CHAPTER 15

Section 15.1

1. We test $H_0 : m = 100$ vs. $H_a : m \neq 100$. The test statistic is s_+ = sum of the ranks associated with the positive values of $(x_i - 100)$, and we reject H_o at significance level .05 if $s_+ \geq 64$. (from Table A.13, n = 12, with $a / 2 = .026$, which is close to the desired value of . 025), or if $s_+ \leq \frac{12(13)}{2} - 64 = 78 - 64 = 14$.

x_i	$(x_i - 100)$	ranks
105.6	5.6	7*
90.9	-9.1	12
91.2	-8.8	11
96.9	-3.1	3
96.5	-3.5	5
91.3	-8.7	10
100.1	0.1	1*
105	5	6*
99.6	-0.4	2
107.7	7.7	9*
103.3	3.3	4*
92.4	-7.6	8

S_+ = 27, and since 27 is neither ≥ 64 nor ≤ 14, we do not reject H_o. There is not enough evidence to suggest that the mean is something other than 100.

2. We test $H_0 : m = 25$ vs. $H_a : m > 25$. With n = 5 and $a \approx .03$, reject H_o if $s_+ \geq 15$. From the table below we arrive at s_+ =1+5+2+3 = 11, which is not ≥ 15, so do not reject H_o. It is still plausible that the mean = 25.

x_i	$(x_i - 25)$	ranks
25.8	0.8	1*
36.6	11.6	5*
26.3	1.3	2*
21.8	-3.2	4
27.2	2.2	3*

3. We test $H_0 : m = 7.39$ vs. $H_a : m \neq 7.39$, so a two tailed test is appropriate. With n = 14 and $a / 2 = .025$, Table A.13 indicates that H_o should be rejected if either $s_+ \geq 84 or \leq 21$. The $(x_i - 7.39)$'s are -.37, -.04, -.05, -.22, -.11, .38, -.30, -.17, .06, -.44, .01, -.29, -.07, and -.25, from which the ranks of the three positive differences are 1, 4, and 13. Since $s_+ = 18 \leq 21$, H_o is rejected at level .05.

4. The appropriate test is $H_0 : m = 30$ vs. $H_a : m < 30$. With n = 15, and $a = .10$, reject H_o if $s_+ \leq \frac{15(16)}{2} - 83 = 37$.

x_i	$(x_i - 30)$	ranks	x_i	$(x_i - 30)$	ranks
30.6	0.6	3*	31.9	1.9	5*
30.1	0.1	1*	53.2	23.2	15*
15.6	-14.4	12	12.5	-17.5	13
26.7	-3.3	7	23.2	-6.8	11
27.1	-2.9	6	8.8	-21.2	14
25.4	-4.6	8	24.9	-5.1	10
35	5	9*	30.2	0.2	2*
30.8	0.8	4*			

$S_+ = 39$, which is not ≤ 37, so H_o cannot be rejected. There is not enough evidence to prove that diagnostic time is less than 30 minutes at the 10% significance level.

5. The data is paired, and we wish to test $H_0 : m_D = 0$ vs. $H_a : m_D \neq 0$. With n = 12 and $a = .05$, H_o should be rejected if either $s_+ \geq 64$ or if $s_+ \leq 14$.

d_i	-.3	2.8	3.9	.6	1.2	-1.1	2.9	1.8	.5	2.3	.9	2.5
rank	1	10*	12*	3*	6*	5	11*	7*	2*	8*	4*	9*

$s_+ = 72$, and $72 \geq 64$, so H_o is rejected at level .05. In fact for $a = .01$, the critical value is c = 71, so even at level .01 H_o would be rejected.

6. We wish to test $H_0 : m_D = 5$ vs. $H_a : m_D > 5$, where $m_D = m_{black} - m_{white}$. With n = 9 and $a \approx .05$, H_o will be rejected if $s_+ \geq 37$. As given in the table below, $s_+ = 37$, which is ≥ 37, so we can (barely) reject H_o at level approximately .05, and we conclude that the greater illumination does decrease task completion time by more than 5 seconds.

d_i	$d_i - 5$	rank	d_i	$d_i - 5$	rank
7.62	2.62	3*	16.07	11.07	9*
8	3	4*	8.4	3.4	5*
9.09	4.09	8*	8.89	3.89	7*
6.06	1.06	1*	2.88	-2.12	2
1.39	-3.61	6			

7. $H_0 : m_D = .20$ vs. $H_a : m_D > .20$, where $m_D = m_{outdoor} - m_{indoor}$. $a = .05$, and because n = 33, we can use the large sample test. The test statistic is $Z = \frac{s_+ - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}}$, and we reject H_o if $z \geq 1.96$.

d_i	$d_i - .2$	rank	d_i	$d_i - .2$	rank	d_i	$d_i - .2$	rank
0.22	0.02	2	0.15	-0.05	5.5	0.63	0.43	23
0.01	-0.19	17	1.37	1.17	32	0.23	0.03	4
0.38	0.18	16	0.48	0.28	21	0.96	0.76	31
0.42	0.22	19	0.11	-0.09	8	0.2	0	1
0.85	0.65	29	0.03	-0.17	15	-0.02	-0.22	18
0.23	0.03	3	0.83	0.63	28	0.03	-0.17	14
0.36	0.16	13	1.39	1.19	33	0.87	0.67	30
0.7	0.5	26	0.68	0.48	25	0.3	0.1	9.5
0.71	0.51	27	0.3	0.1	9.5	0.31	0.11	11
0.13	-0.07	7	-0.11	-0.31	22	0.45	0.25	20
0.15	-0.05	5.5	0.31	0.11	12	-0.26	-0.46	24

$s_+ = 434$, so $z = \frac{424 - 280.5}{\sqrt{3132.25}} = \frac{143.5}{55.9665} = 2.56$. Since $2.56 \geq 1.96$, we reject H_o at significance level .05.

8. We wish to test $H_0 : m = 75$ vs. $H_a : m > 75$. Since n = 25 the large sample approximation is used, so H_o will be rejected at level .05 if $z \ge 1.645$. The $(x_i - 75)'s$ are –5.5, -3.1, -2.4, -1.9, -1.7, 1.5, -.9, -.8, .3, .5, .7, .8, 1.1, 1.2, 1.2, 1.9, 2.0, 2.9, 3.1, 4.6, 4.7, 5.1, 7.2, 8.7, and 18.7. The ranks of the positive differences are 1, 2, 3, 4.5, 7, 8.5, 8.5, 12.5, 14, 16, 17.5, 19, 20, 21, 23, 24, and 25, so s_+ = 226.5 and $\frac{n(n+1)}{4} = 162.5$. Expression (15.2) for s^2 should be used (because of the ties): $t_1 = t_2 = t_3 = t_4 = 2$, so $s_{s_+}^2 = \frac{25(26)(51)}{24} - \frac{4(1)(2)(3)}{48} = 1381.25 - .50 = 1380.75$ and $s = 37.16$. Thus $z = \frac{226.5 - 162.5}{37.16} = 1.72$. Since $1.72 \ge 1.645$, H_o is rejected. $p - value \approx 1 - \Phi(1.72) = .0427$. The data indicates that true average toughness of the steel does exceed 75.

9.

r_1	1	1	1	1	1	1	2	2	2	2	2	2
r_2	2	2	3	3	4	4	1	1	3	3	4	4
r_3	3	4	2	4	2	3	3	4	1	4	1	3
r_4	4	3	4	2	3	2	4	3	4	1	3	1
D	0	2	2	6	6	8	2	4	6	12	10	14

r_1	3	3	3	3	3	3	4	4	4	4	4	4
r_2	1	1	2	2	4	4	1	1	2	2	3	3
r_3	2	4	1	4	1	2	2	3	1	3	1	2
r_4	4	2	4	1	2	1	3	2	3	1	2	1
D	6	10	8	14	16	18	12	14	14	18	18	20

When H_o is true, each of the above 24 rank sequences is equally likely, which yields the distribution of D when H_o is true as described in the answer section (e.g., P(D = 2) = P(1243 or 1324 or 2134) = 3/24). Then c = 0 yields $a = \frac{1}{24} = .042$ while c = 2 implies $a = \frac{4}{24} = .167$.

Section 15.2

10. The ordered combined sample is 163(y), 179(y), 213(y), 225(y), 229(x), 245(x), 247(y), 250(x), 286(x), and 299(x), so w = 5 + 6 + 8 + 9 + 10 = 38. With m = n = 5, Table A.14 gives the upper tail critical value for a level .05 test as 36 (reject H_o if W ≥ 36). Since $38 \ge 36$, H_o is rejected in favor of H_a.

11. With X identified with pine (corresponding to the smaller sample size) and Y with oak, we wish to test $H_0: m_1 - m_2 = 0$ vs. $H_a: m_1 - m_2 \neq 0$. From Table A.14 with m = 6 and n = 8, H_o is rejected in favor of H_a at level .05 if either $w \geq 61$ or if $w \leq 90 - 61 = 29$ (the actual α is 2(.021) = .042). The X ranks are 3 (for .73), 4 (for .98), 5 (for 1.20), 7 (for 1.33), 8 (for 1.40), and 10 (for 1.52), so w = 37. Since 37 is neither ≥ 61 nor ≤ 29, H_o cannot be rejected.

12. The hypotheses of interest are $H_0: m_1 - m_2 = 1$ vs. $H_a: m_1 - m_2 > 1$, where 1(X) refers to the original process and 2 (Y) to the new process. Thus 1 must be subtracted from each x_I before pooling and ranking. At level .05, H_o should be rejected in favor of H_a if $w \geq 84$.

x – 1	3.5	4.1	4.4	4.7	5.3	5.6	7.5	7.6
rank	1	4	5	6	8	10	15	16
y	3.8	4.0	4.9	5.5	5.7	5.8	6.0	7.0
rank	2	3	7	9	11	12	13	14

Since w = 65, H_o is not rejected.

13. Here m = n = 10 > 8, so we use the large-sample test statistic from p. 663. $H_0: m_1 - m_2 = 0$ will be rejected at level .01 in favor of $H_a: m_1 - m_2 \neq 0$ if either $z \geq 2.58$ or $z \leq -2.58$. Identifying X with orange juice, the X ranks are 7, 8, 9, 10, 11, 16, 17, 18, 19, and 20, so w = 135. With $\frac{m(m+n+1)}{2} = 105$ and $\sqrt{\frac{mn(m+n+1)}{12}} = \sqrt{175} = 13.22$, $z = \frac{135-105}{13.22} = 2.27$. Because 2.27 is neither ≥ 2.58 nor ≤ -2.58, H_o is not rejected. $p - value \approx 2(1 - \Phi(2.27)) = .0232.$

14.

x	8.2	9.5	9.5	9.7	10.0	14.5	15.2	16.1	17.6	21.5
rank	7	9	9	11	12.5	16	17	18	19	20
y	4.2	5.2	5.8	6.4	7.0	7.3	9.5	10.0	11.5	11.5
rank	1	2	3	4	5	6	9	12.5	14.5	14.5

The denominator of z must now be computed according to (15.6). With $t_1 = 3$, $t_2 = 2$, $t_3 = 2$, $s^2 = 175 - .0219[2(3)(4) + 1(2)(3) + 1(2)(3)] = 174.21$, so $z = \frac{138.5 - 105}{\sqrt{174.21}} = 2.54$. Because 2.54 is neither ≥ 2.58 nor ≤ -2.58, H_o is not rejected.

15. Let m_1 and m_2 denote true average cotanine levels in unexposed and exposed infants, respectively. The hypotheses of interest are $H_0 : m_1 - m_2 = -25$ vs. $H_a : m_1 - m_2 < -25$. With m = 7, n = 8, H_o will be rejected at level .05 if $w \le 7(7+8+1) - 71 = 41$. Before ranking, -25 is subtracted from each x_I (i.e. 25 is added to each), giving 33, 36, 37, 39, 45, 68, and 136. The corresponding ranks in the combined set of 15 observations are 1, 3, 4, 5, 6, 8, and 12, from which w = 1 + 3 + … + 12 = 39. Because $39 \le 41$, H_o is rejected. The true average level for exposed infants appears to exceed that for unexposed infants by more than 25 (note that H_o would not be rejected using level .01).

16.

a.

X	rank	Y	rank
0.43	2	1.47	9
1.17	8	0.8	7
0.37	1	1.58	11
0.47	3	1.53	10
0.68	6	4.33	16
0.58	5	4.23	15
0.5	4	3.25	14
2.75	12	3.22	13

We verify that w = sum of the ranks of the x's = 41.

b. We are testing $H_0 : m_1 - m_2 = 0$ vs. $H_a : m_1 - m_2 < 0$. The reported p-value (significance) is .0027, which is < .01 so we reject H_o. There is evidence that the distribution of good visibility response time is to the left (or lower than) that response time with poor visibility.

Section 15.3

17. n = 8, so from Table A.15, a 95% C.I. (actually 94.5%) has the form $\left(\bar{x}_{(36-32+1)}, \bar{x}_{(32)}\right) = \left(\bar{x}_{(5)}, \bar{x}_{(32)}\right)$. It is easily verified that the 5 smallest pairwise averages are $\frac{5.0+5.0}{2} = 5.00$, $\frac{5.0+11.8}{2} = 8.40$, $\frac{5.0+12.2}{2} = 8.60$, $\frac{5.0+17.0}{2} = 11.00$, and $\frac{5.0+17.3}{2} = 11.15$ (the smallest average not involving 5.0 is $\bar{x}_{(6)} = \frac{11.8+11.8}{2} = 11.8$), and the 5 largest averages are 30.6, 26.0, 24.7, 23.95, and 23.80, so the confidence interval is (11.15, 23.80).

18. With n = 14 and $\frac{n(n+1)}{2} = 105$, from Table A.15 we se that c = 93 and the 99% interval is $\left(\bar{x}_{(13)}, \bar{x}_{(93)}\right)$. Subtracting 7 from each x_I and multiplying by 100 (to simplify the arithmetic) yields the ordered values –5, 2, 9, 10, 14, 17, 22, 28, 32, 34, 35, 40, 45, and 77. The 13 smallest *sums* are –10, -3, 4, 4, 5, 9, 11, 12, 12, 16, 17, 18, and 19 (so $\bar{x}_{(13)} = \frac{14.19}{2} = 7.095)$ while the 13 largest sums are 154, 122, 117, 112, 111, 109, 99, 91, 87, and 86 (so $\bar{x}_{(93)} = \frac{14.86}{2} = 7.430)$. The desired C.I. is thus (7.095, 7.430).

19. The ordered d_i's are –13, -12, -11, -7, -6; with n = 5 and $\frac{n(n+1)}{2} = 15$, Table A.15 shows the 94% C.I. as (since c = 1) $\left(\bar{d}_{(1)}, \bar{d}_{(15)}\right)$. The smallest average is clearly $\frac{-13-13}{2} = -13$ while the largest is $\frac{-6-6}{2} = -6$, so the C.I. is (-13, -6).

20. For n = 4 Table A.13 shows that a two tailed test can be carried out at level .124 or at level .250 (or, of course even higher levels), so we can obtain either an 87.6% C.I. or a 75% C.I. With $\frac{n(n+1)}{2} = 10$, the 87.6% interval is $\left(\bar{x}_{(1)}, \bar{x}_{(10)}\right) = (.045, .177)$.

21. m = n = 5 and from Table A.16, c = 21 and the 90% (actually 90.5%) interval is $\left(d_{ij(5)}, d_{ij(21)}\right)$. The five smallest $x_i - y_j$ differences are –18, -2, 3, 4, 16 while the five largest differences are 136, 123, 120, 107, 86 (construct a table like Table 15.5), so the desired interval is $(16,86)$.

22. m = 6, n = 8, mn = 48, and from Table A.16 a 99% interval (actually 99.2%) requires c = 44 and the interval is $\left(d_{ij(5)}, d_{ij(44)}\right)$. The five largest $x_i - y_j$'s are 1.52 - .48 = 1.04, 1.40 - .48 = .92, 1.52 - .67 = .85, 1.33 - .48 = .85, and 1.40 - .67 = .73, while the five smallest are –1.04, -.99, -.83, -.82, and -.79, so the confidence interval for $m_1 - m_2$ (where m_1 refers to pine and m_2 refers to oak) is (-.79, .73).

Section 15.4

23. Below we record in parentheses beside each observation the rank of that observation in the combined sample.

1:	5.8(3)	6.1(5)	6.4(6)	6.5(7)	7.7(10)	$r_{1.} = 31$
2:	7.1(9)	8.8(12)	9.9(14)	10.5(16)	11.2(17)	$r_{2.} = 68$
3:	5.191)	5.7(2)	5.9(4)	6.6(8)	8.2(11)	$r_{3.} = 26$
4:	9.5(13)	1.0.3(15)	11.7(18)	12.1(19)	12.4(20)	$r_{4.} = 85$

H_o will be rejected at level .10 if $k \geq c^2_{.10,3} = 6.251$. The computed value of k is $k = \frac{12}{20(21)}\left[\frac{31^2 + 68^2 + 26^2 + 85^2}{5}\right] - 3(21) = 14.06$. Since $14.06 \geq 6.251$, reject H_o.

24. After ordering the 9 observation within each sample, the ranks in the combined sample are

1:	1	2	3	7	8	16	18	22	27	$r_{1.} = 104$
2:	4	5	6	11	12	21	31	34	36	$r_{2.} = 160$
3:	9	10	13	14	15	19	28	33	35	$r_{3.} = 176$
4:	17	20	23	24	25	26	29	30	32	$r_{4.} = 226$

At level .05, $H_0 : m_1 = m_2 = m_3 = m_4$ will be rejected if $k \geq c^2_{.05,3} = 7.815$. The computed k is $k = \frac{12}{36(37)}\left[\frac{104^2 + 160^2 + 176^2 + 226^2}{5}\right] - 3(37) = 7.587$. Since 7.587 is not ≥ 7.815, H_o cannot be rejected.

25. $H_0 : m_1 = m_2 = m_3$ will be rejected at level .05 if $k \geq c^2_{.05,2} = 5.992$. The ranks are 1, 3, 4, 5, 6, 7, 8, 9, 12, 14 for the first sample; 11, 13, 15, 16, 17, 18 for the second; 2, 10, 19, 20, 21, 22 for the third; so the rank totals are 69, 90, and 94.

$k = \frac{12}{22(23)}\left[\frac{69^2}{10} + \frac{90^2}{6} + \frac{94^2}{5}\right] - 3(23) = 9.23$. Since $9.23 \geq 5.992$, we reject H_o.

26.

	1	2	3	4	5	6	7	8	9	10	r_i	r_i^2
A	2	2	2	2	2	2	2	2	2	2	20	400
B	1	1	1	1	1	1	1	1	1	1	10	100
C	4	4	4	4	3	4	4	4	4	4	39	1521
D	3	3	3	3	4	3	3	3	3	3	31	961
												2982

The computed value of F_r is $\frac{12}{4(10)(5)}(2982) - 3(10)(5) = 28.92$, which is $\geq \chi^2_{.01,3} = 11.344$, so H_o is rejected.

27.

	1	2	3	4	5	6	7	8	9	10	r_i	r_i^2
I	1	2	3	3	2	1	1	3	1	2	19	361
H	2	1	1	2	1	2	2	1	2	3	17	289
C	3	3	2	1	3	3	3	2	3	1	24	576
												1226

The computed value of F_r is $\frac{12}{10(3)(4)}(1226) - 3(10)(4) = 2.60$, which is not $\geq \chi^2_{.05,2} = 5.992$, so don't reject H_o.

Supplementary Exercises

28. The Wilcoxon signed-rank test will be used to test $H_0 : \mu_D = 0$ vs. $H_0 : \mu_D \neq 0$, where $\mu_D =$ the difference between expected rate for a potato diet and a rice diet. From Table A.11 with n = 8, H_o will be rejected if either $s_+ \geq 32$ or $s_+ \leq \frac{8(9)}{2} - 32 = 4$. The $d_i's$ are (in order of magnitude) .16, .18, .25, -.56, .60, .96, 1.01, and –1.24, so $s_+ = 1 + 2 + 3 + 5 + 6 + 7 = 24$. Because 24 is not in the rejection region, H_o is not rejected.

29. Friedman's test is appropriate here. At level .05, H_o will be rejected if $f_r \ge \chi^2_{.05,3} = 7.815$. It is easily verified that $r_{1.} = 28$, $r_{2.} = 29$, $r_{3.} = 16$, $r_{4.} = 17$, from which the defining formula gives $f_r = 9.62$ and the computing formula gives $f_r = 9.67$. Because $f_r \ge 7.815$, $H_0 : \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0$ is rejected, and we conclude that there are effects due to different years.

30. The Kruskal-Wallis test is appropriate for testing $H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$. H_o will be rejected at significance level .01 if $k \ge \chi^2_{.01,3} = 11.344$

Treatment			**ranks**			r_i
I	4	1	2	3	5	15
II	8	7	10	6	9	40
III	11	15	14	12	13	65
IV	16	20	19	17	18	90

$k = \frac{12}{420}\left[\frac{225 + 1600 + 4225 + 8100}{5}\right] - 63 = 17.86$. Because $17.86 \ge 11.344$, reject H_o.

31. From Table A.16, m = n = 5 implies that c = 22 for a confidence level of 95%, so $mn - c + 1 = 25 - 22 = 1 = 4$. Thus the confidence interval extends from the 4th smallest difference to the 4th largest difference. The 4 smallest differences are –7.1, -6.5, -6.1, -5.9, and the 4 largest are –3.8, -3.7, -3.4, -3.2, so the C.I. is (-5.9, -3.8).

32.

a. $H_0 : m_1 - m_2 = 0$ will be rejected in favor of $H_a : m_1 - m_2 \neq 0$ if either $w \geq 56$ or $w \leq 6(6+7+1) - 56 = 28$.

Gait	D	L	L	D	D	L	L
Obs	.85	.86	1.09	1.24	1.27	1.31	1.39
Gait	D	L	L	L	D	D	
obs	1.45	1.51	1.53	1.64	1.66	1.82	

$w = 1+4+5+8+12+13 = 43$. Because 43 is neither ≥ 56 nor ≤ 28, we don't reject H_o. There appears to be no difference between m_1 and m_2.

b.

Differences

		Lateral Gait						
		.86	1.09	1.31	1.39	1.51	1.53	1.64
	.85	.01	.24	.46	.54	.66	.68	.79
Diagonal	1.24	-.38	-.15	.07	.15	.27	.29	.40
gait	1.27	-.41	-.18	.04	.12	.24	.26	.37
	1.45	-.59	-.36	-.14	-.06	.06	.08	.19
	1.66	-.80	-.57	-.35	-.27	-.15	-.13	-.02
	1.82	-.96	-.73	-.51	-.43	-.31	-.29	-.18

From Table A.16, c = 35 and $mn - c + 1 = 8$, giving (-.41, .29) as the C.I.

33.

a. With "success" as defined, then Y is a binomial with n = 20. To determine the binomial proportion "p" we realize that since 25 is the hypothesized median, 50% of the distribution should be above 25, thus p = .50. From the Binomial Tables (Table A.1) with n = 20 and p = .50, we see that
$a = P(Y \geq 15) = 1 - P(Y \leq 14) = 1 - .979 = .021$.

b. From the same binomial table as in **a**, we find that
$P(Y \geq 14) = 1 - P(Y \leq 13) = 1 - .942 = .058$ (a close as we can get to .05), so c = 14. For this data, we would reject H_o at level .058 if $Y \geq 14$. Y = (the number of observations in the sample that exceed 25) = 12, and since 12 is not ≥ 14, we fail to reject H_o.

34.

a. Using the same logic as in Exercise 33, $P(Y \le 5) = .021$, and $P(Y \ge 15) = .021$, so the significance level is $\alpha = .042$.

b. The null hypothesis will not be rejected if the median is between the 6th smallest observation in the data set and the 6th largest, exclusive. (If the median is less than or equal to 14.4, then there are at least 15 observations above, and we reject H_o. Similarly, if any value at least 41.5 is chosen, we have 5 or less observations above.) Thus with a confidence level of 95.8% the median will fall between 14.4 and 41.5.

35.

Sample:	y	x	y	y	x	x	x	y	y
Observations:	3.7	4.0	4.1	4.3	4.4	4.8	4.9	5.1	5.6
Rank:	1	3	5	7	9	8	6	4	2

The value of W' for this data is $w' = 3 + 6 + 8 + 9 = 26$. At level .05, the critical value for the upper-tailed test is (Table A.14, m = 4, n = 5) c = 27 ($\alpha = .056$). Since 26 is not ≥ 27, H_o cannot be rejected at level .05.

36. The only possible ranks now are 1, 2, 3, and 4. Each rank triple is obtained from the corresponding X ordering by the "code" 1 = 1, 2 = 2, 3 = 3, 4 = 4, 5 = 3, 6 = 2, 7 = 1 (so e.g. the X ordering 256 corresponds to ranks 2, 3, 2).

X ordering	ranks	w'	X ordering	ranks	w'	X ordering	ranks	w'
123	123	6	156	132	66	267	221	5
124	124	7	157	131	5	345	343	10
125	123	6	167	121	4	346	342	9
126	122	5	234	234	9	347	341	8
127	121	4	235	233	8	356	332	8
134	134	8	236	232	7	357	331	7
135	133	7	237	231	6	367	321	6
136	132	6	245	243	9	456	432	9
137	131	5	246	242	8	457	431	8
145	143	8	247	241	7	467	421	7
146	142	7	256	232	7	567	321	6
147	141	6	257	231	6			

Since when H_o is true the probability of any particular ordering is 1/35, we easily obtain the null distribution and critical values given in the answer section.