Homework 6

1. Recall Scarlett the OB-GYN from class. Suppose the probability that she is sued is .1 and her income is 10000. In the case she is sued she will lose 7500 in the settlement. She may purchase malpractice insurance at a rate of $\$r$ per $\$1$ of coverage. Finally assume that her utility of income is $U(\$) = (\$)^{1/2}$. What is Scarlett's demand curve for insurance (that is find her demand for insurance for all $r \geq .1$)?

2. In the state of Oklahoma the Sooner Insurance Company has noticed the following two differences between men and women drivers:

   1) Men are more likely to have driving accidents: the probability that a man has a car accident is 25% and the probability that a woman has an accident is 19%.
   2) When a man has an accident the damage is likely to be higher: the average damage to a $15,000 car driven by a man is $7500 (so the car is only worth $7500 after the accident) and only $3600 if driven by a woman (so the car is worth only $11,400).

In addition the firm knows that men are willing to pay **at most** $2343.75 for a full insurance policy. Women are willing to pay **at most** $745.50 for a full insurance policy. Drivers are equally divided between men and women.

   a) What is the cost of providing full fair insurance for men? Given the information provided above will men purchase this policy? What is the cost of providing full fair insurance for women? Will women purchase this policy?
   b) What is the expected value of a $15,000 car driven by a man? Given their willingness to pay for full insurance what is the certainty equivalent value of the car? What is the risk premium that they are willing to pay? Illustrate the expected value, the certainty equivalent and the risk premium in a diagram.
   c) What is the expected value of a $15,000 car driven by a woman? Given their willingness to pay for full insurance what is the certainty equivalent value of the car? What is the risk premium that they are willing to pay? Illustrate the expected value, the certainty equivalent and the risk premium in a diagram.

Suppose that the state government passes anti-discrimination legislation and requires that insurance companies offer policies at the same price per dollar of coverage to both men and women.

   d) Suppose that the firm offers only a single price per dollar of coverage and allows individuals to buy insurance to fully insure against their losses. Show that the firm cannot offer a single price for insurance that does not lose money and is acceptable to both men and women.
   e) Given your answer to (c) what policy will the firm offer? Who will be covered men or women?

3. Helen is a small business owner. She has one employee, Gene, whom she pays $1000. Gene's salary is the only cost of running her business. Her revenues vary with market conditions. If demand is HIGH then her revenues are $R_H = 4600$. If demand is LOW then her revenues fall to $R_L = 1900$. Each state (high or low demand) is equally likely. Finally Helen is a risk averse individual and her utility function can be written as $u(\pi) = ln(\pi)$ where π is her profits from the business.

   a) What is her profit if demand is high? What is her profit if demand is low? What is her expected profit?

b) What is the certainty equivalent to her situation? What is the risk premium associated with her situation?

c) Illustrate her expected utility, expected profit, certainty equivalent, and the risk premium in a diagram .

Suppose that Helen offers Gene a profit sharing contract. Hence, instead of paying him a flat wage of 1000, she offers to pay him an amount that varies with the profitability of the company. Specifically, she will pay him W_H when revenues are high and W_L when revenues are low.

d) If $W_H > 1000 > W_L$ and $4600 - W_H > 1900 - W_L$ then in what way is Helen's profit sharing offer like an insurance contract (and who is getting the insurance)? Briefly explain.

For parts (e) through (g) below, assume that the contract that Helen offers Gene is of the form $W_J = 350 + 0.2R_J$ where J is either H or L.

e) What is Gene's expected wage from this contract? If Gene accepts this contract then what is Helen's expected profit?

f) Assuming that Gene accepts the contract then illustrate Helen's expected utility in your diagram above. Will Helen be better or worse off if Gene accepts this contract?

g) If Gene is risk averse then will he accept this contract (ie does he prefer this contract to his flat wage of 1000)? Briefly explain.

Suppose that Gene rejects Helen's contract offer. Gene counteroffers a contract where she pays him W_H and W_L such that Helen is fully insured. In other words, Gene offers a pair of wages such that $R_H - W_H = 4600 - W_H = 1900 - W_L = R_L - W_L$.

h) What would Helen's expected profit be if she accepted this proposal (the answer is a function of the wages)?

i) What is the highest wage demands (W_H and W_L) that Gene can make that Helen will accept? Remember that the wages must satisfy the equation $4600 - W_H = 1900 - W_L$.

j) What is Gene's expected wage at the wages that you determined in part (j)? You can write his expected wages as equal to the sum of his original flat wage and a value, x. In words what is x?

4. Maria has a house worth $202,500 which represents all of her wealth. Unfortunately there is a chance that there will be a fire and that her house will suffer $112,500 in damages. Thus leaving her a house valued at only $90,000. The probability of the fire is .1. Finally assume that her preferences over dollar values of wealth can be represented by the utility function $u(x) = \sqrt{x}$

a) What is her expected wealth? What is Maria's expected utility?

b) What is the certainty equivalent to her situation?

c) What is the risk premium associated with her situation?

d) What is the maximum that Maria would be willing to pay for a full insurance policy?

e) Illustrate her expected utility, expected wealth, certainty equivalent, the risk premium and her maximum willingness to pay for a full insurance policy in the diagram below.

Suppose that she has the option to buy fire insurance. Specifically she can purchase any amount of coverage (C) at a price of $r per dollar of coverage. The maximum coverage that she can purchase is $112,500. If a fire occurs the policy will pay her C. If no fire occurs then the policy pays 0.

f) Write her wealth in each of the 2 states if she purchases $C of coverage. Write her expected utility if she purchases $C of coverage.

g) Using your expected utility from part (f) write the first order conditions that define the coverage C that maximizes her expected utility.

h) What is the price of fair insurance for Maria?

i) If insurance is priced fairly then use your first order conditions from part (g) to find how much insurance Maria will buy.

The local government levies property taxes. In particular the tax on Maria's house is $2,500.

j) If she must pay the tax regardless of whether or not there is a fire then will she change the amount of fair insurance that she purchases? Use your first order conditions from part (g) to justify your answer.

k) If the tax is waived when there is a fire (so she only pays taxes in the no fire state) then will she change the amount of fair insurance that she purchases? Use your first order conditions from part (g) to justify your answer.

5. Lloyd's Limited sells boat insurance. A typical insurance policy costs $r per dollar of coverage which is paid out if an accident occurs. The average probability of a boating accident is 20%.

a) Write the expected payout of an insurance policy. What is the value of r such that the expected payout of the policy is 0?

There are two kinds of individuals. 75% of boaters are *low risk* who have only a 10% chance of an accident. 25% of boaters are *high risk* who have a 50% chance of an accident. Lloyds cannot tell whether someone is high or low risk, and it sells the insurance policy at r = .20 per dollar of coverage to all individuals. For simplicity, assume that all boaters have boats valued at $120,000 and that the damage from an accident is $90,000 (leaving the value of the boat at only $30,000 in the event of an accident). Finally assume that all boaters have a utility function u($) = ln($) where $ represent wealth which is the value of the boat net insurance payments (ie subtract the cost of insurance and add the payout from the policy if applicable).

Luna is a high risk boater so her probability of having an accident is 50%.

b) Assume that she does not have insurance. What is her expected wealth? What is her expected utility?

c) What is the certainty equivalent to her situation without insurance? What is the risk premium associated with her situation without insurance?

d) What is the maximum that Luna would be willing to pay for a full insurance policy?

e) Illustrate her expected utility, expected wealth, certainty equivalent, the risk premium and her maximum willingness to pay for a full insurance policy.

f) At the price of .2 per dollar of coverage, would she be willing to fully insure? Briefly explain your answer.

Helios is a low risk boater so his probability of an accident is 10%.

g) Write his expected utility if he purchases $C of coverage at a price of .2 per dollar of coverage.

h) Using the expected utility from part (h) write the first order conditions that define the coverage C that maximizes his expected utility.

i) Show that full insurance does NOT solve the FOC.

j) Solve the FOC to determine how much insurance that Helios will buy at a price of .2 per dollar of coverage.

A second insurance company, Greenwich LLC , offers another insurance policy. The price of Greenwich's policy is only .10 per dollar of coverage. However, the maximum coverage that Greenwich will sell is 30,000.

k) Will Helios purchase a policy from Greenwich? Briefly explain. Will Luna purchase a policy from Greenwich? Briefly explain.

l) Given the answers above would you Lloyds to continue selling the r=.2 policy? Briefly explain.

6. *Ben's Messenger Service* delivers documents in NYC. Suppose that the only variable input is labor and the relationship between the hours of labor worked and the quantity (measured in documents delivered) produced is given by

$$Q = \sqrt{L}$$

Ben must pay his workers $1 per hour. In addition to labor costs Ben has fixed costs of $64 per day.

a) What is the variable cost curve? What is the total cost curve? Illustrate your answer.

b) What is the marginal cost curve? What is the average total cost curve? What is the quantity that minimizes the per-unit costs of production (the optimal size of the firm)? Illustrate your answer. Be sure to label the optimal size of the firm.

Suppose that at a price of $20 per document Ben can have as much business as he can handle. However at any price higher than $20 he cannot attract any business. In other words Ben's inverse demand curve is constant at $20 per document.

c) What is Ben's revenue curve? What is his marginal revenue?

d) Given your answers to (c) and (b) how many deliveries would Ben like to make?

7. Explain whether each of the following actions would affect the firm's choice of the profit-maximizing output (hint: determine the affect of each on the MR and MC schedules).

a) An increase in the cost of an input such as labor.

b) Introduction of a small fixed fee for a license to do business.

c) Introduction of a 50 percent tax on the firm's profits.

d) Introduction of a per-unit tax on each unit the firm produces.

e) Receipt of a lump sum grant from the government.

f) Receipt of a per-unit subsidy from the government.

g) Receipt of a subsidy per worker hired.