Homework #6

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5.1

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a)

$$
\begin{aligned}
P(X > 3) &= \int_3^\infty \int_0^\infty f(x,y)dydx \\
&= \int_3^\infty \int_0^\infty xe^{-x(1+y)}dydx \\
&= \int_3^\infty xe^{-x} \int_0^\infty e^{-xy}dydx \\
&= \int_3^\infty xe^{-x} \left[\frac{e^{-xy}}{-x}\right]_0^\infty dx \\
&= \int_3^\infty e^{-x}dx \\
&= \left[-e^{-x}\right]_3^\infty \\
&= .05
\end{aligned}
$$

b)

X :

$$
\begin{aligned}
f(x) &= \int_y f(x,y)dy \\
&= \int_0^\infty xe^{-x(1+y)}dy \\
&= xe^{-x} \int_0^\infty e^{-xy}dy \\
&= xe^{-x} \left[\frac{e^{-xy}}{-x}\right]_0^\infty \\
&= e^{-x} \Leftrightarrow x \geq 0
\end{aligned}
$$

Y :

$$\begin{aligned} f(y) &= \int_x f(x,y)dx \\ &= \int_0^\infty xe^{-x(1+y)}dx \\ &= \left[x\left(\frac{e^{-x(1+y)}}{-1-y}\right) - \frac{e^{-x(1+y)}}{(1+y)^2}\right]_0^\infty \\ &= \frac{1}{(1+y)^2} \Leftrightarrow y \geq 0 \end{aligned}$$

No. The joint PDF is not a factor of the product of the marginal PDFs.

0.0.1 c)

$P(\max(X,Y) > 3) = 1 - P(\max(X,Y) \leq 3) = 1 - \int_0^3 xe^{-x}\left(\int_0^3 e^{-xy}dy\right)dx = 1 - \int_0^3 e^{-x}(1-e^{-3x})dx = e^{-3} + 1 - \frac{1}{4}e^{-12} \approxeq .3$

0.1 16

0.1.1 a)

$$\begin{aligned} f(x_1,x_3) &= \int_{-\infty}^{\infty} f(x_1,x_2,x_3)dx_2 \\ &= \int_0^{1-x_1-x_3} kx_1x_2(1-x_3)dx_2 \\ &= 72x_1(1-x_3)(1-x_1-x_3)^2 \Leftrightarrow x_1,x_3 \geq 0; x_1+x_3 \leq 1. \end{aligned}$$

b)

$P(X_1 + X_3 \leq .5) = \int_0^5 \int_0^{5-x_1} 72x_1(1-x_3)(1-x_1-x_3)^2 dx_1 dx_2 = .5312$. Calculated using Wolfram Alpha.

c)

$f_{x_1}(x_1) = \int_{-\infty}^{\infty} f(x_1,x_3)dx_3 = \int 72x_1(1-x_3)(1-x_1-x_3)^2dx_3 = 18x_1 - 48x_1^2 + 36x_1^3 - 6x_1^5 \Leftrightarrow 0 \leq x_1 \leq 1.$

5.2

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a)

$E(X+Y) = \sum_x \sum_y (x+y)p(x,y,) = (0+0)(.02) + (0+5)(.06) + (5+0)(.04) + ... + (10+15)(.01) = 14.1$

b)

$$E(\max(X,Y) = \sum_x \sum_y \max(x+y)p(x,y) = 0 * .02 + 5 * .06 + 5 * .04 + ... + 15 * .01 = 9.6$$

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36)

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