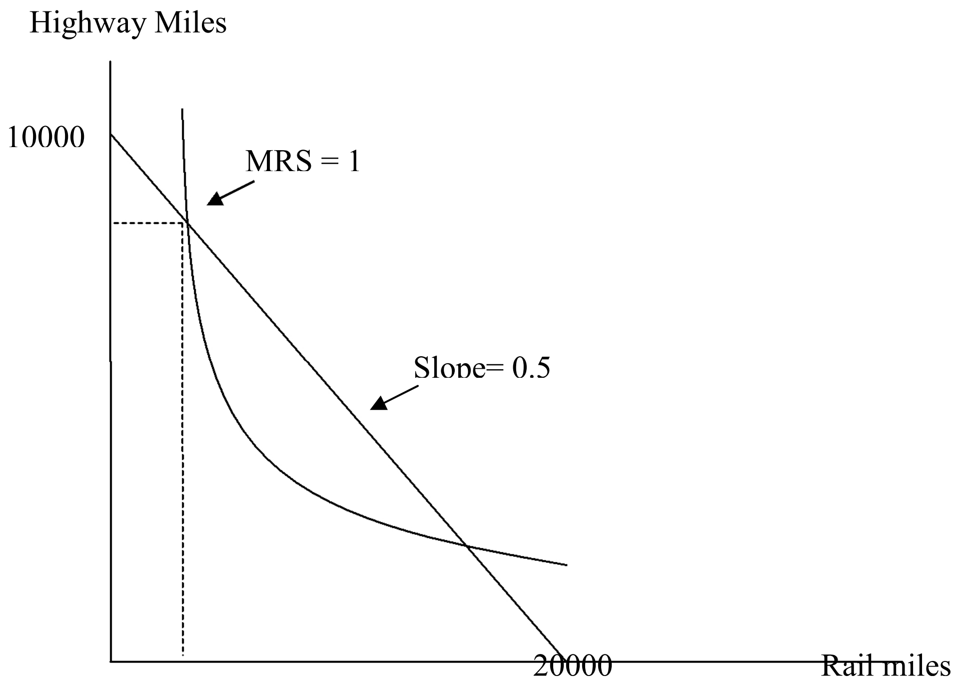
PART I – QUESTION 1

New York State allocates its $1 million transportation budget between maintenance of rail lines and maintenance of highways (both are measured in miles).The cost of maintaining one mile of rail is $50 and the cost of maintaining one mile of highway is $100.

a) **(3 points)** Putting rail lines on the x-axis and highway miles on the y-axis illustrate New York State's budget set. What is the opportunity cost of maintaining an additional rail mile? Show (algebraically) that NY State can afford to maintain 6000 miles of highway and 8000 miles of rail line.

> Opportunity cost = Px/Py = 50/100 = 0.5 highway miles per rail mile
> Cost of (8000, 6000) = (50)*8000 + (100)*6000 = 400000 + 600000 = 1000000
> So (8000,6000) is affordable

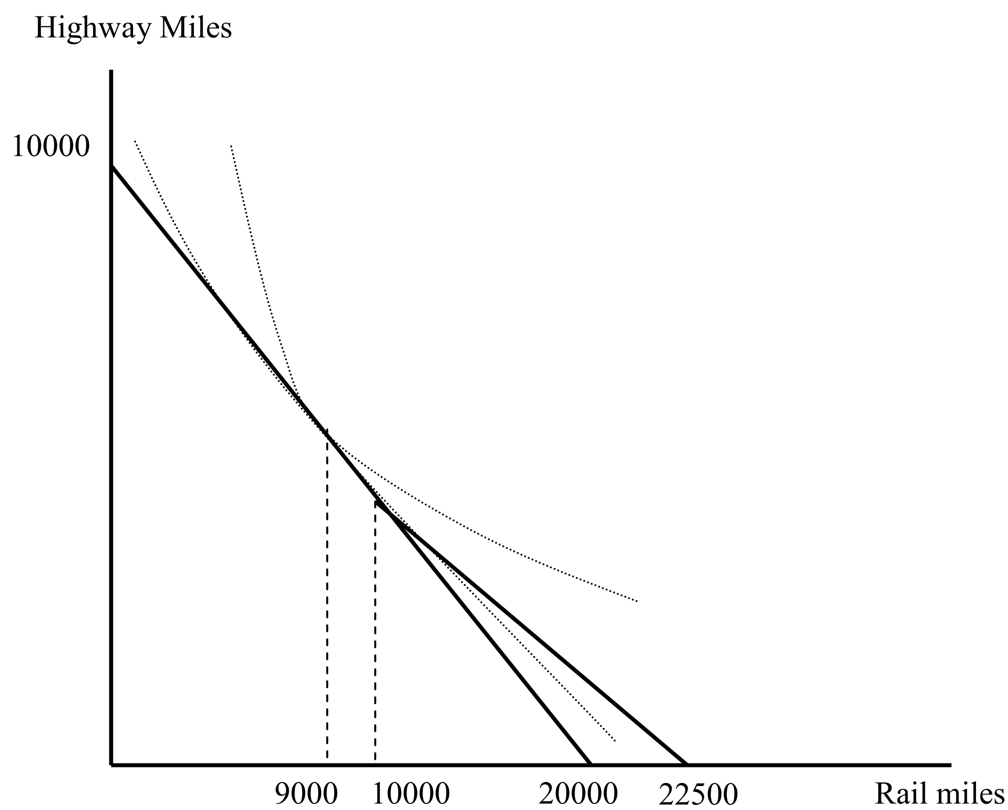


b) **(4 points)** Suppose that NY State's MRS at (8000,6000) is 1. Is (8000,6000) NY State's best bundle? Why or why not? Briefly explain your answer. If not will NY State choose to spend more or less money on rail maintenance? Illustrate the indifference curve through (8000,6000) in your diagram from part (a).

(8000,6000) is not the optimal bundle, because the MRS is greater than the price ratio(slope) at that point. Since MRS>(Px/Py) NY State will choose to spend more money on rail mentainance.

Suppose that the federal government would like to encourage the states to spend more money to maintain rail lines. The federal government is willing to provide a subsidy of $10 per mile of rail line maintained above 10,000 miles. Hence NY State must pay $50 per mile for each mile up to 10,000 and $40 per mile for every mile thereafter.

c) **(4 points)** In a new diagram illustrate NY State's budget set given the Federal government's offer of a subsidy. Be sure to include your budget line from part (a).



d) **(4 points)** If before the subsidy program was announced NY State had planned to maintain 9,000 miles of rail then will it choose to participate in the subsidy program? Use your diagram to explain your answer.

The new budget set does not contain points that are strictly better then the old optimal point (i.e. that have more of both goods than the old optimal point had). The answer thus depends on the particular form of the utility function. As you can see in the diagram, there exists a scenario in which NY State will be better off by participating and there exists one in which they will not be better off by participating.

PART I – QUESTION 2

3. Bruce must decide how to allocate his 24-hour day between non-wage activities, *leisure*, and wage activities, *labor*. For every hour of labor that he supplies he is paid by his employer **\$w**. Bruce can purchase bottles of beer at a price of **\$1** per bottle (and beer is the only consumption good). Bruce has no non-wage income. Finally assume that Bruce's preferences over leisure and beer can be represented by the following utility function: **U(l,b) = b + 120ln(l)**.

a) Find Bruce's demand functions for leisure and beer. What is Bruce's supply curve for labor? Is his supply of labor upward sloping or downward sloping?
b) Suppose that Bruce's wage is **\$10**. Given the demand functions that you found in part (a) what are Bruce's demands for leisure and beer when his wage is \$10? What is his supply of labor? What is the elasticity of Bruce's supply of labor at a wage of \$10?
c) Suppose that Bruce's wage rises to **\$12**. Given the demand functions that you found in part (a) what are Bruce's demands for leisure and beer when his wage is \$12?
d) Putting hours of leisure on the x-axis and bottles of beer of the y-axis illustrate in an indifference curve diagram Bruce's optimal choice of leisure and beer when his wage is \$10. Illustrate in your diagram the effect of the increase in wages on his optimal choice of leisure and beer. Make sure to clearly indicate the income and substitution effects in your diagram. Which effect (the income or substitution) is larger on his demand for leisure?

1. a) Tangency condition:

$$\text{MRS} = \frac{\frac{120}{l}}{1} = \frac{120}{l} = w,$$

so that

$$l = \frac{120}{w}$$

is the demand for leisure.

Feasibility:

$$\begin{aligned} wl + b &= 24w \\ w\frac{120}{w} + b &= 24w \end{aligned}$$

so that

$$b = 24w - 120$$

is the demand for beer.

Supply of labor: $L = 24 - l$:

$$L = 24 - \frac{120}{w},$$

which is upward sloping.

b)

$$\begin{aligned} l &= \frac{120}{10} = 12 \\ b &= (24)(10) - 120 = 120 \\ L &= 24 - 12 = 12. \end{aligned}$$

The elasticity is given by

$$\frac{\mathrm{d}L}{\mathrm{d}w}\frac{w}{L} = \frac{120}{w^2}\frac{w}{L} = \frac{120}{wL} = \frac{120}{(10)(12)} = 1.$$

c) $$l = \frac{120}{12} = 10$$
$$b = 24 \cdot 12 - 120 = 168$$

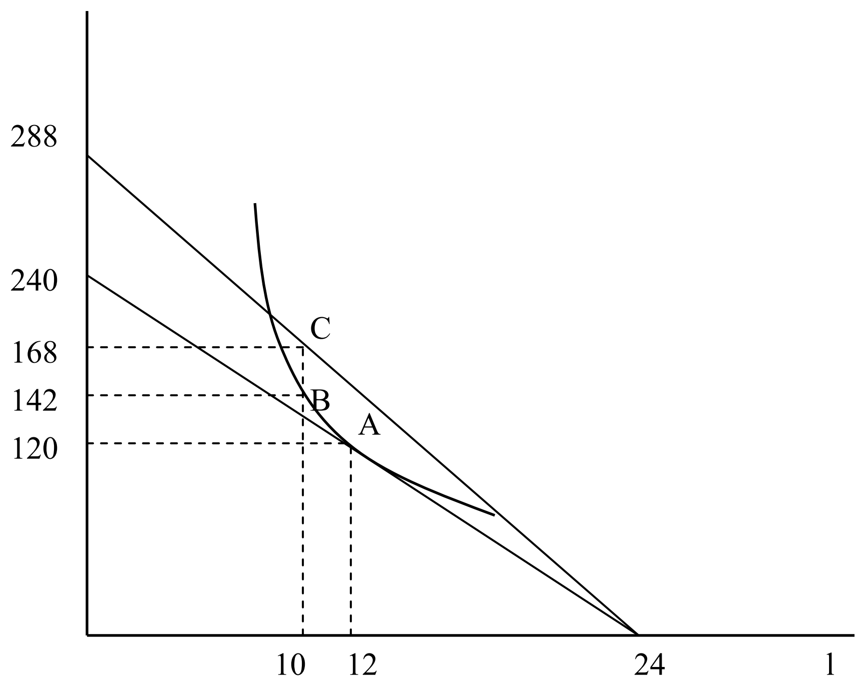
d) At the substitution point, MRS should be equal to 12

$$MRS = 12 \Rightarrow \frac{120}{l} = 12 \Rightarrow l = 10$$

We now need to find the b-coordinate of a point on the initial indifference curve that has l=10.

$$U(10, b) = U(12,120) \Rightarrow b + 120\ln(10) = 120 + 120\ln(12) \Rightarrow b = 120 + 120\ln\frac{12}{10} = 142$$

Graphically:



where A is the initial bundle (12,120), B is the substitution point (10,142) and C is the final bundle (10,168). The substitution effect is the move from A to B and the income effect is the move from B to C. Clearly, there is not income effect for leisure, since B and C have the same l-coordinate, so the substitution effect is larger in magnitude than the income effect is for leisure.

PART I – PROBLEM 3

Flora's Sewing Co has a production function given by $Q = 4L^{\frac{1}{3}}K^{\frac{1}{3}}$ where labor (L) is measured in hours, capital (K) is measured in sewing machine days (available sewing machines per day) and Q is measured in yards hemmed. The wage rate is \$8 per hour (w = 8) and the rental rate on sewing machines is \$2 per machine per day (so r = 2). Flora's has no additional costs.

For parts (a) - (b) below assume that Flora's capital stock is fixed in the short run. In particular Flora's can only use 64 machines per day so the capital is fixed at K = 64. This results in fixed costs in the short run of \$128.

a) (6 points) In the short run what is Flora's (compensated) demand curve for labor? What is the variable cost curve? What is the total cost curve?

ANSWER:

$$Q = 4L^{\frac{1}{3}}(64)^{\frac{1}{3}} = 16L^{\frac{1}{3}}$$
$$L^* = (\frac{Q}{16})^3 = \frac{Q^3}{4096}$$

$$VC = wL = 8L = 8(\frac{Q^3}{4096}) = \frac{Q^3}{512}$$

$$TC = VC + FC = \frac{Q^3}{512} + 128$$

b) (8 points) In the short run what is the marginal cost curve of Flora's Sewing Co? What is the average cost curve? What is the optimal size of the firm? Illustrate the short run marginal and average cost curves on the next page.

ANSWER:

$$MC = \frac{dTC}{dQ} = \frac{3Q^2}{512}$$

$$AC = \frac{TC}{Q} = \frac{\frac{Q^3}{512} + 128}{Q} = \frac{Q^2}{512} + \frac{128}{Q}$$

The optimal size of the firm can be found in two ways: (1) set AC = MC; (2) min AC. Using the first method, we have:

$$
\begin{aligned}
MC &= AC \\
\frac{3Q^2}{512} &= \frac{Q^2}{512} + \frac{128}{Q} \\
\frac{2Q^2}{512} &= \frac{128}{Q} \\
2Q^3 &= 65,536 \\
Q^3 &= 32,768 \\
Q^* &= 32
\end{aligned}
$$

For the graph, the MC curve should be upward sloping and the AC curve should be U-shaped with the two curves intersecting at the minimum point of the AC curve. This point should be labelled as $(Q^*, C^*) = (32, 6)$.

For part (c) below assume that Flora's capital stock is variable in the long run.

c) (6 points) In the long run what is Flora's (compensated) demand curve for labor? What is the (compensated) demand for capital? What is the variable cost curve?

ANSWER:

To solve the long run problem, we want to solve the following constrained minimization problem: minimize cost subject to a given level of production. To find the solution to this problem, we set the MRTS = slope of the isocost line.

$$MRTS = \frac{\frac{\partial Q}{\partial L}}{\frac{\partial Q}{\partial K}} = \frac{\frac{1}{3}L^{\frac{-2}{3}}K^{\frac{1}{3}}}{\frac{1}{3}L^{\frac{1}{3}}K^{\frac{-2}{3}}} = \frac{K}{L}$$

$$slopeofisocostline = \frac{w}{r} = \frac{8}{2} = 4$$

Now, we set the two equal to each other to obtain our <u>tangency condition</u>:

$$
\begin{aligned}
\frac{K}{L} &= 4 \\
K &= 4L
\end{aligned}
$$

Next, we substitute the tangency condition we just found, into the production function to resolve the <u>feasibility condition</u>:

$$Q = 4L^{\frac{1}{3}}(4L)^{\frac{1}{3}}$$
$$Q = 4^{\frac{4}{3}}L^{\frac{2}{3}}$$
$$L^* = \left(\frac{Q}{4^{\frac{4}{3}}}\right)^{\frac{3}{2}} = \frac{Q^{\frac{3}{2}}}{16}$$
$$K^* = 4L = 4\left(\frac{Q^{\frac{3}{2}}}{16}\right) = \frac{Q^{\frac{3}{2}}}{4}$$

Finally, we find variable cost:

$$VC = wL + rK$$
$$VC = 8\left(\frac{Q^{\frac{3}{2}}}{16}\right) + 2\left(\frac{Q^{\frac{3}{2}}}{4}\right) = Q^{\frac{3}{2}}$$

PART II – QUESTION 1

In New Jersey the half of the drivers are women and half of the drivers are men. The probability that the typical woman has a car accident in the upcoming year is 3% and the probability that the typical man will have a car accident is 7%. Hence the probability that the typical driver will have an accident is 5%. Regardless of the gender of the driver the typical car accident will result in $10,000 of damages. The typical NJ resident has an income of $64,000.

The Garden State Insurance Company offers accident liability insurance to New Jersey drivers. Assume that by law the Garden State Insurance Co must offer the same policies at the same prices to both women and men. Garden State offers NJ drivers the opportunity to purchase any amount of coverage, C, up to $10,000 at a price of $r per $1 of coverage.

a) (3 points) If the insurance policy is a fair policy what must be the price per $1 of coverage? What is the cost of a full fair insurance policy?

If the insurance policy is fair then the insurance company should make zero profits:
Expected cost of $1 of coverage = (Average probability of an accident)x($1) = $0.05
Revenue should be equal to expected cost for the company to make zero profits, so the cost of a fair insurance policy for $1 of coverage should be $0.05.
Now, a full insurance policy covers all damages up to $10,000 so it should cost $500.

Dick is a typical male driver and so he faces a 7% chance of having a car accident in the coming year and knows that he faces a 7% chance of an accident. Dick is a risk averse individual.
b) (4 points) If Dick does not purchase insurance then what would be his expected income? If Dick purchases the full fair insurance policy that you priced in part (a) then what would be Dick's expected income?

$$EI_{NO\_INSURANCE} = 0.07 \cdot (64000 - 10000) + 0.93 \cdot (64000) = 63300$$

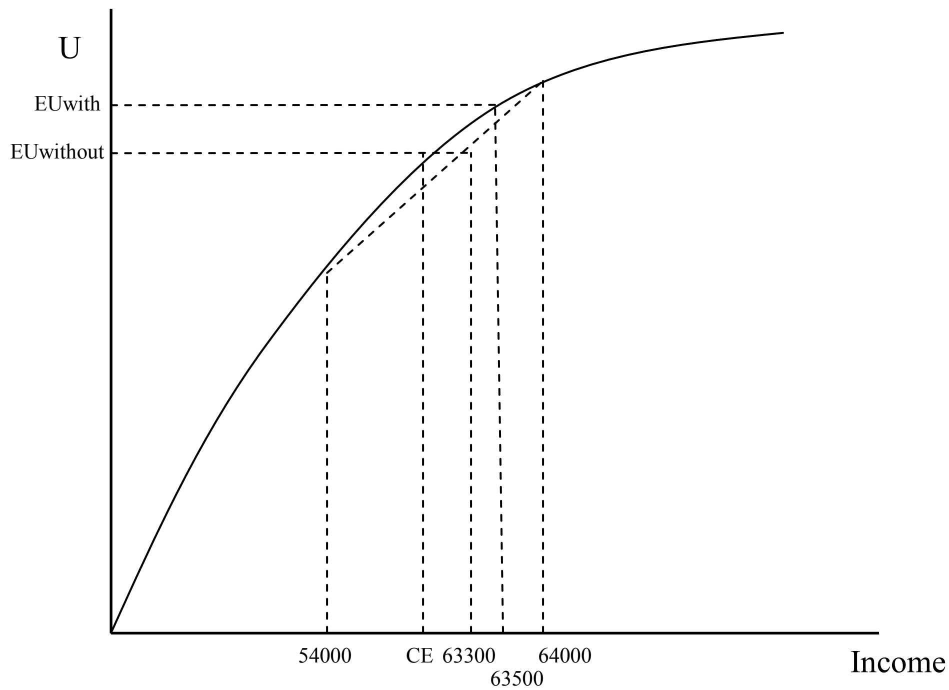
$$EI_{WITH\_INSURANCE} = 0.07 \cdot (64000 - 500 - 10000 + 10000) + 0.93 \cdot (64000 - 500)$$

$$= 0.07 \cdot 63500 + 0.93 \cdot 63500 = 63500$$

Note that with full insurance, he is receiving 63500 with certainty.

c) (4 points) Will Dick purchase the full fair insurance policy that you priced in part (a)? Briefly explain your answer.
Yes he will. If Dick gets no insurance, then his expected income is 63300 and he is also facing uncertainty because if he has a car crash his income will be 54000 and if he does not his income will be 64000. If he buys full insurance then his expected income is 63500 which is higher than his expected income without insurance and he receives that higher income with certainty. A different way to argue this is that the price of the insurance is for the average citizen that has a probability of 0.05 of an accident. Since Dick has a probability of 0.07 of an accident, then the insurance policy is a bargain for him, so he chooses to insure fully.

d) (4 points) In a diagram illustrate Dick's utility function, his expected utility without insurance and his expected utility with insurance. Also indicate in your diagram the certainty equivalent to not having insurance.



Jane is a typical female driver and she faces only a 3% chance of having a car accident in the coming year and knows that she faces a 3% chance of an accident. Jane is also a risk averse individual.

e) (4 points) If Jane does not purchase insurance then what would be her expected income? If Jane purchases the full fair insurance policy that you priced in part (a) then what would be Jane's expected income?

$$EI_{NO\_INSURANCE} = 0.03 \cdot (64000 - 10000) + 0.97 \cdot (64000) = 63700$$

$$EI_{WITH\_INSURANCE} = 0.03 \cdot (64000 - 500 - 10000 + 10000) + 0.97 \cdot (64000 - 500)$$

$$= 0.07 \cdot 63500 + 0.97 \cdot 63500 = 63500$$

f) (4 points) Will Jane purchase the full fair insurance policy that you priced in part (a)? Briefly explain your answer.

Jane's decision can go either way depending on her level of risk aversion. With insurance, her expected income is less than what it is without insurance. If she is sufficiently risk averse, she may prefer this lower expected income (that she receives with certainty) for the sake of avoiding the risk associated with no insurance. If she is risk averse, but not extremely so, she may decide to stay without insurance.

Suppose that Jane's utility function is $u(x) = \sqrt{0.01x}$

g) (3 Points) What is Jane's expected utility if she does not purchase insurance?

$$EU_{NO\_INSURANCE} = 0.03\sqrt{(0.01 \cdot 54000)} + 0.97\sqrt{(0.01 \cdot 64000)} = 25.2364$$

h) (3 Points) What is the certainty equivalent to not having insurance?

$$U(CE) = EU_{NO\_INSURANCE} \Rightarrow \sqrt{(0.01 \cdot CE)} = 25.2364 \Rightarrow CE = \frac{25.2364^2}{0.01} = 63688$$

i) (3 Points) Using your answer from part (h) show that Jane will not purchase the full fair insurance contract**.**
With full insurance, Jane receives 63500 no matter what, so that is her certainty equivalent of purchasing insurance. Since the certainty equivalent of having no insurance is higher than the certainty equivalent of buying full insurance, Jane prefers to stay without insurance

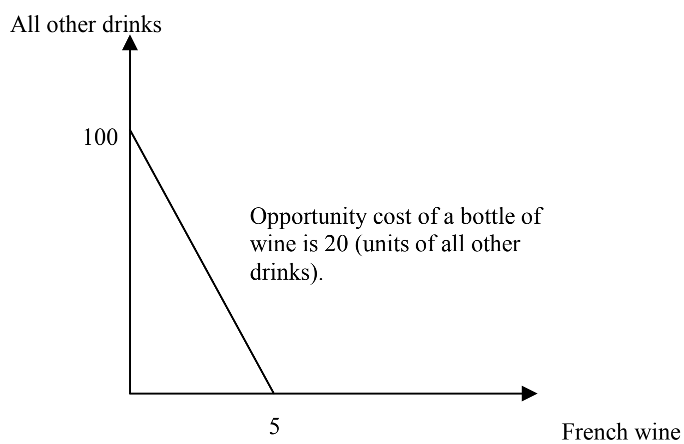
j) (3 Points) If Jane does not purchase insurance and she is a typical female driver in NJ then would you expect the Garden State Insurance Company to offer insurance at the price that you found in part (b)? Briefly explain your answer.
If Jane does not purchase insurance then the insurance company will only be serving men. The expected cost of full coverage only for men is 10000*0.07=$700. At a price of $500 the company is making a loss, so they will have to increase the price to $700 in order to break even.

PART II – QUESTION 2

George enjoys fine French wine. In fact he allocates his $100 beverage budget between French wine and all other drinks. The price of bottle of French wine is $20 and the price of all other beverages is $1 per unit. Assume that French wine and all other beverages are not perfect complements for George.

a) Putting bottles of wine on the x-axis and units of all other beverages on the y-axis illustrate George's budget set. What is the opportunity cost of a bottle of wine?

Answer:



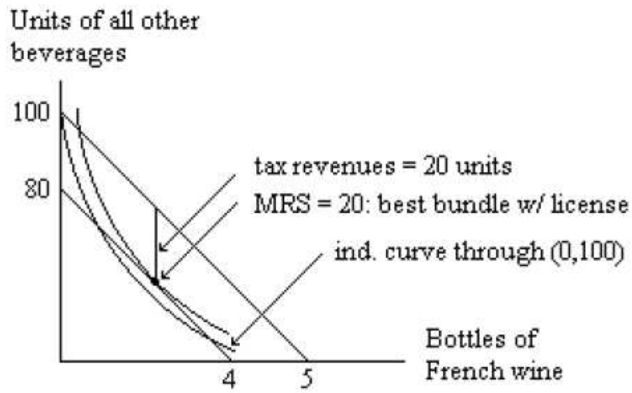
Due to rising federal deficits the federal government needs to raise revenues. One way to do so is to place a tax on French wine consumption. Senator Johns proposes to levy a *fixed fee* on the purchase of French wine. Under the fixed fee plan George must first purchase a license to buy French wine. Once he has the license he can purchase as much French wine as he would like from his local liquor shop at the regular price of $20 per bottle. The cost of a license is $20.

b) In a new diagram illustrate George's budget set if he chose to purchase the license. Be sure to include the original budget line from part (a).
PLEASE SEE DIAGRAM

c) In your diagram from part (b) illustrate preferences (indifference curves) for George such that he would choose to purchase the license to buy French wine. Illustrate in the same diagram George's best bundle after the imposition of the fixed fee and an indifference curve through that bundle.
If George does not purchase the license, then he cannot buy any bottles of French wine so his indifference curve would have to pass through (0,100). If he purchases the license, then he achieves a higher utility since the optimal indifference curve with the license lies above the indifference curve through (0,100)

d) In your diagram from part (b) indicate the tax revenues (denominated in units of other food) generated by the imposition of the fixed fee.

Answer:

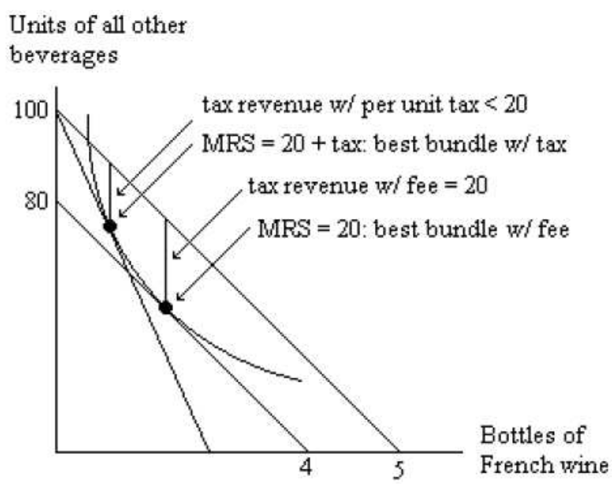


□

Senator Thomas dislikes the idea of a fixed fee and he proposes instead to simply tax each bottle of French wine purchased. Senator Thomas claims that so long as the correct tax rate is chosen the per-unit tax can be used to raise as much money as the fixed fee and will leave George neither better off nor worse off than the fixed fee plan.

e) Suppose that the government chooses the value of the per-unit tax so that George is indifferent between the bundle that he chooses under the fixed fee plan and the bundle that he chooses under per-unit tax plan. In a new diagram show that at this tax rate the government will raise strictly less revenue under the per unit tax plan than it did under the fixed fee plan. In your diagram be sure to include all three budget lines (no-tax line, fixed fee line and the per-unit tax line).

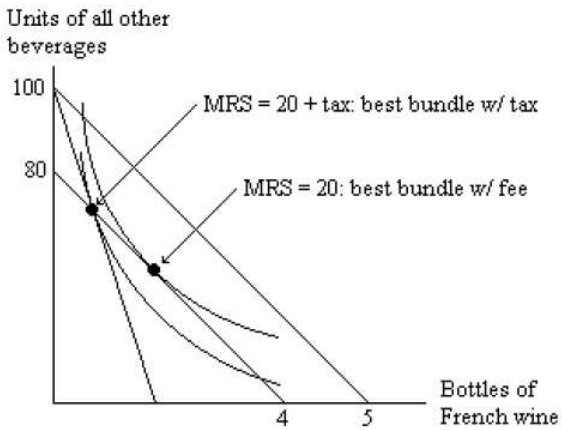
Answer:



□

f) Suppose that the government chooses a value for the per-unit tax so that it will raise exactly the same revenue as the fixed fee plan. Briefly explain why George will be strictly worse off with the bundle that he chooses under the per-unit tax plan than he was with bundle that he chose under the fixed feel plan. Illustrate your answer in a new diagram.

Answer:



□

g) Suppose George's preferences can be represented by the utility function $U(x, y) = xy$. Write the 3 equations that can be used to determine the value of the per unit tax rate that would raise the same revenue as the fixed fee.

Answer:

$$MRS = 20 + \text{tax} \;\Rightarrow\; \frac{y}{x} = 20 + \text{tax}$$

$$20x + y = 80$$

$$(20 + \text{tax})x + y = 100$$

□