Homework 7

1. A firm produces a soft drink using two ingredients, sugar (S) and bubbly water (B) in fixed proportions: 6 tablespoons of sugar per 12 oz of bubbly water.

a) What is the production function? Does this production function exhibit constant, increasing or decreasing returns to scale?
b) Write down the firm's cost minimization problem and solve for the compensated factor demands.

2. *Sam's Trucking Co* hauls freight in the US. *Sam's Trucking* is a perfectly competitive firm in the trucking industry where the price of hauling a ton of freight from NY to Boston is $250. One of the most expensive inputs in the trucking business is the price of diesel fuel, P_D. For each ton carried from NY to Boston 100 gallons of diesel is used so the fuel costs of hauling Q are $(100Q)P_D$. *Sam's Trucking Co* has a variable cost curve $VC(Q)= Q^2+(100Q)P_D$ ($= Q^2$ + fuel costs). In the questions below specific values of P_D will be set. *Sam's Trucking Co* has no fixed costs of production.

a) Given that Sam's is perfectly competitive and the market price is $250 what is *Sam's* revenue curve? What is its marginal revenue curve?
b) If the price of diesel fuel, P_D, is $2 per gallon then what is *Sam's* variable cost? What is the marginal cost?
c) What is the profit maximizing level of Q at these prices? What are the profits of *Sam's Trucking Co* at the profit maximizing level of freight?

Suppose that the diesel price of $2 was an expected price. The actual price of diesel will be either $1.50 or $3 per gallon. The probability that diesel will be $1.50 is 2/3 and the probability that it will be $3 is 1/3. Assume that *Sam's Trucking* must commit to a level freight Q before it knows what the price of diesel will be. Thus the profit maximizing level of freight will remain at 25 tons (as you found in part (c)).

d) Given that *Sam's* will still set Q=25 what will be the profits if the price of diesel is $1.5? What will be the profits if $P_D = 3$?

Sam's has the opportunity of buying an asset that will allow it to "hedge" against high diesel prices. Specifically this asset provides the option to buy one gallon of diesel at its expected price of $2. If the price of diesel turns out to be $3 then the asset is worth $1 (the difference in prices). However, if the price of diesel turns out to be $1.50 then the asset is worthless (since it would be cheaper to buy the diesel in the market at $1.50). *Sam's* may buy as many units of this asset as it would like.

e) What is the expected value of the asset?

Suppose that the asset is priced at the expected value that you found in (f).

f) At this price how many units of the asset would *Sam's* need to buy in order to be perfectly hedged (i.e. *Sam's* profits would be the same regardless of the price of diesel? What are *Sam's* profits in each state if it is perfectly hedged?

3. Bob's Basil Farm uses both premium organic manure (x) and compost (y) to fertilize the basil plants. The production function of Bob's is given by $Q = 4x^{\frac{1}{3}}y^{\frac{1}{3}}$ where both manure (x) and

compost are measured in cubic yards and Q is measured in pounds of basil. The price of manure is $64 per cubic yard and the price of compost is $1 per cubic yard. Bob's has no overhead (fixed costs).

For parts (a) – (b) below assume that Bob's stock of compost is fixed in the short run. In particular Bob has only 8 cubic yards of compost. This results in fixed costs of $8.

a) In the short run what is Bob's (compensated) demand curve for manure? What is his variable cost curve? What is his total cost curve?
b) In the short run what is the marginal cost curve of Bob's Basil Farm? What is the average cost curve? What is the optimal size of the firm? Illustrate the short run marginal and average cost curves below.

For parts (c)- (d) below assume that Bob's stock of compost is variable in the long run.

c) In the long run what is Bob's (compensated) demand curve for manure? What is the (compensated) demand for compost? What is the variable cost curve? What is the total cost curve?
d) In the long run what is Bob's marginal cost curve? What is the average cost curve?

Finally let's compare the average cost curves in the short and long runs. You may assume that at every quantity the long run average cost is less than or equal to the short run average cost.

e) At what quantity is the demand for compost equal to 8? What must be true about the short and long run average costs at this quantity?
f) Given that long run average costs are always less than or equal to the short run average costs and your answers to part (e), illustrate the long run average cost curve in your diagram for part (b).

4. *Hands* is a typical nail salon in Gotham. The production function for *Hands* is given by the following: $Q = f(L,K) = 20L^{\frac{1}{4}}K^{\frac{1}{4}}$ where L represents man-hours, K represents machine hours and output Q is measured in manicures per day. The price of an hour of labor is $16 and the price of an hour of machine time is $4.

a) Given the input prices and the production function set up the cost minimization problem. Solve the cost minimization problem to determine the (compensated) factor demands for labor and machines. Finally use the factor demands to show that the cost of the least-cost input bundle $0.04Q^2$.

In addition to labor and capital costs, *Hands* pays fixed costs of $64 per day. Thus the total cost curve is $C(Q) = 0.04Q^2 + 64$

b) What is the average cost curve of *Hands*? What is the marginal cost curve of *Hands?* What is the optimal scale of the firm? Illustrate the two cost curves.For what values of Q do *Hands* cost curves exhibit "scale" economies?

Nail salons are a perfectly competitive business in Gotham. Market demand for manicures is given by $Q = 9200 - 400P$. Assume that all nail salons have the same cost curve as *Hands*.

c) If the price in the market is $4 per manicure then how many manicures will *Hands* supply? At a price of $4 per manicure, will *Hands* earn supernormal, normal or subnormal profits? In your diagram illustrate the profits of *Hands* at $4.
d) Is $4 the long run market clearing price for a manicure? If not then what is the long run market clearing price? How many manicures will *Hands* supply in the long run? What will be the total number of manicures supplied in Gotham in the long run? How many nail salons will there be?

5. You are asked to do an analysis of the textile industry. Given the current production techniques of a typical manufacturer the number of rolls of fabric produced by a man-hour of labor (the only variable input to production) provided the total number of hours of labor is less than or equal to 10 per day is given by the following production function: $Q = L$. For hours of labor in excess of 10 per day the production function is: $Q = 10 + (L - 10)^{.5}$

a) Illustrate the (daily) production function for a textile manufacturer. In your diagram indicate the region of constant returns to scale production and the region of decreasing returns to scale.

The cost of labor is $2 per hour. In addition to labor costs there are materials costs of $100 per roll and fixed costs of operation of $230 per day. Labor and materials are the only variable costs of production.

b) What is the demand curve for labor? Given your demand for labor what is the cost curve of labor? What is the variable cost curve of a textile manufacturer? What is the total cost curve? What is the marginal cost of any unit up to the 10^{th} unit? What is the marginal cost of any unit after the 10^{th} unit? What is the average cost of any unit up to the 10^{th} unit? What is the average cost of any unit after the 10^{th} unit?
c) Illustrate the marginal and average cost curves. Show that the quantity of 15 rolls minimizes the per-unit costs of production. What is the average cost of 15 rolls? Indicate the minimum of the average cost curve and the optimal size of the firm in your diagram.
d) What is the supply curve for the typical firm? If the price in the market is $160 then how much would the firm want to supply to the market? Would the firm be earning super-normal, normal or sub-normal profits at this price? Illustrate the profits in an average cost and marginal cost curve diagram.
e) You estimate that the typical firm in this industry is producing 12 rolls of fabric per day. What do you predict will happen in this industry? Will firms enter or exit the industry? Will the price rise or fall?
f) If the estimated demand for fabric is $Q^D(P) = 390 - P$ where the quantity is measured in numbers of rolls per day then what will be the long run price and number of rolls traded of fabric? How many firms will there be in the industry?
g) If the price of materials rises by 10% then what will be the impact of this price rise on the variable costs of the firm? What will be the impact on the marginal and average costs of the firm? Illustrate the new marginal and average cost curves in a diagram. Include the original average and marginal cost curves in your diagram. What will be the new optimal size of the firm? Indicate the optimal size of the firm in your diagram.
h) What do you predict will be the new long-run price and quantity traded in the market? Will there be entry or exit into the industry?

6. Suppose that the daily total cost curve for a typical lens grinder is $C(Q) = .2Q^2 + 8Q + 80$. Quantity (Q) is measured in lenses per day. Suppose that demand per day for lenses is $Q^D(P) = 220 – 2P$.

a) What is the average cost curve for a typical lens grinder? What is the marginal cost curve? What is the optimal scale of the grinder? Illustrate the average and marginal cost curves.
b) What is the supply curve of a typical lens grinder? If there are 6 firms in the industry each of which has the typical cost curve then what is the industry supply curve of lenses?
c) If the market is perfectly competitive then what will be the market clearing price and quantity traded in the lens market in the short run? Illustrate your answer in a supply and demand curve diagram.
d) How many lenses will each firm supply? What is the elasticity of the demand curve that an individual firm faces? Illustrate your answer in a diagram of individual firm supply curve and individual inverse firm demand curve.
e) In your diagram for part (a) illustrate the daily profits of the typical lens grinder.

7. Suppose that the daily market demand curve for movers in Hatfield is $Q(P) = 487.5 – 7.5P$. Price (P) is measured in dollars per hour and quantity (Q) is measured in hours per day. The variable cost curve of a typical moving company is $VC(Q) = .2Q^2 + 5Q$ and the fixed cost is 500 .

a) What is the average cost curves for a typical moving company? What is the marginal cost curve? What is the optimal scale of the moving company? What is the long run shut down point? Illustrate the AC and the MC in a diagram and indicate the optimal scale.
b) If the industry in long-run competitive equilibrium then what is the price of an hour of moving services and the number of hours purchased per day in Hatfield? Show that there are 6 firms in Hatfield.

Suppose that price of gas rises resulting in a $3 per hour increase in the costs of the firm.

c) What is the new marginal cost curve for a typical moving company? What is the new average cost curve? What is the optimal scale of the moving company? Illustrate the new cost curves in your diagram for part (a).
d) What is the new SHORT RUN supply curve for a typical moving company? If there are only 6 firms in the industry then what will be the short run market supply curve (6 firms all operating n their short run supply curves)?
e) Given the demand for movers what will be the new short-run market clearing price and quantity traded in Hatfield?
f) Illustrate the two short run market supply curves (before and after the change in gas prices) and the demand curve in a new diagram. Include in your diagram the two market clearing quantities and prices.
g) Do you expect that there will be entry of exit from this market. Briefly explain your answer.

8. *Bonus* Suppose the production function for a firm is given by $f(L,K) = (LK)^{1/4}$ and the input prices are given by $w = 10$ and $r = 2.5$. Use the Lagrangian method to find the compensated factor demands for L and K and the value of the Lagrange multiplier (as a function of Q). Use the compensated factor demands to find the variable cost curve and the marginal cost curve. What is the relationship between the marginal cost curve and the Lagrange multiplier?