

# HW4 - Past Linearity [MACS 30100]

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## Setup - Reading and Cleaning Data

```
gss_train_dfmin <- read_csv("../data/gss_train.csv") %>%
  select(egalit_scale, income06) %>%
  #mutate(egalit_scale = as.factor(egalit_scale)) %>%
  na.omit()

gss_test_dfmin <- read_csv("../data/gss_test.csv") %>%
  select(egalit_scale, income06) %>%
  #mutate(egalit_scale = as.factor(egalit_scale)) %>%
  na.omit()
```

## Q1: Polynomial Regression

```
polyreg_mse <- function(degree_val){
  set.seed(02162020)

  poly_control <- trainControl(method = "cv", number = 10)

  polyreg_mod <- train(form = egalit_scale ~ poly(income06, degree = degree_val),
    data = gss_train_dfmin %>% mutate(degree_val = degree_val), method = "lm", prePr
    metric = "RMSE", trControl = poly_control)

  #summary(polyreg_mod)

  train_mse <- mean((predict(polyreg_mod, newdata = gss_train_dfmin) - gss_train_dfmin$egalit_scale)^2)
  test_mse <- mean((predict(polyreg_mod, newdata = gss_test_dfmin) - gss_test_dfmin$egalit_scale)^2)

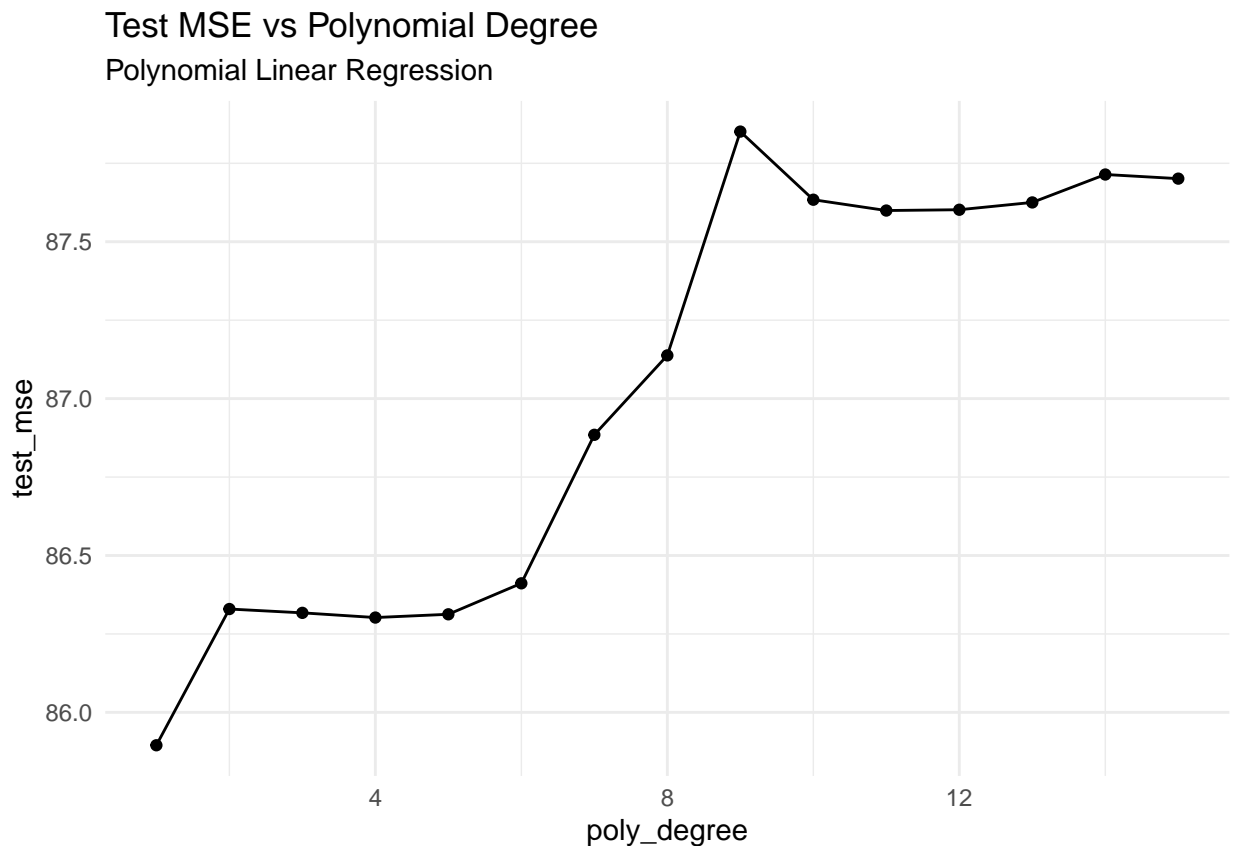
  out_tbl <- tibble(train_mse = train_mse,
    test_mse = test_mse)

  return(out_tbl)
}

polyreg_df <- c(1:15) %>%
  enframe(name = NULL, value = "poly_degree") %>%
  mutate(mse_values = map(poly_degree, ~polyreg_mse(.x))) %>%
  unnest(mse_values)

polyreg_df %>%
```

```
ggplot(aes(x = poly_degree, y = test_mse)) +
  geom_line() +
  geom_point() +
  theme_minimal() +
  ggtitle("Test MSE vs Polynomial Degree", subtitle = "Polynomial Linear Regression")
```



The best MSE is from degree = 1.

```
polymod_best <- lm(egalit_scale ~ income06, data = gss_train_dfmin)
summary(polymod_best)
```

```
##
## Call:
## lm(formula = egalit_scale ~ income06, data = gss_train_dfmin)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -22.5484  -6.7059   0.4677   7.2394  18.7828
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  25.86350    0.72845  35.505  <2e-16 ***
## income06     -0.38585    0.04119  -9.368  <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 9.351 on 1479 degrees of freedom
## Multiple R-squared:  0.05602,    Adjusted R-squared:  0.05538
## F-statistic: 87.76 on 1 and 1479 DF,  p-value: < 2.2e-16

gss_test_dfmin_polyaug <- gss_test_dfmin %>%
  bind_cols(pred_egalit_scale = predict(polymod_best, newdata = gss_test_dfmin)) %>%
  pivot_longer(cols = c("egalit_scale", "pred_egalit_scale"), names_to = "val_type", values_to = "egalit_scale")

gss_test_dfmin_polyaug %>% filter(str_detect(val_type, "pred")) %>%
  ggplot() +
  geom_line(aes(x = income06, y = egalit_scale, colour = val_type)) +
  geom_point(data = gss_test_dfmin_polyaug %>% filter(str_detect(val_type, "pred") == FALSE),
            aes(x = income06, y = egalit_scale, colour = val_type)) +
  theme_minimal() +
  ggtitle("Fit of Polynomial Regression against data",
          subtitle = "degree = 1")
```



```
margins::margins(polymod_best)
```

```
## Average marginal effects
```

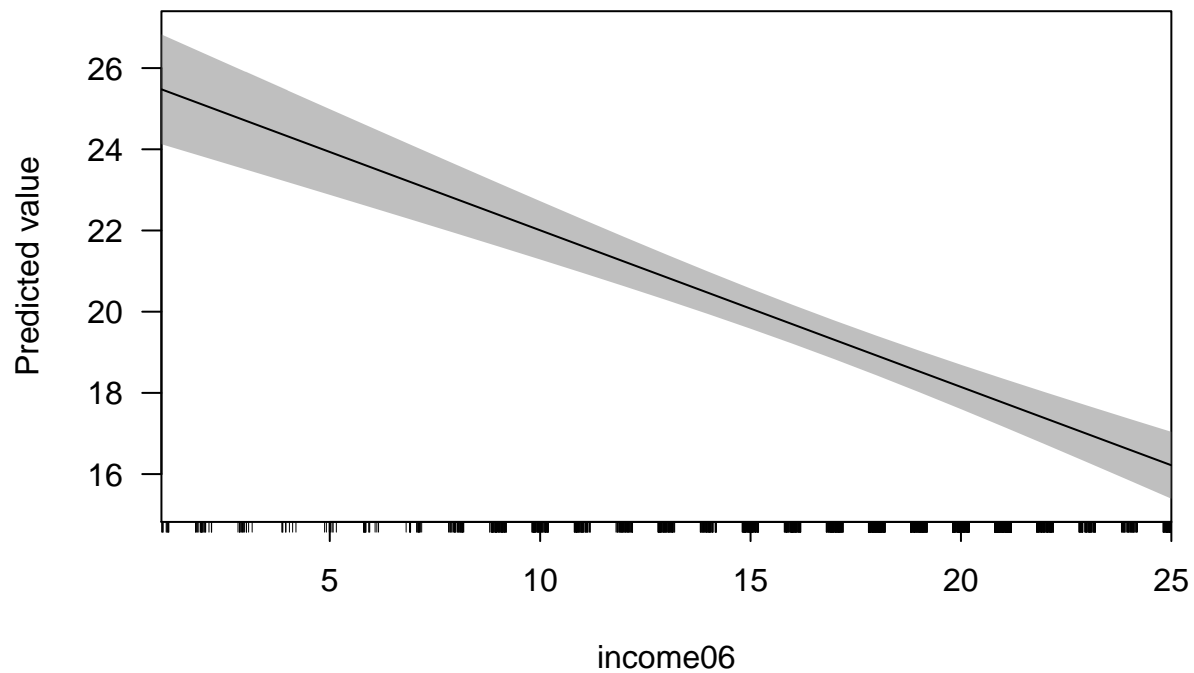
```
## lm(formula = egalit_scale ~ income06, data = gss_train_dfmin)
```

```
## income06
```

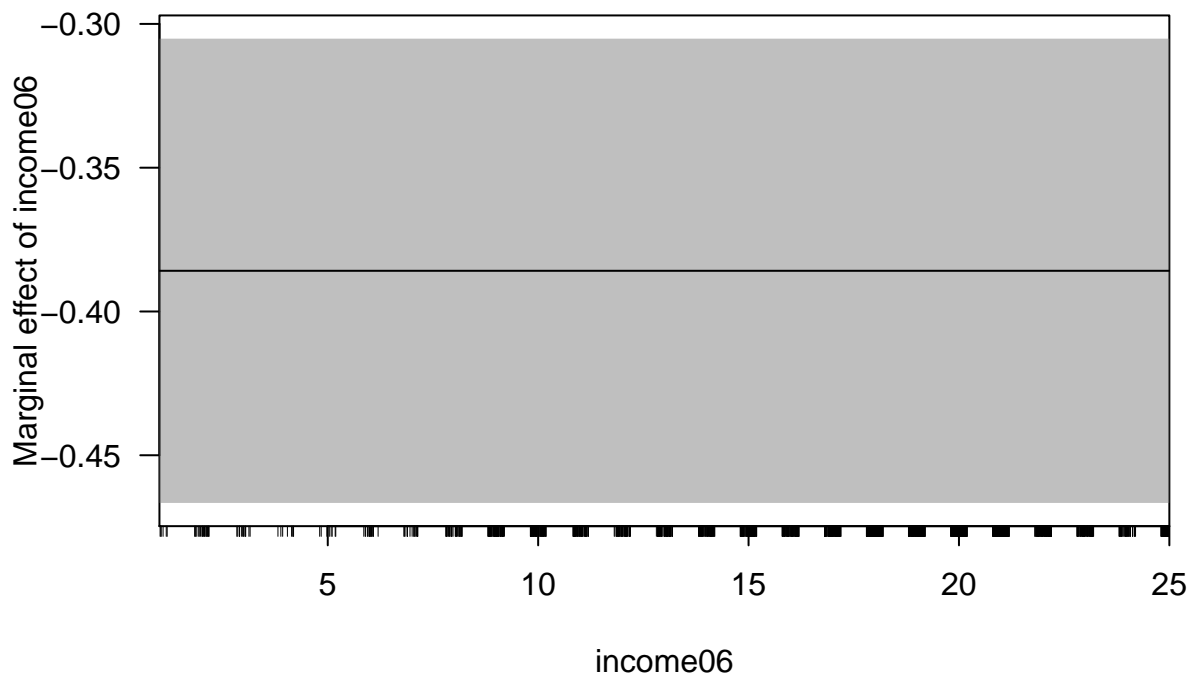
```
## -0.3859
```

```
cplot(polymod_best, "income06")
```

##	xvals	yvals	upper	lower
## 1	1	25.47765	26.82954	24.12575
## 2	2	25.09179	26.36846	23.81513
## 3	3	24.70594	25.90809	23.50380
## 4	4	24.32009	25.44857	23.19161
## 5	5	23.93424	24.99008	22.87840
## 6	6	23.54839	24.53286	22.56392
## 7	7	23.16253	24.07719	22.24788
## 8	8	22.77668	23.62346	21.92991
## 9	9	22.39083	23.17218	21.60948
## 10	10	22.00498	22.72402	21.28594
## 11	11	21.61913	22.27985	20.95840
## 12	12	21.23328	21.84084	20.62571
## 13	13	20.84742	21.40844	20.28640
## 14	14	20.46157	20.98444	19.93871
## 15	15	20.07572	20.57076	19.58068
## 16	16	19.68987	20.16921	19.21052
## 17	17	19.30402	19.78100	18.82703
## 18	18	18.91816	19.40632	18.43001
## 19	19	18.53231	19.04427	18.02035
## 20	20	18.14646	18.69322	17.59970



```
cplot(polymod_best, "income06", what = "effect")
```



Since the best-fit degree is 1, the final model is a linear one-variable fit. `income06` has a negative effect on `egalit_scale`, and the AME is constant at  $-0.386$ . As an individual moves into a higher income slab, their score on the constructed egalitarian scale decreases.

## Q2: Step Regression

```
egalit_levels <- as.character(sort(unique(gss_train_dfmin$egalit_scale)))

stepreg2_mse <- function(splits, ncuts){
  # estimate the model on each fold
  model <- glm(egalit_scale ~ cut(income06, ncuts),
    data = analysis(splits))

  test_mse <- mean((predict(model, newdata = gss_test_dfmin) - gss_test_dfmin$egalit_scale)^2)

  return(test_mse)

  # model_acc <- broom::augment(model, newdata = assessment(splits)) %>%
  #   accuracy(truth = factor(egalit_scale, levels = egalit_levels), estimate = factor(round(.fitted),
  #   #
  #   # mean(model_acc$.estimate)
  # }

# estimate CV error for knots in 0:25
```

```

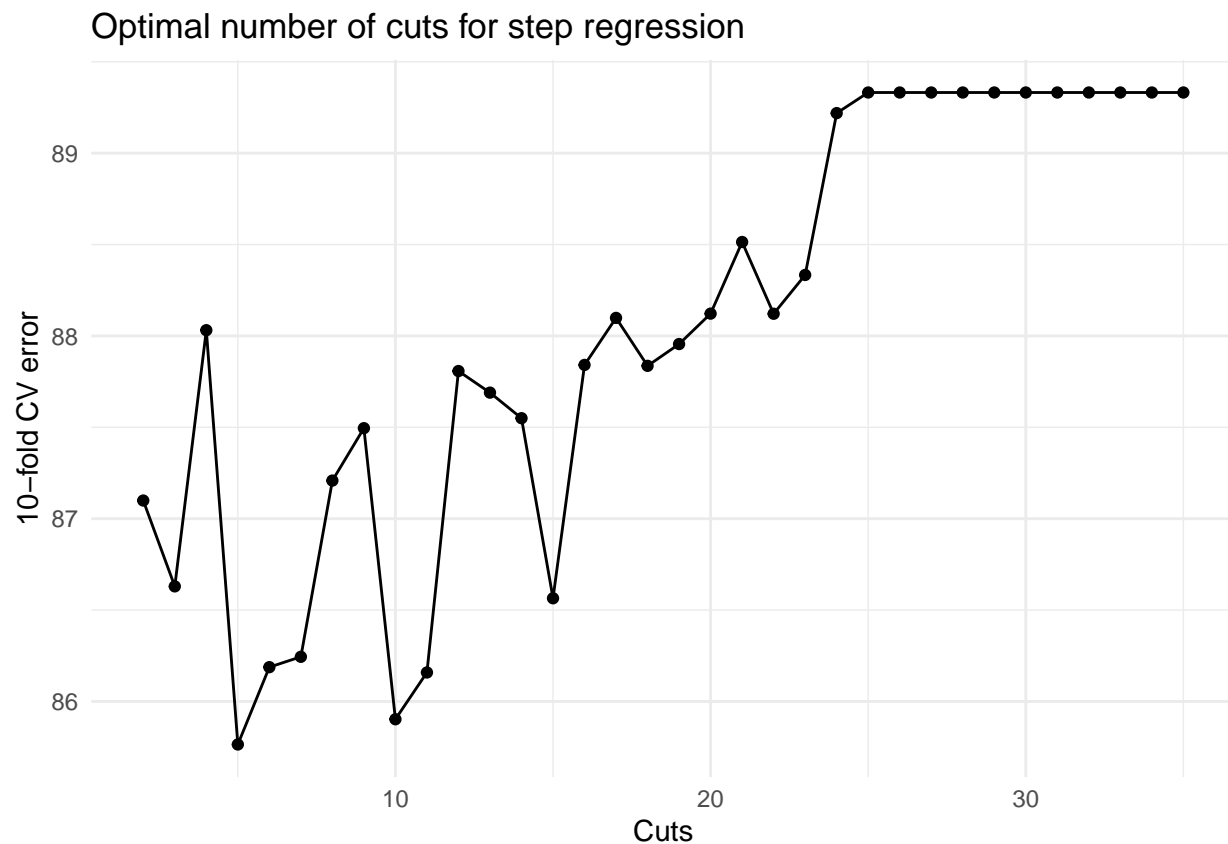
results_step <- vfold_cv(gss_train_dfmin, v = 10)

stepreg_df <- expand(results_step, id, ncuts = 2:35) %>%
  left_join(results_step) %>%
  mutate(test_mse = map2_dbl(splits, ncuts, stepreg2_mse)) %>%
  group_by(ncuts) %>%
  summarize(mean_test_mse = mean(test_mse))

## Joining, by = "id"

stepreg_df %>%
  ggplot(aes(ncuts, mean_test_mse)) +
  geom_point() +
  geom_line() +
  theme_minimal() +
  #scale_y_continuous(labels = scales::percent) +
  labs(title = "Optimal number of cuts for step regression",
       x = "Cuts",
       y = "10-fold CV error")

```



Optimal number of cuts: 5.

```

stepreg_best <- glm(egalit_scale ~ cut(income06, 5),
  data = gss_train_dfmin)

gss_test_dfmin_stepaug <- gss_test_dfmin %>%
  bind_cols(pred_egalit_scale = predict(stepreg_best, newdata = gss_test_dfmin)) %>%

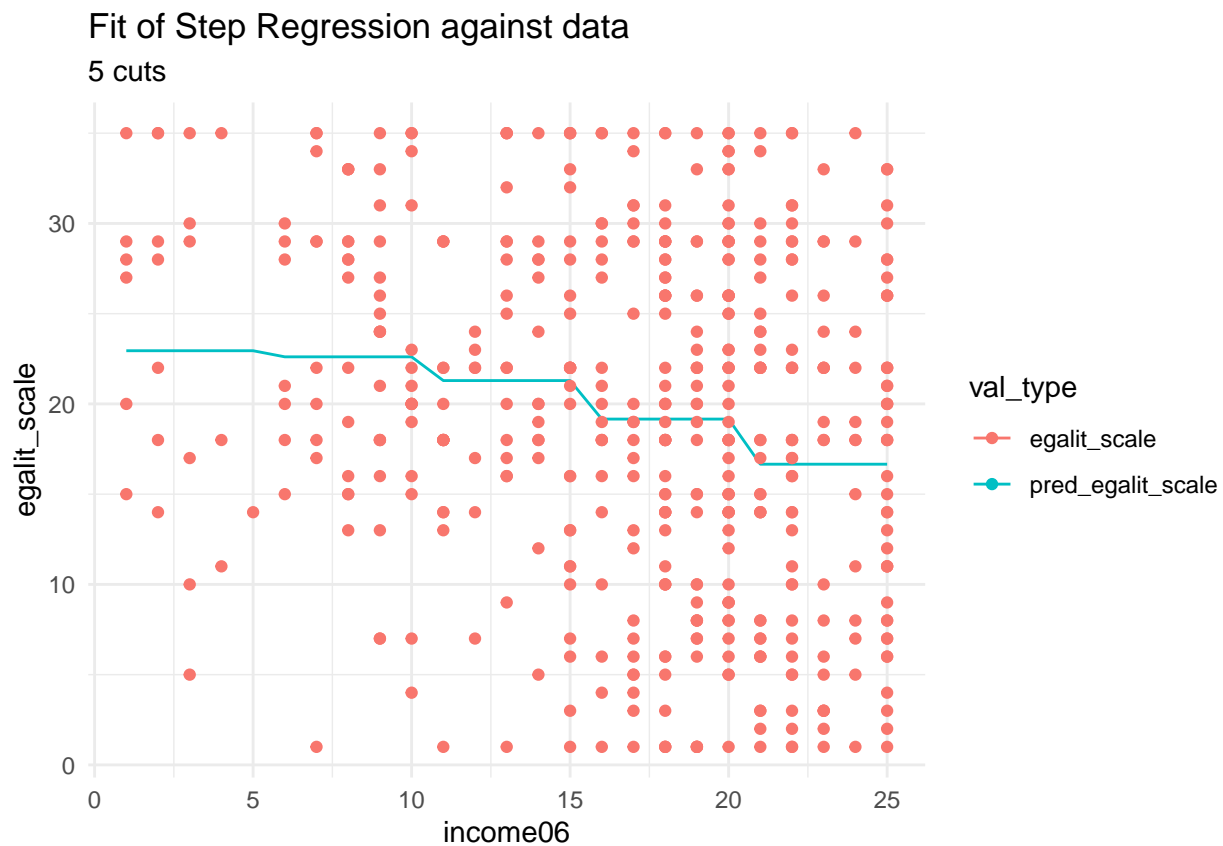
```

```

pivot_longer(cols = c("egalit_scale", "pred_egalit_scale"), names_to = "val_type", values_to = "egalit_scale")

gss_test_dfmin_stepaug %>% filter(str_detect(val_type, "pred")) %>%
  ggplot() +
  geom_line(aes(x = income06, y = egalit_scale, colour = val_type)) +
  geom_point(data = gss_test_dfmin_stepaug %>% filter(str_detect(val_type, "pred") == FALSE),
            aes(x = income06, y = egalit_scale, colour = val_type)) +
  theme_minimal() +
  ggtitle("Fit of Step Regression against data",
          subtitle = "5 cuts")

```



This is a strange model to fit, given the categorical nature of the data, and the fit seems arbitrary.

### Q3: Natural Spline Regression

```

# ## Failed Method 1
#
# nsplinerreg_mse <- function(degree_val, cut_val){
#   set.seed(02162020)
#
#   nspline_control <- trainControl(method = "cv", number = 10)
#
#   knots_len <- length(gss_train_dfmin$income06)
#   knots_val <- sort(gss_train_dfmin$income06)[2:(knots_len-1)]

```

```

#
#   nsplinerreg_mod <- train(form = egalit_scale ~ ns(income06, df = (degree_val*(1+cut_val))), knots = k,
#                           data = gss_train_dfmin %>% mutate(degree_val = degree_val,
#                                                           cut_val = cut_val),
#                           method = "glm",
#                           preProcess = c("center", "scale"),
#                           metric = "RMSE", trControl = nspline_control)
#
#   #summary(polyreg_mod)
#
#   train_mse <- mean(predict(nsplinerreg_mod, newdata = gss_train_dfmin) - gss_train_dfmin$egalit_scale)
#   test_mse <- mean(predict(nsplinerreg_mod, newdata = gss_test_dfmin) - gss_test_dfmin$egalit_scale)
#
#   out_tbl <- tibble(train_mse = train_mse,
#                     test_mse = test_mse)
#
#   return(out_tbl)
#   # if (mse_type == "train") return(train_mse) else return(test_mse)
# }
#
# nsplinerreg_df <- expand_grid(poly_degree = seq(1, 15, by = 1),
#                               n_cuts = seq(1, 15, by = 1)) %>%
#   mutate(mse_values = map2(poly_degree, n_cuts, nsplinerreg_mse)) %>%
#   unnest(mse_values)
#
# nsplinerreg_df %>%
#   ggplot(aes(x = n_cuts, y = test_mse)) +
#   geom_line() +
#   geom_point() +
#   theme_minimal() +
#   ggtitle("Test MSE vs Number of Cuts", subtitle = "Natural Spline Regression")

```

```

## FAILED METHOD 2 - Code from Lab
#
# # function to simplify things
# splinerreg_fun <- function(splits, df = NULL){
#
#   # estimate the model on each fold
#   model <- glm(egalit_scale ~ ns(income06, df),
#               data = analysis(splits))
#
#   model_mse <- broom::augment(model, newdata = assessment(splits)) %>%
#     rcfss::mse(truth = egalit_scale, estimate = .fitted)
#
#
#
#   return(mean(model_mse$.estimate))
# }
#
# tune_over_knots <- function(splits, knots){
#   splinerreg_fun(splits, df = (knots + 3))
# }
#
# # estimate CV error for knots in 0:25

```



```

# results <- vfold_cv(gss_train_dfmin, v = 10)
#
# splinereg_df <- expand(results, id, knots = 1:10) %>%
#   left_join(results) %>%
#   mutate(mse_values = map2(splits, knots, tune_over_knots))
#
# splinereg_df %>%
#   group_by(knots) %>%
#   summarize(mean_test_mse = mean(mse_values, na.rm = T)) %>%
#   ggplot(aes(knots, mean_test_mse)) +
#     geom_point() +
#     geom_line() +
#     #scale_y_continuous(labels = scales::percent) +
#     labs(title = "Optimal number of knots for natural cubic spline regression",
#           x = "Knots",
#           y = "10-fold CV error")

```

```

Error in qr.default(t(const)) : NA/NAN/Inf in foreign function call (arg 1)
24. qr.default(t(const))
23. qr(t(const))
22. ns(income06, df)
21. eval(predvars, data, env)
20. eval(predvars, data, env)
19. model.frame.default(formula = egalit_scale ~ ns(income06, df), data = analysis(splits), drop.unused.levels = TRUE)
18. stats::model.frame(formula = egalit_scale ~ ns(income06, df), data = analysis(splits), drop.unused.levels = TRUE)
17. eval(mf, parent.frame())
16. eval(mf, parent.frame())
15. glm(egalit_scale ~ ns(income06, df), data = analysis(splits))
14. splinereg_fun(splits, df = (knots + 3))
13. .f(.x[[1]], .y[[1]], ...)
12. map2(splits, knots, tune_over_knots)
11. mutate_impl(.data, dots, caller_env())
10. mutate.tbl_df(., mse_values = map2(splits, knots, tune_over_knots))
9. mutate(., mse_values = map2(splits, knots, tune_over_knots))
8. function_list[[k]](value)
7. withVisible(function_list[[k]](value))
6. reduce(value, `_function_list`)
5. `_fseq`(`_lhs`)
4. eval(quote(`_fseq`(`_lhs`)), env, env)
3. eval(quote(`_fseq`(`_lhs`)), env, env)
2. withVisible(eval(quote(`_fseq`(`_lhs`)), env, env))
1. expand(results, id, knots = 1:10) %>% left_join(results) %>% mutate(mse_values = map2(splits, knots, tune_over_knots))

```

Figure 1: Error traceback for the above chunk

## Q4: Estimating Egalitarianism with all Predictors

### Reading and Cleaning Data

```
gss_train_df <- read_csv("../data/gss_train.csv") %>%
  mutate_if(is.numeric, ~as.integer(.x)) %>%
  mutate_if(is.character, ~as.factor(.x))

gss_test_df <- read_csv("../data/gss_test.csv") %>%
  mutate_if(is.numeric, ~as.integer(.x)) %>%
  mutate_if(is.character, ~as.factor(.x))
```

### Helper Functions

```
model_gen <- function(model_type){
  set.seed(02142020)
  tr_control <- trainControl(method = "cv", number = 10)

  gss_model <- train(form = egalit_scale ~ ., data = gss_train_df,
    method = model_type, preProcess = c("center", "scale"),
    metric = "RMSE", trControl = tr_control)

  return(gss_model)
}
```

Implementing it in a tibble object:

```
result_df <- c("lm", "glmnet", "pcr", "pls") %>%
  enframe(name = NULL, value = "model_type") %>%
  mutate(model_obj = map(model_type, ~model_gen(.x)),
    predictor_obj = map(model_obj,
      ~Predictor$new(model = .x,
        data = gss_train_df %>%
          select(-egalit_scale),
        y = gss_train_df$egalit_scale)),
    featureimp_obj = map(predictor_obj, ~FeatureImp$new(.x, loss = "mse")),
    interaction_obj = map(predictor_obj, ~Interaction$new(.x)),
    all_interaction_obj = map(predictor_obj, ~FeatureEffects$new(.x)))
```

### Linear Regression

```
print("Training MSE for lm:")
```

```
## [1] "Training MSE for lm:"
```

```
mean((predict((result_df %>%
  filter(model_type == "lm") %>%
  dplyr::select(model_obj))$model_obj[[1]], newdata = gss_train_df) - gss_train_df$egalit.
```

```
## [1] 54.42554
```

```
print("Testing MSE for lm:")
```

```
## [1] "Testing MSE for lm:"
```

```

mean((predict((result_df %>%
  filter(model_type == "lm") %>%
  select(model_obj))$model_obj[[1]], newdata = gss_test_df) - gss_test_df$egalit_scale)^2)

## [1] 63.37644

featureimp_plot_lm <- plot((result_df %>%
  filter(model_type == "lm") %>%
  select(featureimp_obj))$`featureimp_obj`[[1]]) +
  theme_minimal() +
  ggtitle("Linear Regression",
    subtitle = "Feature Importance")

interaction_plot_lm <- plot((result_df %>%
  filter(model_type == "lm") %>%
  select(interaction_obj))$`interaction_obj`[[1]]) +
  theme_minimal() +
  ggtitle("Linear Regression",
    subtitle = "Overall Interaction Strength")

all_interaction_plot_lm <- plot((result_df %>% filter(model_type == "lm") %>%
  select(all_interaction_obj))$all_interaction_obj[[1]])

```

## Elastic Net

```

print("Training MSE for Elastic Net:")

## [1] "Training MSE for Elastic Net:"

mean((predict((result_df %>%
  filter(model_type == "glmnet") %>%
  dplyr::select(model_obj))$model_obj[[1]], newdata = gss_train_df) - gss_train_df$egalit_s

## [1] 55.9775

print("Testing MSE for Elastic Net:")

## [1] "Testing MSE for Elastic Net:"

mean((predict((result_df %>%
  filter(model_type == "glmnet") %>%
  dplyr::select(model_obj))$model_obj[[1]], newdata = gss_test_df) - gss_test_df$egalit_s

## [1] 61.38986

featureimp_plot_enet <- plot((result_df %>%
  filter(model_type == "glmnet") %>%
  select(featureimp_obj))$`featureimp_obj`[[1]]) +
  theme_minimal() +
  ggtitle("Elastic Net",
    subtitle = "Feature Importance")

interaction_plot_enet <- plot((result_df %>%
  filter(model_type == "glmnet") %>%
  select(interaction_obj))$interaction_obj[[1]]) +

```

```

theme_minimal() +
ggtitle("Elastic Net",
        subtitle = "Overall Interaction Strength")

all_interaction_plot_enet <- plot((result_df %>%
                                filter(model_type == "glmnet") %>%
                                select(all_interaction_obj))$all_interaction_obj[[1]])

```

## Principal Component Regression

```

print("Training MSE for pcr:")

## [1] "Training MSE for pcr:"
mean((predict((result_df %>%
               filter(model_type == "pcr") %>%
               dplyr::select(model_obj))$model_obj[[1]], newdata = gss_train_df) - gss_train_df$egalit_s

## [1] 70.99323
print("Testing MSE for pcr:")

## [1] "Testing MSE for pcr:"
mean((predict((result_df %>%
               filter(model_type == "pcr") %>%
               dplyr::select(model_obj))$model_obj[[1]], newdata = gss_test_df) - gss_test_df$egalit_s

## [1] 69.07672

featureimp_plot_pcr <- plot((result_df %>%
                            filter(model_type == "pcr") %>%
                            select(featureimp_obj))$`featureimp_obj`[[1]]) +
  theme_minimal() +
  ggtitle("Principal Component Regression",
          subtitle = "Feature Importance")

interaction_plot_pcr <- plot((result_df %>%
                             filter(model_type == "pcr") %>%
                             select(interaction_obj))$interaction_obj[[1]]) +
  theme_minimal() +
  ggtitle("Principal Component Regression",
          subtitle = "Overall Interaction Strength")

all_interaction_plot_pcr <- plot((result_df %>%
                                filter(model_type == "pcr") %>%
                                select(all_interaction_obj))$all_interaction_obj[[1]])

```

## Partial Least Squares Regression

```

print("Training MSE for pls:")

## [1] "Training MSE for pls:"
mean((predict((result_df %>%
               filter(model_type == "pls") %>%

```

```

dplyr::select(model_obj))$model_obj[[1]], newdata = gss_train_df) - gss_train_df$egalit,

## [1] 56.74024
print("Testing MSE for pls:")

## [1] "Testing MSE for pls:"
mean(predict((result_df %>%
  filter(model_type == "pls") %>%
  dplyr::select(model_obj))$model_obj[[1]], newdata = gss_test_df) - gss_test_df$egalit_s

## [1] 62.17684
featureimp_plot_pls <- plot((result_df %>%
  filter(model_type == "pls") %>%
  select(featureimp_obj))$`featureimp_obj`[[1]]) +
  theme_minimal() +
  ggtitle("Partial Least Squares Regression",
    subtitle = "Feature Importance")

interaction_plot_pls <- plot((result_df %>%
  filter(model_type == "pls") %>%
  select(interaction_obj))$interaction_obj[[1]]) +
  theme_minimal() +
  ggtitle("Partial Least Squares Regression",
    subtitle = "Overall Interaction Strength")

all_interaction_plot_pls <- plot((result_df %>%
  filter(model_type == "pls") %>%
  select(all_interaction_obj))$all_interaction_obj[[1]])

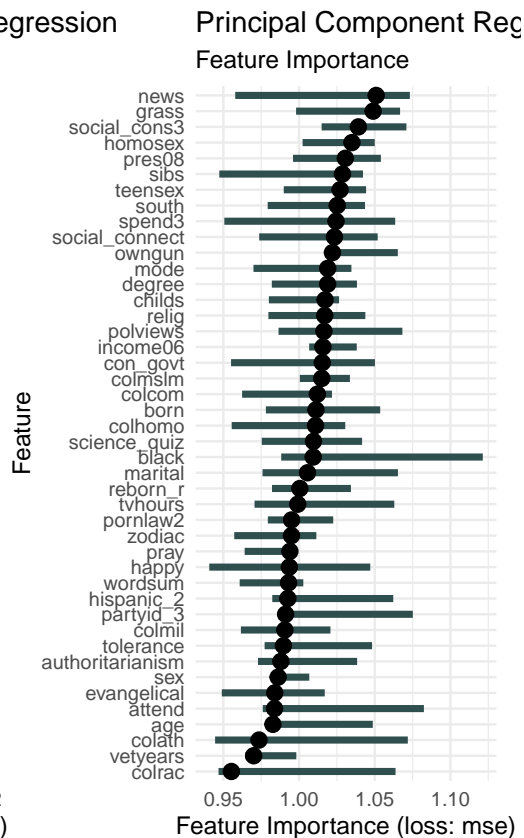
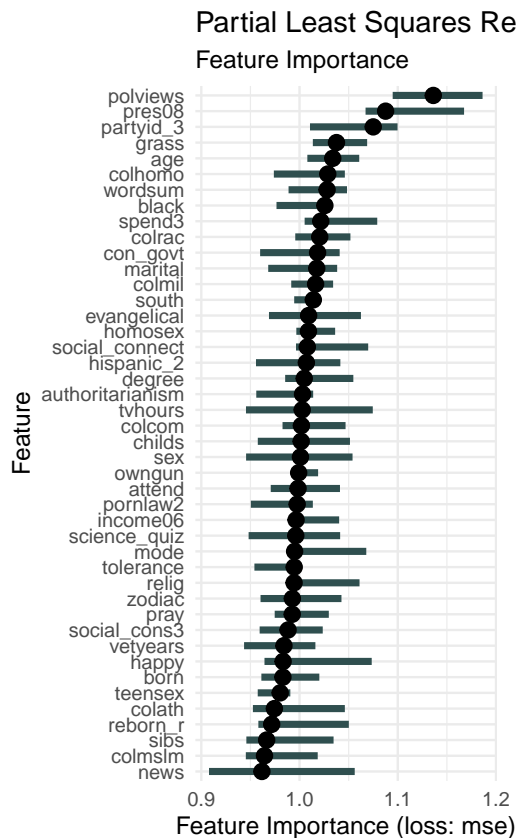
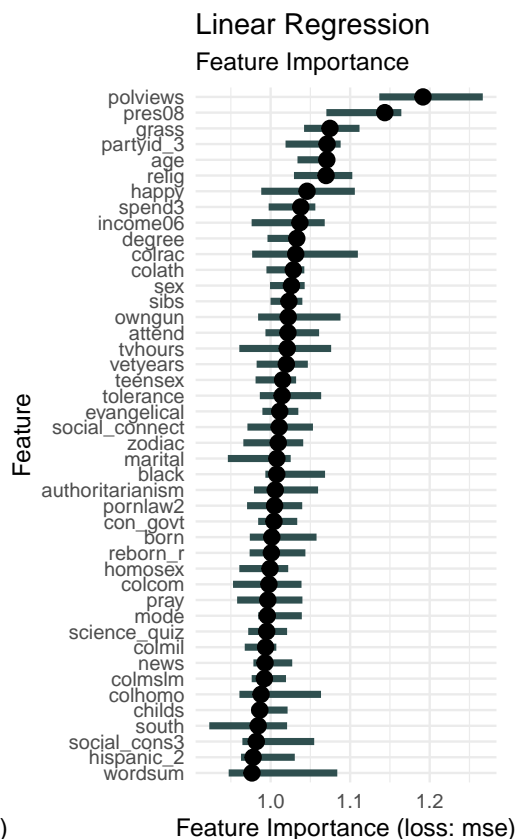
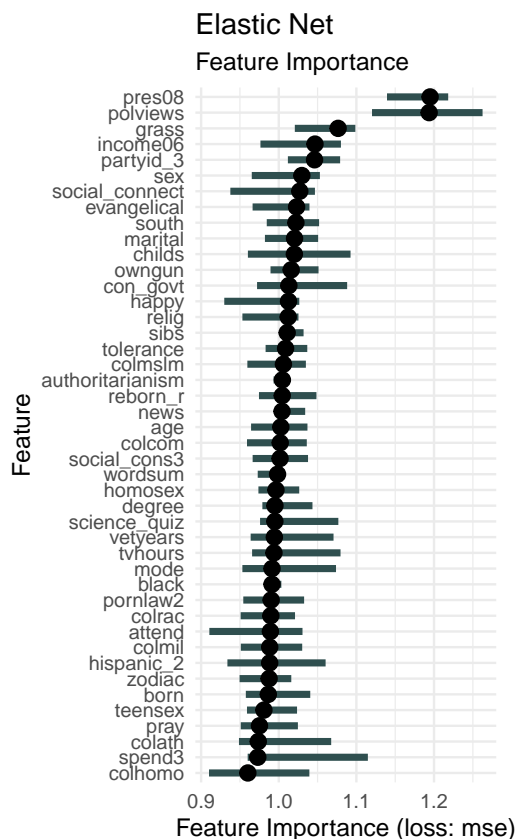
```

## Q5: Feature Importance & Interaction Effects

### Feature Importance

pres08, polviews, grass, partyid\_3 seem to show up as the top features across models, which is in line with expectations – redistribution and opportunity play an important role in political narratives in the US. PCR seems to weight other variables as more important though, and much larger error-bars.

```
(featureimp_plot_enet + featureimp_plot_lm + featureimp_plot_pls + featureimp_plot_pcr)
```



```

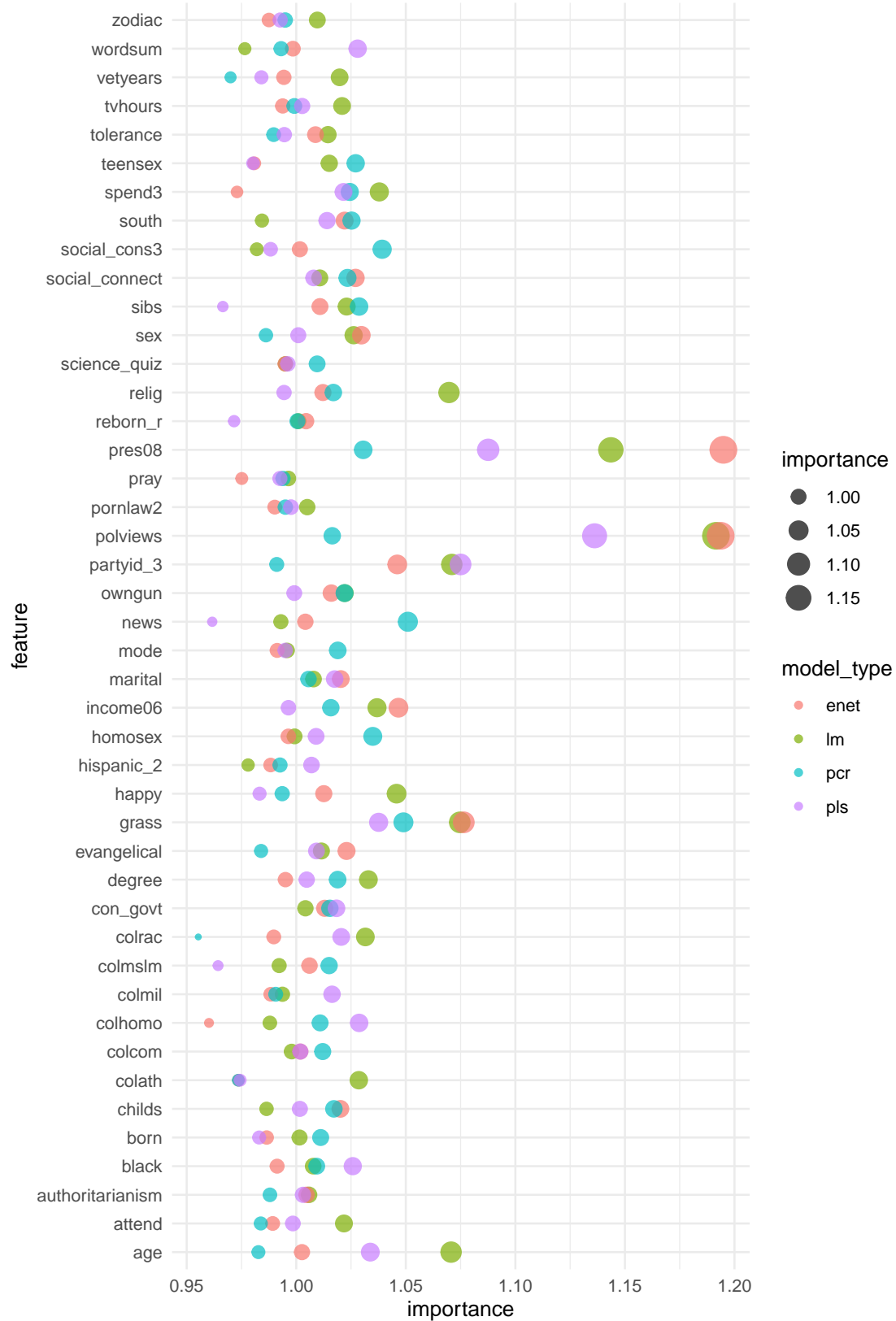
featureimp_values <- bind_rows((result_df %>%
  filter(model_type == "lm") %>%
  select(featureimp_obj)$featureimp_obj[[1]]$results %>%
  as_tibble() %>%
  mutate(model_type = "lm"),
(result_df %>%
  filter(model_type == "glmnet") %>%
  select(featureimp_obj)$featureimp_obj[[1]]$results %>%
  as_tibble() %>%
  mutate(model_type = "enet"),
(result_df %>%
  filter(model_type == "pcr") %>%
  select(featureimp_obj)$featureimp_obj[[1]]$results %>%
  as_tibble() %>%
  mutate(model_type = "pcr"),
(result_df %>%
  filter(model_type == "pls") %>%
  select(featureimp_obj)$featureimp_obj[[1]]$results %>%
  as_tibble() %>%
  mutate(model_type = "pls")))

featureimp_values %>%
  ggplot(aes(x = feature, y = importance, colour = model_type, size = importance)) +
  geom_point(alpha = 0.7) + coord_flip() +
  theme_minimal() +
  ggtitle("Comparison of Feature Importance across Models",
    subtitle = "Elastic Net, Linear Regression, Principal Component Regression, Partial Least Squares")

```

## Comparison of Feature Importance across Models

Elastic Net, Linear Regression, Principal Component Regression, Partial Least

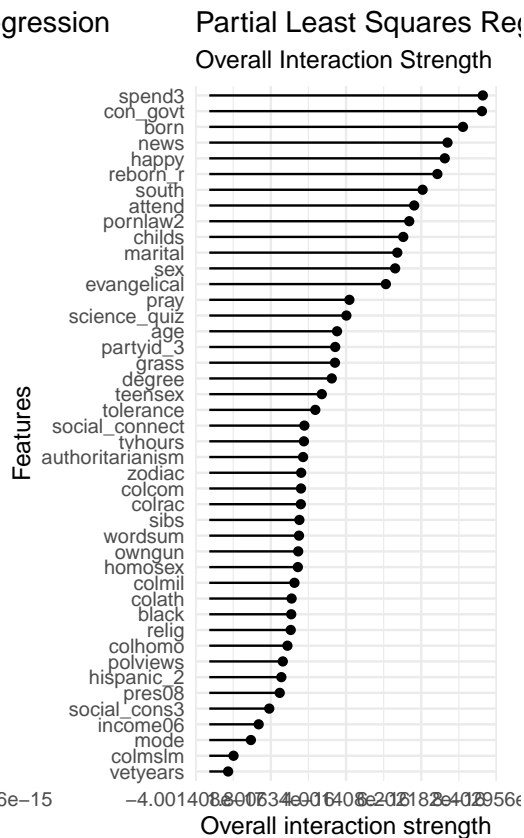
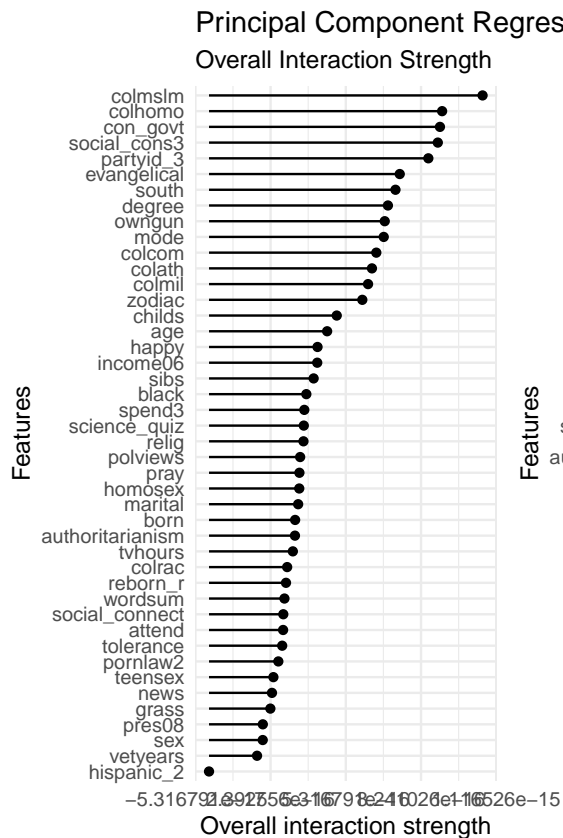
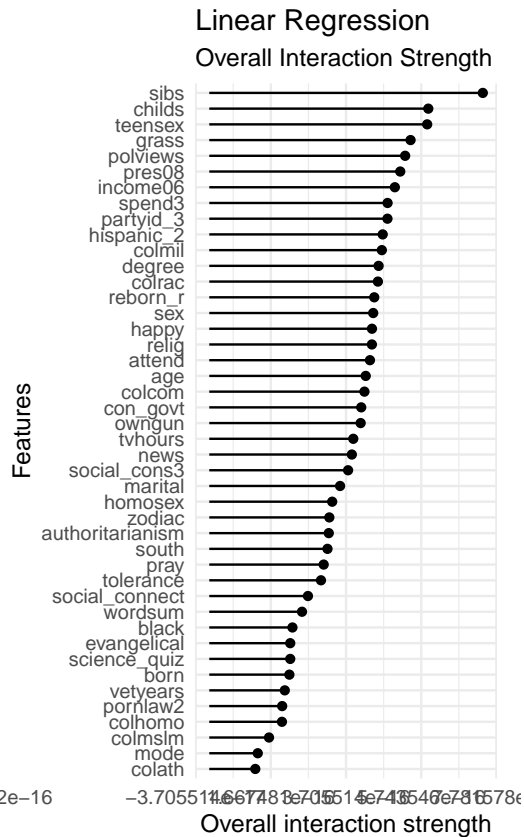
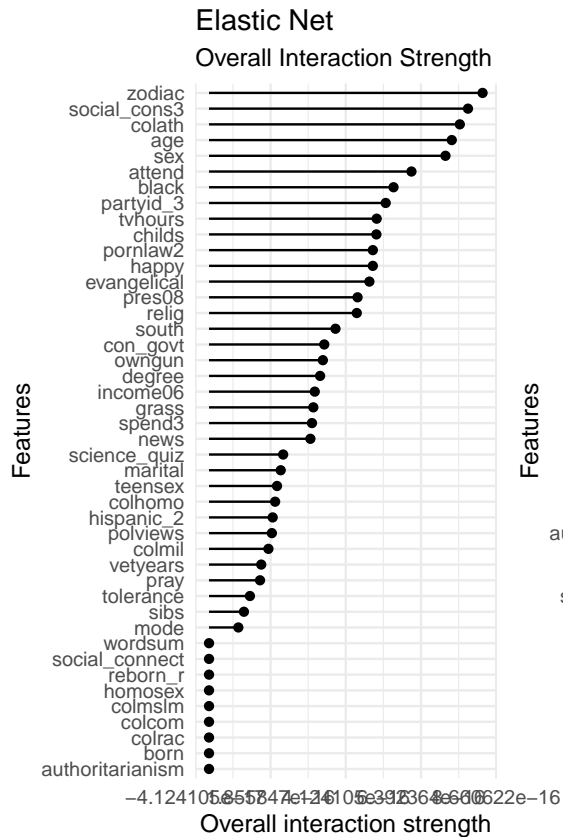




## Feature Interaction

All the interaction effects are extremely small, to the order of  $10^{-15}$ . And as such, there is no consistency in the rank order of these interaction effects.

```
(interaction_plot_enet + interaction_plot_lm + interaction_plot_pcr + interaction_plot_pls)
```



```

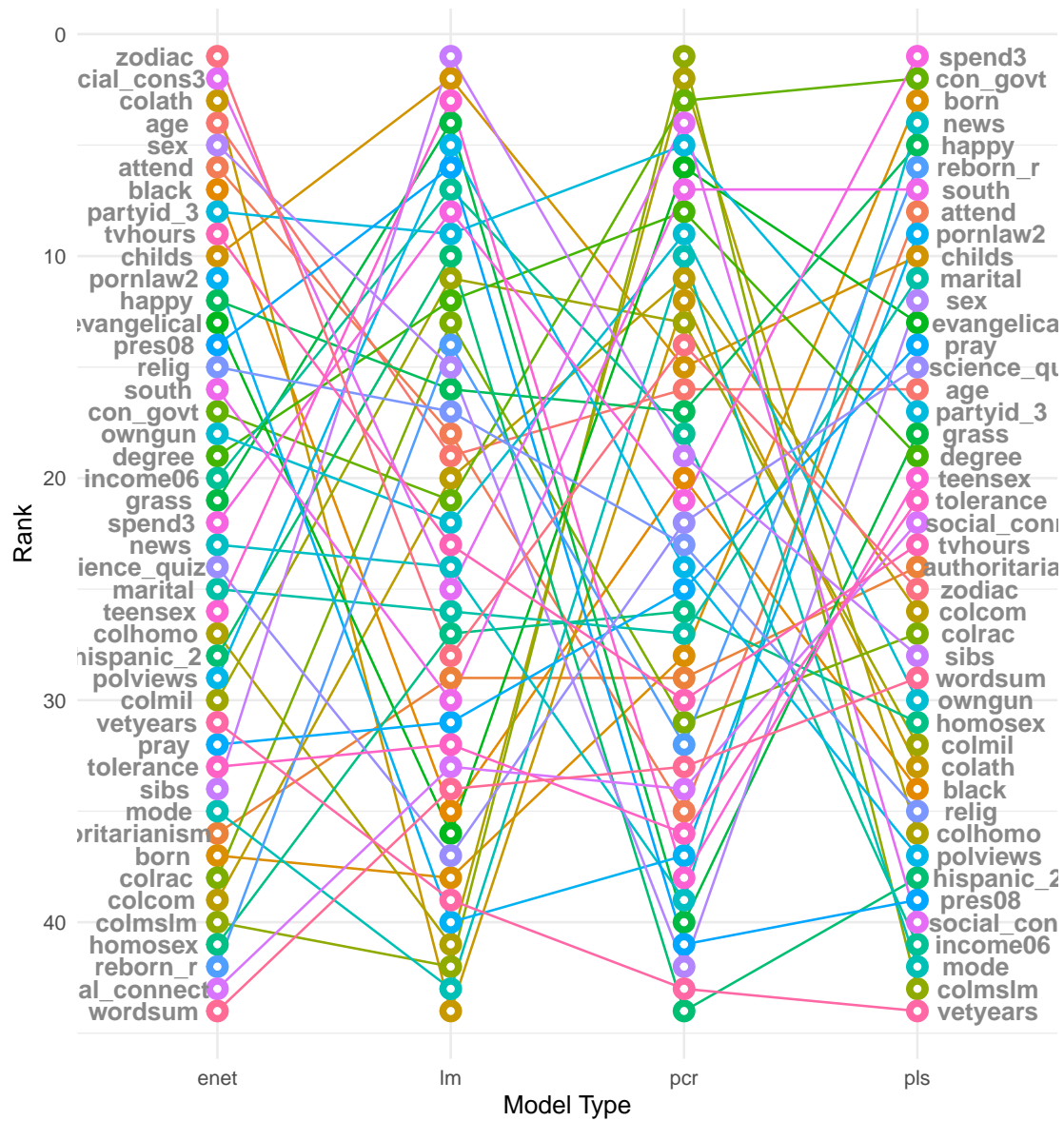
interaction_ranks <- bind_rows((result_df %>%
  filter(model_type == "lm") %>%
  select(interaction_obj)$interaction_obj[[1]]$results %>%
  as_tibble() %>%
  mutate(rank = row_number(desc(.interaction)),
    model_type = "lm"),
  (result_df %>%
    filter(model_type == "glmnet") %>%
    select(interaction_obj)$interaction_obj[[1]]$results %>%
    as_tibble() %>%
    mutate(rank = row_number(desc(.interaction)),
      model_type = "enet"),
  (result_df %>%
    filter(model_type == "pcr") %>%
    select(interaction_obj)$interaction_obj[[1]]$results %>%
    as_tibble() %>%
    mutate(rank = row_number(desc(.interaction)),
      model_type = "pcr"),
  (result_df %>%
    filter(model_type == "pls") %>%
    select(interaction_obj)$interaction_obj[[1]]$results %>%
    as_tibble() %>%
    mutate(rank = row_number(desc(.interaction)),
      model_type = "pls"))

ggplot(data = interaction_ranks, aes(x = model_type, y = rank, group = .feature)) +
  geom_line(aes(color = .feature), size = 0.5, alpha = 1) +
  geom_point(aes(color = .feature), size = 4, alpha = 1) +
  geom_point(color = "#FFFFFF", size = 1) +
  # scale_x_continuous(breaks = 1:16, minor_breaks = 1:16, expand = c(.05, .05)) +
  geom_text(data = interaction_ranks %>% filter(model_type == "enet"),
    aes(label = .feature, x = 0.85) , hjust = .85, fontface = "bold", color = "#888888", size = 10) +
  geom_text(data = interaction_ranks %>% filter(model_type == "pls"),
    aes(label = .feature, x = 4.15) , hjust = 0.15, fontface = "bold", color = "#888888", size = 10) +
  # coord_cartesian(ylim = c(1, show.top.n)) +
  theme_minimal() +
  scale_y_reverse() +
  labs(x = "Model Type",
    y = "Rank",
    title = "Change in Interaction Strength Ranks across Models",
    subtitle = "Elastic Net, Linear Regression, \nPrincipal Component Regression, Partial Least Squares",
    theme(legend.position = "bottom"))

```

## Change in Interaction Strength Ranks across Models

Elastic Net, Linear Regression,  
Principal Component Regression, Partial Least Squares Regression



```

interaction_ranks %>%
  ggplot(aes(x = .feature, y = .interaction, colour = model_type, size = .interaction)) +
  geom_point(alpha = 0.7) + coord_flip() +
  theme_minimal() +
  ggtitle("Comparison of Interaction Strengths across Models",
          subtitle = "Elastic Net, Linear Regression, Principal Component Regression, Partial Least Squares")

```

## Comparison of Interaction Strengths across Models

Elastic Net, Linear Regression, Principal Component Regression, Partial Least

