Homework 7 Solutions Fall 2011

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1. The *Relative Bagel Shoppe* makes bagels in NYC. The shop uses both labor and raw materials (ingredients) to make bagels. The relationship between the hours of labor worked per day and the quantity of bagels (measured in dozens of bagels) produced per day is given by

$$Q = 50\sqrt{L}$$

The bagel shop must pay its employees \$10 per hour and it must use \$2 worth of ingredients to produce each dozen bagels. Finally there are overhead (fixed) costs of \$10 per day of operating the shop.

a) What is the bagel shop's compensated demand curve for labor (labor demand as a function of the quantity produced)? What is the labor cost curve?

Answer To find demand for labor we solve for L,

$$Q = 50L^{1/2} \implies \boxed{L^\star = \frac{Q^2}{2500}}$$

The labor cost curve is

$$\boxed{wL^\star = \frac{Q^2}{250}}$$

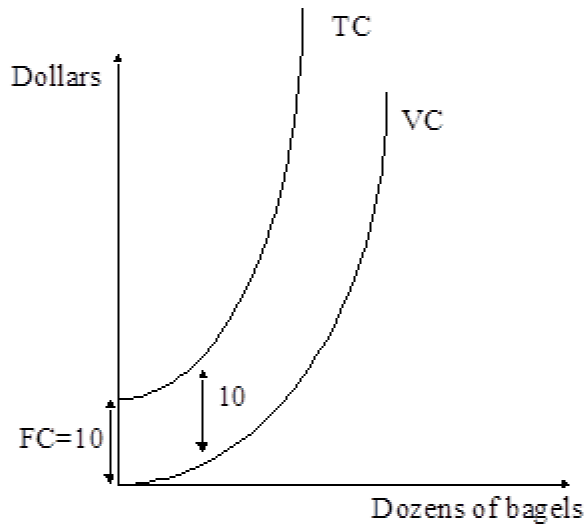
b) Given that the shop must use \$2 worth of ingredients for each dozen bagels what is the cost of ingredients as a function of the number of dozens of bagels produced? If labor and ingredients are the only variable inputs to bagel production then what is the variable cost curve of the firm? What is the total cost curve? Illustrate your answer.

Answer The cost of ingredients is $\boxed{2Q}$. Variable cost is

$$\boxed{VC = \frac{Q^2}{250} + 2Q}$$

The total cost curve is

$$VC + FC = \boxed{TC = \frac{Q^2}{250} + 2Q + 10}$$



c) What is the marginal cost curve? What is the average total cost curve? What is the quantity that minimizes the per-unit costs of production (the optimal size of the firm)? Illustrate your answer. Be sure to label the optimal size of the firm.

Answer To find the marginal cost we differentiate with respect to Q, hence

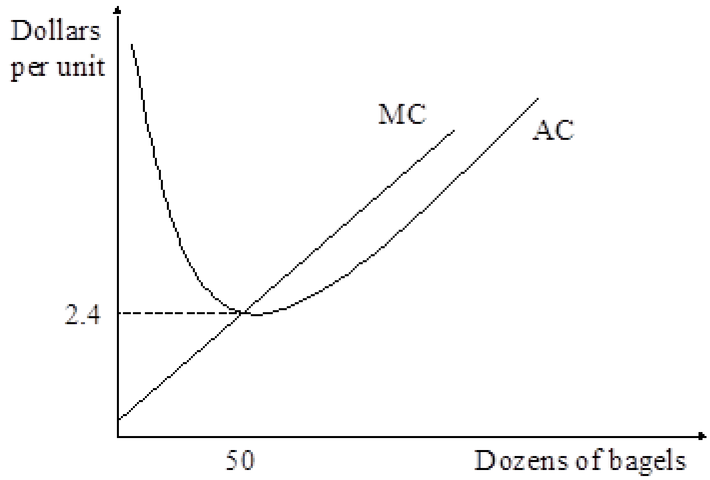
$$\boxed{MC = \frac{2Q}{250} + 2}$$

Average cost is total cost divided by Q

$$\boxed{AC = \frac{Q}{250} + 2 + \frac{10}{Q}}$$

At optimal size $AC = MC$ so

$$\begin{aligned}
\frac{2Q}{250} + 2 &= \frac{Q}{250} + 2 + \frac{10}{Q} \\
\frac{Q}{250} &= \frac{10}{Q} \\
Q^2 &= 2500 \\
Q^{OPT} &= 50
\end{aligned}$$



Suppose that at a price of \$6 per dozen the Relative Bagel Shoppe can sell as many bagels as it can handle. However at any price higher than \$6 it cannot sell any bagels. In other words the bagel shop's inverse demand curve is constant at \$6 per dozen.

d) What is the bagel shop's revenue curve? What is its marginal revenue? How many bagels would the shop like to make?

Answer Revenue is $\boxed{R = 6Q}$. Marginal revenue is $\boxed{MR = 6}$. The shop will set $MR = MC$,

$$\frac{2Q}{250} + 2 = 6 \implies \boxed{Q^{\star} = 500}$$

2. Given the following production functions determine whether the production functions exhibit increasing, decreasing or constant returns to scale for all levels of output.

a) $f(L, K) = L^{1/4}K^{1/3}$

Answer $f(\lambda L, \lambda K) = (\lambda L)^{1/4}(\lambda K)^{1/3} = \lambda^{7/12}L^{1/4}K^{1/3}$. Decreasing returns to scale.

b) $f(L, K) = L + K/3$

Answer $f(\lambda L, \lambda K) = \lambda L + \lambda K/3 = \lambda(L + K/3)$. Constant returns to scale.

c) $f(L,K) = \min[2L, K]$

Answer $f(\lambda L, \lambda K) = \min[2\lambda L, \lambda K] = \lambda \min[2L, K]$. Constant returns to scale.

d) $f(L,K) = 10 + L^{1/2}K^{1/2}$

Answer $f(\lambda L, \lambda K) = 10 + (\lambda L)^{1/2}(\lambda K)^{1/2} < \lambda\left(10 + L^{1/2}K^{1/2}\right)$. Decreasing returns to scale.

3. Explain whether each of the following actions would affect the firm's choice of the profit-maximizing output (hint: determine the affect of each on the MR and MC schedules).

a) An increase in the cost of an input such as labor.

Answer This is a change in marginal cost (increase) thus it will decrease profit-maximizing output.

b) Introduction of a small fixed fee for a license to do business.

Answer This is a fixed cost so it will not influence MC or MR.

c) Introduction of a 50 percent tax on the firm's profits.

Answer This will not influence marginal cost or revenue and thus will not change the profit-maximizing level.

d) Introduction of a per-unit tax on each unit the firm produces.

Answer The per-unit tax will increase marginal cost and thus the profit-maximizing level will decrease.

e) Receipt of a lump sum grant from the government.

Answer No change, does has an impact on marginal cost/revenue.

f) Receipt of a per-unit subsidy from the government.

Answer Decreases marginal cost, thus the profit-maximizing level of output will increase.

g) Receipt of a subsidy per worker hired.

Answer This will decrease the marginal cost of labor and thus the profit-maximizing level of output will increase.

4. The processing of payroll for the workers of a major corporation can be done to varying degrees by clerks and computers. Suppose that we can represent the trade-offs between labor and capital (as measured by the computer processing time) by the following production function $Q = K^{.75}L^{.25}$

Q is measured in thousands of paychecks issued, K is measured in hours of processing time and L is measured in man-hours.

a) Does the production function exhibit constant, increasing, or decreasing returns to scale?

Answer $Q(\lambda K, \lambda L) = (\lambda K)^{.75}(\lambda L)^{.25} = \lambda K L^{.25}$. Thus it displays constant returns to scale.

For the remainder of the question you may assume that the wage rate is \$16 per man-hour and the rental rate of capital is \$3 per hour.

b) Given the prices of the inputs and the production function find the compensated factor demands for labor and capital.

Answer The $MRTS$ is Q_L/Q_K.That is

$$MRTS = \frac{\frac{1}{4}L^{-3/4}K^{3/4}}{\frac{3}{4}L^{1/4}K^{1/4}} = \frac{K}{3L} = \frac{16}{3} = \frac{w}{r} \implies K = 16L$$

$$Q = K^{3/4}L^{1/4} \implies Q = 8L \implies \boxed{L^\star = \frac{Q}{8}}$$

$$Q = K^{3/4}L^{1/4} \implies Q = \frac{K}{2} \implies \boxed{K^\star = 2Q}$$

c) What is the variable cost curve of the firm? If fixed costs are 10 then what is the total cost curve of the firm.

Answer Variable costs are $VC = wL^\star + rK^\star$, hence

$$VC = 16\left(\frac{Q}{8}\right) + 3(2Q) \implies \boxed{VC = 8Q}$$

Total costs are

$$\boxed{TC = 8Q + 10}$$

d) Without calculating the derivative are marginal costs constant, upward sloping, or downward sloping? Briefly explain your answer.

Answer The marginal cost function is constant because the production function exhibits constant returns to scale.

5. Julie's Clip Shop is a dog grooming salon. The number of dogs that Julie's can groom in a day is a function of the number of hours worked (L) in the salon as well the number of scissors (K) owned by the salon. In particular the production function for Julie's Clip Shop is given by the following $Q = 4L^{1/2}K^{1/3}$ where Q represents the number of dogs groomed per day.

a) Does this production function exhibit constant, decreasing or increasing returns to scale?

Answer $Q(\lambda L, \lambda K) = 4(\lambda L)^{1/2}(\lambda K)^{1/3} = \lambda^{5/6}\left(4L^{1/2}K^{1/3}\right) < \lambda 4L^{1/2}K^{1/3}$ so decreasing returns to scale.

For the remainder of the question assume that Julie's Clip Shop has a fixed supply of 8 scissors (so K is fixed at 8).

b) Given the fixed supply of scissors what is the short run production function of Julie's Clip Shop? What is the compensated demand curve for labor?

Answer Short run production function is

$$Q = 4L^{1/2}8^{1/3} \implies \boxed{Q = 8L^{1/2}}$$

Thus demand for labor is

$$\boxed{L^\star = \frac{Q^2}{64}}$$

c) If the wage rate is \$16 per hour then what is the variable cost curve? If the fixed costs of Julie's Clip Shop is \$30 per day then what is the total cost curve? What is the marginal cost curve?

Answer Variable cost curve is

$$\boxed{VC = \frac{Q^2}{4}}$$

$$\boxed{TC = \frac{Q^2}{4} + 30}$$

$$\boxed{MC = \frac{Q}{2}}$$

Suppose that at a price of \$20 per dog the Julie's Clip Shop can groom as many dogs as it can handle. However at any price higher than \$20 it cannot find any customers. In other words the inverse demand curve of Julie's Clip Shop is perfectly elastic at \$20.

d) At a price of \$20 how many dogs will Julie's Clip Shop groom in a day? How many hours will be worked in the shop in a day?

Answer That is, $MR = 20$, thus

$$20 = \frac{Q}{2} \implies \boxed{Q^\star = 40}$$

so she will demand

$$L^\star = \frac{40^2}{64} \implies \boxed{L^\star = 25}$$