Lab 10 Report
Halfquadrotor

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1 Objective

The aim of this experiment is to estimate viscous damping coefficients , the cross thrust gain parameters using LQR(Linear-Quadratic Regulator) optimization and simulate PD control system.

2 Apparatus

MATLAB, Halfquadrotor

3 Setup

In the halfquadrotor setup, both the front and back rotors are horizontal to the ground and only motions about the yaw axis are enabled and the pitch axis is locked. By changing the direction and speed of the rotors, users can change the yaw axis angle.

4 Modelling

4.1 Transfer function and state-space representation:

$$F = ma$$
$$\tau = J \ddot{\psi}$$

$$F = m\ddot{x} + b\dot{x} + kx$$
$$J_p \ddot{\theta} + D_p \dot{\theta} + K_{sp}\theta = \tau_p$$

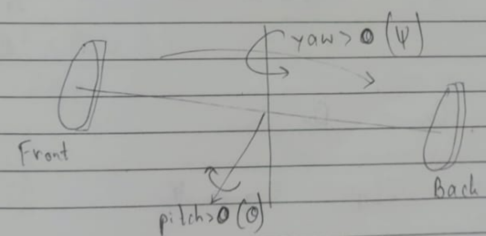
$$J_y \ddot{\psi} + D_y \dot{\psi} = \tau_y$$

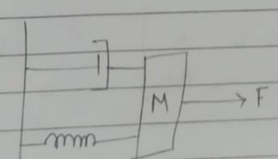
$$\tau_p = K_{pp} V_p + K_{py} V_y$$
$$\tau_y = K_{yp} V_p + K_{yy} V_y$$

$$J_y \ddot{\psi} + D_y \dot{\psi} = K_{yp} V_p + K_{yy} V_y$$

Only yaw, $u = V_p = V_y$

$$\boxed{J_y \ddot{\psi} + D_y \dot{\psi} = 2K_{yp} u}$$





Taking Laplace transform of EOM

$$J_y \Psi(s) s^2 + D_y \Psi(s) s = 2K_{yp} U(s)$$

$$\boxed{\therefore \frac{\Psi(s)}{U(s)} = \frac{2K_{yp}}{J_y s^2 + D_y s}}$$

$x_1 = \psi$

$x_2 = \dot{\psi}$

$\dot{x}_1 = x_2$

$$\dot{x}_2 = \frac{2K_{yp} u}{J_y} - \frac{D_y}{J_y} x_2$$

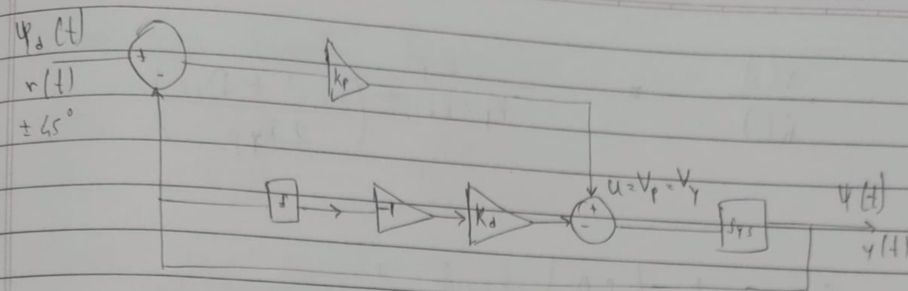
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & -\frac{D_y}{J_y} \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \frac{2K_{yp}}{J_y} \end{bmatrix}}_{B} u$$

$y = Cx + Du$

$y = \psi$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} u$$

4.2 Estimating parameters and controller design:



$$u = K_p \left(r(t) - y(t)\right) - K_d \, \dot{y}(t)$$

$\downarrow$ LT

$$U(s) = K_p \left(R(s) - Y(s)\right) - K_d \, s \, Y(s)$$

$$Y(s) = \Psi(s) = \frac{2 K_{yp}}{J_y s^2 + D_y s} \left[K_p \left(R(s) - Y(s)\right) - K_d \, s \, Y(s) \right]$$

$$\text{CLTF} = \frac{\Psi(s)}{R(s)} = \frac{Y(s)}{R(s)}$$

$$\rightarrow \left(\frac{J_y s^2 + D_y s}{2 K_{yp}}\right) Y(s) = K_p R(s) - K_p Y(s) - K_d \, s \, Y(s)$$

∴ Open loop transfer function :-

$$OLTF = \frac{\Psi(s)}{U(s)} = \frac{2K_y P}{J_y s^2 + D_y s}$$

Closed loop transfer function

$$\frac{\Psi(s)}{R(s)} = \frac{K K_p / \tau}{s^2 + (1+K K_d)/\tau \, s + K K_p/\tau}$$

Where $K = \dfrac{2K_y P}{D_y}$ & $\tau = \dfrac{J_y}{D_y}$

Comparing with $CLTF = \dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

$$\therefore K_p = \frac{\tau\omega_n^2}{K} = \frac{J_y \omega_n^2}{D_y K} = \frac{J_y \omega_n^2}{2K_y P}$$

$$K_d = \frac{2\tau\xi\omega_n - 1}{K} = \frac{2J_y \xi \omega_n - D_y}{K_y P}$$

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$$\frac{Y(s)}{R(s)} = \frac{K_p}{K_p + K_d + \left(\frac{J_y s^2 + D_y s}{2 K_{yp}}\right)}$$

$$\xi = -\ln\left(\frac{P.O}{100}\right) \frac{1}{\sqrt{\left[\ln\left(\frac{P.O}{100}\right)\right]^2 + \pi^2}}$$

$$\omega_n = \frac{\pi}{t_p \sqrt{1 - \xi^2}}$$

$P.O = 7.5 \%$

$t_p = 1.25$ sec.

$$\therefore \xi = (2.590) \times \frac{1}{\sqrt{6.709 + 9.87}}$$

$$= (2.590) \times \frac{1}{4.071}$$

$$\xi = 0.636$$

$$\omega_n = \frac{2.513}{\sqrt{1 - (0.636)^2}} = \frac{2.513}{\sqrt{1 - (0.4045)^2}}$$

$$= 3.26 \text{ rad/s}$$

$\tau_{max} = 3.3 \ s$

$\tau = 1.35$

$$D_y = \frac{J_y}{\tau} = \frac{0.022}{1.135} = 0.019$$

$$K_{yp} = \frac{J_y \frac{\Delta \omega_y}{\Delta t} + D_y \Delta \omega_p}{V_p}$$

$V_p = 20$, $\Delta \omega_p = 0$ (no motion around pitch axis)

$\Delta t = 31 \ s$

$\therefore K_{yp} = -0.00132$

$$\therefore K_p = \frac{J_y \omega_n^2}{2 K_{yp}} = 43.2128$$

$$\boxed{K_p = 43.2128}$$

$$K_d = \frac{2 \tau \omega_n - 1}{K}$$

$$\boxed{K_d = 12.8045}$$

5 Observations and Results:

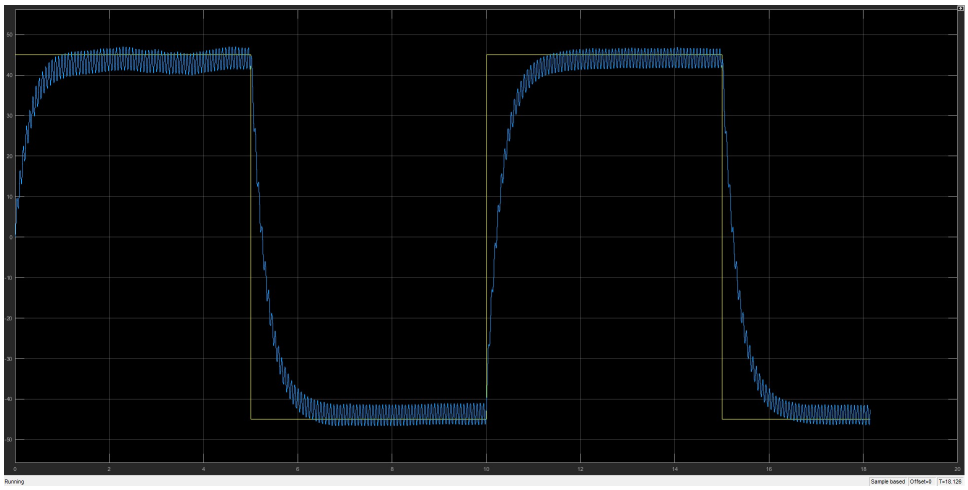

Figure 1: Yaw Position

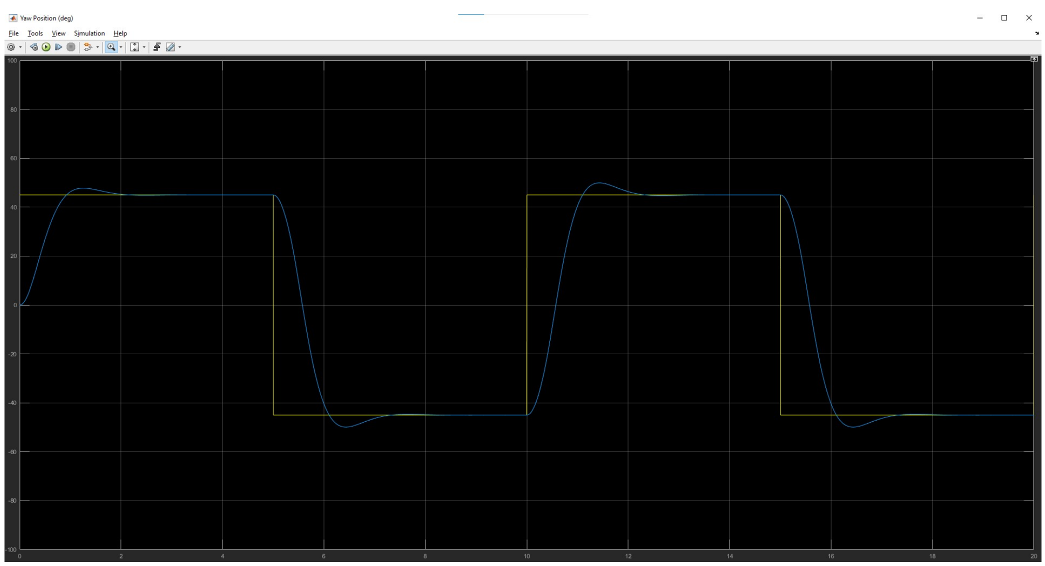

Figure 2: Yaw Angle

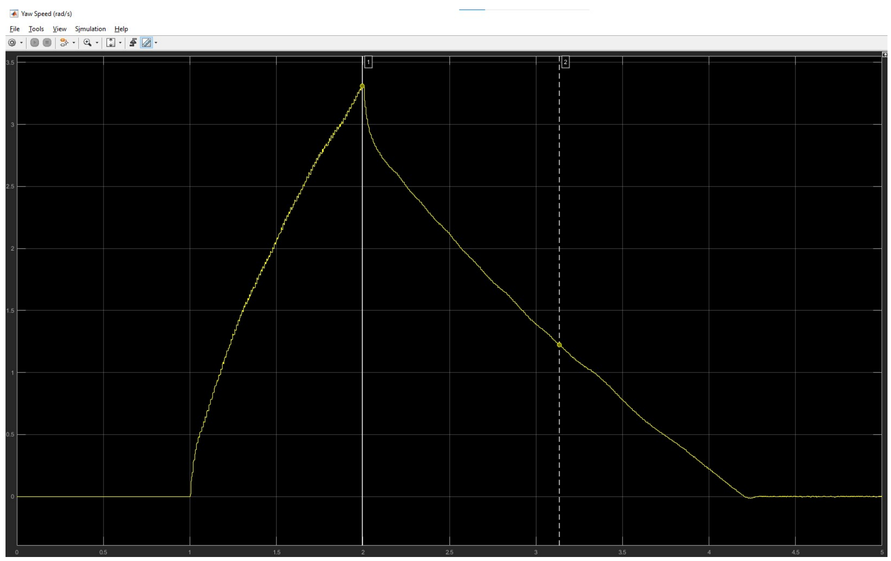

Figure 3: Plot used to calculate speed and time constant

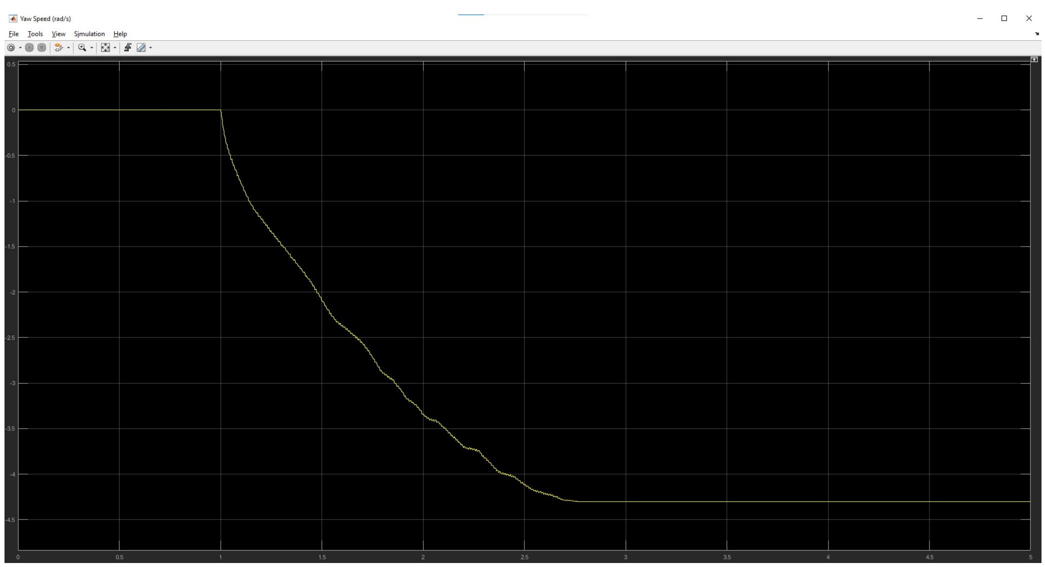

Figure 4: Plot used to calculate Kyp

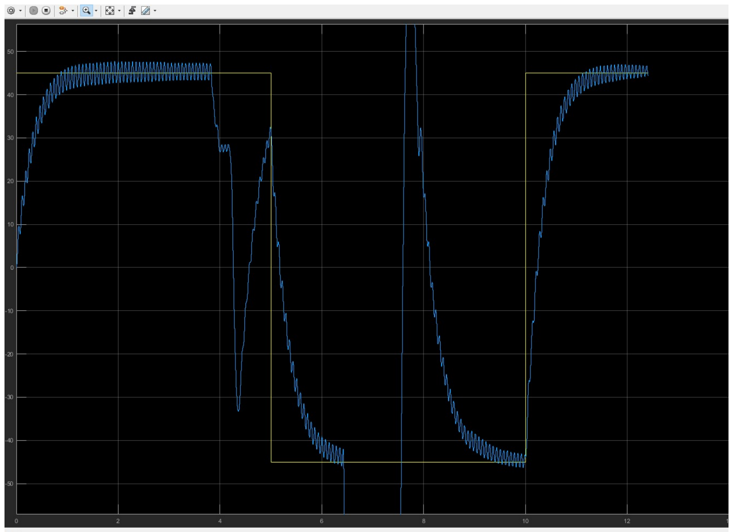

Figure 5: Effect of external disturbances

From the simulation we observe that :

Steady state error => 0 < 2 degrees
Peak time => 11.5 - 10 = 1.5 sec < 2 sec
Percent overshoot => (49.9-45)/90 = 5.4 percent < 7.5 percent
No actuator saturation , Vy < 24V and Vp < 24V