EEL3040 : Control Systems

LAB 10 Quadcopter: Dynamics, Simulation, and Control

November 1, 2022

1 Objective

To use state-feedback using LQR(Linear-Quadratic Regulator) optimization to control the yaw position and implement it on the set-up of half-quadrotor.

2 Apparatus/software required

Half-quadrotor, MATLAB

3 Theory

3.1 Description

- In this set up, both the front and back rotors are horizontal to the ground and only motions about the yaw axis are enabled (i.e. the pitch axis is locked). By changing the direction and speed of the rotors, users can change the yaw axis angle.

Figure 1: Quanser Aero Experiment in half-quadrotor configuration

- A quadrotor helicopter (quadcopter) has four evenly spaced rotors at the corners of a square body. The helicopters help provide damping to stop the motion.

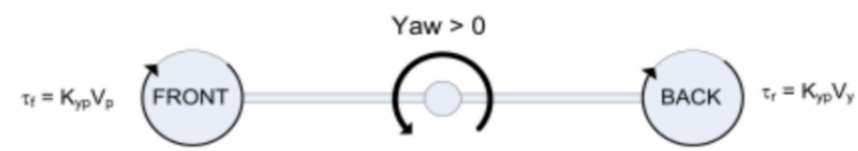
- There are 6 degrees of freedom, 3 translational and 3 rotational. The four rotor speeds are the inputs and are independent of each other as they are perpendicular.

- The torque from the rotors causes the system to rotate about the yaw axis. Equation of yaw motion :

$$\tau = \mathrm{J_y}\ddot{\psi} + \mathrm{D_y}\dot{\psi} = -\ \mathrm{K_{yp}V_p}\ -\ \mathrm{K_{py}V_y}$$

  where J_y is the moment of inertia about the yaw axis, D_y, is the viscous damping coefficient about the yaw axis, K_{yp} is the cross-torque thrust gain, V_p is the voltage applied to the front (pitch) motor, and V_y is the voltage applied to the back (yaw) motor.

- Free Body Diagram of Quadcopter:



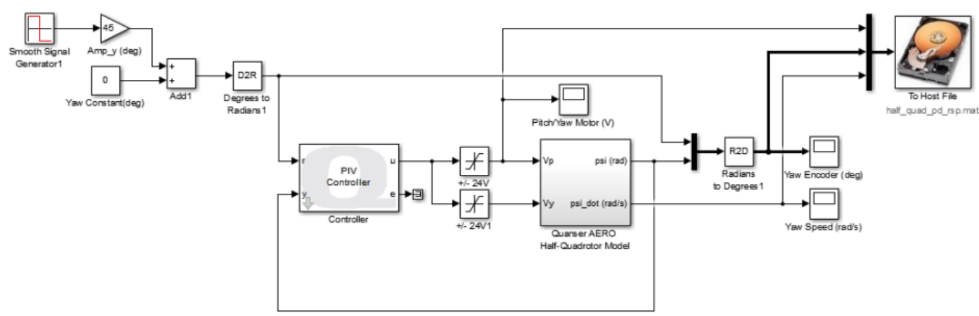
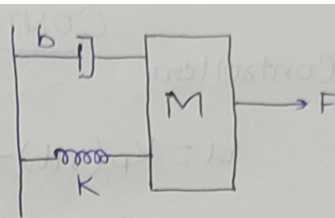


Figure 2: Simulink model used to simulate closed-loop PD response of half-quadrotor system

4 Calculation

$$F = ma$$
$$\tau = J\ddot{\psi}$$



$$F = m\ddot{x} + b\dot{x} + kx$$

$$J_p\ddot{\theta} + D_p\dot{\theta} + K_{sp}\theta = \tau_p \qquad \tau_p = K_{pp}V_p + K_{py}V_y$$
$$J_y\ddot{\psi} + D_y\dot{\psi} = \tau_y \qquad \tau_y = K_{yp}V_p + K_{yy}V_y$$

$$\therefore\ J_y\ddot{\psi} + D_y\dot{\psi} = K_{yp}V_p + K_{yy}V_y$$

Now, taking only yaw, $u = V_p = V_y$

$$\boxed{J_y\ddot{\psi} + D_y\dot{\psi} = 2K_{yp}u}$$

To find state space representation

$$x_1 = \psi$$
$$x_2 = \dot{\psi} = \dot{x}_1$$
$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{2K_{yp}u}{J_y} - \frac{D_y}{J_y}x_2$$

$$\therefore \begin{bmatrix}\dot{x}_1 \\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1 \\ 0 & \frac{-D_y}{J_y}\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix} + \begin{bmatrix}0 \\ \frac{2K_{yp}}{J_y}\end{bmatrix}u$$

$$y = \psi \qquad x = \begin{bmatrix}\psi \\ \dot{\psi}\end{bmatrix}$$

$$\therefore\ C = [1 \quad 0] \qquad D = [0]$$

CONTROLLER DESIGN

PD Controller:

$$u = K_p(r(t) - y(t)) - K_d \dot{y}(t)$$

Taking Laplace Transform,

$$U(s) = K_p(R(s) - Y(s)) - K_d s Y(s)$$

$$\Psi(s) = \frac{2K_{yp}}{J_y s^2 + D_y s}\left[K_p(R(s) - Y(s)) - K_d s \Psi(s)\right]$$

Open loop Transfer function

$$OLTF = \frac{\Psi(s)}{U(s)} = \frac{2K_{yP}}{J_y s^2 + D_y s}$$

Closed loop Transfer function

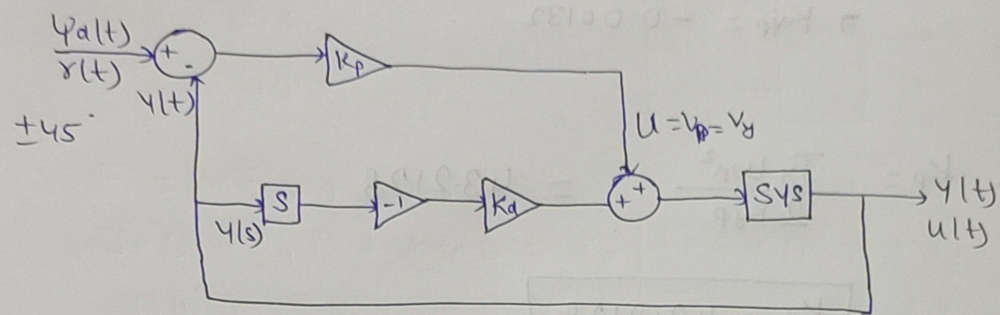
$$\frac{\Psi(s)}{R(s)} = \frac{KK_p/\tau}{s^2 + (1+KK_d)/\tau s + KK_p/\tau}$$

where $K = \frac{2K_{yp}}{D_y}$ and $\tau = \frac{J_y}{D_y}$

Comparing with $CLTF = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$$\therefore K_p = \frac{\tau\omega_n^2}{K} = \frac{J_y \omega_n^2}{D_y K} = \frac{J_y \omega_n^2}{2K_{yp}}$$

$$K_d = \frac{2\tau\zeta\omega_n - 1}{K} = \frac{2J_y\zeta\omega_n - D_y}{K_{yp}}$$



Given:- $PO = 7.5\%$; $t_p = 1.25$

$$\zeta = -\log\left(\frac{PO}{100}\right)\frac{1}{\sqrt{\log\left(\frac{PO}{100}\right)^2+\pi^2}} = 0.636.$$

$$\omega_n = \frac{\pi}{t_p\sqrt{1-\zeta^2}} = 3.26 \text{ rad/s.}$$

Also, $T_{max} = 3.3s$

$\tau = 1.35$

$$\therefore D_y = \frac{J_y}{\tau} = \frac{0.022}{1.135} = 0.019$$

To find K_{yp} $\Rightarrow$

$$K_{yp} = \frac{J_y \frac{\Delta\omega_y}{\Delta t} + D_y \Delta\omega_p}{V_p}$$

$V_p = 20$, $\Delta\omega_p = 0$ [$\because$ no motion around pitch axis]

$\Delta t = 31s$

$\Rightarrow K_{yp} = -0.00132$

$$\therefore K_p = \frac{J_y \omega_n^2}{2 K_{yp}} = 43.2128$$

$$\boxed{K_p = 43.2128}$$

$$K_d = \frac{2 \tau \zeta \omega_n - 1}{K}$$

$$\Rightarrow \boxed{K_d = 12.8045}$$

5 Plots

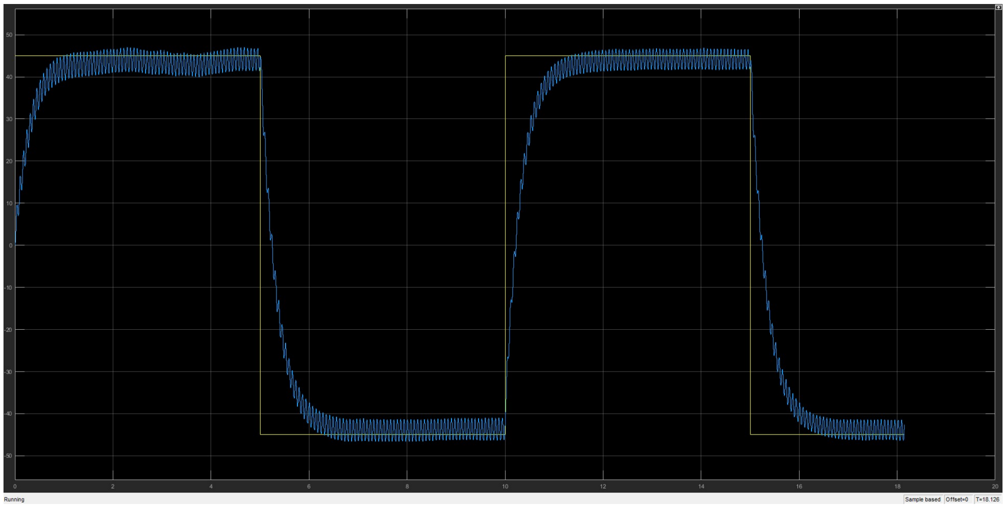


Figure 3: Yaw Position

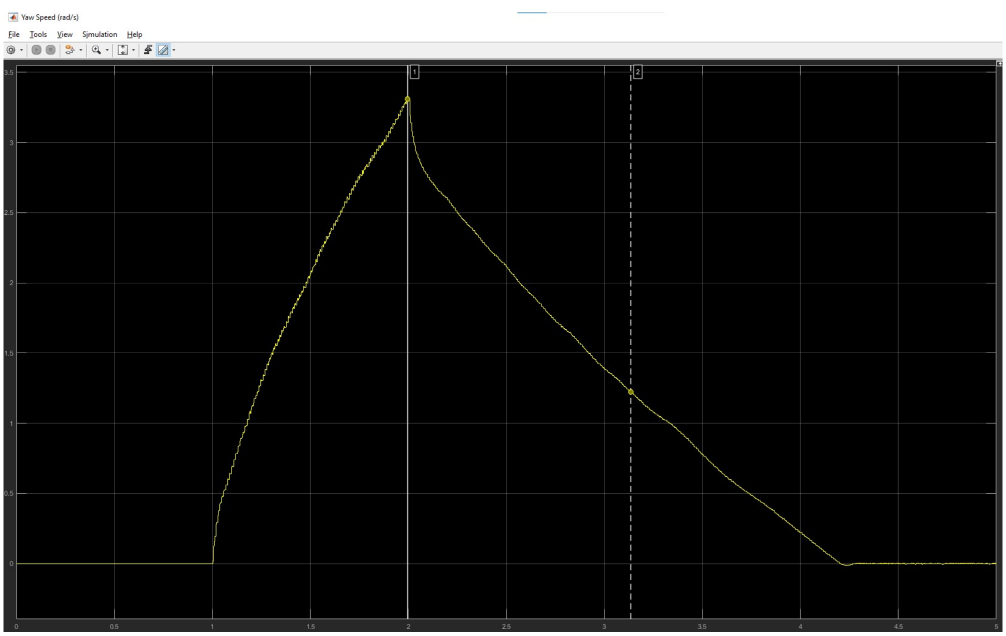


Figure 4: Yaw Speed

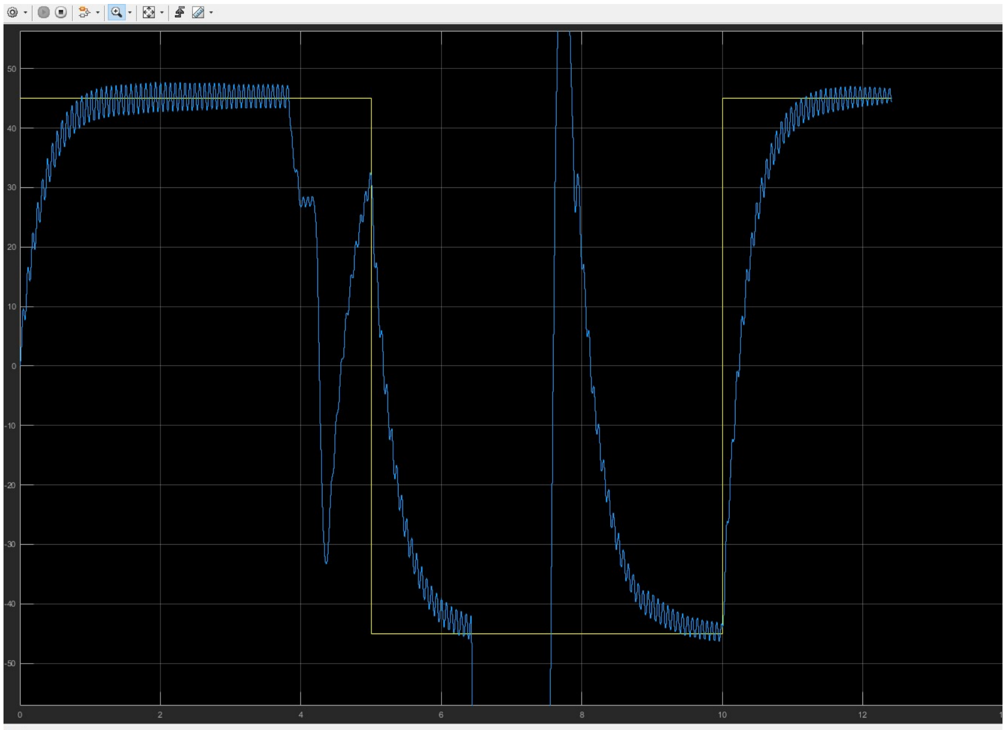


Figure 5: Effect of Disturbances

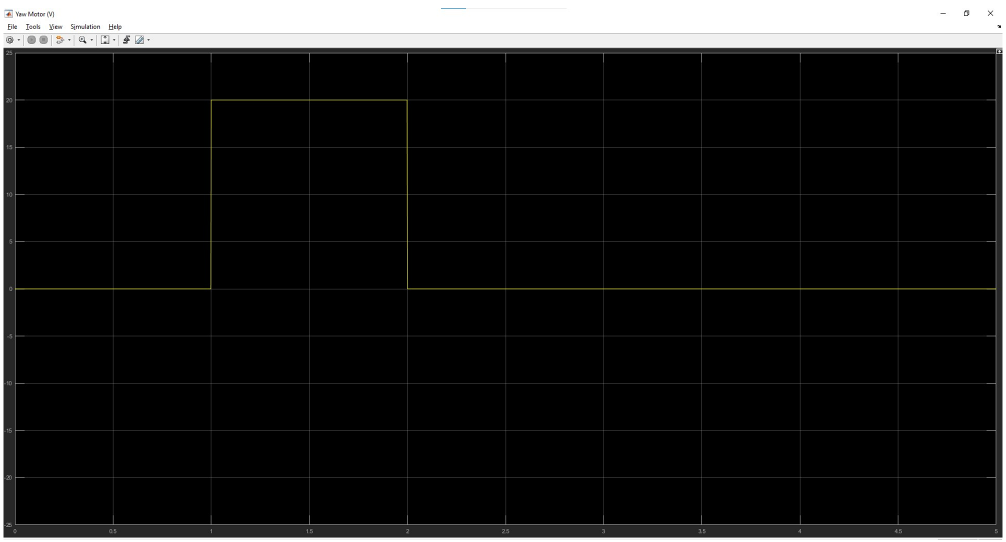


Figure 6: Impulse Voltage

6 Conclusion

Hence, we observed and verified the effect of input waveform, loop gain, and system type upon steady-state errors.