Hiba Safnaz

# Module 1

# OSCILLATIONS AND WAVES

A motion is repeated at regular interval of time is called periodic motion.

Eg. Oscillation of simple pendulum.
Motion of earth around the sun

Motion described functions of sine and cosine is called Harmonic oscillation.
A motion in which a particle moves to and fro about a fixed point and repeat the motion after regular interval of time is called oscillatory.

**Free Oscillations**

If no forces are acting on the particle, the oscillation is continued for indefinite period without change in amplitude.

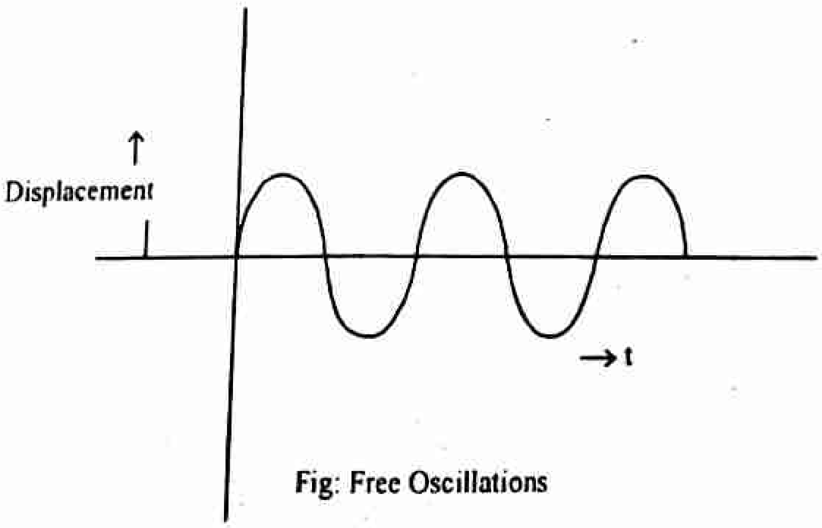



Fig: Free Oscillations

**Damped Oscillation**

A harmonic oscillator in which the motion is damped by the action of additional force is called damped harmonic oscillator.

When a body oscillates in a medium, it will experience a resistive force depending upon the nature of the medium. A part of energy utilized in overcoming these forces. Hence, amplitude goes on decreasing. These oscillations are called damped oscillation

For small amplitude, the damping force is proportional to velocity $\frac{-dx}{dt}$

**Differential equation of damped harmonic oscillation**

Consider a particle, executing damped harmonic oscillations

Restoring force = -kx

Damping force = $-b\frac{dx}{dt}$

$\therefore m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$

$$m\frac{d^2x}{dt^2} + kx + b\frac{dx}{dt} = 0$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x + \frac{b}{m}\frac{dx}{dt} = 0$$

$$\frac{d^2x}{dt^2} + 2\gamma\frac{dx}{dt} + \omega_0^2 x = 0 \text{ -----------------(1)}$$

Where $\frac{k}{m} = \omega_0^2$

$$\frac{b}{m} = 2\gamma$$

Where $\gamma$ is the damping coefficient

**Solution**

Suppose the solution of the form $x=Ae^{\alpha t}$

$$\frac{dx}{dt} = Ae^{\alpha t}.\alpha$$

$= \alpha x$

$$\frac{d^2x}{dt^2} = \alpha.Ae^{\alpha t}.\alpha$$

$= \alpha^2 x$

(1)=>

$$\alpha^2 x + 2\gamma\,\alpha x + \omega_0^2 x = 0$$

$$\alpha^2 + 2\gamma\,\alpha + \omega_0^2 = 0$$

$$\therefore \alpha = \frac{-2\gamma \pm \sqrt{4\gamma^2 - 4\omega_0^2}}{2}$$

$$= \frac{-2\gamma \pm 2\sqrt{\gamma^2 - \omega_0^2}}{2}$$

$$\alpha = -\gamma \pm \sqrt{\gamma^2 - \omega_0^2}$$

$\therefore$ Solution is $Ae^{\alpha t}$

$$x = Ae^{\left(-\gamma \pm \sqrt{\gamma^2 - \omega_0^2}\right)t}$$

General solution is

$$\boxed{x = A_1 e^{\left(-\gamma + \sqrt{\gamma^2 - \omega_0^2}\right)t} + A_2 e^{\left(-\gamma - \sqrt{\gamma^2 - \omega_0^2}\right)t}} \quad \text{——— (a)}$$

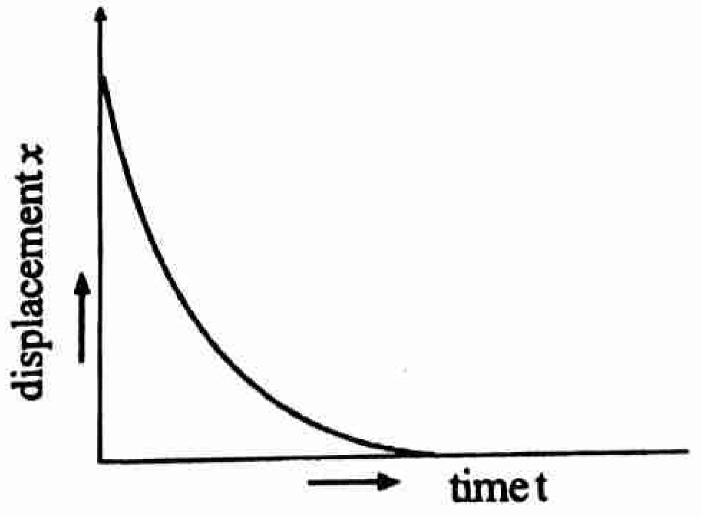
**Case 1 – Over damped**

If the damping is high, $\gamma > \omega_0$ then, $\sqrt{\gamma^2 - \omega_0^2}$ is real but it is less than r

$\therefore \left(-\gamma + \sqrt{\gamma^2 - \omega_0^2}\right) t$ *and* $\left(-\gamma - \sqrt{\gamma^2 - \omega_0^2}\right) t$, both are negative,

So, the displacement x decays exponentially to zero without any oscillation. This oscillation is called over damped or dead beat.

Eg. Dead beat galvanometer, door closing mechanism.



**Case 2 Critically Damped**

$\omega_0 = \gamma$, then $x = A_1 e^{-\gamma t} + A_2 e^{-\gamma t}$

x= $(A_1 + A_2)e^{-\gamma t}$

$=ce^{-\gamma t}$ where $A_1 + A_2 = c$

The above equation contains only one constant. Hence, this solution is not possible. Letus consider a case that,

$\sqrt{\gamma^2 - \omega_0^2}$ doesn't vanish but equal to very small quantity h

$$\sqrt{\gamma^2 - \omega_0^2} \cong h > 0$$

$$x = A_1 e^{-\gamma t} e^{ht} + A_2 e^{-\gamma t} e^{-ht}$$

$$= e^{-\gamma t}[A_1 e^{+ht} + A_2 e^{-ht}]$$

$$= e^{-\gamma t}[A_1(1 + ht) + A_2(1 - ht)]$$

Neglecting the higher power of h due to small amplitude.

$$= e^{-\gamma t}[(A_1 + A_2)ht + (A_1 - A_2)ht]$$

$$x = e^{-\gamma t}[P + Qt]$$

Where, $A_1 + A_2 = P$ and $(A_1 - A_2)h = Q$

As t increases, (P+Qt) increases. But the exponential term increases more than P+Qt.

∴Displacement decreases from maximum value to zero quickly. This motion is neither damped nor oscillatory. This motion is called critically damped. Here the particle suddenly acquires the equilibrium position.

**Applications**

1. Automobile shock absorber
2. Door closer mechanism
3. Recoil mechanism in guns.

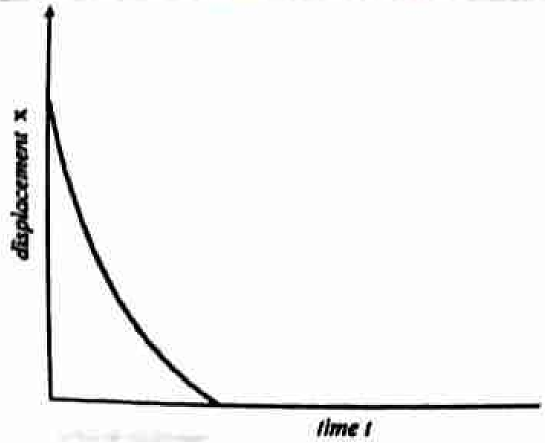



Fig. Critically damped.

**Case 3: Under Damped**

If damping is low, $\gamma < \omega_0$ (Oscillatory condition)

$$\sqrt{\gamma^2 - \omega_0^2} = \sqrt{-(\omega_0^2 - \gamma^2)} = i\omega$$

Where w=$\sqrt{\omega_0^2 - \gamma^2}$

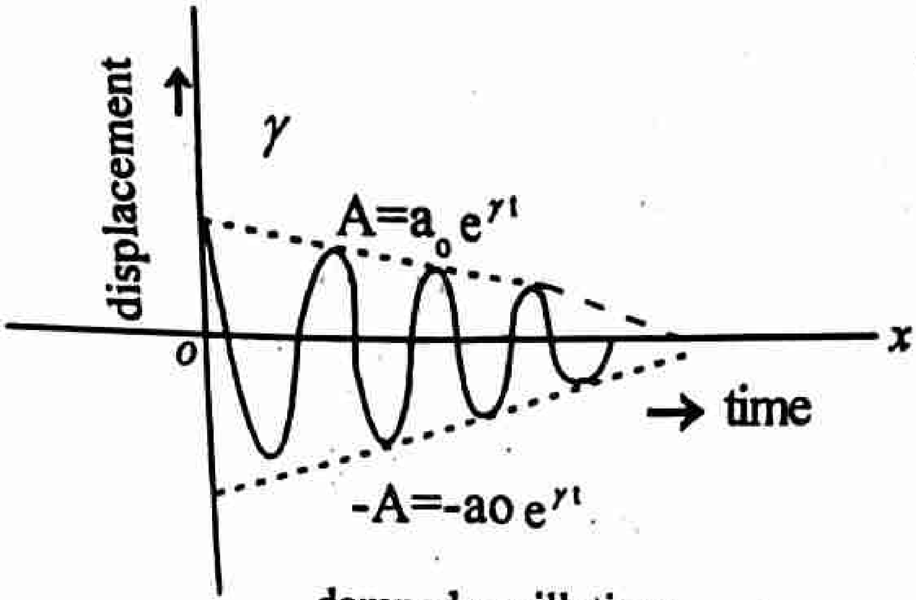
$$x = A_1 e^{(-\gamma+i\omega)t} + A_2 e^{(-\gamma-i\omega)t}$$

$$x = e^{-\gamma t}\left(A_1 e^{i\omega t} + A_2 e^{-i\omega t}\right)$$
$$x = e^{-\gamma t}[A_1(Cos\omega t + i\,Sin\omega t) + A_2(Cos\omega t - i\,Sin\omega t)]$$
$$x = e^{-\gamma t}[Cos\omega t(A_1 + A_2) + i\,Sin\omega t(A_1 - A_2)]$$
$$x = e^{-\gamma t}[Cos\omega t\, a_0\, sin\Phi + Sin\omega t a_0 \cos\Phi]$$

Where$A_1 + A_2 = a_0\, sin\Phi$ and $(A_1 - A_2)i = a_0 \cos\Phi$

$$x = e^{-\gamma t}[a_0\, sin\Phi Cos\omega t + a_0 \cos\Phi\, Sin\omega t]$$
$$x = a_0 e^{-rt}[Sin(\omega t + \Phi)]$$

The amplitude $a_o e^{-\gamma t}$ is not a constant. But decreases with time. The period of oscillation $T = \frac{2\pi}{\sqrt{\omega_0^2 - \gamma^2}}$




damped oscillations

**Effect of damping**

1. Amplitude of oscillation decreases exponentially with time
2. The frequency of oscillation of damped oscillator is less than frequency of undamped oscillation.

**Forced Harmonic Oscillator**

If an external force acts on a damped oscillator, the oscillating system is called forced harmonic oscillator.

The energy of a damped oscillation decreases on a passage of time. It is possible to compensate the energy loss by applying suitable external force. This supplies energy to the oscillator. When a particle is executing oscillation under an external forces there are 3 forces acting on it.

1. Restoring force ∝ displacement
2. Damping force ∝ velocity
3. External periodic force, which is represented by $F_0 Sin\omega_F t$

∴ Total force F = $F_1 + F_2 + F_3$

$$\frac{md^2x}{dt^2} = {}^-kx - b\frac{dx}{dt} + F_0 Sin\omega_f t$$

$$\frac{md^2x}{dt^2} + kx + b\frac{dx}{dt} = F_0 Sin\omega_f t$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x + \frac{b}{m}\frac{dx}{dt} = \frac{F_0}{m} Sin\omega_f t$$

$$\frac{d^2x}{dt^2} + \omega_0^2 x + 2\gamma \frac{dx}{dt} = F_0 Sin\omega_f t \quad -(1)$$

Where $\frac{k}{m} = \omega_0^2, \frac{b}{m} = 2\gamma, \frac{F_0}{m} = f_0$

This is the differential equation for forced harmonic oscillatior.

When external periodic force is applied both damping and forced terms contribute for the motion of oscillator and try to balance between the contribution of both. After sometime the system reaches a steady state. The resultant oscillations are called forced oscillations.

Let us consider the solution of above equation when steady state is reached.

$$x = A\sin(\omega_f t - \theta)$$

$$\frac{dx}{dt} = A\cos(\omega_f t - \theta)\,\omega_f$$

$$\frac{d^2x}{dt^2} = A\,\omega_f - sin(\omega_f t - \theta)\,\omega_f$$

$$= -A\,\omega_f^2\, Sin(\omega_f t - \theta)$$

Substitutingin (1)

$$-A\,\omega_f^2\, Sin(\omega_f t - \theta) + \omega_0^2\, A\, Sin(\omega_f t - \theta) + 2\,\gamma\, A\omega_f\, Cos\,(\omega_f t - \theta)$$

$$= F_0\, \sin(\omega_f t - \theta + \theta)$$

$$-A\,\omega_f^2\, Sin(\omega_f t - \theta) + \omega_0^2\, A\, Sin(\omega_f t - \theta) + 2\,\gamma\, A\omega_f\, Cos\,(\omega_f t - \theta)$$

$$= F_0\,[\sin(\omega_f t - \theta)cos\theta + cos(\omega_f t - \theta)\sin\theta\,]$$

$$[-A\,\omega_f^2 + \omega_0^2 A - F_0\, Cos\theta]Sin(\omega_f t - \theta) + [2\gamma\, A\omega_f - F_0\, Sin\theta]\, Cos\,(\omega_f t - \theta) = 0 \text{------} (2)$$

To find A,

The above relation satisfy for all value of t. The coefficient of terms

$Sin(\omega_f t - \theta)$ and $Cos(\omega_f t - \theta)$ must be zero separately

$-A\,\omega_f^2 + \omega_0^2 A - F_0\,Cos\theta = 0$

$A[\omega_0^2 - \omega_f^2] = F_0\,Cos\theta$ -------------------(3)

$2\gamma\, A\omega_f - F_0\,Sin\,\theta = 0$

$2\gamma\, A\omega_f = F_0\,Sin\,\theta$ -------------------(4)

Squaring and adding (3) and (4)

$A^2[\omega_0^2 - \omega_f^2]^2 + 4\gamma^2 A^2\omega_f^2 \quad = \quad {F_0}^2 Cos^2\theta + {F_0}^2 Sin^2\theta$

$A^2[(\omega_0^2 - \omega_f^2)^2 + 4\gamma^2\omega_f^2\,] \quad = \quad {F_0}^2\ (\because Cos^2\theta + Sin^2\theta = 1)$

$$\boxed{A = \frac{F_0}{\sqrt{(\omega_0^2 - \omega_f^2)^2 + 4\gamma^2\omega_f^2}}}$$

Which is the amplitude of forced oscillation,

dividing (4) by (3)

$\Rightarrow \tan\theta = \frac{2\gamma\omega_f}{\omega_0^2 - \omega_f^2}$

This gives the phase difference between forced oscillation and applied force,

$\therefore x = A\,Sin(\omega_f t - \theta)$

$$x = \frac{F_0}{\sqrt{(\omega_0^2 - \omega_f^2)^2 + 4\gamma^2\omega_f^2}} Sin\left[\omega_f t - \left(tan^{-1}\frac{2\gamma\omega_f}{\omega_0^2 - \omega_f^2}\right)\right]$$

**Amplitude Resonance**

$If\ \omega_0 = \omega_f$

If $\omega_f$ is increased the amplitude increase and reaches a maximum value. The particular frequency of applied force at which the amplitude of oscillation is maximum called resonant frequency.

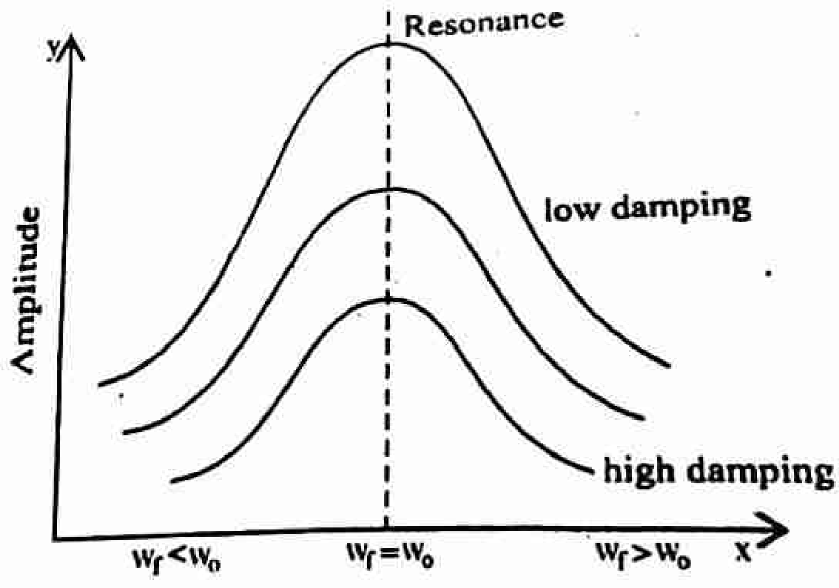
$$\boxed{A_{max} = \frac{F_0}{2\gamma\omega_f}}$$

$\tan\theta = \frac{2\gamma\omega_f}{\omega_0^2 - \omega_f^2} = \infty$

$\tan\theta = \infty$

$\theta = \frac{\pi}{2}$

Variation of amplitude A with frequency $\omega_f$ of applied force




**Sharpness of Resonance**

It is the rate of decrease of amplitude with change in frequency of applied periodic force on either side of resonance frequency.

**Quality factor**

It represents the efficiency of oscillator. It is $2\pi$ times the energy stored to the energy dissipated per the cycle.

$$Q = 2\pi\ x \frac{Energy\ stored\ per\ cycle}{Energy\ dissipated\ per\ cycle}$$

$$2\pi\ x \frac{Energy\ stored\ per\ cycle}{Energy\ dissipated\ per\ second\ x\ period\ T}$$

$$= \frac{2\pi E}{\left(\frac{-dE}{dt}\right)xt} \qquad = \frac{2\pi E}{PT}$$

$$\therefore \frac{-dE}{dt} = Power\ P$$

P= $\gamma$E

$$Q = \frac{2\pi E}{\gamma E\ x\ T}$$

$$= \frac{2\pi}{\gamma T}$$

$$= \frac{2\pi}{\gamma x \frac{2\pi}{\omega}} \qquad = \frac{\omega}{\gamma}$$

$Q = \frac{\omega}{\gamma} = \frac{2m\omega}{b}$ Where $\gamma = \frac{b}{2m}$

Q will be large if damping coefficient, $\gamma$ is low ie, efficiency is high.

**LCR CIRCUIT**

It is an electrical analogue of mechanical oscillator.

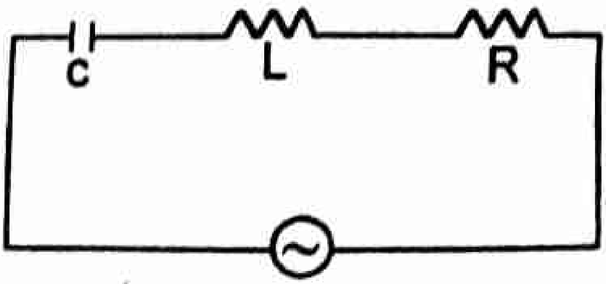
**Oscillations in L-C Circuit**

A pure L-C circuit is electrical analogue of simple pendulum. In simple pendulam energy alternate between potential energy and kinetic energy. In L-C circuit the energy is alternately stored in the capacitor as electric field and in the inductor as magnetic field.

The frequency of oscillation in the L-C circuit

$$n = \frac{1}{2\pi\sqrt{LC}}$$

**Forced Oscillations in a LCR Circuit**




$v = v_0 \sin \omega t$

Applying Kirchhoff's law to the circuit

$V_L + V_C + IR = V_0 \text{Sin } \omega t$

$L\frac{dI}{dt} + \frac{q}{C} + IR = V_0 \text{Sin } \omega t$

$L\frac{dI}{dt} + \frac{q}{C} + \frac{Rdq}{dt} = V_0 \text{Sin } \omega t$

$L\frac{d^2q}{dt^2} + \frac{q}{C} + \frac{dq}{dt}R = V_0 \text{Sin } \omega t$  $\left[\because I = \frac{dq}{dt}, \ \frac{dI}{dt} = \frac{d}{dt}\left(\frac{dq}{dt}\right) = \frac{d^2q}{dt^2}\right]$

$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{CL} = \frac{V_0}{L}\text{Sin } \omega t$

This is the differential equation in electrical oscillator

Physical quantities in mechanical oscillator and electrical oscillator.

| | Mechanical Oscillator | Electrical Oscillator |
|---|---|---|
| 1 | Displacement, x | Charge q |
| 2 | Velocity $\frac{dx}{dT}$ | Current $\frac{dq}{dt}$ |
| 3 | Mass m | Inductance L |
| 4 | Damping Co-efficient $\gamma$ | Resistance R |
| 5 | Force amplitude $f_0$ | Voltage amplitude $V_0$ |
| 6 | Driving frequency $\omega_f$ | Oscillator frequency $\omega_0$ |

# WAVES

Wave motion is a kind of disturbance or mode of momentum or energy transfer which is due to repeated periodic vibrations of particles of medium about equilibrium position

**Mechanical Wave**

It requires a material medium for their propagation. They are characterized by transport of energy through matter. Eg. Sound wave

**Electromagnetic wave**

It requires no medium for their propagation. They travel through vacuum. They are produced by oscillating electric charges. Electric field and magnetic field acts perpendicular to direction of propagation. Eg. Radio wave, micro wave.

| Transverse wave | Longitudinal wave |
|---|---|
| 1. Particles of medium vibrate in a direction perpendicular to the direction of propagation<br>2. Consist of crests and troughs Eg. Light wave. | 1. Particles of the medium vibrate parallel to the direction of propagation<br>2. Consists of compressions and rare faction. Eg.Sound wave. |

**One dimensional wave equation.**

The equation of wave motion is given by

$u(x,t) = f(x - vt)$

Differentiating equation (1) with respect to x and t twice

$\frac{\partial u}{\partial x} = f^{1}(x - vt)$ --(2)

$\frac{\partial^2 u}{\partial x^2} = f^{11}(x - vt)$ --(3)

Then,

$$\frac{\partial u}{\partial t} = f^{1}(x - vt) - v \qquad \text{--(4)}$$

$$\frac{\partial^2 u}{\partial t^2} = f^{11}(x - vt)v^2 \qquad \text{--(5)}$$

From the eqn (3)

$$\frac{\partial^2 u}{\partial t^2} = v^2 \frac{\partial^2 u}{\partial x^2}$$

$\frac{\partial^2 u}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2}$ This is one dimensional differential equation of wave motion

Eqn (4) =>

$\frac{\partial u}{\partial t} = -v\frac{\partial u}{\partial x}$ (From equn 2)

**3-Dimensional wave equation**

One dimensional wave equation is,

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{v^2} \cdot \frac{\partial^2 u}{\partial t^2}$$

In 3D,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{v^2} \cdot \frac{\partial^2 u}{\partial t^2}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)u = \frac{1}{v^2} \cdot \frac{\partial^2 u}{\partial t^2}$$

$\nabla^2 . u = \frac{1}{v^2} \cdot \frac{\partial^2 u}{\partial t^2}$

Where$\nabla^2 = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$, It is called Laplacian operator

**Solution of one dimensional equation**

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{v^2} \cdot \frac{\partial^2 u}{\partial t^2} \quad \text{------------------(1)}$$

Suppose, the solution of the form

$$u(x,t) = X(x)\,T(t) \quad \text{------------------(2)}$$

X(x) is a function of x and T(t) is a function of t.

Differentiating (2) with respect to x and t twice

$$\frac{d^2u}{dx^2} = T\frac{d^2X}{dx^2}$$

$$\frac{d^2u}{dt^2} = X\,\frac{d^2T}{dt^2}$$

Eqn (1) =>

$$T\,\frac{d^2X}{dx^2} = \frac{1}{v^2}\,X\,\frac{d^2T}{dt^2}$$

Dividing by X T.

$$\frac{1}{X}\frac{d^2X}{dx^2} = \frac{1}{v^2}\frac{1}{T}\frac{d^2T}{dt^2}$$

LHS and RHS contains only one variable.

Each side separately equate to common constant $-k^2$

$$\therefore \frac{1}{X}\frac{d^2X}{dx^2} = -k^2$$

$$\frac{d^2X}{dx^2} = -k^2X$$

$$\frac{d^2X}{dx^2} + k^2X = 0 \quad \text{-----------------------------(3)}$$

Then,

$$\frac{1}{v^2} \cdot \frac{1}{T} \cdot \frac{d^2T}{dt^2} = -k^2$$

$$\frac{d^2T}{dt^2} + k^2v^2\,T = 0$$

$$\frac{d^2T}{dt^2} + \omega^2 T = 0 \qquad \text{———————} (4)$$

Where $k^2v^2 = \omega^2$ $(\omega = kv)$

$X(x) = Ce^{\pm ikx}$

$T(t) \; = Ce^{\pm i\omega t}$

$u(x,t) = X(x).T(t)$

$u = Ce^{i(\pm kx \pm \omega t)}$

**Solution of 3 dimensional wave equation**

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{V^2}\frac{\partial^2 u}{\partial t^2} \qquad \text{———————} (1)$$

$\therefore$ *solution is*

$u(x, y, x, t) = X(x), Y(y), Z(z)T(t)$

$X = ce^{\pm ik_x x}$

$Y = ce^{\pm ik_y y}$

$Z = ce^{\pm ik_z z}$

$T = ce^{\pm i\omega t}$

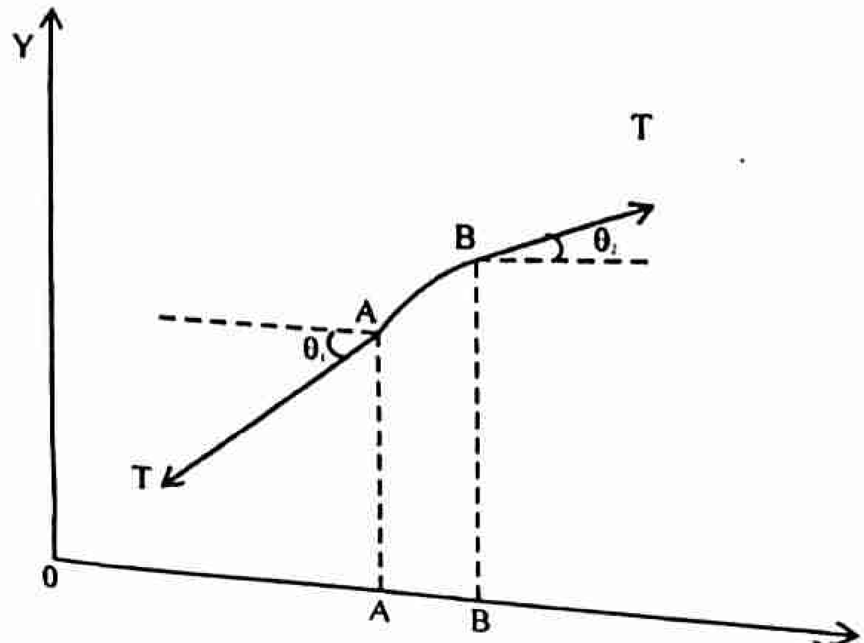
**Solution is**

$u(x,y,z,t) \quad = \quad X(x), Y(y), Z(z)\, T(t)$

$\quad = \quad ce^{\pm i(K_x x + K_y y + K_z z + \omega t)}$

$u(x,y,z,t) \quad = \quad ce^{\pm i(\vec{K}.\,\vec{r} + \omega t + \phi)}$

**Transverse waves in a stretched string.**




Consider a perfectly uniform string stretched between two points by a constant tension T. Let $\mu$ be mass per unit length of the wire. Let the string be slightly displaced along y-axis and released. Then, transverse vibrations set up in the string. The string is made up of large no.of small elements of length $\delta x$, $\theta_1$ and $\theta_2$ are the angles made by the tension T with x-axis at A and B

Then,

$F_x = T\cos\theta_2 - T\cos\theta_1$

$F_y = T\sin\theta_2 - T\sin\theta_1$

For small $\theta$, $F_x = 0$, [$\cos\theta = 1$]

$$F_y = T\tan\theta_2 - T\tan\theta_1 \quad [\sin\theta \cong \tan\theta]$$

Tan $\theta$ is the slop of the wave at any point

$\tan\theta = \frac{\partial y}{\partial x}$

$\tan\theta_2 = \left(\frac{\partial y}{\partial x}\right)_{x+\delta x}$

$\text{Tan}\ \theta_1 = \left(\frac{\partial y}{\partial x}\right)_x$

Then,

$$Fy = T\left[\left(\frac{\partial y}{\partial x}\right)_{x+\delta x} - \left(\frac{\partial y}{\partial x}\right)_x\right]$$

$$\mu\delta x\frac{\partial^2 y}{\partial t^2} = T\left[\left(\frac{\partial y}{\partial x}\right)_{x+\delta x} - \left(\frac{\partial y}{\partial x}\right)_x\right]$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{T}{\mu}\left[\frac{\left(\frac{\partial y}{\partial x}\right)_{x+\delta x} - \left(\frac{\partial y}{\partial x}\right)_x}{\delta x}\right]$$

We have $Lt\ \delta x \to 0\left[\frac{\left(\frac{\partial y}{\partial x}\right)_{x+\delta x} - \left(\frac{\partial y}{\partial x}\right)_x}{\delta x}\right] = \frac{\partial^2 y}{\partial x^2}$

$\therefore \frac{\partial^2 y}{\partial t^2} = \frac{T}{\mu}\frac{\partial^2 y}{\partial x^2}$ ------- (A)

This is the one dimensional differential equation of a transverse waves in a stretched string.

$\frac{\partial^2 y}{\partial t^2} = v^2\frac{\partial^2 y}{\partial x^2}$ ------ (B)

Comparing (A) and (B)

$$\therefore V = \sqrt{\frac{T}{\mu}}$$

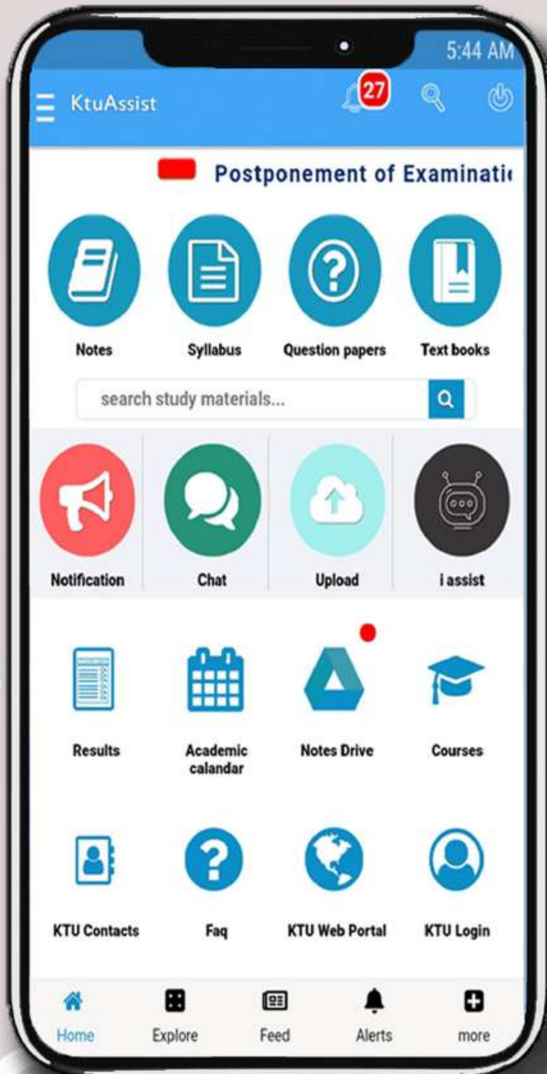
Frequency $\upsilon = \frac{v}{\lambda}$

$$\boxed{\upsilon = \frac{1}{\lambda}\sqrt{\frac{T}{\mu}}}$$

Where T is tension and $\mu$ is mass per unit length

**Law of Vibration**

The frequency of transverse waves is directly proportional to square root of tension an inversely proportional to wavelength and square root of linear density of the string.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Faq
KTU Web Portal
KTU Login
Home
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more

# MODULE –2
# WAVE OPTICS

**Huygen's wave theory**

Light is propagated through a medium in the form of transverse wave. Each point of source of light is a centre of disturbance fromwhich waves are propagated in all direction. Each point of this wave front is a source of secondary disturbance and wavelet emitting from these points spread out in all directions. The displacement of any particle at any instant, can be represented as,

y= A Sin $\omega$t

A is the amplitude and $\omega$t is the phase

**Period (T)**

It is the time taken by the particle to make one complete vibration

**Wavelength ($\lambda$)**

It is the distance between two successive crest or trough. It is the distance travelled by the wave during T seconds.

**Frequency($\upsilon$)**

Number of vibrations made by the particle in one second

$$\upsilon = \frac{1}{T}$$

Velocity V is equal to distance travelled in one second

$$V = \frac{\lambda}{T} = \upsilon\,\lambda$$

**Interference**

It is the remodification of light energy due to the super position of two or more light waves having nearly same amplitude same frequency and constant phase difference.

**Principle of super position**

Resultant displacement of the particle acted by two or more waves is equal to algebraic sum of the displacement due to each wave.

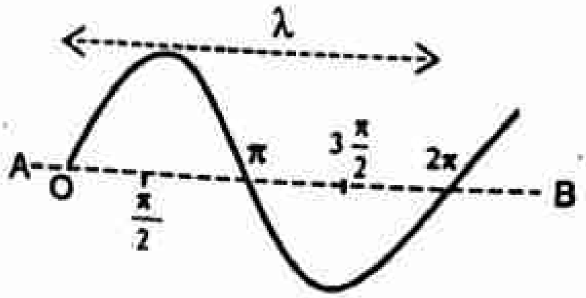
The superposition of waves resulting maximum intensity is called constructive interference and give minimum intensity called destructive interference.

There is no loss of energy due to interference, total energy remains constant only remodification of energy takes place.

**Path difference and Phase difference**

Path difference $\lambda$ corresponds to phase difference $2\pi$

$\lambda \rightarrow 2\pi$




Path difference $x$ corresponds to phase difference $\phi$

$x \rightarrow \phi$

$$\frac{\lambda}{x} = \frac{2\pi}{\phi}$$

$$\boxed{\phi = \left(\frac{2\pi}{\lambda}\right) x}$$

**COHRENT SOURCE**

The two source are said to be coherent if they emit light wave of nearly same amplitude, same frequency and constant phase difference.

Eg.:- Two slit illuminated by Monochromatic source of light.

**CONDITIONS FOR INTERFERENCE**

- Sources should be coherent
- Light wave from two coherent source should superimpose at the same time and same place
- The sources should be very close to each other.

**OPTICAL PATH**

A light travelling through a medium of refractive index $\mu$ and length d with a velocity v.


The time of travel in medium, $t = \frac{d}{v}$

During this time the distance travelled by the light in air = ct

$= \quad c \times \frac{d}{v}$

$= \quad \frac{c}{v} \times d$

$= \quad \mu d \left[\because \frac{c}{v} = \mu\right]$

μd is the optical path.

**CONDITION FOR MAXIMUM INTENSITY AND MINIMUM INTENSITY**

Path difference between the wave is integral multiple of $\lambda$

Path difference = n $\lambda$ where n=0,1,2,3,....

i.e path path difference is = 0, $\lambda$, 2 $\lambda$......

Condition for minimum intensity is half integral multiple of $\lambda$

i.e path difference = $(2n+1)\frac{\lambda}{2}$ where n=0,1,2,3,....

i.e path difference = $\frac{\lambda}{2}$, $3\frac{\lambda}{2}$, $5\frac{\lambda}{2}$, etc

The amplitude of two interfering waves must be nearly the same or equal

If $a_1 = a_2 = a$

$I_{max} = (a+a)^2 = 4a^2 = 4I_0$

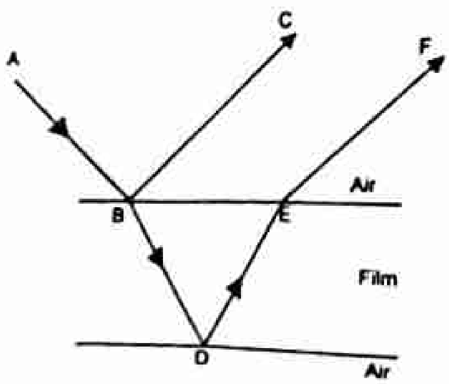
$I_0$ is intensity due to single wave.

$I_{min} = (a-a)^2 - 0$

For constructive interference, the resultant intensity is 4 times the intensity of incident wave.

**Path difference due to dissimilar reflection**

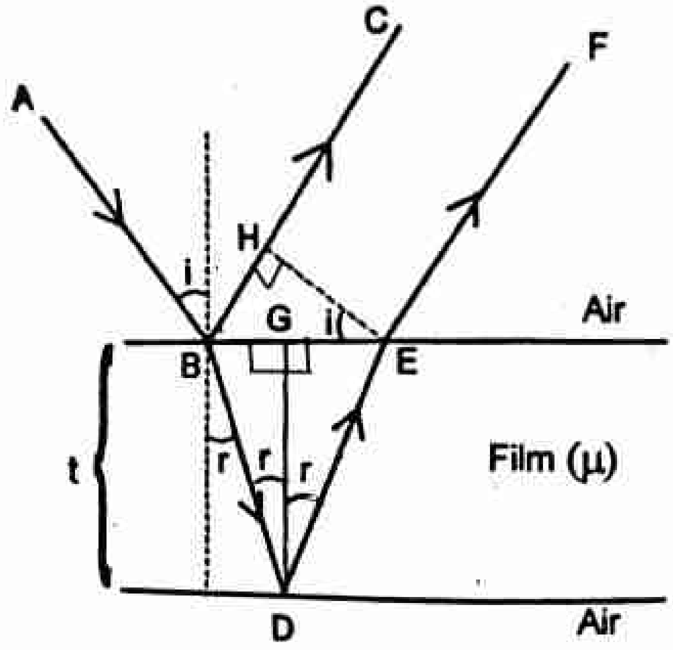
When reflection takes place at the surface of denser medium, there is a phase change of $\pi$ or additional path difference $\frac{\lambda}{2}$ with respect to the reflection takes place at rarer medium.




BC undergo phase change $\pi$ or additional path difference $\frac{\lambda}{2}$( Actual path difference is BC+$\frac{\lambda}{2}$ )

**Colour of thin films by reflected light**

Consider a thin transparent film of thickness, t and refractive index $\mu$. The optical path difference between the light rays reflected from top to bottom surface of the film is given by,




Path difference = $(BD+DE)_{film} - BH_{air}$

(Since BD and DE are in film equivalent length in air is (BD+DE) $\mu$)

Optical path difference = $\mu$ (BD+DE)-BH -(1)

Triangles BGD and ΔEGD are similar

∴ BD=DE

From the Δ BGD,

$$\text{Cos r} = \frac{GD}{BD}$$

$$= \frac{t}{BD}$$

$$BD = \frac{t}{\cos r} \quad -(2)$$

$$DE = \frac{t}{\cos r} \quad [\therefore BD = DE] \quad -(3)$$

Consider Δ BHE,

$$\text{Sin i} = \frac{BH}{BE} = \frac{BH}{BG+GE} = \frac{BH}{2BG} \; [\, BG = GE]$$

$$BH = 2\,BG \sin i$$

Consider Δ BGD,

$$\tan r = \frac{BG}{DG}$$

$$\tan r = \frac{BG}{t}$$

$$BG = t \times \tan r$$

$\therefore$ BH = 2BG Sin i

BH = 2t x tan r x Sin i -(4)

(2), (3) and (4) are substituted in (1), optical path difference is

$$\mu\left[\left(\frac{t}{\cos r}\right) + \left(\frac{t}{\cos r}\right)\right] - 2t \times \tan r \,\text{Sin i}$$

(in film) ( in air)

$$\text{opd} = \mu\frac{2t}{\cos r} - 2t \times \frac{Sin\, r}{\cos r} \times \mu\, Sin\, r \qquad \left[\frac{Sin\, i}{\cos r} = \mu\right]$$

$$= \frac{2\mu t}{\cos r}[1 - Sin^2 r]$$

$$= \frac{2\mu t}{\cos r} \times Cos^2 r \qquad Sin^2 r + Cos^2 r = 1$$

Opd $=2\,\mu$t Cos r

$\therefore$ Actual path difference between reflected rays BC and EF is,

$$\boxed{2\,\mu t \,\text{Cos r} - \frac{\lambda}{2}}$$

- **Condition for brightness (constructive interference)**

Path difference = $n\lambda$

$$2\,\mu t\,\text{Cos r} - \frac{\lambda}{2} = n\lambda$$

$$2\,\mu t\,\text{Cos r} = n\lambda + \frac{\lambda}{2}$$

$$2\,\mu t\,\text{Cos r} = (2n+1)\frac{\lambda}{2} \qquad n = 1,2,3,\ldots\ldots$$

- **Condition for darkness**

Path difference = $(2n+1)\frac{\lambda}{2}$

$$2\,\mu t\,\text{Cos r} - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$2\,\mu t\,\text{Cos r} = (2n+1)\frac{\lambda}{2} + \frac{\lambda}{2}$$

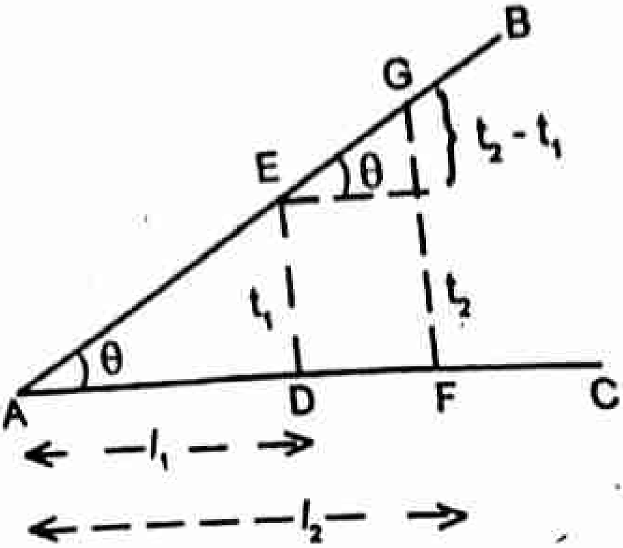
$$2\,\mu t\,\text{Cos r} = n\,\lambda$$

n = 0,1,2,3,.......

**Colour of thin films**

A thin transparent film viewed with white light shows different colour, the condition for darkness, $2\,\mu$t Cos r $= n\,\lambda$. A portion of the film satisfies the condition by substituting $\mu$, t and r for wavelength $\lambda$. Then, the colour of that wavelength will be absent. In reflected light mixing of remaining colour will be seen. Eg. If red light satisfies above condition, that colour is absent and that film appears in blue-violet region.

Air Wedge




- 2 glass plate AB and AC are placed, they are contact in one end and separated by small distance at the other end.
- A wedge shaped air film formed between them
- When a beam of monochromatic light is incident on a glass plate normally the reflected rays from top and bottom surface of air film interfere each other.
- Equidistant, II^el dark and bright bands are observed
- Angle between glass plate are called angle of air wedge $\theta$, which is in radian
- Thickness of $n^{th}$ and $(n+1)^{th}$ dark band $t_1$ and $t_2$, and they are at distance $l_1$ and $l_2$ from the point of contact respectively.

$$tan\ \theta = \frac{t_1}{l_1} = \frac{t_2}{l_2} = \frac{t_2 - t_1}{l_2 - l_1}$$

$\theta$ is very small, tan $\theta \cong \theta$

$\theta = \frac{t_2 - t_1}{\beta}$ --(1)

Where, $\beta = l_2 - l_1$

Band width, $\beta$ is the distance between 2 consecutive dark band or bright band.

- **Condition for $n^{th}$ dark band**

  $2\mu t_1 \text{ Cos r} = n\lambda$

For normal incidence cos r =1

For air $\mu = 1$

$2t_1 = n\lambda$ --(2)

- Condition for $(n+1)^{th}$

$2t_2 = (n+1)\lambda$ --(3)

(3)-(2) :-

$2(t_2-t_1) = \lambda$

$t_2 - t_1 = \frac{\lambda}{2}$, substituting this in equation (1)

(1) $\Rightarrow \theta = \frac{\lambda}{2\beta}$

If the medium between glass plate have refractive index $\mu$, Then

$$\theta = \frac{\lambda}{2\mu\beta}$$

**Applications**

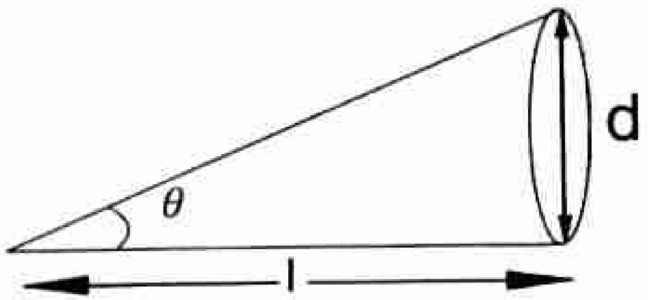
To find the diameter of wire

$$tan\theta = \frac{d}{l}$$

$$\theta = \frac{d}{l}$$

$$\frac{\lambda}{2\beta} = \frac{d}{l}$$

$$\boxed{d = \frac{\lambda l}{2\beta}}$$



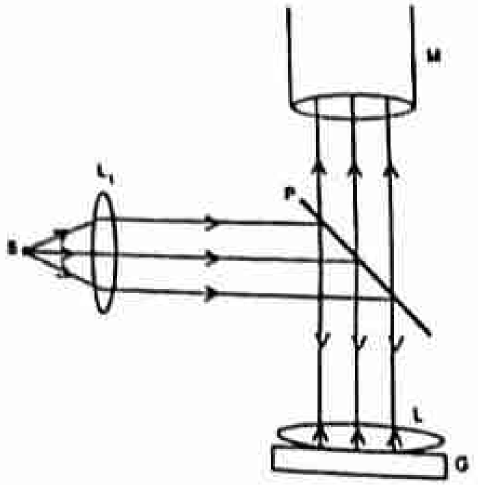

- **Optical planeness of surface**

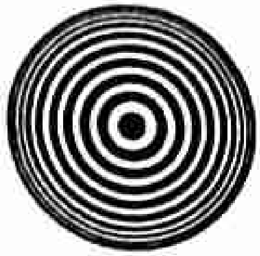
The planeness of a surface can be tested by observingthenature of band obtainec by air wedge experiment. If the surfaces are optically plane, the bands are parallel and equidistant. The bands are not have equal distant and parallel, the surfaces are not optically plane.

The given surface AB to be tested is placed on a optically plane glass plate AC which is called reference plate. A beam of monochromatic light allow to fall o air wedge. Then, if the interference pattern regular AB is optically plan otherwise it is not optically plane

**Newton's Ring**

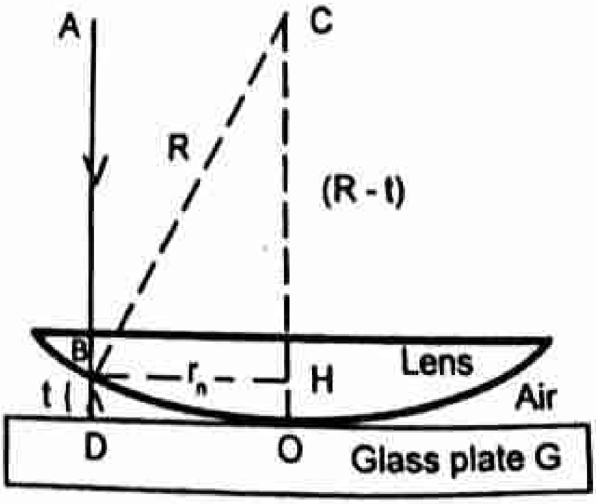
Circular interference fringes can be observed, if a very thin film of air or some other transparent medium of varying thickness is enclosed between a plane glass plate and plano convex lens of large radius of curvature.





The thickness of air film is zero at the point of contact between the lens glass plate and gradually increases towards the edge of the lens. The light from monochromatic source is made parallel by a convex lens and it is made to fall on a glass plate, inclined at $45^0$ to the incident beam. The light is reflected normally on to the system of plate and lens light reflected from top and bottom surface of air film superimpose each other and corresponding bright and dark rings are observed due to their path differences. The fringes are circular, because the thickness of the film is symmetric about the point of contact.(is constant over a circle)

To find the radious of $n^{th}$ dark ring




| | | |
|---|---|---|
| OH | = | t |
| OC | = | R |
| CH | = | R-t |
| BH | = | $r_n$ |

From $\Delta CHB$,

$R^2 = r_n^2 + (R-t)^2$

$R^2 = r_n^2 + R^2 - 2Rt + t^2$

t is very small, $t^2$ is neglected

$r_n^2 = 2Rt$ --(1)

Condition for dark ring,

For a medium of R-I $\mu$ $r_n = \sqrt{\frac{nRP}{\mu}}$

$2\mu t \cos r = n\lambda$

$\cos r = 1$ [for normal incidence]

For air $\mu = 1$

$2t = n\lambda$

(1) => $rn^2 = nR\lambda$ $r_n \alpha \sqrt{n}$

$$\boxed{r_n = \sqrt{nR\lambda}}$$

Radius of dark ring is proportional to square root of natural numbers.

Find the radius of bright ring.

$2\mu t \cos r = (2n+1)\frac{\lambda}{2}$

For normal incidence, cos r =1, for air $\mu = 1$

Incidence,

$2t = (2n+1)\frac{\lambda}{2}$

(1) => $r_n^2 = 2Rt$

$r_n^2 = \frac{(2n+1)R\lambda}{2}$

$r_n^2 = \sqrt{\frac{(2n+1)R\lambda}{2}}$

<u>To find the wavelength of monochromatic source of light</u>

$r_n^2 = nR\lambda$

We have, $r_n = \frac{D_n}{2}$

$\therefore \left(\frac{D_n}{2}\right)^2 = nR\lambda$

$D_{n^2} = 4nR\lambda$

$D^2{}_{n+k} = 4(n+k)R\lambda$

$D^2{}_{n+k} - D_n{}^2 = 4(n+k)R\lambda - 4nR\lambda$

$= 4nR\lambda + 4KR\lambda - 4nR\lambda$

$= 4KR\lambda$

$$\boxed{\lambda = \frac{{D_{n+k}}^2 - {D_n}^2}{4KR}}$$

In the case of bright ring

$${r_n}^2 = \frac{(2n+1)R\lambda}{2}$$

We have, $r_n = \frac{D_n}{2}$

$$\left(\frac{{D_n}^2}{2}\right) = \frac{(2n+1)R\lambda}{2}$$

$${D_n}^2 = \frac{4}{2}(2n+1)R\lambda$$

$${D_n}^2 = 2(2n+1)R\lambda$$

$$D^2{}_{n+k} = 2[2(n+k)+1]R\lambda$$

$$D^2{}_{n+k} - D^2{}_n = 2[2(n+k)+1]R\lambda - 2(2n+1)R\lambda$$

$$= 4KR\lambda$$

$$\lambda = \frac{D^2{}_{n+k} - D^2{}_n}{4KR}$$

**To find the refractive index of the medium (liquid)**

Let a liquid of refractive index $\mu$ be introduced between the glass plate and the lens.

Let $d_{n+k}$ and $d_n$ be the diameter of (n+k) and n[th] dark rings.

$$r_n = \sqrt{\frac{nR\lambda}{\mu}}$$

$$\left(\frac{d_n}{2}\right)^2 = \sqrt{\frac{nR\lambda}{\mu}}$$

$${d_n}^2 = \frac{4nR\lambda}{\mu}$$

$$d^2{}_{n+k} = \frac{4(n+k)R\lambda}{\mu}$$

$$d^2{}_{n+k} - d^2{}_n = \frac{4KR\lambda}{\mu}$$

$$\therefore \lambda = \mu\,\frac{d^2{}_{n+k} - dn^2}{4KR}$$

$$d^2{}_{n+k} - dn^2 = \frac{4KR\lambda}{\mu} \qquad \text{-(1)}$$

We know that, for air, medium

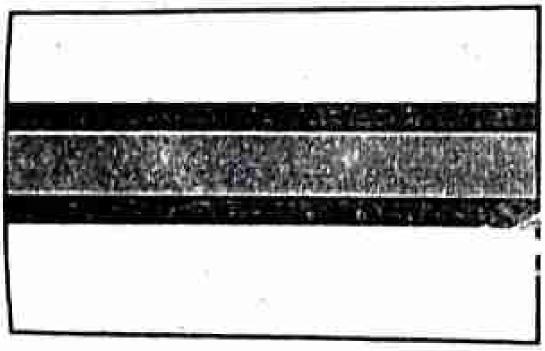
$$D^2_{n+k} - D^2{}_n = 4KR\lambda \quad -(2)$$

$$\boxed{\frac{(2)}{(1)} \Rightarrow \frac{D^2_{n+k} - D^2{}_n}{d^2_{n+k} - d^2{}_n} = \mu}$$

**Interference Filter**

The arrangement for filtering out a monochromatic beam from the incident white light is called interference filter.

Interference filter consists of a very thin film of transparent dielectric material like magnesium fluoride, cryoliteetc, placed in between two optically plane and flat glass plates. A very thin metallic coating of silver or aluminum is deposited by evaporation on the inner surfaces if the glass plates where the dielectric film is in contact.




When a narrow beam of white light is incident normally on this filter, it undergoes multiple reflections in the dielectric, it loses a major part of its intensity and finally emerge from the filter as a very narrow band. Reducing the thickness of the dietctricfilm, a particular wavelength (colour) can be filtered out.

**Anti reflection coating.**

This is an important application of thin film interference and it is used to reduce the loss of intensity of the incident beam of light by reflection. In so many optical instruments like telescopes, microscopes, camera etc, the incident light undergoes several reflections from many reflecting surfaces.

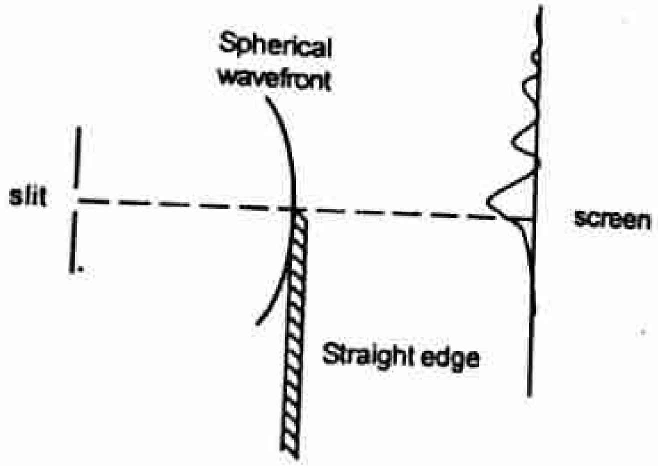
Such a loss of intensity dye to reflection can be reduced to a large extend or even can be eliminated by coating the reflecting surfaces of prisms, lenses etc. with a suitable transparent dielectric material such as calcium fluoride, magnesium fluoride, cryolite etc. The refractive index of such dielectric material must be in between that of air and glass. Such a film is called non-reflecting film or an

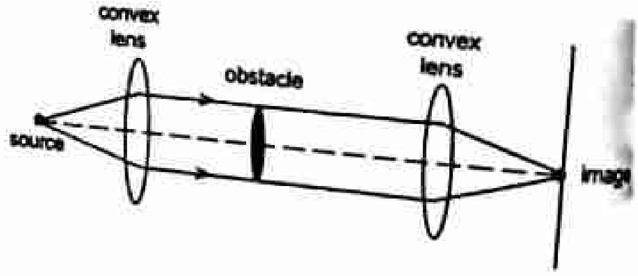
# DIFFRACTION

It is the phenomena of bending of light around the corners of an obstacle or encroachment of light into geometrical shadow region. It occurs when the size of the object comparable with wavelength of light used.

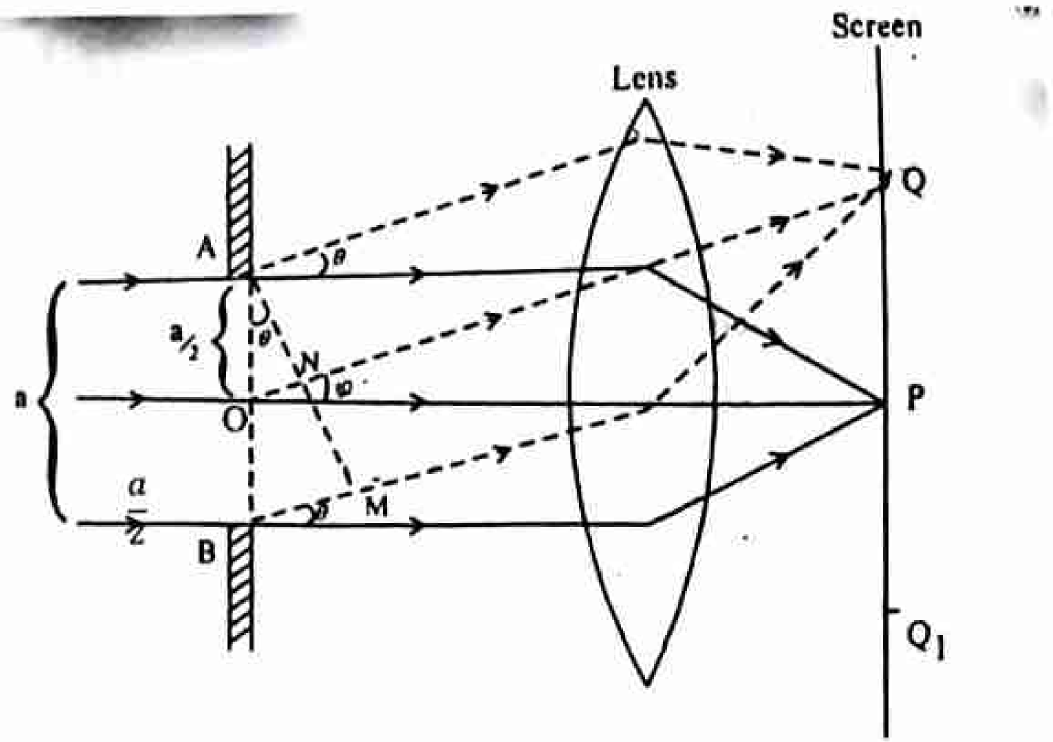
There are 2 classes of diffraction depending on the position of source and screen with respect to obstacle.

| **Fresnel diffraction** | **Fraunhofer diffraction** |
|---|---|
| Either the source or the screen are at finite distance from the obstacle | Source or the screen are at infinite distance |
| Wave front is spherical or cylindrical | Wave front is plane |
| Lenses are not used | Lenses are used to produce parallel rays. |
| Eg: Diffraction at straight edge | Plane transmission grating. |



**Fraunhofer diffraction at single slit**




Let AB be a narrow slit of width $a$ . A plane wave front of monochromatic light of wavelength $\lambda$ is falling normally on the slit. According to Huygen's principle each point on the wave front behaves like secondary waves. Almost all the waves proceeding from the source are travelling parallel to OP and focus at P. These rays are covering equal path and super impose constructively and producing maximum brightness at P. This is called central maxima. Consider another point Q above P where the waves are travelling at an angle $\theta$ with OP. The waves producing from A and B of the slit are reach at Q. Then AM is the perpendicular. BM is the path difference between the waves.

From the $\Delta\, AMB$,

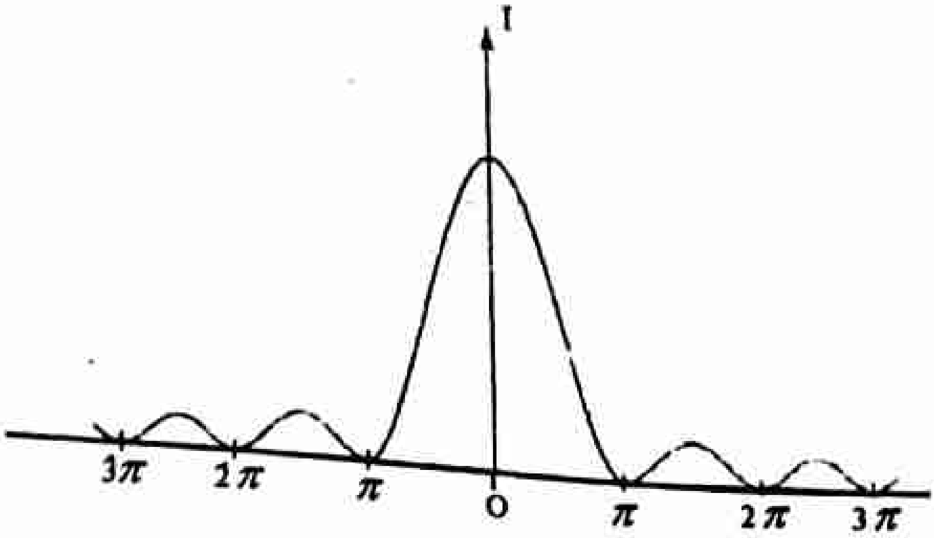
$$Sin\ \theta = \frac{BM}{AB}$$

BM = AB Sin $\theta$
BM = a Sin $\theta$

The point Q will be bright or dark depending on the pathdifference.
The general condition for minima,
$a\ Sin\theta_n = n\lambda$

The condition for secondary maxima,
$a\ Sin\theta_n = (2n+1)\frac{\lambda}{2}$

Where $\theta$ is the diffracting angle. The intensity distribution for diffraction at a single slit is shown.



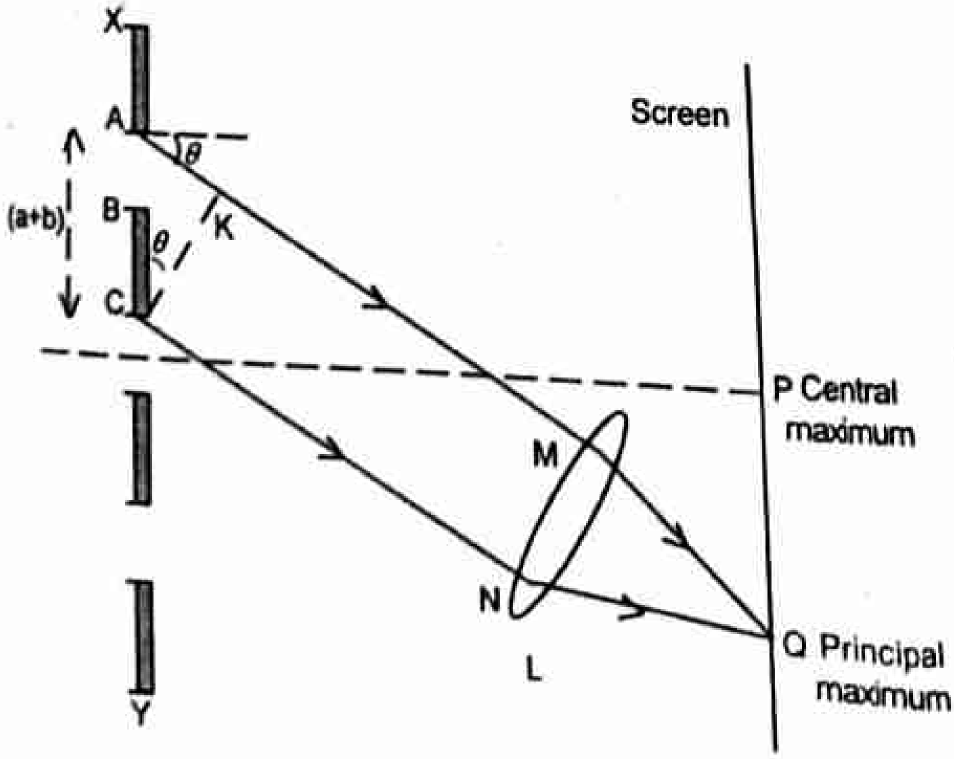

**Plane transmission grating**

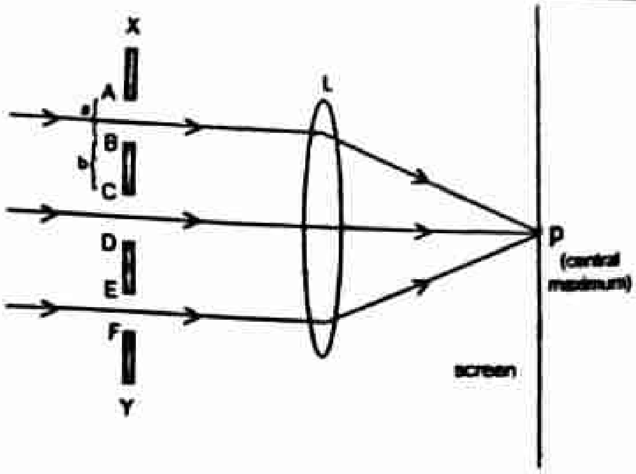
A plane transmission grating is a plane glass plate containing a large number of equidistant parallel lines drawn using fine diamond point. The space between the lines act asa narrow slit through which the light is transmitted. The lines are opaque to light. In the visible region grating contains 5000 lines to 12000 lines per cm.

**Conditions for good quality grating.**

1. Number of lines per cm should be large.
2. Spacing between the lines should be equal.

**Theory of diffraction grating.**






XY – represents a plane transmission grating, AB represents a slit width ä BC represents a line width $\ddot{b}$. AC represents (a+b) grating element or grating constant. Let a plane wavefront incident normally on a grating. Most of the waves travel in the direction of incident light and focused at P, and give a line of maximum intensity which has the colour of incident light. It is called central maximum. When the width of the slit is in the order of wavelength $\lambda$, light gets diffracted.

Consider two waves diffracted from A and C and $\theta$ be the angle of diffraction. Path difference is AK. From the $\Delta\, AKC$

$$Sin\ \theta = \frac{AK}{AC}$$

$$Sin\ \theta = \frac{AK}{a+b}$$

AK = (a+b)Sin $\theta$

The path difference equal to n $\lambda$, the 2 waves interfere constructively.

ie, (a+b)Sin$\theta$ $= n\lambda$ -(1)

where n=0,1,2,3……

All the waves starting from different corresponding points superimpose each other and give a bright line Q called principle maxima. If n=1, called first order maxima or first order spectrum, n=2 called 2$^{nd}$order spectrum. On either side of central maxima, number of principal maxima are obtained. If there are N lines/unit length, Then,

(a+b)N = 1 (unit length)

$$\therefore N = \frac{1}{a+b}$$

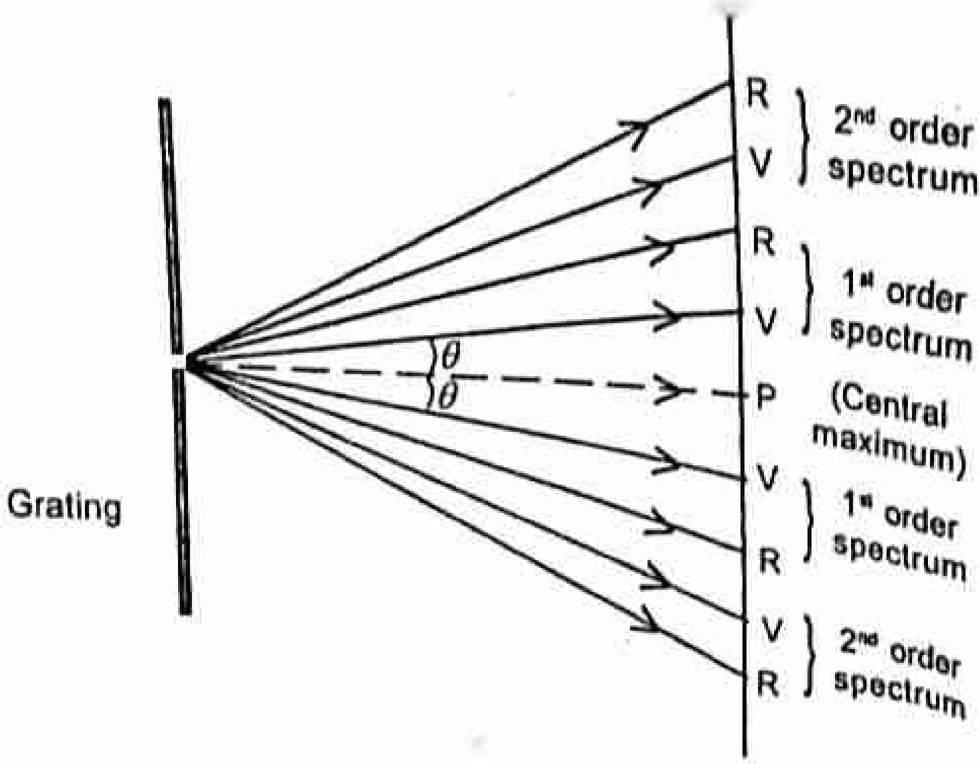
Eqn–(1) $\Rightarrow \frac{Sin\theta}{N} = n\lambda$

$\boxed{Sin\ \theta = Nn\lambda}$ Grating equation or grating law.

Where

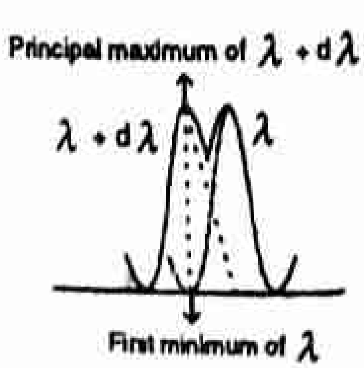
N – no.of lines / unit length
n – order of spectrum.

**Grating spectrum with white light.**
For grating spectrum with white light, $\theta$ is different for different colour, according to the equationSin $\theta$ = Nn$\lambda$




If the white light is used, it splits into different colour. For each value of n different colurs are diffracted at different angle.

**Rayleigh's criteria for resolution of spectral lines**
According to this two neighboring spectral lines will be just resolved when the principle maximum of one falls on minima of the other with the same order.




$\lambda$ and $\lambda + d\lambda$ be the wave length of 2 neighboring spectral lines with the same order. These two lines will be visible as separate when principle maxima of $\lambda + d\lambda$ falls on a minimum of $\lambda$

**Resolving power of grating**

It is the ability to show two neighbouring spectral lines as separate. It is the ratio of wavelength $\lambda$ to change in wavelength d$\lambda$

Resolving power $= \frac{\lambda}{d\lambda}$ where $\lambda = \frac{\lambda_1+\lambda_2}{2}$, d $\lambda = \lambda_1 - \lambda_2$

Suppose, there are Nslits in a grating path difference between the waves from the extreme slit change by $\lambda$. Therefore the path difference between the waves from adjacent slits change by $\frac{\lambda}{N}$.

According to Rayleigh's criteria two spectral lines will be resolved when principle maximum of $\lambda + d\lambda$ falls on minimum of $\lambda$

Consider, maximum condition for $\lambda + d\lambda$

$$(a+b)Sin\theta = n(\lambda + d\lambda)$$

Consider minimum condition for $\lambda$

$$(a+b)Sin\theta = n\,\lambda + \frac{\lambda}{N}$$

$$\therefore n(\lambda + d\lambda) = n\lambda + \frac{\lambda}{N}$$

$$nd\lambda = \frac{\lambda}{N}$$

$$\frac{\lambda}{d\lambda} = nN$$

$$\boxed{R.P = \frac{\lambda}{d\lambda} = nN}$$

**Dispersive power of grating**

It is the ratio of change in angle of diffraction to change in wavelength.

$$DP = \frac{d\theta}{d\lambda}$$

Consider 2 waves $\lambda$ and $\lambda + d\lambda$ diffracted through $\theta$ and $\theta + d\theta$

$$(a+b)Sin\theta = n\,\lambda$$

Differentiating on both sides

$$(a+b)Cos\theta\, d\theta = n\,\lambda\, d\lambda$$

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)cos\theta} = \frac{nN}{Cos\theta}$$

$$\left[\because N = \frac{1}{a+b}\right]$$

$$\boxed{D.P = \frac{d\theta}{d\lambda} = \frac{nN}{Cos\theta}}$$

| Interference band | Diffractive band |
| --- | --- |
| 1. Formed by superposition of waves from two coherent sources. | 1. Formed by super position of waves from different part of same wave front |
| 2. Bands may be of equal width | 2. Are never equal width |
| 3. Bands of minimum intensity are almost dark | 3. Bands of minimum intensity are not dark |
| 4. Intensity of bright band is almost same | 4. Not same |