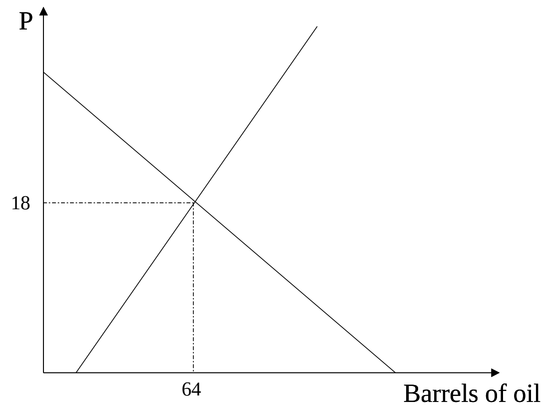
Problem 1

Suppose that the domestic supply of oil and domestic demand for oil are given by the following functions: $Q^S = 10 + 3P$ and $Q^D = 100 - 2P$ where quantity is measured in millions of barrels and price in dollars per barrel.

a) What is the domestic market-clearing price per barrel of oil? What is the quantity associated with this price? Illustrate your answer in a diagram.

$$Q^s = Q^d$$
$$10 + 3P = 100 - 2P$$
$$P = \frac{90}{5} = 18$$
$$P^* = 18$$
$$Q = 100 - 2(18) = 64$$
$$Q^* = 64$$

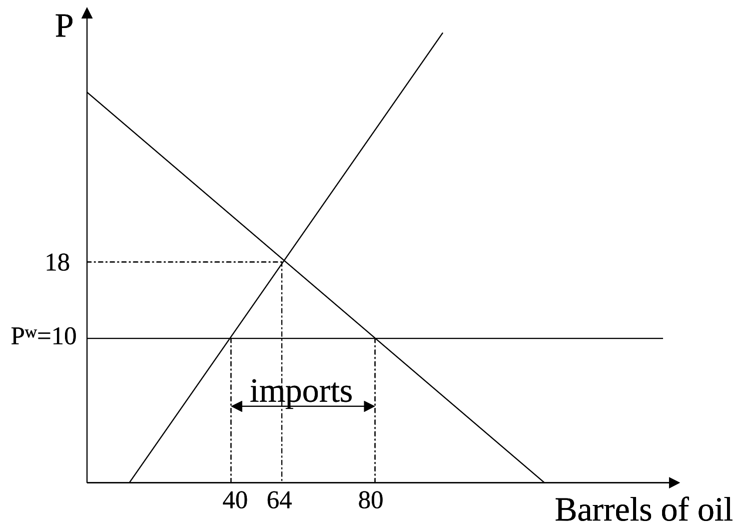


Suppose that the world price of oil is $10 per barrel. If there are no barriers to trade then the domestic price will equal the world price of $10.

b) At this price what is the domestic demand? What is the domestic supply? What is the demand for imports? Illustrate your answer in a diagram.

$$Q^d(P = 10) = 100 - 2(10)$$
$$Q^d(P = 10) = 80$$
$$Q^s(P = 10) = 10 + 3(10)$$
$$Q^s(P = 10) = 40$$

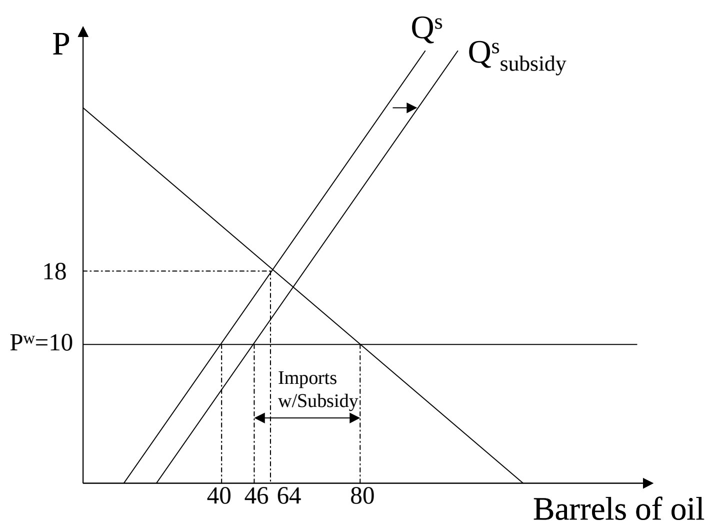
$$\text{Demand for imports } Q_I{}^d = Q^d - Q^s = 100 - 2P - 10 - 3P$$
$$Q_I{}^d = 90 - 5P$$
$$Q_I{}^d = 40$$



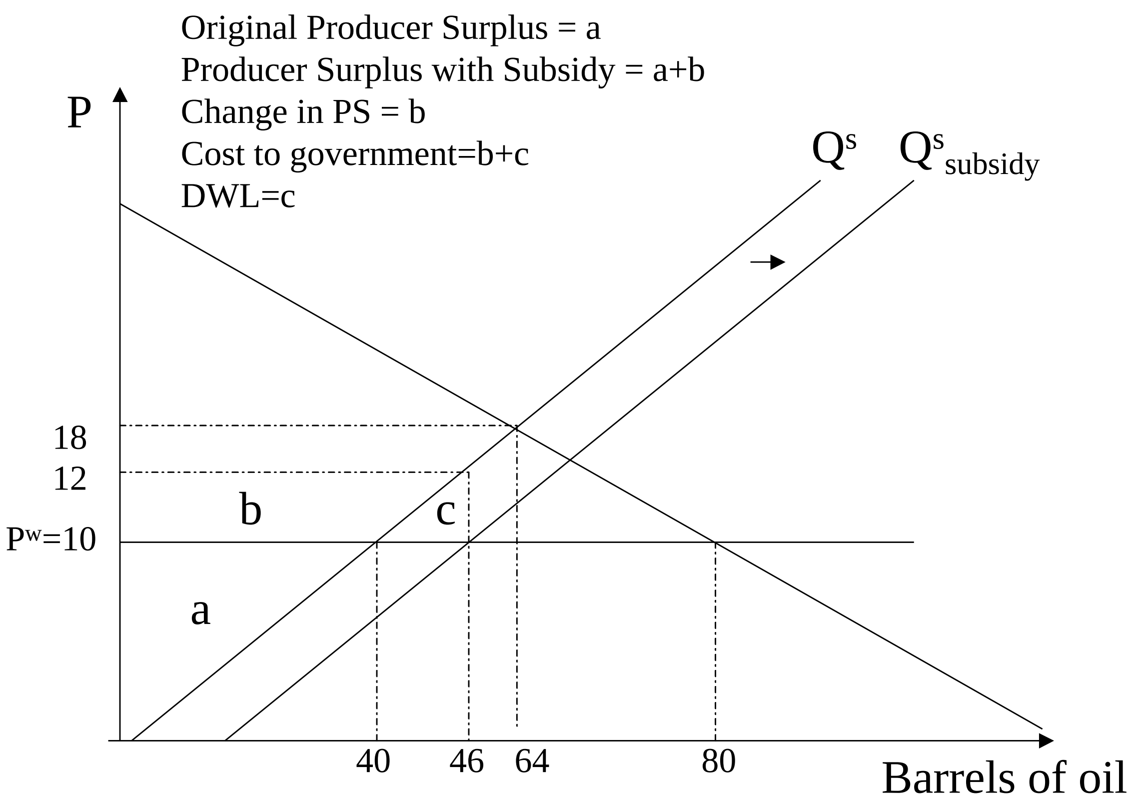
Suppose that the government decides to subsidize domestic oil producers at $2 per barrel.

c) Given the subsidy how much oil will domestic suppliers provide? What is the change in the demand for imports?

$$Q^s = 10 + 3P$$
$$Subsidy = 2$$
$$Q^s{}' = 10 + 3(P^s + 2)$$
$$Q^s{}' = 16 + 3P^s$$
$$if \quad P^w = 10 \Rightarrow Q^s{}' = 16 + 3(10)$$
$$Q^s{}' = 46$$



d) What will be the gain to domestic suppliers (that is what is the change in producer surplus)? What is the cost of the subsidy program to the government? What is the deadweight loss? Illustrate the change in producer surplus, the cost of the program and the deadweight loss in a diagram.



$$\Delta PS : b = \frac{(B+b)h}{2} = \frac{(46+40)2}{2} = 86$$

$$Cost\ \ to\ \ Government : b + c = 2 * 46 = 92$$

$$DWL : c = 86 - 92 = 6$$

2. In class we discussed the market for natural gas represented by the following supply and demand curves: $Q^D(P_G,P_O) = -5P_G + 3.75\ P_O$ and $Q^S(P_G,P_O) = 14 + 2P_G + .25P_O$ where P_G and P_O are the prices of natural gas (in \$ per mcf) and oil (in \$ per barrel) and quantity of natural gas is measured in tcf. Adapt the procedure developed in class used to calculate the incidence of a tax to find an expression for the derivative of the market clearing price of gas with respect to the price of oil in terms of the derivatives of supply and demand for natural gas with respect to both prices. Using the specific values for the derivatives of supply and demand with respect to prices from the supply and demand curves given above what is the value of the derivative of market clearing price of gas with respect to the price of oil? If the price of a barrel of oil rises by \$1 then what do you predict will be the change in price of natural gas? Illustrate your answer in a diagram.

Excess demand is $Q^D(P_G,P_O) - Q^S(P_G,P_O) = E(P_G,P_O)$

In equilibrium excess demand is zero: $E(P_G,P_O) = 0)$

Now totally differentiate the excess demand function:

$$dE(P_G,P_O)_{E=0} = \frac{\partial E(P_G,P_O)}{\partial P_G}dP_G + \frac{\partial E(P_G,P_O)}{\partial P_O}dP_O = 0$$

$$or$$

$$\left(\frac{\partial Q^D(P_G,P_O)}{\partial P_G} - \frac{\partial Q^S(P_G,P_O)}{\partial P_G}\right)dP_G + \left(\frac{\partial Q^D(P_G,P_O)}{\partial P_O} - \frac{\partial Q^S(P_G,P_O)}{\partial P_O}\right)dP_O = 0$$

$$or$$

$$\frac{dP_G}{dP_O} = -\frac{\dfrac{\partial Q^D(P_G,P_O)}{\partial P_O} - \dfrac{\partial Q^S(P_G,P_O)}{\partial P_O}}{\dfrac{\partial Q^D(P_G,P_O)}{\partial P_G} - \dfrac{\partial Q^S(P_G,P_O)}{\partial P_G}}$$

Plugging in the specific values for the derivatives:

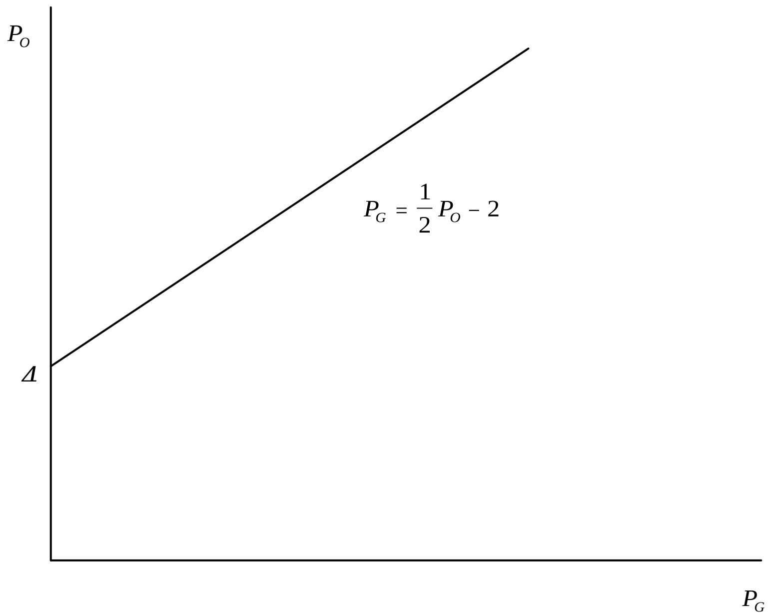
$$\frac{\partial Q^D(P_G,P_O)}{\partial P_G} = -5, \frac{\partial Q^S(P_G,P_O)}{\partial P_G} = 2$$

$$\frac{\partial Q^D(P_G,P_O)}{\partial P_O} = 3.75, \frac{\partial Q^S(P_G,P_O)}{\partial P_O} = .25$$

$$and\ hence:$$

$$\frac{dP_G}{dP_O} = -\frac{3.75 - .25}{-5-2} = \frac{1}{2}$$

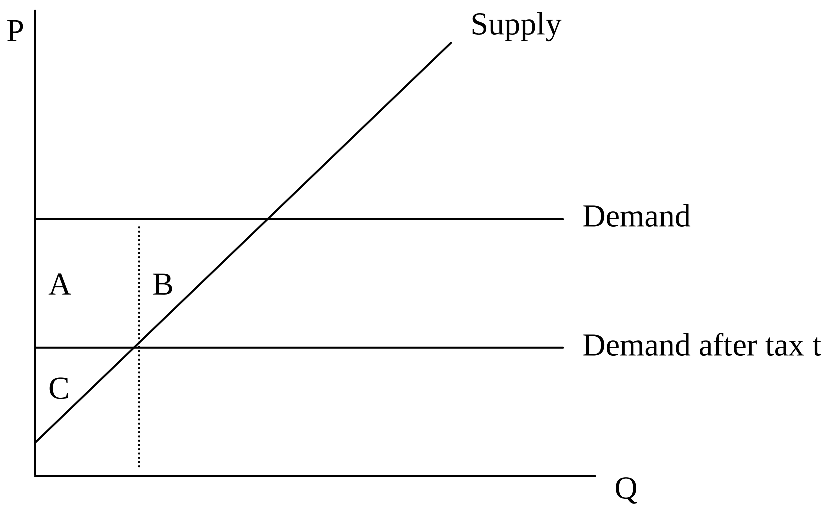
Graphically:



3. Briefly explain how the following statements could be true. In your explanations you should include a diagram and a brief discussion of how elasticity is relevant (or is not relevant) to the truth of the statement.

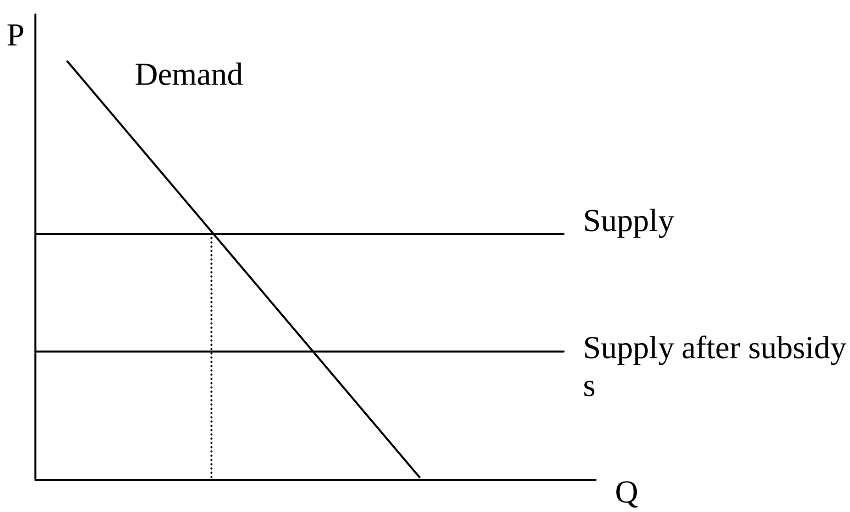
a) The government imposes a per-unit tax on the consumers of a product, gives all of the tax revenues to the producers and the producers are worse off than they were before the imposition of the tax.

Demand perfectly elastic



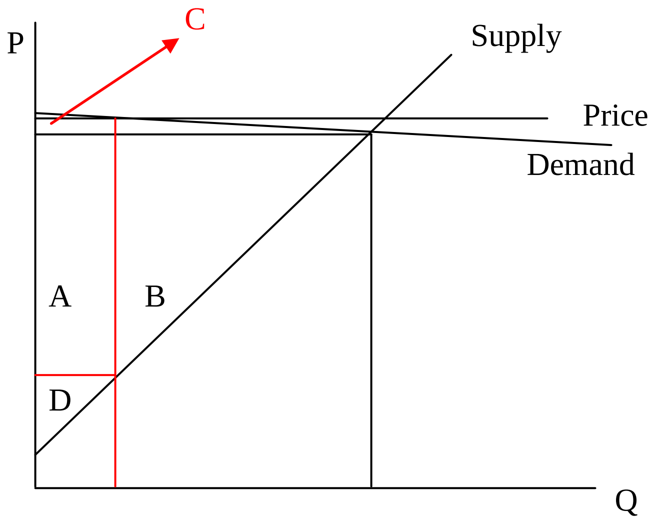
Before tax : PS = A+B+C
After Tax: PS = C, Tax revenue = A: producer gets A+C and loses B.

b) The government provides a per-unit subsidy to the producers of a good and the cost to the government of the subsidy exceeds the benefit received by the producers.



Supply perfectly elastic (constant MC) then there is zero benefit for producer. After subisidy s: p=MC-s

c) The government imposes a price floor in a market to raise the price of the product and producers are made worse off.



Demand elastic, high price floor . Before pricefloor: PS = A+B+D, After price Floor : PS = A+C+D

6. Suppose that in the Bruce and Sheila exchange economy there are only 6 feasible allocations of the goods. The **utilities** of Bruce and Sheila for each of the 6 allocations is listed in the table below:

Allocation	**1**	**2**	**3**	**4**	**5**	**6**
Sheila's utility	20	25	30	18	35	22
Bruce's utility	15	18	10	15	5	12

a) Which allocations are Pareto efficient?
Allocations 2, 3, and 5 are PE. The criterion for an allocation to be PE is that there's no other allocation where one is better off without the other person being worse off.

The remaining questions discuss alternative criteria for selecting allocations (alternative to Pareto efficiency).

b) An alternative is the *maxmin* criteria whereby an allocation is selected if it gives the highest possible utility to the least well-off individual (for example in allocation 1 Bruce is the least well-off individual since 15 < 20). Which allocations are selected by the maxmin criteria? Are those allocations Pareto efficient in this example? Would those allocations be Pareto efficient in general? Does every Pareto efficient allocation satisfy the maxmin criteria? Do you think that every competitive equilibrium allocation will satisfy the maxmin criteria?

Allocation 2 will be selected by the maxmin criterion. This is a PE allocation according to part a). In general these allocations will not be PE. Not every PE allocation satisfies the maxmin criteria, for example: allocation 5 is PE and is not selected by the maxmin criterion. Not every competitive equilibrium allocation will satisfy the maxmin criterion.

c) Another alternative is the *utilitarian* criteria whereby an allocation is selected if the sum of the utilities of the two individuals is the highest possible. Which allocations are selected by the utilitarian criteria? Are those allocations Pareto efficient in this example? Would those allocations be Pareto efficient in general? Does every Pareto efficient allocation satisfy the utilitarian criteria? Do you think that every competitive equilibrium allocation will satisfy the utilitarian criteria?

Allocation 2 will be selected by the utilitarian criterion; this is also a PE allocation. In general, such allocations will be PE, since otherwise we would be able to find another allocation where the sum of the utilities would be higher, and this would contradict the fact that the allocation chosen had the highest sum of utilities. The PE allocations need not satisfy the utilitarian criterion (like 3 and 5 for example). There are competitive equilibriums that do not satisfy the utilitarian criterion.

d) Another is an *equality* criteria whereby an allocation is selected if the absolute value of the difference of the two utilities is as small as possible. Which allocations are selected by the equality criteria? Are those allocations Pareto efficient in this example? Would those allocations be Pareto efficient in general? Does every Pareto efficient allocation satisfy the equality criteria? Do you think that every competitive equilibrium allocation will satisfy the equality criteria?

Allocation 4 will be selected by the equality criterion. It’s not a PE allocation, and in general allocations that satisfy equality criterion are not necessary to be a PE allocation. The PE allocations need not satisfy criterion either. There are competitive equilibria that do not satisfy the equality criterion.

7.

2. After work Melvyn comes home to his wife Jane with 6 bottles of beer.

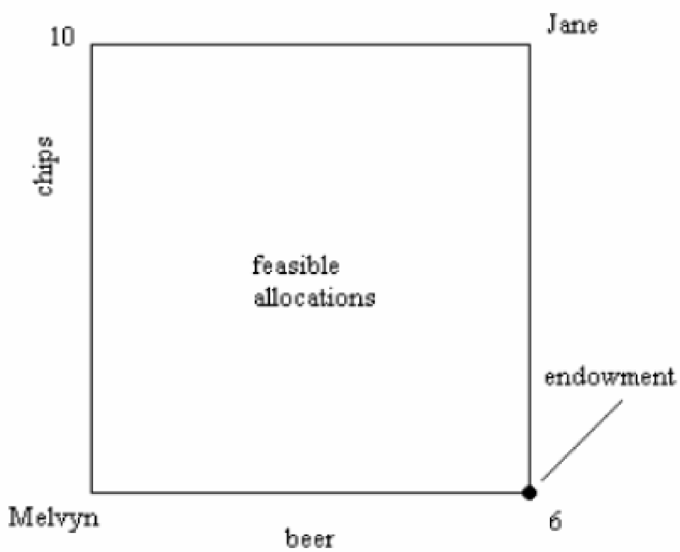
a) If both Melvyn and Jane prefer more beer to less then what allocation(s) of the 6 bottles of beer between them will be Pareto efficient? [NB in this part of question you are only allocating the one good, beer, between them]

Every allocation is Pareto efficient: since both prefer more beer to less, any trade between the two will necessarily make one of them worse off. So any given allocation cannot be Pareto improved.

Suppose that in addition to Melvyn's 6 bottles of beer, Jane has 10 bags of chips.

b) Illustrate the feasible allocations of the two goods between Melvyn and Jane. Make sure that you include the initial allocation (endowment) in your diagram.

b)



c) Suppose Melvyn's preferences over beer (x^M) and chips (y^M) and Jane's preferences over beer (x^J) and (y^J) are given by the following;

$$U^M(x^M,y^M) = (x^M+1)(y^M+1) \text{ and } U^J(x^J,y^J) = (x^J+1)(y^J+1)$$

Is the initial allocation efficient? What are the efficient allocations of beer and chips for these preferences? Illustrate the contract curve in an Edgeworth box diagram.

c)

$$\begin{aligned} \mathrm{MRS}^M &= \frac{y^M+1}{x^M+1} \\ \mathrm{MRS}^J &= \frac{y^J+1}{x^J+1} \end{aligned}$$

At initial endowment:

$$\begin{aligned} \mathrm{MRS}^M(6,0) &= \frac{1}{7} \\ \mathrm{MRS}^J(0,10) &= \frac{11}{1} = 11 \end{aligned}$$

so $\mathrm{MRS}^J(0,10) > \mathrm{MRS}^M(6,0)$ and endowment is not efficient: at this point, Jane is willing to give up 11 bags of chips to get one bottle of beer, and Melvyn would trade a bottle for only $\frac{1}{7}$ of a bag.

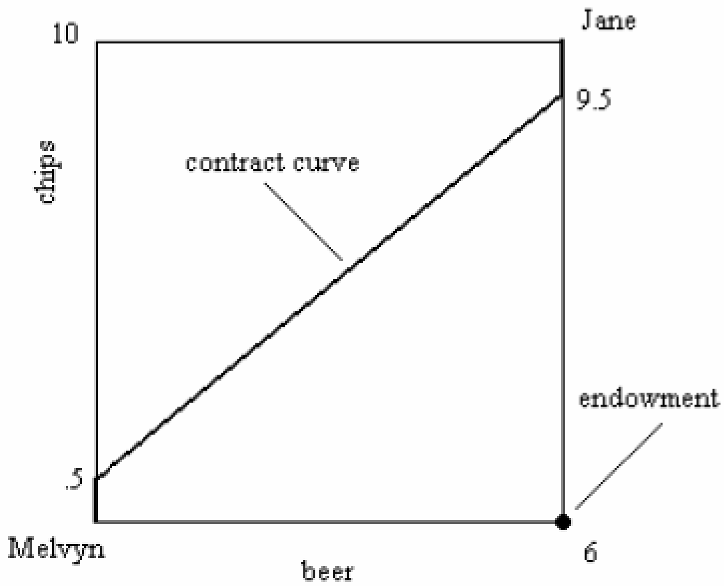
Efficient allocations solve the following equations:

$$\begin{aligned} \mathrm{MRS}^M &= \mathrm{MRS}^J \\ x^M + x^J &= 6 \\ y^M + y^J &= 10 \end{aligned}$$

Hence

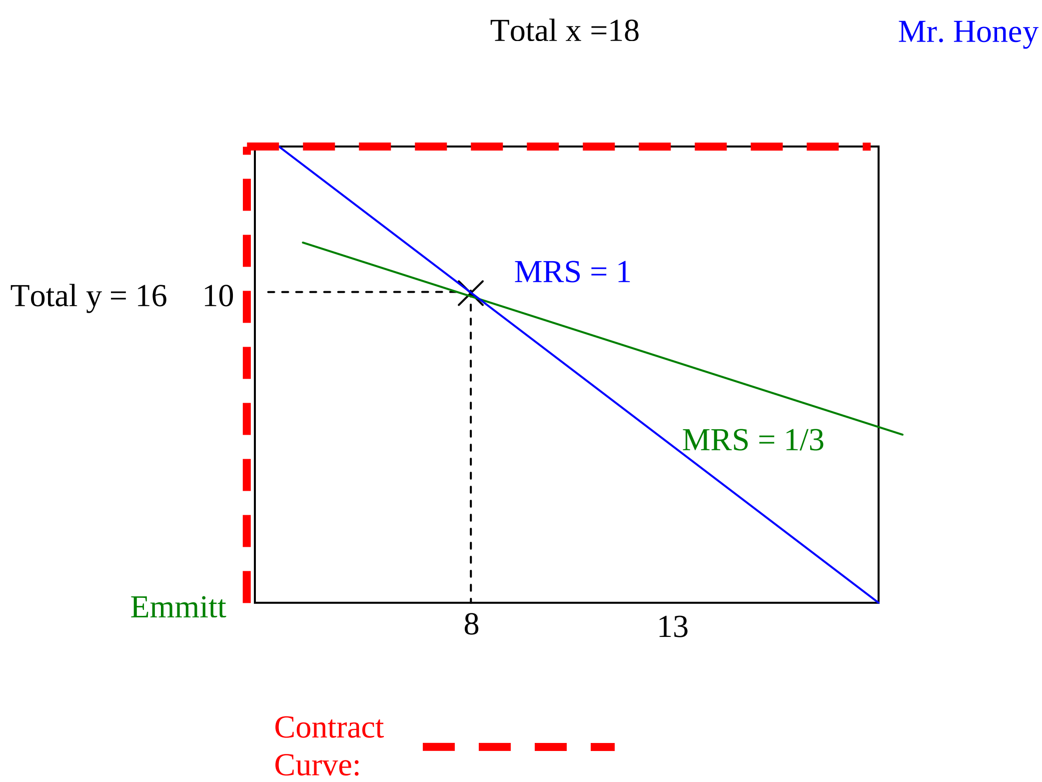
$$\begin{aligned} \frac{y^M+1}{x^M+1} &= \frac{10-y^M+1}{6-x^M+1} \\ (y^M+1)(7-x^M) &= (11-y^M)(x^M+1) \\ 7y^M - x^My^M + 7 - x^M &= 11x^M + 11 - x^My^M - y^M \\ y^M &= \frac{1}{2} + \frac{3}{2}x^M, \end{aligned}$$

which is the equation of the contract curve.


10
Jane
9.5
chips
contract curve
endowment
.5
Melvyn
beer
6

Question 9

Emmitt's preferences over cans of tuna (x) and cans of salmon (y) can be represented by the utility function, $U(x,y) = x + 3y$. Mr Honey's preferences over the same two goods can be represented by the utility function, $U(x,y) = x + y$. Suppose that Emmitt is endowed with the bundle (8,10) and Mr Honey is endowed with (10,6). Illustrate the contract curve for the exchange economy of Emmitt and Mr Honey.



The endowment point is not pareto efficient, since the region between the indifference curves to the upper left is preferred by both. Such a region will exit anywhere in the interior of the box and on the right and the bottom border of the box. However on the upper border and the left border of the box this region is entirely outside of the box (or more precisely it doesn't exist, as preferences are not defined for negative consumption). Thus for these points there is no pareto superior region and they are thus efficient. Therefore the upper and the left border constitute the contract curve.

8 a) Define a Pareto Efficient Allocation for a 2 person, 2 good exchange economy.

For the remainder of the question you will discuss the following two person, two good exchange economy where Sheila is endowed with 10 pens (x) and 10 pads of paper (y) and Bruce is endowed with 16 pens and 4 pads of paper. Sheila's utility function is $U^S(x, y) = x^s y^s$ and Bruce's is $U^B(x, y) = \ln x^B + 2\ln y^B$.

b) What are Bruce and Sheila's marginal rates of substitution at the endowment point? Draw an Edgeworth box diagram that illustrates this Sheila and Bruce economy. Draw a sample set of indifference curves through the endowment point.

c) Is the endowment Pareto efficient? Why or why not?

d) Can the endowment be an allocation associated with a general equilibrium set of prices if both markets are perfectly competitive? Why or why not?

e) If Sheila and Bruce trade what will be the pattern of mutually beneficial trade? What will be the range of the terms of trade that are associated with mutually beneficial trade?

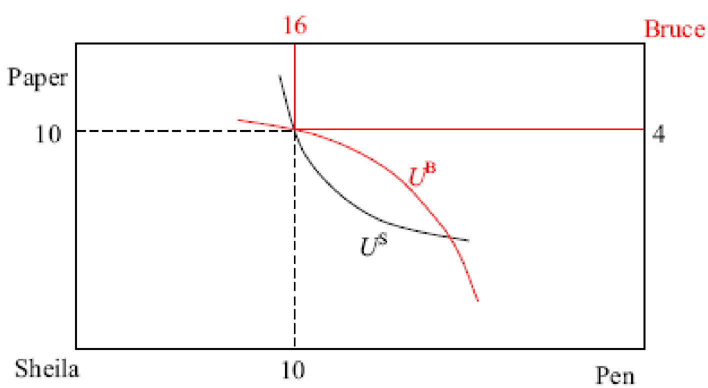
f) Give an example of one trade that would make both Sheila and Bruce strictly better off. Illustrate that trade in your Edgeworth box diagram.

Answer:

a) It is an allocation with the property that there does not exist any other feasible allocation that both individuals prefer and at least one of them strictly prefers.

b) $MRS^S = \dfrac{y^S}{x^S} = \dfrac{10}{10} = 1$ at (10, 10)

$$MRS^B = \frac{y^B}{2x^B} = \frac{4}{2 \cdot 16} = \frac{1}{8} \text{ at } (16, 4)$$



c) No, the endowment is not Pareto efficient because the marginal rates of substitution at the endowment point are not equal.

d) No, it cannot because a general equilibrium allocation is Pareto efficient when the markets are perfectly competitive.

e) Sheila will give up pads of paper and get pens because she is will to give up one pad of paper to get one pen while Bruce is willing to receive only 1/8 of a pad to give her a pen. The range of terms of trade is [1/8, 1].

f) If Sheila gives up 1 pad of paper and gets 2 pens (the term of trade is ½ in this example), she ends up with (12, 9) and Bruce will have (14, 5). Both of them are strictly better off:

10. Sheila and Bruce are going to the park. They are bringing a total of 18 ounces of chips (x) and 18 ounces of pretzels (y). Sheila and Bruce have preferences that can be represented by the following utility functions: $U^S(x^S, y^S) = 3\ln x^S + \ln y^S$ and $U^B(x^B, y^B) = \ln x^B + \ln y^B$.

a) Consider the equal division of the endowment where each receives 9 ounces of each good. Is this allocation envy free? Is the equal division a Pareto efficient allocation of chips and pretzels?

It is envy free since both consume the same bundle so why would anyone envy the others' bundle.

Not an efficient allocation.

$$MRS^S = \frac{3y^S}{x^S} = \frac{3 \times 9}{9} = 3$$

$$MRS^B = \frac{y^B}{x^B} = \frac{9}{9} = 1$$

b) Given their marginal rates of substitution at the endowment point who values chips more highly? What will be the pattern of trade associated with every mutually beneficial trade?

Sheila does

c) Letting P represent the terms of trade (ounces of pretzels per ounce of chips) then what is the range of the terms of trade associated with mutually beneficial trades?

Anything between 1 and 3

d) Find one trade that will make both Sheila and Bruce strictly better off. To receive full credit you must verify that it makes both Sheila and Bruce better off.

Let Sheila give 0.15 of y for 0.1 of x. The new bundle of Sheila will be (9.1 , 8.85) and Bruce gets the rest. Verify by plugging in utility function.

e) Illustrate in an Edgeworth box diagram the endowment point, a pair of indifference curves passing through the equal division and the trade that makes both better off.

It's the usual figure