Control System
Lab 9

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B20EE086

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1. **Objective:** The objective of this laboratory is to design and implement a state-feedback control system that will balance the pendulum in the upright, vertical position.

2. **Apparatus/software required:** MATLAB, Simulink, and Inverted Pendulum experiment setup.

3. **Modeling:**

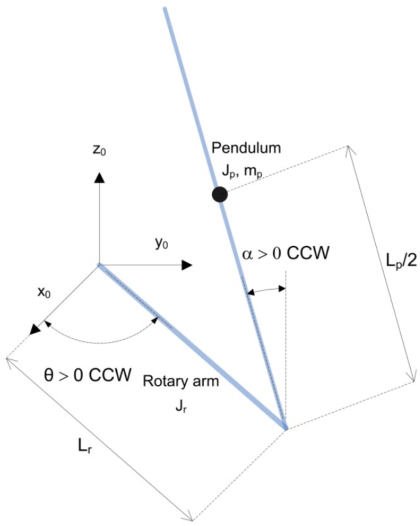

Figure 1: Inverted pendulum conventions

Non-linear equations of motion:
The Euler-Lagrange method is used to determine the equations of motion. The calculations and obtained results are given by:

$$Q_1 = \tau - B_r\dot{\theta}$$
$$Q_2 = -B_p\dot{\alpha}$$

Substituting the values of L and Q and rearranging the equation we get non-linear equation of motion.

$$\left(m_pL_r^2+\frac{1}{4}m_pL_p^2-\frac{1}{4}m_pL_p^2\cos(\alpha)^2+J_r\right)\ddot{\theta}-\left(\frac{1}{2}m_pL_pL_r\cos(\alpha)\right)\ddot{\alpha}$$
$$+\left(\frac{1}{2}m_pL_p^2\sin(\alpha)\cos(\alpha)\right)\dot{\theta}\dot{\alpha}+\left(\frac{1}{2}m_pL_pL_r\sin(\alpha)\right)\dot{\alpha}^2 \quad =\tau-B_r\dot{\theta}$$
$$-\frac{1}{2}m_pL_pL_r\cos(\alpha)\ddot{\theta}+\left(J_p+\frac{1}{4}m_pL_p^2\right)\ddot{\alpha}-\frac{1}{4}m_pL_p^2\cos(\alpha)\sin(\alpha)\dot{\theta}^2$$
$$-\frac{1}{2}m_pL_pg\sin(\alpha) \quad =-B_p\dot{\alpha}.$$

4. **Linear State Space Model:**

$$f_{lin}=f(z_0)+\left(\frac{\partial f(z)}{\partial z_1}\right)\Big|_{z=z_0}(z_1-a)+\left(\frac{\partial f(z)}{\partial z_2}\right)\Big|_{z=z_0}(z_2-b)$$

After using the above linearization equations, we obtain the linear state-space model of the form:

$$\dot{x}=Ax+Bu$$
$$y=Cx+Du$$

$$x=\begin{bmatrix}0\\ \alpha\\ \dot{\theta}\\ \dot{\alpha}\end{bmatrix}$$

Replacing given values of pendulum mass and dimensions the above matrices are given by:

$$A=\begin{bmatrix}0&0&1&0\\0&0&0&1\\0&80.3&-45.8&-0.93\\0&122&-44.1&-1.4\end{bmatrix},B=\begin{bmatrix}0\\0\\83.4\\80.3\end{bmatrix}$$

$$C=\begin{bmatrix}1&0&0&0\\0&1&0&0\end{bmatrix},D=\begin{bmatrix}0\\0\end{bmatrix}$$

5. **Balance Control:**
   1. Pole placement using companion matrix method:

18-10-22

CONTROL LAB

INVERTED PENDULUM

$\xi = 0.7$ - Damping Ratio

$\omega_n = 4$ rad/sec

$\alpha < 15^\circ$

$V_m < 10V$

$\dot{x} = Ax + Bu$

$y = Cx + Du$

$x = [\theta \ \alpha \ \dot{\theta} \ \dot{\alpha}]^T$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 80.3 & -45.8 & -0.93 \\ 0 & 122 & -44.1 & -1.4 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 83.4 \\ 80.3 \end{bmatrix}$$

$p_1 = -\sigma + j\omega_d$

$p_2 = -\sigma - j\omega_d$

$p_3 = -30$

$p_4 = -40$

~~$W_d = W_n\sqrt{}$~~

$DP = [p_1 \ p_2 \ p_3 \ p_4]^T$

$W_d = \omega_n\sqrt{1-(\xi)^2} = W_n\sqrt{1-(0.7)^2}$

$W_d = 4\sqrt{1-0.49} = \underline{\underline{2.85}}$

$\sigma = \omega_n \xi = 4 \times (0.7) = 2.8$

$P_1 = -2.8 + 2.85j$ $\quad P_2 = -2.8 - 2.85j$

$DP = [-2.8+2.85j \quad -2.8-2.85j \quad -30 \quad -40]$

Dominant Poles

$K = [$

```
A = [0 0 1 0;
    0 0 0 1;
    0 80.3 -45.8 -0.93;
    0 122 -44.1 -1.4]
B = [0; 0; 83.4; 80.3]

a = poly(A);
Ac = [0 1 0 0;
      0 0 1 0;
      0 0 0 1;
      -a(5) -a(4) -a(3) -a(2) ];

Bc = [0; 0; 0; 1];

T = ctrb(A,B);
Tc = ctrb(Ac, Bc);

W = T*inv(Tc);
Kc = acker(Ac, Bc, dp);
K1 = Kc*inv(W);
```

2. From scratch to find the gain matrix:

```
dp = [-2.8+2.85i -2.8-2.85i -30 -40];

Qc = [B A*B (A^2)*B (A^3)*B];
det(Qc)

P = [0 0 0 1]*inv(Qc);

Pc = [P; P*A; P*(A)^2; P*(A)^3];
b = poly(dp);

a = a(2:5);
b = b(2:5);

K2 = flip(b-a)*Pc
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It is observed that, K = K1 = K2 = [5.1399 28.0233 2.7117 3.1701]

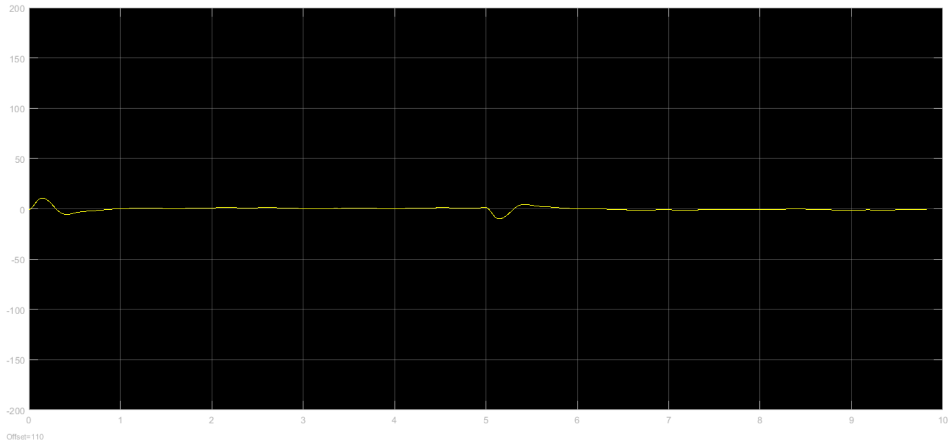

Figure 2: α

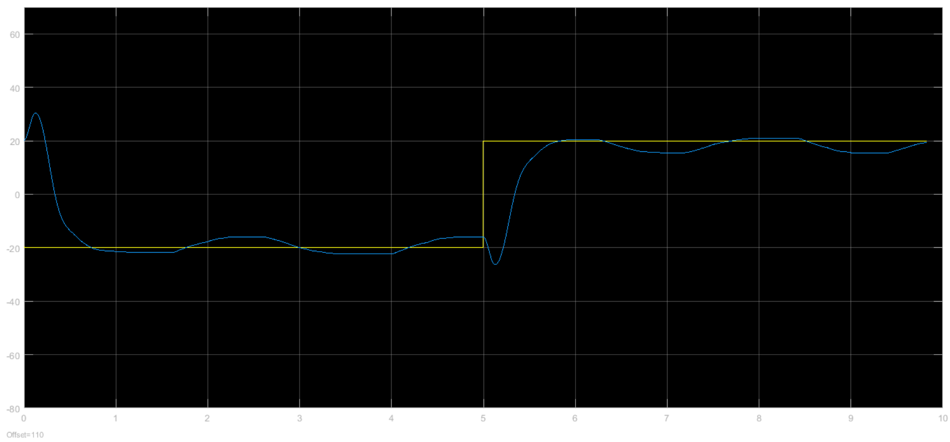

Figure 3: θ

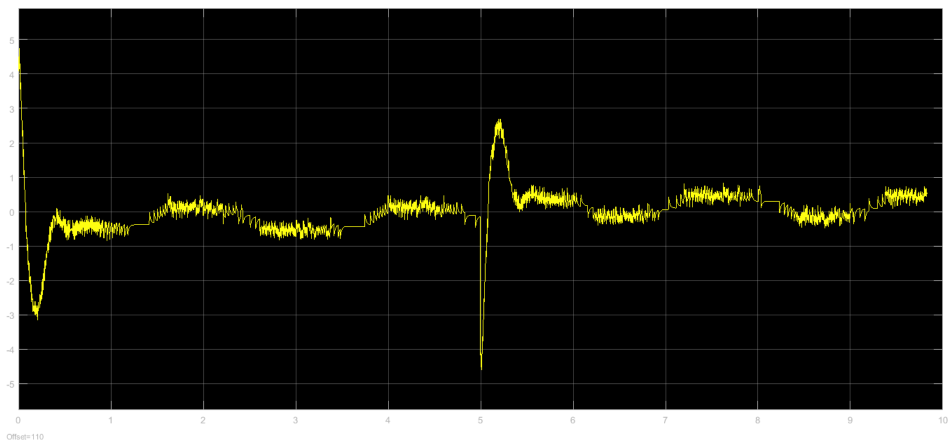

Figure 4: Motor voltage

6. **Swing up control:**

$\boxed{\text{Swing up control}}$ - non linear control

$\dot{\alpha} =$

$E_p = \frac{1}{2} m_p g L_p (1-\cos\alpha)$ $\quad E_k = \frac{1}{2} J_p \dot{\alpha}^2$

Total Energy $= E_p + E_k = \frac{1}{2} m_p g L_p (1-\cos\alpha) + \frac{1}{2} J_p \dot{\alpha}^2$

$$\tau = \frac{\eta_g K_g \eta_m k_t (V_m - K_g k_m \dot{\theta})}{R_m}$$

$V_m - K_g k_m \dot{\theta} = 5$

$\eta_g = 0.69$

$K_g = 70$

$\eta_m = 0.9$

$k_t = 0.00768$

$R_m = 2.6$

$\tau = \frac{0.69 \times 70 \times 0.9 \times 0.00768 \times 5}{2.6}$

$\tau = \cancel{0.18}\ 0.642$

$u_{max} = \frac{\tau}{m_r l_r} = \frac{0.6442}{(0.257)(0.216)}$ $\quad m_r = 0.25$, $l_r = 0.2$

$u_{max} = 11.565 = 11.565\ m/s^2$

Upward Energy $\rightarrow$ $\alpha = 0$

Upward Energy $= \frac{1}{2} J_p \dot{\alpha}^2 = 0$

Downwards Energy $\rightarrow$ $\alpha = 180^\circ$

$$= \underbrace{\frac{1}{2} J_p \dot{\alpha}^2}_{0} + \frac{1}{2} \times (0.127)(9.8)(0.337)(1-(-1))$$

$$= \underline{\underline{0.4894}}$$

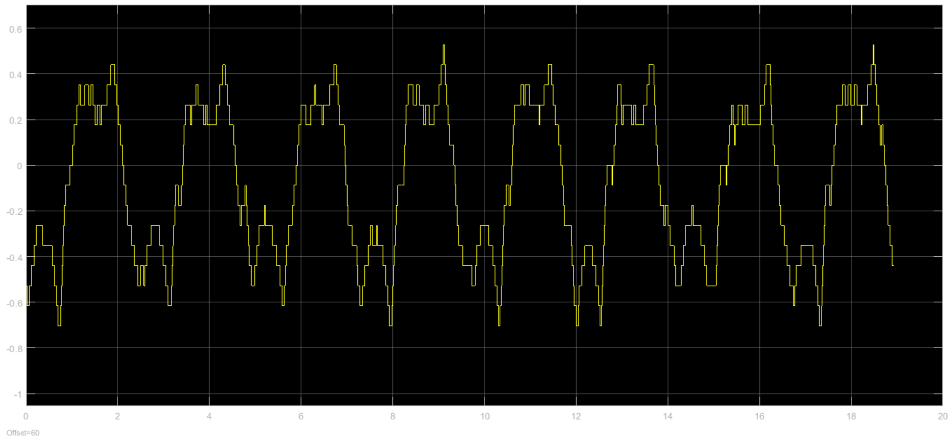

Figure 5: α

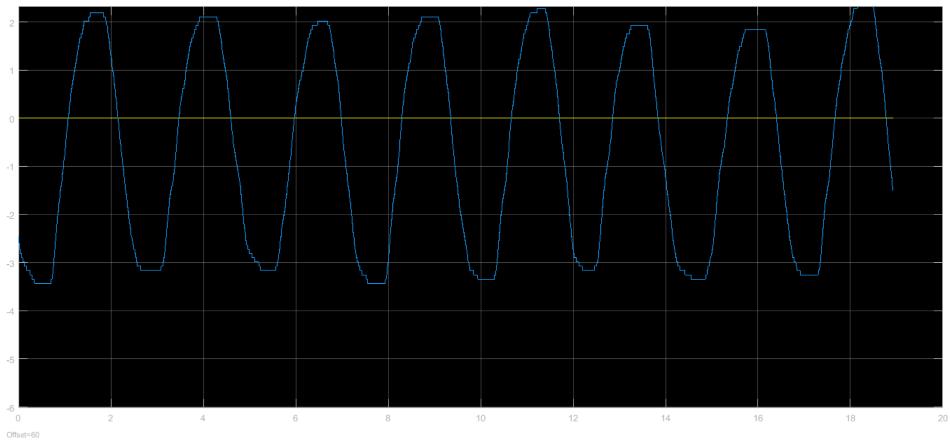

Figure 6: θ

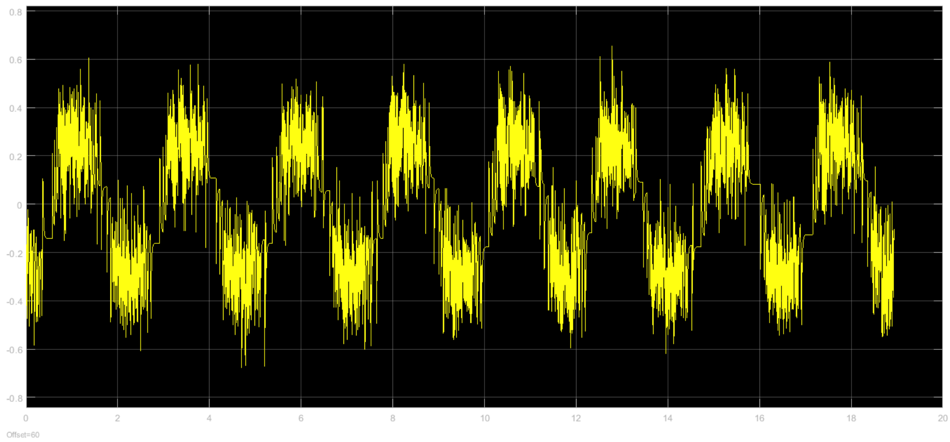

Figure 7: Motor voltage