

# **EE101: Assignment no. 2**

Due on 4 August 2020

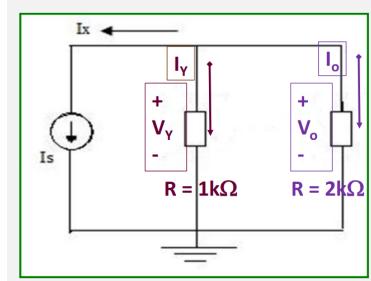
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**Pranjal**

## Problem 1

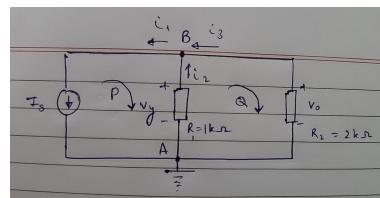
- (a) Identify number of nodes, loops in the circuit (Figure Q-01)  
 (b) Number of independent KCL, KVL equations for the circuit  
 (c) Kirchhoff's law to solve for  $I_x, I_y, I_o, V_Y, V_o$ , given  $I_s = 1mA$

Figure 1: Q-01



### Solution:

- (a) Node: point of connection between 2 or more branches.



**Number of node in this circuit (n)=2**

Number of 2 terminal elements (E)=3

Therefore **number of loops= E-n+1 =3-2+1 =2**

(b) Therefore **number of independent KCL equations= n-1 = 2-1 =1**

And **the number of independent KVL equations= E-n+1 = 3-2+1 =2**

(c) KCL @ Node B:

$$-i_1 + i_2 + i_3 = 0$$

given  $I_s = 1mA$

$$\text{hence } i_2 + i_3 = 1mA \text{ ---eq(1)}$$

Using KVL in loop Q:

$$i_3 * R_2 + (-i_2) * R_1$$

$$\text{i.e., } i_3(R_2) = i_2(R_1)$$

$$\text{i.e., } i_3(R_2) = (1mA - i_3)(R_1)$$

$$\text{i.e., } i_3 = 1mA * (R_1) / (R_1 + R_2)$$

substituting the values ,we get,

$$\text{hence } i_3 = 1 * 1 / (1+2) \text{ mA} = 0.33$$

$$i_3 = 0.33 \text{ mA}$$

hence,  $i_2 = 0.67 \text{ mA}$

$$\text{Answers: } I_x = I_s = i_1 = 1 \text{ mA}$$

$$I_y = i_2 = 0.67 \text{ mA}$$

$$I_o = i_3 = 0.33 \text{ mA}$$

$$V_Y = I_y * R_1 = (2/3) * 1 = 0.67 \text{ V}$$

$$V_o = I_o * R_2 = (1/3) * 2 = 0.67 \text{ V}$$

## Problem 2

(a) Identify number of nodes, loops in the circuit (Fig. Ques-2)

(b) Number of independent KCL, KVL equations for the circuit

(c) Kirchhoff's law to solve for  $V_o$  and  $I_o$   $V_s=4\text{V}$ ,  $R=1\text{k}\Omega$

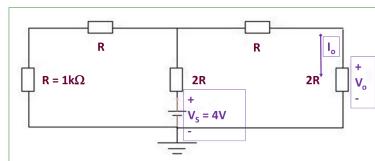


Figure 2: Ques-2

### Solution:

(a) Refer to Fig. Soln-2

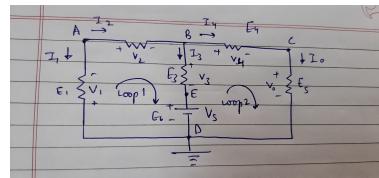


Figure 3: Soln-2

Number of elements (E) = 6 [Labelled as  $E_1, E_2, E_3, E_4, E_5$  and  $E_6$ ]

Number of nodes (N) = 5 [Labelled as A, B, C, D and E]

Number of loops (L) =  $E-N+1 = 6-5+1 = 2$

(b) Refer Fig. Soln-2

Number of branches (B) = 6

Number of Independent KCL equations =  $N-1 = 5-1 = 4$

Number of Independent KVL equations =  $B-N+1 = 6-5+1 = 2$

(c) Applying KVL on both the loops -

By KVL on Loop-1:

$$V_1 + V_2 + V_3 + V_s = 0 \quad (1)$$

By KVL on Loop-2:

$$(-V_s) + (-V_3) + V_4 + V_o = 0 \quad (2)$$

Elements:

$$-V_1 = I_1 * R \quad (3)$$

$$V_2 = I_2 * R \quad (4)$$

$$V_3 = I_3 * 2R \quad (5)$$

$$V_4 = I_4 * R \quad (6)$$

$$V_o = I_o * 2R \quad (7)$$

Applying KCL on nodes -

By KCL on Node-A:

$$I_1 + I_2 = 0 \quad (8)$$

By KCL on Node-B:

$$I_2 = I_3 + I_4 \quad (9)$$

By KCL on Node-C:

$$I_4 = I_o \quad (10)$$

Sustituting values of voltages obtained in terms of current and resistors in equation (1)

$$\begin{aligned} V_1 + V_2 + V_3 + V_s &= 0 \\ (-I_1 * R) + I_2 * R + I_3 * 2R + V_s &= 0 \\ (I_2 * R) + I_2 * R + I_3 * 2R + V_s &= 0 \\ 2 * (I_2 * R) + 2 * (I_3 * R) + V_s &= 0 \\ 2 * ((I_3 + I_4) * R) + 2 * (I_3 * R) + V_s &= 0 \\ 2 * ((I_3 + I_o) * R) + 2 * (I_3 * R) + V_s &= 0 \\ 4 * (I_3 * R) + 2 * (I_o * R) + V_s &= 0 \end{aligned} \quad (11)$$

Sustituting values of voltages obtained in terms of current and resistors in equation (2)

$$\begin{aligned} (-V_s) + (-V_3) + V_4 + V_o &= 0 \\ -V_s - (I_3 * 2R) + (I_4 * R) + (I_o * 2R) &= 0 \\ -V_s - (I_3 * 2R) + (I_o * R) + 2 * (I_o * R) &= 0 \\ -V_s - 2 * (I_3 * R) + 3 * (I_o * R) &= 0 \\ -V_s + 3 * (I_o * R) &= 2 * (I_3 * R) \end{aligned} \quad (12)$$

Now substituting value from equation (11) into equation (12)

$$\begin{aligned}
 2 * (2 * (I_3 * R)) + 2 * (I_o * R) + V_s &= 0 \\
 2 * (-V_s + 3 * (I_o * R)) + 2 * (I_o * R) + V_s &= 0 \\
 -2 * V_s + 6 * (I_o * R) + 2 * (I_o * R) + V_s &= 0 \\
 8 * (I_o * R) - V_s &= 0 \\
 8 * (I_o * R) &= V_s \\
 I_o &= \frac{V_s}{8 * R} \\
 I_o &= \frac{4V}{8 * 1000\Omega} \\
 I_o &= \frac{0.5}{1000} A \\
 I_o &= 0.5mA
 \end{aligned}
 \tag{13}$$

As we know relation between  $I_o$  and  $V_o$  by equation (7), we calculate  $V_o$

$$\begin{aligned}
 V_o &= I_o * 2R \\
 V_o &= (0.5mA) * (2 * 1000\Omega) \\
 V_o &= 1V
 \end{aligned}
 \tag{14}$$

### Problem 3

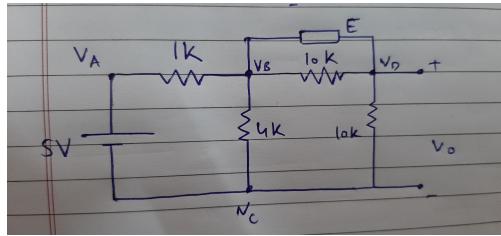


Figure 4: Q3 circuit

In order to solve for the output voltage using N-V analysis, we must convert the voltage source into an equivalent current source using source transformation.  $i_s * R_s = V_s$  and  $R_s$  is added parallel to the current source. This replaces the part of the circuit comprising the voltage  $V_s = 5V$  and the resistance  $R_s = 10K$  connected in series. Therefore, here,  $i_s = 5mA$  and  $R_s = 10K$ .

The circuit transforms into the following -

First of all, the nodes have to be identified. There are 3 nodes in this circuit marked as A,B and C. We know that there are  $N-1$  independent equations in node voltage analysis in case of  $N$  nodes. Therefore, here we will have 2 independent equations and the node voltage matrices will also be of order = 2.

Let us choose the nodes B and C for our analysis as we are interested in obtaining the final output voltage that is

$$V_o = V_b - V_c \tag{15}$$

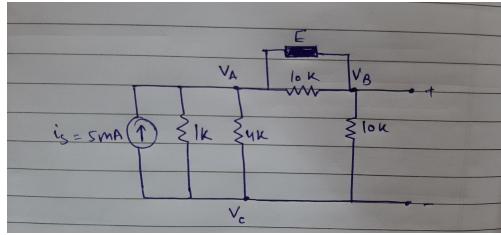


Figure 5: Q3 circuit after source transformation

Following the rules according to which, L.H.S =  $\sum G_x * V_x$  with  $V_x$  signifies the positive voltage for node x and negative for the nodes other than the node for which the equation is being written, under consideration and G is the conductance of all branches connected to  $V_x$  and the node for which the equation is being written and R.H.S =  $i_s$  if  $i_s$  enters the node, R.H.S = -  $i_s$  if  $i_s$  exits the node and 0 otherwise.

Let  $R = 1K$  and  $G = 1/R$ .

The following equations are obtained on analysing the circuit in case 1 when E is a 10 K  $\Omega$  resistor.  
For node C;

$$V_C * G * (0.1 + 0.25 + 1) - V_B * G * (0.1) = -i_s \quad (16)$$

For node B;

$$-V_C * G * (0.1) + V_B * G * (0.1 + 0.1 + 0.1) = 0 \quad (17)$$

After multiplying both equations by R on both sides, the N-V matrices are as follows:

$$\begin{bmatrix} 1.35 & -0.1 \\ -0.1 & 0.3 \end{bmatrix} \begin{bmatrix} V_C \\ V_B \end{bmatrix} = \begin{bmatrix} -i_s * R \\ 0 \end{bmatrix} \quad (18)$$

Therefore the voltages  $V_B$  and  $V_C$  come out to be

$$V_C = \frac{\begin{vmatrix} -i_s * R & -0.1 \\ 0 & 0.3 \end{vmatrix}}{\begin{vmatrix} 1.35 & -0.1 \\ -0.1 & 0.3 \end{vmatrix}} \quad (19)$$

$$V_B = \frac{\begin{vmatrix} 1.35 & -i_s * R \\ -0.1 & 0 \end{vmatrix}}{\begin{vmatrix} 1.35 & -0.1 \\ -0.1 & 0.3 \end{vmatrix}} \quad (20)$$

Thus,

$$V_B = -1.26V \quad (21)$$

$$V_C = -3.78V \quad (22)$$

Thus by equation 15,

$$V_o = 2.53V. \quad ('Answer for Q3 (a)')$$

The following equations are obtained on analysing the circuit in case 1 when E is a  $i_x = 4mA$  independent current source. For node C;

$$V_C * G * (0.1 + 0.25 + 1) - V_B * G * (0.1) = -i_s \quad (23)$$

For node B;

$$-V_C * G * (0.1) + V_B * G * (0.1 + 0.1) = -i_x \quad (24)$$

After multiplying both equations by R on both sides, the N-V matrices are as follows:

$$\begin{bmatrix} 1.35 & -0.1 \\ -0.1 & 0.2 \end{bmatrix} \begin{bmatrix} V_C \\ V_B \end{bmatrix} = \begin{bmatrix} -i_s * R \\ -i_x * R \end{bmatrix} \quad (25)$$

Putting the values of  $i_s * R = 5V$  and  $i_x * R = 4V$ ; the voltages  $V_B$  and  $V_C$  come out to be

$$V_C = \frac{\begin{vmatrix} -5 & -0.1 \\ -4 & 0.2 \end{vmatrix}}{\begin{vmatrix} 1.35 & -0.1 \\ -0.1 & 0.2 \end{vmatrix}} \quad (26)$$

$$V_B = \frac{\begin{vmatrix} 1.35 & -5 \\ -0.1 & -4 \end{vmatrix}}{\begin{vmatrix} 1.35 & -0.1 \\ -0.1 & 0.2 \end{vmatrix}} \quad (27)$$

Thus,

$$V_B = -22.69V \quad (28)$$

$$V_C = -5.38V \quad (29)$$

Thus by equation 15,

$$V_o = -17.31V. \quad ('Answer for Q3 (b)')$$

## Problem 4

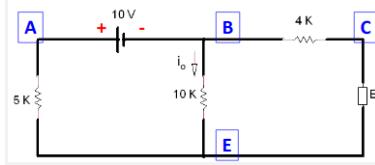
Use Mesh-Current (M-C) analysis to find current  $i_o$  in Fig. Q-4 when the element E is of two types:

(a) 5-V source (positive reference at Node C), and (b) 10-kΩresistor.

(ii) Identify number of independent (M-C) equations

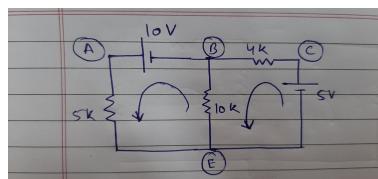
(iii) Create mesh-current matrix and determine mesh currents

Figure 6: Q-4

**Solution:**

there are a total of 2 mesh current equations.

a) given: E is a **5-V source**.



here left left loop is referred to as the first one.

$$\begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad (30)$$

where

$$R_{11} = 10 + 5 = 15\Omega$$

$$R_{22} = 10 + 4 = 14\Omega$$

$$R_{12} = R_{21} = -10\Omega$$

$$v_1 = +10V$$

$$v_2 = 5V$$

$$\begin{bmatrix} 15 & -10 \\ -10 & 14 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

multiplying by inverse of impedance matrix on both sides, we get :

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 15 & -10 \\ -10 & 14 \end{bmatrix}^{-1} * \begin{bmatrix} 10 \\ 5 \end{bmatrix} = \begin{bmatrix} 7/55 & 1/11 \\ 1/11 & 3/22 \end{bmatrix} * \begin{bmatrix} 10 \\ 5 \end{bmatrix} = \begin{bmatrix} \frac{19}{11} \\ \frac{35}{22} \end{bmatrix}$$

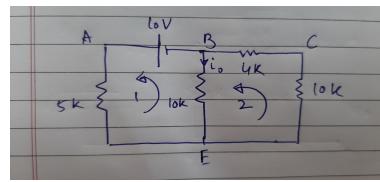
therefore:  $i_1 = 19/11mA$ ;  $i_2 = 35/22mA$

the current  $i_o = i_2 - i_1 = -3/22 = -0.136mA$

b) given: E is a **10-k resistor**.

here left left loop is referred to as the first one.

$$\begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad (31)$$



where

$$R_{11} = 10 + 5 = 15\Omega$$

$$R_{22} = 10 + 10 + 4 = 24\Omega$$

$$R_{12} = R_{21} = -10\Omega$$

$$v_1 = +10V$$

$$v_2 = 0V$$

$$\begin{bmatrix} 15 & -10 \\ -10 & 24 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

multiplying by inverse of impedance matrix on both sides, we get :

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 15 & -10 \\ -10 & 24 \end{bmatrix}^{-1} * \begin{bmatrix} 10 \\ 0 \end{bmatrix} = \begin{bmatrix} 6/65 & 1/26 \\ 1/26 & 3/52 \end{bmatrix} * \begin{bmatrix} 10 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{12}{13} \\ \frac{5}{13} \end{bmatrix}$$

therefore:  $i_1 = 12/13mA$ ;  $i_2 = 5/13mA$

the current  $i_o = i_2 - i_1 = -7/13 = -0.538mA$

## Problem 5

The circuit shown below has three parts I, II and III. Part I is the practical voltage source circuit, Part II is the practical current source circuit and Part III is the load resistor circuit.

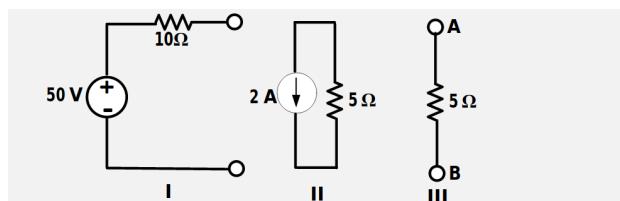


Figure 7: ques 5

- (a) Illustrate Part I, II and III graphically.
- (b) Connect Part I and Part II and use source transformation technique to generate an equivalent practical voltage source circuit.
- (c) Connect the load resistor (Part III) circuit and determine the signal levels at the load(voltage, current and

power).

**Solution:(a)**

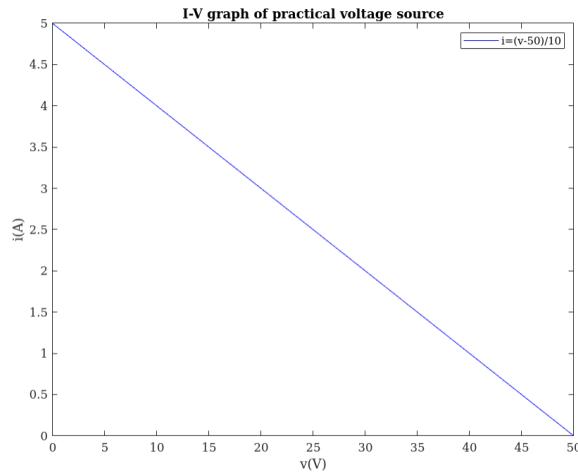


Figure 8: Part 1

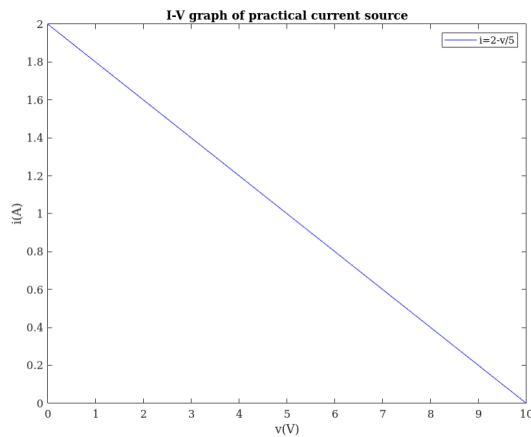


Figure 9: Part 2

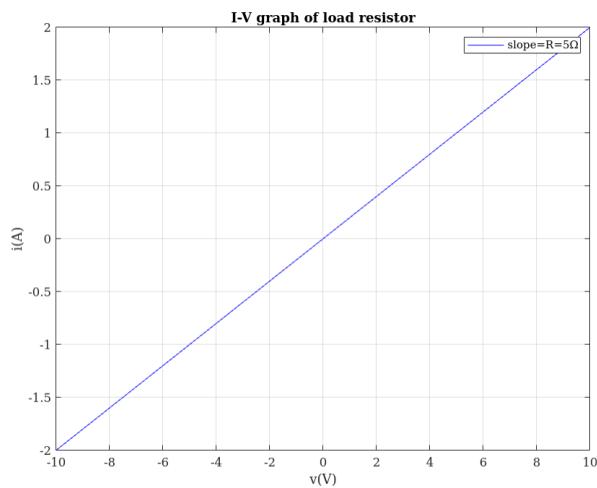


Figure 10: Part 3

(b) by transforming the source circuit into its Thévenin or Norton equivalent circuit

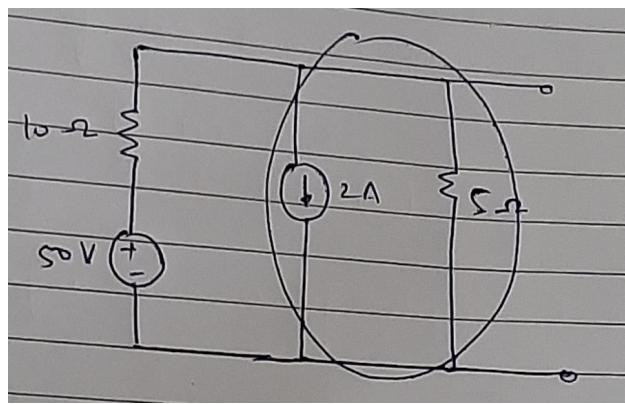


Figure 11: Step 1

Source transformation of voltage source to current source

$$i_c = V_s/R - s$$

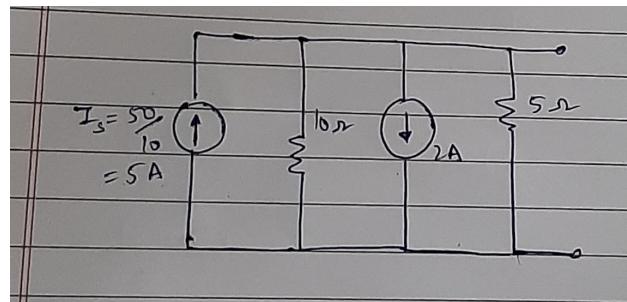


Figure 12: Step 2

Combining both current sources; and resistances in parallel

$$R_e = R_1 * R_2 / (R_1 + R_2) = 5 * 10 / 15 = 10/3 \text{ ohm}$$

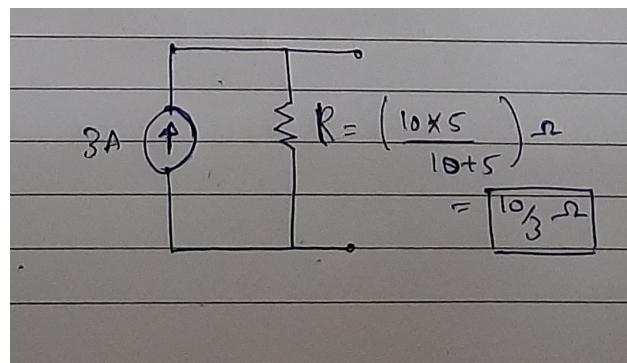


Figure 13: Step 3

Source transformation of current source to voltage source

$$V = I_s * R_s = 3 * 10/3 = 3V$$

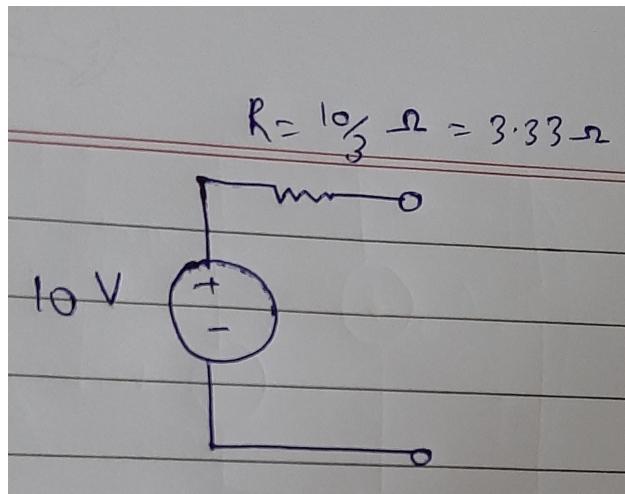


Figure 14: Step 4

(c)

by transforming the source circuit into its Thévenin or Norton equivalent circuit

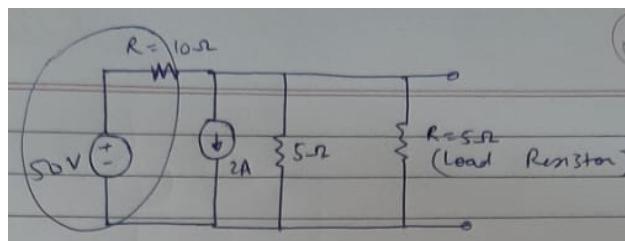


Figure 15: Step 1

Source transformation of voltage source to current source

$$i_c = V_s/R - s$$

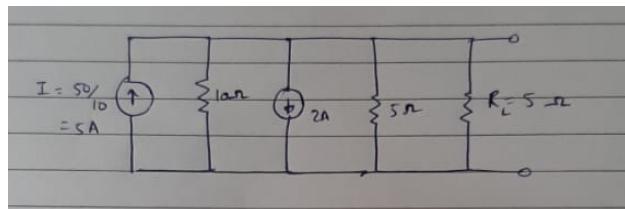


Figure 16: Step 2

Combining both current sources; and resistances in parallel

$$R_e = R_1 * R_2 / (R_1 + R_2) = 5 * 10 / 15 = 10 / 3 \text{ ohm}$$

Source transformation of current source to voltage source

$$V = I_s * R_s = 3 * 10 / 3 = 3V$$

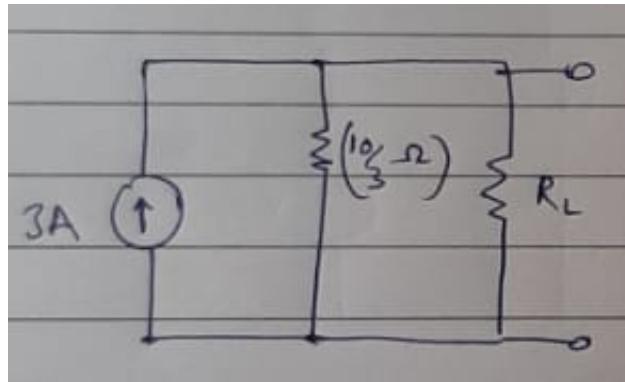


Figure 17: Step 3

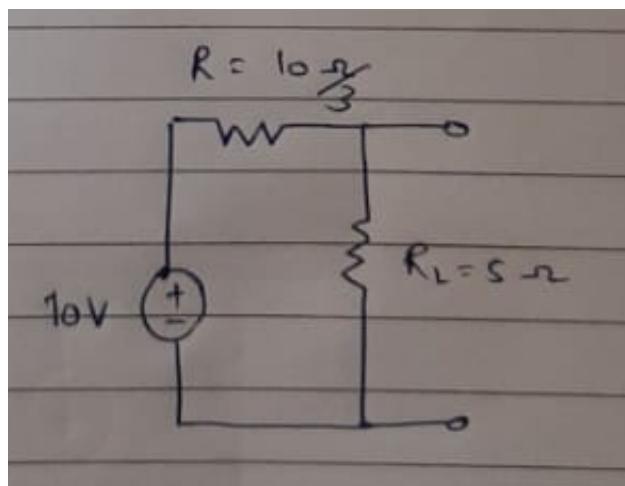


Figure 18: Step 4

therefore;

$$I_L = V/(R + R_L)$$

$$\text{i.e., } I_L = 10/(5+10/3) \text{ A} = 1.2 \text{ A}$$

and

$$V_L = I_L * R_L = 1.2 * 5 \text{ V} = 6 \text{ V}$$

## Problem 6

(a) In circuit shown below, determine Thévenin equivalent ( $V_T, R_T$ ) of the circuit using Node-Voltage (N-V) analysis.

(b) Connect a load resistor of 50 across the terminals 'ab' and determine the signal levels at the load (voltage, current and power).

**Solution:** (a)

$$i_4 + i_3 = i_1 \quad i_1 = i_2$$

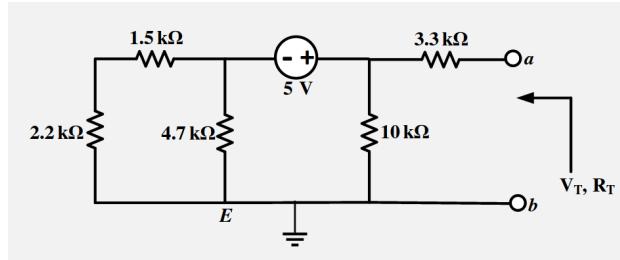


Figure 19: Ques-6

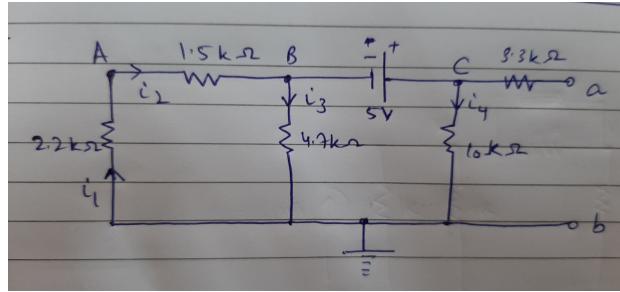


Figure 20: Soln-6

$$G = 1/R, \text{ where } R = 1\text{k}\Omega$$

$$\begin{aligned} -V_c \times G/10 &= i_4 \\ -V_B \times G/4.7 &= i_e \\ (V_B - V_A)G/1.5 &= i_2 \\ \frac{V_A G}{2.2} &= i_1 \end{aligned}$$

Combining these equations, we will get,

$$\begin{aligned} 3.7V_A - 2.2V_B &= 0 \\ 69.05V_B - 47V_A &= -35.25 \end{aligned}$$

$$\begin{aligned} \begin{bmatrix} 3.7 & -2.2 \\ 47 & -69.05 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} &= \begin{bmatrix} 0 \\ 35.25 \end{bmatrix} \\ V_B &= \frac{\begin{vmatrix} 3.7 & 0 \\ 47 & 35.25 \end{vmatrix}}{\begin{vmatrix} 3.7 & -2.2 \\ 47 & -69.05 \end{vmatrix}} = -0.857 \\ V_T &= V_C = 4.14V \end{aligned}$$

$$R_T = 5.01\text{k}\Omega$$

(b) Using backward method ie short circuiting the voltage source

Applying Thevenin theorem,

$$\begin{aligned} I_L &= \frac{V_T}{R_T + R_L} \\ &= \frac{4.14}{5.01 + 0.05} \times 10^{-3} \text{mA} \\ &= 0.81 \text{mA} \end{aligned}$$

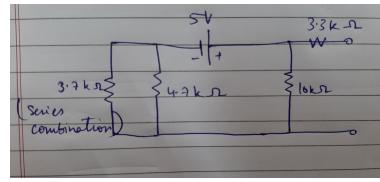


Figure 21: Series calculation of 2.2kΩ and 1.5kΩ

$$\begin{aligned} V_L &= I_L R_L \\ &= 0.81 \times 50 \times 10^{-3} V \\ &= 0.04 V \end{aligned}$$

## Problem 7

(a) The number of circuit elements in this circuit(E) is 6 which includes resistors of  $20\Omega$ ,  $20\Omega$ ,  $40\Omega$ ,  $30\Omega$ ,  $60\Omega$  and a voltage source of  $50V$  and the number of nodes(N) is 4 which have been marked as A, B, C and D. Therefore, the number of mesh equations are  $E - N + 1 = 6 - 4 + 1 = 3$ .

There are 3 independent mesh equations.

(b) Considering the meshes A, B and C for mesh current analysis, we get the following equations using the rules according to which the L.H.S. is equal to  $\sum R_x * i_x$  where  $i_x$  is positive for the mesh under consideration and negative for the circuited), the mesh analysis matrices are as follows : -

$$\begin{bmatrix} 80 & -40 & 0 \\ -40 & 70 & -30 \\ 0 & -30 & 90 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} = \begin{bmatrix} 0 \\ -50 \\ 0 \end{bmatrix} \quad (32)$$

We are interested in finding the current  $i_C$  which can be calculated as follows :-

$$i_C = \frac{\begin{vmatrix} 80 & -40 & 0 \\ -40 & 70 & -50 \\ 0 & -30 & 0 \end{vmatrix}}{\begin{vmatrix} 80 & -40 & 0 \\ -40 & 70 & -30 \\ 0 & -30 & 90 \end{vmatrix}} \quad (33)$$

Therefore,  $i_C = -0.42A$ . The norton current is  $-0.42A$ .

(c) When ab is an open circuit, the number of circuit elements (E) reduce to 5 and the number of nodes(N) remains equal to 4 in number . So, the number of independent mesh equations changes to  $E - N + 1 = 5 - 4 + 1 = 2$ . The mesh analysis matrices are as follows :-

$$\begin{bmatrix} 80 & -40 \\ -40 & 70 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} 0 \\ -50 \end{bmatrix} \quad (34)$$

We are interested in finding  $i_B$ . This can be calculated as follows :-

$$i_B = \frac{\begin{vmatrix} 80 & 0 \\ -40 & -50 \end{vmatrix}}{\begin{vmatrix} 80 & -40 \\ -40 & 70 \end{vmatrix}} \quad (35)$$

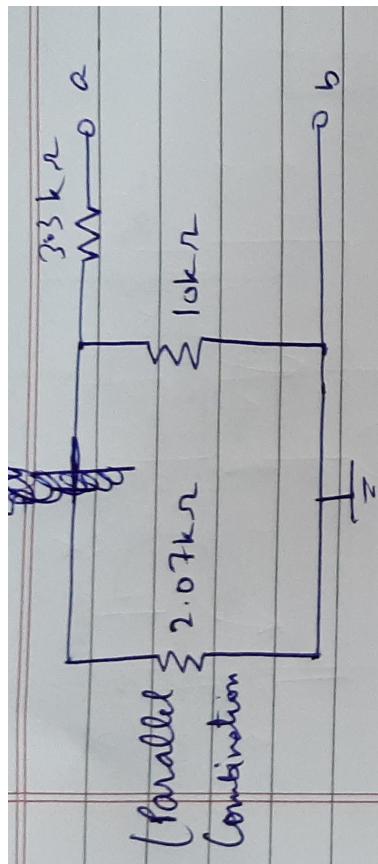


Figure 22: Parallel calculation of  $3.7\text{k}\Omega$  and  $4.7\text{k}\Omega$

Therefore  $V_{ab} = i_B * 40 = 30V$  by Ohm's law. The thevenin voltage is 30V

(d) The norton resistance is given as the thevenin voltage divided by the norton current  $= 30/0.42 = 72\Omega$ . The norton resistance is  $72\Omega$ . The norton equivalent circuit is as follows :-

(e) In order to determine the resistance by lookback method ( as there are no active current or voltage sources ); we can short circuit the voltage source (equivalent to "switching off" a voltage source ) and find out the resistance across ab in the resulting circuit. This should be equal to the norton resistance found in part (d). The  $20\Omega$  resistances in mesh A are in series which is equivalent to a  $40\Omega$  resistance which is in parallel with the  $40\Omega$  resistance between B and D. This gives an equivalent of  $\frac{40 * 40}{40 + 40} = 20\Omega$  which is then in parallel with  $30\Omega$  resistance between C and D which gives an equivalent of  $\frac{20 * 30}{20 + 30} = 12\Omega$ . This is finally in series with the resistance  $60\Omega$  between C and a which gives an equivalent of  $72\Omega$ .

(f) Using current division method, according to which current

$$\text{through resistance } R_x : -i_x = \frac{R_y * i_t}{R_x + R_y}$$

where  $i_t$  is the total current through  $R_x$  and  $R_y$ . Here  $i_t = 0.42$ . Therefore, for resistance of  $50\Omega$ , the current through it is  $\frac{72 * 0.42}{50 + 72} = 0.245A$ .

The voltage across the load resistance of  $50\Omega$  will then be equal to  $V_x = i_x * R_x = 0.245 * 5 = 12.2750$  by Ohm's Law. The power across the load is equal to  $P_x = V_x * i_x = 12.275 * 0.245 = 3W$

Current through load = 0.245 A

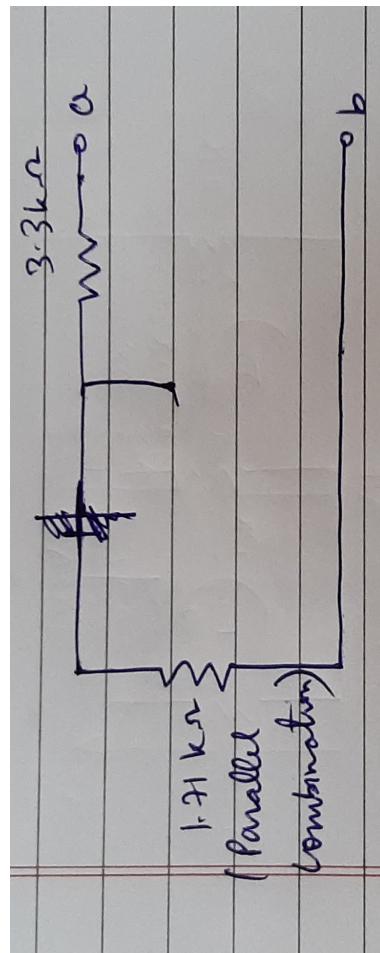


Figure 23: Parallel calculation of  $2.07\text{k}\Omega$  and  $10\text{k}\Omega$

Voltage across load =  $12.275\text{ V}$

Load power =  $3\text{ W}$

## Problem 8

Using super position principle for the circuit shown below ,determine voltage across  $2.5\text{k}\Omega$  resistor. Hint:apply mesh analysis

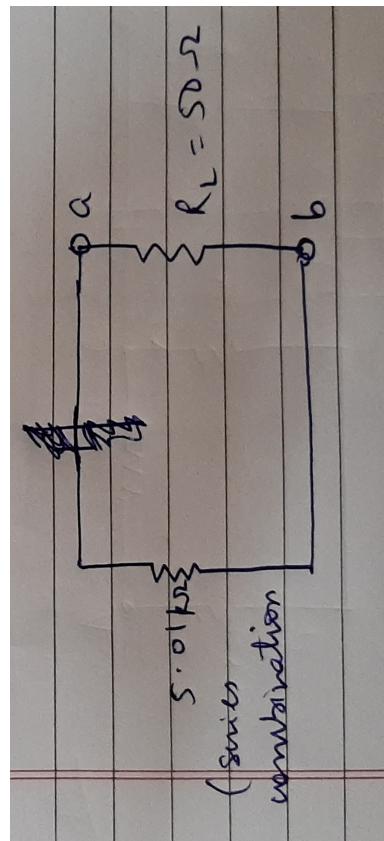
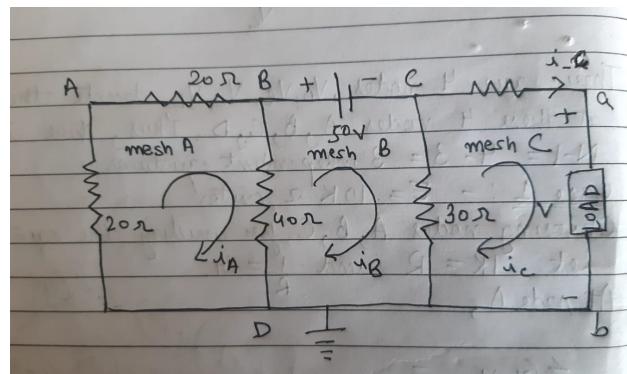
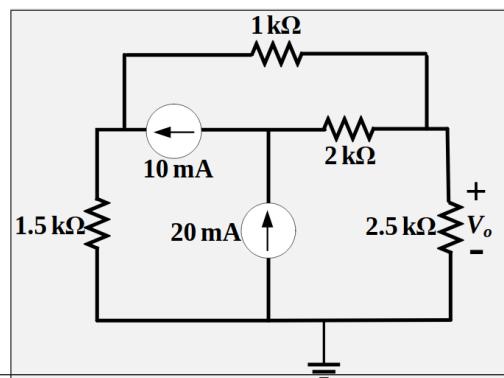
Figure 24: Series calculation of  $1.71\text{k}\Omega$  and  $3.3\text{k}\Omega$ 

Figure 25: Q7



Problem 8 continued on next page...

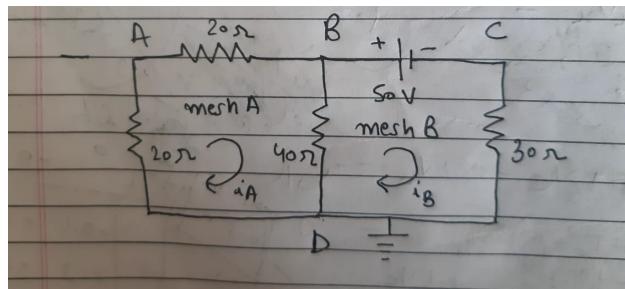


Figure 26: Open circuit at ab

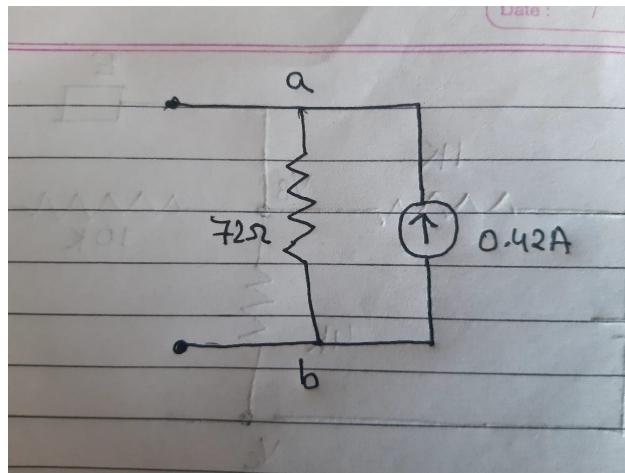


Figure 27: Norton equivalent circuit

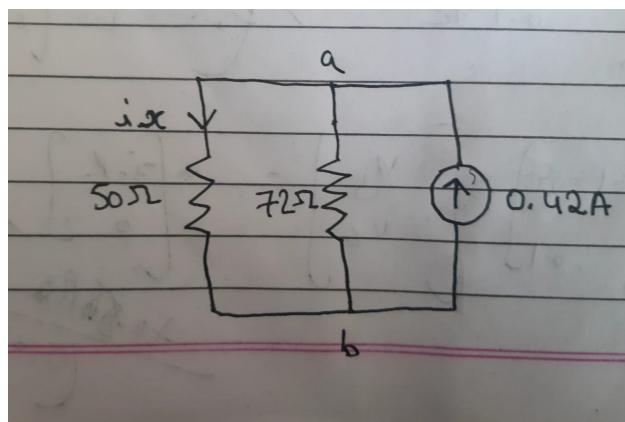


Figure 28: 50Ω load across ab

**solution:** to use super position principle to find the total voltage drop across load resistance, we replace the constant current source by an open circuit one by one, and add the effective voltage drop in the two cases.

case1: the current source pointing left is replaced by an open circuit.  
the circuit obtained by the replacement is:

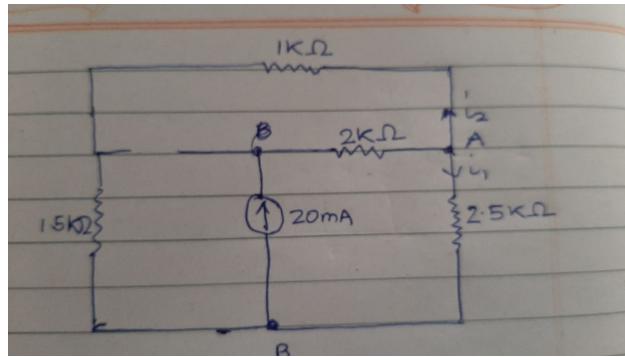


Figure 30: ques 8-a

Using KCL at node A,  $i_1 + i_2 = i_o$

Voltage Difference between node A and node B:  $V_A - V_B = (1.5k + 1k)\Omega * i_1 = 2.5k\Omega * i_2$

$$\rightarrow i_1 = i_2 = i_o/2$$

$$\text{Voltage across } 2.5k \text{ resistor} = V_1 = i_o/2 * 2.5k\Omega = 25V$$

case2: the current source pointing upwards is replaced by an open circuit.  
the circuit obtained by this is:

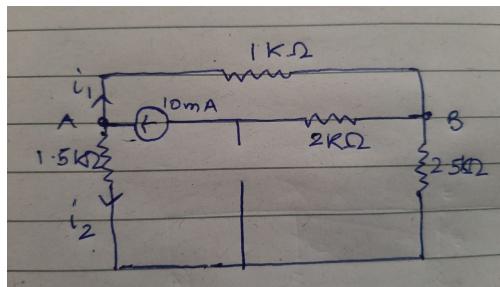


Figure 31: ques 8-b

Using KCL at node A:  $i_1 + i_2 = i_o$

according to KVL between nodes A and B:  $V_A - V_B = (1.5k + 2.5k)\Omega * i_1 + 1k\Omega * i_2 \rightarrow i_1 = i_2/4 = i_o/5$   
voltage drop across  $2.5k\Omega$  is  $i_o/5 * 2.5k\Omega = 5V$  which is in the reverse direction of the one given in the problem. therefore the drop to be considered is  $-5V$ .

Combining the results from the two cases shown above using superposition, we get:  
the total drop obtained is  $25V + (-5V) = 20V$

## Problem 9

A practical voltage source circuit connects to variable load resistor  $R_L$

- a) Determine expression for signal levels at the load (voltage, current, power).
- b) Determine expression for maximum current and voltage at the load.
- c) Derive the maximum power condition
- d) Assuming (c) is satisfied, determine signal levels (current, voltage, power) at the load for  $V_T=2.5V$ ,  $R_T=60\Omega$ .
- e) Plot the graphs normalized output voltage( $V/V_{(oc)}$ ), output current( $I/I_{(sc)}$ ), and output power( $P_o/P_{(max)}$ )

versus normalized resistance ( $R_L/R_S$ )

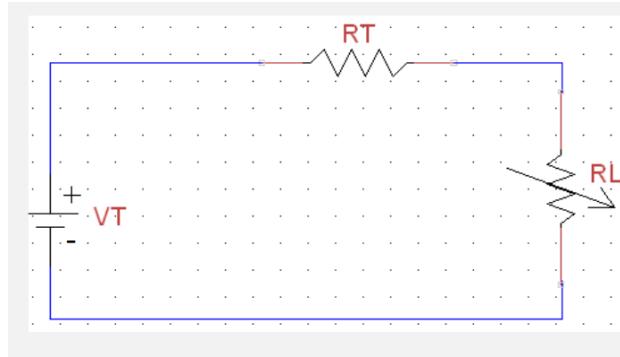


Figure 32: ques 9

**Solution:**

(a)

let  $i_L$  and  $V_L$  be the current and voltage across the load respectively

$$i_L = V_T / (R_T + R_L)$$

$$V_L = i_L * R_L = V_T * R_L / (R_T + R_L)$$

$$\text{power} = (w_L = V_L * i_L = (V_T / (R_L + R_T))^2 * R_L)$$

(b)

$$i_L = V_T / (R_T + R_L)$$

hence; as  $R_L$  decreases  $i_L$  increases

therefore maximum value of  $i_L$  occurs when  $R_L \rightarrow 0$

$$i_{L\max} = V_T / R_T$$

$$V_L = V_T / (1 + R_T / R_L)$$

hence as  $R_L$  increases  $V_T$  also increases

maximum value of  $V_L$  obtained when  $R_L \rightarrow \infty$

$$V_{L\max} = \lim(R_L \rightarrow \infty) [V_T / (1 + R_T / R_L)] = V_T$$

(c)

for maximum power:

derivative of  $w_L$  w.r.t  $R_L$  should be 0

$$\Rightarrow (V_T / R_L + R_T)^2 + (V_T)^2 / (R_L + R_T)^3 * R_L (-2) = 0$$

$$\Rightarrow 2 * R_L = R_T + R_L$$

$$\Rightarrow R_L = R_T$$

(d)

given (c) is satisfied ;  $\Rightarrow R_L = R_T$

therefore ;

$$V_L = V_T * R_L / (R_T + R_L) = V_T / 2$$

$$\Rightarrow V_L = 2.5 / 2 \text{ V} = 1.25 \text{ V}$$

therefore ;

$$i_L = V_T / (R_T + R_L) = V_T / (2 * R_T)$$

$$\Rightarrow V_L = 2.5 / (2 * 60) = 20.8 \text{ mA}$$

therefore ;

$$w_L = (V_T / R_L + R_T)^2 * R_L = (V_T)^2 / (4 * R_T)$$

$$\Rightarrow V_L = (2.5)^2 / (4 * 60) = 26.04 \text{ mW}$$

(e)

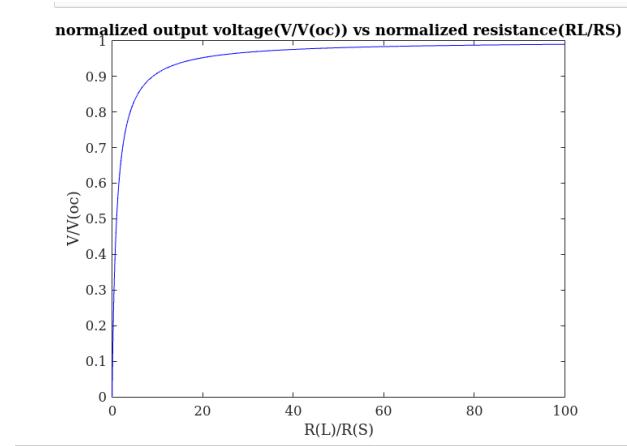
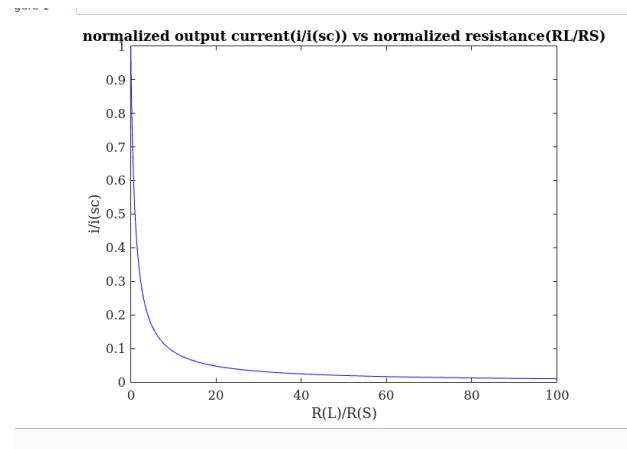
$$V_{(oc)} = V_T$$

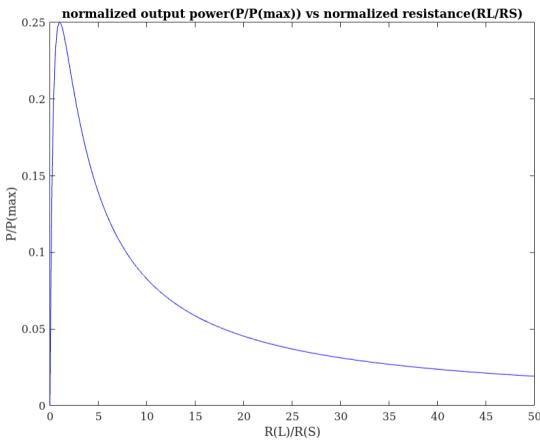
$$V(\text{scaled}) = V_L / V_T = R_L / (R_L + R_T) = (R_L / R_T) / (1 + R_L / R_T)$$

$$i_{(sc)} = V_T / R_T$$

$$i(\text{scaled}) = i_L / i_{(sc)} = R_T / (R_L + R_T) = 1 / (1 + R_L / R_T)$$

$$w(\text{scaled}) = i(\text{scaled}) * V(\text{scaled}) = (R_L / R_T) / (1 + R_L / R_T)^2$$





## Problem 10

- A practical current source circuit ( $I_s=300mA, R_s=50\Omega$ ) is connected to a variable load resistor  $R_L=200\Omega$ . Design an interface circuit, 5V is delivered to the load.
- If the interface is a series resistor as in 'a' section of figure below slide, design the interface to maintain the desired voltage.
  - If the interface is a parallel (shunt) resistor as in 'b' section of figure in below slide, design the interface to maintain the desired voltage.
  - From part (a) and (b), calculate the power consumed at the (i) interface ( $X, X'$ ) and at (ii) load terminals ( $Y, Y'$ ).

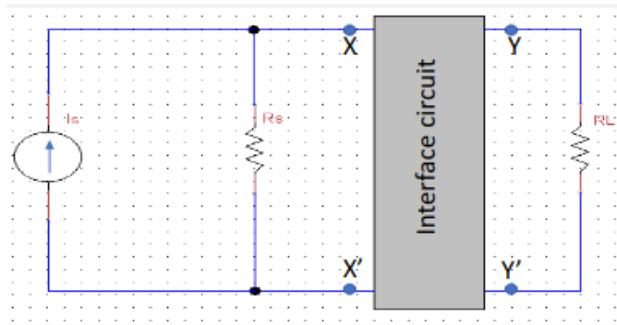


Figure 33: question 10

### Solution

a)

given: the interface is a resistor in parallel.

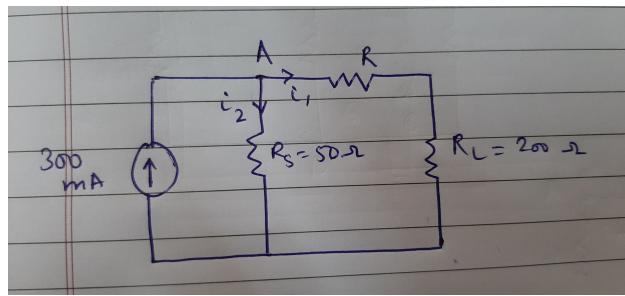


Figure 34: question 10: a

the required voltage across the  $R_L$  is 5V. By ohms law, we get, the current through the resistor is  $5 \frac{mA}{200=25} = 25mA$ . Using KCL at node A, we get :

$$i_1 + i_2 = i_o = 300mA$$

$$\rightarrow i_2 = 275mA$$

the current through the  $R_s$  is thus 275mA also, by KVL

$$Ri_1 + R_L i_1 - R_s i_2 = 0$$

$$R = \frac{R_s i_2 - R_L i_1}{i_1} = R_s * \frac{i_2}{i_1} - R_L = 350\Omega$$

b)

given: The resistance R is in parallel to  $R_L$ :

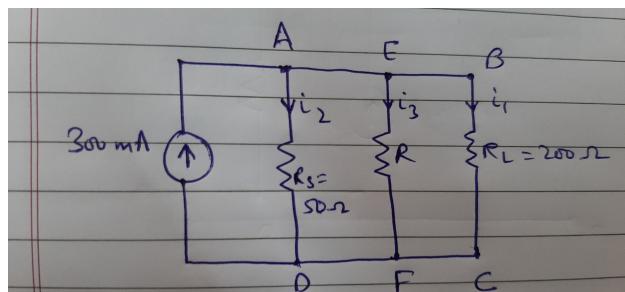


Figure 35: question 10: b

the required voltage across the  $R_L$  is 5V. By ohms law, we get, the current through the resistor is  $\frac{5}{200}=25mA$ . Using KVL in loop ABCDA, we get  $i_1 R_L - i_2 R_s = 0$

$$\rightarrow i_2 = 100mA$$

Also, using KCL on node A, we get  $i_1 + i_2 + i_3 = 300mA \rightarrow i_3 = 175mA$

finally, using KVL around loop AEFDA,  $i_3 R - i_2 R_s = 0 \rightarrow R = 200/7\Omega = 28.57\Omega$

c)

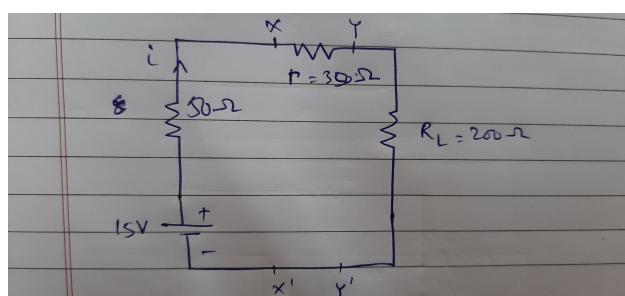


Figure 36: question 10: c

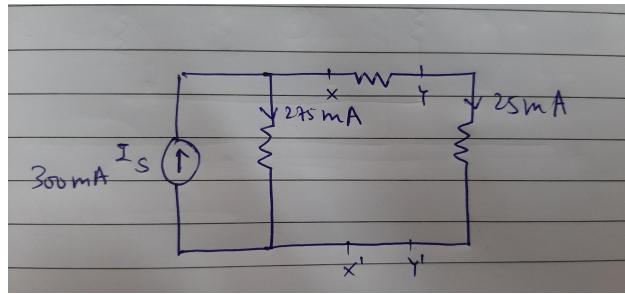


Figure 37: question 10: c

Since voltage across load resistor is 5V, so current across it is-

$$i_L = \frac{5V}{200\Omega}$$

$$i_L = 25\text{mA}$$

$$P_{YY'} = \frac{V^2}{R_L}$$

$$P_{YY'} = \frac{5^2}{200}$$

$$P_{YY'} = \frac{125}{1000}$$

$$P_{YY'} = 125\text{mW}$$

$P_{XX'}$  for (a) part:

$$P_{XX'} = (i_L^2) * (R + R_L)$$

$$P_{XX'} = (25\text{mA}^2) * (350\Omega + 200\Omega)$$

$$P_{XX'} = 625 * 10^{-6} * 550$$

$$P_{XX'} = 343.75\text{mW}$$

$$P_{XX'} = 344\text{mW}$$

$P_{XX'}$  for (b) part:

$$P_{XX'} = \frac{V^2}{R_L} + \frac{V^2}{R}$$

$$P_{XX'} = \frac{5^2}{200} + \frac{5^2}{28.57}$$

$$P_{XX'} = 0.125 + 0.875$$

$$P_{XX'} = 1\text{W}$$