Homework Assignment 4

1. Charles likes to drink sparkling waters. His preferences on San Pelligrino water (x) and Vintage seltzer water (y) can be represented by the utility function $U(x,y) = 4\ln x + \ln y$ where both are measured in liters.

   a) If the price of San Pelligrino is P_x per liter, the price of Vintage is P_y per liter and he has $I to spend each month on sparkling waters then find his demand functions for San Pelligrino and Vintage.
   b) Suppose that $P_x = \$1.50$, $P_y = \$.50$ and $I = \$60$. Using your answer to (a) find his demands for San Pelligrino and Vintage. Illustrate your answer in an indifference curve diagram. Include in your diagram an indifference curve through his best bundle.

The domestic sparkling water industry would like to boost the sales of a domestic sparkling waters. As a consequence the industry (which includes the makers of Vintage) lobbies congress to place a tariff on the imports of foreign sparkling water (including San Pelligrino). The effect of the tariff is to raise the price of San Pelligrino to $2 per liter.

   c) After the price increase what are Charles's new demands for San Pelligrino and Vintage? Illustrate the effect of the subsidy in your diagram for part (b). In your illustration be sure to clearly label the substitution and income effects.
   d) Did the tariff on San Pelligrino achieve the ends of domestic sparkling water producers to boost the demand for domestic sparkling water? Use your substitution and income effects to explain your answer.

2. Edith's preferences over donuts (x) and croissants (y) can be represented by the function $u(x,y) = \min[2x,y]$.

   a) Find Edith's Marshallian demand functions for donuts and croissants.
   b) If $P_x = 2$, $P_y = 3$ and $I = 80$ then how many donuts and croissants does she buy? Illustrate your answer in a diagram.
   c) If income rises to 88, then what will be her new best bundle? Illustrate the new budget line and best bundle in your diagram.
   d) Calculate the income and the own price elasticity of Edith's demand for donuts. Write your answer as functions of prices and income only

   In class we derived the equality $\theta_x \eta_x + \theta_y \eta_y = 1$ where $\theta_x = \dfrac{P_x x}{I}$ is the share of income

   spent on the x good and η_x is the income elasticity of the x good, donuts (similarly θ_y and η_y are the share of income and the income elasticity of the y good, croissants.

   e) Use your income elasticities above to verify that this expression holds for Edith's demands.
   f) In general (not just for Edith's demands) how does θ_x vary when I changes (i.e. find $\dfrac{\partial \theta_x}{\partial I}$)?
   g) If $\eta_x < 1$ then what is the sign of the derivative in part (f)? If $\eta_x = 1$ then what is the sign of the derivative in part (f)? If a good is a necessity then as income rises does the share of income spent on that good rise or fall?
   h) Given the income elasticity that you found for donuts in part (d) what will be the

derivative of θ_x with respect to I for Edith's demands?

i) Write θ_x as a function of only prices and income for Edith's demand for donuts. Verify your answer to part h (ie calculate the derivative directly to show that it equals your answer)?

j) The income-consumption curve (icc) shows how Edith's consumption of donuts and croissants increases as her income increases. Illustrate the icc in your diagram above.

3. Bart purchases food (x) and lottery tickets (y). His preferences on units of food (x) and lottery tickets (y) can be represented by the utility function $U(x,y) = \min[2x,y]$.

a) If the price of a unit of food is P_x, the price of a lottery ticket is P_y and he has $I to spend then find his demand functions for food and lottery tickets.

b) Suppose that $P_x = \$2$, $P_y = \$1$ and $I = \$24$. Using your answer to (a) find his demands for food and lottery tickets. Illustrate your answer in an indifference curve diagram on the next page.

The government would like to encourage Bart to eat more but not to buy more lottery tickets As a consequence congress decides to subsidize the purchase of food by $1 per unit. Thus the price of a unit of food falls to $1.

c) After the price decrease what are Bart's new demands for food and lottery tickets? Illustrate the effect of the subsidy in your diagram for part (b). In your illustration be sure to clearly label the substitution and income effects.

d) Did the subsidy achieve the goal of only increasing Bart's consumption of food? Use your substitution and income effects to explain your answer.

e) Using your answer to part (a), show that the cross price elasticity of lottery tickets with respect to the price of food is negative.

f) Does a negative cross price elasticity imply that the substitution effect is negative? Briefly explain why or why not.

g) Show directly that the substitution effect in this case is 0 by calculating the derivative of the Hicksian demand for lottery tickets with respect to the price of food.

4. In the state of New Francia a typical worker earns a wage of $10 per hour. She can work up to 20 hours per week tax free that is she can earn $200 per week before she must pay taxes on her earnings. For every dollar that she earns above 200 per week she must pay a 10% tax on her earnings. Hence her after tax wages are only $9 for all hours above 20 hours per week. The typical worker in New Francia has no outside income (no income other than wage income) and she uses her income to purchase a consumption good priced at $1 per unit. There are 168 hours in a week.

a) Illustrate the budget set of a typical worker in New Francia.

b) What is the opportunity cost of leisure if she is working 10 hours per week? What is the opportunity cost of leisure if she is working 30 hours per week?

The typical worker in New Francia works 45 hours per week.

c) In your diagram in part (a) illustrate the indifference curve for the typical worker through her best bundle (at 45 hours per week of labor). In your diagram assume that leisure and the consumption good are not perfect complements.

d) What is the (approximate) average hourly wage of the typical worker who is working 45 hours per week? What is the average tax rate?

New Francia is considering a "flat tax" system. Specifically the new tax system will charge the same tax rate on every single dollar earned so the typical worker will have to pay taxes on every dollar earned (including the first dollar earned). However, New Francia will lower the tax rate to the average tax rate that the typical worker was paying under the old tax system (the dollar figure you found in part (d)).

e) In your diagram in part (a) illustrate the new budget line associated with the new flat tax.
f) Will the typical worker still work 45 hours per week under the new contract? Briefly explain your answer.

5. Barry must decide how to allocate his 24-hour day between non-wage activities, *leisure*, and wage activities, *labor*. For every hour of labor that he supplies he is paid by his employer $w. Barry can purchase bottles of wine (y) at a price of $10 per bottle (and wine is the only consumption good). Barry has non-wage income of $I and so his total income is equal to the sum of his wage income and his non-wage income. Finally assume that Barry's preferences over leisure and wine can be represented by the following utility function: $U(\ell, y) = \ln(\ell) + \ln(y)$

a) If Barry consumes 24 hours of leisure then how many bottles of wine can he purchase? If Barry works 24 hours then how many bottles of wine can he purchase. Illustrate his budget set in a diagram below and write the equation for his budget line.
b) Find Barry's demand for leisure as a function of w and I. What is Barry's supply of labor as a function of w and I?

For parts (c) through (e) below set $I = $40.

c) Given your answer to (b) above what will be Barry's supply curve of labor? Illustrate it below.
d) Suppose that Barry's wage is **$20**. Given the demand functions that you found above what are Barry's demands for leisure and wine at this wage? What is his supply of labor? Illustrate your answer in your diagram in part (a). Be sure to include an indifference curve through his best bundle.
e) Suppose that Barry's wage rises. Illustrate in your indifference curve diagram the effect of the increase in wages on his optimal choice of leisure and wine. Make sure to clearly indicate the income and substitution effects in your diagram. Given the slope of the labor supply curve which effect (the income or substitution) is larger on his demand for leisure?
f) Hold wage constant at $20 per hour. Suppose that his non-wage income were to increase. Given your labor supply function from part (b) what will be the effect of an increase in non-wage income on his labor supply. Why did a change in non-wage income have a different effect on labor supply than the change in wage that you analyzed in part (e)?

6. There are 168 hours in a week. Assume that Betty earns $10 per hour and has no other source of income. She can use her income to purchase food priced at $1 per unit.

a) Illustrate Betty's budget set.

The government would like to subsidize Betty's wages. For the first 20 hours that she works in a week the government will pay her an additional $1 per hour (thus raising her wage to $11 for each hour worked up to 20). However, starting at 20 hours of work, the government will no longer subsidize her wages and in fact will start taxing them. In particular the government will levy a tax of $1 per hour on all hours worked above 20 hours (thus lowering her wages to $9 for every hour of work over 20).

b) In your diagram for part (a) illustrate the effect of the subsidy/tax program on the Betty's budget set. In the diagram label the intercepts of the new budget line.
c) Find the bundle that is common to both budget lines.

In parts (d)- (f) below you will examine 3 different best bundles on the after subsidy/tax budget line. For each of them you will need a separate diagram.

d) Suppose that after the subsidy/tax plan is implemented Betty chose to work 30 hours per week. Show that Betty is a net recipient of government funds (i.e. her total subsidy – total taxes > 0). Illustrate her choice in your diagram from part (a). In your diagram indicate the net income that she receives from the government.
e) Suppose that after the implementation of the subsidy/tax plan Betty chooses the bundle such that she nets 0 from the government's program (total subsidy = total taxes). Briefly explain why Betty will be worse off under the new subsidy/tax program than she was before the program. You might find it useful to use a diagram in your explanation (in particular first locate the bundle such that she nets 0 from the program).
f) Finally suppose Betty is indifferent between the two programs (so she is indifferent between her best bundle when there is no program and her best bundle when there is a program). Show that Betty will work less and be a net recipient of government income in this case. Again a diagram may be helpful.

7. On homework 2 you found the demand functions for donuts (x) and bagels (y) and the utility function $U(x,y) = x + 2 \ln y$ where x represents the number of donuts and y represents the number of bagels. You also found the best bundle when prices and income were $P_x=1$, $P_y=1$, and $I = 12$ and when the price of x rose to 1.5.

a) Redraw your diagram illustrating the change in the best bundle due to the price change. On your new diagram illustrate the substitution and income effects of the price change.
b) Calculate the Hicksian demand functions for this utility function.
c) In your diagram for part (a), indicate the Hicksian demands for both goods at the original prices and original utility level and at the new prices and original utility level.

Bonus: On homework assignment 2 you calculated the uncompensated demand curves for the utility function $U(x,y) = \ln x + 3 \ln y$.

a) For this utility function calculate the compensated demand curves for x and y.
b) Using your uncompensated and compensated demand curves calculate the partial derivative of both uncompensated and compensated demand for x with respect to p_x and the partial derivative of uncompensated demand with respect to income.
c) Using your answers for (b) verify the Slutsky equation.