Homework 10

1. Sheila and Bruce are taking a long plane flight. Sheila brought 10 bags of peanuts (x) and 10 bags of chips (y). Sheila's utility function is $U^S(x,y) = \ln x^S + \ln y^S$. Bruce brought 12 bags of peanuts and 8 bags of chips. Bruce's utility function is $U^B(x,y) = 3x^B + y^B$

a) What are Bruce and Sheila's marginal rates of substitution at the endowment point? In an Edgeworth box diagram illustrate the endowment point and draw a sample set of indifference curves through the endowment point.
b) If Sheila and Bruce trade what will be the pattern of mutually beneficial trade? What will be the range of the terms of trade that are associated with mutually beneficial trade?
c) Provide an example of one trade that DOES NOT make both Sheila and Bruce strictly better off. Illustrate the allocation after the trade in your Edgeworth box.
d) Provide an example of one trade that DOES make both Sheila and Bruce strictly better off. Illustrate the allocation after the trade in your Edgeworth box diagram.
e) Find the contract curve for this exchange economy and illustrate it in your diagram.

2. Sheila and Bruce are going to the park. They are bringing a total of 18 ounces of chips (x) and 18 ounces of pretzels (y). Sheila and Bruce have preferences that can be represented by the following utility functions: $U^S(x^S,y^S) = 3\ln x^S + \ln y^S$ and $U^B(x^B,y^B) = \ln x^B + \ln y^B$.

a) Consider the equal division of the endowment where each receives 9 ounces of each good. Is this allocation envy free?
b) Is the equal division a Pareto efficient allocation of chips and pretzels?
c) Given their marginal rates of substitution at the endowment point who values chips more highly? What will be the pattern of trade associated with every mutually beneficial trade?
d) Letting P represent the terms of trade (ounces of pretzels per ounce of chips) then what is the range of the terms of trade associated with mutually beneficial trades?
e) Find one trade that will make both Sheila and Bruce strictly better off (you must verify that it makes both Sheila and Bruce better off).
f) Illustrate in an Edgeworth box diagram the endowment point, a pair of indifference curves passing through the equal division and the trade that makes both of them better off (from part (e)).

3. Sheila and Bruce are having a party. Sheila has brought with her 20 cans of beer **(x)** and 12 bags of nuts **(y)**. Bruce has brought 20 cans of beer and 10 bags of nuts. Sheila and Bruce have preferences that can be represented by the following utility functions:

$$U^S(x^S,y^S) = \ln x^S + \ln y^S \text{ and } U^B(x^B,y^B) = 4\ln x^B + \ln y^B$$

a) Given their preferences is the endowment a Pareto efficient allocation of nuts and beer?
b) Given their marginal rates of substitution at the endowment point who values beer more highly? What will be the pattern of trade associated with every mutually beneficial trade? What is the range of the terms of trade associated with mutually beneficial trade?
c) Suppose that Sheila proposes to give Bruce 1 can of beer in exchange for 3 bags of nuts. Without calculating their utilities, can you determine if this trade is mutually beneficial or not? Briefly explain. Use their utility functions to verify your answer.
d) Could the price vector = (3,1) be a General Equilibrium price vector for this economy? Briefly explain.

e) In the Edgeworth box diagram below illustrate the endowment point and a pair of indifference curves through the endowment. Illustrate the trade that Sheila proposed in part (c).

4. Suppose that there are two goods and two individuals in an exchange economy. Melvyn is endowed with 10 bottles of beer (x) and 5 bottles of wine (y). His roommate Martin has only 15 bottles of beer (i.e. his endowment of wine, is 0). Melvyn's preferences are represented by the utility function $\ln x + \ln y$ then and Martin's preferences are represented by $2\ln x + \ln y$.

a) Find the competitive price ratio and allocation for this economy.
b) Graph the competitive outcome in an Edgeworth box Be sure to include the budget line and sample indifference curves at the competitive allocation.
c) Use Walras' law to check that your answer to (a) was correct.

5. Sheila and Bruce are going to the movies. Sheila is bringing 20 ounces of soda (x) and 8 ounces of chocolate (y). Bruce is bringing 10 ounces of soda and 12 ounces of chocolate. Sheila's and Bruce's preferences over soda and chocolate can be represented by the utility functions: $U^S(x^S,y^S) = 27\ln x^S + 7\ln y^S$ and $U^B(x^B,y^B) = 6\ln x^B + \ln y^B$.

a) What is Sheila's MRS at her endowment? What is Bruce's MRS at his endowment? Is the endowment point Pareto efficient? Illustrate in an Edgeworth box diagram the endowment point and a pair of indifference curves through the endowment point.

Instead of trading directly with each other Sheila and Bruce trade with a market. Sheila and Bruce are both price takers on both the soda and chocolate markets. Each can buy and sell a good at the same price. Let P_x be the price of an ounce of soda and P_y be the price of an ounce of chocolate.

b) Write the two equations that define the best bundle for Sheila given that the prices are P_x and P_y. Do the same for Bruce.
c) Suppose that the price of soda (P_x) is equal to 13 cents and that the price of chocolate (P_y) is equal to 10 cents. Given the MRS's that you found in part (a) explain why these prices could not represent a general equilibrium pair of prices.
d) Using your answer for part (b) find Sheila's best bundle when $P_x = 13$ and $P_y = 10$. Is Sheila a net demander or a net supplier of chocolate (y)? Find Bruce's best bundle at the same prices. Is Bruce a net demander or supplier of chocolate?
e) Using your answer for part (d) show that when $P_x = 13$ and $P_y = 10$ neither market clears. Which market has excess supply? Which market has excess demand? Illustrate the budget line associated with these prices in your diagram for part (a).
f) Using your answer from part (b) show that if $P_x = 3$ and $P_y = 1$ then both markets clear.
g) Illustrate the budget line associated with the general equilibrium prices in your diagram.

6. For US cattle ranchers who want to enter the Japanese beef market, it is crucial to be able to raise the Wagyu breed of cattle. Unfortunately it is generally impossible to import the Wagyu breed from Japan. However, a Texan named Don Lively managed to obtain a quantity of Wagyu bull semen. This was the only source of the semen outside Japan, and ranchers from all over the world paid Mr Lively $250 per vial.

a) Assuming that the marginal cost of Wagyu semen is zero and that Mr Lively was maximizing profit, illustrate his price and output decisions. What can you say about the price elasticity of demand for the semen when the price is $250 per vial?

Now suppose that the government decided to levy a 10 percent tax on the sale of bull semen. That is Mr Lively would have to pay 10 percent of his gross revenues to the government.

b) Graphically illustrate Mr Lively's price and output decisions in the presence of the tax. Who bears the burden of the tax?

7. The taxation of a monopoly can sometimes produce results different from those that arise in the competitive case. Consider a per unit tax on the monopolist's good. For every unit sold the monopolist must pay the government $t per unit. In the following assume that the firm has constant marginal costs of production c.

a) Write the first order conditions for the profit maximizing level of output given the tax.
b) The first order condition in part (a) establishes an implicit relationship between the quantity chosen by the monopolist and the tax. Use the first order condition to find the derivative of the profit maximizing level quantity with respect to the tax. (Hint: refer to your notes on the incidence of a per unit tax in the competitive case and adapt that analysis to find the derivative of the implicitly defined relationship). Use this answer to write the derivative of the monopoly price with respect to the tax.
c) Suppose that the firm faces a linear demand curve. Use the expression for the derivative in part (b) to determine how much the price rises as a function of the tax. Illustrate your answer in a diagram. Contrast the answer here with the incidence of a tax in the perfectly competitive case.
d) Suppose instead that the demand curve in part (c) were a constant elasticity curve. Show that for some elasticities the price would now increase by more than the tax. Was this possible in the perfectly competitive market?

8. *Horizon* provides broadband service in Gotham. The production function for *Horizon* is given by the following: $Q = f(L,K) = 5L^{\frac{1}{5}}K^{\frac{4}{5}}$ where L represents man-hours, K represents machine hours and output Q is measured in households served. The price of an hour of labor is $16 and the price of an hour of machine time is $2.

a) Given the input prices and the production function set up the cost minimization problem. Solve the cost minimization problem to determine the (compensated) factor demands for labor and machines. Finally use the factor demands to show that the variable cost function (the cost of the least-cost bundle) is VC(Q) = Q.

Assume for the remainder of the question that the fixed costs of *Horizon* are 4000.

b) Given the fixed costs what are the average costs of *Horizon*? What is the marginal cost curve of *Horizon*. Illustrate the two cost curves in the diagram below. For what values of Q do *Horizon*'s cost curves exhibit "scale" economies?

Because of *Horizon*'s cost structure it is a monopolist in Gotham. Demand for broadband in Gotham is given by **Q = 510- 10P** where Q is again measured in thousands of households hours and P is the price of serving a household.

c) Given its cost structure and the market demand, how many households will *Horizon* choose to serve? What price will it charge?

d) In your diagram for part (b) illustrate the demand curve and marginal revenue curve for this market. Indicate the profit maximizing price and quantity that you found in part (d). In your diagram for part (b) illustrate the profits of *Horizon* at the profit maximizing price and quantity. Illustrate the producer surplus at this same price and quantity.
e) The profits and the producer surplus are different in your diagram. What explains the difference?

The government is aware that as a monopolist *Horizon* is earning supernormal profits. A regulator is considering setting a price so that *Horizon* earns normal profits.

f) What price should the regulator set to achieve zero profits for *Horizon*? What quantity would be traded at this price (assuming that consumers can buy as many units at this price as they would desire). At this price would *Horizon* be willing to supply all of the units that consumers wish to buy? Briefly explain.
g) Illustrate the price and quantity that results in 0 profits in your diagram.

9. A video game producer has costs of $25,000 per month that are fixed with regard to output. The firm's marginal cost is $5 per unit of output for output between 1 and 15,000 units. The firm cannot produce more that 15,000 units. Information from the market research group indicates that the demand for the video game can be represented by the following $P = 9.8 - .0002Q$.

a) One officer of the firm feels that price should be set at the level that would maximize the revenue of the firm. At what price would this objective be accomplished? What would be the price elasticity and marginal revenue at this price? What are the profits of the firm at this price?
b) Other officers are concerned with profit. What price should be set to maximize profit? What output is sold? What are the profits of the firm?

The firm has the opportunity to sell in a second market that is separated from the first. For the second market the market research group has estimated the demand relationship to be $P_2 = 7 - .0001Q_2$.

c) Some officers of the firm believe that this second market offers an opportunity for additional profit. Should the firm sell any units in this market? Should it sell only units that would not be absorbed in the primary market at the profit-maximizing price? Should it divert some units from the primary to the secondary market? What price would you set in each market? What is the profit of the firm?