Lab 5 Report
Magnetic Levitation

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1 Objective

To model the magnetic levitation (MAGLEV) plant and to design a controller that levitates the ball from post and ball position tracks a desired trajectory. MATLAB, Magnetic Levitation kit, Q8-USB, UPM-2405 amplifier will be used for this experiment

2 Theory

The "MAGLEV" experiment consists of an electromagnet encased in a rectangular enclosure. One electromagnet pole faces a black post upon which a 2.54 cm diameter steel ball rests. The ball elevation from the post is measured using a sensor embedded in the post. The post is designed such that with the ball at rest on its surface, it is 14 mm from the face of the electromagnet. Electrical systems The coil used in the electromagnet has an inductor and a resistance. The voltage applied to the coil results in a current governed by the differential equation.
The actual system is equipped with resistor Rs in series with the coil whose voltage V can be measured using the A/D. The measured voltage can be used to compute the current in the coil. The sense resistor in the circuit results in the equation

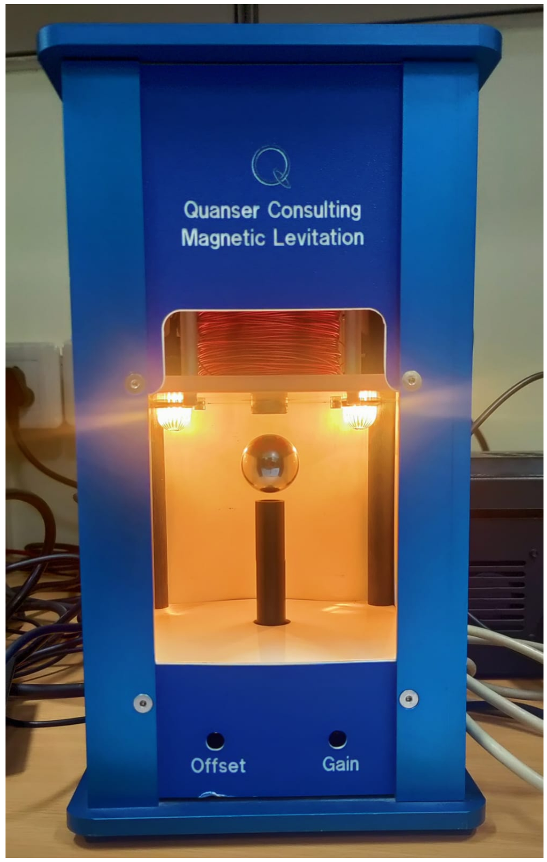

Figure 1: Magnetic Levitation kit

Magnetic Levitation is largely used in a lot of applications and proves to be very useful since it provides contact-less support that eliminates friction. All practical magnetic levitation systems are inherently open-loop unstable and rely on feedback control for producing the desired actions.

3 Methodology

3.1 Mathematical Model

We need to evaluate both mechanical and electrical systems.

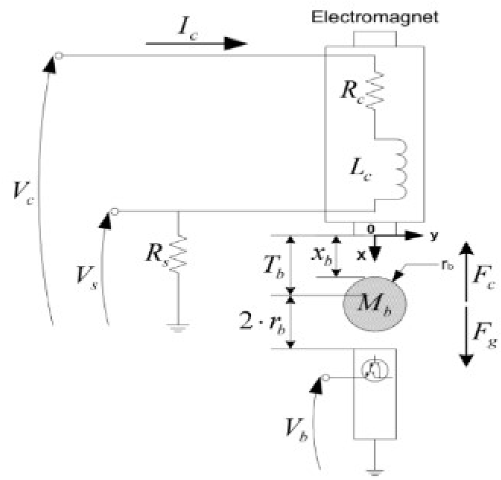

Figure 2: Mathematical Model

Symbol	*Description*	*Value*	*Unit*
I_{c_max}	Maximum Continuous Coil Current	3	A
L_c	Coil Inductance	412.5	mH
R_c	Coil Resistance	10	Ω
N_c	Number Of Turns in the Coil Wire	2450	
l_c	Coil Length	0.0825	m
r_c	Coil Steel Core Radius	0.008	m
K_m	Electromagnet Force Constant	6.5308E-005	$N.m^2/A^2$
R_s	Current Sense Resistance	1	Ω
r_b	Steel Ball Radius	1.27E-002	m
M_b	Steel Ball Mass	0.068	kg
T_b	Steel Ball Travel	0.014	m
g	Gravitational Constant on Earth	9.81	m/s^2
i_0	Magnetic Permeability Constant	4ð E-007	H/m
K_B	Ball Position Sensor Sensitivity (Assuming a User-Calibrated Sensor Measurement Range from 0 to 5 V)	2.83E-003	m/V

Figure 3: Given values

3.2 Electrical System

The coil used in the electromagnet has an inductor and a resistance. The voltage applied to the coil results in a current governed by the differential equation given below:

For the given system :-

$$V(t) = i(t)\,(R_c + R_s) + L\,\frac{d\,i(t)}{dt}$$

Taking Laplace transform on both sides :-

$$V(s) = I(s)\,(R_c + R_s) + L_c\left[s\,I(s)\right]$$

$$\frac{I(s)}{V(s)} = \frac{\left(\frac{1}{R_c+R_s}\right)}{\frac{L_c s + (R_c+R_s)}{(R_c+R_s)}} = \frac{1}{sL_c + R_c + R_s}$$

$$\frac{I(s)}{V(s)} = \frac{K_{c\,dc}}{T_c s + 1}$$

$$K_{c\,dc} = \frac{1}{R_c + R_s}$$

$$\tau = \frac{L_c}{R_s + R_c}$$

We observe that the system is of order 1 and its only pole is negative. Since the pole is situated in the Left Half Plane, the electrical system is stable.

3.3 Mechanical System

For mechanical system :-

$$F_g = M_b g$$

$$F_c + F_g = -\frac{1}{2}\left(\frac{K_m I_c^2}{x_b^2}\right) + M_b g$$

$$F_g = M_b g$$
$$= 0.068 \times 9.81$$
$$F_g = 0.66708\,N$$

Now,

$$F_c + F_g = M_b \ddot{x}_b$$

$$\ddot{x}_b = g - \frac{1}{2}\left(\frac{K_m I_c^2}{x_b^2 M_b}\right)$$

EOM should be linearized around a quiescent point of operation. In the case of the levitated ball, the operating range corresponds to small departure positions, xb1, small departure currents, Ic1, from the desired equilibrium point (xb0, Ic0).

3.4 EOM Linearization and Transfer function

$\Rightarrow$ 2^{nd} order diff. eq^n

The eq^n of motion should linearized, around the point of operation (say equilibrium)

Let equilibrium point be (x_{b_0}, I_{c_0}) – (constant)

Around this point, we have

$$x_b = x_{b_0} + x_{b_1} \quad ; \quad I_c = I_{c_0} + I_{c_1}$$

$\because$ x_{b_0} and I_{c_0} are constants :-
the derivatives of x_{b_0} and x_{b_1} and I_c and I_{c_1} will be same.

$\therefore$ At equilibrium, the above eq^n will be :-

$$g = \frac{1}{2}\left(\frac{K_m I_c^2}{M_b x_b^2}\right)$$

$$\therefore \quad M_b g = \frac{1}{2}\left(\frac{K_m I_c^2}{M_b x_b^2}\right)$$

Now, $I_c = \sqrt{\dfrac{2 M_b^2 g x_b^2}{K_m}} = I_{c0}$ (at eq.)

$$\therefore \quad I_{c0} = \left(\sqrt{\frac{2g}{k_m}}\right) M_b \, x_{b0} = 0.8575$$

Using Taylor seriers of linearization of EOM :-

$$F(x,y) = F(x_0, y_0) + \frac{\partial F(x,y)}{\partial x}(x_1) + \frac{\partial F(x,y)}{\partial y}(y_1)$$

$$F = g - \frac{1}{2} \frac{K_m I_c^2}{x_b^2 M_b}$$

$$\frac{dF}{dx} = \frac{-1}{2} \frac{K_m I_c^2}{M_b} \times \frac{(-2)}{x_b^3}$$

$$\frac{dF}{dy} = \frac{dF}{I_c} = \frac{-K_m I_c}{x_b^2 M_b}$$

$$\ddot{x}_b = g - \frac{1}{2} \frac{K_m I_{c0}^2}{x_{b0}^2 M_b} + \frac{K_m I_{c0}^2 x_{b1}}{M_b x_b^3} - \frac{K_m I_{c0} I_{c1}}{x_b^2 M_b}$$

$$\ddot{x}_b = \frac{2g x_{b1}}{x_{b0}} - \frac{2g I_{c1}}{I_{c0}}$$

$$\frac{X(s)}{I(s)} = \frac{K_{b\_dc} \, \omega_b^2}{s^2 - \omega_b^2}$$

$$K_{b\_dc} = \frac{x_{b0}}{I_{c0}} = 0.00699 = 7 \times 10^{-3}$$

$$\omega_b = \sqrt{\frac{2g}{x_{b0}}} = \sqrt{\frac{2 \times 9.81}{6 \times 10^{-3}}} = 57.18$$

$$G(s) = -\frac{K_{b\_dc}\ \omega_b^2}{s^2 - \omega_b^2}$$

$$= \frac{-23.05}{s^2 - (57.18)^2}$$

This is a second-order system of type zero. Two poles are located on the real axis. The open loop system is thus unstable and requires a proper feedback control.

4 Controller Design

4.1 Coil current controller design:

Prior to control the steel ball position, the current flowing through the electromagnet needs to be controlled. The electromagnet current control loop consists of a Proportional-plus-Integral (PI) closed-loop scheme.

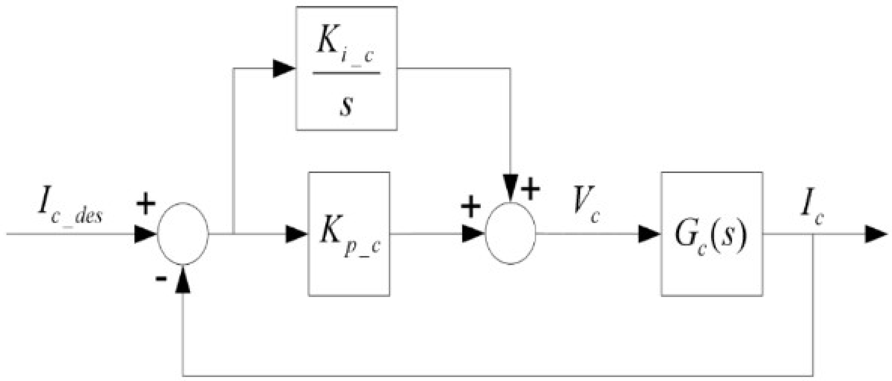

Figure 4: Coil Current Controller

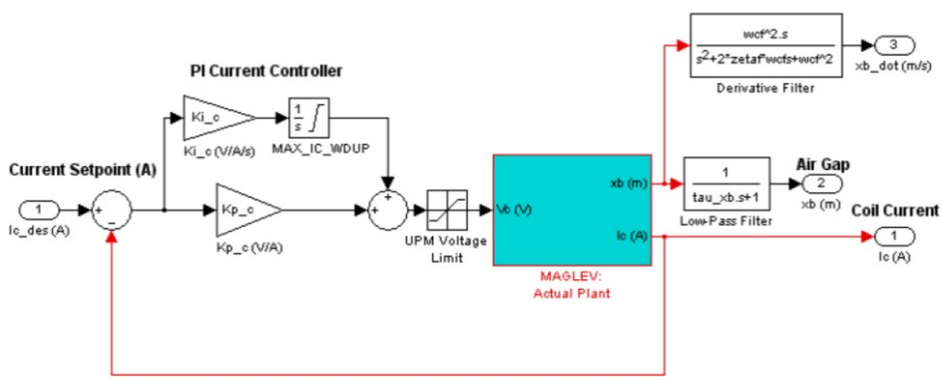

Figure 5: Simulink model for Coil Current Controller

Transfer function :-

$$\frac{\left(K_p + \frac{K_I}{s}\right) G(s)}{1 + \left(K_p + \frac{K_I}{s}\right) G(s)}$$

For poles -

$$1 + \left(K_p + \frac{K_I}{s}\right) G(s) = 0$$

$$s(0.0375s + 1) + (K_ps + K_I)(0.0909) = 0$$

$$s^2 + \frac{(1 + 0.0909 K_p)}{0.0375} s + \frac{(0.0909) K_I}{0.0375} = 0 \quad - (1)$$

Desired poles :-

$$pc_{1,2} = -235 \pm 70i$$

$$s^2 + (-pc_1 - pc_2)s + (pc_1)(pc_2) = 0$$

$$s^2 + 470s + 60125 = 0 \quad - (2)$$

Comparing (1) and (2) (Design charactersictic eq^n and Desired characteristic eq^n)

$$\boxed{K_p = 182.89}$$

$$\boxed{K_I = 24804.04}$$

4.2 Ball Position Controller Design:

The steel ball position is controlled by means of a Proportional-plus-Integral-plus-Velocity (PIV or PID) closed-loop scheme with the addition of a feed-forward action.

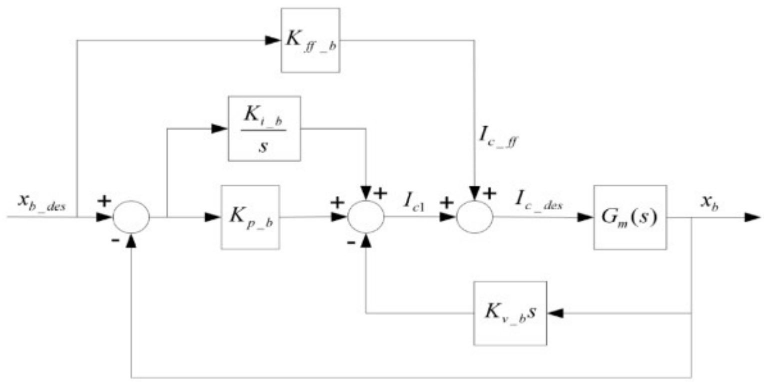

Figure 6: Ball Position Controller

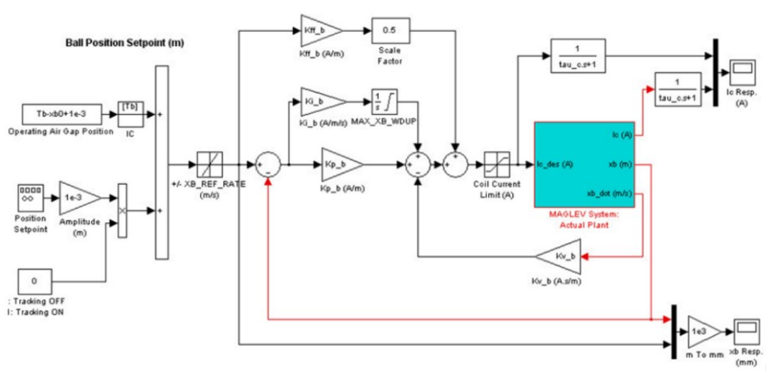

Figure 7: Simulink model for Ball Position Controller

For ball position controller design :-

poles = $\{-2.5, -44, -51.6\}$

$$s^3 + (98.1)s^2 + (2509.4)s + 5676 = 0$$

$$s^3 - \frac{2gk_v}{I_{c0}} s^2 + \left(\frac{-2g}{x_{b0}} - \frac{2gk_p}{I_{c0}}\right) s - \frac{2gk_i}{I_{c0}} = 0$$

$$\frac{-2gk_v}{I_{c0}} = 98.1$$

$$\boxed{k_v = -4.2875}$$

$$\frac{-2g}{x_{b0}} - \frac{2gk_p}{I_{c0}} = 2509.4$$

$$\boxed{k_p = -252.591}$$

$$\frac{-2gk_i}{I_{c0}} = +5676$$

$$\boxed{k_i = -248.07}$$

$$k_{ff} = \frac{I_{c0}}{x_{b0}} = \frac{0.8575}{6 \times 10^{-3}}$$

$$\boxed{k_{ff} = 142.916}$$

5 Observations and Results

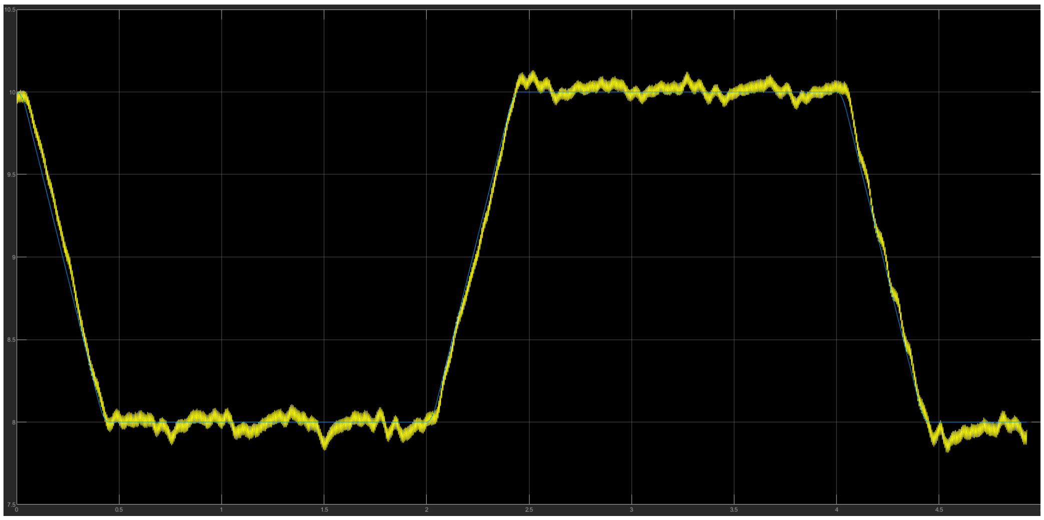

Figure 8: Positon Response

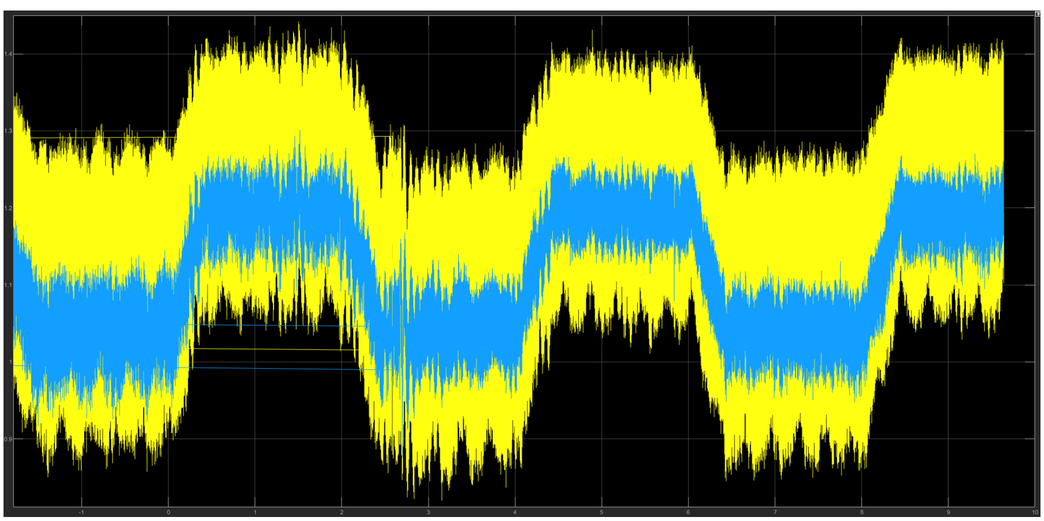

Figure 9: Current Response

The fluctuations are observed due to various factors like linear approximation using Taylor series, mechanical dealys and errors in the system, etc.

Controller Gains	**Values**
$K_{P,C}$	182.89
$K_{I,C}$	24804.08
$K_{FF,B}$	142.916
$K_{P,B}$	-252.591
$K_{I,B}$	-248.07
$K_{VV,B}$	-4.2875