bundle on his new (post-transfer) budget line.
d) We can value the in-kind transfer by thinking of the dollar change in income that is equivalent in its effect to the in-kind transfer. What would be the value of the in-kind transfer of 20 gallons of heating oil?

Representative Mock dislikes government handouts and would much prefer to manipulate the market to make Felix better off. So Representative Mock proposes to get rid of the in-kind transfer of heating oil and in its place use a per-unit subsidy to lower the price of heating oil.

His first proposal is to set the subsidy per gallons of heating oil so that the Felix could continue to afford the same bundle that he chose when he received the in-kind transfer).

- e) What would the price of heating oil have to be set at so that Felix could just afford his best bundle from part (c)?
f) Illustrate in your diagram from (b) the budget line associated with the per gallon subsidy.
g) Will Felix buy more or less heating oil than in part (c)? Will the cost to the government be more or less than the \$80 that the government spent on the in-kind transfer? Briefly explain your answer.

His second proposal is to set the value of the subsidy so that Felix is no better off with the subsidy than he was with the in-kind transfer. In particular he proposes to set the price of heating oil at \$2.56 per gallon.

- h) Using your demand functions from part (a) find Felix's best bundle at the new price of \$2.56 per gallon. Verify that Felix is equally happy with this bundle as he was with the bundle from part (c).
i) In a new diagram illustrate the budget line associated with a price of \$2.56 per gallon of heating oil and indicate the best bundle from part (h). Include in your diagram the budget line and best bundle associated with the in kind transfer.
j) What is the equivalent variation of this price change (the drop from \$4 to \$2.56)? What is the cost to the government?

2. Sergei enjoys caviar. In fact he allocates his \$500 food budget between caviar and all other foods. The price of caviar is \$10 per teaspoon and the price of all other food is \$1 per unit. Assume that caviar and all other foods are not perfect complements for Sergei.

- a) Putting teaspoons of caviar on the x-axis and units of all other food on the y-axis illustrate Sergei's budget set. What is the opportunity cost of a teaspoon of caviar?

Due to rising federal deficits the federal government needs to raise revenues. One way to do so is to place a tax on caviar consumption. Senator Johns proposes to levy a *fixed fee* on the purchase of caviar. Under the fixed fee plan Sergei must first purchase a license to buy caviar. Once he has the license he can purchase as much caviar as he would like from his local gourmet shop at the regular price of \$10 per teaspoon. The cost of a license is \$20.

- b) In your diagram for part (a) illustrate Sergei's budget set if he chose to purchase the license. Illustrate preferences (indifference curves) for Sergei such that he would choose to purchase the license to buy caviar. Illustrate in the same diagram Sergei's best bundle after the imposition of the fixed fee and an indifference curve through that bundle. Finally, indicate the tax revenues (denominated in units of other food) generated by the imposition of the fixed fee.

Senator Thomas dislikes the idea of a fixed fee and he proposes instead to simply tax each unit purchased of a teaspoon of caviar. Senator Thomas claims that so long as the correct tax rate is chosen the per-unit tax can be used to raise as much money as the fixed fee and will leave Sergei neither better off nor worse off than the fixed fee plan.

- c) Suppose that the government chooses the value of the per-unit tax so that Sergei is indifferent between the bundle that he chooses under the fixed fee plan and the bundle that he chooses under per-unit tax plan. In a new diagram show that at this tax rate the government will raise strictly less revenue under the per unit tax plan than it did under the fixed fee plan. In your diagram be sure to include all three budget lines (no-tax line, fixed fee line and the per-unit tax line).
- d) Suppose that the government chooses a value for the per-unit tax so that it will raise exactly the same revenue as the fixed fee plan. Briefly explain why Sergei will be strictly worse off with the bundle that he chooses under the per-unit tax plan than he was with bundle that he chose under the fixed fee plan. Illustrate your answer in a new diagram.
- e) Suppose Sergei's preferences can be represented by the utility function $U(x,y) = xy$. Write the 3 equations that can be used to determine the value of the per unit tax rate that would raise the same revenue as the fixed fee.

3. In September 2000 Laura went to college. Her parents gave her \$120 per semester to spend on beer and movies. Laura's preferences can be represented by the utility function $U(x,y) = x + 8 \ln y$ where x is the number of cans of beer she purchases and y is the number of movies she sees. In 2000 the price of a can of beer was \$3 and the price of a movie was \$8.

- a) How many cans of beer would Laura choose to purchase? How many movies would she see? Illustrate Laura's budget set. Include in your diagram Laura's best bundle. What is the opportunity cost of a can of beer?

In September of 2004 Laura went to graduate school. Her parents have agreed to continue to subsidize her leisure activities. In the four years since she first went to college the prices of both of the goods, beer and movies have risen. The price of a can of beer in 2004 was \$5 and the price of a movie was \$10.

- b) Using the bundle that you found in part (a) as your fixed basket and 2000 as your base year, calculate the value of the fixed weight price index in 2004. What was the rate of inflation between 2000 and 2004?
- c) If her parents used the fixed weight price index as a cost of living index (so that they adjust the income they give her to compensate her for inflation) then how much money will her parents give her in 2004? Using this income level and the prices in 2004 illustrate Laura's budget set in 2004. Include in your diagram Laura's original 2000 budget line. In your diagram indicate the location of Laura's 2000 best bundle.

Suppose that Laura's parents know Laura's preferences. In this case they can calculate an ideal cost of living adjustment. In particular Laura's parents can find the least cost way of maintaining her 2000 utility level.

- d) Set up her parents cost minimization problem. Find the solution to the cost minimization problem (in other words find Laura's compensated demands).
- e) What is the (approximate) cost of the cost-minimizing bundle? Is it higher or lower than the amount of money that her parents gave her in part c? Does the fixed weight index over-compensate or under-compensate consumers?
- f) Illustrate the relationship between the bundle that you found in part d and the best bundle in 2000. Include in your diagram an indifference curve and the budget lines from 2000 and 2004.

4. In order to aid the poor the government introduces a scheme whereby the first 2 lb of butter a family buys is subsidized and the remaining amounts are taxed. Illustrate budget sets before and after the government program. Consider a family that consumes butter and is made neither better nor worse off as a result of this scheme. Show that the total amount of tax it pays cannot exceed the subsidy it receives.

5. Suppose that Sheila has preferences represented by $U(x,y) = \min [3x, 2y]$ where x is the number of jars of peanut butter she consumes and y is the number of jars of jelly.

- a) Calculate her demand functions for peanut butter and jelly.
- b) Suppose a per-unit tax, t , is imposed on peanut butter. Illustrate the change in her budget set and the choice of her best bundle.
- c) Instead of a per-unit tax, suppose the government decides to raise the same revenue via a lump-sum tax. Illustrate Sheila's new budget constraint and best bundle. Does her choice depend on the way the tax is collected?
- d) Using the demand functions in part (a) verify your conclusion in (c) by following these steps;
 - i. Find the demands when the price of peanut butter is $P_x + t$.
 - ii. Find the total revenue R as a result of the tax.
 - iii. Find Sheila's demand when the amount R is deducted directly from her income and goods are untaxed.

6. Suppose that an individual consumes 3 goods, clothing (x), shelter (y) and food (z). The preferences over these three goods can be represented by the utility function

$$U(x, y, z) = \frac{1}{3} \ln x + \frac{1}{6} \ln y + \frac{1}{2} \ln z. \text{ The prices of the three goods are } P_x, P_y \text{ and } P_z \text{ respectively.}$$

Find the uncompensated demands for the three goods. Hint: you can solve this two ways. One way would be to set up the Lagrangian and maximize it to solve for the 4 variables (x,y,z,λ). Alternatively you could use the budget line and calculate the MRS between each pair of goods and set each equal to the price ratio (this are our usual tangency and feasibility conditions).

7. Suppose that we represent Herman's preferences for gallons of gasoline (the x-good) and burritos (y-good) with the utility function $u(x, y) = xy^2$. The Marshallian demand functions

associated with this utility function are given by the following demand functions: $x = \frac{I}{3P_x}$ and

$$y = \frac{2I}{3P_y} \text{ where } P_x \text{ and } P_y \text{ are the prices of gallons of gasoline and burritos respectively and } I \text{ is}$$

the income he will spend per week.

- a) If $P_x = \$1$, $P_y = \$16$ and $I = \$2880$ then how many gallons of gasoline and burritos does he demand? Illustrate his best bundle below and label the bundle **A**.

The government decides to levy a per unit tax on gallons of gasoline. The tax rate is set at \$7 per gallon.

- b) What will be his best bundle after the tax raises the price to \$8? Illustrate his new budget line and best bundle in your diagram. Label the best bundle **B**. What is the level of his utility at bundle B?
- c) How much revenue did the per unit tax raise? Illustrate these tax revenues measured in burritos in your diagram.

Suppose instead of a per unit tax the government decides to levy a lump sum tax.

- d) Assume that the government sets the lump sum tax equal to the tax revenues of the per unit tax. Illustrate in your diagram the effect of the lump sum tax on Herman's choice of best bundle. You do not need to calculate his best bundle. Label this best bundle **C**.
- e) Write the expenditure minimization problem for Herman's utility function.
- f) Solve for his Hicksian demands for gasoline and burritos as a function of prices and utility level.
- g) Use your Hicksian demands to find a bundle, D, such that 1) Herman is indifferent between D and B; 2) Herman would choose bundle D at the original prices. What is the cost of D at the original prices? Illustrate bundle D in your indifference curve diagram.
- h) What is the equivalent variation to the price changed caused by the per unit tax on gasoline? What is the excess burden of the tax on gasoline? Illustrate these values in your indifference curve diagram.
- i) What bundle of goods will Herman purchase if prices remain at their original value and his income was reduced by the equivalent variation that you calculated in part (h) above?