Homework #7

Ben Drucker

5.3

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a)

The first row (P_{t_0}) represents the PMF of T_0:

t_0	0	1	2	3	4	$\sum$
$P(t_0)$	.04	.2	.37	.3	.09	1
$t_0P(t_0)$	0	.2	.74	.9	.36	2.2
$t_0^2P(t_0)$	0	.2	1.48	2.7	1.44	5.82

b)

$\mu_{T_0} = \sum t_0 P(t_0) = 2.2; \quad \mu_{T_0} = 2\mu$

c)

$\sigma^2_{T_0} = \sum t_0^2 P(t_0) - \mu^2_{T_0} = 5.82 - 2.2^2 = .98; \quad \sigma^2_{T_0} = 2\sigma^2$

d)

$E(T_0) = 4.4; \quad VT_0 = 1.96$

e)

$P(T_0 = 8) = .09^2 = .0081; \quad P(T_0 \geq 7) = P(T_0 = 8) + 2 * .3 * .09 = .0081 + .054 = .0621$

42)

a)

$\overline{x}$	27.75	28	29.7	29.95	31.65	21.9	33.6
$p(\overline{x})$	$\frac{4}{30}$	$\frac{2}{30}$	$\frac{6}{30}$	$\frac{4}{30}$	$\frac{8}{30}$	$\frac{4}{30}$	$\frac{2}{30}$

b)

$\overline{x}$	27.75	31.65	31.9
$p(\overline{x})$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

c)

$E(X)_a = E(X)_b = \mu = 30.4\overline{3}$

5.4)

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a)

$E(\overline{X}) = \mu = 12$
$\sigma_{\overline{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{.04}{4} = .01$

b)

$E(\overline{X}) = \mu = 12$
$\sigma_{\overline{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{.04}{8} = .005$

c)

The second sample, which has less variability due to the larger n.

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a)

Yes. Given that the population is men 18 years of age, it is plausible to assume a symmetric distribution around an unknown mean with decreasing frequency with increasing distance from the mean. If the population were all 18 year olds, the population would likely be bi-model because of sex differences.

b)

$\mu = 85; \quad \sigma_{\overline{x}} = \frac{15}{\sqrt{277}} \approxeq .901262$
$P(\overline{X} \geq 86.3) = 1 - P(\overline{X} \leq 86.3) = 1 - P\left(Z < \Phi\left[\frac{86.3-85}{.901262}\right]\right) = 1 - P(Z < 1.442) = 1 - .425 = .575$

c)

$\mu = 82; \quad \sigma_{\overline{x}} = .901262$
$P(\overline{X} \geq 86.3) = 1 - P(\overline{X} \leq 86.3) = 1 - P\left(Z < \Phi\left[\frac{86.3-82}{.901262}\right]\right) = 1 - P(Z < 4.77) \approx 0.$
82 cm would not be reasonable.

56)

a)

$\mu = np = 1000 * .1 = 100; \quad \sigma = \sqrt{\mu(1-p)} = \sqrt{100 * .9} = 9.486832981.$

$$\begin{aligned} P(X \leq 125) &= P\left(\frac{X-\mu}{\sigma} \leq \frac{125-\mu}{\sigma}\right) \\ &= P\left(Z \leq \frac{125-100}{9.486832981}\right) \\ &= P(Z < 2.64) \\ &= .996 \end{aligned}$$

b)

Let Δ be the difference in errors between the two independent transmissions:

$$\begin{aligned} P(|\Delta| \leq 50 &= P(-50 \leq \Delta \leq 50) \\ &= P\left(\frac{-50}{\sigma} \leq Z \leq \frac{50}{\sigma}\right) \\ &\approx P\left(\frac{50}{\sqrt{2np(1-p)}} = \frac{-50}{\sqrt{2 * 1000 * .1 * (1-.1)}} = -3.7268 \leq Z \leq 3.7268\right) \\ &= .999903 - .00009697 = .99980603 \end{aligned}$$

1 5.5

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$\mu_Y = \frac{1}{2}(\mu_1 + \mu_2) - \frac{1}{3}(\mu_3 + \mu_4 + \mu_5) = -1$
$\sigma_Y^2 = \frac{1}{4}\sigma_1^2 + \frac{1}{4}\sigma_2^2 + \frac{1}{9}\sigma_3^2 + \frac{1}{9}\sigma_4^2 + \frac{1}{9}\sigma_5^2 = 3.167; \ \sigma_Y = \sqrt{3.167} = 1.7795$
$P(Y \geq 0) = P\left(Z \geq \frac{1}{1.7795}\right) = P(Z \geq .56) = 1 - P(Z \leq .56) = .2877$
$P(-1 \leq Y \leq 1) = P\left(0 \leq Z \leq \frac{2}{1.7795}\right) = P(Z < 1.12) = .369$

64)

$X_1..X_5$ for morning, $X_6..X_{10}$ for evening.

a)

$E(X_1 + ... + X_10) = 5E(X_1) + 5(X_6) = 5 * 4 + 5 * 5 = 45$

b)

$\sigma^2 = 5Var(X_1) + 5(X_6) = 5\left(\frac{8^2}{12} + \frac{10^2}{12}\right) = \frac{820}{12} = 68.\overline{3}$

c)

$E(X_1 - X_6) = E(X_1) - E(X_6) = 4 - 5 = -1$
$Var(X_1 - X_6) = Var(X_1) + Var(X_6) = \frac{64}{12} + \frac{100}{12} = 13.\overline{6}$

d)

$E[(X_1 + ... + X_5) - (X_6 + ... + X_{10})] = 5 * 4 - 5 * 5 = -5$
$Var[(X_1 + ... + X_5) - (X_6 + ... + X_{10})] = Var(X_1 + ... + X_5) + Var(X_6 + ... + X_{10}) = 68.\overline{3}$

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a)

$$E(Y_i) = \frac{1}{2} \to E(W) = \sum_{i=1}^{n} iE(Y_i) = \frac{1}{2}\sum_{i=1}^{n} i = \frac{n^2+n}{2^2}$$

b)

$$Var(Y_i) = \frac{1}{4} \to Var(W) = \sum_{i=1}^{n} i^2 Var(Y_i) = \frac{1}{4}\sum_{i=1}^{n} i^2 = \frac{(n^2+n)(2n+1)}{4 * 2^2}$$