Homework #8

Ben Drucker

5.5

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$\mu_X = 50 * .7 = 35; \sigma_X^2 = 50 * .7 * .3 = 10.5$
$\mu_Y = 50 * .6 = 30; \sigma_Y^2 = 50 * .6 * .4 = 12$
$\mu_{X-Y} = 35 - 30 = 5; \sigma_{X-Y}^2 = 10.5 + 12 = 22.5$
$P(-5 < X < 5) \approxeq P\left(\frac{-10}{\sqrt{22.5}} < Z < \frac{0}{\sqrt{22.5}}\right) = .483$

5 (Supplementary)

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No, only if either σ is 0. When taking the percentile of sums, there will be a $\sqrt{\sigma_1^2 + \sigma_2^2}$ term. In order for this to equal the $\sigma_1 + \sigma_2$ term from the sum of the percentiles, at least one σ would need to be 0. It would also hold true at $Z = 0$, the center of the distribution.

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$$E\left[(X + Y - t)^2\right] = \int_0^1 \int_0^1 (x + y - t)^2 * f(x, y) dxdy$$
$$\frac{dE\left[(X + Y - t)^2\right]}{dt} = 0 = -2 \int_0^1 \int_0^1 2(x + y - t) f(x, y)$$

$$\int_0^1 \int_0^1 t f(x, y) dxdy = t$$
$$= \int_0^1 \int_0^1 (x + y) f(x, y) dxdy = E(X + Y) = 1.1\overline{6}$$

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Tried, couldn't answer it.

6.1

8

a)

$\hat{p} = \frac{12}{80} = .15$

b)

P(system works) $= \hat{p}^2 = \left(\frac{80-12}{80}\right)^2 = \frac{289}{400}$

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a)

$E(\overline{X}^2 = Var\overline{X} + [E(\overline{X})]^2 = \frac{\sigma^2}{n} + \mu^2$. The bias is $\frac{\sigma^2}{n}$.

b)

$E(\overline{X}^2 - kS^2) = E(\overline{X}^2) - kE(S^2) = \frac{\sigma^2}{n} + \mu^2 - k\sigma^2$. The estimator is unbiased where $k = \frac{1}{n}$.

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a)

$E[\delta\overline{X} + \overline{Y}(1-\delta)] = \delta E(\overline{X} + (1-\delta)E(\overline{Y}) = \delta\mu + \mu(1-\delta) = \mu$

b)

$Var[\delta\overline{X} + (1-\delta)\overline{Y}] = \delta^2 * Var(\overline{X}) + (1-\delta)^2 * Var(\overline{Y}) = \frac{\delta^2\sigma^2}{m} + \frac{4(1-\delta)^2\sigma^2}{n} = F$
$\frac{dF}{d\delta} = 0 = \frac{2\delta\sigma^2}{m} + \frac{8(1-\delta)\sigma^2}{n} \Rightarrow \delta = \frac{4m}{4m+n}$

6.2

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a)

$L = \ln\left[\begin{pmatrix} n \\ x \end{pmatrix} p^x(1-p)^{n-x}\right] = \ln\begin{pmatrix} n \\ x \end{pmatrix} + x\ln p + \ln(1-p)(n-x)$
$\frac{dL}{dp} = 0 = \frac{x}{p} - \frac{n-x}{1-p} \Rightarrow \hat{p} = \frac{x}{n}$
Given $n = 20, x = 3 : \hat{p} = \frac{3}{20}$.

b)

Yes. $E(\hat{p}) = E\left(\frac{x}{n}\right) = \frac{E(X)}{n} = \frac{np}{p} = p$.

c)

$(1-p)^5 = \left(1 - \frac{3}{20}\right)^5 \approxeq .44371$

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a)

$E(X) = \int_0^1 f(x;\theta)dx = \frac{\theta+1}{\theta+2}$
$\overline{X} = \frac{\hat{\theta}+1}{\hat{\theta}+2} \Rightarrow \hat{\theta} = \frac{1}{1-\overline{X}} - 2 = \frac{1}{1-.8} - 2 = 3$

b)

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