Electroweak Symmetry Breaking

Adarsh Pyarelal

I have attempted to provide a brief introduction to Electroweak symmetry breaking. This paper will take the reader through different cases of symmetry breaking, each with progressively more complicated symmetries.

Introduction

Electroweak symmetry breaking is the cornerstone of the Standard Model of particle physics. It describes how the particles that mediate the weak force gain mass.

Why do we need such a mechanism at all? We know that the weak force has an extremely short range. However, Weinberg's original model of leptons had massless vector bosons, giving the weak force an infinite range. Electroweak symmetry breaking was introduced by Glashow, Weinberg and Salam to address this weakness of the previous theory. Through this mechanism, the W and Z bosons gain mass, thus limiting their range. In this paper, I will attempt to briefly summarize electroweak symmetry breaking, building up to it by analyzing a few simpler examples.

Global Symmetry

Single real field

THE SIMPLEST CASE of spontaneous symmetry breaking can be seen in systems with global symmetry. Consider for example the following Lagrangian for a single real field ϕ.

$$\mathcal{L}(\phi) = \frac{1}{2}\partial^{\mu}\phi\partial_{\mu}\phi - V(\phi)$$
$$V(\phi) = \frac{\lambda}{4}\left(\phi^2 - \frac{m^2}{\lambda}\right)^2$$

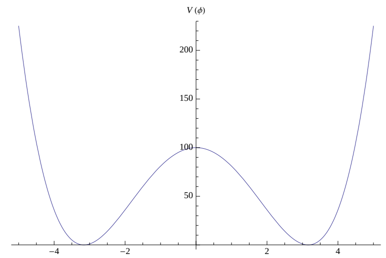

Figure 1: Plot of $V(\phi)$ versus ϕ.

This Lagrangian has global reflection symmetry under the reflection of the field, $\phi \to -\phi$.

With this form of the potential, the theory is unstable at $\phi = 0$. Perturbing about this point will give us tachyons, particles with imaginary mass, that travel faster than the speed of light. Instead, we will choose one of the stable vacua, at $\pm\sqrt{m^2/\lambda}$. Since it does not matter which minimum we choose, we can arbitrarily choose the positive value. This is the vacuum expectation value (VEV). If we define the field $\phi' = \phi - \sqrt{m^2/\lambda}$, then our Lagrangian can be rewritten as

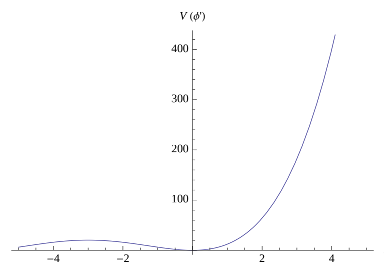

Figure 2: Plot of $V(\phi')$ versus ϕ'. We can clearly see that there is no longer a reflection symmetry.

$$\mathcal{L}(\phi') = \frac{1}{2}\partial^{\mu}\phi'\partial_{\mu}\phi' - V(\phi')$$
$$V(\phi') = \frac{\lambda}{4}\phi'^4 + \sqrt{\frac{m^2}{\lambda}}\phi'^3 + m^2\phi'^2$$

Note that we can do this because the physics remains the same under this translational redefinition of the field. The advantage is that we can perturb about $\phi' = 0$. As you can see in Fig. 2, it is a stable equilibrium point. It is just that while the field ϕ had an imaginary mass since its mass term (term quadratic in ϕ) in the potential $V(\phi)$ was negative, the potential $V(\phi')$ contains such a term, making the ϕ' a particle with mass $\sqrt{2}m$. Note that we have not done anything to the system to break this symmetry. This is why this mechanism is known as *spontaneous* symmetry breaking, as opposed to *explicit* symmetry breaking, where add terms to the Lagrangian to explicitly break the symmetry. We are basically interpreting the system in a different way, looking at it through a different lens.

The masses of the fields do indeed depend upon where the theory 'lives'. Nature prefers configurations with lower potentials. In this case, the potential $V(\phi)$ is minimized at a non-zero value of the field (Fig. 1). You can imagine the potential as a surface along which a ball rolls, and the mass to be related to the 'resistance' the ball faces from the surface if it tries to roll. This resistance is given by the curvature of the surface - the second derivative of the potential function at a point equals the square of the mass of the field. At $\phi = 0$ there is 'negative resistance', corresponding to tachyons with imaginary mass, whereas if it resides in one of the troughs, then it would face some resistance, corresponding to particles with positive mass.

Keeping the above in mind, it might be more helpful to think about the symmetry as being *hidden* by choosing a particular VEV. The original symmetry is still present, it is just hidden away in complicated relationships between the parameters.

Single complex field

What if the ϕ was complex, instead of real? It would then have an additional degree of freedom from its imaginary component. This makes it possible for the Lagrangian to have a *continuous symmetry*, as opposed to the *discrete* reflection symmetry. The potential has the same structure

$$V(\phi) = \frac{\lambda}{4}\left(\phi^{\dagger}\phi - \frac{m^2}{\lambda}\right)^2$$

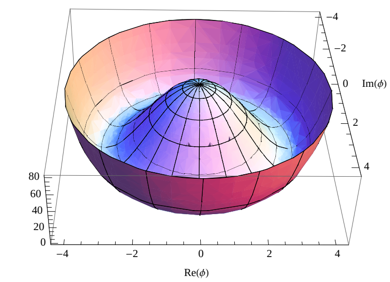

Figure 3: Plot of $V(\phi)$, with $\mathrm{Re}(\phi)$ and $\mathrm{Im}(\phi)$ on the Cartesian axes - the infamous 'Mexican hat' potential.

In this case, the Lagrangian has an additional symmetry, known as global $U(1)$ symmetry. This means that it is invariant under transformations of the form $\phi \rightarrow e^{i\theta}\phi$, where θ is a parameter independent of space-time. The form of transformation suggests rotations, and indeed we can see that this potential will be invariant under rotations about its symmetry axis.[1]

[1] The reflection symmetry from the previous example is incorporated here - it is nothing but a $U(1)$ transformation with $\theta = \pi$.

We can pick any of the points at the minimum of this potential as a vacuum expectation value to perturb about. For convenience, we will pick the positive purely real value. Once we do this, we break symmetry in a manner similar to the previous example.

Notice that if we carry our rolling ball analogy from the previous example to this one, we see that the curvature in the radial direction will provide some resistance, but there is no curvature in the angular direction - the ball can roll around in a circle without any resistance. Since the curvature in the angular direction is zero, the mass of the particle will be zero.

Basically, we can write our single complex field in terms of two real fields, one corresponding to the radial degree of freedom and the other corresponding to the angular degree of freedom. We can interpret these two real fields as particles. The field that can only change in the radial direction now has positive mass, whereas the field that can change in the angular direction is massless. This field corresponds to a massless particle, termed the *Goldstone boson*.

There is an important theorem that states that for every spontaneous breaking of a continuous symmetry, there appears a Goldstone boson. This is known as *Goldstone's Theorem*.

Local symmetry

$U(1)$

A MORE NUANCED DISCUSSION is called for when we consider local symmetries, or *gauge symmetries*. An example of a system with such a symmetry would be the previous example, except with θ no longer independent of space-time. However, introducing this dependence means that the Lagrangian is no longer invariant under the transformation $\phi \to e^{i\theta(x)}\phi$. To remedy this, we will introduce a new field, the *gauge field* A_μ and the *gauge-covariant derivative* $D_\mu \equiv \partial_\mu + ieA_\mu$. If we pick a gauge $\theta(x)$, A_μ transforms as follows

e is a constant - in quantum electrodynamics it represents the charge of the electron.

$$A_\mu \to A_\mu - \frac{1}{e}\partial_\mu \theta(x)$$

It turns out that if we replace the normal partial derivatives ∂_μ in the Lagrangian with the covariant derivatives, then the Lagrangian is invariant under gauge transformations. Our Lagrangian is now

$$\mathcal{L}(\phi) = \frac{1}{2}D^\mu\phi D_\mu\phi - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$V(\phi) = \lambda\left(\phi^*\phi - \frac{v^2}{2}\right)^2$$

We have added the third term in the Lagrangian to make A_μ a dynamical variable[2]. We can now proceed with spontaneously breaking this symmetry. To do so, we choose a vacuum expectation value $\langle\phi\rangle = v/\sqrt{2}$. We can write our original complex field as $\phi = e^{i\frac{\xi}{v}}\left(v+\rho\right)/2$, where ξ and ρ are two real fields corresponding to the angular and radial degrees of freedom respectively. When we pick the vacuum expectation value as $v/\sqrt{2}$, this sets $\langle\xi\rangle = \langle\rho\rangle = 0$. We can then write the Lagrangian in terms of ξ and ρ as follows.

[2] $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

We cannot add terms of the form $A_\mu A^\mu$ explicitly, as they would make the theory non-renormalizable.

$$\mathcal{L}(\xi,\rho) = \frac{1}{2}\partial_\mu\rho\partial^\mu\rho + \frac{(v+\rho)^2}{2}\left(eA_\mu + \frac{\partial_\mu\xi}{v}\right)^2 - \frac{\lambda}{4}(\rho^2+2v\rho)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

In this form, we see that there is a mass term of the form $A_\mu A^\mu$ for the gauge field. However, there is also a mixed term $evA_\mu\partial^\mu\xi$ whose interpretation is not quite clear.[3] We can fix this by choosing a gauge that makes ϕ always real.

[3] This also means that the mass matrix M^2 is not diagonalized.

$$\theta(x) = -\frac{\xi(x)}{ev}$$

When we do so, the terms with ξ disappear. Thus, gathering the terms in the Lagrangian quadratic in the fields, we get

$$\mathcal{L}^{\text{unitary gauge}}_{\text{quadratic}} = \frac{1}{2}\partial_\mu\rho\partial^\mu\rho + e^2v^2A_\mu A^\mu - \lambda v^2\rho^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

We have mass terms for the gauge field A_μ and the field that changes along the radial direction, ρ. We term this field the physical *Higgs boson*. We seem to have gotten rid of the Goldstone boson that should appear due to Goldstone's theorem! However, upon closer inspection, we see that the Goldstone boson is just hidden in the theory as an extra degree of freedom. Before spontaneous symmetry breaking, we had three fields, two massive scalar fields and a massless gauge field. After symmetry breaking, the massless gauge field combines with one of the scalar fields to form a massive vector field. We can shed some light on the situation by examining the degrees of freedom. The gauge field originally had two degrees of freedom corresponding to the two possible polarization states, and the ξ and ρ had one degree of freedom each. The ρ retained its degree of freedom, but the vector boson field now has three degrees of freedom (three possible polarization states) after 'eating' the ξ field. The ξ field is called a 'would-be' Goldstone boson. This is the Higgs mechanism for the Abelian case.

$SU(2) \times U(1)$

Now, instead of a single complex field, let us consider a doublet

The + and 0 refer to the hypercharge of the fields.

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$$

Our Lagrangian is similar in structure to the earlier case

We cannot visualize $V(H)$ like we could $V(\phi)$, since there are two complex fields, each with real and imaginary components. We would have to think in five dimensions. Alas, we are only human, and can visualize a maximum of three dimensions! But whereas our spatial intuition fails us at this point, the math will carry us forward.

$$\mathcal{L}(H) = \frac{1}{2}(D_\mu H)^\dagger (D_\mu H) - V(H) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$V(H) = \left(H^\dagger H - \frac{v^2}{2}\right)$$

This Lagrangian is invariant under the familiar $U(1)$ transformation. However, it is also invariant under $SU(2)$ transformations of the form $H \to e^{i\epsilon_a(x)T^a}H$. Here, $\epsilon_a(x)$ is a space-time dependent parameter and $T_a = \frac{\tau_a}{2}$ where τ_a are the familiar Pauli matrices.

Since we have three generators (T_a) for the $SU(2)$ symmetries, and one (Y) for the $U(1)$ symmetry[4], we will introduce four gauge fields to construct the covariant derivative D_μ.

[4] The field H transforms as follows

$$YH = \frac{1}{2}H$$
$$T_a H = \frac{\tau_a}{2}H$$

$$SU(2) \to W^1_\mu, W^2_\mu, W^3_\mu$$
$$U(1) \to B_\mu$$

The gauge covariant derivative takes the form

$$D_\mu = \partial_\mu + igW^a_\mu \frac{\tau^a}{2} + ig'YB_\mu$$

The potential $V(H)$ reaches its minimum when $H^\dagger H = v^2/2$. We can pick a vacuum expectation value for H that breaks the neutral sector symmetry (corresponding to H^0) but not the charged symmetry (corresponding to H^+), since we know that electromagnetic gauge invariance is a good symmetry to keep because photons are massless.

$$H = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

We combine W^1_μ and W^2_μ to form two composite fields, $W^\pm$.

$$W^\pm_\mu = \frac{W^1_\mu \pm iW^2_\mu}{\sqrt{2}}$$

We use this redefinition and plug in our value for the vacuum expectation value into the kinetic term of the Lagrangian.[5]

[5] Remember, we are interested in finding the masses of the vector boson fields - this is the term that will contain their mass terms.

$$(D_\mu H)^\dagger (D_\mu H) = \begin{pmatrix} 0 & \frac{v}{\sqrt{2}} \end{pmatrix} \left(\partial_\mu - \frac{ig}{\sqrt{2}} \begin{pmatrix} \frac{W^3_\mu}{\sqrt{2}} & W^+_\mu \\ W^-_\mu & -\frac{W^3_\mu}{\sqrt{2}} \end{pmatrix} - \frac{ig'}{2} B_\mu \right) \left(\partial_\mu + \frac{ig}{\sqrt{2}} \begin{pmatrix} \frac{W^3_\mu}{\sqrt{2}} & W^+_\mu \\ W^-_\mu & -\frac{W^3_\mu}{\sqrt{2}} \end{pmatrix} + \frac{ig'}{2} B_\mu \right) \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

If simplify the above expression and isolate the mass terms, we get

Also remember that $\partial_\mu \equiv \partial_\mu \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $B_\mu \equiv B_\mu \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, satisfying size requirements for matrix-vector multiplication.

$$\frac{g^2 v^2}{4} W^-_\mu W^+_\mu + \frac{1}{2}\frac{v^2}{4}(g^2 + g'^2) \left(\frac{gW^3_\mu}{\sqrt{g^2 + g'^2}} - \frac{g' B_\mu}{\sqrt{g^2 + g'^2}} \right)^2$$

We can make the following definitions

$$\frac{g}{\sqrt{g^2 + g'^2}} = \cos\theta$$
$$\frac{g'}{\sqrt{g^2 + g'^2}} = \sin\theta$$

We also make composite fields out of W^3_μ and B_μ, namely the Z-boson, Z_μ and the photon, A_μ, with Z_μ defined as follows.

$$\frac{gW^3_\mu}{\sqrt{g^2 + g'^2}} - \frac{g' B_\mu}{\sqrt{g^2 + g'^2}} = Z_\mu$$

The product $W^+_\mu W^-_\mu$ can be written as a mass term for a generic W_μ boson that comes in two charges.[6] Rewriting our mass terms for the vector boson fields with the above redefinitions, we get

[6] This is because W^+_μ and W^-_μ are Hermitian conjugates of each other.

$$\mathcal{L}_{\text{mass term}} = \frac{e^2 v^2}{4 \sin^2\theta} W_\mu W^\mu + \frac{v^2 e^2}{8 \sin^2\theta \cos^2\theta} Z_\mu Z^\mu$$

After accounting for symmetry factors, we get the masses of the W and Z bosons as

$$m_W = \frac{ev}{2\sin\theta}$$
$$m_Z = \frac{ev}{\sin 2\theta}$$

We see that there is no mass term for A_μ, which means that this gauge field remains massless.

Thus we see that the W and Z bosons gain mass, limiting their range, while the photon remains massless, corresponding to the behavior of the physical world we inhabit.

Couplings

To find out how these gauge particles interact with spin-1/2 fermions, we will take the familiar Lagrangian that leads to the Dirac equation:

$$\mathcal{L} = i\bar{\psi}_L \gamma^\mu D_\mu \psi_L$$

Here, ψ_L represents a left-handed fermion field. It could correspond to leptons or quarks. In this case, let us consider the first generation of leptons, the electron and the electron neutrino. These fields are represented by the fermion doublet field

$$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

For this fermion doublet, both components have the same hypercharge Y[7]. So now we have the interaction term of the Lagrangian that involves the force carrier particles as shown below

[7] The fields are eigenvectors of T_3, Q, and Y with eigenvalues as shown below.

Field	T_3	Y	Q
ν_L	1/2	-1/2	0
e_L	-1/2	-1/2	1

We see that this definition of Q recovers the familiar charges of the electron and the neutrino.

$$\mathcal{L}_{\text{interaction terms}} = \frac{-g}{\sqrt{2}} \begin{pmatrix} \bar{\nu}_L & \bar{e}_L \end{pmatrix} \gamma^\mu \begin{pmatrix} \frac{W^3_\mu}{\sqrt{2}} & W^+_\mu \\ W^-_\mu & -\frac{W^3_\mu}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} - g'Y \begin{pmatrix} \bar{\nu}_L & \bar{e}_L \end{pmatrix} \gamma^\mu B_\mu \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

Charged sector

We will first consider the charged fields, $W^\pm$. To do this, we will isolate the terms in the covariant derivative D_μ that correspond to them, that is, the terms containing W_1^μ and W_2^μ.[8]

[8] Since we defined $W^\pm$ in terms of these fields, and the other terms, W_3^μ and B_μ do not contribute.
Since $\gamma^j = i\tau^j$ for $j \in \{1, 2, 3\}$

Writing this out, we get

$$\frac{-g}{\sqrt{2}} \bar{\nu}_L \gamma^\mu W^+_\mu e_L - \frac{g}{\sqrt{2}} \bar{e}_L \gamma^\mu W^-_\mu \nu_L$$

These give us Feynman diagram vertices for the weak interactions.

Neutral sector

We can apply a similar treatment to the neutral gauge particles, A_μ and Z_μ. Isolating the coupling terms in the covariant derivative, we have

$$gW^3_\mu T_3 + g' B_\mu Y$$

Remembering our previous definitions of g and g' in terms of the electroweak mixing angle θ, we can write the expressions for Z_μ and its orthogonal field A_μ.

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} = \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$$

We can invert this matrix to find Z and A in terms of W^3 and B. Doing so, we get

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$$

So we get our coupling terms in the following form

$$\begin{aligned} &g\left(Z\cos\theta + A\sin\theta\right)T_3 + g'\left(-Z\sin\theta + A\cos\theta\right)Y \\ =\ &A\left(g\sin\theta T_3 + g'\cos\theta Y\right) + Z\left(g\cos\theta T_3 - g'\sin\theta Y\right) \end{aligned}$$

But we know that

$$g\sin\theta = g'\cos\theta = \frac{gg'}{\sqrt{g^2 + g'^2}}$$

So our photon coupling term becomes

$$A\left(\frac{gg'}{\sqrt{g^2 + g'^2}}\right)(T_3 + Y)$$

We know that the strength of the electromagnetic interaction depends upon the magnitude of the electric charge. We label the constant $gg'/\sqrt{g^2 + g'^2}$ as e, and the operator $T_3 + Y$ as Q, the charge matrix. Thus the electron will have a charge of $-e/2$ and the up quark will have a charge of $2e/3$, and so on.

So now we have g and g' in terms of θ and e, and we can substitute those definitions into the Z coupling to get the Z coupling term.

$$Z\frac{e}{\sin\theta\cos\theta}(T_3 - \sin^2\theta Q)$$

We can apply the same treatment to right handed lepton fields, as well as quarks.

Questions

1. Where do we get the specific number of left handed and right handed fields?

2. How exactly do we get e? How do we say that A must couple to eQ? Is my explanation correct? Somewhat?

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