INDIAN INSTITUTE OF TECHNOLOGY ROORKEE

EEN-300 INDUSTRY ORIENTED PROBLEM

MID Term Evaluation – WriteUp

C++ Implementation of various Matrix Operations as a Template Library

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ACCOMPLISHMENTS

Three algorithms for matrix multiplication and three algorithms for matrix inversion have been completely implemented and their run times have been analysed.
All algorithms have been written in C++, and are standalone, meaning that they can be ran individually.

1. MATRIX MULTIPLICATION

- Brute Force Multiplication.
- Strassen's Multiplication.
- Matrix Exponentiation (positive powers)
- Algorithm for positive fractional powers.

Brute Force Multiplication

The brute force multiplication of a matrix is the algorithm that follows the definition of matrix multiplication.

Following is the implementation of the brute force approach.

```
#include <bits/stdc++.h>

using namespace std;

#define ll long long
#define N 10004

ll a[N][N], b[N][N], c[N][N] = {0};

int main(){
     int n, m1, m2, m, p;
     cout<<"Enter dimensions\n";
     cin>>n>>m1>>m2>>p;
     if(m1 != m2){
          cout<<"Matrix multiplication not possible!\n";
          return 0;
     }
     m = m1;
     cout<<"Enter Matrices in order, row first\n";
     int i, j, k;
     for(i=1; i<=n; i++){
          for(j=1; j<=m; j++){
```

```
                cin>>a[i][j];
            }
        }
        for(i=1; i<=m; i++){
            for(j=1; j<=p; j++){
                cin>>b[i][j];
            }
        }
        for(i=1; i<=n; i++){
            for(j=1; j<=m; j++){
                for(k=1; k<=p; k++){
                    c[i][k] = c[i][k] + a[i][j]*b[j][k];
                }
            }
        }
        cout<<"Multiplication result\n";
        for(i=1; i<=n; i++){
            for(j=1; j<=p; j++){
                cout<<c[i][j]<<" ";
            }
            cout<<"\n";
        }
        return 0;
}
```

The implementation contains taking input from user, and using the following code, run times can be clocked.

```
int seeds[] = {41661, 82263, 31243, 70828, 46412, 25622, 67921,
74947, 78627, 84862, 66061, 70145, 95453, 43355, 37710, 99532,
59418, 11642, 36212, 82328}; // random seeds for rand() function.

clock_t clk;
srand(seeds[t-1]);

clk = clock();
// operation
clk = clock() - clk;
cout<<fixed<<setprecision(6)<<n<<""<<((double)clk)/CLOCKS_PER_SEC<
<"\n";
```

Strassen's Algorithm for Matrix Multiplication

We now turn toward Strassen's algorithm, such that we will be able to reduce the number of sub-calls to matrix-multiplies to 7, using just a bit of algebra.
In this way, we bring the work down to $\tilde{O}(n^{log7})$.
How do we do this? We use the following factoring scheme.
We write down *Cij* 's in terms of block matrices *Mk*'s. Each *Mk* may be calculated simply from products and sums of sub-blocks of *A* and *B*.
That is, we let

$$
\begin{aligned}
M_1 &= (A_{11} + A_{22})\,(B_{11} + B_{22}) \\
M_2 &= (A_{21} + A_{22})B_{11} \\
M_3 &= A_{11}(B_{12} - B_{22}) \\
M_4 &= A_{22}(B_{21} - B_{11}) \\
M_5 &= (A_{11} + A_{12})B_{22} \\
M_6 &= (A_{21} - A_{11})(B_{11} + B_{12}) \\
M_7 &= (A_{12} - A_{22})(B_{21} + B_{22})
\end{aligned}
$$

And thus, we calculate C as

$$
\begin{aligned}
C_{11} &= M_1 + M_4 - M_5 + M_7 \\
C_{12} &= M_3 + M_5 \\
C_{21} &= M_2 + M_4 \\
C_{22} &= M_1 - M_2 + M3 + M_6
\end{aligned}
$$

The implementation of the algorithm is given below.
(PTO)

```
#include <bits/stdc++.h>

using namespace std;

int seeds[] = {41661, 82263, 31243, 70828, 46412, 25622, 67921,
74947, 78627, 84862, 66061, 70145, 95453, 43355, 37710, 99532,
59418, 11642, 36212, 82328};

vector < vector <int> > add(vector < vector <int> > x, vector <
vector <int> > y, int s, int f){
     vector < vector <int> > ans(f-s+1);
     for(int i=s; i<=f; i++){
          for(int j=s; j<=f; j++){
               ans[i-s].push_back(x[i][j] + y[i][j]);
          }
     }
     return ans;
}

vector < vector <int> > sub(vector < vector <int> > x, vector <
vector <int> > y, int s, int f){
     vector < vector <int> > ans(f-s+1);
     for(int i=s; i<=f; i++){
          for(int j=s; j<=f; j++){
               ans[i-s].push_back(x[i][j] - y[i][j]);
          }
     }
     return ans;
}

vector < vector <int> > strassens_multiply(vector < vector <int> >
x, vector < vector <int> > y, int s, int f){
     if(s + 1 == f){
          vector < vector <int> > c(2);
          c[0].push_back(x[s][s]*y[s][s] + x[s][f]*y[f][s]);
          c[0].push_back(x[s][s]*y[s][f] + x[s][f]*y[f][f]);
          c[1].push_back(x[f][s]*y[s][s] + x[f][f]*y[f][s]);
          c[1].push_back(x[f][s]*y[s][f] + x[f][f]*y[f][f]);
          return c;
     }
     int sz = (s + f)>>1, i, j;
     vector < vector <int> > a11(sz+1), a12(sz+1), a21(sz+1),
a22(sz+1), b11(sz+1), b12(sz+1), b21(sz+1), b22(sz+1);
     for(i=s; i<=sz; i++){
          for(j=s; j<=sz; j++){
               a11[i-s].push_back(x[i][j]);
               b11[i-s].push_back(y[i][j]);
          }
     }
```

```
     for(i=s; i<=sz; i++){
          for(j=sz+1; j<=f; j++){
               a12[i-s].push_back(x[i][j]);
               b12[i-s].push_back(y[i][j]);
          }
     }
     for(i=sz+1; i<=f; i++){
          for(j=s; j<=sz; j++){
               a21[i-sz-1].push_back(x[i][j]);
               b21[i-sz-1].push_back(y[i][j]);
          }
     }
     for(i=sz+1; i<=f; i++){
          for(j=sz+1; j<=f; j++){
               a22[i-sz-1].push_back(x[i][j]);
               b22[i-sz-1].push_back(y[i][j]);
          }
     }
     vector < vector <int> > m1(sz+1), m2(sz+1), m3(sz+1),
m4(sz+1), m5(sz+1), m6(sz+1), m7(sz+1);
     m1 = strassens_multiply(add(a11, a22, 0, sz), add(b11, b22, 0,
sz), 0, sz);
     m2 = strassens_multiply(add(a21, a22, 0, sz), b11, 0, sz);
     m3 = strassens_multiply(a11, sub(b12, b22, 0, sz), 0, sz);
     m4 = strassens_multiply(a22, sub(b21, b11, 0, sz), 0, sz);
     m5 = strassens_multiply(add(a11, a12, 0, sz), b22, 0, sz);
     m6 = strassens_multiply(sub(a21, a11, 0, sz), add(b11, b12, 0,
sz), 0, sz);
     m7 = strassens_multiply(sub(a12, a22, 0, sz), add(b21, b22, 0,
sz), 0, sz);
     vector < vector <int> > c11(sz+1), c12(sz+1), c21(sz+1),
c22(sz+1);
     c11 = add(m1, add(m4, sub(m7, m5, 0, sz), 0, sz), 0, sz);
     c12 = add(m3, m5, 0, sz);
     c21 = add(m2, m4, 0, sz);
     c22 = add(m1, add(m3, sub(m6, m2, 0, sz), 0, sz), 0, sz);
     vector < vector <int> > c(f - s + 1);
     for(i=s; i<=sz; i++){
          for(j=s; j<=sz; j++){
               c[i-s].push_back(c11[i-s][j-s]);
          }
     }
     for(i=s; i<=sz; i++){
          for(j=sz+1; j<=f; j++){
               c[i-s].push_back(c12[i-s][j-sz-1]);
          }
     }
     for(i=sz+1; i<=f; i++){
          for(j=s; j<=sz; j++){
               c[i-sz+((f-s)>>1)].push_back(c21[i-sz-1][j-s]);
```

```
            }
        }
        for(i=sz+1; i<=f; i++){
            for(j=sz+1; j<=f; j++){
                c[i-sz+((f-s)>>1)].push_back(c22[i-sz-1][j-sz-1]);
            }
        }
        return c;
}

int main(){
        freopen("Strassen's2_times.txt", "w", stdout);
        clock_t clk;
        int n, i, j;
        for(int t=1; t<=9; t++){
            n = (1<<t);
            srand(seeds[t-1]);
            vector < vector <int> > a(n), b(n);
            for(i=0; i<n; i++){
                for(j=0; j<n; j++){
                    a[i].push_back(rand()%1000);
                    b[i].push_back(rand()%1000);
                }
            }
            clk = clock();
            vector < vector <int> > c = strassens_multiply(a, b, 0,
n-1);
            clk = clock() - clk;
            cout<<fixed<<setprecision(6)<<n<<"
"<<((double)clk)/CLOCKS_PER_SEC<<"\n";
        }
        return 0;
}
```

The runtime analysis is done using the in-code analysis as written.

Matrix Exponentiation Algorithm for finding poitive integral powers of a matrix

Suppose we have a matrix A of order N, and we want to find the matrix A^P, where P is a non-negative integer number.

Naïve method:
Initialise answer matrix as the Identity matrix. Now, we multiply A to it P times. The method takes N^3 time for multiplication and we multiply P times. Thus, the overall time complexity of the algorithm is $O(N^3P)$.

Logarithmic exponentiation
As the name suggests the method does matrix multiplication for order of $log_2 P$ times instead of the linear time in naïve method. The working can be explained as follows:

If P is even:

$$A^P = (A^{\frac{P}{2}})^2 = (A^2)^{\frac{P}{2}}$$

If P is odd:

$$A^P = (A)(A^{P-1})$$

Here, in every two iterations the power to be raised is reduced to at least half of the initial power. Thus, the overall time complexity of the algorithm is $O(N^3 log_2 P)$.

The C++ code:

```
#include <bits/stdc++.h>
using namespace std;

#define ld long double

const int mod = 1e9 + 7;

int sz;
const int NN = 101;

class matrix{
public:
    ld mat[NN][NN];
    matrix(){
        for(int i = 0; i < NN; i++)
            for(int j = 0; j < NN; j++)
                mat[i][j] = 0;
    }
    inline matrix operator * (const matrix &a){
        matrix temp;
        for(int i = 0; i < sz; i++)
            for(int j = 0; j < sz; j++){
                for(int k = 0; k < sz; k++){
                temp.mat[i][j] += (mat[i][k] * a.mat[k][j]);
```

```
            }
        }
        return temp;
    }
    inline matrix operator + (const matrix &a){
        matrix temp;
        for(int i = 0; i < sz; i++)
            for(int j = 0; j < sz; j++)
                temp.mat[i][j] = mat[i][j] + a.mat[i][j];
        return temp;
    }
    inline matrix operator - (const matrix &a){
        matrix temp;
        for(int i = 0; i < sz; i++)
            for(int j = 0; j < sz; j++)
                temp.mat[i][j] = mat[i][j] - a.mat[i][j];
        return temp;
    }
    inline void operator = (const matrix &b){
        for(int i = 0; i < sz; i++)
            for(int j = 0; j < sz; j++)
                mat[i][j] = b.mat[i][j];
    }
    inline void print(){
        for(int i = 0; i < sz; i++){
            for(int j = 0; j < sz; j++){
                cout << setprecision(15) << mat[i][j] << " ";
            }
            cout << endl;
        }
    }
};

matrix pow(matrix a, int k){
    matrix ans;
    for(int i = 0; i < sz; i++)
        ans.mat[i][i] = 1;
    while(k){
        if(k & 1)
            ans = ans * a;
        a = a * a;
        k >>= 1;
    }
    return ans;
}

signed main() {
     matrix a;
     int p;
     cout << "Enter size of matrix:\n";
```

```
	cin >> sz;
	cout << "Enter elements of matrix:\n";
	for(int i = 0; i < sz; i++)
		for(int j = 0; j < sz; j++)
			cin >> a.mat[i][j];
	cout << "Enter power to be raised:\n";
	cin >> p;
	a = pow(a, p);
	cout << "Resultant matrix:\n";
	a.print();
	return 0;
}
```

The run time analysis is done later.

Algorithm for finding poitive fractional power of a matrix

Diagonalization:
In linear algebra, a square matrix *A* is called diagonalizable or nondefective if it is similar to a diagonal matrix, i.e., if there exists an invertible matrix *P* such that $P^{-1}AP$ is a diagonal matrix. Diagonalization is the process of finding a corresponding diagonal matrix for a diagonalizable matrix. Diagonalizable matrices and maps are of interest because diagonal matrices are especially easy to handle; once their eigenvalues and eigenvectors are known, one can raise a diagonal matrix to a power by simply raising the diagonal entries to that same power, and the determinant of a diagonal matrix is simply the product of all diagonal entries.

For a matrix *A* which is a *N***N* matrix, to be diagonalizable, it should have *n* distinct eigenvalues.

These eigenvalues are the values that will appear in the diagonalized form of matrix *A*, so by finding the eigenvalues of *A* we have diagonalized it. The matrix formed with column as the eigenvectors of the given matrix is the invertible matrix *P*.
Diagonalization can be used to compute the not only integral power but also fractional power.

We have,

$$A = PDP^{-1}$$

Let,

$$d^m = D$$

Where d can be computed by taking the *mth* root of each of the diagonal elements of D.

Then,

$$A = Pd^mP^{-1}$$

Thus,

$$A = \underbrace{(PdP^{-1})(PdP^{-1}).....(PdP^{-1})}_{m\ times}$$

Let,

$$a = PdP^{-1}$$

Then,

$$a^m = A$$

And thus a is the *mth* root of matrix A.

Matrix Inversion using LU Decomposition

Working of LU Decomposition can be explained in the following way.

Consider the system of linear equations $Ax = B$. If we are able to write the A matrix as a product of lower triangular matrix and an upper triangular matrix, then we may do forward substitution and back substitution to get the solution. Also, forward substitution and back substitution takes $O(n^2)$ to compute, where n is the size of the matrix. So if given a set of linear equations, $Ax = B1$, $Ax = B2$, ... $Ax = Bk$, then we can first do the LU decomposition of the matrix A in $O(n^3)$, then solve for each of the linear equation systems in $O(n^2)$. So the total complexity of would be $O(n^3 + k*n^2)$.

Furthermore, traditional method of computation for inverse of matrix A is very costly in terms of number of operation.

Types of LU Decomposition:

1. Doolittle Decomposition

The diagonal entries of L are taken to be one.
For eg:

$$A = \begin{bmatrix} A11 & A12 & A13 \\ A21 & A22 & A23 \\ A31 & A32 & A33 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ L21 & 1 & 0 \\ L31 & L32 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} U11 & U12 & U13 \\ 0 & U22 & U23 \\ 0 & 0 & U33 \end{bmatrix}$$

Multiplying L and U, we get

$$\begin{bmatrix} U11 & U12 & U13 \\ L21 * U11 & L21 * U12 + U22 & L21 * U13 + U23 \\ L31 * U11 & L31 * U12 + L32 * U22 & L31 * U13 + L32 * U23 + U33 \end{bmatrix}$$

Comparing this matrix with matrix A, the general expression for solution of each value of L and U,

$$L(i,k) = (A(i,k) - \sum_{j=0}^{i} L(i,j) * U(j,k)) / U(k,k)$$

and,

$$U(i,k) = A(i,k) - \sum_{j=0}^{i} L(i,j) * U(j,k)$$

.

2. Crout Decomposition

In this type of decomposition, we assume that the diagonal entries of U matrix is 1. The computation of L and U is similiar to Doolittle Decomposition.

3. Cholesky Decompostion

In this type of decomposition, we have $U = L^T$. For cholesky decomposition, we must have A as a symmetric matrix and positive definite.

Using these, we can compute the inverse of the matrix.
Consider the equation $Ax = B$. The equation $Ax = I$ would give inverse of A as the solution. After replacing $A = LU$, we get $LUx = I$. Now we can treat Ux as a separate matrix Y. Then we get $LY = I$. Y can be calculated in $O(n^2)$ using forward substitution. After solving for Y, we can equate it to $Ux = Y$ and solve for x using back substitution.

The C++ implemenatation of the above methods is discussed below.

```
// Performs LU decomposition - Dolittle, Crout, Cholesky and
computes inverse of a matrix using these decompositions.
#include<bits/stdc++.h>
using namespace std;
typedef long double f80;
void print(vector<vector<f80>> A) {
    int n = A.size();
    for(int i = 0; i < n; i++) {
        for(int j = 0; j < n; j++) {
            cout << A[i][j] << " ";
        }
        cout << endl;
    }
    cout << endl;
}
vector<vector<f80>> operator *(vector<vector<f80>> A,
vector<vector<f80>> B){
    int n = A.size();
    vector<vector<f80>> temp(n);
    for(int i = 0; i < n; i++) {
        temp[i].resize(n, 0);
    }
    for(int i = 0; i < n; i++) {
        for(int j = 0; j < n; j++) {
```

```
            for(int k = 0; k < n; k++) {
                temp[i][j] += A[i][k] * B[k][j];
            }
        }
    }
    return temp;
}
class Matrix{
public:
    vector<vector<f80>> A, L, U, IA;
    bool decomposed;
    int n;
    Matrix(int n) : n(n) {
        decomposed = 0;
        A.resize(n);
        for(int i = 0; i < n; i++){
            A[i].resize(n, 0);
        }
    }
    void reset(){
        L.clear();
        U.clear();
        L.resize(n);
        U.resize(n);
        for(int i = 0; i < n; i++){
            L[i].resize(n, 0);
            U[i].resize(n, 0);
        }
    }
    void LU_Crout() { // U[i][j] = 1 for 0 <= i < n
        reset();
        decomposed = 1;
       for (int j = 0; j < n; j++) {
        U[j][j] = 1;
            for (int i = j; i < n; i++) {
                L[i][j] = A[i][j];
                for (int k = 0; k < j; k++) {
                    L[i][j] -= L[i][k] * U[k][j];
                }
            }
            for (int i = j; i < n; i++) {
                U[j][i] = A[j][i];
                for(int k = 0; k < j; k++) {
                    U[j][i] -= L[j][k] * U[k][i];
                }
                U[j][i] /= L[j][j];
            }
        }
    }
    void LU_Dolittle() { // L[i][i] = 1 for 0 <= i < n
```

```
        reset();
        decomposed = 1;
        for(int i = 0; i < n; i++) {
            L[i][i] = 1;
            for(int j = i; j < n; j++) {
                U[i][j] = A[i][j];
                for(int k = 0; k < i; k++) {
                    U[i][j] -= L[i][k] * U[k][j];
                }
            }
            for(int j = i + 1; j < n; j++) {
                L[j][i] = A[j][i];
                for(int k = 0; k < i; k++) {
                    L[j][i] -= L[j][k] * U[k][i];
                }
                L[j][i] /= U[i][i];
            }
        }
    }
    void LU_Cholesky() { // U = L^T, possible only if A is
symmetric
        reset();
        decomposed = 1;
        for(int i = 0; i < n; i++) {
            L[i][i] = A[i][i];
            for(int j = 0; j < i; j++) {
                L[i][i] -= L[i][j] * L[i][j];
            }
            assert(L[i][i] >= 0);
            L[i][i] = sqrtl(L[i][i]);
            for(int j = i + 1; j < n; j++) {
                L[j][i] = A[j][i];
                for(int k = 0; k < i; k++) {
                    L[j][i] -= L[j][k] * L[i][k];
                }
                L[j][i] /= L[i][i];
            }
        }
        for(int i = 0; i < n; i++) {
            for(int j = 0; j < n; j++) {
                U[i][j] = L[j][i];
            }
        }
    }
    void operator = (const Matrix &M){
        A = M.A, L = M.L, U = M.U, IA = M.IA, n = M.n;
    }
    void sub(vector<vector<f80>> &a, int r2, int r1, f80 k){ // r2
= r2 - k * r1
        for(int i = 0; i < n; i++) {
```

```
            a[r2][i] -= k * a[r1][i];
        }
    }
    void rowmul(vector<vector<f80>> &a, int r, f80 k){ // r = r *
k;
        for(int i = 0; i < n; i++) {
            a[r][i] *= k;
        }
    }
    void inverse() { // Call any LU - decomposition method first,
otherwise DoLittle by default
        if(!decomposed) LU_Dolittle(); // by default
        IA.clear();
        IA.resize(n);
        for(int i = 0; i < n; i++) {
            IA[i].resize(n, 0);
            IA[i][i] = 1;
        }
        auto T = L;
        for(int i = 0; i < n; i++){ // forward substitution
            f80 val = 1.0 / T[i][i];
            rowmul(T, i, val);
            rowmul(IA, i, val);
            for(int j = i + 1; j < n; j++) {
                f80 val = T[j][i];
                sub(T, j, i, val);
                sub(IA, j, i, val);
            }
        }
        T = U;
        for(int i = n - 1; i >= 0; i--) { // back substitution
            f80 val = 1.0 / T[i][i];
            rowmul(T, i, val);
            rowmul(IA, i, val);
            for(int j = i - 1; j >= 0; j--) {
                f80 val = T[j][i];
                sub(T, j, i, val);
                sub(IA, j, i, val);
            }
        }
    }
};

int main() {
    Matrix M(3);
    M.A = {{25, 15, -5}, {15, 18, 0}, {-5, 0, 11}};
    M.LU_Cholesky();
    cout << "L - Matrix: " << endl;
    print(M.L);
    cout << "U - Matrix " << endl;
```

```
    print(M.U);
    M.inverse();
    cout << "Inverse of Matrix : " << endl;
    print(M.IA);
    return 0;
}
```

Algorithm for finding inverse of a matrix using adjoint determinant method

The C++ implementation of this famous algorithm is done below.

```
#include <bits/stdc++.h>

using namespace std;

const long double eps=1e-9;
typedef long double ld;


class Matrix
{
     public:

          int N,M;
          vector<vector<ld>>mat;
          bool inverse_exists;
          ld det;

          Matrix(int n,int m)
          {
               N=n;
               M=m;
               mat.clear();
               mat.resize(N,vector<ld>(M,0));
               if(N!=M)
                    inverse_exists=0;
          }

          void reset()
          {
               mat.clear();
               N=0;
          }

          void Print()
          {
               cout<<"Matrix:"<<endl;
```

```
		for(int i=0;i<N;i++)
		{
			for(int j=0;j<M;j++)
			{
				cout<<fixed<<setprecision(5)<<mat[i][j]<<"
";
			}
			cout<<endl;
		}
	}

	ld Determinant(Matrix X)
	{
		ld res=1;
		for(int i=0;i<X.N;i++)
		{
			int select=i;
			while(select<X.N&&fabs(X.mat[select][i])<eps)
			{
				select++;
			}
			if(select==X.N)
				return 0;
			if(select!=i)
			{
				res*=-1;
				for(int j=0;j<X.N;j++)
				{
					swap(X.mat[i][j],X.mat[select][j]);
				}
			}
			res*=X.mat[i][i];
			for(int j=i+1;j<X.N;j++)
			{
				ld tmp=X.mat[j][i]/X.mat[i][i];
				for(int k=0;k<X.N;k++)
				{
					X.mat[j][k]-=X.mat[i][k]*tmp;
				}
			}
		}
		return res;
	}

	Matrix GetCofactor(Matrix X,int I,int J)
	{
		Matrix res(X.N-1,X.N-1);
		int curi=0;
		for(int i=0;i<X.N;i++)
		{
```

```
                    if(i==I)
                        i++;
                    if(i>=X.N)
                        break;
                    int curj=0;
                    for(int j=0;j<X.N;j++)
                    {
                        if(j==J)
                            j++;
                        if(j>=X.N)
                            break;
                        res.mat[curi][curj]=X.mat[i][j];
                        curj++;
                    }
                    curi++;
                }
                return res;
            }

            Matrix Adjoint(Matrix X)
            {
                Matrix res(X.N,X.N);
                for(int i=0;i<X.N;i++)
                {
                    for(int j=0;j<X.N;j++)
                    {
                        res.mat[i]
[j]=Determinant(GetCofactor(X,i,j));
                        if((i+j)%2==1)
                            res.mat[i][j]*=-1;
                    }
                }
                return res;
            }

            Matrix Inverse()
            {
                Matrix res(N,N);
                res.inverse_exists=0;
                if(N!=M)
                    return res;
                Matrix B(N,N);
                B.mat=mat;
                ld det=Determinant(B);
                if(fabs(det)<eps)
                    return res;
                res=Adjoint(B);
                res.inverse_exists=1;
                for(int i=0;i<N;i++)
                {
```

```
                    for(int j=0;j<N;j++)
                    {
                         res.mat[i][j]/=det;
                    }
               }
               for(int i=0;i<N;i++)
               {
                    for(int j=i+1;j<N;j++)
                    {
                         swap(res.mat[i][j],res.mat[j][i]);
                    }
               }
               return res;
          }


};

int main()
{
     Matrix A(3,3);
     A.mat={{5,7,9},{4,3,8},{7,5,6}};
     Matrix B=A.Inverse();
     if(B.inverse_exists==0)
     {
          cout<<"Inverse Does Not Exist!!"<<endl;
     }
     else
     {
          B.Print();
     }
     return 0;
}
```

Algorithm for finding inverse of a matrix using adjoint determinant method

The C++ implementation of this famous algorithm is done below.

```
#include <bits/stdc++.h>

using namespace std;

const long double eps=1e-9;
typedef long double ld;
```

```
class Matrix
{
     public:

          int N,M;
          vector<vector<ld>>mat;
          bool inverse_exists;

          Matrix(int n,int m)
          {
               N=n;
               M=m;
               mat.clear();
               mat.resize(N,vector<ld>(M,0));
               if(N!=M)
                    inverse_exists=0;
          }

          void reset()
          {
               mat.clear();
               N=0;
          }

          void Print()
          {
               cout<<"Matrix:"<<endl;
               for(int i=0;i<N;i++)
               {
                    for(int j=0;j<M;j++)
                    {
                         cout<<fixed<<setprecision(5)<<mat[i][j]<<"
";
                    }
                    cout<<endl;
               }
          }

          Matrix Gauss_Jordan_Elimination(Matrix X)
          {
               X.inverse_exists=0;
               for(int i=0;i<N;i++)
               {
                    int select=i;
                    for(int j=i;j<N;j++)
                    {
                         if(fabs(X.mat[j][i])>fabs(X.mat[select]
[i]))
                         {
                              select=j;
```

```
                }
            }
            if(fabs(X.mat[select][i])<eps)
                return X;
            for(int j=i;j<2*N;j++)
                swap(X.mat[i][j],X.mat[select][j]);
            for(int j=0;j<N;j++)
            {
                if(j!=i)
                {
                    ld tmp=X.mat[j][i]/X.mat[i][i];
                    for(int k=0;k<2*N;k++)
                    {
                        X.mat[j][k]-=tmp*X.mat[i][k];
                    }
                }
            }
            ld tmp=X.mat[i][i];
            for(int j=0;j<2*N;j++)
            {
                X.mat[i][j]/=tmp;
            }
        }
        X.inverse_exists=1;
        return X;
}

Matrix Inverse()
{
        Matrix res(N,N);
        res.inverse_exists=0;
        if(N!=M)
            return res;
        Matrix B(N,N+N);
        for(int i=0;i<N;i++)
        {
            for(int j=0;j<N;j++)
            {
                B.mat[i][j]=mat[i][j];
            }
            B.mat[i][N+i]=1;
        }
        B=Gauss_Jordan_Elimination(B);
        if(B.inverse_exists==0)
            return res;
        res.inverse_exists=1;
        for(int i=0;i<N;i++)
            for(int j=0;j<N;j++)
                res.mat[i][j]=B.mat[i][j+N];
        return res;
```

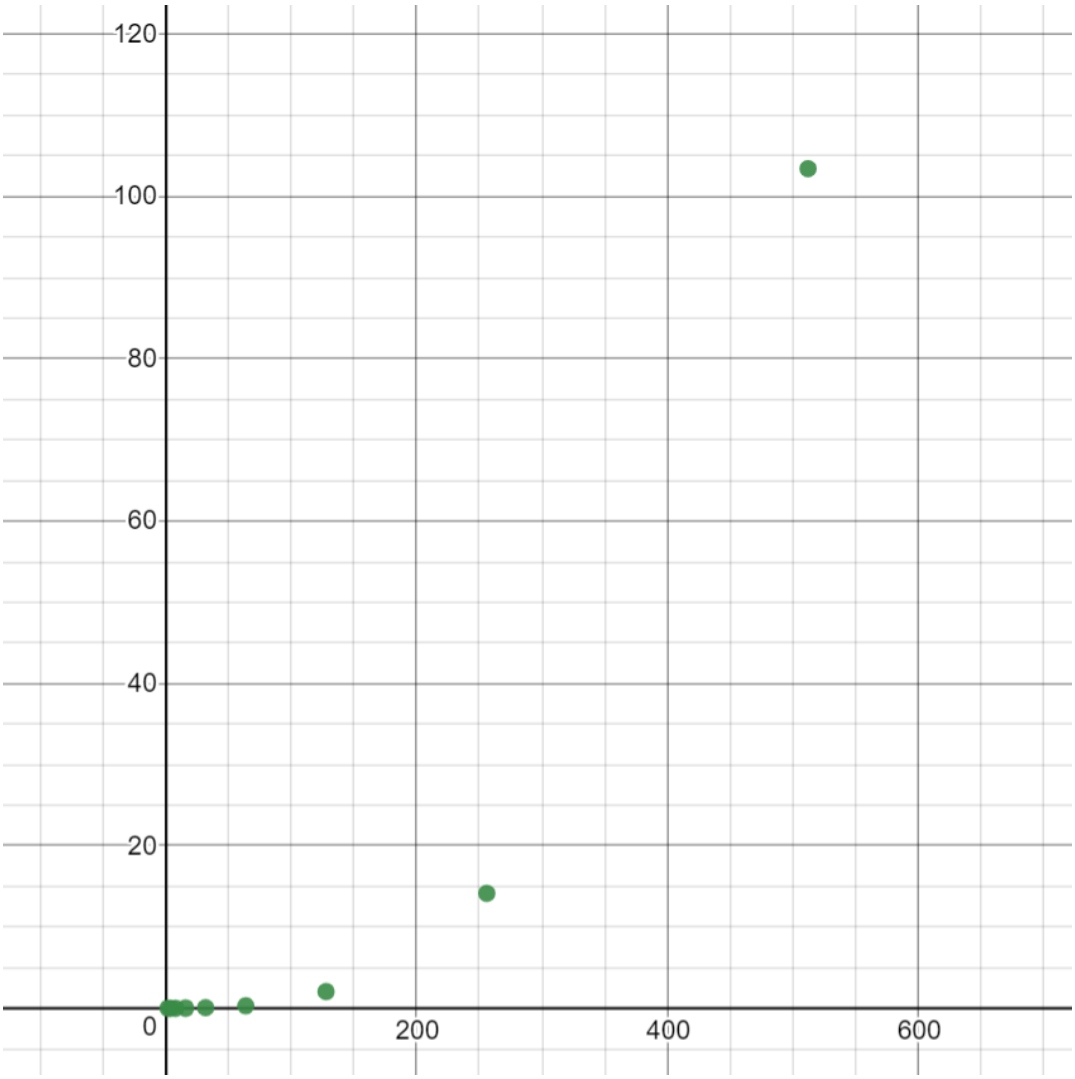
```
            }
};

int main()
{
     Matrix A(3,3);
     A.mat={{5,7,9},{4,3,8},{7,5,6}};
     Matrix B=A.Inverse();
     if(B.inverse_exists==0)
     {
          cout<<"Inverse Does Not Exist!!"<<endl;
     }
     else
     {
          B.Print();
     }
     return 0;
}
```

RunTime Ananlysis of various Algorithms

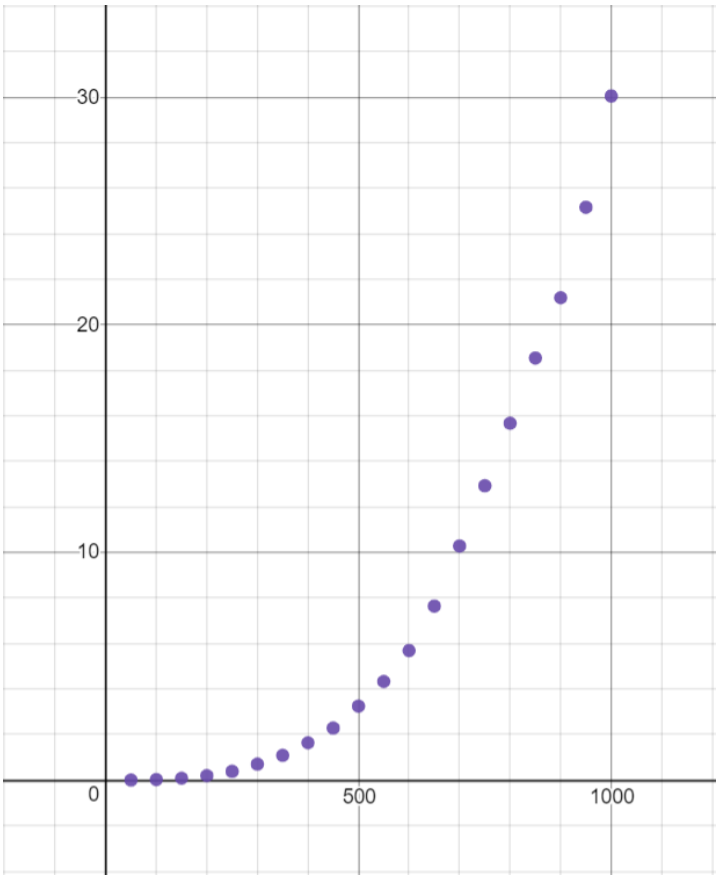
The runtime analysis of each code has been done using **input size *n* on the horizontal axis** and **time taken in seconds in the vertical axis**. This is same for all the graphs below.

All these run times have been recorded using Intel(R) Core™ i7-8550U CPU at 1.8GHz.

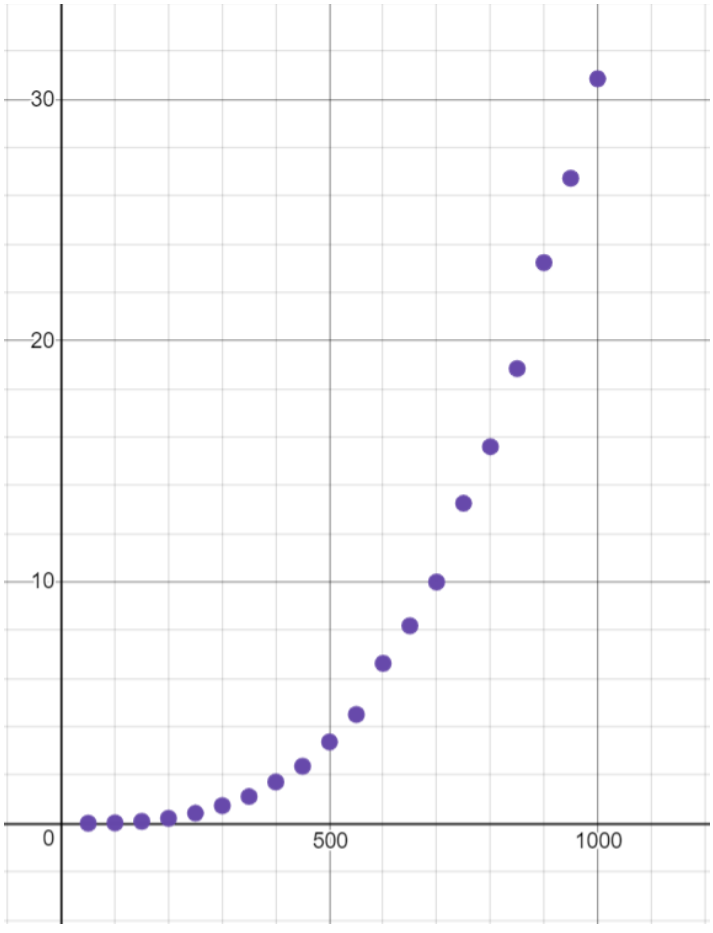
Strassen's Multiplication:



Dolittle LU Decomposition:

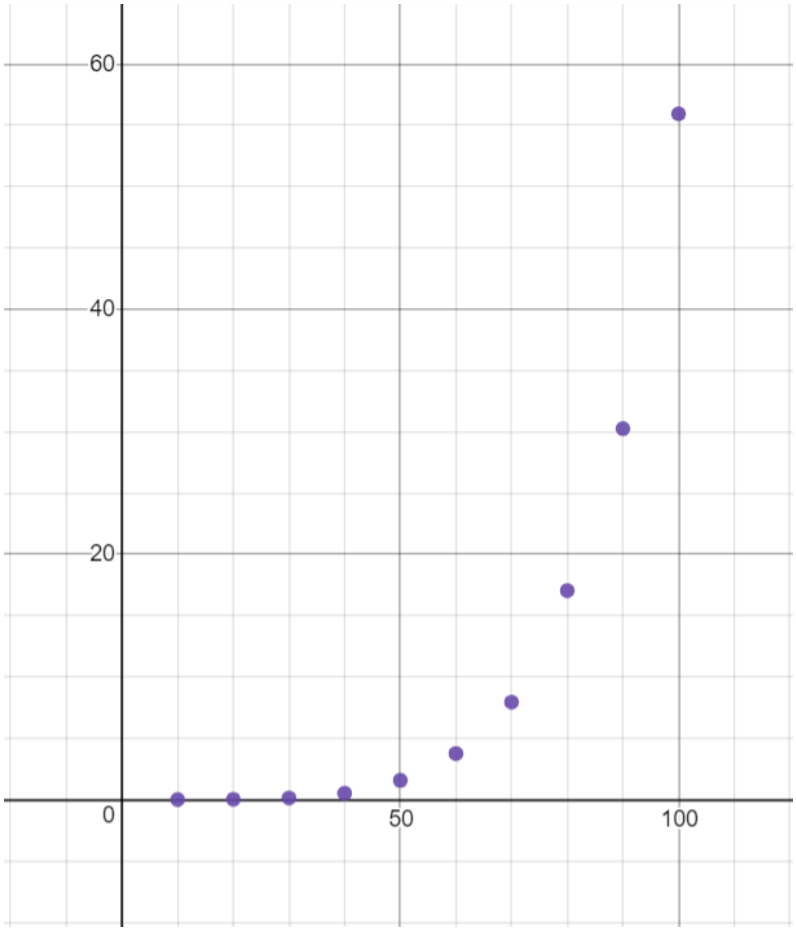


Crout LU Decomposition:

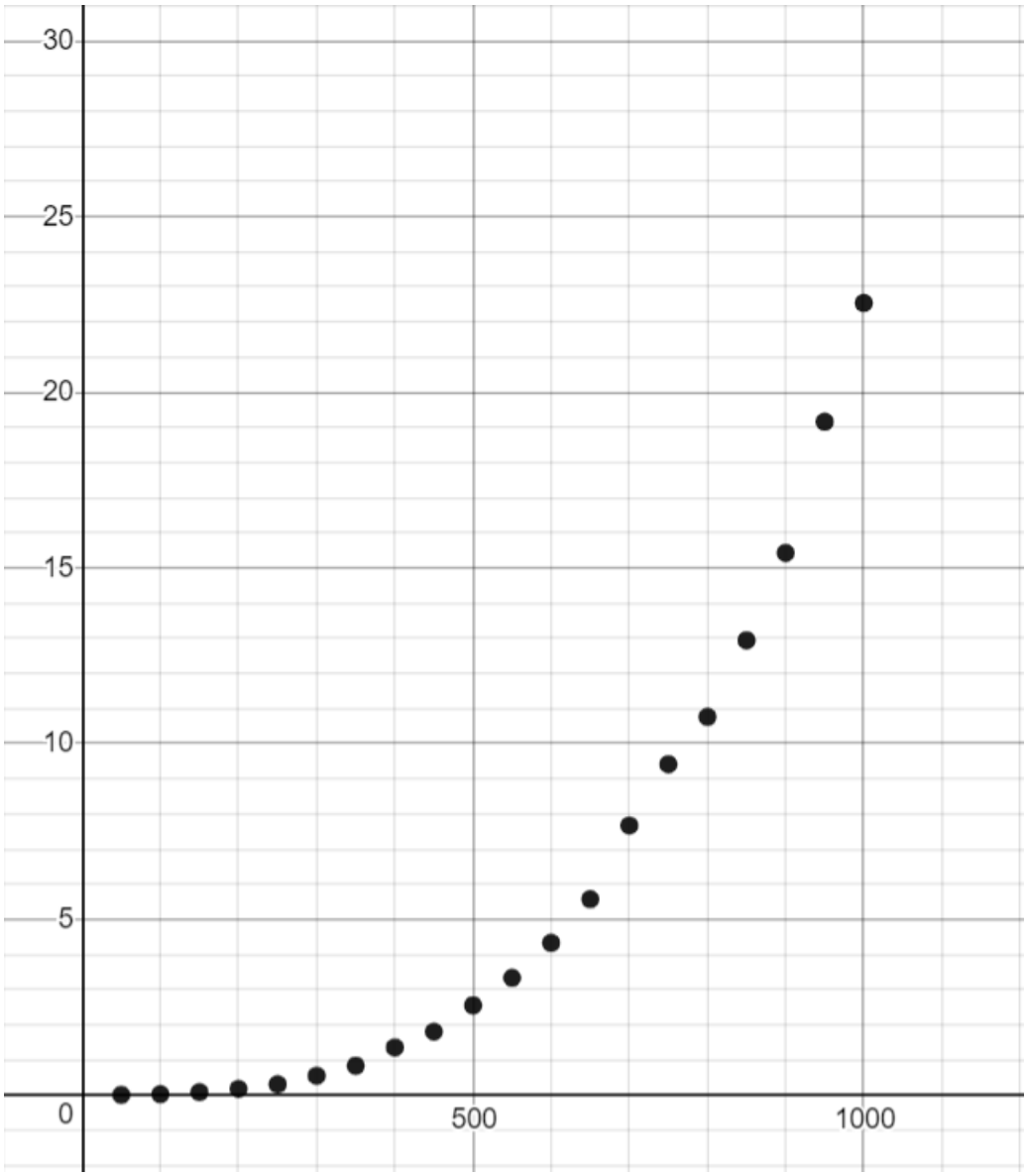


Cholesky LU Decomposition:

Inversion using Adjoint Determinant Method:



Inverse using Guass Elimination Method



Remaining Tasks

Future Goals and targets are listed below:

- Develop implementation for calculating eigenvalues and eigenvectors.
- Implement the algorithm for fractional power of matrix.
- Implement the algorithm for Normalization of a square matrix.
- Using Cayley Hamilton Theorem to find Inverse of matrix.

All codes and implementations can be found at
https://github.com/AayushChaturvedi/IOP2019