Solutions for Sample Final Questions Fall 2008

Problem 1

Suppose that the following production function represents a firm's ability to manufacture pencils: $\mathbf{f(L,K) = 3^{1/2}K^{1/4}L^{1/4}}$.

a) Show that this production function exhibits decreasing returns to scale.

$$f(\lambda L, \lambda K) = 3^{1/2}(\lambda K)^{1/4}(\lambda L)^{1/4} = \lambda^{1/2}3^{1/2}(K)^{1/4}(L)^{1/4} = \lambda^{1/2}f(L,K) < \lambda f(L,K)$$

So, we have decreasing returns to scale.

b) Assume that the wage rate is equal to 9 and the rental rate on capital is equal to 4. In addition assume that the firm has a fixed cost of production equal to 16. Find the compensated factor demands for labor and capital. Find the cost of the cost minimizing input bundle (ie find the variable cost curve) and the total cost curve.

(I) Tangency condition

$$\frac{1/4 * 3^{1/2}K^{1/4}L^{-3/4}}{1/4 * 3^{1/2}K^{-3/4}L^{1/4}} = \frac{9}{4}$$

$$\frac{K}{L} = \frac{9}{4} \Rightarrow K = \frac{9}{4}L$$

(II) Feasibility condition

$$\begin{aligned} q &= 3^{1/2}K^{1/4}L^{1/4} \\ &= 3^{1/2}\left(\frac{9}{4}L\right)^{1/4}L^{1/4} \\ &= \left(\frac{9}{2}L\right)^{1/2} \Rightarrow L(q) = \frac{2}{9}q^2 \Rightarrow K(q) = \frac{1}{2}q^2 \end{aligned} \qquad \text{Conditional Demands} \quad (8)$$

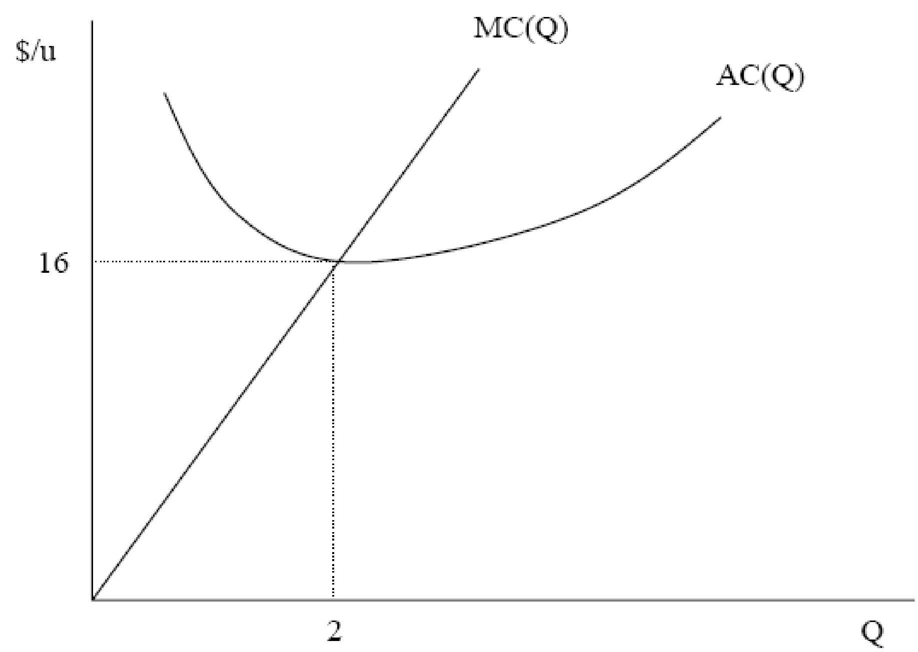
$$\begin{aligned} VC(q) &= wL(q) + rK(q) \\ &= 9 * \frac{2}{9}q^2 + 4 * \frac{1}{2}q^2 \\ &= 4q^2 \end{aligned} \qquad \text{Variable Cost} \quad (9)$$

$$TC(q) = 4q^2 + 16 \qquad \text{Total Cost} \quad (10)$$

c) Find the marginal cost function and the average cost function. Graph the marginal cost function and the average cost function in a single diagram. Show that the optimal size of the firm is 2. What is the minimum value of the average cost curve?

$$MC(q) = 8q \qquad \text{Marginal Cost} \qquad (11)$$

$$AC(q) = 4q + \frac{16}{q} \qquad \text{Average Cost} \qquad (12)$$



d) If this firm is a price taker then what is the firm's supply curve?

$$P(q) = 8q \Rightarrow q(P) = \frac{1}{8}P \qquad \text{Firm's supply} \qquad (13)$$

e) Suppose that there is a second firm in the pencil business, which has an identical total cost function and that these two firms are the only two firms in the industry then what is the industry supply curve?

$$Q(P) = 2q(P) = \frac{1}{4}P \qquad \text{Industry supply} \qquad (14)$$

f) If the demand for pencils is given by the inverse market demand curve **P = 100 – Q** then what is the market clearing price and quantity for the pencil market?

$$\frac{1}{4}P = 100 - P$$
$$P^* = 80 \Rightarrow Q^* = 20$$

g) Do you expect that there will be entry or exit in the long run? What will be the long run price of pencils? How many units will be traded in the long run?

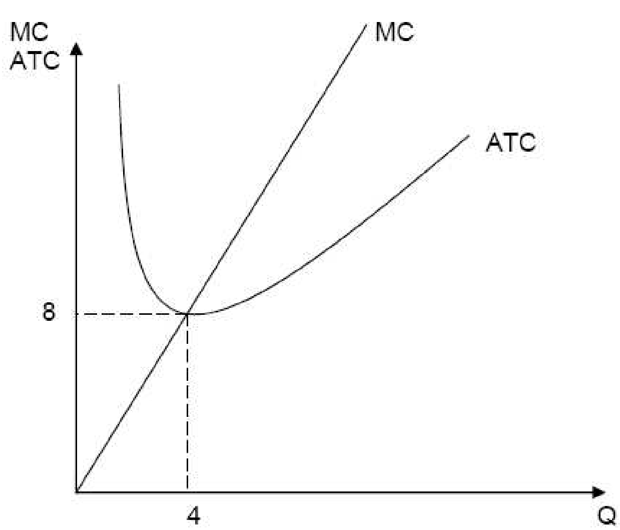
There will be entry since the existing firms are earning super-normal profits. In the long run, the price will be 16 and 84 units will be traded; thus, 40 additional firms will enter the industry.

Problem 2

2. The bolt industry currently consists of 20 producers, all of whom operate with identical total cost functions given by $TC(Q) = 16 + Q^2$. Quantity is measured in tons of bolts.

a) What is the marginal cost curve associated with this total cost curve? What is the average cost curve? What is the optimal size of the firm? What is the minimum value of the average cost curve? Illustrate your answer in a diagram.

$$TC(Q) = 16 + Q^2$$
$$MC = 2Q$$
$$ATC = \frac{16}{Q} + Q$$
$$MC = AC$$
$$2Q = \frac{16}{Q} + Q$$
$$Q^* = 4$$
$$\min ATC = \frac{16}{4} + 4 = 8$$



b) What is the individual firm's supply curve? What is the market supply curve?

$$P = MC$$
$$P = 2Q$$
$$Q_i^s = \frac{P}{2}$$
$$20Q_i = Q_{MKT}^s = 10P$$

Suppose that demand for bolts is given by $Q^D = 132 - P$. Price is measured in dollars per ton.

c) What will be the market clearing price and quantity traded in the short-run? How many units will each firm supply to the market? Will firms be earning normal, super-normal or sub-normal profits? Briefly explain your answer without calculating the firm's level of profits.

$$Q^D = 132 - P$$
$$Q^s = 10P$$
$$132 - P = 10P$$
$$P^* = 12$$
$$Q^* = 120$$
$$Q_i = \frac{120}{20} = 6$$

Firms are earning super normal profits as they are selling 6 units at a price of \$12 each. We know that the Price in the long run is equal to the minimum of the ATC, \$8; so in this case P > minATC.

d) What is the long-run price of bolts? Do you predict that there will be entry or exit from this industry?

$P^{LR} = \min ATC = 8$

There will be entry in this industry as there are incentives for other firms to make positive profits (at least in the short run)

e) What is the long-run quantity traded in this market? How many units will each firm produce in the long-run? How many firms will there be in the long-run?

$$Q^D = 132 - 8 = 124$$
$$Q_i = \frac{P}{2} = \frac{8}{2} = 4$$
$$N = \frac{124}{4} = 31$$

Problem 3

5. Given the elasticities and the current price and quantity, we compute the slopes of the supply and demand functions:

$$\begin{aligned} m^D \frac{P}{Q} &= \epsilon^D \Rightarrow m^D \frac{1}{9} = -\frac{4}{9} \Rightarrow m^D = -4 \\ m^S \frac{P}{Q} &= \epsilon^S \Rightarrow m^S \frac{1}{9} = \frac{4}{3} \Rightarrow m^S = 12. \end{aligned}$$

Deficiency Payment Program: the government promises that $P = 1.1$, so farmers supply as if they faced this price in the market. This means that they supply the quantity associated with $P = 1.1$, which is found by

$$\begin{aligned} m^S &= \frac{\Delta Q}{\Delta P} \Rightarrow 12 = \frac{\Delta Q}{.1} \\ \Delta Q &= 1.2, \end{aligned}$$

so that they supply 10.2 (1,020 million pounds of peanuts). Consumers are not willing to pay 1.1 per unit to get 10.2: the price they will pay for 10.2 is found by

$$\begin{aligned} m^D &= \frac{\Delta Q}{\Delta P} \Rightarrow -4 = \frac{1.2}{\Delta P} \\ \Delta P &= -.3, \end{aligned}$$

that is, it is given by 70 cents per unit. This means that the government has to make up for 40 cents of the price.

Peanut Purchase Program: the government acts as a consumer and increases the market demand for peanuts up to the point that the equilibrium price is \$1.10. This means that consumers face this price, which leads them to demand less:

$$\begin{aligned} m^D &= \frac{\Delta Q}{\Delta P} \Rightarrow -4 = \frac{\Delta Q}{.1} \\ \Delta Q &= -.4, \end{aligned}$$

so that they demand only 8.6 (860 million pounds). The government has to make up the difference: it demands the additional 1.6 (160 million pounds) so that quantity traded is 10.2 and the equilibrium price is indeed 1.1.

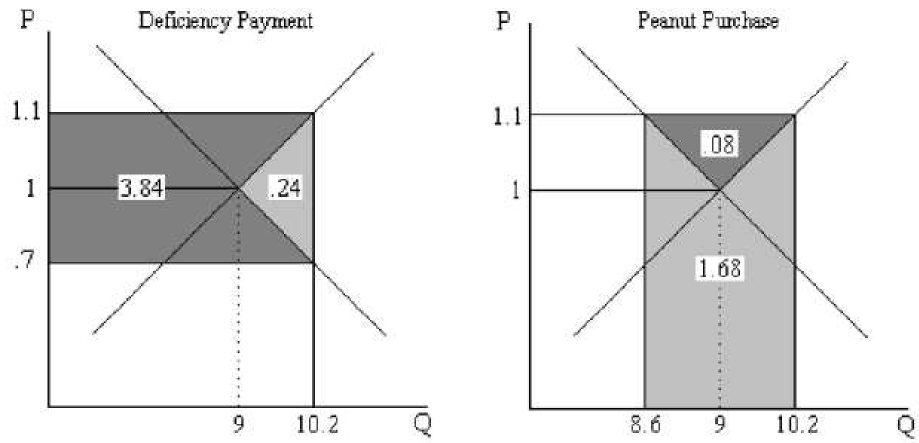
a) Producers supply 10.2 under both programs, and their surplus increases by the same amount (.96) in both programs, so they are indifferent.

b) The cost of the Deficiency Payment Program is $(.4)(10.2) = 4.08$ and that of Peanut Purchase Program is $(1.6)(1.1) = 1.76$, so the government prefers the Purchase Program.

c) The social cost of the Deficiency Program is .24: PS increases .96 and CS increases 2.88; since the cost is 4.08, $4.08 - .96 - 2.88 = .24$ is lost.

The social cost of the Purchase Program is 1.68: CS decreases .88 and PS increases .96, so consumers and farmers together gain only .08. Since the government spends 1.76, we get that $1.76 - .08 = 1.68$ is lost.

Graphically: the cost of the programs are the large rectangles below; the dark grey area represents the part of the cost that is captured by either producers or consumers; the light grey area is what is lost.



Problem 4

4. The demand and supply of soybeans in the United States is given by the following: $Q^D_{US} = 218 - 4P$ and $Q^S_{US} = -3 + .25P$ where quantity is measured in millions of tons and price is measured in dollars per ton.

a) What is the market-clearing price and quantity of soybeans in the US? Illustrate your answer in a diagram.

In an effort to raise the income of farmers the government would like to raise the price of soybeans. One way to raise the price of soybeans would be to restrict the supply of soybeans. In order to decrease the supply of soybeans the government forces farmers to cut back on their production of soybeans to only 8 million tons.

b) In a new diagram illustrate (no numbers) the impact of the quota on the price charged in the market and the quantity traded. At this price is there excess supply or excess demand?
c) In your diagram for (b) illustrate the changes in producer surplus caused by the quota. What are the two changes in producer surplus? Which of these changes increase the income of farmers? Which of these changes decreases the income of the farmers?
d) What is the new price and quantity traded for soybeans after the quota is imposed?
e) What is the dollar value of the transfer from consumers to farmers? What is the deadweight loss of farmers? What is the net effect of the quota on the income of farmers? Was the governments program successful at raising the income of farmers?

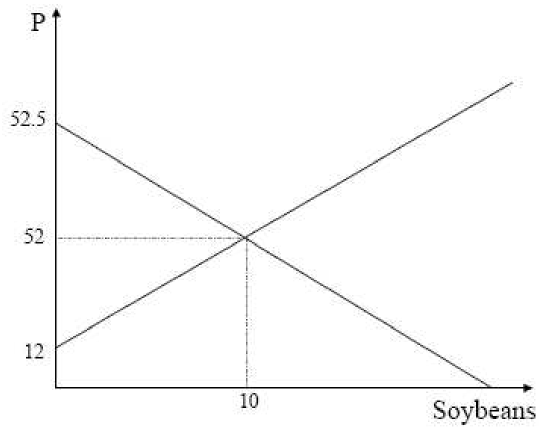
Instead of imposing a quota on the farmers the government decides to levy a tax on the consumers of soybeans and to distribute all of the revenues from that tax to the farmers.

f) Suppose that the tax is \$8.5 per ton of soybeans. Illustrate the impact of this tax in a supply and demand curve diagram. Will this tax plan raise the income of farmers more that the quota? Briefly explain your answer.

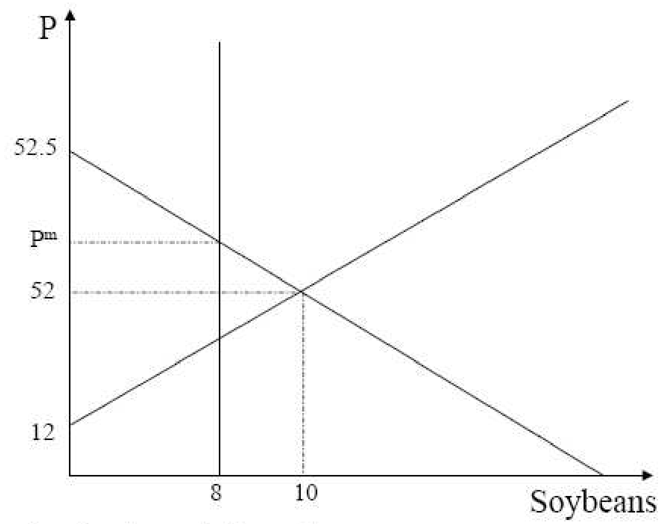
Answers:

a)

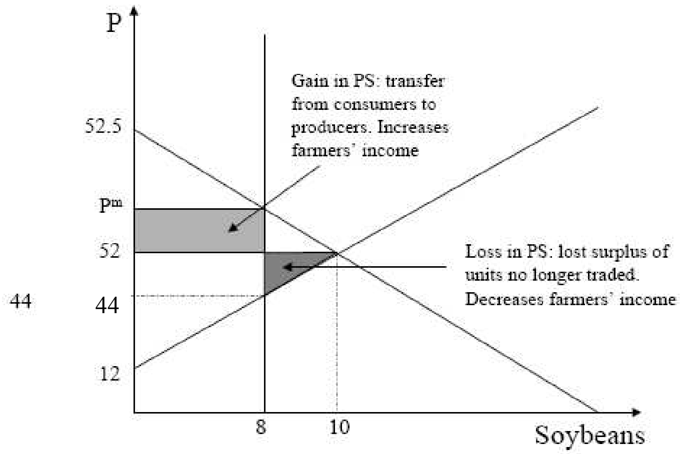
$$Q^D_{US} = 218 - 4P$$
$$Q^S_{US} = -3 + .25P$$
$$Q^D_{US} = Q^S_{US}$$
$$218 - 4P = -3 + .25P$$
$$P^* = 52, \quad Q^* = 10$$



b)



At P^m there is excess supply of soybeans in the market.



d)

$$Q^* = 8$$
$$8 = 218 - 4P^m$$
$$P^m = 52.5$$

e)

$$Transfer \Rightarrow .5(Q^*) = .5 * 8 = \$4$$
$$DWL = \frac{8*2}{2} = \$8$$
$$Net \quad effect = +4 - 8 = -\$4$$

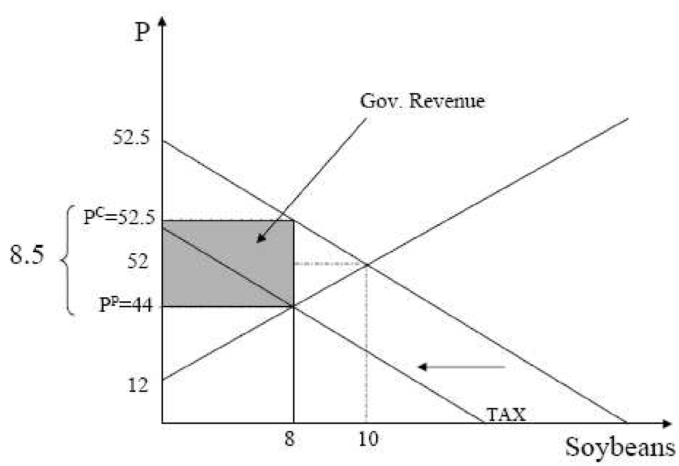
No, the government program wasn't successful in raising farmers' income.

f)

$$Q_{US}^D = 218 - 4(P^C + 8.5)$$
$$Q_{US}^D{}' = 184 - 4P$$
$$Q_{US}^S = -3 + .25P$$
$$184 - 4P = -3 + .25P$$

This program doesn't raise farmers' income more than quota.

$$P = 44, \quad Q = 8$$



Problem 5

Sheila and Bruce are going to the beach. Sheila is bringing 12 ounces of soda (x) and 10 bags of chips(y). Bruce is bringing 8 ounces of soda and 8 bags of chips. Sheila 's and Bruce's preferences over soda and chips can be represented by the utility functions:

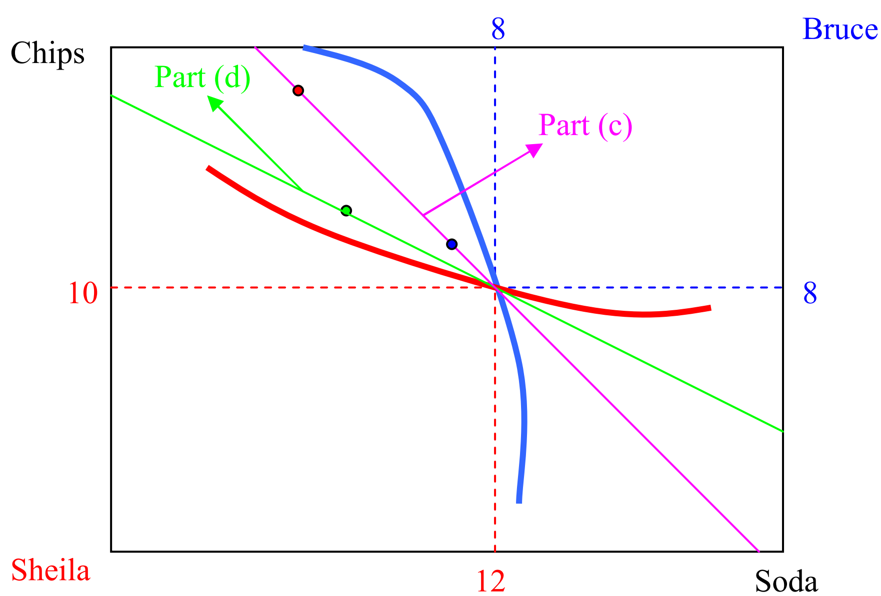
$$U^S(x^S, y^S) = \ln x^S + 3\ln y^S \text{ and } U^B(x^B, y^B) = \ln x^B + \ln y^B.$$

a) What is Sheila's MRS at her endowment? What is Bruce's MRS at his endowment? Is the endowment point Pareto efficient? Illustrate in the Edgeworth box diagram the endowment point and a pair of indifference curves through the endowment point.

$$MRS^S = \frac{1/x^S}{3/y^S} = \frac{y^S}{3x^S} = \frac{10}{3 \cdot 12} = \frac{10}{36}$$

$$MRS^B = \frac{1/x^B}{1/y^B} = \frac{y^B}{x^B} = \frac{8}{8} = 1$$

Since the MRSs are not equal then the endowment point is not PE.



Instead of trading directly with each other Sheila and Bruce trade with a market. Sheila and Bruce are both price takers on both the soda and chip markets. Each can buy and sell a good at the same price. Let P_x be the price of an ounce of soda and P_y be the price of an bag of chips.

b) Write the two equations that define the best bundle for Sheila given that the prices are P_x and P_y. Do the same for Bruce.

Sheila

Feasibility: $P_x \cdot x^S + P_y \cdot y^S = 12P_x + 10P_y$

Tangency: $\frac{y^S}{3x^S} = \frac{P_x}{P_y}$

Bruce

Feasibility: $P_x \cdot x^B + P_y \cdot y^B = 8P_x + 8P_y$

Tangency: $\frac{y^B}{x^B} = \frac{P_x}{P_y}$

c) Using your answer for part (b) find Sheila's best bundle when $P_x = 5$ and $P_y = 6$. Is Sheila a net demander or a net supplier of chips (y)? Find Bruce's best bundle at the same prices. Show that when $P_x = 5$ and $P_y = 6$ neither market clears. Which market has excess supply? Illustrate the budget line associated with these prices in your diagram for part (a). Indicate the best bundles for Sheila and Bruce on this budget line.

Sheila

Feasibility: $5x^S + 6y^S = 120$

Tangency: $\frac{y^S}{3x^S} = \frac{5}{6}$

Combining the two, we get:

$5x^S + 15x^S = 120 \Rightarrow x^S = 6 \Rightarrow y^S = \frac{5}{6} \cdot 3x^S = 15$ so Sheila is a net demander of chips

Bruce

Feasibility: $5x^B + 6y^B = 88$

Tangency: $\frac{y^B}{x^B} = \frac{5}{6}$

Combining the two, we get:

$5x^B + 5x^B = 88 \Rightarrow x^B = 8.8 \Rightarrow y^B = \frac{5}{6} \cdot x^B = \frac{44}{6} = 7.3$

$x^S + x^B = 6 + 8.8 = 14.8 < 20$ Excess supply of soda

$y^S + y^B = 15 + 7.3 = 22.3 > 16$ Excess demand for chips

d) Using your answer for part (b) find Sheila's best bundle when $P_x = 1$ and $P_y = 2$.Show that when $P_x = 1$ and $P_y = 2$ both markets clear. Illustrate the budget line associated with the general equilibrium prices in a new Edgeworth box diagram. Indicate the best bundles on the budget line and the endowment point.

Sheila

Feasibility: $x^S + 2y^S = 32$

Tangency: $\frac{y^S}{3x^S} = \frac{1}{2}$

Combining the two, we get:

$$x^S + 3x^S = 32 \Rightarrow x^S = 8 \Rightarrow y^S = \frac{1}{2} \cdot 3x^S = 12$$

Bruce

Feasibility: $x^B + 2y^B = 24$

Tangency: $\frac{y^B}{x^B} = \frac{1}{2}$

Combining the two, we get:

$$x^B + x^B = 24 \Rightarrow x^B = 12 \Rightarrow y^B = \frac{1}{2} \cdot x^B = 6$$

$x^S + x^B = 8 + 12 = 20$ Soda market clears

$y^S + y^B = 12 + 6 = 18$ Chips market clears

Problem 6

6. Sheila and Bruce are on a hike. Sheila has brought with her 8 bottles of water (x) and 5 granola bars (y). Bruce has brought 4 bottles of water and 8 granola bars. Sheila and Bruce have preferences that can be represented by the following utility functions:

$U^S(x^S,y^S) = \ln x^S + \ln y^S$ and $U^B(x^B,y^B) = \ln x^B + 2 \ln y^B$.

a) Given their preferences is the endowment point a Pareto efficient allocation of water and granola bars?

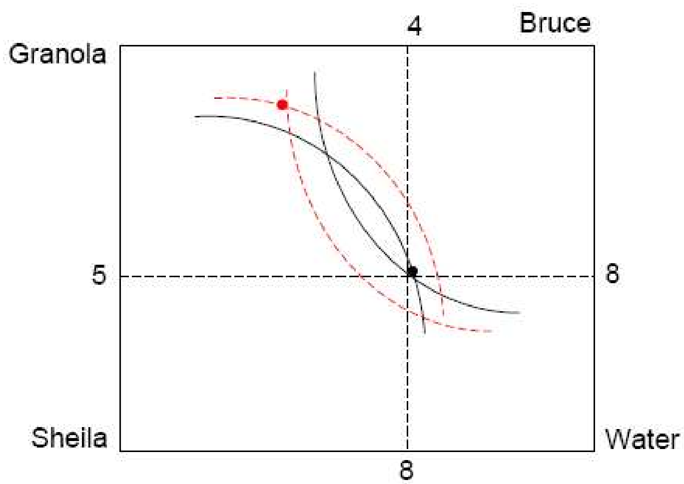
$$MRS^{SHEILA} = \frac{y^S}{x^S} = \frac{5}{8} \qquad MRS^{BRUCE} = \frac{y^B}{2x^B} = \frac{8}{8} = 1$$

$$MRS^{SHEILA} \neq MRS^{BRUCE} \therefore endowment \neg PE.$$

b) Given their marginal rates of substitution at the endowment point who values water more highly? What will be the pattern of trade associated with every mutually beneficial trade? Find one trade that will make both Sheila and Bruce strictly better off.

Answer: Bruce is willing to give up 1 granola for 1 bottle of water. Sheila is willing to give up 0.625 granola bars in exchange for 1 bottle of water. Hence, Bruce values water more highly. The pattern of trade associated with every mutually beneficial trade will be any amount of granola bars between 0.625 and 1 in exchange for 1 bottle of water. Sheila should trade water for granola bars and Bruce should give up granola bars and receive water. Example of trade: ¾ of granola bar in exchange for 1 bottle of water.

c) Illustrate in an Edgeworth box diagram the endowment point, a pair of indifference curves passing through the endowment point and the trade that makes both of them better off (from part (b)).



d) An allocation is envy free if Bruce prefers his bundle to Sheila's bundle and Sheila prefers her bundle to his. Show (algebraically) that the endowment point is an envy free allocation. Illustrate graphically in your Edgeworth box diagram that the endowment is envy free. If you cannot illustrate it in the diagram that you have already drawn then sketch a new Edgeworth box below.

$U^S(8,5) = \ln 8 + \ln 5 = \ln 40$

$U^S(4,8) = \ln 4 + \ln 8 = \ln 32$

$U^B(4,8) = \ln 4 + 2\ln 8 = \ln 256$

$U^B(8,5) = \ln 8 + 2\ln 5 = \ln 200$

Both Bruce and Sheila prefer their own endowments to the other's endowment.

Problem 7

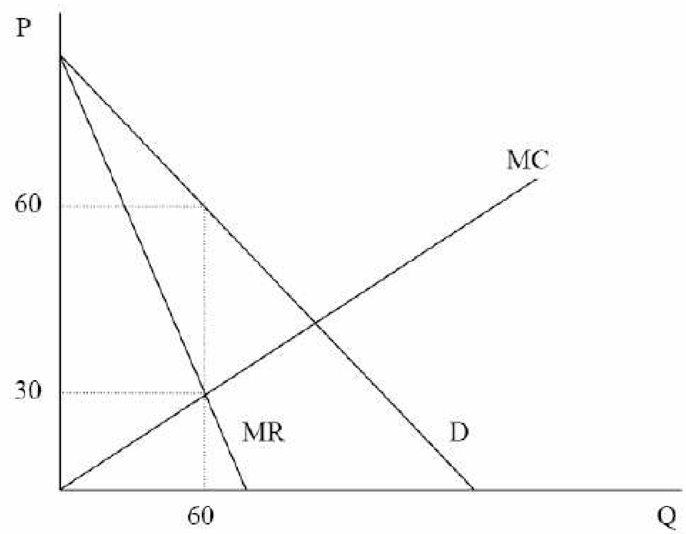
7. The country of Utopia is a closed economy so it does not trade with the world economy. HD Chemicals is the only producer of Dexinitrite, an important additive to fertilizers, in Utopia. The demand for Dexinitrite in Utopia is given by the demand curve: Q = 180 - 2P where P is the price per ton of Dexinitrite and Q is measured in tons. The cost of producing Dexinitrite is given by C = .25Q2 + 400.

a) Given that HD Chemicals is a monopolist what price will it charge for Dexinitrite? What quantity will it sell? What will be its profits? Illustrate your answer in a diagram.

$$MC = MR$$
$$.5Q = 90 - Q$$
$$Q^* = 60 \Rightarrow P^* = 60$$

$$AC(60) = .25 * 60 + 400/60$$
$$AC(60) = 21.67$$

$$\Pi = (60 - 21.67) * 60 = 2300$$



Utopia decides to open some of its markets to trade. To protect HD Chemicals the market for Dexinitrite will not be opened to trade. However, HD Chemicals will be allowed to sell Dexinitrite on the world market. The world market for Dexinitrite is a perfectly competitive market and the current world price of Dexinitrite is \$50 per ton. HD Chemicals can sell as much Dexinitrite as it would wish to the world market at the price of \$50 per ton and can choose any price that it would wish in its home market in Utopia (so it can price discriminate).

b) Given that HD Chemicals can price discriminate between its domestic and foreign customers then what price will it charge to its home customers? How much Dexinitrite will it sell at home? How much

Dexinitrite will it export? Are domestic customers better or worse off now that HD Chemicals can expo Dexinitrite? What are the profits of HD Chemicals? Is HD Chemicals better off now that it can export?

$MR_1 = 90 - Q_1$

$MR_2 = P^{world} = 50$

Profit Maximizing Condition :

$$\mathrm{MR}_1 = MR_2 = MC \Rightarrow \begin{cases} 90 - Q_1 = 0.5Q \\ 50 = 0.5Q \end{cases}$$

$\Rightarrow Q_1 = 40, Q_2 = 60$

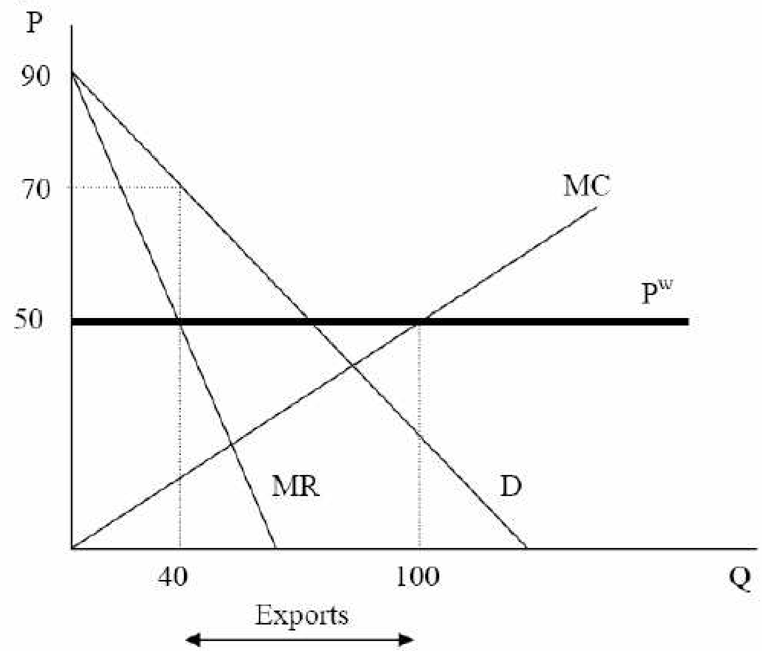
Q_1 is the amount sold at home. Q_2 is the amount sold in the world market (exports).

Since the price in the domestic market increases to \$70, domestic costumers are worse off (the consumer surplus is reduced).

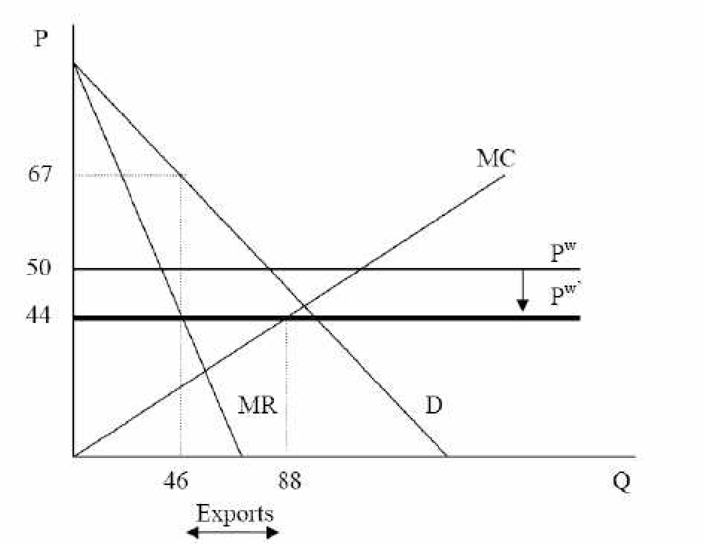
$\Pi = 70 * 40 + 50 * 60 - 0.25 * 100^2 - 400 = 2900$

HD Chemicals is better off since its profits increase.

c) Illustrate your answer to (b) in a diagram similar to the one in (a) that is a diagram showing the domesti demand for Dexinitrite and the marginal cost of production. Indicate the domestic sales of HD Chemica and the foreign sales of HD Chemicals.



d) If the world price of Dexinitrite falls to only \$44 per ton then will HD Chemicals change the domestic price of Dexinitrite? How much will it sell at home? How much will it export at this price? Illustrate this answer in the diagram in part (c).

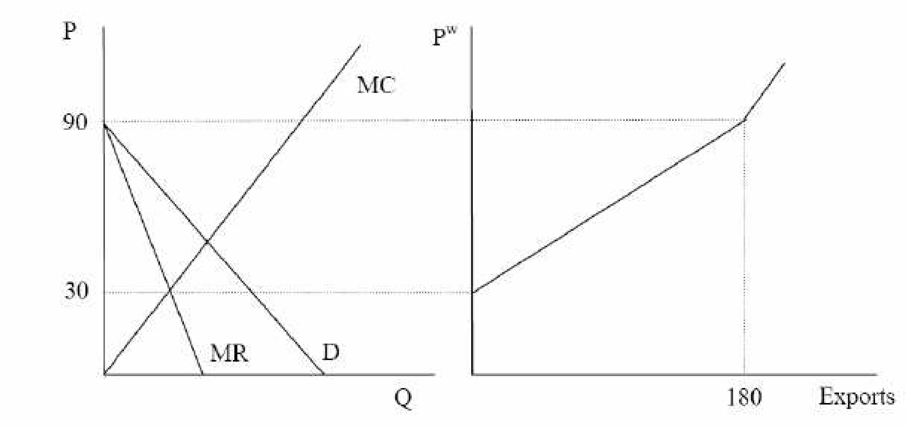


e) What is the lowest world price of Dexinitrite at which HD Chemicals will choose to export Dexinitrite? Is there a world price such that at that world price HD Chemicals will supply no Dexinitrite to its Utopia market?

When the MR=30 we are back in the situation a) where HD is a monopolist selling only in the domestic market, at that world price there will not be any exports (so the minimum price should be slightly above 30)
If the world price is above the maximum that the firm can charge domestically (90), the firm will export all its production and will not supply the good to its Utopia market.

f) Illustrate the export supply curve of HD Chemicals.

It's the horizontal distance between the MR and MC curves above P=30.



Problem 8

8. Suppose that the total cost curve for a typical firm in the olive oil industry is $TC(Q) = .05Q^2 + 15$. Quantity (Q) is measured in millions of gallons per year.

a) What is the supply curve of a typical firm in the olive oil industry? If there are 10 firms in the industry each of which has the typical cost curve then what is the industry supply curve of olive oil?

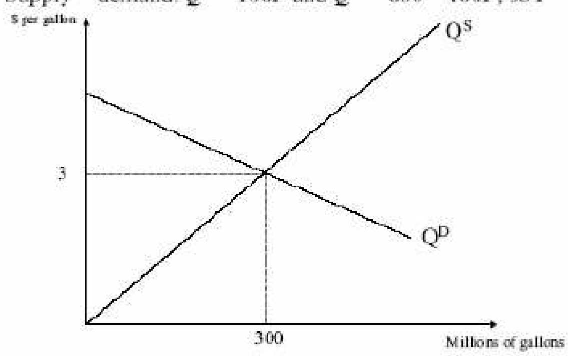
Demand per year for olive oil is $Q^D = 600 - 100P$. Quantity (Q) is measured in millions of gallons per year and price (P) is measured in dollars per gallon.

b) If the market is perfectly competitive then what will be the market clearing price and quantity traded in the olive oil market? Illustrate your answer in a supply and demand curve diagram.
c) How many gallons will each firm supply? Illustrate your answer in a diagram of individual firm supply curve and individual inverse firm demand curve.
d) If the olive oil producers formed a cartel and decided to choose a price and quantity that would maximize industry profits then what price and quantity will the cartel choose? Illustrate your answer in your diagram from part (b).

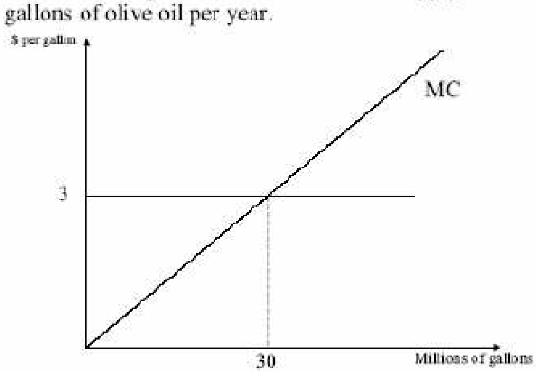
Answer:

a) $MC(Q) = 0.1Q$, the supply curve of a typical firm is $Q(P) = 10P$, and the industry supply curve of olive oil is $Q^S = 100P$.

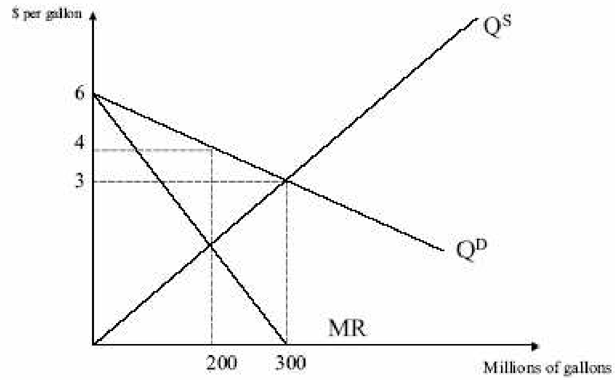
b) Supply = demand: $Q^S = 100P$ and $Q^D = 600 - 100P$, so $P = 3$, and $Q = 300$.



c) Each firm will provide 1/10th of the total supply, so a firm produces 30 millions of gallons of olive oil per year.



d) The market demand function is $Q^D = 600 - 100P$, so the inverse demand function is $P = 6 - 0.01Q$. Therefore, the marginal revenue is $MR = 6 - 0.02Q$. The industry supply curve is $Q^S = 100P$, thus $MC = 0.01Q$.
$MR = MC \Rightarrow 6 - 0.02\,Q = 0.01Q \Rightarrow Q = 200$, $P = 4$.



Problem 9

9. Suppose that there are two firms, upstream and downstream, such that the upstream firm is the sole supplier of an input, K, to the downstream firm and the downstream firm is the only buyer of the input from the upstream firm. The downstream firm sells its output in a perfectly competitive market at price p = 16. The downstream firm also acts as a price taker on the input market for K. The upstream firm on the other hand acts as a monopolist in the market for K. Suppose that the downstream firm's production function is given by $x = K^{1/2}$ and it has no fixed costs of production. Suppose that the upstream firm's cost function is given by C(K) = K.

a) Find the downstream firm's demand curve for the input K as a function of the price of K. (Hint you may first find the compensated demand for K and the cost function for the downstream firm. Using this cost function find the profit-maximizing level of output. Finally substitute the level of output into the compensated factor demand).
b) Given the downstream firm's demand curve find the quantity of K that maximizes the upstream firm's profits. Find the price associated with this quantity of K.
c) Find the profits of each firm.
d) If the downstream firm had the opportunity to purchase the upstream firm at a price 16, would it do so?

Answer:

a) Let's call the price of *K* as *r*. The downstream maximize its profit with price of the good *x* and input *K* all given.

$$\max_{x,K} 16x - rK$$

st. $x = K^{1/2}$

$K = x^2 \Rightarrow \max_x 16x - rx^2$

FOC = 0

$$16 - 2rx = 0 \Rightarrow x = \frac{8}{r} \Rightarrow K = \frac{64}{r^2}$$

b) The upstream firm can sell *K* at price *r*, therefore their profit maximizing question is

$$\max_K \; rK - C(K)$$

$$\Rightarrow \max_r \; r \cdot \frac{64}{r^2} - \frac{64}{r^2},$$

the question is thus to find the price of *K* which maximize the above equation. We solve it by finding the *r* satisfies FOC = 0.

$$-\frac{64}{r^2} + \frac{128}{r^3} = 0, \; r = 2.$$

$$K = \frac{64}{r^2} = \frac{64}{2^2} = 16$$

c) Upstream firm's profit = 2*16 - 16 = 16
To solve for the downstream firm's profit, we have to find how much they produce first. Since *K* = 16, *x* = 4. And
Downstream firm's profit = 16*4 – 2*16 = 32

d) If the downstream firm can purchase the upstream firm, then it becomes the price maker of *K* instead of the price taker. Its profit maximizing question becomes

$$\max(16x - rK) + (rK - K) = \max 16x - K$$

st. $x = K^{1/2}$

This question can be reorganized as

$$\max_x 16x - x^2$$

When $x = 8$, the firm maximizes its profit.

$$\pi = 16 \cdot 8 - 8^2 = 64$$

The downstream firm increases its profit by \$16 after paying \$16 to purchase the upstream firm, so it would be glad to do so.