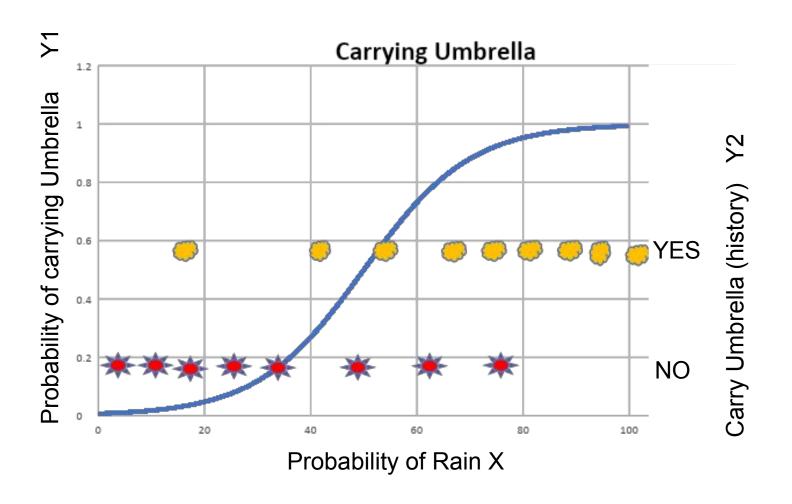
Logistic Regression

Probabilistic classification

- Most real-life prediction scenarios with discreet outputs, such as *Yes/No*, are probabilistic:
 - Will you carry an umbrella if it is raining?
 - Will you carry an umbrella if it is sunny?
 - Will you carry an umbrella if it drizzles?
- Logistic Regression gives the probability of an event occurring, given historical data to train-test the model

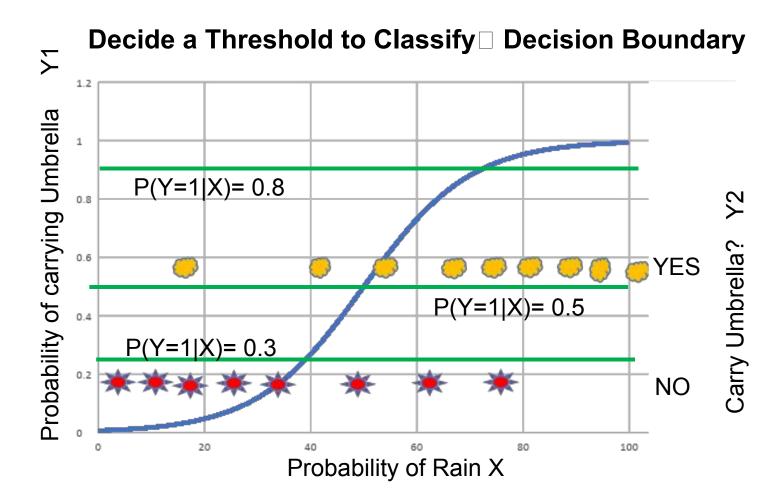






Historical data (Y2 Axis)

Prediction (Y1 Axis)







Historical data (Y2 Axis)

Prediction (Y1 Axis)

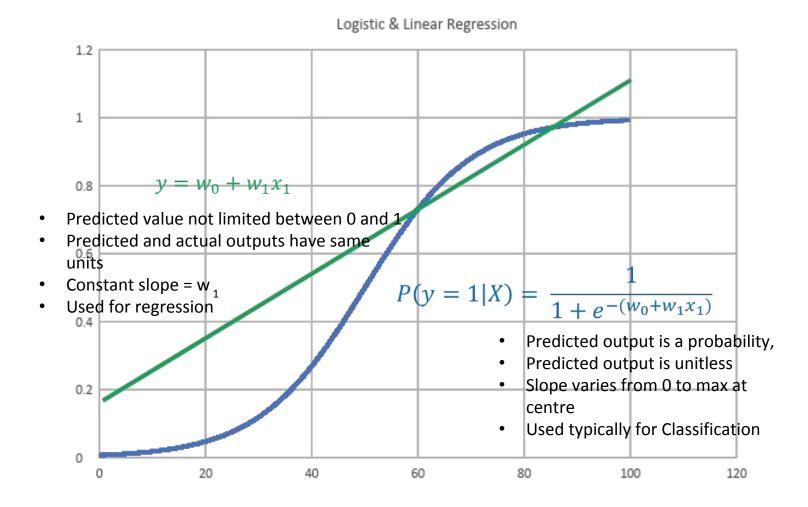
Logistic Regression

 Logistic regression gives the probability of an event occurring, given historical data {y,X}:

$$P(y=1|\overrightarrow{X},\overrightarrow{w}) = \frac{1}{1+e^{-\overrightarrow{w}\cdot\overrightarrow{X}}}$$

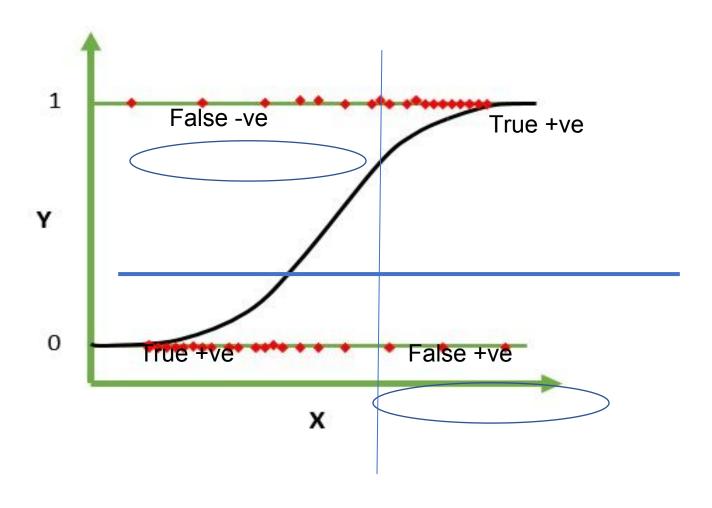
- Where: $\overrightarrow{w} \cdot \overrightarrow{X} = w_0 + w_1 x_1 + \cdots + w_k x_k$ w are the adjustable weight parameters
- P(Y= This is the Sigmoid function sition

A Comparison



Midpoint & Slope

Performance Tallies



Log odds or Logit

Assume there are two classes, y = 0 and y = 1 and

$$p_1 = \frac{1}{1 + e^{-wx}} \quad 1 - p_1 = 1 - \frac{1}{1 + e^{-wx}}$$

- Odds:
- Log Odds:
- That is, the log odds of class 1 w.r.t class 2, is a linear function of x

Model Fitting

```
Let p_1 be P(y=1|x,w)
```

```
Sequence n: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

Actual Data y: 1 1 1 0 0 0 1 1 1 1 1 0 0 0 0 1 1 1

Prediction p: p<sub>1</sub> p<sub>1</sub> p<sub>1</sub> 1-p<sub>1</sub> 1-p<sub>1</sub> 1-p<sub>1</sub> p<sub>1</sub> p<sub>1</sub> p<sub>1</sub> p<sub>1</sub> p<sub>1</sub> p<sub>1</sub> 1-p<sub>1</sub> 1-p<sub>1</sub> 1-p<sub>1</sub> p<sub>1</sub> p<sub>1</sub>
```

Likelihood of a match?

Sequence n: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

Actual Data y: 1 1 1 0 0 0 1 1 1 1 1 0 0 0 0 1 1 1

Prediction p: p₁ p₁ p₁ 1-p₁ 1-p₁ 1-p₁ p₁ p₁ p₁ p₁ p₁ p₁ 1-p₁ 1-p₁ 1-p₁ p₁ p₁

Likelihood of a match? Note y_n can be either 1 or 0

$$\mathcal{L}(w) = \prod_{n} p_1^{y_n} (1 - p_1)^{1 - y_n}$$

Log Likelihood:

$$l(\mathbf{w}) = \sum_{l} y^{l} \ln P(y^{l} = 1 \mid \mathbf{x}^{l}, \mathbf{w}) + (1 - y^{l}) \ln P(y^{l} = 0 \mid \mathbf{x}^{l}, \mathbf{w})$$

Training

Maximum Likehood Estimation MLE.

$$\mathbf{w} = \arg\max_{\mathbf{w}} \prod_{l} P(y^{l} \mid \mathbf{x}^{l}, \mathbf{w})$$

- Note:
 - Here x^I and y^I are pre-determined from training data.
 - Intercept w₀, and coefficients w_i calculated so as to maximize probability
 - So, how many w should we try out?

Computing the Log-Likelihood

 We can re-express the log of the conditional likelihood as:

$$l(\mathbf{w}) = \sum_{l} y^{l} \ln P(y^{l} = 1 \mid \mathbf{x}^{l}, \mathbf{w}) + (1 - y^{l}) \ln P(y^{l} = 0 \mid \mathbf{x}^{l}, \mathbf{w})$$

$$= \sum_{l} y^{l} \ln \frac{P(y^{l} = 1 | \mathbf{x}^{l}, \mathbf{w})}{P(y^{l} = 0 | \mathbf{x}^{l}, \mathbf{w})} + \ln P(y^{l} = 0 | \mathbf{x}^{l}, \mathbf{w})$$

$$= \sum_{i=1}^{n} y^{i}(w_{0} + \sum_{i=1}^{n} w_{i}x_{i}^{l}) - \ln(1 + \exp(w_{0} + \sum_{i=1}^{n} w_{i}x_{i}^{l}))$$
• Need to maximize $l(w)$

Fitting LogR by Gradient Ascent

- Unfortunately, there is no closed form solution to maximizing I(w) with respect to w. Therefore, one common approach is to use gradient ascent
- The i th component of the vector gradient has the form

$$\frac{\partial}{\partial w_i} l(\mathbf{w}) = \sum_l x_i^l (y^l - \hat{P}(y^l = 1 \mid \mathbf{x}^l, \mathbf{w}))$$

Fitting LogR by Gradient Ascent

 Use standard gradient ascent to optimize w. Begin with initial weights = zero

$$w_i \leftarrow w_i + \eta \sum_l x_i^l (y^l - \hat{P}(y^l = 1 \mid \mathbf{x}^l, \mathbf{w}))$$

Regularization in Logistic Regression

 Overfitting the training data is a problem that can arise in Logistic Regression, especially when data has very high dimensions and is sparse.

 One approach to reducing overfitting is regularization, in which we create a modified "penalized log likelihood function," which penalizes large values of w.

$$\mathbf{w} = \arg\max_{\mathbf{w}} \sum_{l} \left(\ln P(y^{l} \mid \mathbf{x}^{l}, \mathbf{w}) - \frac{\lambda}{2} ||\mathbf{w}||^{2} \right)$$

Regularization in Logistic Regression

 The derivative of this penalized log likelihood function is similar to our earlier derivative, with one additional penalty term

$$\frac{\partial}{\partial w_i} l(\mathbf{w}) = \sum_{i} x_i^l (y^l - \hat{P}(y^l = 1 | \mathbf{x}^l, \mathbf{w})) - \lambda w_i$$
• which give $\hat{\theta}$ us the modified gradient descent rule

$$w_i \leftarrow w_i + \eta \sum_l x_i^l (y^l - \hat{P}(y^l = 1 \mid \mathbf{x}^l, \mathbf{w})) - \eta \lambda w_i$$

Summary of Logistic Regression

- Learns the Conditional Probability Distribution P(y|x)
- Local Search.
 - Begins with initial weight vector.
 - Modifies it iteratively to maximize an objective function.
 - The objective function is the conditional log likelihood of the data so the algorithm seeks the probability distribution P(y|x) that is most likely given the data.

What you should know LogR

- In general, NB and LR make different assumptions
 - NB: Features independent given class -> assumption on P(X|Y)
 - LR: Functional form of P(Y|X), no assumption on P(X|Y)
- LogR can be used as a linear classifier
 - decision rule is a hyperplane
- LogR optimized by conditional likelihood
 - no closed-form solution
 - concave -> global optimum with gradient ascent