CHAPTER 11

Section 11.1

1.

a. $MSA = \frac{30.6}{4} = 7.65$, $MSE = \frac{59.2}{12} = 4.93$, $f_A = \frac{7.65}{4.93} = 1.55$. Since 1.55 is not $\geq F_{.05,4,12} = 3.26$, don't reject H_{oA}. There is no difference in true average tire lifetime due to different makes of cars.

b. $MSB = \frac{44.1}{3} = 14.70$, $f_B = \frac{14.70}{4.93} = 2.98$. Since 2.98 is not $\geq F_{.05,3,12} = 3.49$, don't reject H_{oB}. There is no difference in true average tire lifetime due to different brands of tires.

2.

a. $x_{1\bullet} = 163$, $x_{2\bullet} = 152$, $x_{3\bullet} = 142$, $x_{4\bullet} = 146$, $x_{\bullet 1} = 215$, $x_{\bullet 2} = 188$, $x_{\bullet 3} = 200$, $x_{\bullet\bullet} = 603$, $\Sigma\Sigma x_{ij}^2 = 30{,}599$, $CF = \frac{(603)^2}{12} = 30{,}300.75$, so SST = 298.25, $SSA = \frac{1}{3}\left[(163)^2 + (152)^2 + (142)^2 + (146)^2\right] - 30{,}300.75 = 83.58$,
$SSB = 30{,}392.25 - 30{,}300.75 = 91.50$,
$SSE = 298.25 - 83.58 - 91.50 = 123.17$.

Source	Df	SS	MS	F
A	3	83.58	27.86	1.36
B	2	91.50	45.75	2.23
Error	6	123.17	20.53	
Total	11	298.25		

$F_{.05,3,6} = 4.76$, $F_{.05,2,6} = 5.14$. Since neither f is greater than the appropriate critical value, neither H_{oA} nor H_{oB} is rejected.

b. $\hat{m} = \bar{x}_{\bullet\bullet} = 50.25$, $\hat{a}_1 = \bar{x}_{1\bullet} - \bar{x}_{\bullet\bullet} = 4.08$, $\hat{a}_2 = .42$, $\hat{a}_3 = -2.92$, $\hat{a}_4 = -1.58$, $\hat{b}_1 = \bar{x}_{\bullet 1} - \bar{x}_{\bullet\bullet} = 3.50$, $\hat{b}_2 = -3.25$, $\hat{b}_3 = -.25$.

3. $x_{1\bullet} = 927$, $x_{2\bullet} = 1301$, $x_{3\bullet} = 1764$, $x_{4\bullet} = 2453$, $x_{\bullet 1} = 1347$, $x_{\bullet 2} = 1529$, $x_{\bullet 3} = 1677$, $x_{\bullet 4} = 1892$, $x_{\bullet\bullet} = 6445$, $\Sigma\Sigma x_{ij}^2 = 2{,}969{,}375$,

$$CF = \frac{(6445)^2}{16} = 2{,}596{,}126.56,\ SSA = 324{,}082.2,\ SSB = 39{,}934.2,$$

$SST = 373{,}248.4$, $SSE = 9232.0$

a.

Source	Df	SS	MS	F
A	3	324,082.2	108,027.4	105.3
B	3	39,934.2	13,311.4	13.0
Error	9	9232.0	1025.8	
Total	15	373,248.4		

Since $F_{.01,3,9} = 6.99$, both H_{oA} and H_{oB} are rejected.

b. $Q_{.01,4,9} = 5.96$, $w = 5.96\sqrt{\frac{1025.8}{4}} = 95.4$

i:	1	2	3	4
$\bar{x}_{i\bullet}$:	231.75	325.25	441.00	613.25

All levels of Factor A (gas rate) differ significantly except for 1 and 2

c. $w = 95.4$, as in **b**

i:	1	2	3	4
$\bar{x}_{\bullet j}$:	336.75	382.25	419.25	473

Only levels 1 and 4 appear to differ significantly.

4.

a. After subtracting 400, $x_{1\bullet} = 151$, $x_{2\bullet} = 137$, $x_{3\bullet} = 125$, $x_{4\bullet} = 124$, $x_{\bullet 1} = 183, x_{\bullet 2} = 169$, $x_{\bullet 3} = 185$, $x_{\bullet\bullet} = 537$, $SSA = 159.98$, $SSB = 38.00$, $SST = 238.25$, $SSE = 40.67$.

Source	Df	SS	MS	f	$F_{.05}$
A	3	159.58	53.19	7.85	4.76
B	2	38.00	19.00	2.80	5.14
Error	6	40.67	6.78		
Total	11	238.25			

b. Since $7.85 \geq 4.76$, reject H_{oA}: $a_1 = a_2 = a_3 = a_4 = 0$: The amount of coverage depends on the paint brand.

c. Since 2.80 is not ≥ 5.14, do not reject H_{oA}: $b_1 = b_2 = b_3 = 0$. The amount of coverage does not depend on the roller brand.

d. Because H_{oB} was not rejected. Tukey's method is used only to identify differences in levels of factor A (brands of paint). $Q_{.05,4,6} = 4.90$, w = 7.37.

i:	4	3	2	1
$\bar{x}_{i\bullet}$:	41.3	41.7	45.7	50.3

Brand 1 differs significantly from all other brands.

5.

Source	Df	SS	MS	f
Angle	3	58.16	19.3867	2.5565
Connector	4	246.97	61.7425	8.1419
Error	12	91.00	7.5833	
Total	19	396.13		

$H_0 : a_1 = a_2 = a_3 = a_4 = 0$; H_a : at least one a_i is not zero.

$f_A = 2.5565 < F_{.01,3,12} = 5.95$, so fail to reject H_o. The data fails to indicate any effect due to the angle of pull.

6.

a. $MSA = \frac{11.7}{2} = 5.85$, $MSE = \frac{25.6}{8} = 3.20$, $f = \frac{5.85}{3.20} = 1.83$, which is not significant at level .05.

b. Otherwise extraneous variation associated with houses would tend to interfere with our ability to assess assessor effects. If there really was a difference between assessors, house variation might have hidden such a difference. Alternatively, an observed difference between assessors might have been due just to variation among houses and the manner in which assessors were allocated to homes.

7.

a. CF = 140,454, SST = 3476,

$$SSTr = \frac{(905)^2 + (913)^2 + (936)^2}{18} - 140{,}454 = 28.78,$$

$SSBl = \frac{430{,}295}{3} - 140{,}454 = 2977.67$, SSE = 469.55, MSTr = 14.39, MSE = 13.81, $f_{Tr} = 1.04$, which is clearly insignificant when compared to $F_{.05,2,51}$.

b. $f_{Bl} = 12.68$, which is significant, and suggests substantial variation among subjects. If we had not controlled for such variation, it might have affected the analysis and conclusions.

8.

a. $x_{1\bullet} = 4.34$, $x_{2\bullet} = 4.43$, $x_{3\bullet} = 8.53$, $x_{\bullet\bullet} = 17.30$, $SST = 3.8217$, $SSTr = 1.1458$, $SSBl = \frac{32.8906}{3} - 9.9763 = .9872$, $SSE = 1.6887$, $MSTr = .5729$, $MSE = .0938$, f = 6.1. Since $6.1 \geq F_{.05,2,18} = 3.55$, H_{oA} is rejected; there appears to be a difference between anesthetics.

b. $Q_{.05,3,18} = 3.61$, w = .35. $\bar{x}_{1\bullet} = .434$, $\bar{x}_{2\bullet} = .443$, $\bar{x}_{3\bullet} = .853$, so both anesthetic 1 and anesthetic 2 appear to be different from anesthetic 3 but not from one another.

9.

Source	Df	SS	MS	f
Treatment	3	81.1944	27.0648	22.36
Block	8	66.5000	8.3125	6.87
Error	24	29.0556	1.2106	
Total	35	176.7500		

$F_{.05,3,24} = 3.01$. Reject H_o. There is an effect due to treatments.

$$Q_{.05,4,24} = 3.90\,;\; w = (3.90)\sqrt{\frac{1.2106}{9}} = 1.43$$

1	4	3	2
8.56	9.22	10.78	12.44

10.

Source	Df	SS	MS	f
Method	2	23.23	11.61	8.69
Batch	9	86.79	9.64	7.22
Error	18	24.04	1.34	
Total	29	134.07		

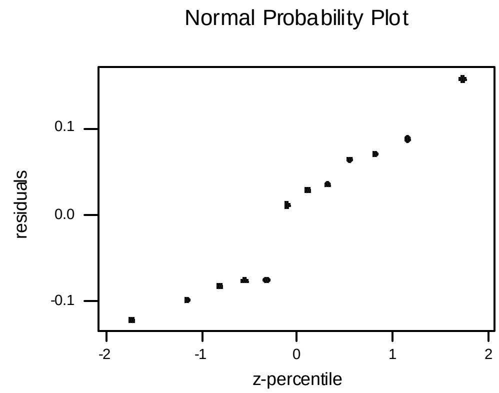
$F_{.01,2,18} = 6.01 < 8.69 < F_{.001,2,18} = 10.39$, so .001 < p-value < .01, which is significant. At least two of the curing methods produce differing average compressive strengths. (With p-value < .001, there are differences between batches as well.)

$$Q_{.05,3,18} = 3.61\,;\; w = (3.61)\sqrt{\frac{1.34}{10}} = 1.32$$

Method A	Method B	Method C
29.49	31.31	31.40

Methods B and C produce strengths that are not significantly different, but Method A produces strengths that are different (less) than those of both B and C.

11. The residual, percentile pairs are (-0.1225, -1.73), (-0.0992, -1.15), (-0.0825, -0.81), (-0.0758, -0.55), (-0.0750, -0.32), (0.0117, -0.10), (0.0283, 0.10), (0.0350, 0.32), (0.0642, 0.55), (0.0708, 0.81), (0.0875, 1.15), (0.1575, 1.73).



The pattern is sufficiently linear, so normality is plausible.

12. $MSB = \frac{113.5}{4} = 28.38$, $MSE = \frac{25.6}{8} = 3.20$, $f_B = 8.87$, $F_{.01,4,8} = 7.01$, and since $8.87 \geq 7.01$, we reject H_o and conclude that $s_B^2 > 0$.

13.

a. With $Y_{ij} = X_{ij} + d$, $\overline{Y}_{i\bullet} = \overline{X}_{i\bullet} + d$, $\overline{Y}_{\bullet j} = \overline{X}_{\bullet j} + d$, $\overline{Y}_{\bullet\bullet} = \overline{X}_{\bullet\bullet} + d$, so all quantities inside the parentheses in (11.5) remain unchanged when the Y quantities are substituted for the corresponding X's (e.g., $\overline{Y}_{i\bullet} - \overline{Y}_{\bullet\bullet} = \overline{X}_{i\bullet} - \overline{X}_{\bullet\bullet}$, etc.).

b. With $Y_{ij} = cX_{ij}$, each sum of squares for Y is the corresponding SS for X multiplied by c^2. However, when F ratios are formed the c^2 factors cancel, so all F ratios computed from Y are identical to those computed from X. If $Y_{ij} = cX_{ij} + d$, the conclusions reached from using the Y's will be identical to those reached using the X's.

14.
$$E\left(\overline{X}_{i\bullet} - \overline{X}_{\bullet\bullet}\right) = E\left(\overline{X}_{i\bullet}\right) - E\left(\overline{X}_{\bullet\bullet}\right) = \frac{1}{J}E\left(\sum_j X_{ij}\right) - \frac{1}{IJ}E\left(\sum_i\sum_j X_{ij}\right)$$
$$= \frac{1}{J}\sum_j\left(m + a_i + b_j\right) - \frac{1}{IJ}\sum_i\sum_j\left(m + a_i + b_j\right)$$
$$= m + a_i + \frac{1}{J}\sum_j b_j - m - \frac{1}{I}\sum_i a_i - \frac{1}{J}\sum_j b_j = a_i, \text{ as desired.}$$

15.

a. $\Sigma a_i^2 = 24,$ so $\Phi^2 = \left(\frac{3}{4}\right)\left(\frac{24}{16}\right) = 1.125$, $\Phi = 1.06$, $n_1 = 3,\ n_2 = 6,$ and from figure 10.5, power $\approx .2$. For the second alternative, $\Phi = 1.59$, and power $\approx .43$.

b. $\Phi^2 = \left(\frac{1}{J}\right)\sum \frac{b_j^2}{s^2} = \left(\frac{4}{5}\right)\left(\frac{20}{16}\right) = 1.00$, so $\Phi = 1.00$, $n_1 = 4,\ n_2 = 12,$ and power $\approx .3$.

Section 11.2

16.

a.

Source	Df	SS	MS	f
A	2	30,763.0	15,381.50	3.79
B	3	34,185.6	11,395.20	2.81
AB	6	43,581.2	7263.53	1.79
Error	24	97,436.8	4059.87	
Total	35	205,966.6		

b. $f_{AB} = 1.79$ which is not $\geq F_{.05,6,24} = 2.51$, so H_{oAB} cannot be rejected, and we conclude that no interaction is present.

c. $f_A = 3.79$ which is $\geq F_{.05,2,24} = 3.40$, so H_{oA} is rejected at level .05.

d. $f_B = 2.81$ which is not $\geq F_{.05,3,24} = 3.01$, so H_{oB} is not rejected.

e. $Q_{.05,3,24} = 3.53$, $w = 3.53\sqrt{\frac{4059.87}{12}} = 64.93$.

3	1	2
3960.02	4010.88	4029.10

Only times 2 and 3 yield significantly different strengths.

17.

a.

Source	Df	SS	MS	f	$F_{.05}$
Sand	2	705	352.5	3.76	4.26
Fiber	2	1,278	639.0	6.82*	4.26
Sand&Fiber	4	279	69.75	0.74	3.63
Error	9	843	93.67		
Total	17	3,105			

There appears to be an effect due to carbon fiber addition.

b.

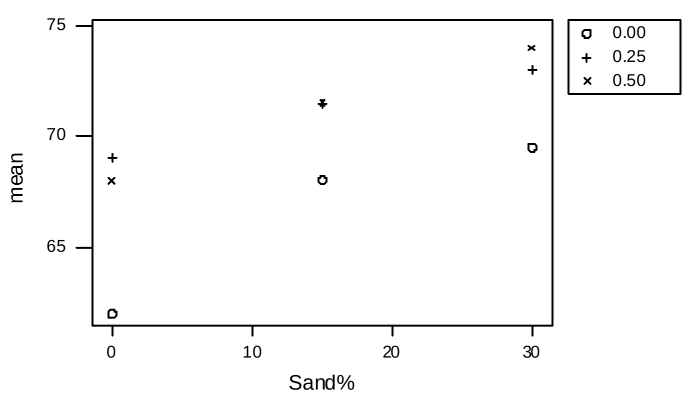
Source	Df	SS	MS	f	$F_{.05}$
Sand	2	106.78	53.39	6.54*	4.26
Fiber	2	87.11	43.56	5.33*	4.26
Sand&Fiber	4	8.89	2.22	.27	3.63
Error	9	73.50	8.17		
Total	17	276.28			

There appears to be an effect due to both sand and carbon fiber addition to casting hardness.

c.

Sand%	0	15	30	0	15	30	0	15	30
Fiber%	0	0	0	0.25	0.25	0.25	0.5	0.5	0.5
$\bar{x}$	62	68	69.5	69	71.5	73	68	71.5	74

The plot below indicates some effect due to sand and fiber addition with no significant interaction. This agrees with the statistical analysis in part **b**



18.

Source	Df	SS	MS	f	$F_{.05}$	$F_{.01}$
Formulation	1	2,253.44	2,253.44	376.2**	4.75	9.33
Speed	2	230.81	115.41	19.27**	3.89	6.93
Formulation & Speed	2	18.58	9.29	1.55	3.89	6.93
Error	12	71.87	5.99			
Total	17	2,574.7				

a. There appears to be no interaction between the two factors.

b. Both formulation and speed appear to have a highly statistically significant effect on yield.

c. Let formulation = Factor A and speed = Factor B.

For Factor A: $m_{1\bullet} = 187.03 \quad m_{2\bullet} = 164.66$

For Factor B: $m_{\bullet 1} = 177.83 \quad m_{\bullet 2} = 170.82 \quad m_{\bullet 3} = 178.88$

For Interaction: $m_{11} = 189.47 \quad m_{12} = 180.6 \quad m_{13} = 191.03$

$m_{21} = 166.2 \quad m_{22} = 161.03 \quad m_{33} = 166.73$

overall mean: $m = 175.84$

$a_i = m_{i\bullet} - m$: $\quad a_1 = 11.19 \quad a_2 = -11.18$

$b_j = m_{\bullet j} - m$: $\quad b_1 = 1.99 \quad b_2 = -5.02 \quad b_3 = 3.04$

$y_{ij} = m_{ij} - \left(m + a_i + b_j\right)$:

$y_{11} = .45 \quad y_{12} = -1.41 \quad y_{13} = .96$

$y_{21} = -.45 \quad y_{22} = 1.39 \quad y_{23} = -.97$

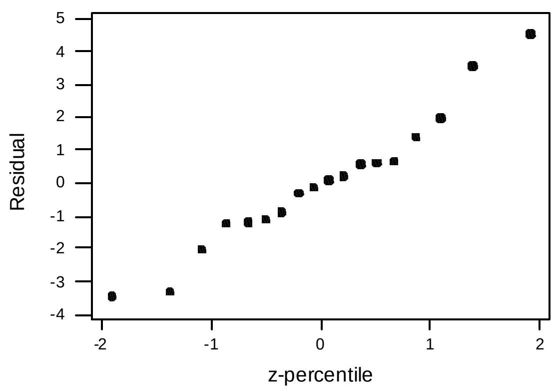
d.

Observed	Fitted	Residual	Observed	Fitted	Residual
189.7	189.47	0.23	161.7	161.03	0.67
188.6	189.47	-0.87	159.8	161.03	-1.23
190.1	189.47	0.63	161.6	161.03	0.57
165.1	166.2	-1.1	189.0	191.03	-2.03
165.9	166.2	-0.3	193.0	191.03	1.97
167.6	166.2	1.4	191.1	191.03	0.07
185.1	180.6	4.5	163.3	166.73	-3.43
179.4	180.6	-1.2	166.6	166.73	-0.13
177.3	180.6	-3.3	170.3	166.73	3.57

e.

i	Residual	Percentile	z-percentile
1	-3.43	2.778	-1.91
2	-3.30	8.333	-1.38
3	-2.03	13.889	-1.09
4	-1.23	19.444	-0.86
5	-1.20	25.000	-0.67
6	-1.10	30.556	-0.51
7	-0.87	36.111	-0.36
8	-0.30	41.667	-0.21
9	-0.13	47.222	-0.07
10	0.07	52.778	0.07
11	0.23	58.333	0.21
12	0.57	63.889	0.36
13	0.63	69.444	0.51
14	0.67	75.000	0.67
15	1.40	80.556	0.86
16	1.97	86.111	1.09
17	3.57	91.667	1.38
18	4.50	97.222	1.91

Normal Probability Plot of ANOVA Residuals



The residuals appear to be normally distributed.

19.

a.

	$x_{ij\bullet}$	j=1	2	3	$x_{i\bullet\bullet}$
i	1	16.44	17.27	16.10	49.81
	2	16.24	17.00	15.91	49.15
	3	16.80	17.37	16.20	50.37
	$x_{\bullet j\bullet}$	49.48	51.64	48.21	$x_{\bullet\bullet\bullet} = 149.33$
					CF = 1238.8583

Thus SST = 1240.1525 – 1238.8583 = 1.2942,

$$SSE = 1240.1525 - \frac{2479.9991}{2} = .1530,$$

$$SSA = \frac{(49.81)^2 + (49.15)^2 + (50.37)^2}{6} - 1238.8583 = .1243,\ SSB = 1.0024$$

Source	Df	SS	MS	f	$F_{.01}$
A	2	.1243	.0622	3.66	8.02
B	2	1.0024	.5012	29.48*	8.02
AB	4	.0145	.0036	.21	6.42
Error	9	.1530	.0170		
Total	17	1.2942			

H_{oAB} cannot be rejected, so no significant interaction; H_{oA} cannot be rejected, so varying levels of NaOH does not have a significant impact on total acidity; H_{oB} is rejected: type of coal does appear to affect total acidity.

b. $Q_{.01,3,9} = 5.43$, $w = 5.43\sqrt{\frac{.0170}{6}} = .289$

j:	3	1	2
$\bar{x}_{\bullet j\bullet}$	8.035	8.247	8.607

Coal 2 is judged significantly different from both 1 and 3, but these latter two don't differ significantly from each other.

20. $x_{11\bullet} = 855$, $x_{12\bullet} = 905$, $x_{13\bullet} = 845$, $x_{21\bullet} = 705$, $x_{22\bullet} = 735$, $x_{23\bullet} = 675$, $x_{1\bullet\bullet} = 2605$, $x_{2\bullet\bullet} = 2115$, $x_{\bullet1\bullet} = 1560$, $x_{\bullet2\bullet} = 1640$, $x_{\bullet3\bullet} = 1520$, $x_{\bullet\bullet\bullet} = 4720$, $\Sigma\Sigma\Sigma x_{ijk}^2 = 1{,}253{,}150$, CF = 1,237,688.89, $\Sigma\Sigma x_{ij\bullet}^2 = 3{,}756{,}950$, which yields the accompanying ANOVA table.

Source	Df	SS	MS	f	$F_{.01}$
A	1	13,338.89	13,338.89	192.09*	9.93
B	2	1244.44	622.22	8.96*	6.93
AB	2	44.45	22.23	.32	6.93
Error	12	833.33	69.44		
Total	17	15,461.11			

Clearly, $f_{AB} = .32$ is insignificant, so H_{oAB} is not rejected. Both H_{oA} and H_{oB} are both rejected, since they are both greater than the respective critical values. Both phosphor type and glass type significantly affect the current necessary to produce the desired level of brightness.

21.

a. $SST = 12{,}280{,}103 - \frac{(19{,}143)^2}{30} = 64{,}954.70$,

$SSE = 12{,}280{,}103 - \frac{(24{,}529{,}699)}{2} = 15{,}253.50$,

$SSA = \frac{122{,}380{,}901}{10} - \frac{(19{,}143)^2}{30} = 22{,}941.80$, $SSB = 22{,}765.53$,

$SSAB = 64{,}954.70 - [22{,}941.80 + 22{,}765.53 + 15{,}253.50] = 3993.87$

Source	Df	SS	MS	f
A	2	22,941.80	11,470.90	$\frac{11{,}470.90}{499.23} = 22.98$
B	4	22,765.53	5691.38	$\frac{5691.38}{499.23} = 11.40$
AB	8	3993.87	499.23	.49
Error	15	15,253.50	1016.90	
Total	29	64,954.70		

b. $f_{AB} = .49$ is clearly not significant. Since $22.98 \geq F_{.05,2,8} = 4.46$, H_{oA} is rejected; since $11.40 \geq F_{.05,4,8} = 3.84$, H_{oB} is also rejected. We conclude that the different cement factors affect flexural strength differently and that batch variability contributes to variation in flexural strength.

22. The relevant null hypotheses are $H_{0A}: \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0$; $H_{0B}: \sigma_B^2 = 0$; $H_{0AB}: \sigma_G^2 = 0$.

$$SST = 11{,}499{,}492 - \frac{(16{,}598)^2}{24} = 20{,}591.83,$$

$$SSE = 11{,}499{,}492 - \frac{(22{,}982{,}552)}{2} = 8216.0,$$

$$SSA = \left[\frac{(4112)^2 + (4227)^2 + (4122)^2 + (4137)^2}{6}\right] - \frac{(16{,}598)^2}{24} = 1387.5,$$

$$SSB = \left[\frac{(5413)^2 + (5621)^2 + (5564)^2}{8}\right] - \frac{(16{,}598)^2}{24} = 2888.08,$$

$$SSAB = 20{,}591.83 - [8216.0 + 1387.5 + 2888.08] = 8216.25$$

Source	Df	SS	MS	f	$F_{.05}$
A	3	1,387.5	462.5	$\frac{MSA}{MSAB} = .34$	4.76
B	2	2,888.08	1,444.04	$\frac{MSB}{MSAB} = 1.07$	5.14
AB	6	8,100.25	1,350.04	$\frac{MSAB}{MSE} = 1.97$	3.00
Error	12	8,216.0	684.67		
Total	23	20,591.83			

Interaction between brand and writing surface has no significant effect on the lifetime of the pen, and since neither f_A nor f_B is greater than its respective critical value, we can conclude that neither the surface nor the brand of pen has a significant effect on the writing lifetime.

23. Summary quantities include $x_{1\bullet\bullet} = 9410$, $x_{2\bullet\bullet} = 8835$, $x_{3\bullet\bullet} = 9234$, $x_{\bullet 1\bullet} = 5432$, $x_{\bullet 2\bullet} = 5684$, $x_{\bullet 3\bullet} = 5619$, $x_{\bullet 4\bullet} = 5567$, $x_{\bullet 3\bullet} = 5177$, $x_{\bullet\bullet\bullet} = 27{,}479$, $CF = 16{,}779{,}898.69$, $\Sigma x_{i\bullet\bullet}^2 = 251{,}872{,}081$, $\Sigma x_{\bullet j\bullet}^2 = 151{,}180{,}459$, resulting in the accompanying ANOVA table.

Source	Df	SS	MS	f
A	2	11,573.38	5786.69	$\frac{MSA}{MSAB} = 26.70$
B	4	17,930.09	4482.52	$\frac{MSB}{MSAB} = 20.68$
AB	8	1734.17	216.77	$\frac{MSAB}{MSE} = 1.38$
Error	30	4716.67	157.22	
Total	44	35,954.31		

Since $1.38 < F_{.01,8,30} = 3.17$, H_{oG} cannot be rejected, and we continue: $26.70 \ge F_{.01,2,8} = 8.65$, and $20.68 \ge F_{.01,4,8} = 7.01$, so both H_{oA} and H_{oB} are rejected. Both capping material and the different batches affect compressive strength of concrete cylinders.

24.

a. $$E\left(\overline{X}_{i..} - \overline{X}_{...}\right) = \frac{1}{JK}\sum_j\sum_k E\left(X_{ijk}\right) - \frac{1}{IJK}\sum_i\sum_j\sum_k E\left(X_{ijk}\right)$$
$$= \frac{1}{JK}\sum_j\sum_k\left(m + a_i + b_j + g_{ij}\right) - \frac{1}{IJK}\sum_i\sum_j\sum_k\left(m + a_i + b_j + g_{ij}\right) = m + a_i - m = a_i$$

b. $$E\left(\hat{g}_{ij}\right) = \frac{1}{K}\sum_k E\left(X_{ijk}\right) - \frac{1}{JK}\sum_j\sum_k E\left(X_{ijk}\right) - \frac{1}{IK}\sum_i\sum_k E\left(X_{ijk}\right) + \frac{1}{IJK}\sum_i\sum_j\sum_k E\left(X_{ijk}\right)$$
$$= m + a_i + b_j + g_{ij} - \left(m + a_i\right) - \left(m + b_j\right) + m = g_{ij}$$

25. With $q = a_i - a_i'$, $\hat{q} = \overline{X}_{i..} - \overline{X}_{i'..} = \frac{1}{JK}\sum_j \sum_k \left(X_{ijk} - X_{i'jk}\right)$, and since $i \neq i'$, $X_{ijk} and X_{i'jk}$ are independent for every j, k. Thus $Var(\hat{q}) = Var(\overline{X}_{i..}) + Var(\overline{X}_{i'..}) = \frac{s^2}{JK} + \frac{s^2}{JK} = \frac{2s^2}{JK}$ (because $Var(\overline{X}_{i..}) = Var(\overline{e}_{i..})$ and $Var(e_{ijk}) = s^2$) so $\hat{s}_{\hat{q}} = \sqrt{\frac{2MSE}{JK}}$. The appropriate number of d.f. is IJ(K – 1), so the C.I. is $\left(\overline{x}_{i..} - \overline{x}_{i'..}\right) \pm t_{a/2, IJ(K-1)} \sqrt{\frac{2MSE}{JK}}$. For the data of exercise 19, $\overline{x}_{2..} = 49.15$, $\overline{x}_{3..} = 50.37$, MSE = .0170, $t_{.025,9} = 2.262$, J = 3, K = 2, so the C.I. is

$$(49.15 - 50.37) \pm 2.262\sqrt{\frac{.0370}{6}} = -1.22 \pm .17 = (-1.39, -1.05).$$

26.

a. $\frac{E(MSAB)}{E(MSE)} = 1 + \frac{Ks_G^2}{s^2} = 1$ if $s_G^2 = 0$ and > 1 if $s_G^2 > 0$, so $\frac{MSAB}{MSE}$ is the appropriate F ratio.

b. $\frac{E(MSA)}{E(MSAB)} = \frac{s^2 + Ks_G^2 + JKs_A^2}{s^2 + Ks_G^2} = 1 + \frac{JKs_A^2}{s^2 + Ks_G^2} = 1$ if $s_A^2 = 0$ and > 1 if $s_A^2 > 0$, so $\frac{MSA}{MSAB}$ is the appropriate F ratio.

Section 11.3

27.

a.

Source	Df	SS	MS	f	$F_{.05}$
A	2	14,144.44	7072.22	61.06	3.35
B	2	5,511.27	2755.64	23.79	3.35
C	2	244,696.39	122.348.20	1056.24	3.35
AB	4	1,069.62	267.41	2.31	2.73
AC	4	62.67	15.67	.14	2.73
BC	4	331.67	82.92	.72	2.73
ABC	8	1,080.77	135.10	1.17	2.31
Error	27	3,127.50	115.83		
Total	53	270,024.33			

b. The computed f-statistics for all four interaction terms are less than the tabled values for statistical significance at the level .05. This indicates that none of the interactions are statistically significant.

c. The computed f-statistics for all three main effects exceed the tabled value for significance at level .05. All three main effects are statistically significant.

d. $Q_{.05,3,27}$ is not tabled, use $Q_{.05,3,24} = 3.53$, $w = 3.53\sqrt{\dfrac{115.83}{(3)(3)(2)}} = 8.95$. All three levels differ significantly from each other.

28.

Source	Df	SS	MS	f	$F_{.01}$
A	3	19,149.73	6,383.24	2.70	4.72
B	2	2,589,047.62	1,294,523.81	546.79	5.61
C	1	157,437.52	157,437.52	66.50	7.82
AB	6	53,238.21	8,873.04	3.75	3.67
AC	3	9,033.73	3,011.24	1.27	4.72
BC	2	91,880.04	45,940.02	19.40	5.61
ABC	6	6,558.46	1,093.08	.46	3.67
Error	24	56,819.50	2,367.48		
Total	47	2,983,164.81			

The statistically significant interactions are AB and BC. Factor A appears to be the least significant of all the factors. It does not have a significant main effect and the significant interaction (AB) is only slightly greater than the tabled value at significance level .01

29. I = 3, J = 2, K = 4, L = 4; $SSA = JKL\sum\left(\bar{x}_{i...} - \bar{x}_{....}\right)^2$; $SSB = IKL\sum\left(\bar{x}_{.j..} - \bar{x}_{....}\right)^2$;
$SSC = IJL\sum\left(\bar{x}_{..k.} - \bar{x}_{....}\right)^2$.

For level A: $\bar{x}_{1...} = 3.781$ $\bar{x}_{2...} = 3.625$ $\bar{x}_{3...} = 4.469$

For level B: $\bar{x}_{.1..} = 4.979$ $\bar{x}_{.2..} = 2.938$

For level C: $\bar{x}_{..1.} = 3.417$ $\bar{x}_{..2.} = 5.875$ $\bar{x}_{..3.} = .875$ $\bar{x}_{..4.} = 5.667$

$\bar{x}_{....} = 3.958$

SSA = 12.907; SSB = 99.976; SSC = 393.436

a.

Source	Df	SS	MS	f	F.05*
A	2	12.907	6.454	1.04	3.15
B	1	99.976	99.976	16.09	4.00
C	3	393.436	131.145	21.10	2.76
AB	2	1.646	.823	.13	3.15
AC	6	71.021	11.837	1.90	2.25
BC	3	1.542	.514	.08	2.76
ABC	6	9.805	1.634	.26	2.25
Error	72	447.500	6.215		
Total	95	1,037.833			

*use 60 df for denominator of tabled F.

b. No interaction effects are significant at level .05

c. Factor B and C main effects are significant at the level .05

d. $Q_{.05,4,72}$ is not tabled, use $Q_{.05,4,60} = 3.74$, $w = 3.74\sqrt{\frac{6.215}{(3)(2)(4)}} = 1.90$.

Machine:	3	1	4	2
Mean:	.875	3.417	5.667	5.875

30.

a. See ANOVA table

b.

Source	Df	SS	MS	f	$F_{.05}$
A	3	.22625	.075417	77.35	9.28
B	1	.000025	.000025	.03	10.13
C	1	.0036	.0036	3.69	10.13
AB	3	.004325	.0014417	1.48	9.28
AC	3	.00065	.000217	.22	9.28
BC	1	.000625	.000625	.64	10.13
ABC	3	.002925	.000975		
Error	--	--	--		
Total	15	.2384			

The only statistically significant effect at the level .05 is the factor A main effect: levels of nitrogen.

c. $Q_{.05,4,3} = 6.82$; $w = 6.82\sqrt{\frac{.002925}{(2)(2)}} = .1844$.

1	2	3	4
1.1200	1.3025	1.3875	1.4300

31.

$x_{ij.}$	B_1	B_2	B_3
A_1	210.2	224.9	218.1
A_2	224.1	229.5	221.5
A_3	217.7	230.0	202.0
$x_{.j.}$	652.0	684.4	641.6

$x_{i.k}$	A_1	A_2	A_3
C_1	213.8	222.0	205.0
C_2	225.6	226.5	223.5
C_3	213.8	226.6	221.2
$x_{i..}$	653.2	675.1	649.7

$x_{.jk}$	C_1	C_2	C_3
B_1	213.5	220.5	218.0
B_2	214.3	246.1	224.0
B_3	213.0	209.0	219.6
$x_{..k}$	640.8	675.6	661.6

$\Sigma\Sigma x_{ij.}^2 = 435{,}382.26$ $\quad\Sigma\Sigma x_{i.k}^2 = 435{,}156.74$ $\quad\Sigma\Sigma x_{.jk}^2 = 435{,}666.36$

$\Sigma x_{.j.}^2 = 1{,}305{,}157.92$ $\quad\Sigma x_{i..}^2 = 1{,}304{,}540.34$ $\quad\Sigma x_{..k}^2 = 1{,}304{,}774.56$

Also, $\Sigma\Sigma\Sigma x_{ijk}^2 = 145{,}386.40$, $x_{...} = 1978$, CF = 144,906.81, from which we obtain the ANOVA table displayed in the problem statement. $F_{.01,4,8} = 7.01$, so the AB and BC interactions are significant (as can be seen from the p-values) and tests for main effects are not appropriate.

32.

a. Since $\dfrac{E(MSABC)}{E(MSE)} = \dfrac{s^2 + Ls_{ABC}^2}{s^2} = 1$ if $s_{ABC}^2 = 0$ and > 1 if $s_{ABC}^2 > 0$, $\dfrac{MSABC}{MSE}$ is the appropriate F ratio for testing $H_0 : s_{ABC}^2 = 0$. Similarly, $\dfrac{MSC}{MSE}$ is the F ratio for testing $H_0 : s_C^2 = 0$; $\dfrac{MSAB}{MSABC}$ is the F ratio for testing $H_0 : all$ $g_{ij}^{AB} = 0$; and $\dfrac{MSA}{MSAC}$ is the F ratio for testing $H_0 : all \ \ a_i = 0$.

b.

Source	**Df**	**SS**	**MS**	**f**	**F.01**
A	1	14,318.24	14,318.24	$\frac{MSA}{MSAC} = 19.85$	98.50
B	3	9656.4	3218.80	$\frac{MSB}{MSBC} = 6.24$	9.78
C	2	2270.22	1135.11	$\frac{MSC}{MSE} = 3.15$	5.61
AB	3	3408.93	1136.31	$\frac{MSAB}{MSABC} = 2.41$	9.78
AC	2	1442.58	721.29	$\frac{MSAC}{MSABC} = 2.00$	5.61
BC	6	3096.21	516.04	$\frac{MSBC}{MSE} = 1.43$	3.67
ABC	6	2832.72	472.12	$\frac{MSABC}{MSE} = 1.31$	3.67
Error	24	8655.60	360.65		
Total	47				

At level .01, no H_0's can be rejected, so there appear to be no interaction or main effects present.

33.

Source	**Df**	**SS**	**MS**	**f**
A	6	67.32	11.02	
B	6	51.06	8.51	
C	6	5.43	.91	.61
Error	30	44.26	1.48	
Total	48	168.07		

Since .61 < $F_{.05,6,30} = 2.42$, treatment was not effective.

34.

	1	**2**	**3**	**4**	**5**	**6**
$x_{i..}$	144	205	272	293	85	98
$x_{.j.}$	171	199	147	221	177	182
$x_{..k}$	180	161	186	171	169	230

Thus $x_{...} = 1097$, $CF = \frac{(1097)^2}{36} = 33{,}428.03$, $\Sigma\Sigma x_{ij(k)}^2 = 42{,}219$, $\Sigma x_{i..}^2 = 239{,}423$, $\Sigma x_{.j.}^2 = 203{,}745$, $\Sigma x_{..k}^2 = 203.619$

Source	**Df**	**SS**	**MS**	**f**
A	5	6475.80	1295.16	
B	5	529.47	105.89	
C	5	508.47	101.69	1.59
Error	20	1277.23	63.89	
Total	35	8790.97		

Since 1.59 is not $\geq F_{.05,5,20} = 2.71$, H_{oC} is not rejected; shelf space does not appear to affect sales.

35.

	1	**2**	**3**	**4**	**5**	
$x_{i..}$	40.68	30.04	44.02	32.14	33.21	$\Sigma x_{i..}^2 = 6630.91$
$x_{.j.}$	29.19	31.61	37.31	40.16	41.82	$\Sigma x_{.j.}^2 = 6605.02$
$x_{..k}$	36.59	36.67	36.03	34.50	36.30	$\Sigma x_{..k}^2 = 6489.92$

$x_{...} = 180.09$, CF = 1297.30, $\Sigma\Sigma x_{ij(k)}^2 = 1358.60$

Source	**Df**	**SS**	**MS**	**f**
A	4	28.89	7.22	10.78
B	4	23.71	5.93	8.85
C	4	0.69	0.17	0.25
Error	12	8.01	.67	
Total	24	61.30		

$F_{4,12} = 3.26$, so both factor A (plant) and B(leaf size) appear to affect moisture content, but factor C (time of weighing) does not.

36.

Source	Df	SS	MS	f	$F_{.01}$*
A (laundry treatment)	3	39.171	13.057	16.23	3.95
B (pen type)	2	.665	.3325	.41	4.79
C (Fabric type)	5	21.508	4.3016	5.35	3.17
AB	6	1.432	.2387	.30	2.96
AC	15	15.953	1.0635	1.32	2.19
BC	10	1.382	.1382	.17	2.47
ABC	30	9.016	.3005	.37	1.86
Error	144	115.820	.8043		
Total	215	204.947			

*Because denominator degrees of freedom for 144 is not tabled, use 120.
At the level .01, there are two statistically significant main effects (laundry treatment and fabric type). There are no statistically significant interactions.

37.

Source	Df	MS	f	$F_{.01}$*
A	2	2207.329	2259.29	5.39
B	1	47.255	48.37	7.56
C	2	491.783	503.36	5.39
D	1	.044	.05	7.56
AB	2	15.303	15.66	5.39
AC	4	275.446	281.93	4.02
AD	2	.470	.48	5.39
BC	2	2.141	2.19	5.39
BD	1	.273	.28	7.56
CD	2	.247	.25	5.39
ABC	4	3.714	3.80	4.02
ABD	2	4.072	4.17	5.39
ACD	4	.767	.79	4.02
BCD	2	.280	.29	5.39
ABCD	4	.347	.355	4.02
Error	36	.977		
Total	71			

*Because denominator d.f. for 36 is not tabled, use d.f. = 30

SST = (71)(93.621) = 6,647.091. Computing all other sums of squares and adding them up = 6,645.702. Thus SSABCD = 6,647.091 – 6,645.702 = 1.389 and

$$MSABCD = \frac{1.389}{4} = .347.$$

At level .01 the statistically significant main effects are A, B, C. The interaction AB and AC are also statistically significant. No other interactions are statistically significant.

Section 11.4

38.

a.

Treatment Condition	$x_{ijk.}$	1	2	Effect Contrast	$SS = \frac{(contrast)^2}{16}$
$(1) = x_{111.}$	404.2	839.2	1991.0	3697.0	
$a = x_{211.}$	435.0	1151.8	1706.0	164.2	1685.1
$b = x_{121.}$	549.6	717.6	83.4	583.4	21,272.2
$ab = x_{221.}$	602.2	988.4	80.8	24.2	36.6
$c = x_{112.}$	339.2	30.8	312.6	-285.0	5076.6
$ac = x_{212.}$	378.4	52.6	270.8	-2.6	.4
$bc = x_{122.}$	473.4	39.2	21.8	-41.8	109.2
$abc = x_{222.}$	515.0	41.6	2.4	-19.4	23.5

$$\Sigma\Sigma\Sigma\Sigma x_{ijkl}^2 = 882{,}573.38; \quad SST = 882{,}573.38 - \frac{(3697)^2}{16} = 28{,}335.3$$

b. The important effects are those with small associated p-values, indicating statistical significance. Those effects significant at level .05 (i.e., p-value < .05) are the three main effects and the speed by distance interaction.

39.

Condition	Total	1	2	Contrast	$SS = \frac{(contrast)^2}{24}$
111	315	927	2478	5485	
211	612	1551	3007	1307	A = 71,177.04
121	584	1163	680	1305	B = 70,959.38
221	967	1844	627	199	AB = 1650.04
112	453	297	624	529	C = 11,660.04
212	710	383	681	-53	AC = 117.04
122	737	257	86	57	BC = 135.38
222	1107	370	113	27	ABC = 30.38

a. $\hat{b}_1 = \bar{x}_{.2..} - \bar{x}_{....} = \dfrac{584+967+737+1107-315-612-453-710}{24} = 54.38$

$\hat{g}_{11}^{AC} = \dfrac{315-612+584-967-453+710-737+1107}{24} = 2.21$;

$\hat{g}_{21}^{AC} = -\hat{g}_{11}^{AC} = 2.21$.

b. Factor SS's appear above. With $CF = \dfrac{5485^2}{24} = 1{,}253{,}551.04$ and $\Sigma\Sigma\Sigma\Sigma x_{ijkl}^2 = 1{,}411{,}889$, SST = 158,337.96, from which SSE = 2608.7. The ANOVA table appears in the answer section. $F_{.05,1,16} = 4.49$, from which we see that the AB interaction and al the main effects are significant.

40.

a. In the accompanying ANOVA table, effects are listed in the order implied by Yates' algorithm. $\Sigma\Sigma\Sigma\Sigma\Sigma x_{ijklm}^2 = 4783.16$, $x_{.....} = 388.14$, so

$SST = 4783.16 - \frac{368.14^2}{32} = 72.56$ and SSE = 72.56 – (sum of all other SS's) = 35.85.

Source	Df	SS	MS	f
A	1	.17	.17	< 1
B	1	1.94	1.94	< 1
C	1	3.42	3.42	1.53
D	1	8.16	8.16	3.64
AB	1	.26	.26	< 1
AC	1	.74	.74	< 1
AD	1	.02	.02	< 1
BC	1	13.08	13.08	5.84
BD	1	.91	.91	< 1
CD	1	.78	.78	< 1
ABC	1	.78	.78	< 1
ABD	1	6.77	6.77	3.02
ACD	1	.62	.62	< 1
BCD	1	1.76	1.76	< 1
ABCD	1	.00	.00	< 1
Error	16	35.85	2.24	
Total	31			

b. $F_{.05,1,16} = 4.49$, so none of the interaction effects is judged significant, and only the D main effect is significant.

41. $\Sigma\Sigma\Sigma\Sigma\Sigma x_{ijklm}^2 = 3{,}308{,}143$, $x_{.....} = 11{,}956$, so $CF = \frac{(11{,}956)^2}{48} = 2{,}979{,}535.02$, and SST = 328,607.98. Each SS is $\frac{(effectcontrast)^2}{48}$ and SSE is obtained by subtraction. The ANOVA table appears in the answer section. $F_{.05,1,32} \approx 4.15$, a value exceeded by the F ratios for AB interaction and the four main effects.

42. $\Sigma\Sigma\Sigma\Sigma\Sigma x_{ijklm}^2 = 32{,}917{,}817$, $x_{.....} = 39{,}371$, $SS = \dfrac{(contrast)^2}{48}$, and error d.f. = 32.

Effect	MS	f	Effect	MS	f
A	16,170.02	3.42	**BD**	3519.19	< 1
B	332,167.69	70.17	**CD**	4700.52	< 1
C	43,140.02	9.11	**ABC**	1210.02	< 1
D	20,460.02	4.33	**ABD**	15,229.69	3.22
AB	1989.19	< 1	**ACD**	1963.52	< 1
AC	776.02	< 1	**BCD**	10,354.69	2.19
AD	16,170.02	3.42	**ABCD**	1692.19	< 1
BC	3553.52	< 1	**Error**	4733.69	

$F_{.01,1,32} \approx 7.5$, so only the B and C main effects are judged significant at the 1% level.

43.

Condition/Effect	$SS = \frac{(contrast)^2}{16}$	f	Condition/Effect	$SS = \frac{(contrast)^2}{16}$	f
(1)	--		**D**	414.123	1067.33
A	.436	1.12	**AD**	.017	< 1
B	.099	< 1	**BD**	.456	< 1
AB	.497	1.28	**ABD**	.055	--
C	.109	< 1	**CD**	2.190	5.64
AC	.078	< 1	**ACD**	1.020	--
BC	1.404	3.62	**BCD**	.133	--
ABC	.051	--	**ABCD**	.681	--

SSE = .051 + .055 + 1.020 + .133 + .681 = 1.940, d.f. = 5, so MSE = .388. $F_{.05,1,5} = 6.61$, so only the D main effect is significant.

44.

a. The eight treatment conditions which have even number of letters in common with abcd and thus go in the first (principle) block are (1), ab, ac, bc, ad, bd, cd, and abd; the other eight conditions are placed in the second block.

b. and **c.**

$x_{....} = 1290$, $\Sigma\Sigma\Sigma\Sigma x_{ijkl}^2 = 105{,}160$, so SST = 1153.75. The two block totals are 639 and 651, so $SSBl = \frac{639^2}{8} + \frac{651^2}{8} - \frac{1290^2}{16} = 9.00$, which is identical (as it must be here) to SSABCD computed from Yates algorithm.

Condition/Effect	Block	$SS = \frac{(contrast)^2}{16}$	f
(1)	1	--	
A	2	25.00	1.93
B	2	9.00	< 1
AB	1	12.25	< 1
C	2	49.00	3.79
AC	1	2.25	< 1
BC	1	.25	< 1
ABC	2	9.00	--
D	2	930.25	71.90
AD	1	36.00	2.78
BD	1	25.00	1.93
ABD	2	20.25	--
CD	1	4.00	< 1
ACD	2	20.25	--
BCD	2	2.25	--
ABCD=Blocks	1	9.00	--
Total		1153.75	

$SSE = 9.0 + 20.25 + 20.25 + 2.25 = 51.75$; d.f. = 4, so MSE = 12.9375, $F_{.05,1,4} = 7.71$, so only the D main effect is significant.

45.

a. The allocation of treatments to blocks is as given in the answer section, with block #1 containing all treatments having an even number of letters in common with both ab and cd, etc.

b. $x_{.....} = 16{,}898$, so $SST = 9{,}035{,}054 - \frac{16{,}898^2}{32} = 111{,}853.88$. The eight $block \times replication$ totals are 2091 (= 618 + 421 + 603 + 449, the sum of the four observations in block #1 on replication #1), 2092, 2133, 2145, 2113, 2080, 2122, and 2122, so $SSBl = \frac{2091^2}{4} + ... + \frac{2122^2}{4} - \frac{16{,}898^2}{32} = 898.88$. The remaining SS's as well as all F ratios appear in the ANOVA table in the answer section. With $F_{.01,1,12} = 9.33$, only the A and B main effects are significant.

46. The result is clearly true if either defining effect is represented by either a single letter (e.g., A) or a pair of letters (e.g. AB). The only other possibilities are for both to be "triples" (e.g. ABC or ABD, all of which must have two letters in common.) or one a triple and the other ABCD. But the generalized interaction of ABC and ABD is CD, so a two-factor interaction is confounded, and the generalized interaction of ABC and ABCD is D, so a main effect is confounded.

47. See the text's answer section.

48.

a. The treatment conditions in the observed group are (in standard order) (1), ab, ac, bc, ad, bd, cd, and abcd. The alias pairs are {A, BCD}, {B, ACD}, {C, ABD}, {D, ABC}, {AB, CD}, {AC, BD}, and {AD, BC}.

b.

	A	**B**	**C**	**D**	**AB**	**AC**	**AD**
(1) = 19.09	-	-	-	-	+	+	+
Ab = 20.11	+	+	-	-	+	-	-
Ac = 21.66	+	-	+	-	-	+	-
Bc = 20.44	-	+	+	-	-	-	+
Ad = 13.72	+	-	-	+	-	-	+
Bd = 11.26	-	+	-	+	-	+	-
Cd = 11.72	-	-	+	+	+	-	-
Abcd = 12.29	+	+	+	+	+	+	+
Contrast	5.27	-2.09	1.93	-32.31	-3.87	-1.69	.79
SS	3.47	.55	.47	130.49	1.87	.36	.08
f	4.51	< 1	< 1	169.47	SSE=2.31		MSE=.770

$F_{.05,1,3} = 10.13$, so only the D main effect is judged significant.

49.

		A	B	C	D	E	AB	AC	AD	AE	BC	BD	BE	CD	CE	DE
a	70.4	+	-	-	-	-	-	-	-	-	+	+	+	+	+	+
b	72.1	-	+	-	-	-	-	+	+	+	-	-	-	+	+	+
c	70.4	-	-	+	-	-	+	-	+	+	-	+	+	-	-	+
abc	73.8	+	+	+	-	-	+	+	-	-	+	-	-	-	-	+
d	67.4	-	-	-	+	-	+	+	-	+	+	-	+	-	+	-
abd	67.0	+	+	-	+	-	+	-	+	-	-	+	-	-	+	-
acd	66.6	+	-	+	+	-	-	+	+	-	-	-	+	+	-	-
bcd	66.8	-	+	+	+	-	-	-	-	+	+	+	-	+	-	-
e	68.0	-	-	-	-	+	+	+	+	-	+	+	-	+	-	-
abe	67.8	+	+	-	-	+	+	-	-	+	-	-	+	+	-	-
ace	67.5	+	-	+	-	+	-	+	-	+	-	+	-	-	+	-
bce	70.3	-	+	+	-	+	-	-	+	-	+	-	+	-	+	-
ade	64.0	+	-	-	+	+	-	-	+	+	+	-	-	-	-	+
bde	67.9	-	+	-	+	+	-	+	-	-	-	+	+	-	-	+
cde	65.9	-	-	+	+	+	+	-	-	-	-	-	-	+	+	+
abcde	68.0	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

Thus $SSA = \frac{(70.4 - 72.1 - 70.4 + ... + 68.0)^2}{16} = 2.250$, SSB = 7.840, SSC = .360, SSD = 52.563, SSE = 10.240, SSAB = 1.563, SSAC = 7.563, SSAD = .090, SSAE = 4.203, SSBC = 2.103, SSBD = .010, SSBE = .123, SSCD = .010, SSCE = .063, SSDE = 4.840, Error SS = sum of two factor SS's = 20.568, Error MS = 2.057, $F_{.01,1,10} = 10.04$, so only the D main effect is significant.

Supplementary Exercises

50.

Source	Df	SS	MS	f
Treatment	4	14.962	3.741	36.7
Block	8	9.696		
Error	32	3.262	.102	
Total	44	27.920		

$H_0 : a_1 = a_2 = a_3 = a_4 = a_5 = 0$ will be rejected if $f = \frac{MSTr}{MSE} \geq F_{.05,4,32} = 2.67$.

Because $36.7 \geq 2.67$, H_o is rejected. We conclude that expected smoothness score does depend somehow on the drying method used.

51.

Source	Df	SS	MS	f
A	1	322.667	322.667	980.38
B	3	35.623	11.874	36.08
AB	3	8.557	2.852	8.67
Error	16	5.266	.329	
Total	23	372.113		

We first test the null hypothesis of no interactions ($H_0 : g_{ij} = 0$ for all I, j). H_o will be rejected at level .05 if $f_{AB} = \frac{MSAB}{MSE} \ge F_{.05,3,16} = 3.24$. Because $8.67 \ge 3.24$, H_o is rejected. Because we have concluded that interaction is present, tests for main effects are not appropriate.

52. Let $X_{ij} =$ the amount of clover accumulation when the ith sowing rate is used in the jth plot = $m + a_i + b_j + e_{ij}$. $H_0 : a_1 = a_2 = a_3 = a_4 = 0$ will be rejected if

$$f = \frac{MSTr}{MSE} \ge F_{a,I-1,(I-1)(J-1)} = F_{.05,3,9} = 3.86$$

Source	Df	SS	MS	f
Treatment	3	3,141,153.5	1,040,751.17	2.28
Block	3	19,470,550.0		
Error	9	4,141,165.5	460,129.50	
Total	15	26,752,869.0		

Because $2.28 < 3.86$, H_o is not rejected. Expected accumulation does not appear to depend on sowing rate.

53. Let A = spray volume, B = belt speed, C = brand.

Condition	Total	1	2	Contrast	$SS = \frac{(contrast)^2}{16}$
(1)	76	129	289	592	21,904.00
A	53	160	303	22	30.25
B	62	143	13	48	144.00
AB	98	160	9	134	1122.25
C	88	-23	31	14	12.25
AC	55	36	17	-4	1.00
BC	59	-33	59	-14	12.25
ABC	101	42	75	16	16.00

The ANOVA table is as follows:

Effect	**Df**	**MS**	**f**
A	1	30.25	6.72
B	1	144.00	32.00
AB	1	1122.25	249.39
C	1	12.25	2.72
AC	1	1.00	.22
BC	1	12.25	2.72
ABC	1	16.00	3.56
Error	8	4.50	
Total	15		

$F_{.05,1,8} = 5.32$, so all of the main effects are significant at level .05, but none of the interactions are significant.

54. We use Yates' method for calculating the sums of squares, and for ease of calculation, we divide each observation by 1000.

Condition	Total	1	2	Contrast	$SS = \frac{(contrast)^2}{8}$
(1)	23.1	66.1	213.5	317.2	-
A	43.0	147.4	103.7	20.2	51.005
B	71.4	70.2	24.5	44.6	248.645
AB	76.0	33.5	-4.3	-12.0	18.000
C	37.0	19.9	81.3	-109.8	1507.005
AC	33.2	4.6	-36.7	-28.8	103.68
BC	17.0	-3.8	-15.3	-118.0	1740.5
ABC	16.5	-.5	3.3	18.6	43.245

We assume that there is no three-way interaction, so the MSABC becomes the MSE for ANOVA:

Source	df	MS	f
A	1	51.005	1.179
B	1	248.645	5.750*
AB	1	18.000	< 1
C	1	1507.005	34.848*
AC	1	103.68	2.398
BC	1	1740.5	40.247*
Error	1	43.245	
Total	8		

With $F_{.05,1,8} = 5.32$, the B and C main effects are significant at the .05 level, as well as the BC interaction. We conclude that although binder type (A) is not significant, both amount of water (B) and the land disposal scenario (C) affect the leaching characteristics under study., and there is some interaction between the two factors.

55.

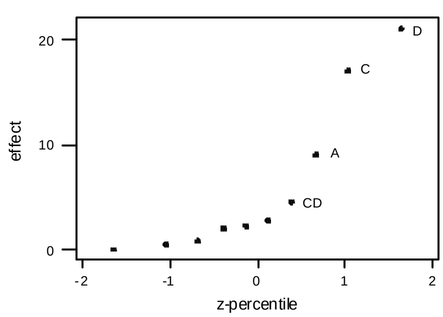
a.

Effect	%Iron	1	2	3	Effect Contrast	SS
	7	18	37	174	684	
A	11	19	137	510	144	1296
B	7	62	169	50	36	81
AB	12	75	341	94	0	0
C	21	79	9	14	272	4624
AC	41	90	41	22	32	64
BC	27	165	47	2	12	9
ABC	48	176	47	-2	-4	1
D	28	4	1	100	336	7056
AD	51	5	13	172	44	121
BD	33	20	11	32	8	4
ABD	57	21	11	0	0	0
CD	70	23	1	12	72	324
ACD	95	24	1	0	-32	64
BCD	77	25	1	0	-12	9
ABCD	99	22	-3	-4	-4	1

We use $estimate = \frac{contrast}{2^p}$ when n = 1 (see p 472 of text) to get

$\hat{a}_1 = \frac{144}{2^4} = \frac{144}{16} = 9.00$, $\hat{b}_1 = \frac{36}{16} = 2.25$, $\hat{d}_1 = \frac{272}{16} = 17.00$,
$\hat{g}_1 = \frac{336}{16} = 21.00$. Similarly, $\left(\widehat{ab}\right)_{11} = 0$, $\left(\widehat{ad}\right)_{11} = 2.00$, $\left(\widehat{ag}\right)_{11} = 2.75$,
$\left(\widehat{bd}\right)_{11} = .75$, $\left(\widehat{bg}\right)_{11} = .50$, and $\left(\widehat{dg}\right)_{11} = 4.50$.

b.



The plot suggests main effects A, C, and D are quite important, and perhaps the interaction CD as well. (See answer section for comment.)

56. The summary quantities are:

$x_{ij\bullet}$	j = 1	2	3	$x_{i\bullet\bullet}$
i = 1	6.2	4.0	5.8	16.0
2	7.6	6.2	6.4	20.2
$x_{\bullet j\bullet}$	13.8	10.2	12.2	$x_{\bullet\bullet\bullet} = 36.2$

$$CF = \frac{(36.2)^2}{30} = 43.6813,\ \Sigma\Sigma\Sigma x_{ijk}^2 = 45.560, \text{ so}$$
$$SST = 45.560 - 43.6813 = 1.8787,$$
$$SSE = 45.560 - \frac{225.24}{5} = .5120,\ SSA = \frac{(16.0)^2 + (20.2)^2}{15} - CF = .5880,$$
$$SSB = \frac{(13.8)^2 + (10.2)^2 + (12.2)^2}{10} - CF = .6507,$$

and by subtraction, SSAB = .128

```
Analysis of Variance for Average Bud Rating
Source        DF        SS        MS        F
Health         1    0.5880    0.5880    27.56
pH             2    0.6507    0.3253    15.25
Interaction    2    0.1280    0.0640     3.00
Error         24    0.5120    0.0213
Total         29    1.8787
```

Since 3.00 is not $\geq F_{.05,2,24} = 3.40$, we fail to reject the no interactions hypothesis, and we continue: $27.56 \geq F_{.05,1,24} = 4.26$, and $15.25 \geq F_{.05,2,24} = 3.40$, so we conclude that both the health of the seedling and its pH level have an effect on the average rating.

57. The ANOVA table is:

Source	df	SS	MS	f	$F_{.01}$
A	2	34,436	17,218	436.92	5.49
B	2	105,793	52,897	1342.30	5.49
C	2	516,398	258,199	6552.04	5.49
AB	4	6,868	1,717	43.57	4.11
AC	4	10,922	2,731	69.29	4.11
BC	4	10,178	2,545	64.57	4.11
ABC	8	6,713	839	21.30	3.26
Error	27	1,064	39		
Total	53	692,372			

All calculated f values are greater than their respective tabled values, so all effects, including the interaction effects, are significant at level .01.

58.

Source	**df**	**SS**	**MS**	**f**	$\mathbf{F_{.05}}$
A(pressure)	1	6.94	6.940	11.57*	4.26
B(time)	3	5.61	1.870	3.12*	3.01
C(concen.)	2	12.33	6.165	10.28*	3.40
AB	3	4.05	1.350	2.25	3.01
AC	2	7.32	3.660	6.10*	3.40
BC	6	15.80	2.633	4.39*	2.51
ABC	6	4.37	.728	1.21	2.51
Error	24	14.40	.600		
Total	47	70.82			

There appear to be no three-factor interactions. However both AC and BC two-factor interactions appear to be present.

59. Based on the p-values in the ANOVA table, statistically significant factors at the level .01 are adhesive type and cure time. The conductor material does not have a statistically significant effect on bond strength. There are no significant interactions.

60.

Source	df	SS	MS	f	$F_{.05}$
A (diet)	2	18,138	9.69.0	28.9*	≈ 3.32
B (temp.)	2	5,182	2591.0	8.3*	≈ 3.32
Interaction	4	1,737	434.3	1.4	≈ 2.69
Error	36	11,291	313.6		
Total	44	36,348			

Interaction appears to be absent. However, since both main effect f values exceed the corresponding F critical values, both diet and temperature appear to affect expected energy intake.

61. $SSA = \sum_i \sum_j \left(\overline{X}_{i...} - \overline{X}_{....}\right)^2 = \frac{1}{N}\Sigma X_{i...}^2 - \frac{X_{....}^2}{N}$, with similar expressions for SSB, SSC, and SSD, each having N – 1 df.

$SST = \sum_i \sum_j \left(X_{ij(kl)} - \overline{X}_{....}\right)^2 = \sum_i \sum_j X_{ij(kl)}^2 - \frac{X_{....}^2}{N}$ with $N^2 - 1$ df, leaving $N^2 - 1 - 4(N-1)$ df for error.

	1	2	3	4	5	Σx^2
$x_{i...}$:	482	446	464	468	434	1,053,916
$x_{.j..}$:	470	451	440	482	451	1,053,626
$x_{..k.}$:	372	429	484	528	481	1,066,826
$x_{...l}$:	340	417	466	537	534	1,080,170

Also, $\Sigma\Sigma x_{ij(kl)}^2 = 220{,}378$, $x_{....} = 2294$, and CF = 210,497.44

Source	df	SS	MS	f	$F_{.05}$
A	4	285.76	71.44	.594	3.84
B	4	227.76	56.94	.473	3.84
C	4	2867.76	716.94	5.958*	3.84
D	4	5536.56	1384.14	11.502*	3.84
Error	8	962.72	120.34		
Total	24				

H_{oA} and H_{oB} cannot be rejected, while while H_{oC} and H_{oD} are rejected.