Control System Lab
Experiment 8:Halfquadrotor

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1 Objective

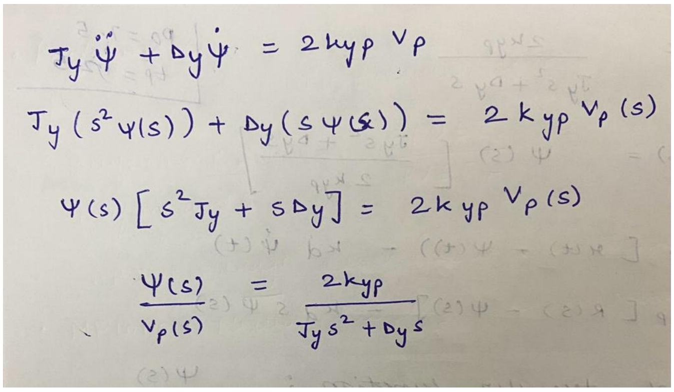
To estimate viscous damping coefficients , the corss thrust gain parameters and to design and simulate PD control system.

2 Theory

The Quanser Aero Experiment experiment can be configured as a half-quadrotor system. By changing the direction and speed of the rotors, users can change the yaw axis angle.Unmanned quardrotor vehicles are used for wide-variety of applications. Using a tethered half-quardrotor systems allows students and researchers to focus on the modeling, control, and parameter estimation in yaw-axis motion of quardrotors, which can then be applied to full quardrotor system.

3 Modeling

3.1 Transfer function model



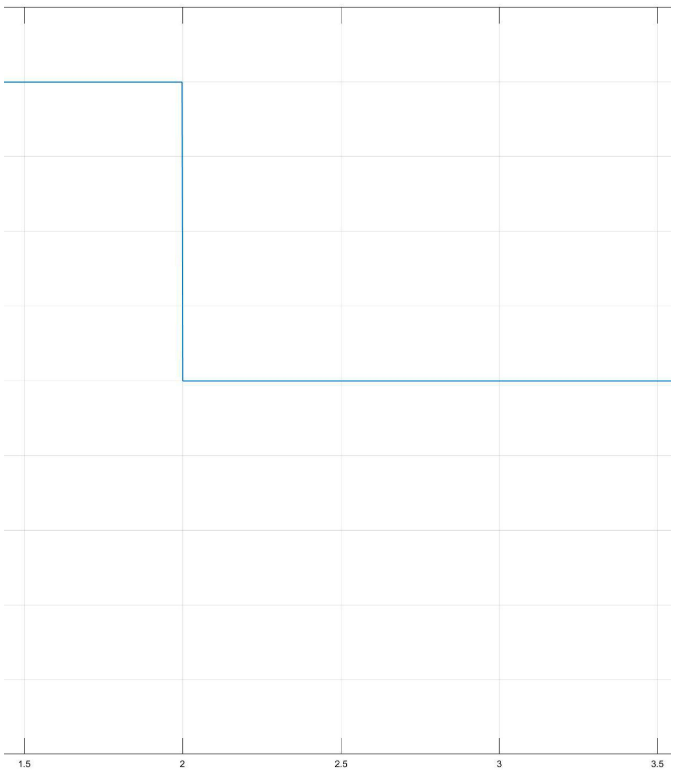
$$J_y \ddot{\psi} + D_y \dot{\psi} = 2 k_{yp} V_p$$

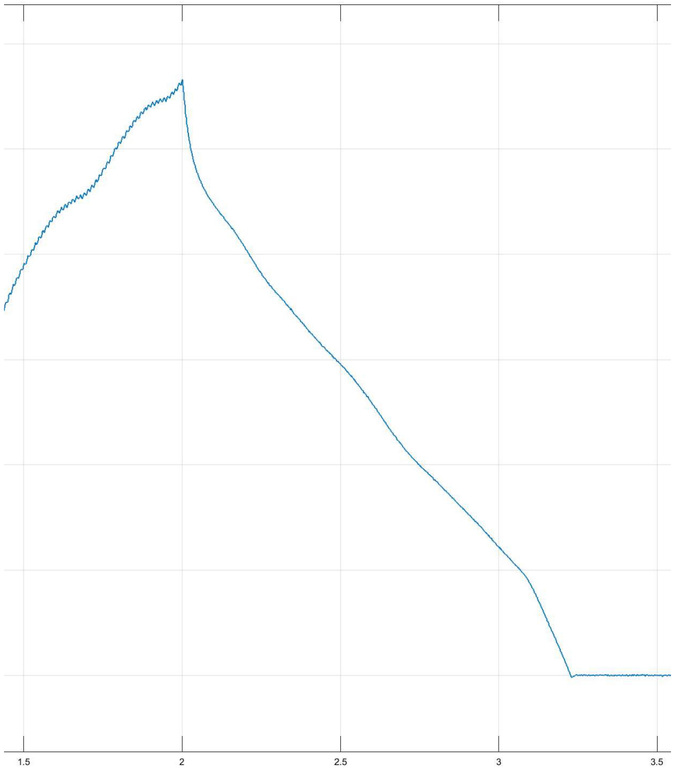
$$J_y (s^2 \psi(s)) + D_y (s \psi(s)) = 2 k_{yp} V_p(s)$$

$$\psi(s) [s^2 J_y + s D_y] = 2 k_{yp} V_p(s)$$

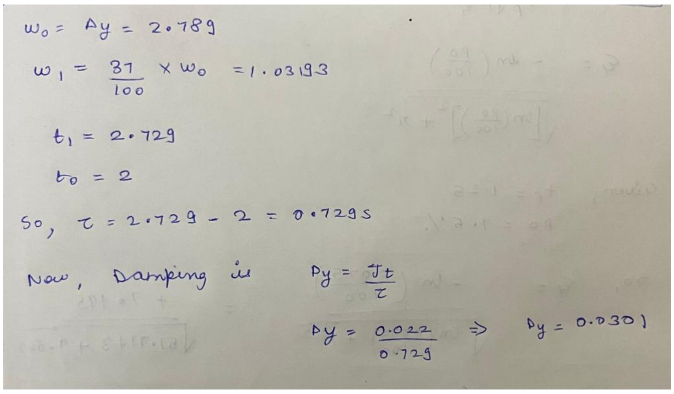
$$\frac{\psi(s)}{V_p(s)} = \frac{2 k_{yp}}{J_y s^2 + D_y s}$$

3.2 Estimating viscous damping coefficients





From the above plot we note the Speed and time to calculate the time constant.



$w_0 = D_y = 2.789$

$w_1 = \frac{37}{100} \times w_0 = 1.03193$

$t_1 = 2.729$

$t_0 = 2$

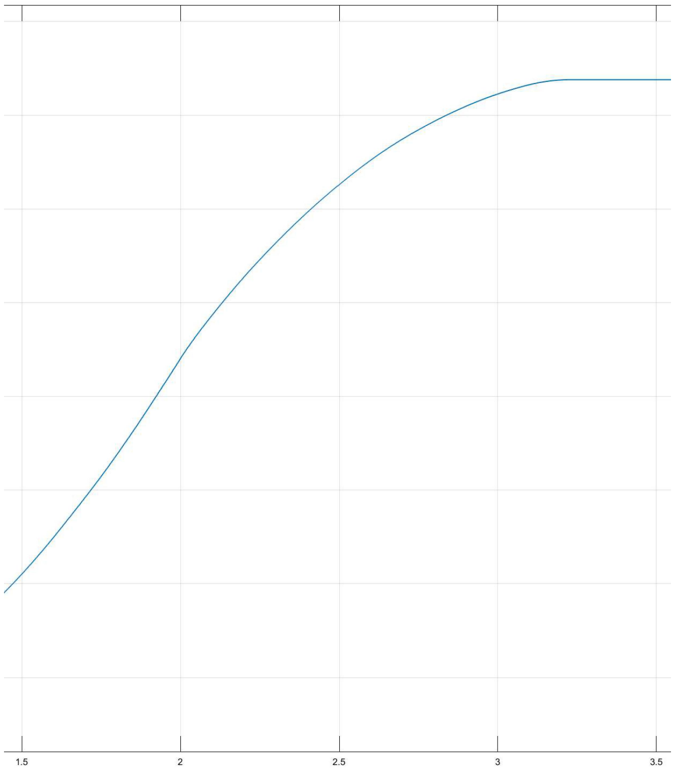
So, $\tau = 2.729 - 2 = 0.729s$

Now, Damping is $D_y = \frac{J_t}{\tau}$

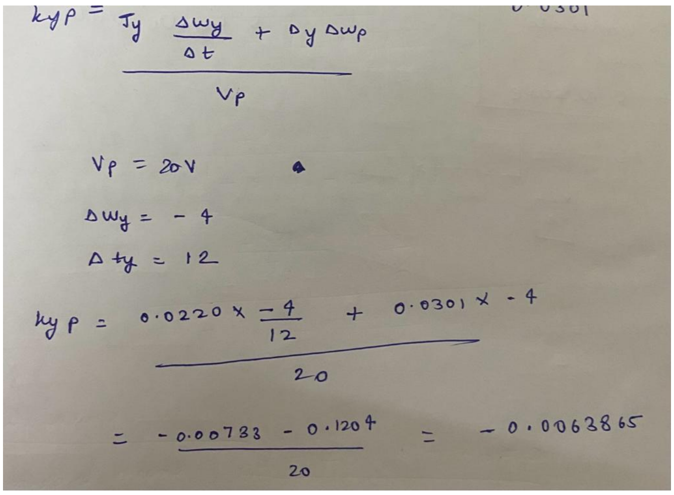
$D_y = \frac{0.022}{0.729} \Rightarrow D_y = 0.0301$

3.3 Estimating the Cross-thrust gain parameters

Plots of yaw step response from step voltage.



the cross thrust gain of the yaw due to the pitch rotor is .



$$k_{yp} = \frac{J_y \frac{\Delta \omega_y}{\Delta t} + D_y \Delta \omega_p}{V_p}$$

$V_p = 20V$

$\Delta \omega_y = -4$

$\Delta t_y = 12$

$$k_{yp} = \frac{0.0220 \times \frac{-4}{12} + 0.0301 \times -4}{20}$$

$$= \frac{-0.00733 - 0.1204}{20} = -0.0063865$$

4 PD Control

4.1 Background

Now,

$$\dot{X} = AX + BU$$

$$\dot{Y} = CX + DU$$

$$X_1 = \psi \qquad X_2 = \dot{\psi}$$

$$\dot{X}_1 = \dot{\psi} \qquad \dot{X}_2 = \ddot{\psi}$$

$$\dot{X}_1 = X_2$$

$$\dot{X}_2 = -\frac{D_y}{J_y}\dot{\psi} + \frac{2k_{yp}}{J_y} V_p$$

$$\dot{X}_2 = \frac{-D_y}{J_y} X_2 + \frac{2k_{yp}}{J_y} V_p$$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{D_y}{J_y} \end{bmatrix} \begin{bmatrix} \psi \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{2k_{yp}}{J_y} \end{bmatrix} V_p.$$

$$A = \begin{bmatrix} 0 & 0 \\ 0 & -D_y/J_y \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 2k_{yp}/J_y \end{bmatrix}$$

$$Y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \psi \\ \dot{\psi} \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 \end{bmatrix}$$

$$\frac{\psi(s)}{u(s)} = \frac{2k_{yp}}{J_y s^2 + D_y s}$$

$P_o = 7.5$
$t_p = 1.25$

$$\Rightarrow U(s) = \psi(s)\left[\frac{J_y s^2 + D_y s}{2k_{yp}}\right]$$

$$u = k_p\left[r(t) - \psi(t)\right] - k_d\,\dot{\psi}(t)$$

$$U(s) = k_p\left[R(s) - \psi(s)\right] - k_d\, s\, \psi(s)$$

Closed loop transfer function : $\frac{\psi(s)}{R(s)}$

$$\psi(s)\left[\frac{J_y s^2 + D_y s}{2k_{yp}}\right] = \psi(s)\left[-k_p - sk_d\right] + k_p R(s)$$

$$T.F = \left(\frac{K}{Ts+1}\right)$$

$$\frac{\psi(s)}{R(s)} = \frac{k_p}{k_p + k_d s + \frac{D_y s}{2k_{yp}} + \frac{J_y s^2}{2k_{yp}}}$$

$$\frac{\psi(s)}{R(s)} = \frac{2k_{yp}k_p}{J_y s^2 + s\left(D_y + 2k_{yp}k_d\right) + 2k_{yp}k_p}$$

$$= \frac{\frac{2k_{yp}k_p}{J_y} \times \frac{D_y}{D_y}}{s^2 + s\left(\frac{D_y}{J_y} + \frac{2k_{yp}k_d}{J_y}\right) + \frac{2k_{yp}k_p}{J_y}}$$

Comparing with : $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

standard second order transfer $f^n = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

$$2\xi\omega_n = \frac{D_y}{J_y} + \frac{2k_{yp}k_d}{J_y} \quad -①$$

$$\omega_n^2 = \frac{2k_{yp}k_p}{J_y} \quad -②$$

Also, $\omega_n = \dfrac{\pi}{t_p\sqrt{1-\xi^2}}$

$$\xi = \frac{-\ln\left(\frac{PO}{100}\right)}{\sqrt{\left[\ln\left(\frac{PO}{100}\right)\right]^2 + \pi^2}}$$

Given, $t_p = 1.25$

$PO = 7.5\%$

So, $\xi = \dfrac{-\ln\left(\frac{0.075}{100}\right)}{\sqrt{\ln^2\left(\frac{0.075}{100}\right) + \pi^2}} = \dfrac{+7.195}{\sqrt{51.7743 + 9.869}}$

$\xi = 0.636$

$\omega_n = \dfrac{\pi}{1.25\sqrt{1-(0.636)^2}} = 3.26$ rad/s

Using ②

$$(3.26)^2 = \frac{2 \times k_{yp} \times k_p}{J_y}$$

$$\Rightarrow k_p = 5.3138 \frac{J_y}{k_{yp}}$$

Using ① $\quad 2(0.636)(3.26) = \dfrac{D_y}{J_y} + \dfrac{2k_{yp}k_d}{J_y}$

$$k_p = \frac{\tau \omega_n^2}{k} = \frac{J_y}{D_y} \times \omega_n^2 \times \frac{D_y}{2k_{yp}}$$

$$= \frac{J_y \omega_n^2}{2k_{yp}}$$

$$k_d = \frac{2 \times \frac{J_y}{D_y} \xi \omega_n - 1}{\frac{2k_{yp}}{D_y}}$$

$$\Rightarrow k_p = \frac{0.022 \times (3.26)^2}{2 \times (-0.0063865)} = -18.3$$

$$\text{and } k_d = \frac{2 \times \frac{0.022}{0.0301} \times 0.636 \times 3.26 - 1}{\frac{2 \times -0.0063865}{0.0301}}$$

$$= 4.785$$

$$\Rightarrow 4.14672 J_y = D_y + 2k_{yp}k_d$$

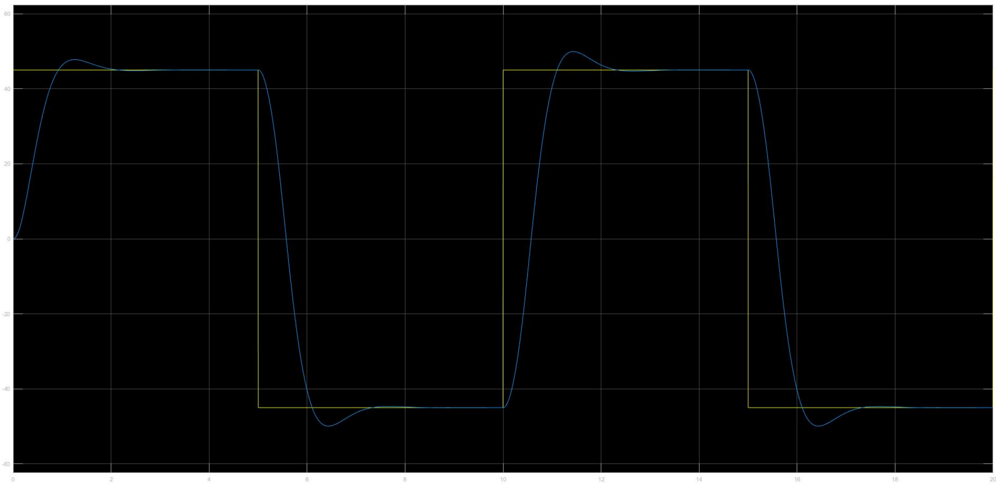
$$\Rightarrow k_d = \frac{4.14672 J_y - D_y}{2k_{yp}}$$

4.2 PD COntrol design and Simulation

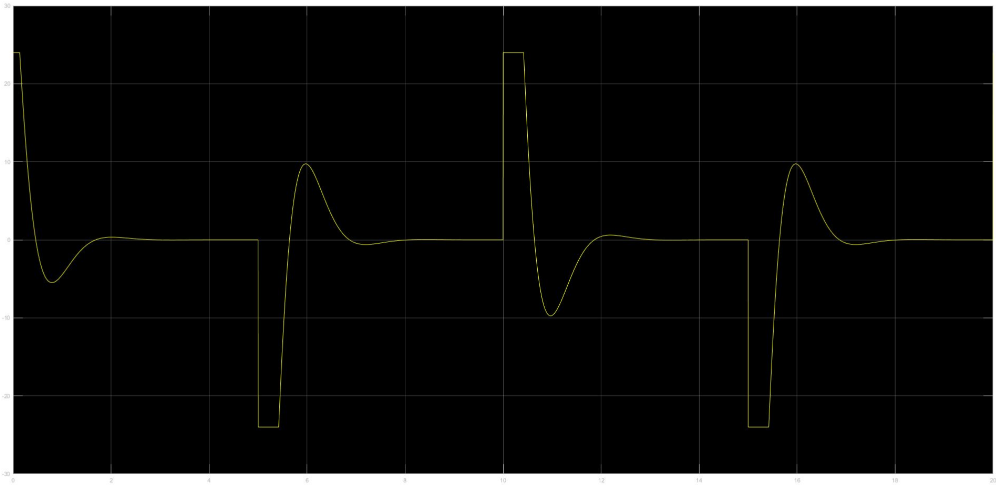
Desired closed loop specification:
1.Steady state error ≤ 2deg
2.Peak time ≤ 2s
3.Percent overshoot ≤ 7.5percent
4.No actuator saturation , |Vy| ≤ 24V and |Vp| ≤ 24V
This model uses the QUARC PID block to implement the PV control for the pitch and yaw axes. The Quanser AERO Half-Quadrotor Model subsystem implements the transfer function described earlier. We will design the PV gains for the peak time tp = 1.25 rad/s and the overshoot of PO = 7.5percent. We use a lower peak time specification to generate the PV gain to ensure the peak time criteria above is satisfied.

Respnse analysis

Yaw angle



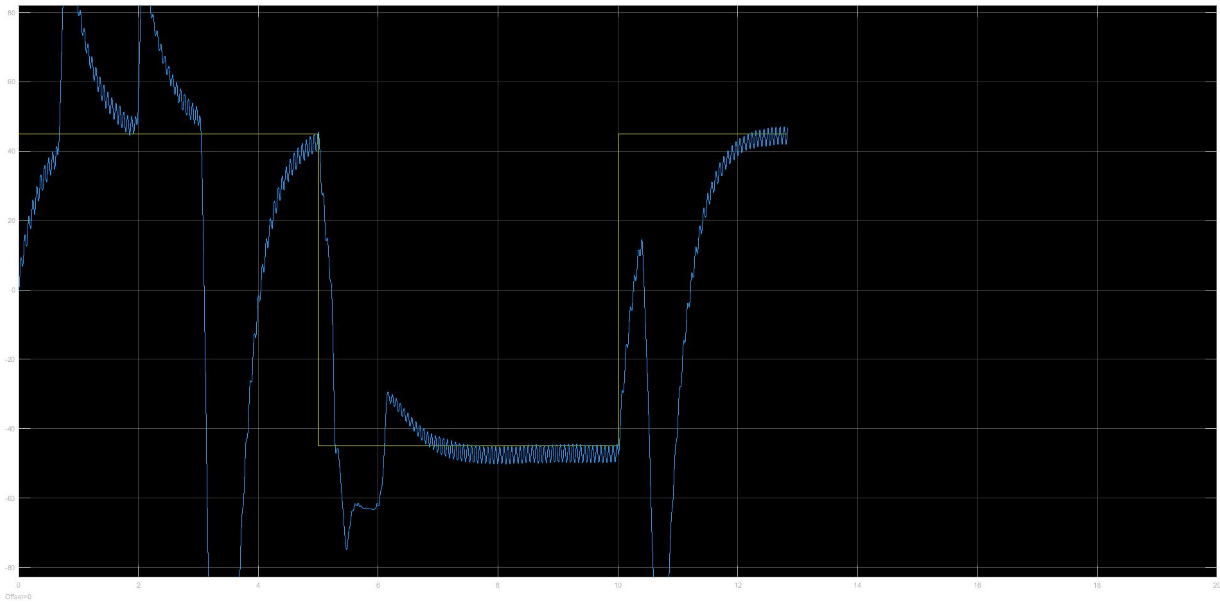
Pitch-yaw motor voltage



The peak time, overshoot, and steady-state error of the simulated response are:
1.Steady state error =0 ≤ 2deg
2.Peak time = 11.4-10 = 1.4s ≤ 2s
3.Percent overshoot = (49.9-45)/90 = 5.4percent ≤ 7.5percent
4.No actuator saturation , ∣Vy∣ ≤ 24V and ∣Vp∣ ≤ 24V

4.3 Running PD on system

yaw position(degree)



Here, we notice that as we try to move the motor manually it again try to rotate in the opposite direction to chase its path.The yellow line represents its desired path whereas the blue line represents its actual path.

5 Results and Discussions

The observations are probably due to the unmodeled friction (e.g. Coulomb friction) about the yaw axis. Increasing the proportional gain, introducing an integrator, or using a more advanced friction-compensation scheme could minimize this. Lastly, the motors do get saturated but only for a short instant so this is still acceptable. The saturation blocks prevent higher voltages as well.