Lab 4 Report

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B20EE087

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1 Observer Design

Aim: To design a full order state observer, given system matrices A,B,C and input u.

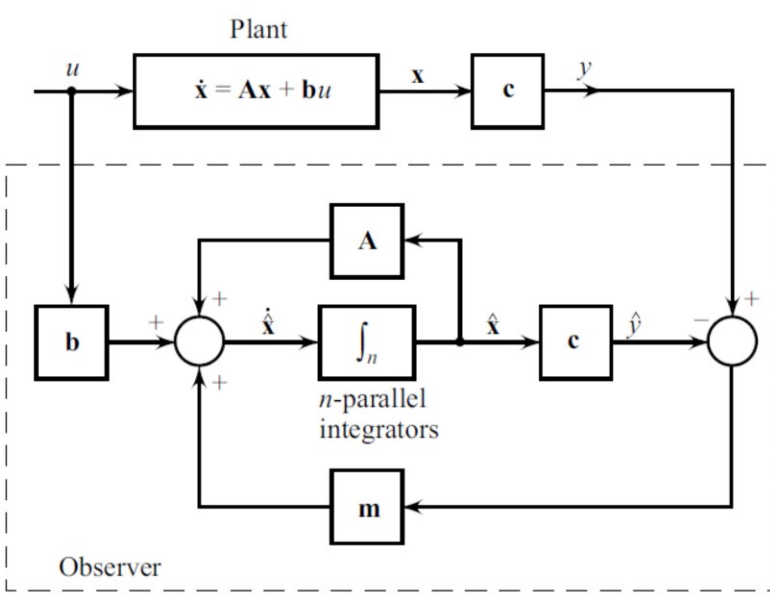

Figure 1: Full order state observer

Consider a system with a state model:

$$\frac{dx}{du} = Ax + Bu$$
$$y = Cx$$

The estimated state equation is given by:

$$\dot{\hat{x}} = A\hat{x} + Bu$$

Our goal is to estimate the state values with minimum error. If the difference between the measured output and the estimated output is fed back to the system, we can speed up the estimation process, which can help to provide a useful state estimate. This method is known as Luenberger state observer. The equation can be given by:

$$\dot{\hat{x}} = A\hat{x} + Bu + m(y - \hat{y})$$

The error can be shown as:

$$e = x - \hat{x}$$

The above equation can be simplified into:

$$\dot{e} = (A - mC)e$$

The transposed auxiliary system is given by:

$$\dot{\zeta} = A^T\zeta + C^T\eta$$

$$\eta = -m^T\zeta$$

Figure 2: Equations for the transpose auxiliary system

The controllability matrix U is given as:

$$U = [C^T A^T C^T (A^T)^2 C^T ....(A^T)^{(n-1)} C^T]$$

$$\mathbf{P} = \begin{bmatrix} \mathbf{p}_1 \\ \mathbf{p}_1\mathbf{A} \\ \vdots \\ \mathbf{p}_1\mathbf{A}^{n-1} \end{bmatrix}$$

Figure 3: Transformation matrix P where p1 is the last row of P

The characteristic equation of matrix A^T and the characteristic equation of the desired observer poles can be given by:

$$|s\mathbf{I} - \mathbf{A}^T| = s^4 + \alpha_1 s^3 + \alpha_2 s^2 + \alpha_3 s + \alpha_4$$

$$(s - \lambda_1)(s - \lambda_2) \cdots (s - \lambda_n) = s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n$$

The selection of m can be approached in exactly the same fashion as the selection of k in the control law design.

$$\mathbf{m}^T = [a_4 - \alpha_4 \quad a_3 - \alpha_3 \quad a_2 - \alpha_2 \quad a_1 - \alpha_1]\,\mathbf{P}$$

To verify the value of m, we can use the duality principle and substitute the values in the state feedback controller design equation to check whether K = m^T.

Control	*Estimation*
$\mathbf{A}$	$\mathbf{A}^T$
$\mathbf{b}$	$\mathbf{c}^T$
$\mathbf{k}$	$\mathbf{m}^T$

Figure 4: Duality Principle

Hand calculation for the given question are as follows:

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For the given system :-

$$A = \begin{bmatrix} 1 & 0.5 & 0 & 0 \\ 0 & 0.2 & 0.2 & 0 \\ 0 & 0.5 & 0.2 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 0.5 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0.5 & 0.2 & 1 & 0 \end{bmatrix}$$

Desired observer poles :-

$$P = \begin{bmatrix} -0.3 \pm j0.5 & -1 & -1.5 \end{bmatrix}$$

The transposed auxillary system is given by

$$\dot{\rho}(t) = A^T \rho(t) + C^T \eta(t)$$

$$\eta(t) = -m^T \rho(t)$$

$\therefore$ The controllability matrix U :-

$$U = \left[C^T \quad A^T C^T \quad (A^T)^2 C^T \quad (A^T)^3 C^T \right]$$

Calculating, we get -

$$U = \begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0.2 & 0.79 & 0.528 & 0.4586 \\ 1 & 0.24 & 0.206 & 0.1468 \\ 0 & 1.0 & 1.24 & 1.4460 \end{bmatrix}$$

Similary,

$$U^{-1} = \begin{bmatrix} -5.0731 & 0.6574 & 3.4051 & 1.2 \\ -31.3268 & 8.2267 & 14.0181 & 6.8 \\ 117.4728 & -22.4254 & -54.2513 & -28 \\ -79.0729 & 13.5414 & 36.8282 & 20 \end{bmatrix}$$

Using U^{-1}

$$p' = \left[-79.0729 \quad 13.5414 \quad 36.8282 \quad 20 \right]$$

Now, $P = \begin{bmatrix} p_1 \\ p_1 (A^T) \\ p_1 (A^T)^2 \\ p_1 (A^T)^3 \end{bmatrix}$

Calculating the above matrix, we get -

$$P = \begin{bmatrix} 0 & -0.9401 & 0 & 0 \\ -0.9401 & 0 & 0 & 0 \\ 0 & -15.3333 & 0 & 1 \\ -15.3333 & 0 & 1 & 0 \end{bmatrix}$$

Calculating characteristic eq^n for A^T

$$|sI - A^T| = 0$$

$$\begin{vmatrix} s-1 & 0 & 0 & 0 \\ 0.5 & s-0.2 & 0.5 & 0 \\ 0 & 0.2 & s-0.2 & 0 \\ 0 & 0 & 1 & s-1 \end{vmatrix} = 0$$

$$s^4 - (2.4)s^3 + (1.74)s^2 + (-0.28)s + (-0.06) = 0 \quad -(1)$$

For characteristic eq^n of poles -

$$(s - (-0.3 + j0.5))(s - (-0.3 - j0.5))(s+1)(s+1.5) = 0$$

$$(s + 0.3 + 0.5j)(s + 0.3 - 0.5j)(s+1)(s+1.5) = 0$$

$$(s+0.3)^2\ (0.25)\ (s^2+2.5s+1.5)=0$$

$$s^4+(3.1)s^3+(3.34)s^2+(1.75)s+0.51=0$$

Now,

$$m^T=\begin{bmatrix}a_4-\alpha_4 & a_3-\alpha_3 & a_2-\alpha_2 & a_1-\alpha_1\end{bmatrix}P$$

$$=\begin{bmatrix}0.57 & 2.03 & 1.6 & 5.5\end{bmatrix}P$$

(Using both characteristic eq^{n}s (1) and (2))

$$\therefore m^T=\begin{bmatrix}-645.1128 & 87.0928 & 310.6378 & 194\end{bmatrix}$$

$$\therefore m=\begin{bmatrix}-645.1128\\ 87.0928\\ 310.6378\\ 194.0000\end{bmatrix}$$

To verify m, we can the use the principle of duality by substituting

$A = A^T$

$B = C^T$

And we will get $K = m^T$,

Where K is the feedback matrix.

Using the method for state feedback controller design :-

We get $K = \begin{bmatrix} -645.1128 & 87.0928 & 310.6378 & 194 \end{bmatrix}$

$\therefore \underline{K = m^T}$

$\therefore$ The value of matrix m is verified using the principle of duality.

The Matlab code for the above calculations is as follows:

```
A = [1 0.5 0 0;
 0 0.2 0.2 0;
 0 0.5 0.2 1;
 0 0 0 1]

B = [0.5; 1; 0; 1]

C = [0.5 0.2 1 0]

desired_ob_poles = [complex(-0.3,0.5) complex(-0.3,-0.5) -1 -1.5]

Qc = ctrb(A,B)
det(Qc)
%system is completely controllable

U = ctrb(A',C')
I = eye(4,4)
U_inverse = I/U

p1 = U_inverse(4,:)

P = [p1;
     p1*(A');
     p1*(A')^2;
     p1*(A')^3;]

coeff_1_alpha = charpoly(A')
coeff_2_a = poly(desired_ob_poles)

coeff_1 = flip(coeff_1_alpha(1, 2:5))
coeff_2 = flip(coeff_2_a(1, 2:5))

m_transpose = (coeff_2 - coeff_1)*P
m = m_transpose'
```

Output:

```
U = 4×4
      0.5000    0.5000    0.5000    0.5000
      0.2000    0.7900    0.5280    0.4586
      1.0000    0.2400    0.2060    0.1468
           0    1.0000    1.2400    1.4460

U_inverse = 4×4
     -5.0731    0.6574    3.4051    1.2000
    -31.3268    8.2267   14.0181    6.8000
    117.4728  -22.4254  -54.2513  -28.0000
    -79.0729   13.5414   36.8282   20.0000

p1 = 1×4
    -79.0729   13.5414   36.8282   20.0000

P = 4×4
    -79.0729   13.5414   36.8282   20.0000
    -72.3022   10.0739   34.1363   20.0000
    -67.2653    8.8420   31.8642   20.0000
    -62.8442    8.1413   30.7939   20.0000

coeff_1_alpha = 1×5
      1.0000   -2.4000    1.7400   -0.2800   -0.0600

coeff_2_a = 1×5
      1.0000    3.1000    3.3400    1.7500    0.5100

m_transpose = 1×4
   -645.1128   87.0928  310.6378  194.0000

m = 4×1
   -645.1128
     87.0928
    310.6378
    194.0000
```

Matlab code for verifying m using Duality:

```
A = [1 0 0 0;
     0.5 0.2 0.5 0;
     0 0.2 0.2 0;
     0 0 1 1]
B = [0.5; 0.2; 1; 0]

Qc = ctrb(A,B)
det(Qc)
%since Qc != 0 system is controllable

ch_eqn_coeff = charpoly(A)
ch_eqn_coeff_2_4 = ch_eqn_coeff(1,2:5)


poles = [complex(-0.3,0.5) complex(-0.3,-0.5) -1 -1.5]
poles_eqn_coeff = poly(poles)
poles_eqn_coeff_2_4 = poles_eqn_coeff(1,2:5)

P1 = [0 0 0 1] / (Qc)

Pc = [P1; P1*A; P1*A^2; P1*A^3]

ch_eqn_coeff_2_4_rev = flip(ch_eqn_coeff_2_4)

poles_eqn_coeff_2_4_rev = flip(poles_eqn_coeff_2_4)

K = (poles_eqn_coeff_2_4_rev - ch_eqn_coeff_2_4_rev)*Pc
```

Plot for errors of state variables:
We observe that the error converges to zero for all the state variables.

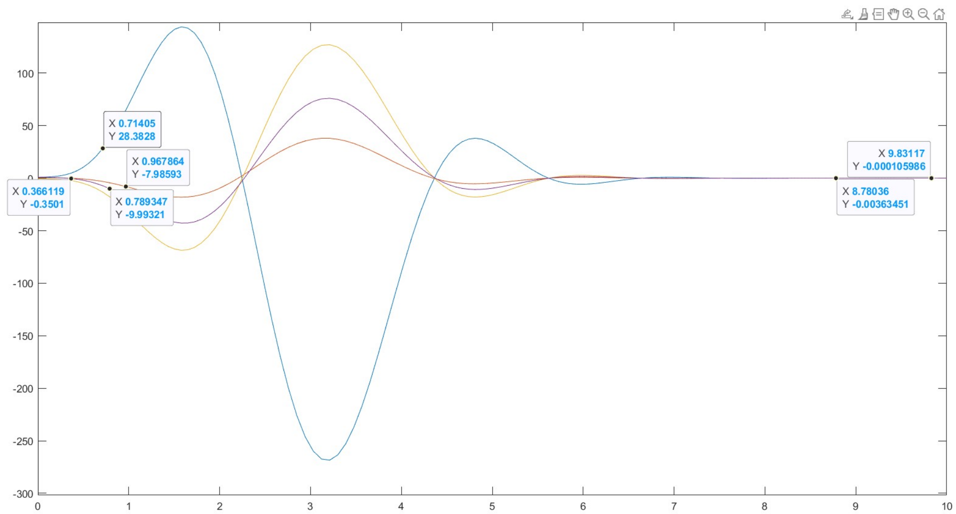

Figure 5: Plot of error v/s time