Pricing the term structure with linear regressions: Adrian, Crump, Moench (JFE 2013)

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Overview

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About the paper

Limitations of earlier literature:

- Maximum likelihood (ML) methods to estimate coefficients and pricing factors–subject to constraints across maturities.
- Serially uncorrelated yield pricing errors in ML methods imply excess return predictability not captured by the pricing factors.

Contribution:

- Propose an Affine model of the term structure of interest rates that includes five principal components of yields as factors.
- Can incorporate unspanned factors and estimate term structure models without observing a zero-coupon yield curve.

Finding:

- Their model outperforms the [Cochrane and Piazzesi(2009)] four-factor specification.

Theoretical background

Affine models of the term structure of interest rates have three assumptions:

- the pricing kernel is exponentially affine in the shocks that drive the economy
- prices of risk are affine in the state variables
- innovations to state variables and log yield observation errors are conditionally Gaussian

Dynamic evolution of the state variables: VAR

$$X_{t+1} = \mu + \Phi X_t + \nu_{t+1} \tag{1}$$

where, $\nu_{t+1} \mid \{X_s\}_{s=0}^{t} \sim N(0, \Sigma)$ and $\{X_s\}_{s=0}^{t}$ denotes the history of X_t

Theoretical background

- Zero coupon Treasury bond price with maturity n at time t

$$P_t^{(n)} = E_t\left[M_{t+1}P_{t+1}^{(n-1)}\right] \tag{2}$$

where, pricing kernel M_{t+1} is assumed to be exponentially affine

$$M_{t+1} = \exp\left(-r_t - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\Sigma^{-1/2}v_{t+1}\right) \tag{3}$$

where $r_t = \ln P_t^{(1)}$ is the continuously compounded risk-free rate

- Market prices of risk are assumed to be affine as suggested in [Duffee(2002)]

$$\lambda_t = \Sigma^{-1/2}\left(\lambda_0 + \lambda_1 X_t\right) \tag{4}$$

Theoretical background

- $rx_{t+1}^{(n-1)}$ is the log excess holding return of a bond maturing in n periods:

$$rx_{t+1}^{(n-1)} = \ln P_{t+1}^{(n-1)} - \ln P_t^{(n)} - r_t \tag{5}$$

Using Eq. (3) and Eq. (4) in Eq. (2) yields

$$1 = E_t \left[\exp \left(rx_{t+1}^{(n-1)} - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\Sigma^{-1/2}v_{t+1} \right) \right] \tag{6}$$

- Upon assuming that $\left\{ rx_{t+1}^{(n-1)}, v_{t+1} \right\}$ are jointly normally distributed,

$$E_t \left[rx_{t+1}^{(n-1)} \right] = Cov_t \left[rx_{t+1}^{(n-1)}, v_{t+1}'\Sigma^{-1/2}\lambda_t \right] - \frac{1}{2} Var_t \left[rx_{t+1}^{(n-1)} \right] \tag{7}$$

here, let $\beta_t^{(n-1)'} = \mathrm{Cov}_t \left[rx_{t+1}^{(n-1)}, v_{t+1}' \right] \Sigma^{-1}$

Theoretical background

- The return generating process for log excess holding period returns is then

$$
\begin{aligned}
rx_{t+1}^{(n-1)} = \underbrace{\beta^{(n-1)\prime}\left(\lambda_0 + \lambda_1 X_t\right)}_{\text{Expected Return}} - \underbrace{\frac{1}{2}\left(\beta^{(n-1)\prime}\Sigma\beta^{(n-1)} + \sigma^2\right)}_{\text{Convexity adjustment}} \\
+ \underbrace{\beta^{(n-1)\prime} v_{t+1}}_{\text{Priced return innovation}} + \underbrace{e_{t+1}^{(n-1)}}_{\text{Return pricing error}}
\end{aligned}
\tag{8}
$$

- Stacking this system across maturities and time periods, we rewrite it as

$$
rx = \beta'\left(\lambda_0 \iota_T' + \lambda_1 X_-\right) - \frac{1}{2}\left(B^* \operatorname{vec}(\Sigma) + \sigma^2 \iota_N\right)\iota_T' + \beta' V + E \tag{9}
$$

Empirical approach

Three step regression based estimator for the parameters of the model:

- Decompose X_{t+1} into a predictable component and estimate of the innovation $\hat{v}_{t+1}$ (see eq.(1)). Stack these innovations in matrix $\hat{V}$.
- Regress excess returns on a constant, lagged pricing factors and contemporaneous pricing factor innovations according to

$$rx = \mathbf{a}\iota_T' + \beta'\hat{V} + \mathbf{c}X_- + E \tag{10}$$

- Estimate the price of risk parameters λ_0 and λ_1 via cross-sectional regression. We know from Eq. (9) that $\mathbf{a} = \beta'\lambda_0 - \frac{1}{2}\left(B^\star \operatorname{vec}(\Sigma) + \sigma^2\iota_N\right)$ and $\mathbf{c} = \beta'\lambda_1$.
  Use these expressions to obtain the estimators for λ_0 and λ_1 :
  $\widehat{\lambda}_0 = \left(\hat{\beta}\hat{\beta}'\right)^{-1}\hat{\beta}\left(\hat{\mathbf{a}} + \frac{1}{2}\left(\hat{B}^\star \operatorname{vec}(\hat{\Sigma}) + \hat{\sigma}^2\iota_N\right)\right)$ and $\widehat{\lambda}_1 = \left(\hat{\beta}\hat{\beta}'\right)^{-1}\hat{\beta}\hat{\mathbf{c}}$

Empirical approach

- Affine Yields:

$$\ln P_t^{(n)} = A_n + B_n' X_t + u_t^{(n)} \tag{11}$$

From eq. (11), (5) and (8),

$$A_n = A_{n-1} + B_{n-1}'\left(\mu - \lambda_0\right) + \frac{1}{2}\left(B_{n-1}'\Sigma B_{n-1} + \sigma^2\right) + A_1 \tag{12}$$

$$B_n' = B_{n-1}'\left(\Phi - \lambda_1\right) + B_1' \tag{13}$$

$$A_0 = 0, B_0' = 0; \beta^{(n)\prime} = B_n' \tag{14}$$

$$\underbrace{u_{t+1}^{(n-1)} - u_t^{(n)} + u_t^{(1)}}_{\text{Log yield pricing error}} = \underbrace{e_{t+1}^{(n-1)}}_{\text{return pricing error}} \tag{15}$$

Setting λ_0 and λ_1 to zero in Eq.(12) (13) generates the risk-adjusted bond pricing parameters A_n^{RF} and B_n^{RF}.

Literature Review

- [Joslin et al.(2011)Joslin, Singleton, and Zhu] and [Hamilton and Wu(2012)] assume that the yield pricing errors are conditionally independent of lagged values of yield pricing errors. Serially uncorrelated yield pricing errors imply excess return predictability not captured by the pricing factors.
- [Cochrane and Piazzesi(2009)] includes the first three principal components of Treasury yields and a linear combination of forward rates designed to predict Treasury returns (the CP factor) as pricing factors.

Data

- The sample period is 1987:01 - 2011:12, which provides a total of T = 300 monthly observations.
- Factors extracted from the constant maturity yields from the Federal Reserve Board's H.15 release
- Excess returns of Fama maturity-sorted bond portfolios and the Fama one-month risk-free rate are from the Center for Research in Security Prices (CRSP).

Results

Table 3
Five-factor model: market prices of risk.

This table summarizes the estimates of the market price of risk parameters λ_0 and λ_1 for the five factor specification. t-Statistics are reported in parentheses. The standard errors have been computed according to the formulas from Section 2.3. Wald statistics for tests of the rows of Λ and of λ_1 being different from zero are reported along each row, with the corresponding p-values in parentheses below. PC1,…, PC5 denote the first through fifth principal components of Treasury yields. Bolded coefficients represent significance at the 5% level.

Factor	λ_0	$\lambda_{1.1}$	$\lambda_{1.2}$	$\lambda_{1.3}$	$\lambda_{1.4}$	$\lambda_{1.5}$	W_Λ	W_{λ_1}
PC1	**−0.019**	−0.003	**−0.016**	−0.005	0.012	**0.030**	**30.367**	**23.705**
(t-statistic)	(−2.566)	(−0.443)	(−2.160)	(−0.648)	(1.605)	(3.987)	(0.000)	(0.000)
PC2	0.013	0.027	−0.011	−0.003	−0.011	0.015	7.097	6.167
(t-statistic)	(0.951)	(1.914)	(−0.818)	(−0.213)	(−0.792)	(1.077)	(0.312)	(0.290)
PC3	−0.030	**−0.077**	−0.001	**−0.093**	**−0.132**	−0.056	**37.170**	**36.277**
(t-statistic)	(−0.951)	(−2.466)	(−0.029)	(−2.987)	(−4.244)	(−1.783)	(0.000)	(0.000)
PC4	0.042	0.064	−0.007	0.015	−0.058	**−0.086**	10.444	9.374
(t-statistic)	(1.062)	(1.594)	(−0.189)	(0.367)	(−1.461)	(−2.147)	(0.107)	(0.095)
PC5	0.005	**−0.105**	0.012	−0.004	−0.073	**−0.324**	**45.691**	**45.688**
(t-statistic)	(0.097)	(−2.028)	(0.243)	(−0.070)	(−1.431)	(−6.287)	(0.000)	(0.000)

Figure: Five-factor model: market prices of risk.

Results

Table 5
Four-factor model: market prices of risk.

This table summarizes the estimates of the market price of risk parameters λ_0 and λ_1 for the four factor specification. *t*-Statistics are reported in parentheses. The standard errors have been computed according to the formulas from Section 2.3. Wald statistics for tests of the rows of Λ and of λ_1 being different from zero are reported along each row, with the corresponding *p*-values in parentheses below. PC1,…, PC5 denote the first through fifth principal components of Treasury yields. Bolded coefficients represent significance at the 5% level.

Factor	λ_0	$\lambda_{1.1}$	$\lambda_{1.2}$	$\lambda_{1.3}$	$\lambda_{1.4}$	W_Λ	W_{λ_1}
PC1	**−0.019**	−0.001	0.003	0.003	**−0.043**	**37.239**	**30.393**
(*t*-statistic)	(−2.597)	(−0.195)	(0.325)	(0.420)	(−5.002)	(0.000)	(0.000)
PC2	0.012	0.026	0.000	0.002	−0.025	6.746	5.981
(*t*-statistic)	(0.865)	(1.801)	(0.005)	(0.113)	(−1.524)	(0.240)	(0.201)
PC3	−0.015	−0.063	−0.051	**−0.109**	**0.111**	**18.326**	**18.110**
(*t*-statistic)	(−0.454)	(−1.862)	(−1.366)	(−3.186)	(2.861)	(0.003)	(0.001)
CP	−0.087	0.019	0.098	−0.010	**−0.291**	**27.851**	**25.546**
(*t*-statistic)	(−1.728)	(0.374)	(1.753)	(−0.188)	(−4.872)	(0.000)	(0.000)

Figure: Four-factor model: market prices of risk.

Results

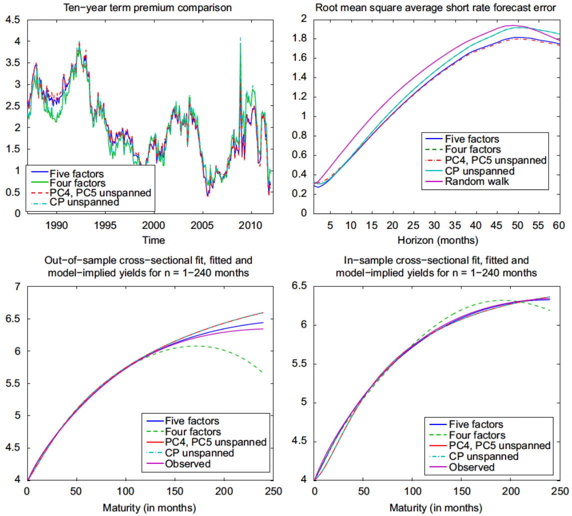

Figure: Comparison of models

Results

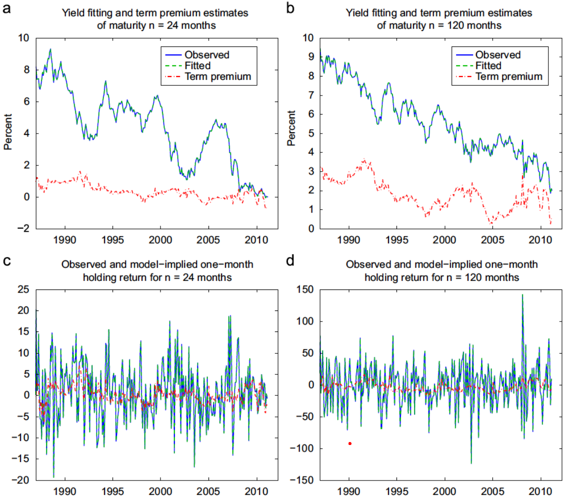

Figure: Five-factor model: observed and model-implied time series.

References

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