CHAPTER 16

Section 16.1

1. All ten values of the quality statistic are between the two control limits, so no out-of-control signal is generated.

2. All ten values are between the two control limits. However, it is readily verified that all but one plotted point fall below the center line (at height .04975). Thus even though no single point generates an out-of-control signal, taken together, the observed values do suggest that there may be a decrease in the average value of the quality statistic. Such a "small" change is more easily detected by a CUSUM procedure (see section 16.5) than by an ordinary chart.

3. P(10 successive points inside the limits) = P(1st inside) x P(2nd inside) x…x P(10th inside) = $(.998)^{10}$ = .9802. P(25 successive points inside the limits) = $(.998)^{25}$ = .9512. $(.998)^{52}$ = .9011, but $(.998)^{53}$ = .8993, so for 53 successive points the probability that at least one will fall outside the control limits when the process is in control is 1 - .8993 = .1007 > .10.

Section 16.2

4. For Z, a standard normal random variable, $P(-c \le Z \le c) = .995$ implies that $\Phi(c) = P(Z \le c) = .995 + \frac{.005}{2} = .9975$. Table A.3 then gives c = 2.81. The appropriate control limits are therefore $m \pm 2.81s$.

5.

a. P(point falls outside the limits when $m = m_0 + .5s$)

$$= 1 - P\left(m_0 - \frac{3s}{\sqrt{n}} < \overline{X} < m_0 + \frac{3s}{\sqrt{n}} \text{ when } m = m_0 + .5s \right)$$
$$= 1 - P\left(-3 - .5\sqrt{n} < Z < 3 - .5\sqrt{n}\right)$$
$$= 1 - P(-4.12 < Z < 1.882) = 1 - .9699 = .0301.$$

b. $$1 - P\left(m_0 - \frac{3s}{\sqrt{n}} < \overline{X} < m_0 + \frac{3s}{\sqrt{n}} \text{ when } m = m_0 - s \right)$$
$$= 1 - P\left(-3 + \sqrt{n} < Z < 3 + \sqrt{n}\right) = 1 - P(-.76 < Z < 5.24) = .2236$$

c. $$1 - P\left(-3 - 2\sqrt{n} < Z < 3 - 2\sqrt{n}\right) = 1 - P(-7.47 < Z < -1.47) = .6808$$

6. The limits are $13.00 \pm \frac{(3)(.6)}{\sqrt{5}} = 13.00 \pm .80$, from which LCL = 12.20 and UCL = 13.80. Every one of the 22 $\bar{x}$ values is well within these limits, so the process appears to be in control with respect to location.

7. $\bar{\bar{x}} = 12.95$ and $\bar{s} = .526$, so with $a_5 = .940$, the control limits are $12.95 \pm 3\frac{.526}{.940\sqrt{5}} = 12.95 \pm .75 = 12.20, 13.70$. Again, every point $(\bar{x})$ is between these limits, so there is no evidence of an out-of-control process.

8. $\bar{r} = 1.336$ and $b_5 = 2.325$, yielding the control limits $12.95 \pm 3\frac{1.336}{2.325\sqrt{5}} = 12.95 \pm .77 = 12.18, 13.72$. All points are between these limits, so the process again appears to be in control with respect to location.

9. $\bar{\bar{x}} = \frac{2317.07}{24} = 96.54$, $\bar{s} = 1.264$, and $a_6 = .952$, giving the control limits $96.54 \pm 3\frac{1.264}{.952\sqrt{6}} = 96.54 \pm 1.63 = 94.91, 98.17$. The value of $\bar{x}$ on the 22nd day lies above the UCL, so the process appears to be out of control at that time.

10. Now $\bar{\bar{x}} = \frac{2317.07 - 98.34}{23} = 96.47$ and $\bar{s} = \frac{30.34 - 1.60}{23} = 1.250$, giving the limits $96.47 \pm 3\frac{1.250}{.952\sqrt{6}} = 96.47 \pm 1.61 = 94.86, 98.08$. All 23 remaining $\bar{x}$ values are between these limits, so no further out-of-control signals are generated.

11.

a. $P\left(m_0 - \frac{2.81s}{\sqrt{n}} < \bar{X} < m_0 + \frac{2.81s}{\sqrt{n}} \; when m = m_0 \right)$

$= P(-2.81 < Z < 2.81) = .995$, so the probability that a point falls outside the limits is .005 and $ARL = \frac{1}{.005} = 200$.

b. P = P(a point is outside the limits)

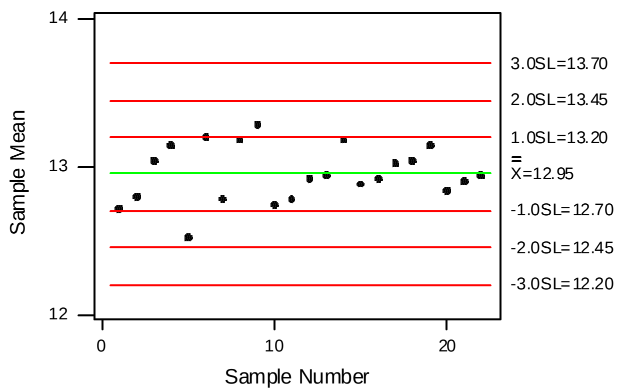
$$= 1 - P\left(m_0 - \frac{2.81s}{\sqrt{n}} < \overline{X} < m_0 + \frac{2.81s}{\sqrt{n}} \, when m = m_0 + s\right)$$

$$= 1 - P\left(-2.81 - \sqrt{n} < Z < 2.81 - \sqrt{n}\right)$$

$$= 1 - P\left(-4.81 < Z < .81\right) = 1 - .791 = .209$$. Thus $ARL = \frac{1}{.209} = 4.78$

c. 1 - .9974 = .0026 so $ARL = \frac{1}{.0026} = 385$ for an in-control process, and when $m = m_0 + s$, the probability of an out-of-control point is $1 - P(-3 - 2 < Z < 1)$ $= 1 - P(Z < 1) = .1587$, so $ARL = \frac{1}{.1587} = 6.30$.

12.



The 3-sigma control limits are from problem 7. The 2-sigma limits are $12.95 \pm .50 = 12.45, 13.45$, and the 1-sigma limits are $12.95 \pm .25 = 12.70, 13.20$. No points fall outside the 2-sigma limits, and only two points fall outside the 1-sigma limits. There are also no runs of eight on the same side of the center line – the longest run on the same side of the center line is four (the points at times 10, 11, 12, 13). No out-of-control signals result from application of the supplemental rules.

13. $\overline{\overline{x}} = 12.95$, IQR = .4273, $k_5 = .990$. The control limits are

$$12.95 \pm 3\frac{.4273}{.990\sqrt{5}} = 12.45, 13.45 = 12.37, 13.53.$$

Section 16.3

14. $\Sigma s_i = 4.895$ and $\bar{s} = \frac{4.895}{24} = .2040$. With $a_5 = .940$, the lower control limit is zero and the upper limit is $.2040 + \frac{3(.2040)\sqrt{1-(.940)^2}}{.940} = .2040 + .2221 = .4261$. Every s_I is between these limits, so the process appears to be in control with respect to variability.

15.

a. $\bar{r} = \frac{85.2}{30} = 2.84$, $b_4 = 2.058$, and $c_4 = .880$. Since n = 4, LCL = 0 and UCL $= 2.84 + \frac{3(.880)(2.84)}{2.058} = 2.84 + 3.64 = 6.48$.

b. $\bar{r} = 3.54$, $b_8 = 2.844$, and $c_8 = .820$, and the control limits are $= 3.54 \pm \frac{3(.820)(3.54)}{2.844} = 3.54 \pm 3.06 = .48, 6.60$.

16. $\bar{s} = .5172$, $a_5 = .940$, LCL = 0 (since n = 5) and UCL = $.5172 + \frac{3(.5172)\sqrt{1-(.940)^2}}{.940} = .5172 + .5632 = 1.0804$. The largest s_I is s_9 = .963, so all points fall between the control limits.

17. $\bar{s} = 1.2642$, $a_6 = .952$, and the control limits are $1.2642 \pm \frac{3(1.2642)\sqrt{1-(.952)^2}}{.952} = 1.2642 \pm 1.2194 = .045, 2.484$. The smallest s_I is s_{20} = .75, and the largest is s_{12} = 1.65, so every value is between .045 and 2.434. The process appears to be in control with respect to variability.

18. $\Sigma s_i^2 = 39.9944$ and $\bar{s}^2 = \frac{39.9944}{24} = 1.6664$, so LCL = $\frac{(1.6664)(.210)}{5} = .070$, and UCL = $\frac{(1.6664)(20.515)}{5} = 6.837$. The smallest s^2 value is $s_{20}^2 = (.75)^2 = .5625$ and the largest is $s_{12}^2 = (1.65)^2 = 2.723$, so all s_i^2's are between the control limits.

Section 16.4

19. $\overline{p} = \Sigma \frac{\hat{p}_i}{k}$ where $\Sigma \hat{p}_i = \frac{x_1}{n} + ... + \frac{x_k}{n} = \frac{x_1 + ... + x_k}{n} = \frac{578}{100} = 5.78$. Thus $\overline{p} = \frac{5.78}{25} = .231$.

a. The control limits are $.231 \pm 3\sqrt{\frac{(.231)(.769)}{100}} = .231 \pm .126 = .105, .357$.

b. $\frac{13}{100} = .130$, which is between the limits, but $\frac{39}{100} = .390$, which exceeds the upper control limit and therefore generates an out-of-control signal.

20. $\Sigma x_i = 567$, from which $\overline{p} = \frac{\Sigma x_i}{nk} = \frac{567}{(200)(30)} = .0945$. The control limits are $.0945 \pm 3\sqrt{\frac{(.0945)(.9055)}{200}} = .0945 \pm .0621 = .0324, .1566$. The smallest x_i is $x_7 = 7$, with $\hat{p}_7 = \frac{7}{200} = .0350$. This (barely) exceeds the LCL. The largest x_i is $x_5 = 37$, with $\hat{p}_5 = \frac{37}{200} = .185$. Thus $\hat{p}_5 > UCL = .1566$, so an out-of-control signal is generated. This is the only such signal, since the next largest x_i is $x_{25} = 30$, with $\hat{p}_{25} = \frac{30}{200} = .1500 < UCL$.

21. LCL > 0 when $\overline{p} > 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$, i.e. (after squaring both sides) $50\overline{p}^2 > 3\overline{p}(1-\overline{p})$, i.e. $50\overline{p} > 3(1-\overline{p})$, i.e. $53\overline{p} > 3 \Rightarrow \overline{p} = \frac{3}{53} = .0566$.

22. The suggested transformation is $Y = h(X) = \sin^{-1}\left(\sqrt{X/n}\right)$, with approximate mean value $\sin^{-1}\left(\sqrt{p}\right)$ and approximate variance $\frac{1}{4n}$. $\sin^{-1}\left(\sqrt{x_1/n}\right) = \sin^{-1}\left(\sqrt{.050}\right) = .2255$ (in radians), and the values of $y_i = \sin^{-1}\left(\sqrt{x_i/n}\right)$ for i = 1, 2, 3, …, 30 are

0.2255	0.2367	0.2774	0.3977
0.3047	0.3537	0.3381	0.2868
0.3537	0.3906	0.2475	0.2367
0.2958	0.2774	0.3218	0.3218
0.4446	0.2868	0.2958	0.2678
0.3133	0.3300	0.3047	0.3835
0.1882	0.3047	0.2475	
0.3614	0.2958	0.3537	

These give $\Sigma y_i = 9.2437$ and $\bar{y} = .3081$. The control limits are $\bar{y} \pm 3\sqrt{\frac{1}{4n}} = .3081 \pm 3\sqrt{\frac{1}{800}} = .3081 \pm .1091 = .2020, .4142$. In contrast ot the result of exercise 20, there I snow one point below the LCL (.1882 < .2020) as well as one point above the UCL.

23. $\Sigma x_i = 102$, $\bar{x} = 4.08$, and $\bar{x} \pm 3\sqrt{\bar{x}} = 4.08 \pm 6.06 \approx (-2.0, 10.1)$. Thus LCL = 0 and UCL = 10.1. Because no x_i exceeds 10.1, the process is judged to be in control.

24. $\bar{x} - 3\sqrt{\bar{x}} < 0$ is equivalent to $\sqrt{\bar{x}} < 3$, i.e. $\bar{x} < 9$.

25. With $u_i = \frac{x_i}{g_i}$, the u_i's are 3.75, 3.33, 3.75, 2.50, 5.00, 5.00, 12.50, 12.00, 6.67, 3.33, 1.67, 3.75, 6.25, 4.00, 6.00, 12.00, 3.75, 5.00, 8.33, and 1.67 for I = 1, …, 20, giving $\bar{u} = 5.5125$. For $g_i = .6$, $\bar{u} \pm 3\sqrt{\frac{\bar{u}}{g_i}} = 5.5125 \pm 9.0933$, LCL = 0, UCL = 14.6. For $g_i = .8$, $\bar{u} \pm 3\sqrt{\frac{\bar{u}}{g_i}} = 5.5125 \pm 7.857$, LCL = 0, UCL = 13.4. For $g_i = 1.0$, $\bar{u} \pm 3\sqrt{\frac{\bar{u}}{g_i}} = 5.5125 \pm 7.0436$, LCL = 0, UCL = 12.6. Several u_i's are close to the corresponding UCL's but none exceed them, so the process is judged to be in control.

26. $y_i = 2\sqrt{x_i}$ and the y_i's are 3/46, 5.29, 4.47, 4.00, 2.83, 5.66, 4.00, 3.46, 3.46, 4.90, 5.29, 2.83, 3.46, 2.83, 4.00, 5.29, 3.46, 2.83, 4.00, 4.00, 2.00, 4.47, 4.00, and 4.90 for I = 1, …, 25, from which $\Sigma y_i = 98.35$ and $\bar{y} = 3.934$. Thus $\bar{y} \pm 3 = 3.934 \pm 3 = .934, 6.934$. Since every y_i is well within these limits it appears that the process is in control.

Section 16.5

27. $m_0 = 16$, $k = \frac{\Delta}{2} = 0.05$, $h = .20$, $d_i = \max\left(0, d_{i-1} + \left(\bar{x}_i - 16.05\right)\right)$, $e_i = \max\left(0, e_{i-1} + \left(\bar{x}_i - 15.95\right)\right)$.

i	$\bar{x}_i - 16.05$	d_i	$\bar{x}_i - 15.95$	e_i
1	-0.058	0	0.024	0
2	0.001	0.001	0.101	0
3	0.016	0.017	0.116	0
4	-0.138	0	-0.038	0.038
5	-0.020	0	0.080	0
6	0.010	0.010	0.110	0
7	-0.068	0	0.032	0
8	-0.151	0	-0.054	0.054
9	-0.012	0	0.088	0
10	0.024	0.024	0.124	0
11	-0.021	0.003	0.079	0
12	-0.115	0	-0.015	0.015
13	-0.018	0	0.082	0
14	-0.090	0	0.010	0
15	0.005	0.005	0.105	0

For no time r is it the case that $d_r > .20$ or that $e_r > .20$, so no out-of-control signals are generated.

28. $m_0 = .75$, $k = \dfrac{\Delta}{2} = 0.001$, $h = .003$, $d_i = \max\left(0, d_{i-1} + \left(\bar{x}_i - .751\right)\right)$,
$e_i = \max\left(0, e_{i-1} + \left(\bar{x}_i - .749\right)\right)$.

i	$\bar{x}_i - .751$	d_i	$\bar{x}_i - .749$	e_i
1	-.0003	0	.0017	0
2	-.0006	0	.0014	0
3	-.0018	0	.0002	0
4	-.0009	0	.0011	0
5	-.0007	0	.0013	0
6	.0000	0	.0020	0
7	-.0020	0	.0000	0
8	-.0013	0	.0007	0
9	-.0022	0	-.0002	.0002
10	-.0006	0	.0014	0
11	.0006	.0006	.0026	0
12	-.0038	0	-.0018	.0018
13	-.0021	0	-.0001	.0019
14	-.0027	0	-.0007	.0026
15	-.0039	0	-.0019	.0045*
16	-.0012	0	.0008	.0037
17	-.0050	0	-.0030	.0067
18	-.0028	0	-.0008	.0075
19	-.0040	0	-.0020	.0095
20	-.0017	0	.0003	.0092
21	-.0048	0	-.0028	.0120
22	-.0029	0	-.0009	.0129

Clearly $e_{15} = .0045 > .003 = h$, suggesting that the process mean has shifted to a value smaller than the target of .75.

29. Connecting 600 on the in-control ARL scale to 4 on the out-of-control scale and extending to the k' scale gives k' = .87. Thus $k' = \dfrac{\Delta/2}{s/\sqrt{n}} = \dfrac{.002}{.005/\sqrt{n}}$ from which $\sqrt{n} = 2.175 \Rightarrow n = 4.73 = s$. Then connecting .87 on the k' scale to 600 on the out-of-control ARL scale and extending to h' gives h' = 2.8, so

$$h = \left(\frac{s}{\sqrt{n}}\right)(2.8) = \left(\frac{.005}{\sqrt{5}}\right)(2.8) = .00626 .$$

30. In control ARL = 250, out-of-control ARL = 4.8, from which

$k' = .7 = \dfrac{\Delta/2}{s/\sqrt{n}} = \dfrac{s\;/2}{s\;/\sqrt{n}} = \dfrac{\sqrt{n}}{2}$. So $\sqrt{n} = 1.4 \Rightarrow n = 1.96 \approx 2$. Then h' = 2.85,

giving $h = \left(\dfrac{s}{\sqrt{n}}\right)(2.85) = 2.0153s$.

Section 16.6

31. For the binomial calculation, n = 50 and we wish

$$P(X \le 2) = \binom{50}{0}p^0(1-p)^{50} + \binom{50}{1}p^1(1-p)^{49} + \binom{50}{2}p^2(1-p)^{48}$$

$= (1-p)^{50} + 50p(1-p)^{49} + 1225p^2(1-p)^{48}$ when p = .01, .02, …, .10. For the hypergeometric calculation

$$P(X \le 2) = \frac{\binom{M}{0}\binom{500-M}{50}}{\binom{500}{50}} + \frac{\binom{M}{1}\binom{500-M}{49}}{\binom{500}{50}} + \frac{\binom{M}{2}\binom{500-M}{48}}{\binom{500}{50}}$$, to be

calculated for M = 5, 10, 15, …, 50. The resulting probabilities appear in the answer section in the text.

32. $P(X \le 1) = \binom{50}{0}p^0(1-p)^{50} + \binom{50}{1}p^1(1-p)^{49} = (1-p)^{50} + 50p(1-p)^{49}$

p	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10
$P(X \le 1)$	.9106	.7358	.5553	.4005	.2794	.1900	.1265	.0827	.0532	.0338

33. $P(X \le 2) = \binom{100}{0}p^0(1-p)^{100} + \binom{100}{1}p^1(1-p)^{99} + \binom{100}{2}p^2(1-p)^{98}$

p	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10
$P(X \le 2)$	.9206	.6767	.4198	.2321	.1183	.0566	.0258	.0113	.0048	.0019

For values of p quite close to 0, the probability of lot acceptance using this plan is larger than that for the previous plan, whereas for larger p this plan is less likely to result in an “accept the lot” decision (the dividing point between “close to zero” and “larger p” is someplace between .01 and .02). In this sense, the current plan is better.

34. $\dfrac{LTPD}{AQL}=\dfrac{.07}{.02}=3.5\approx 3.55$, which appears in the $\dfrac{p_1}{p_2}$ column in the c = 5 row. Then

$$n=\frac{np_1}{p_1}=\frac{2.613}{.02}=130.65\approx 131.$$

$$P(X>5 \text{ when } p=.02)=1-\sum_{x=0}^{5}\binom{131}{x}(.02)^x(.98)^{131-x}=.0487\approx .05$$

$$P(X\le 5 \text{ when } p=.07)=\sum_{x=0}^{5}\binom{131}{x}(.07)^x(.93)^{131-x}=.0974\approx .10$$

35. P(accepting the lot) = P(X_1 = 0 or 1) + P(X_1 = 2, X_2 = 0, 1, 2, or 3) + P(X_1 = 3, X_2 = 0, 1, or 2) = P(X_1 = 0 or 1) + P(X_1 = 2)P(X_2 = 0, 1, 2, or 3) + P(X_1 = 3)P(X_2 = 0, 1, or 2).

p = .01: $=.9106+(.0756)(.9984)+(.0122)(.9862)=.9981$

p = .05: $=.2794+(.2611)(.7604)+(.2199)(.5405)=.5968$

p = .10: $=.0338+(.0779)(.2503)+(.1386)(.1117)=.0688$

36. P(accepting the lot) = P(X_1 = 0 or 1) + P(X_1 = 2, X_2 = 0 or 1) + P(X_1 = 3, X_2 = 0) [since $c_2 = r_1 - 1 = 3$] = P(X_1 = 0 or 1) + P(X_1 = 2)P(X_2 = 0 or 1) + P(X_1 = 3)P(X_2 = 0)

$$=\sum_{x=0}^{1}\binom{50}{x}p^x(1-p)^{50-x}+\binom{50}{2}p^2(1-p)^{48}\cdot\sum_{x=0}^{1}\binom{100}{x}p^x(1-p)^{100-x}$$

$$=\binom{50}{3}p^3(1-p)^{47}\cdot\binom{100}{0}p^0(1-p)^{100}.$$

p = .02: $=.7358+(.1858)(.4033)+(.0607)(.1326)=.8188$

p = .05: $=.2794+(.2611)(.0371)+(.2199)(.0059)=.2904$

p = .10: $=.0338+(.0779)(.0003)+(.1386)(.0000)=.0038$

37.

a. $AOQ = pP(A) = p[(1-p)^{50} + 50p(1-p)^{49} + 1225p^2(1-p)^{48}]$

p	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10
AOQ	.010	.018	.024	.027	.027	.025	.022	.018	.014	.011

b. p = .0447, AOQL = .0447P(A) = .0274

c. ATI = 50P(A) + 2000(1 – P(A))

p	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10
ATI	77.3	202.1	418.6	679.9	945.1	1188.8	1393.6	1559.3	1686.1	1781.6

38. $AOQ = pP(A) = p[(1-p)^{50} + 50p(1-p)^{49}]$. Exercise 32 gives P(A), so multiplying each entry in the second row by the corresponding entry in the first row gives AOQ:

p	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10
AOQ	.0091	.0147	.0167	.0160	.0140	.0114	.0089	.0066	.0048	.0034

ATI = 50P(A) + 2000(1 – P(A))

p	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10
ATI	224.3	565.2	917.2	1219.0	1455.2	1629.5	1753.3	1838.7	1896.3	1934.1

$\frac{d}{dp}AOQ = \frac{d}{dp}\left[pP(A) = p[(1-p)^{50} + 50p(1-p)^{49}]\right] = 0$ gives the quadratic equation $2499p^2 - 48p - 1 = 0$, from which $p = \frac{48 + 110.91}{4998} = .0318$, and $AOQL = .0318P(A) \approx .0167$.

Supplementary Exercises

39. n = 6, k = 26, $\Sigma \bar{x}_i = 10{,}980$, $\bar{\bar{x}} = 422.31$, $\Sigma s_i = 402$, $\bar{s} = 15.4615$, $\Sigma r_i = 1074$, $\bar{r} = 41.3077$

S chart: $15.4615 \pm \dfrac{3(15.4615)\sqrt{1-(.952)^2}}{.952} = 15.4615 \pm 14.9141 \approx .55, 30.37$

R chart: $41.31 \pm \dfrac{3(.848)(41.31)}{2.536} = 41.31 \pm 41.44$, so LCL = 0, UCL = 82.75

$\bar{X}$ chart based on $\bar{s}$: $422.31 \pm \dfrac{3(15.4615)}{.952\sqrt{6}} = 402.42, 442.20$

$\bar{X}$ chart based on $\bar{r}$: $422.31 \pm \dfrac{3(41.3077)}{2.536\sqrt{6}} = 402.36, 442.26$

40. A c chart is appropriate here. $\Sigma x_i = 92$ so $\bar{x} = \dfrac{92}{24} = 3.833$, and

$\bar{x} \pm 3\sqrt{\bar{x}} = 3.833 \pm 5.874$, giving LCL = 0 and UCL = 9.7. Because $x_{22} = 10 >$ UCL, the process appears to have been out of control at the time that the 22nd plate was obtained.

41.

i	$\bar{x}_i$	s_i	r_i
1	50.83	1.172	2.2
2	50.10	.854	1.7
3	50.30	1.136	2.1
4	50.23	1.097	2.1
5	50.33	.666	1.3
6	51.20	.854	1.7
7	50.17	.416	.8
8	50.70	.964	1.8
9	49.93	1.159	2.1
10	49.97	.473	.9
11	50.13	.698	.9
12	49.33	.833	1.6
13	50.23	.839	1.5
14	50.33	.404	.8
15	49.30	.265	.5
16	49.90	.854	1.7
17	50.40	.781	1.4
18	49.37	.902	1.8
19	49.87	.643	1.2
20	50.00	.794	1.5
21	50.80	2.931	5.6
22	50.43	.971	1.9

$\Sigma s_i = 19.706$, $\bar{s} = .8957$, $\Sigma \bar{x}_i = 1103.85$, $\bar{\bar{x}} = 50.175$, $a_3 = .886$, from which an s chart has LCL = 0 and UCL = $.8957 + \dfrac{3(.8957)\sqrt{1-(.886)^2}}{.886} = 2.3020$, and $s_{21} = 2.931 > UCL$. Since an assignable cause is assumed to have been identified we eliminate the 21st group. Then $\Sigma s_i = 16.775$, $\bar{s} = .7998$, $\bar{\bar{x}} = 50.145$. The resulting UCL for an s chart is 2.0529, and $s_i < 2.0529$ for every remaining i. The $\bar{x}$ chart based on $\bar{s}$ has limits $50.145 \pm \dfrac{3(.7988)}{.886\sqrt{3}} = 48.58, 51.71$. All $\bar{x}_i$ values are between these limits.

42. $\bar{p} = .0608$, n = 100, so $UCL = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} = 6.08 + 3\sqrt{6.08(.9392)}$
$= 6.08 + 7.17 = 13.25$ and LCL = 0. All points are between these limits, as was the case for the p-chart. The p-chart and np-chart will always give identical results since

$$\bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} < \hat{p}_i < \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \quad \textbf{iff}$$

$$n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} < n\hat{p}_i = x_i < n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$$

43. $\Sigma n_i = 4(16) + (3)(4) = 76$, $\Sigma n_i \bar{x}_i = 32{,}729.4$, $\bar{\bar{x}} = 430.65$,

$s^2 = \dfrac{\Sigma(n_i - 1)s_i^2}{\Sigma(n_i - 1)} = \dfrac{27{,}380.16 - 5661.4}{76 - 20} = 590.0279$, so s = 24.2905. For variation:

when n = 3, $UCL = 24.2905 + \dfrac{3(24.2905)\sqrt{1-(.886)^2}}{.886} = 24.29 + 38.14 = 62.43$,

when n = 4, $UCL = 24.2905 + \dfrac{3(24.2905)\sqrt{1-(.921)^2}}{.921} = 24.29 + 30.82 = 55.11$.

For location: when n = 3, $430.65 \pm 47.49 = 383.16, 478.14$, and when n = 4, $430.65 \pm 39.56 = 391.09, 470.21$.

44.

a. Provided the $E(\bar{X}_i) = m$ for each i,

$$E(W_t) = aE(\bar{X}_t) + a(1-a)E(\bar{X}_{t-1}) + \ldots + a(1-a)^{t-1}E(\bar{X}_1) + (1-a)^t m$$
$$= m\left[a + a(1-a) + \ldots + a(1-a)^{t-1} + (1-a)^t\right]$$
$$= m\left[a\left(1 + (1-a) + \ldots + (1-a)^{t-1}\right) + (1-a)^t\right]$$
$$= m\left[a\sum_{i=0}^{\infty}(1-a)^i - a\sum_{i=t}^{\infty}(1-a)^i + (1-a)^t\right]$$
$$= m\left[\frac{a}{1-(1-a)} - a(1-a)^t \cdot \frac{1}{1-(1-a)} + (1-a)^t\right] = m$$

b. $V(W_t) = a^2 V(\bar{X}_t) + a^2(1-a)^2 V(\bar{X}_{t-1}) + \ldots + a^2(1-a)^{2(t-1)}V(\bar{X}_1)$

$$= a^2\left[1 + (1-a)^2 + \ldots + (1-a)^{2(t-1)}\right] \cdot V(\bar{X}_1)$$

$= a^2\left[1 + C + \ldots + C^{t-1}\right] \cdot \dfrac{s^2}{n}$ (where $C = (1-a)^2$.)

$= a^2 \dfrac{1-C^t}{1-C} \cdot \dfrac{s^2}{n}$, which gives the desired expression.

c. From Example 16.8, $s = .5$ (or $\bar{s}$ can be used instead). Suppose that we use $a = .6$ (not specified in the problem). Then

$$w_0 = m_0 = 40$$
$$w_1 = .6\bar{x}_1 + .4m_0 = .6(40.20) + .4(40) = 40.12$$
$$w_2 = .6\bar{x}_2 + .4w_1 = .6(39.72) + .4(40.12) = 39.88$$
$$w_3 = .6\bar{x}_3 + .4w_2 = .6(40.42) + .4(39.88) = 40.20$$
$$w_4 = 40.07,\ w_5 = 40.06,\ w_6 = 39.88,\ w_7 = 39.74,\ w_8 = 40.14,$$
$$w_9 = 40.25,\ w_{10} = 40.00,\ w_{11} = 40.29,\ w_{12} = 40.36,\ w_{13} = 40.51,$$
$$w_{14} = 40.19,\ w_{15} = 40.21,\ w_{16} = 40.29$$

$$s_1^2 = \frac{.6\left[1-(1-.6)^2\right]}{2-.6} \cdot \frac{.25}{4} = .0225,\ s_1 = .1500,$$
$$s_2^2 = \frac{.6\left[1-(1-.6)^4\right]}{2-.6} \cdot \frac{.25}{4} = .0261,\ s_2 = .1616,$$
$$s_3 = .1633,\ s_4 = .1636,\ s_5 = .1637 = s_6 \ldots s_{16}$$

Control limits are:

For t = 1, $40 \pm 3(.1500) = 39.55, 40.45$
For t = 2, $40 \pm 3(.1616) = 39.52, 40.48$
For t = 3, $40 \pm 3(.1633) = 39.51, 40.49$.
These last limits are also the limits for t = 4, …, 16.

Because $w_{13} = 40.51 > 40.49 =$ UCL, an out-of-control signal is generated.