Program : **B.Tech**

Subject Name: **Analog and Digital Communication**

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Semester: **4th**

UNIT 3

Syllabus Angle modulation: Introduction and types of angle modulation, frequency modulation, frequency deviation, modulation index, deviation ratio, bandwidth requirement of FM wave, types of FM. Phase modulation, difference between FM and PM, Direct and indirect method of FM generation, FM demodulators- slope detector, Foster seeley discriminator, ratio detector. Introduction to pulse modulation systems.

3.1. Introduction: In Frequency Modulation (FM) the instantaneous value of the information signal (message) controls the frequency of the carrier wave. This is illustrated in the following diagrams.

Notice that as the Amplitude of information signal increases, the frequency of the carrier increases, and as the Amplitude of information signal decreases, the frequency of the carrier decreases.
The frequency f_i of the information signal controls the rate at which the carrier frequency increases and decreases. As with AM, f_i must be very much less than f_c. The amplitude of the carrier remains constant throughout this process.
When the information voltage reaches its maximum value then the change in frequency of the carrier will have also reached its maximum deviation above the nominal value. Similarly when the information reaches a minimum the carrier will be at its lowest frequency below the nominal carrier frequency value. When the information signal is zero, then no deviation of the carrier will occur.
The maximum change that can occur to the carrier from its base value f_c is called the **frequency deviation**, and is given the symbol Δf_c. This sets the *dynamic range* (i.e. voltage range) of the transmission. The *dynamic range* is the ratio of the largest and smallest analogue information signals that can be transmitted.

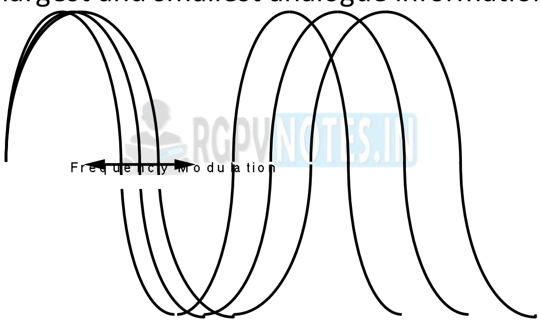


Figure 3.1: Frequency deviation illustration

Notice that frequency modulation looks very much like phase modulation. They are in fact very similar, and many textbooks refer to them both as angle modulation.

3.2. Concept of Frequency Modulation:

Frequency modulation: It is the form of angle modulation in which instantaneous frequency $f_i(t)$ is varied linearly with the information signal m(t)

$$f_i(t)=f_c+k_f\, m(t) \qquad \text{.....................................(1)}$$

where f_c –un-modulated carrier, k_f –Frequency sensitivity of the modulator, m(t)-Information signal
Integrating above equation with respect to time limit 0 to t and multiplying with 2π

$$2\pi \int f_i(t)\, dt = 2\pi f_c \int dt + 2\pi K_f \int m(t)\, dt$$

$$\Theta_i(t) = 2\pi f_c \int dt + 2\pi K_f \int m(t)\, dt$$

$$s(t)=A_c \cos(\Theta_i(t))$$

$$s(t)= A_c \cos(2\pi f_c t + 2\pi K_f \int m(t)\, dt) \qquad \text{.....................................(2)}$$

Phase modulation:

It is that form of Angle modulation in which angle $\phi_i(t)$ is varied linearly with the base band signal m(t) as as shown by $\phi_i(t) = K_p m(t)$

$$S(t)=A_c \cos(\omega_i(t)+\phi_i(t))$$

$$S(t)= A_c \cos(2\pi f_c t+K_p m(t)) \qquad \text{.....................................(3)}$$

3.3. Relationship between PM and FM

PM and FM are closely related in the sense that the net effect of both is variation in total phase angle. In PM, phase angle varies linearly with m(t) where in FM phase angle varies linearly with the integral of m(t). In other words, we can get FM by using PM, provided that at first, the modulating signal is integrated, and then applied to the phase modulator. The converse is also true, i.e. we can generate a PM wave using frequency modulator provided that m(t) is first differentiated and then applied to the frequency modulator.

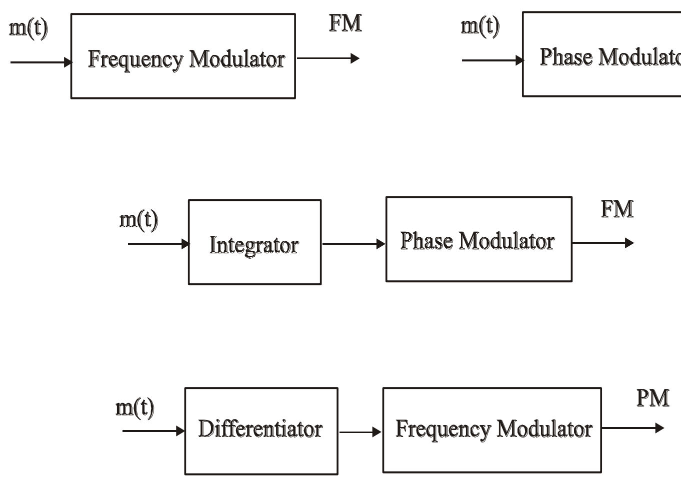


Figure 3.2.Relation between Fm and PM

Recall that a general sinusoid is of the form: $e_c = \sin(\omega_c t + \theta)$

Frequency modulation involves deviating a carrier frequency by some amount. If a sine wave was used to frequency modulate a carrier, the mathematical expression would be:

$$\omega_i = \omega_c + \Delta\omega \sin \omega_m t$$

Where
$$\begin{aligned}\omega_i &= \text{instantaneous frequency}\\ \omega_c &= \text{carrier frequency}\\ \Delta\omega &= \text{carrier deviation}\\ \omega_m &= \text{modulation frequency}\end{aligned}$$

This expression shows a signal varying sinusoidally about some average frequency. However, we cannot simply substitute expression in the general equation for a sinusoid. This is because the sine operator acts upon angles, not frequency. Therefore, we must define the instantaneous frequency in terms of angles. It should be noted that the amplitude of the modulation signal governs the amount of carrier deviation, while the modulation frequency governs the rate of carrier deviation.

The term $\frac{d\theta}{dt}$ is an angular velocity and it is related to frequency and angle by the following relationship:

$$\omega = 2\pi f = \frac{d\theta}{dt}.$$

To find the angle, we must integrate ω with respect to time, we obtain:

$$\int \omega dt = \theta$$

We can now find the instantaneous angle associated with an instantaneous frequency:

$$\theta = \int \omega_i \, dt = \int (\omega_c + \Delta\omega \sin \omega_m t) dt$$

$$= \omega_c t - \frac{\Delta\omega}{\omega_m} \cos \omega_m t = \omega_c t - \frac{\Delta f}{f_m} \cos \omega_m t$$

This angle can now be substituted into the general carrier signal to define FM:

$$e_{fm} = \sin\left(\omega_c t - \frac{\Delta f}{f_m}\cos\omega_m t\right) \qquad \text{.......................................(3)}$$

All FM transmissions are governed by a modulation index, β, which controls the dynamic range of the information being carried in the transmission.

$$\beta = \frac{\Delta f_c}{f_i}$$

3.4. Tone modulation:

Tone modulation is special case when message is sinusoidal as $m(t) = A_m \cos\omega_m t$

For Phase Modulation equation become $s_{PM}(t) = A \cos[\omega_c t + \theta_0 + k_{PM} m(t)]$

$$= A \cos[\omega_c t + \theta_0 + k_{PM} A_m \cos\omega_m t]$$

$$= A \cos[\omega_c t + \theta_0 + m_p \cos\omega_m t]$$

where $m_p = k_{PM} A_m$ is the **phase modulation index**, representing the maximum phase deviation $\Delta\theta$.

3.5. Frequency Modulation

$$s_{FM}(t) = A\cos[\omega_c t + \theta_0 + k_{FM}\int m(t)dt] = A\cos[\omega_c t + \theta_0 + k_{FM}\int A_m \cos\omega_m t dt]$$

$$= A\cos[\omega_c t + \theta_0 + \frac{k_{FM} A_m}{\omega_m}\sin\omega_m t] = A\cos[\omega_c t + \theta_0 + m_f \sin\omega_m t]$$

where $\beta = m_f = k_{FM}A_m / \omega_m = \Delta\omega / \omega_m$, i.e. the ratio of frequency deviation to the modulating frequency, is called the **frequency modulation index**.

$$\beta = \Delta\omega / \omega_m \qquad \text{.......................................(4)}$$

The relationship between phase deviation and frequency deviation in FM is given by

$$\Delta\theta = \beta = \Delta\omega / \omega_m \qquad \text{.......................................(5)}$$

3.6. Types of frequency modulation

The bandwidth of an FM signal depends on the frequency deviation. When the deviation is high, the bandwidth will be large, and vice-versa. According to the equation $\Delta\omega = k_{FM}|m(t)|_{max}$, for a given m(t), the frequency deviation, and hence the bandwidth, will depend on frequency sensitivity k_{FM}. Thus, depending on the value of k_{FM} (or $\Delta\omega$) we can divide FM into two categories: *narrowband FM* and *wideband FM*.

3.6.1. Narrowband FM(NBFM (β<<1))

When k_{FM} is small, the bandwidth of FM is narrow, this type of FM is called narrowband FM.

Since when $x << 1$, $\cos x \approx 1$, $\sin x \approx x$, we have

$$s_{NBFM}(t) = A\cos[\omega_c t + \theta_0 + k_{FM}\int m(t)dt]$$

$$= A\cos(\omega_c t + \theta_0)\cos[k_{FM}\int m(t)dt] - A\sin(\omega_c t + \theta_0)\sin[k_{FM}\int m(t)dt]$$

$$\approx A\cos[\omega_c t + \theta_0] - Ak_{FM}\int m(t)dt \sin[\omega_c t + \theta_0]$$

Narrowband modulation methods:

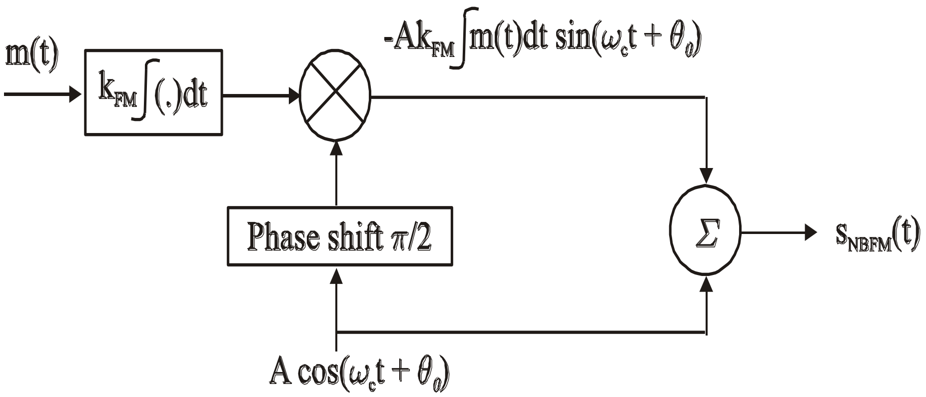


Figure 3.3.NBFM generation

Equation of narrowband frequency modulation (Tone modulation)

The message signal $m(t) = A_m \cos\omega_m t$

Signal waveform (assume $\theta_0 = 0$ for simplicity)

$$s_{NBFM}(t) = A\cos\omega_c t - Ak_{FM}\int m(t)dt\sin\omega_c t = A\cos\omega_c t - Ak_{FM}\int A_m\cos\omega_m t dt\sin\omega_c t$$

$$= A\cos\omega_c t - Ak_{FM}\frac{A_m}{\omega_m}\sin\omega_m t\sin\omega_c t = A\cos\omega_c t - Am_f\sin\omega_m t\sin\omega_c t$$

$$= A\cos\omega_c t + \frac{1}{2}Am_f\cos(\omega_c+\omega_m)t - \frac{1}{2}Am_f\cos(\omega_c-\omega_m)t$$

where $\beta = k_{FM}A_m / \omega_m$ is the FM modulation index.

Signal spectrum $S_{NBFM}(\omega) = \pi A[\delta(\omega - \omega_c) + \delta(\omega + \omega_c)] +$
$(1/2)\pi Am_f[\delta(\omega - \omega_c - \omega_m) + \delta(\omega + \omega_c + \omega_m) - \delta(\omega - \omega_c + \omega_m) - \delta(\omega + \omega_c - \omega_m)]$

Narrowband FM demodulation method

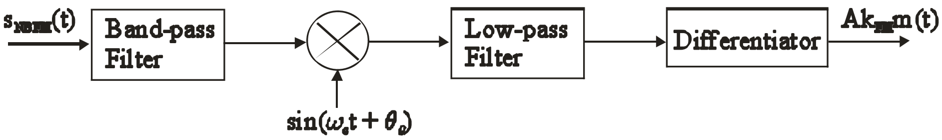


Figure 3.4.NBFM demodulation

3.6.2. Wideband FM (WBFM (β>>1))

When k_{FM} is large, the bandwidth of FM is wide, this type of FM is called *wideband FM*.

It is usually very difficult to analyze a general FM signal, we will restrict our analysis to the wideband FM with sinusoidal signal.

The message signal $m(t) = A_m \cos\omega_m t$

Signal waveform (assume $\theta_0 = 0$ for simplicity)

$$s_{FM}(t) = A\cos[\omega_c t + Ak_{FM}\int m(t)dt] = A\cos[\omega_c t + Ak_{FM}\int A_m\cos\omega_m t dt]$$

$$= A\cos[\omega_c t + m_f\sin\omega_m t] = A\cos\omega_c t\cos(m_f\sin\omega_m t) - A\sin\omega_c t\sin(m_f\sin\omega_m t)$$

$\cos(m_f \sin\omega_m t)$ and $\sin(m_f \sin\omega_m t)$ can be expressed in Fourier series

$$\cos(m_f\sin\omega_m t) = J_0(m_f) + 2\sum_{n=1}^{\infty}J_{2n}(m_f)\cos 2n\omega_m t$$

$$\sin(m_f\sin\omega_m t) = 2\sum_{n=1}^{\infty}J_{2n-1}(m_f)\cos(2n-1)\omega_m t$$

where $J_n(m_f) = \sum_{m=0}^{\infty}\frac{(-1)^m(\frac{m_f}{2})^{2m+n}}{m!(m+n)!}$

is the **Bessel function** of the first kind. Thus,

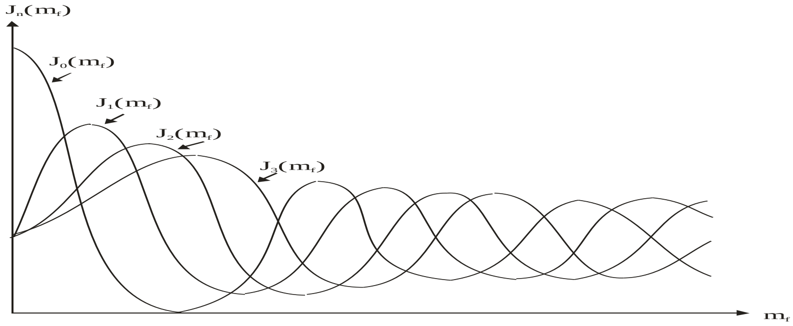
$$s_{FM}(t) = A\cos\omega_c t[J_0(m_f) + 2\sum_{n=1}^{\infty}J_{2n}(m_f)\cos 2n\omega_m t] - A\sin\omega_c t[2\sum_{n=1}^{\infty}J_{2n-1}(m_f)\cos(2n-1)\omega_m t]$$

by using $\cos\alpha\cos\beta = (1/2)[\cos(\alpha + \beta) + \cos(\alpha - \beta)]$,
$\sin\alpha\sin\beta = (1/2)[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$
and the property of Bessel function $J_{-n}(m_f) = (-1)^n J_n(\beta_{FM})$

$s_{FM}(t)$ can be written in the *Bessel function* form

$$s_{FM}(t) = A\sum_{n=-\infty}^{\infty}J_n(m_f)\cos[(\omega_c + n\omega_m)t]$$

The spectrum of $s_{FM}(t)$ $S_{FM}(\omega) = A\pi\sum_{n=-\infty}^{\infty}J_n(m_f)[\delta(\omega-\omega_c-n\omega_m)+\delta(\omega+\omega_c+n\omega_m)]$



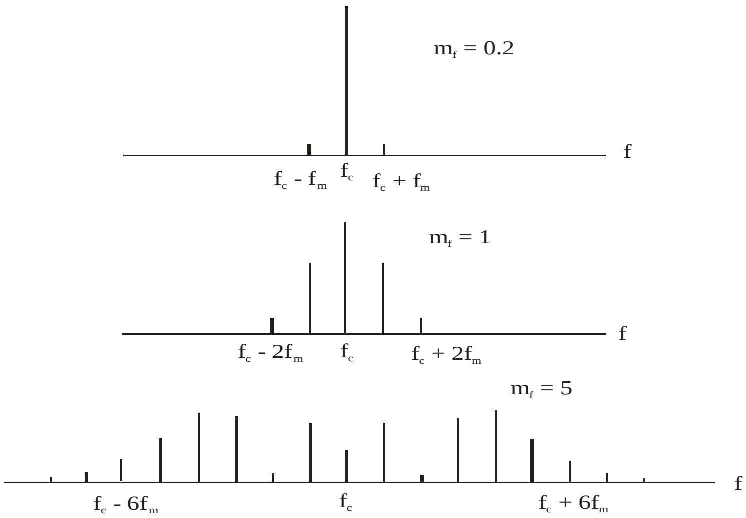


Figure 3.5.Typical plots of $S_{FM}(\omega)$ for different β.

The following observation can be made

- The carrier term $\cos\omega_c t$ has a magnitude of $J_0(m_f)$. The maximum value of $J_0(m_f)$ is 1 when $m_f = 0$, which is equivalent to no modulation.
- Theoretically infinitely number of sidebands are produced, and the amplitude of each sideband is decided by the corresponding *Bessel function* $J_n(m_f)$. The presence of infinite number of sidebands makes the ideal bandwidth of the FM signal infinite.
- When *m_f is small*, there are *few sideband frequencies of large amplitude* and, when *m_f is large*, there are *many sideband frequencies but with smaller amplitudes*. Hence, in practice, to determine the bandwidth, it is only necessary to consider a finite number of *significant sideband* components.
- Thus, the sidebands with small amplitudes can be ignored. The sidebands having amplitudes more than or equal to 1% of the carrier amplitude are known as *significant sidebands*. They are finite in number.

3.7. Bandwidth of a sinusoidally modulated FM signal

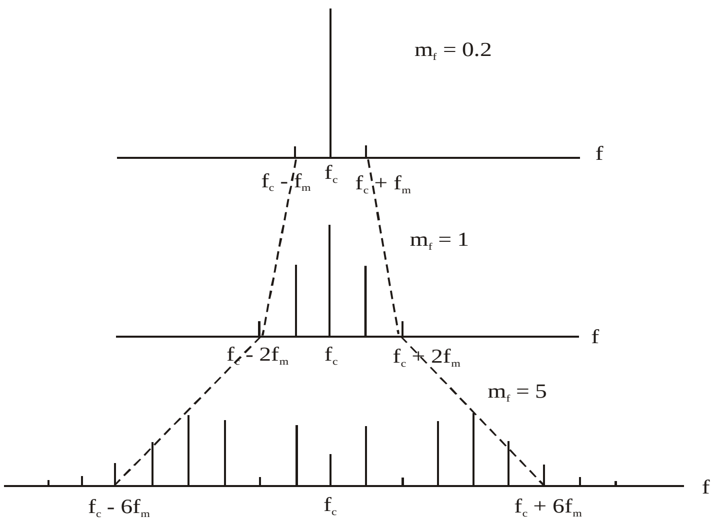


Figure 3.6.Spectrum of FM with different values of m_f

How many sidebands are significant in the FM?

$J_n(m_f)$ diminishes rapidly for n > m_f, particularly as m_f becomes large, the number of significant sideband is $m_f + 1$, i.e. $W_{FM} = 2(m_f + 1)\omega_m = 2(\Delta\omega + \omega_m)$

or

$$B_{FM} = 2(m_f + 1)f_m = 2(\Delta f + f_m)$$

Expressed in words, *the bandwidth is twice the sum of the maximum frequency deviation and the modulating frequency*. This rule for bandwidth is called **Carson's rule**.

Bandwidth of FM signal with arbitrary modulating signals

$$W_{FM} = 2(D_{FM} + 1)\omega_m = 2(\Delta\omega + \omega_m)$$

or $$B_{FM} = 2(D_{FM} + 1)f_m = 2(\Delta f + f_m)$$

Where D_{FM} is the **frequency deviation ratio** defined by

$$D_{FM} = \frac{\Delta\omega}{\omega_m} = \frac{\Delta f}{f_m}$$

3.8. Power of the FM signal

Since the amplitude of FM remains unchanged, the power of FM signal is the same as that of unmodulated carrier.

$$P_{FM} = \overline{s^2{}_{FM}(t)} = A^2 \overline{\sum_{n=-\infty}^{\infty} [J_n(m_f)\cos(\omega_c + n\omega_m)t]^2} = \frac{A^2}{2}\sum_{n=-\infty}^{\infty} J_n{}^2(m_f) = \frac{A^2}{2}$$

where we used one of the properties of Bessel function $\sum_{n=-\infty}^{\infty} J_n{}^2(m_f) = 1$

The total power is independent of the FM modulation process, since the power is related to signal amplitude, which is constant for FM, and not necessarily dependent on the signal's phase.

Comparison between WBFM and NBFM

S. No	**WBFM**	**NBFM**
I.	Modulating index is greater than1	Modulation index is less than 1
II.	Frequency deviation =75 KHz.	Frequency deviation 5 Khz.
III.	Modulating frequency range	Modulation frequency =3 Khz
iV.	From 30 Hz-15 Khz. Bandwidth 15 times NBFM.	Bandwidth =2 FM
V.	Noise is more suppressed.	Less suppressing of noise
Vi.	Use: Entertainment and broadcasting	Use: Mobile communication.

3.9. Wideband modulation methods

There are two methods for generating wideband FM signals: direct and indirect methods.

Direct method, voltage-controlled oscillator (VCO)

The direct method depends on varying the frequency of an oscillator linearly with m(t) for FM.

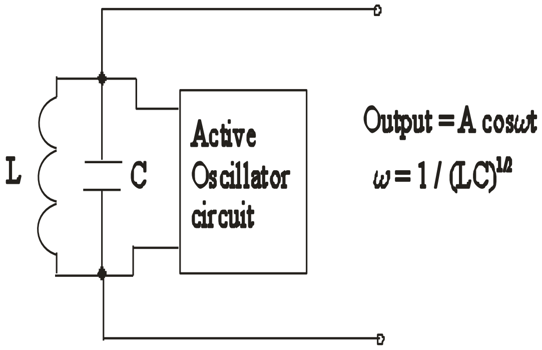


Figure 3.7.Direct method of FM generation

In the VCO, the modulating signal varies the voltage across the capacitor, as a consequence, the capacitance changes and causes a corresponding change in the oscillator frequency, i.e

$C = C_0 + \Delta C = C_0 + k_0 m(t)$

where k_0 is a constant.

Assume that $$\omega_0 = \frac{1}{\sqrt{LC_0}}$$

then $$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{LC_0(1+\frac{\Delta C}{C_0})}} = \frac{1}{\sqrt{LC_0}}\frac{1}{\sqrt{1+\frac{\Delta C}{C_0}}} \approx \omega_0(1-\frac{\Delta C}{2C_0}) = \omega_0(1-\frac{k_0 m(t)}{2C_0}) = \omega_0 - \frac{\omega_0 k_0}{2C_0}m(t) = \omega_0 - km(t)$$

where $k = \omega_0 k_0/2C_0$ is a constant, and the result: $(1 + x)^{-1/2} = 1 - x/2$, when x is small, is used.

Indirect or multiplication method

The indirect method depends on first generating a narrow FM signal and then using a multiplication technique whereby the deviation ratio can be raised to a large value.

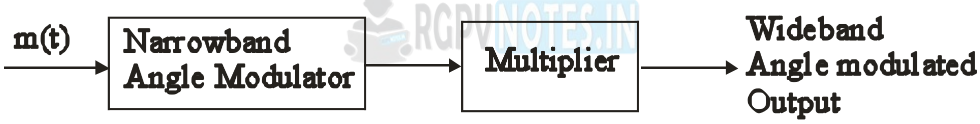


Figure 3.8.WBFM signal using NBFM

The multiplier is a device that multiplies the instantaneous frequency of its input waveform by a factor N.

3.10. Wideband FM demodulation method

Apply $s_{FM}(t)$ to a differentiator, the output is

$$\frac{ds_{FM}(t)}{dt} = \frac{d\{A\cos[\omega_c t + k_{FM}\int m(t)dt]\}}{dt} = -A[\omega_c + k_{FM}m(t)]\sin[\omega_c t + k_{FM}\int m(t)dt]$$

which is similar to a standard AM signal with small deviation ratio. The deviation ratio is small, since usually $\Delta\omega = k_{FM}|m(t)|_{max} << \omega_c$. The response of an envelope detector becomes

$$A[\omega_c + k_{FM}m(t)]$$

Blocking the dc term $A\omega_c$, the output is $s_o(t) = Ak_{FM}m(t)$

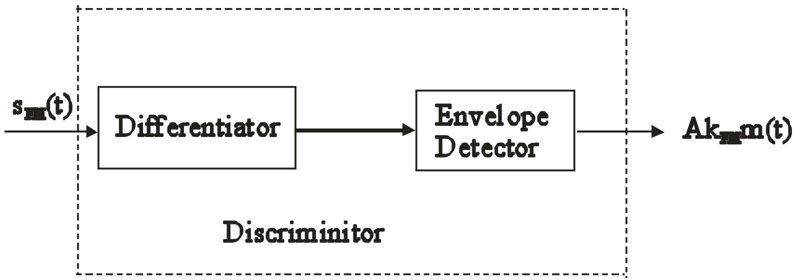


Figure 3.9.WBFM demodulator

The FM detector extracts a modulating signal from a frequency modulated carrier in two steps:

1. It converts the frequency modulated (FM) signal into a corresponding amplitude modulated (AM) signal by using frequency dependent circuits whose output voltage depends on input frequency. Such circuits are called *frequency discriminators*.
2. The original modulating signal m(t) is recovered from this AM signal by using a linear diode envelope detector.

3.11. Comparison between AM and FM

1. Noise performance: Wideband FM has better noise performance than AM. The greater the bandwidth, better is the noise performance. Narrowband FM has a noise performance equivalent to AM.

2. Channel bandwidth
   The wideband FM has a larger bandwidth as compared to AM because wideband FM produces a larger number of sidebands. In a typical broadcast system, each channel bandwidth in AM is 15kHz, whereas, in FM, it is 150kHz. Therefore, FM has a disadvantage over AM.

The modulation index β, is the ratio of the frequency deviation, Δf_c , to the maximum information frequency, f_i , as shown below:

The diagrams opposite show examples of how the modulation index affects the FM output, for a simple sinusoidal information signal of fixed frequency. The carrier signal has a frequency of ten times that of the information signal.

The first graph shows the information signal, the second shows the unmodulated carrier.

This graph shows the frequency modulated carrier when the modulation index = 3.

This graph shows the frequency modulated carrier when the modulation index = 5.

This graph shows the frequency modulated carrier when the modulation index = 7.

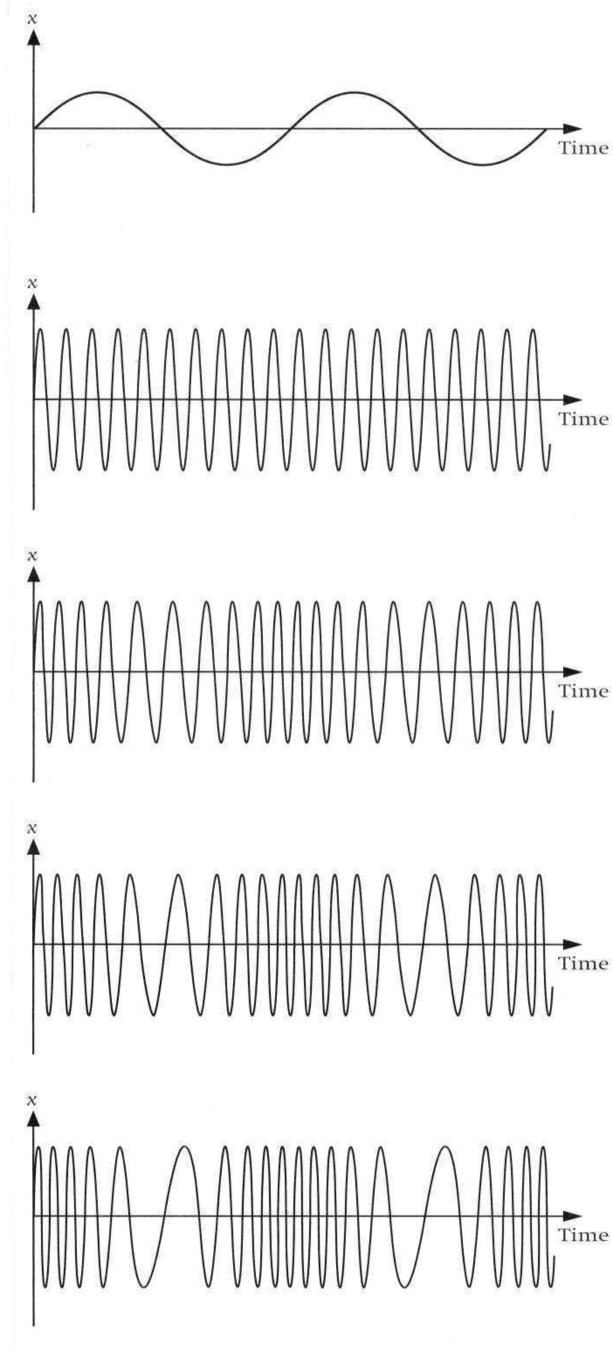


Figure 3.10.FM with different Modulation indices

As the modulation index increases you should notice that the peaks of the high frequency get closer together and low frequency get further apart. For the same information signal therefore, the carrier signal has a higher maximum frequency.
The FM modulation index is defined as the ratio of carrier deviation to modulation frequency:
As a result, the FM equation is generally written as:

$$e_{fm} = \sin\left(\omega_c t - m_{fm} \cos \omega_m t\right)$$

This is a very complex expression and it is not readily apparent what the sidebands of this signal are like. The solution to this problem requires knowledge of Bessel's functions of the first kind and order *p*. In open form, it resembles:

$$J_p(x) = \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{x}{2}\right)^{2k+p}}{k!(k+p)!}$$

$$J_p(x) = \text{magnitude of frequency component}$$

where

$$p = \text{side frequency number}$$

$$x = \text{modulation index}$$

As a point of interest, Bessel's functions are a solution to the following equation:

$$x^2 + \frac{d^2y}{dx^2} + x\frac{dy}{dx} + \left(x^2 - p^2\right) = 0$$

Bessel's functions occur in the theory of cylindrical and spherical waves, much like sine waves occur in the theory of plane waves. It turns out that FM generates an infinite number of side frequencies. Each frequency is an integer multiple of the modulation signal. It should be noted that the amplitude of the higher order sided frequencies drops off quickly. It is also interesting to note that the amplitude of the carrier signal is also a function of the modulation index. Under some conditions, the amplitude of the carrier frequency can actually go to zero. This does not mean that the signal disappears, but rather that all of the broadcast energy is redistributed to the side frequencies.

3.12. Generation of frequency Modulation

3.12.1. Direct FM

(a) Reactance Modulator

The reactance modulator is a voltage controlled capacitor and is used to vary an oscillator's frequency or phase. A simplified circuit resembles:

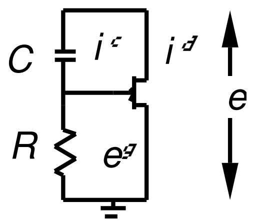


Figure 3.11.Reactance modulator

Since the gate does not draw an appreciable amount of current, applying Ohm's law in the RC branch results in:

$$e_g = i_C$$

$$i_C = \frac{e}{R - jX_C}$$

$$\therefore e_g = \frac{e}{R - jX_C} R$$

The JFET drain current is given by:

$$i_d = g_m e_g = g_m \frac{e}{R - jX_C} R$$

where g_m is the trans-conductance.

The impedance as seen from the drain to ground is given by:

$$Z = \frac{e}{i_d} = e\frac{1}{g_m}\frac{R - jX_C}{e}\frac{1}{R} = \frac{1}{g_m} - j\frac{X_C}{g_m R}$$

Since trans-conductance is normally very large, the impedance reduces to:

$$Z \approx -j\frac{X_C}{g_m R} = \frac{-j}{2\pi f C g_m R}$$

The term in the denominator can be thought of as an equivalent capacitance:

$$C_{eq} = C g_m R$$

Then

$$Z = \frac{-j}{2\pi f C_{eq}}$$

Since the equivalent capacitance is larger than the original capacitor, we have created a capacitance amplifier. Because the value of this capacitance is a function of applied voltage, we actually have a voltage controlled capacitor. This device can be used to control an oscillator frequency, thus producing FM.

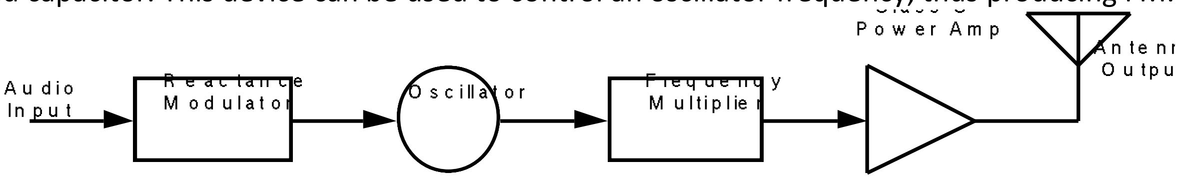


Figure 3.12.FM transmitter

Varactor Diodes

The capacitance of a varactor diode is a function of its' reverse bias voltage.

$$C_d = \frac{C_0}{\sqrt{1 + 2|V_R|}}$$

Where C_0 is the diode capacitance at zero bias, and V_R is the reverse bias voltage. A typical response is:

3.12.2. Indirect Method:

Because crystal oscillators are so stable, it is desirable to use them in modulator circuits. However, their extreme stability makes it difficult to modulate their frequency.

Fortunately, it is possible to vary the phase of a crystal oscillator. However, in order to use this as an FM source, the relationship between frequency and phase needs to be reexamined.

Frequency is the rate of change of angle, its first derivative:

$$\omega = \frac{d}{dt}\phi$$

The instantaneous phase angle is comprised of two components, the number of times the signal has gone through its cycle, and its starting point or offset:

$$\phi(t) = \underbrace{\omega_c t}_{\substack{\text{rotating}\\ \text{angle}}} + \underbrace{\theta}_{\substack{\text{offset}\\ \text{angle}}}$$

The instantaneous frequency is therefore:

$$\omega_i = \frac{d}{dt}\phi(t) = \frac{d}{dt}[\omega_c t + \theta] = \omega_c + \frac{d}{dt}\theta$$

From this we observe that the instantaneous frequency of a signal is its un-modulated frequency plus a change. This is equivalent to frequency modulation. Therefore we may write:

$$\omega_c + \frac{d}{dt}\theta = \omega_c + \omega_{eq}$$

$$\omega_{eq} = \frac{d}{dt}\theta$$

$$f_{eq} = \frac{1}{2\pi}\frac{d}{dt}\theta$$

This means that the output of a phase modulator is proportional to the equivalent frequency modulation. If the angle is proportional to the amplitude of a modulation signal $\theta = k\, e_m$, Then:

$$f_{eq} = \frac{1}{2\pi}\frac{d}{dt}ke_m$$

and by integrating the modulation signal prior to modulation, we obtain:

$$f_{eq} = \frac{1}{2\pi}\frac{d}{dt}\int ke_m dt = \frac{k}{2\pi}e_m$$

This means that the equivalent frequency modulation is directly proportional to the amplitude of a phase modulation signal if the modulation signal is integrated first.
This indirect modulation scheme is the heart of the Armstrong modulator.

3.13. Demodulation / Detector of Frequency Modulation:

3.13.1. Phase Detector [Foster-Seeley]

The Foster-Seeley detector converts the incoming frequency variation to an equivalent phase variation and then to an equivalent amplitude variation. This is accomplished by using the phase angle shift which occurs between the primary and secondary of a transformer tuned circuit.
It is important to recognize that the signal on the primary side gets to the secondary through two distinctly different paths:

- Through the transformer via the primary winding
- bypassing the primary winding and directly into the secondary center tap

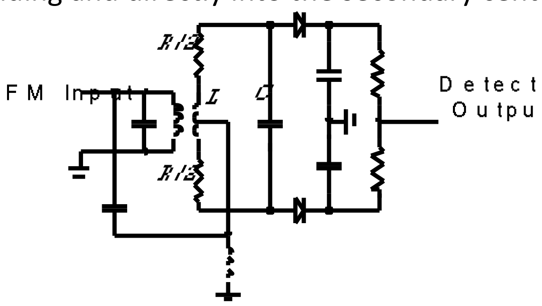


Figure 3.13.Foster seeley detector

The voltage appearing on the secondary side of the transformer is given by:

$$e_s \approx e_p k\sqrt{\frac{L_p}{L_s}}$$

$$e_p = \text{primary voltage}$$
$$k = \text{coupling coefficient}$$
$$L_p = \text{primary inductance}$$
$$L_s = \text{secondary inductance}$$

This is applied to the series resonant circuit in the transformer secondary winding. The impedance of this resonant circuit is given by:

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right) = R\left[1 + j\left(\frac{\omega L}{R} - \frac{1}{\omega RC}\right)\right]$$

$$= R\left[1 + j\left(\frac{\omega}{\omega_o}\frac{\omega_o L}{R} - \frac{\omega_o}{\omega}\frac{1}{\omega_o RC}\right)\right]$$

Where ω_O is the resonant frequency

Since, $Q = \frac{\omega_o L}{R} = \frac{1}{\omega_o RC}$, the impedance can be written as:

$$Z = R\left[1 + j\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)Q\right]$$

Defining a new parameter: $Y = \frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}$, we obtain:

$$Z = R[1 + jYQ]$$

The impedance phase angle is given by: $\varphi = \tan^{-1}(YQ)$.

It is interesting to observe what happens to this angle when the input frequency varies.

Let the input frequency be of the form: $\omega = \omega_o + \Delta\omega$, then:

$$Y = \frac{\omega_o + \Delta\omega}{\omega_o} - \frac{\omega_o}{\omega_o + \Delta\omega} = \frac{2\omega_o\Delta\omega + \Delta\omega^2}{\omega_o^{\ 2} + \omega_o\Delta\omega}$$

But if the deviation is much smaller than the carrier: $\Delta\omega << \omega_o$, then:

$$Y \approx \frac{2\Delta\omega}{\omega_o}$$

Notice that the parameter *Y* varies directly with deviation. For small angle changes $\varphi \approx \tan^{-1}\varphi$. This means that the impedance phase angle varies directly with the frequency deviation. This in turn causes a variation in the currents and voltages in the secondary.

The output of the transformer consists of the vector sum of two components:

- The phase shifted signal passing through the transformer
- The un-shifted signal which has bypassed the transformer

The combination of these two signals results in amplitude variations which are directly proportional to the frequency deviation. This AM signal is then detected through a standard envelope detector.

This circuit is quite sensitive however; any amplitude variations in the signal caused by varying signal strength are also detected.

3.13.2. Ratio Detector

This circuit is a slight modification of the Foster-Seeley detector:

- One diode is reversed
- The output is taken from the combined loads

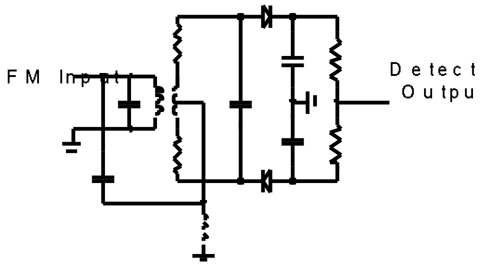


Figure 3.14.Ratio detector

The limiting action of the detector has been improved and variations in signal strength are not as noticeable. This also implies that it is less sensitive.

3.13.3. Phase Locked Loop

Although phase locked loops can be implemented using analog or digital circuitry, the following discussion will be limited to linear circuits since they are much easier to analyze.

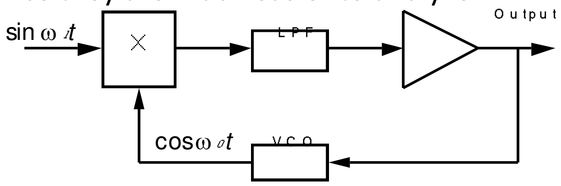


Figure 3.16.Phase lock loop

The loop achieves lock in two steps. First it acquires frequency lock, and then it acquires phase lock.
If the input signal and VCO output are completely different, the output of the multiplier is given by:

$$V_{mult} = \sin(\omega_i t)\cos(\omega_o t) = \frac{1}{2}\sin(\omega_i + \omega_o)t + \frac{1}{2}\sin(\omega_i - \omega_o)t$$

The low pass filter removes the high frequency component before passing the signal to the output amplifier. The output, which also is the input to the VCO, is of the form:

$$V_{output} = \sin(\omega_i - \omega_o)t$$

Notice that if $\omega_i > \omega_o$, the output is positive, but if $\omega_i < \omega_o$, the output is negative.
This change in polarity can be seen by observing the sine function near the origin:

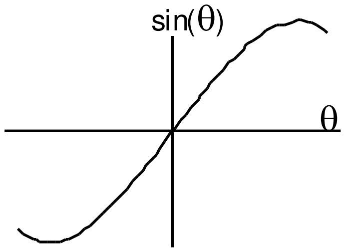


Figure 3.17.sinusoidal variation

This voltage can be used as an error signal to drive the VCO until $\omega_i = \omega_o$, and frequency lock is achieved. The two signals however, are not necessarily in phase at this point.
Once frequency lock occurs, the multiplier output becomes:

$$V_{mult} = \sin(\omega_i t)\cos(\omega_i t - \varphi) = \frac{1}{2}\sin(2\omega_i - \varphi)t + \frac{1}{2}\sin(\varphi)t$$

Again the low pass filter removes the high frequency component and the output is of the form:

$$V_{output} = \sin(\varphi)t$$

Again if $\varphi > 0$ the output is positive; but if $\varphi < 0$ the output is negative. This error signal is used to drive the VCO until $\varphi = 0$, and phase lock is achieved. Notice that at lock, the incoming signal and the VCO output are in quadrature.
A PLL can be used after the IF amplifier in a radio to reproduce the original modulation signal. This allows the free running VCO oscillator frequency to be preset, thus making it easier to acquire lock.

3.14. FM Spectrum:

When the amplitude of the frequency components of FM waveform are plotted as a function of frequency, the resulting spectrum is much more complicated than that of the AM waveform This is because there are now multiple frequencies present in the FM signal.
Theoretically, an FM spectrum has an infinite number of sidebands, spaced at multiples of f_i above and below the carrier frequency f_c. However the size and significance of these sidebands is very dependent on the modulation index, β. If β<1, then the spectrum looks like this:

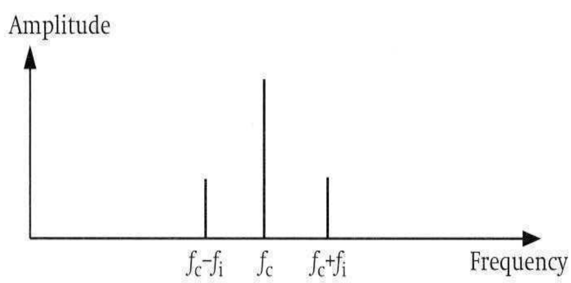


Figure 3.18.FM spectrum for β<1

From the spectrum above it can be seen that there are only two significant sidebands, and thus the spectrum looks very similar to that for an AM carrier.
Ifβ=1, then the spectrum looks like this:

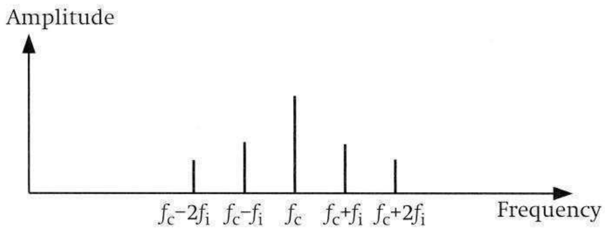


Figure 3.19.FM spectrum for β =1

From the spectrum above we can see that the number of significant sidebands has increased to four.

Ifβ=3, then the spectrum looks like this:

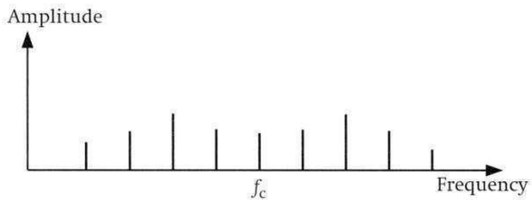


Figure 3.20.FM spectrum for β=3

From the spectrum above we can see that the number of significant sidebands has increased to eight. It can be deduced that the number of significant sidebands in an FM transmission is given by2(β+1). The implication for the bandwidth of an FM signal should now be coming clear. The practical bandwidth is going to be given by the number of significant sidebands multiplied by the width of each sideband (i.e. f_i).

$$
\begin{aligned}
Bandwidth_{FM} &= 2(\beta + 1) f_i \\
&= 2\left(\frac{\Delta f_c}{f_i} + 1\right) f_i \\
&= 2(\Delta f_c + f_i)
\end{aligned}
$$

The bandwidth of an FM waveform is therefore twice the sum of the frequency deviation and the maximum frequency in the information.

3.15. Carson rule:

It states that the bandwidth required transmitting an angle modulated wave as twice the sum of the peak frequency deviation and the highest modulating signal frequency.

Band Width = 2 [Δf + $f_{m(max)}$] Hz

Δf = frequency deviation in Hz

$f_{m(max)}$ = highest modulating signal frequency in Hz

Additional Points to remember.

- An FM transmission is a constant power wave, regardless of the information signal or modulation index, β because it is operated at constant amplitude with symmetrical changes in frequency.
- As β increases, the relative amplitude of the carrier component decreases and may become much smaller than the amplitudes of the individual sidebands. The effect of this is that a much greater proportion of the transmitted power is in the sidebands (rather than in the carrier), which is more efficient than AM.

3.15.1. Determination of Bandwidth for FM Radio

FM radio uses a modulation index, β> 1, and this is called **wideband FM.** As its name suggests the bandwidth is much larger than AM.

In national radio broadcasts using FM, the frequency deviation of the carrier Δf_c, is chosen to be 75 kHz, and the information baseband is the high fidelity range 20 Hz to 15 kHz.
Thus the modulation index, β is 5 (i.e. 75 kHz ÷ 15 kHz), and such a broadcast requires an FM signal bandwidth given by:

$$Bandwidth_{FM\ \ Radio} = 2(\Delta f_c + f_{i(\max)})$$
$$= 2(75+15)$$
$$= 180kHz$$

3.16. Advantages of FM over AM

a) The amplitude of FM is constant. Hence transmitter power remains constant in FM where as it varies in AM.
b) Since amplitude of FM is constant, the noise interference is minimum in FM. Any noise superimposing on modulated carrier can be removed with the help of amplitude limiter.
c) The depth of modulation has limitation in AM. But in FM, the depth of modulation can be increased to any value.
d) Since guard bands are provided in FM, there is less possibility of adjacent channel interference.
e) Since space waves are used for FM, the radius of propagation is limited to line of sight(LOS) . Hence it is possible to operate several independent transmitters on same frequency with minimum interference.
f) Since FM uses UHF and VHF ranges, the noise interference is minimum compared to AM which uses MF and HF ranges.

3.17. Introduction to pulse modulation systems:

Pulse modulation is “the process in which signal is transmitted by pulses (i.e., discontinuous signals) with a special technique”. The pulse modulation is classified as analog pulse modulation and digital pulse modulation. The analog pulse modulation is again classified as,
1. Pulse amplitude modulation
2. Pulse width modulation and
3. Pulse position modulation

3.17.1. Pulse Amplitude Modulation

In Pulse Amplitude Modulation (PAM) technique, the amplitude of the pulse carrier varies, which is proportional to the instantaneous amplitude of the message signal. The width and positions of the pulses are constants in this modulation.

3.17.2. Pulse Width Modulation

Pulse Width Modulation (PWM) or Pulse Duration Modulation (PDM) or Pulse Time Modulation (PTM) is an analog modulating scheme in which the duration or width or time of the pulse carrier varies proportional to the instantaneous amplitude of the message signal.

3.17.3. Pulse Position Modulation

Pulse Position Modulation (PPM) is an analog modulating scheme in which the amplitude and width of the pulses are kept constant, while the position of each pulse, with reference to the position of a reference pulse varies according to the instantaneous sampled value of the message signal.

