HW 6 Solutions- question 3 still missing.
1. Scarlett wants to maximize EU, so we write
EU=.1(10000-7500+C-rC)^(1/2)+.9(10000-rC)^(1/2) =.1(2500+C(1-r))^(1/2)+.9(10000-rC)^(1/2)
Differentiate wrt C, set=0, and then solve for C as a function of r
.1*(1-r)(1/2)(2500+C(1-r))^(-1/2)+.9(-r)(1/2)(10000-rC)^(1-2)=0
(2500+C(1-r))/(10000-rC)=((.9*r*1/2)/(.1*(1-r)(1/2)))^2=(.81r^2)/(.01(1-r)^2)
(10000-rC)(.81r^2)=(2500+C(1-r))(.01(1-r)^2)
8100*r^2-.81Cr^3=25(1-r)^2+.01C(1-r)^3
.01C(1-r)^3+.81Cr^3=8100*r^2-25(1-r)^2
C=(8100*r^2-25(1-r)^2)/(.01(1-r)^3+.81*r^3)
(note we can check we are right by plugging in the fair price, r=.1, and seeing that we get C=7500 or full insurance.)

2. In the state of Oklahoma the Sooner Insurance Company has noticed the following two differences between men and women drivers:

1) Men are more likely to have driving accidents: the probability that a man has a car accident is 25% and the probability that a woman has an accident is 19%.
2) When a man has an accident the damage is likely to be higher: the average damage to a $15,000 car driven by a man is $7500 (so the car is only worth $7500 after the accident) and only $3600 if driven by a woman (so the car is worth only $11,400).

In addition the firm knows that men are willing to pay **at most** $2343.75 for a full insurance policy. Women are willing to pay **at most** $745.50 for a full insurance policy. Drivers are equally divided between men and women.

a) What is the cost of providing full fair insurance for men? Given the information provided will men purchase this policy? What is the cost of providing full fair insurance for women? Will women purchase this policy?

<u>men</u>
coverage for full insurance = total loss in case of accident = $7,500
cost of providing full insurance = (0.25)(7,500) = $1875

men will purchase this policy, because if the insurance is fair, then the insurance company's expected profits will be zero, meaning that it will charge $1875, the cost of providing full insurance, for the policy; men are willing to pay up to $2343.75 for full insurance, so they will purchase this policy

<u>women</u>
coverage for full insurance = total loss in case of accident = $3,600
cost of providing full insurance = (0.19)(3,600) = $684

women will purchase this policy, based on the same reasoning as above

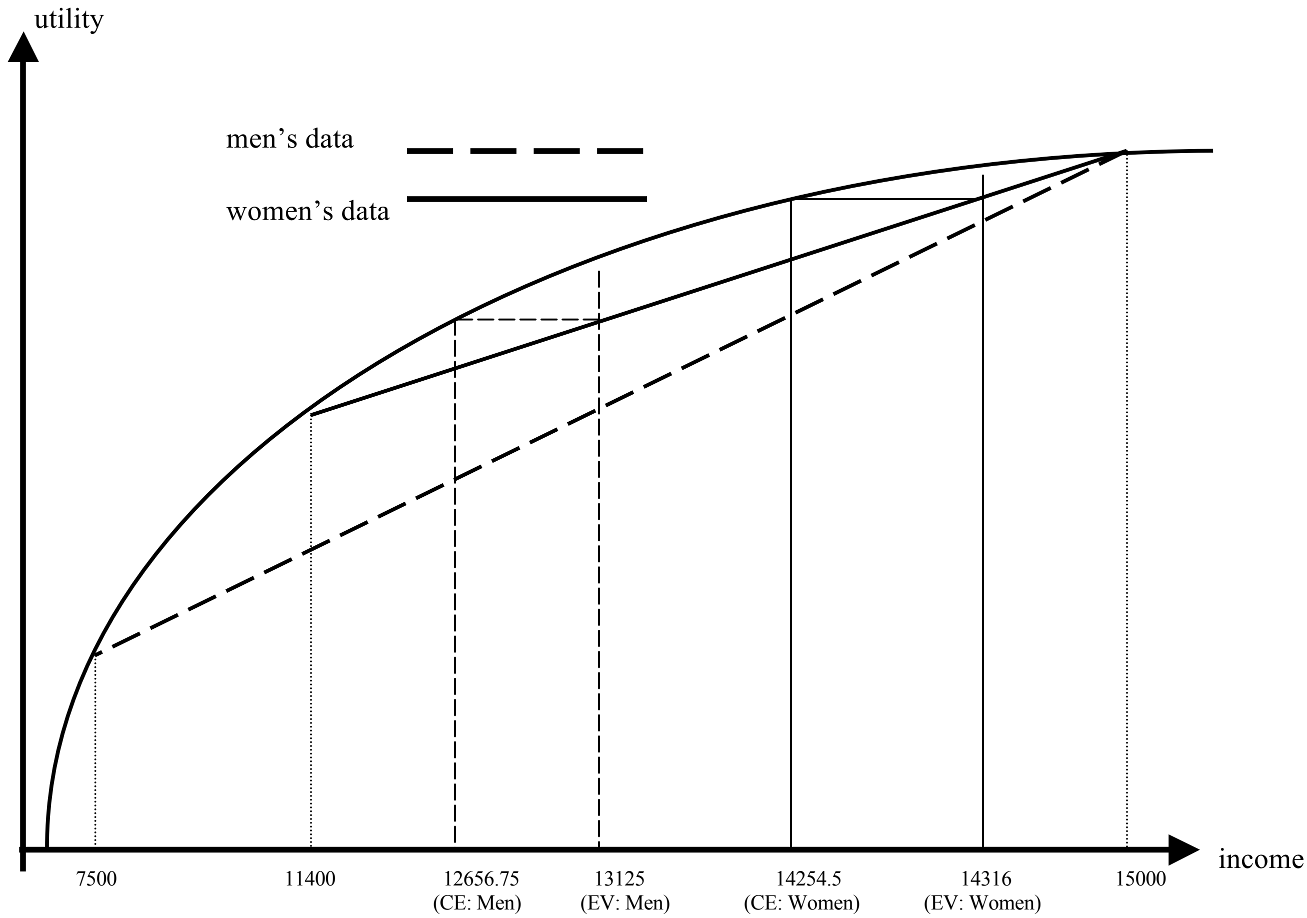
b) What is the expected value of a $15,000 car driven by a man? Given their willingness to pay for full insurance what is the certainty equivalent of the car? What is the risk premium that they are willing to pay? Illustrate the expected value, the certainty equivalent and the risk premium in a diagram.

<u>men</u>

$EV_m = (0.25)(7500)+(0.75)(15000) = \$13,125$
$CE_m = 15,000 - 2343.75 = \$12,656.25$
$RP_m = EV_m - CE_m = \$468.75$

c) What is the expected value of a \$15,000 car driven by a woman? Given their willingness to pay for full insurance what is the certainty equivalent of the car? What is the risk premium that they are willing to pay? Illustrate the expected value, the certainty equivalent and the risk premium in a diagram.

women
$EV_w = (0.19)(11400)+(0.81)(15000) = \$14,316$
$CE_w = 15,000 - 745.50 = \$14,254.5$
$RP_w = EV_w - CE_w = \$61.5$



Suppose that the state government passes anti-discrimination legislation and requires that insurance companies offer policies at the same price per dollar of coverage to both men and women.

d) Suppose that the firm offers only a single price per dollar of coverage and allows individuals to buy insurance to fully insure against their losses. Show that the firm cannot offer a single price for insurance that does not lose money and is acceptable to both men and women.

With 50% men and 50% women in the population, the expected value of such a single price, all encompassing insurance:

.5*(.25*$7500+.75*$15000) + .5(.19*$11400+.81*$15000) = **<u>$13,720.5, i.e. cost of insurance = $1,279.5</u>**

This is the minimum-price the insurance company can offer, without losing money, for full coverage of both men and women. Since women are only willing to pay $745.5 for insurance, or $534 less that the cost of the insurance, women cannot be covered. Men on the other hand are willing to pay $2343.75 for the insurance or $1064 more than the insurance will cost (i.e. a $1064.25 risk-premium) can be covered profitably by the combined policy.

e) Given your answer to (c) what policy will the firm offer? Who will be covered men or women?

The company will not offer the combined policy, despite the opportunity for overcharging men up to $1,064.25 per policy. As women will not sign up to the policy, the true costs of the policy will be $1,875 arising from only male policy holders, representing a loss of $595,5 per policy.

Therefore, the company will cover only men, and will charge at least $1,875 per policy.

4.

Maria has a house worth $202,500 which represents all of her wealth. Unfortunately there is a chance that there will be a fire and that her house will suffer $112,500 in damages. Thus leaving her a house valued at only $90,000. The probability of the fire is .1. Finally assume that her preferences over dollar values of wealth can be represented by the utility function $u(x)=x^{(1/2)}$

a) What is her expected wealth? What is Maria's expected utility?

$EV=0.1\times 90{,}000+0.9\times 202{,}500=191{,}250$

b) What is the certainty equivalent to her situation?

$EU=0.1\times u(90{,}000)+0.9\times u(202{,}500)=435$

$U(CE)=EU(gamble)=435$

$CE=435^2=189{,}225$

c) What is the risk premium associated with her situation?

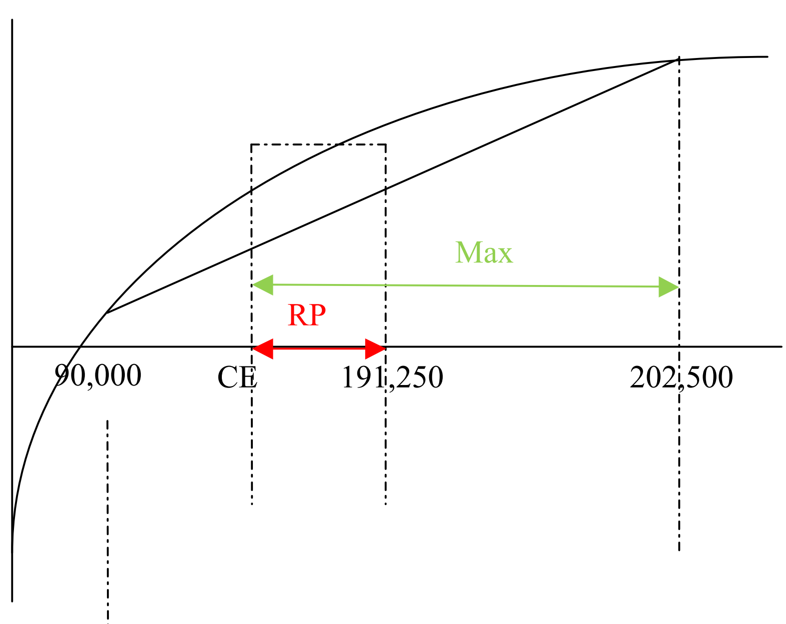
$RP=EV-CE=191{,}250-189{,}225=2025$

d) What is the maximum that Maria would be willing to pay for a full insurance policy?

$Max\ Payment=202{,}500-CE=13{,}275$

e) Illustrate her expected utility, expected wealth, certainty equivalent, the risk premium and her

maximum willingness to pay for a full insurance policy in the diagram below



Suppose that she has the option to buy fire insurance. Specifically she can purchase any amount of coverage (C) at a price of $r per dollar of coverage. The maximum coverage that she can purchase is $112,500. If a fire occurs the policy will pay her C. If no fire occurs then the policy pays 0.
f) Write her wealth in each of the 2 states if she purchases $C of coverage. Write her expected utility if
she purchases $C of coverage.
Wealth Accident: 202,500− 112,500+C−rC
Wealth no accident: 202,500−rC
*EU=0.1 *(90,000+C(1−r)^(1/2)*+0.9*(*202,500−rC)^(1/2)*
g) Using your expected utility from part (f) write the first order conditions that define the coverage C that maximizes her expected utility.
.1(1/2)(1-r)(90,000+C(1-r))^(-1/2)+.9*(1/2)(-r)(202,500-rC)^(-1/2)=0
.1(1/2)(1-r)(90,000+C(1-r))^(-1/2)=.9(1/2)(r)(202,500-rC)^(-1/2)
h) What is the price of fair insurance for Maria?
r=0.1
i). If insurance is priced fairly then use your first order conditions from part (g) to find how much insurance Maria will buy.
Notice that plugging in r=.1 cancels out all the constants, so we can focus on what is inside the parentheses.
202500-.1C=90000+.9C
C=112,500
The local government levies property taxes. In particular the tax on Maria's house is $2,500.
j) If she must pay the tax regardless of whether or not there is a fire then will she change the amount of fair insurance that she purchases? Use your first order conditions from part (g) to justify your answer.

No. It would be
202,500−2,500−0.1C=90,000−2,500+0.9C
Which leads to the same answer
k) If the tax is waived when there is a fire (so she only pays taxes in the no fire state) then will she change the amount of fair insurance that she purchases? Use your first order conditions from part (g) to justify your answer.
Yes. Change the amounts in the square roots accordingly to see this.

5. Lloyd's Limited sells boat insurance. A typical insurance policy costs \$r per dollar of coverage which is paid out if an accident occurs. The average probability of a boating accident is 20%.

a) **(2 points)** Write the expected payout of an insurance policy. What is the value of r such that the expected payout of the policy is 0?

(1pt) EV(payout) = .8*(-rC)+.2*(C-rC) = -rC + .2C

(1pt) Fair insurance => EV(payout)=0 => 0=.2C-rC => r = .2

There are two kinds of individuals.75% of boaters are *low risk* who have only a 10% chance of an accident. 25% of boaters are *high risk* who have a 50% chance of an accident. Lloyds cannot tell whether someone is high or low risk, and it sells the insurance policy at r = .20 per dollar of coverage to all individuals. For simplicity, assume that all boaters have boats valued at \$120,000 and that the damage from an accident is \$90,000 (leaving the value of the boat at only \$30,000 in the event of an accident). Finally assume that all boaters have a utility function u(\$) = *ln*(\$) where \$ represent wealth which is the value of the boat net insurance payments (ie subtract the cost of insurance and add the payout from the policy if applicable).

<u>High Risk</u>

Luna is a high risk boater so her probability of having an accident is 50%.

b) **(2 points)** Assume that she does not have insurance. What is her expected wealth? What is her expected utility?

(1pt) EW = .5*130,000 + .5*30,000 = 75,000

(1pt) EU = .5*ln(120,000) + .5*ln(30,000)

c) **(2 points)** What is the certainty equivalent to her situation without insurance?

(1pt) u(CE) = ln(CE) = EU = .5*ln(120,000) + .5*ln(30,000)

CE = $(120{,}000*30{,}000)^{1/2}$

(1pt) CE = 60,000

d) **(1 point)** What is the risk premium associated with her situation without insurance?

RP = EW – CE = 120,000 – 60,000 = 60,000

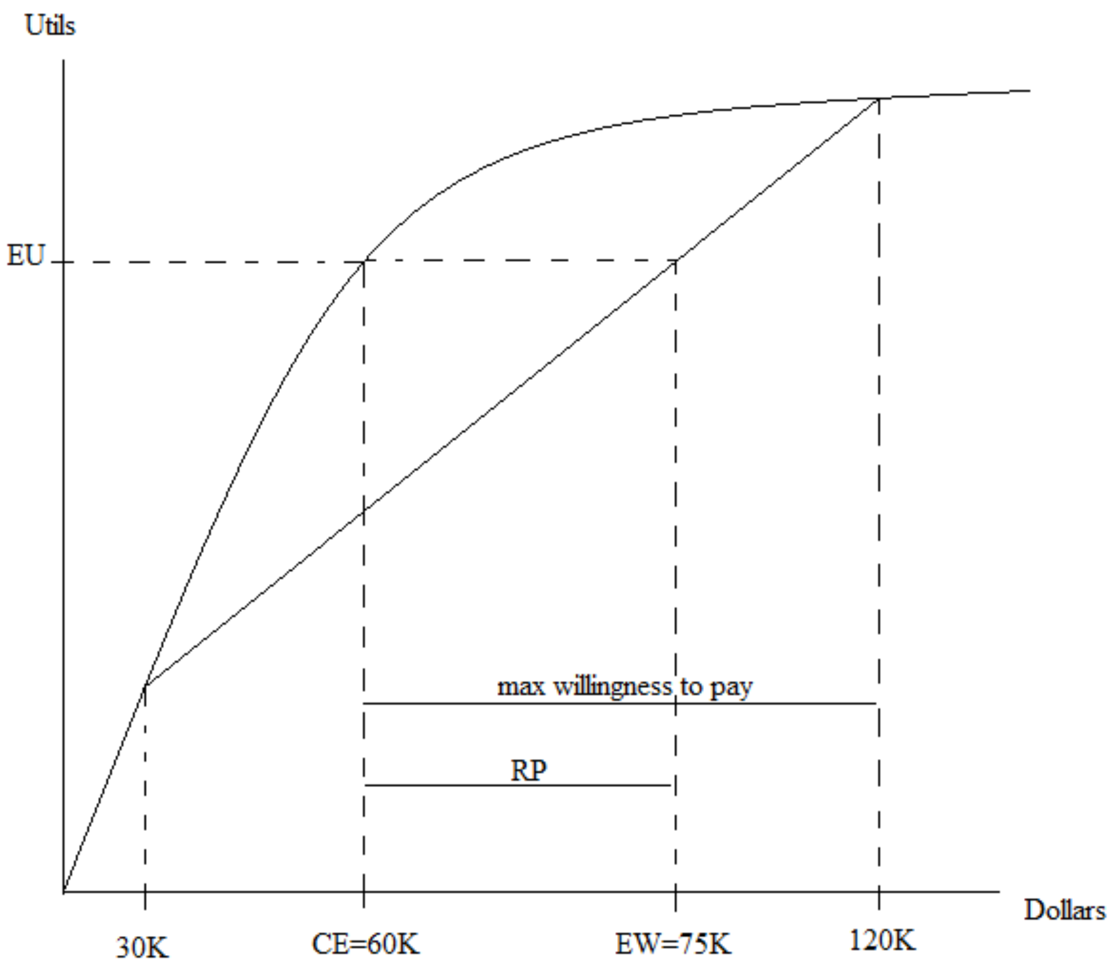
e) **(2 points)** What is the maximum that Luna would be willing to pay for a full insurance policy?

(1pt) max willingness to pay = W – CE

= 120,000 – 60,000

(1pt) = 60,000

f) **(6 points)** Illustrate her expected utility(1pt), expected wealth(1pt), certainty equivalent(1pt), the risk premium(1pt) and her maximum willingness to pay (1pt) for a full insurance policy in the diagram on the next page. (1pt general shape)



g) **(2 points)** At the price of .2 per dollar of coverage, would she be willing to fully insure? Briefly explain your answer.

Yes. Potential explanations:

1) .2*90,000 = $18,000 < $60,000 = maximum willingness to pay
2) Fair insurance would cost .5 per dollar of coverage, so this is *more than fair*. A risk averse individual (which she is because the ln function is concave) will fully insure if offered fair insurance, and this is an even better deal.

Low Risk

Helios is a low risk boater so his probability of an accident is 10%.

h) **(1 point)** Write his expected utility if he purchases $C of coverage at a price of .2 per dollar of coverage.

EU = .9*ln(120,000-.2C) + .1*ln(30,000+.8C)

i) **(3 points)** Using the expected utility from part (h) write the first order conditions that define the coverage C that maximizes his expected utility.

$$.9\left(\frac{1}{120{,}000-.2C}\right)(-.2)+.1\left(\frac{1}{30{,}000+.8C}\right)(.8)=0$$

j) **(2 points)** Show that full insurance does NOT solve the FOC above.

$$.9\left(\frac{1}{120{,}000-.2*90{,}000}\right)(-.2)+.1\left(\frac{1}{30{,}000+.8*90{,}000}\right)(.8)=$$

$$\left(\frac{-.18}{120{,}000-18{,}000}\right)+\left(\frac{.08}{30{,}000+72{,}000}\right)=$$

$$\left(\frac{-.18}{112{,}000}\right)+\left(\frac{.08}{112{,}000}\right)\neq 0$$

k) **(3 points)** Solve the FOC to determine how much insurance that Helios will buy at a price of .2 per dollar of coverage.

$$.9\left(\frac{1}{120{,}000-.2C}\right)(.2)=.1\left(\frac{1}{30{,}000+.8C}\right)(.8)\Rightarrow\left(\frac{9}{120{,}000-.2C}\right)=\left(\frac{4}{30{,}000+.8C}\right)$$

$$\Rightarrow 270{,}000+7.2C=480{,}000-.8C\Rightarrow 8C=480{,}000-270{,}000=210{,}000\Rightarrow C=26{,}250$$

A second insurance company, Greenwich LLC , offers another insurance policy. The price of Greenwich's policy is only .10 per dollar of coverage. However, the maximum coverage that Greenwich will sell is 30,000.

l) **(3 points)** Will Helios purchase a policy from Greenwich? Briefly explain.

He will prefer a policy from Greenwich (1pt). Even if he only bought C=26,250 from Greenwhich (what he bought from Lloyds) he would be better of as it is half the price. He will do even better by purchasing the 30,000 because the policy is fairly priced for him (2pts).

m) **(2 points)** Will Luna purchase a policy from Greenwich? Briefly explain.

We must compare her expected utility from the 2 policies (1pt).

EU(full insurance from Lloyds) = u(120,000-18,000) = ln(102,000)

EU(30,000 from Greenwhich) = .5u(120,000-30,000) + .5u(30,000+.9*30,000)

= .5*ln(117,000) + .5*ln(57,000) = ln(81,644).

She prefers Lloyds (1pt).

n) **(1 points)** Given the answers above would you Lloyds to continue selling the r=.2 policy? Briefly explain.

No (0pts). Only Luna buys from Lloyds and the cost of giving her a policy is .5 per dollar of coverage, so Lloyds will lose money on the policy priced at r=.2.

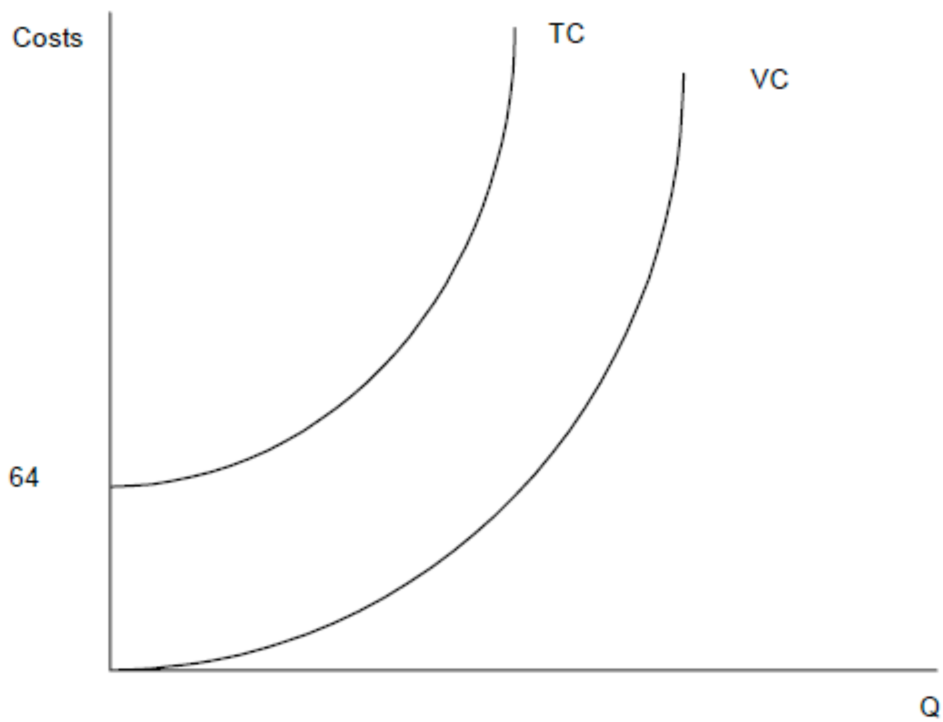
6. *Ben's Messenger Service* delivers documents in NYC. Suppose that the only variable input is labor and the relationship between the hours of labor worked and the quantity (measured in documents delivered) produced is given by

$$Q = \sqrt{L}$$

Ben must pay his workers \$1 per hour. In addition to labor costs Ben has fixed costs of \$64 per day.

a) What is the variable cost curve? What is the total cost curve? Illustrate your answer.

$$
\begin{aligned}
Q &= \sqrt{L} \Rightarrow L = Q^2 \\
VC(Q) &= wL = wQ^2 = Q^2 \\
TC(Q) &= VC(Q) + FC = Q^2 + 64
\end{aligned}
$$



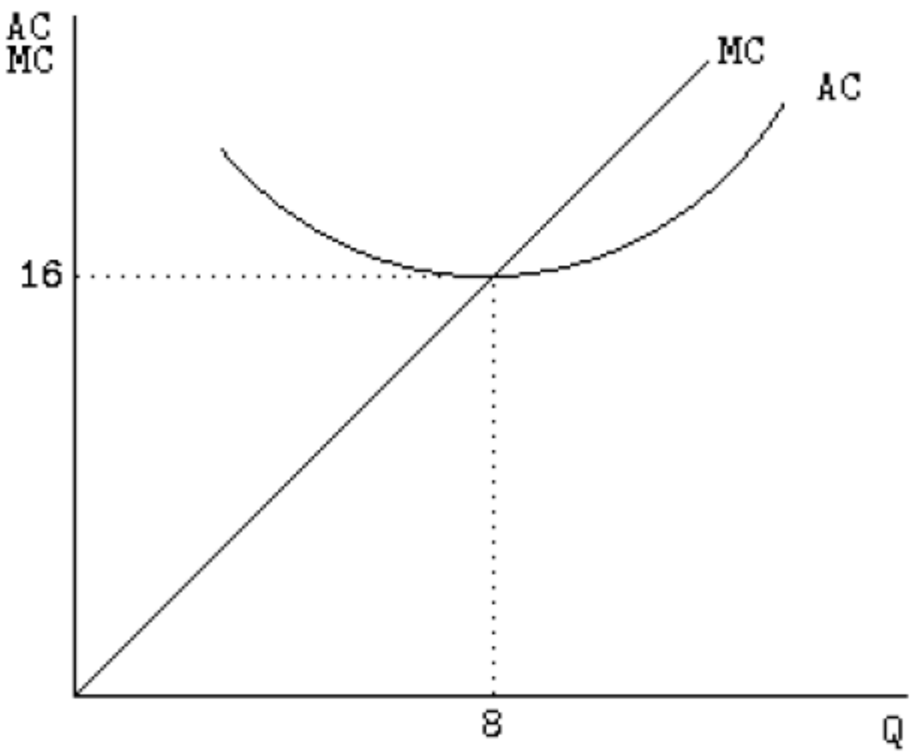
b) What is the marginal cost curve? What is the average total cost curve? What is the quantity that minimizes the per-unit costs of production (the optimal size of the firm)? Illustrate your answer. Be sure to label the optimal size of the firm.

b)

$$
\begin{aligned}
MC(Q) &= \frac{\partial TC(Q)}{\partial Q} = 2Q \\
ATC(Q) &= \frac{TC(Q)}{Q} = Q + \frac{64}{Q}
\end{aligned}
$$

Minimum of $ATC(Q)$ is found by Q such that $ATC(Q) = MC(Q)$:

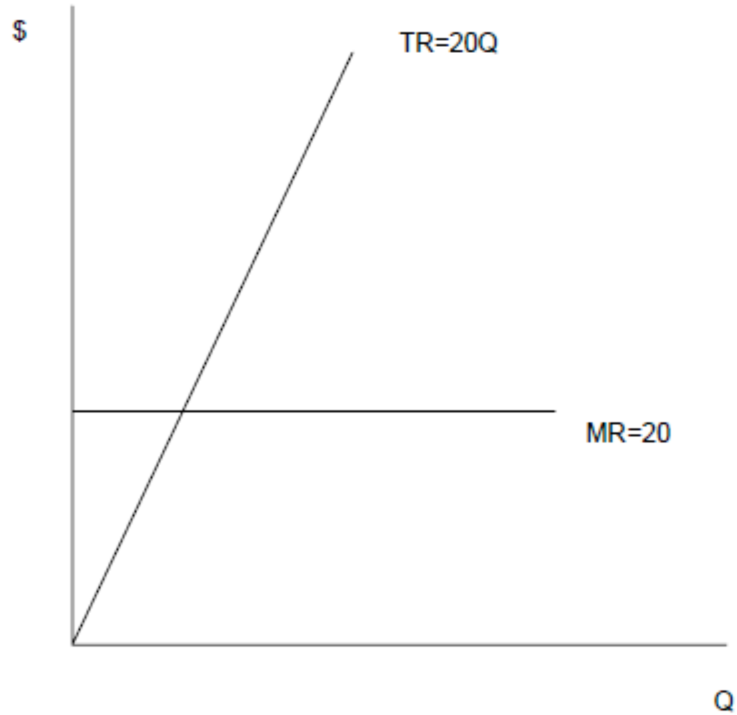
$$
\begin{aligned}
Q + \frac{64}{Q} &= 2Q \Rightarrow 64 = Q^2 \\
\therefore \quad & Q_{\text{opt}} = 8
\end{aligned}
$$



Suppose that at a price of \$20 per document Ben can have as much business as he can handle. However at any price higher than \$20 he cannot attract any business. In other words Ben's inverse demand curve is constant at \$20 per document.

c) What is Ben's revenue curve? What is his marginal revenue?

$$
\begin{aligned}
TR(Q) &= PQ = 20Q \\
MR(Q) &= \frac{\partial TR(Q)}{\partial Q} = 20
\end{aligned}
$$



d) Given your answers to (c) and (b) how many deliveries would Ben like to make?

d) Profit maximizing Q is found by setting $MR(Q) = MC(Q)$:

$$20 = 2Q \Rightarrow Q^* = 10.$$

7.

The profit maximizing choice is the output level that satisfies *MR*=*MC*. It is affected if one of these two marginal functions is affected.

a) Yes, because the cost of one additional input increases, this means *MC* changes.
b) No, because the fee does not affect the cost per unit produced.
c) No, because the tax affects both *MR* and *MC* equally. Profits are given by *R*-*C*, revenue minus cost. Suppose the tax rate is τ, then the after tax profit is $(1-\tau)(R-C)$. This is still maximized when *MR*=*MC*, therefore the tax rate doesn't affect the optimal output level.
d) Yes, because the per-unit tax affects only *MC*, not *MR*. The marginal cost increases.

e) No, the lump-sum grants won't affect the marginal decisions.
f) Yes, it can reduce *MC*.
g) Yes, it's the reduction of *MC*.