Lab 6 Report

Ball and Beam Position Control using QuaRC

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B20EE087

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1 Objective

To stabilize the ball to a desired position along the beam using the remote sensor unit. A cascade control system will be designed to meet a set of specifications, using the proportional-derivative (PD) family. MATLAB, Quanser SRV02 unit, Quanser Ball and Beam module, Q8-USB, UPM-2405 amplifier and remote sensor unit will be used for this experiment.

2 Theory

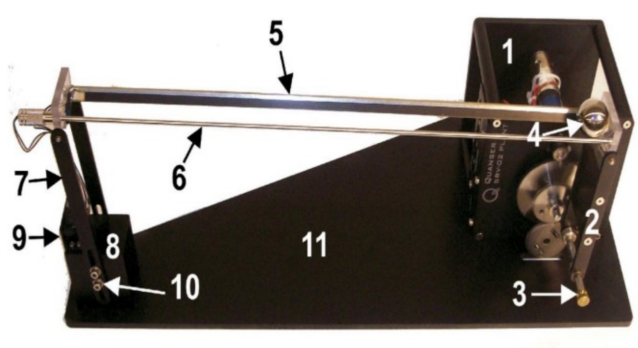

Figure 1: SRV02 Ball and Beam Module

The apparatus consists of a track on which the metal ball is free to roll. It outputs a voltage signal proportional to the position of the ball as the track is fitted with a linear transducer to measure the position of the ball. One side of the beam is attached to a lever arm that can be coupled to the load gear of the Quanser SRV02 unit. By controlling the position of the servo, the beam angle can be adjusted to balance the ball to a desired position.

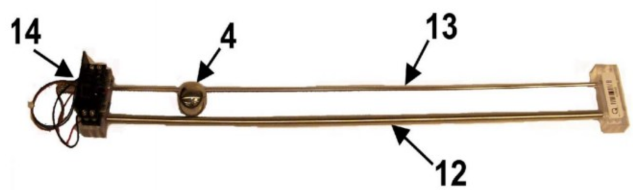

Figure 2: BB01 Module

ID #	Component	ID #	Component
1	SRV02	8	Support base
2	Lever arm	9	Support arm screws
3	Coupling screw	10	Analog ball position sensor connector
4	Steel ball	11	Calibration base
5	BB01 Potentiometer sensor	12	SS01 Potentiometer sensor
6	BB01 Steel rod	13	SS01 Steel rod
7	Support arm	14	Analog remote sensor connector

Figure 3: Components

The track of the BB01 linear transducer module consists of a steel rod in parallel with a nickel-chromium wire-wound resistor on which the metal ball is free to roll. When the ball rolls along the track, it acts as a wiper similar to a potentiometer. Using this mechanism the position of the ball can be tracked by measuring the voltage.

3 Methodology

3.1 Settling time, natural frequency and Cascade control design:

To obtain the desired control response, we should have zero steady state error. The cascade control that will used for the SRV02+BB01 system is illustrated by the block diagram below:

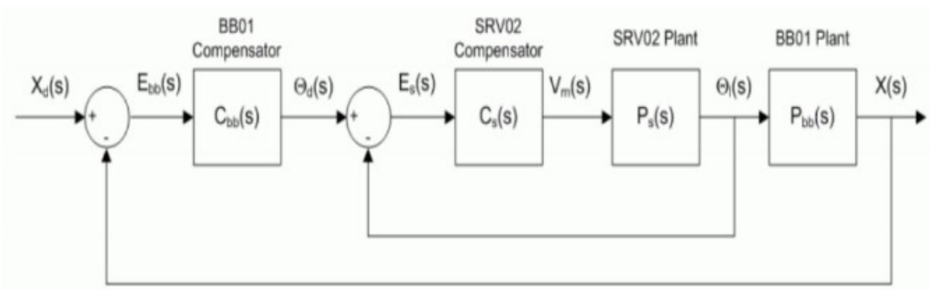

Figure 4: Cascade control design

When tracking the load shaft step reference, the transient response should have a peak time less than or equal to 0.15 seconds, an overshoot less than or equal to 5 %, and no steady-state error. Given a step reference, the peak position of the ball should not overshoot over 10%. After 3.5 seconds, the ball should settled within 4% of its steady-state value (i.e. not the reference) and the steady-state should be within 5 mm of the desired position. Thus the response settles within 5% of its steady-state value, which is between 0.95 and 1.05, in 0.30 seconds.
These can be calculated as follows:

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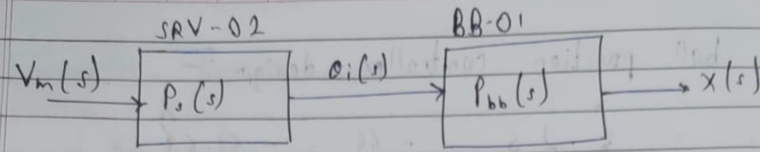

For SRV-02 plant

We know that

$$P.O = 100\, e^{\frac{-\pi \varepsilon}{\sqrt{1-\varepsilon^2}}}$$

$$t_p = \frac{\pi}{\omega_n \sqrt{1-\varepsilon^2}}$$

$$P.O = 5\ \%$$

$$t_p = 0.15\ s$$

$$5 = 100\, e^{-\left(\frac{3.14\, \varepsilon}{\sqrt{1-\varepsilon^2}}\right)}$$

$$-2.9957 = -\left(\frac{3.14\ \varepsilon}{\sqrt{1-\varepsilon^2}}\right)$$

$$\frac{\varepsilon}{\sqrt{1-\varepsilon^2}} = 0.954$$

$$\varepsilon^2 = 0.91 - 0.91\varepsilon^2$$

$$\varepsilon = 0.686$$

$$\boxed{\varepsilon \approx 0.69}$$

$$t_p = \frac{\pi}{\omega_n\sqrt{(1-(0.69)^2)}}$$

$$\omega_n = \frac{20.933}{\sqrt{0.5239}}$$

$$\boxed{\omega_n = 28.92}$$

For BB - 01 Plant

$PO = 10\ \%$

$t_s = 3.5\ s$

$C_{t_s} = 0.04$

$$\therefore 10 = 100\ e^{\frac{-\pi\varepsilon}{\sqrt{1-\varepsilon^2}}}$$

$$-2.3026 = \frac{-\pi\ \varepsilon}{\sqrt{1-\varepsilon^2}}$$

$$\frac{\varepsilon^2}{1-\varepsilon^2} = 0.5372$$

$$\varepsilon^2 = \frac{0.5372}{1.5372}$$

$$\boxed{\varepsilon = 0.59}$$

$$t_s = -\frac{\ln\left[c_{t_s}\sqrt{1-\varepsilon^2}\right]}{\varepsilon\,\omega_n}$$

$$\omega_n = -\frac{\ln\left[0.04\sqrt{1-(0.59)^2}\right]}{(0.59)\times(3.5)} = \frac{-\ln(0.0323)}{(0.59)(3.5)}$$

$$\boxed{\therefore \omega_n = 1.66}$$

3.2 Inner Loop Design: SRV-02 PV Position Controller

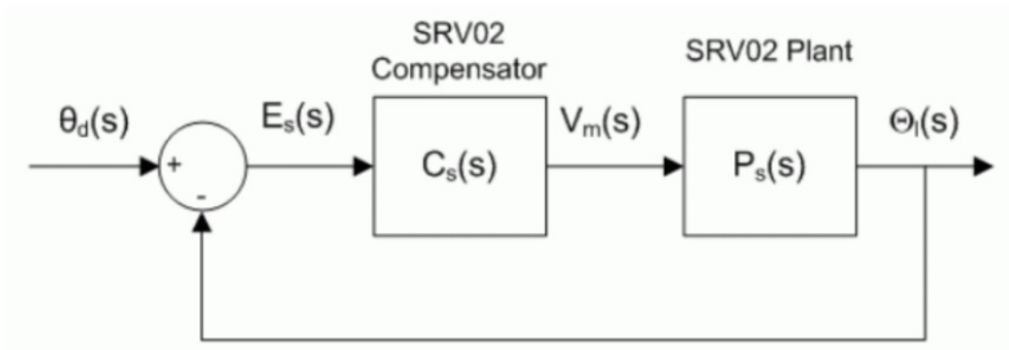

Figure 5: SRV-02 Inner loop

Taking Laplace transform and comparing with the standard equation we can get the desired values of the coefficients. We can also use the block diagram to obtain the characteristic equation for the system.

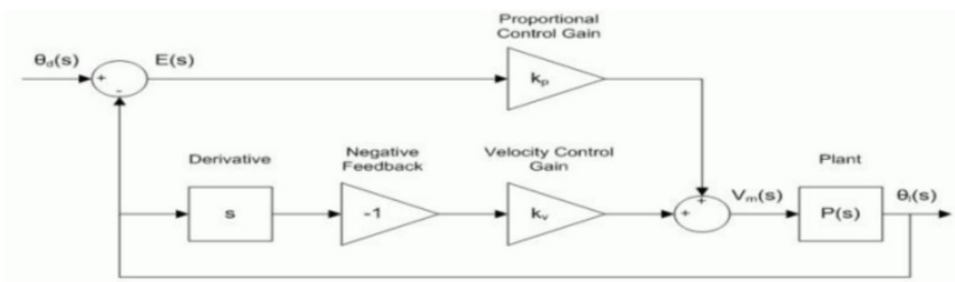

Figure 6: Block diagram fro SRV-02

For SAV 02 Compensator :-

$$V_m(t) = k_p\left(\theta_d(t) - \theta_l(t)\right) - k_v\left(\dot{\theta}_d(t)\right)$$

Taking Laplace transform on both sides :-

$$V_m(s) = k_p\left(\theta_d(s) - \theta_l(s)\right) - k_v s\left(\theta_d(s)\right) \quad - (1)$$

and we known that

$$\frac{\theta_l(s)}{V_m(s)} = \frac{K}{s(1+s\tau)} \quad - (2)$$

Dividing eqn (1) by $\theta_l(s)$

$$\frac{s(1+s\tau)}{K} = kp\left(\frac{\theta_d(s)}{\theta_l(s)} - 1\right) - s\,kv\,\frac{\theta_d(s)}{\theta_l(s)}$$

$$\frac{s(1+s\tau)}{K} = \frac{\theta_d(s)}{\theta_l(s)}\left[kp - skv\right] - kp$$

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$$\frac{\theta_d(s)}{\theta_l(s)} = \frac{s(1+s\tau) + Ks k_v + Kk_p}{K K_p}$$

$$\frac{\theta_l(s)}{\theta_d(s)} = \frac{K k_p}{s^2 \tau + s(K K_v + 1) + K K_p}$$

Comparing with

$$\frac{\theta_l(s)}{\theta_d(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

~~the~~ We have -

$$\frac{\theta_l(s)}{\theta_d(s)} = \frac{K K_p / \tau}{s^2 + s\frac{(KK_v+1)}{\tau} + \frac{KK_p}{\tau}}$$

$$\frac{K K_p}{\tau} = \omega_n^2 \Rightarrow \boxed{K_p = 13.54}$$

$$\frac{(K K_v + 1)}{\tau} = 2\xi\omega_n \Rightarrow \boxed{K_v = 0.078}$$

Where $\tau = 0.0285$ and $K = 1.76$

3.3 Outer Loop Design

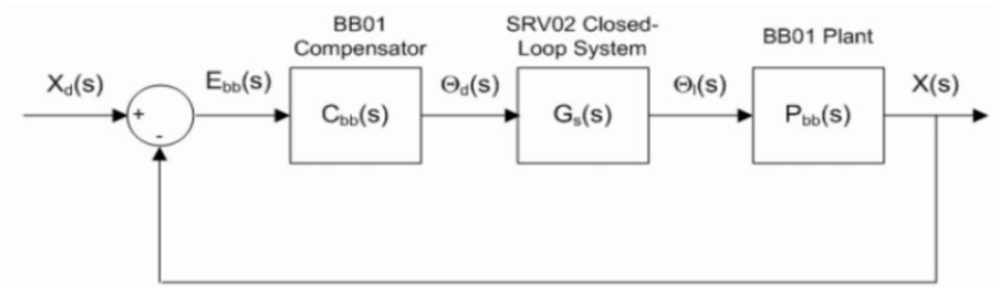

Figure 7: BB-01 Outer loop

Using the block diagram diagram to find out the characteristic equation of the system and comparing with the standard equation, we can find out the desired coefficients.

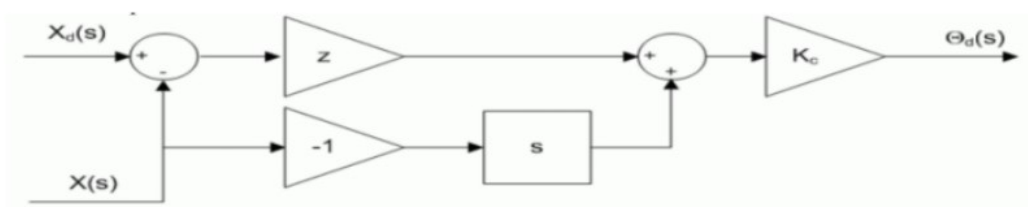

Figure 8: Block diagram for BB-01

For BB - 01 Compensator

Using the block diagram :-

$$\left[\left(X_d(s) - X(s)\right) z + \left(-X(s)\, s\right)\right] K_c = \theta_l(s)$$

$$\therefore \theta_l(s) = K_c z \left(X_d(s) - X(s)\right) - K_c\, s\, X(s) \quad - (1)$$

$$\frac{X(s)}{\theta_l(s)} = \frac{K_{bb}}{s^2} \quad - (11)$$

Dividing (1) by $X(s)$ and using (11) -

$$\frac{s^2}{K_{bb}} = K_c z \left(\frac{X_d(s)}{X(s)} - 1\right) - K_c s$$

$$\frac{\frac{s^2}{K_{bb}} + K_c s + K_c z}{K_c z} = \frac{X_d(s)}{X(s)}$$

$$\therefore \frac{X(s)}{X_d(s)} = \frac{K_c z\, K_{bb}}{s^2 + K_c s K_{bb} + K_c z K_{bb}}$$

Comparing with

$$\frac{X(s)}{X_d(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\therefore \quad K_c \, K_{bb} = 2 \, \xi \, \omega_n \quad \Rightarrow \quad \boxed{K_c = 4.686}$$

$$\text{and} \quad K_c \, K_{bb} \, z = \omega_n^{2} \quad \Rightarrow \quad \boxed{z = 1.407}$$

$$\text{Where} \quad K_{bb} = 0.418 \quad \cancel{\text{and}}$$

4 Observations and Results

4.1 For Square wave position fluctuation:

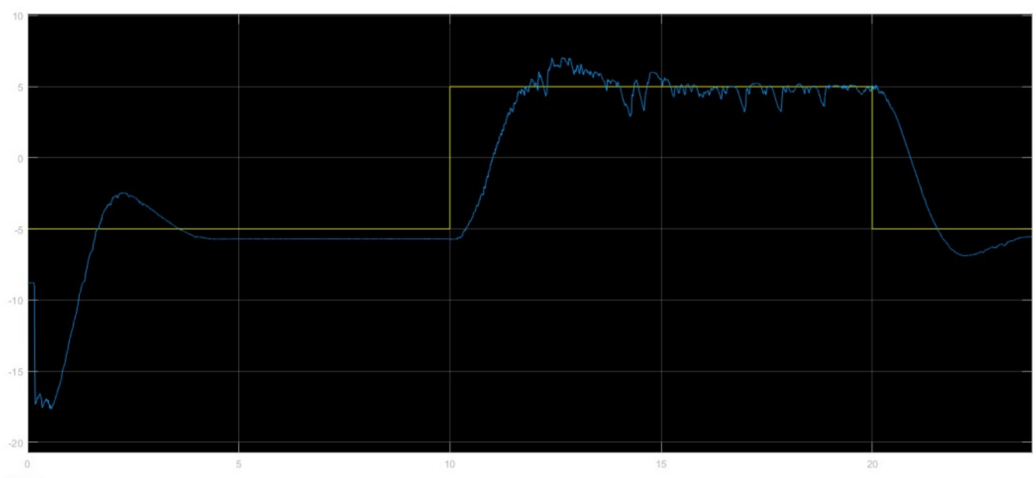

Figure 9: Cascade control ball position

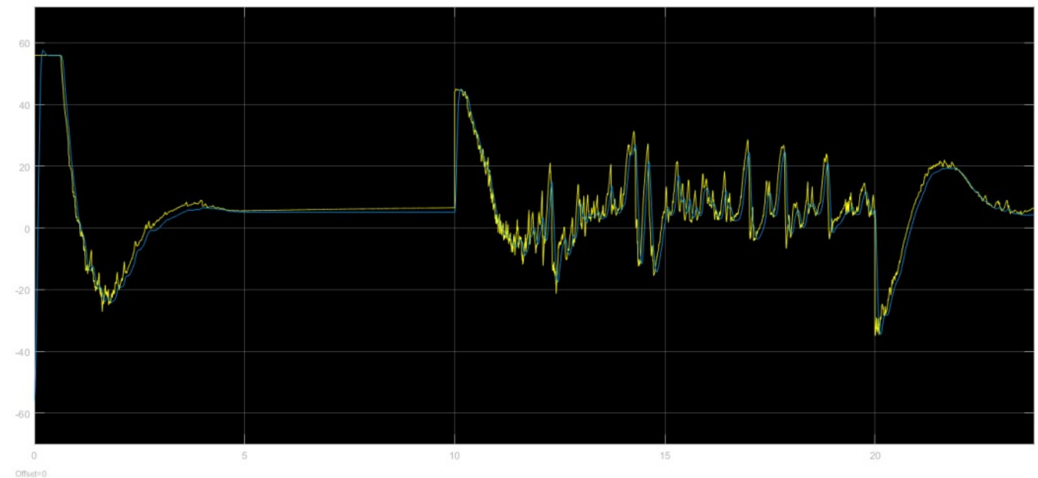

Figure 10: Cascade Control Servo angle

4.2 For Ball Tracking:

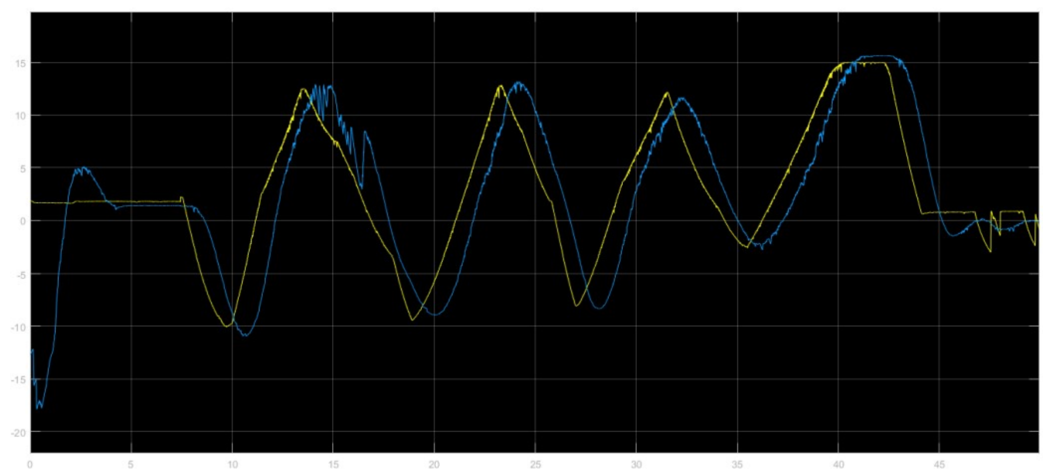

Figure 11: Ball Tracking Ball position Response

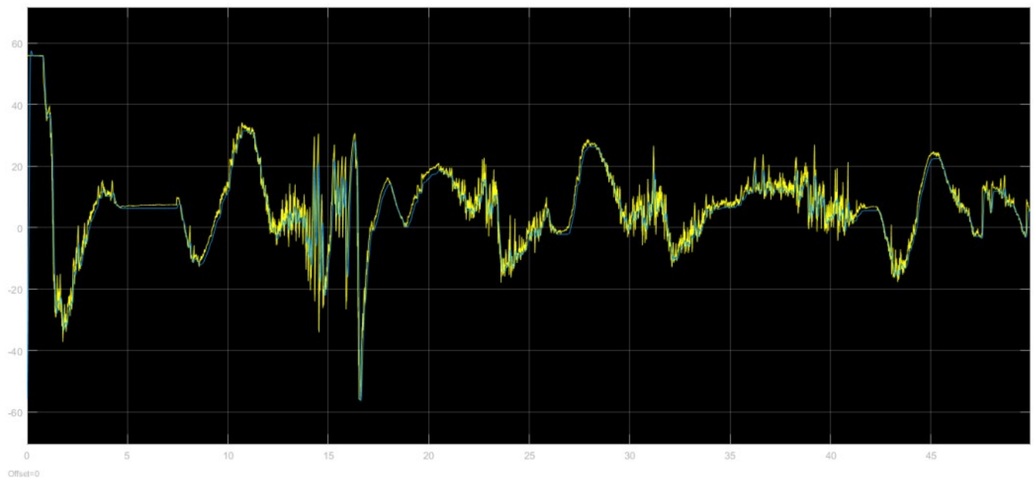

Figure 12: Ball tracking Servo Angle response

4.3 Calculated Values:

Parameter	**Value**
$\zeta 1$	0.69
$\omega 1$	28.92
$\zeta 2$	0.59
$\omega 2$	1.66
K_P	13.54
K_V	0.078
K_C	4.686
Z	1.407