Lab 7 Report
Steady State Error

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1 Objective

To verify the effect of input waveform, loop gain, and system type upon steady-state errors. MATLAB, Simulink and the control system toolbox will be used for this experiment.

2 Theory

The closed loop system is given in the diagram below. The steady state error is defined as the difference between the input and output of the system as time tends to infinity.

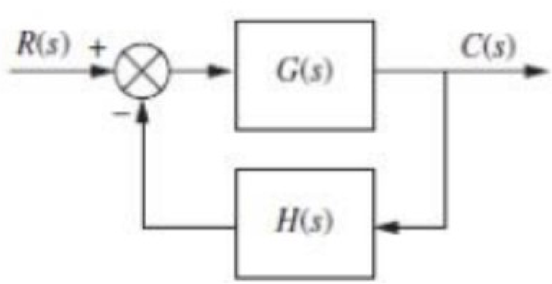

Figure 1: Closed Loop System

For the given system:

$$C(s) = \frac{G(s)C(s)}{1 + H(s)G(s)}$$

and,

$$H(s) = 1$$

For Steady State error,

$$e(s) = R(s) - C(s)$$

$$e(s) = \lim_{s \to 0} \frac{(1/s)R(s)}{1 + G(s)}$$

3 Answers for Prelab Questions:

Answer 1: For step input, system having type 1 or more than one will produce zero steady state error.
Answer 2: For ramp input, system having type 2 or more than two will produce zero steady state error.
Answer 3: For ramp input, system having type 0 will produce infinite steady state error.
Answer 4: For parabolic input, system having type 3 or more will produce zero steady state error.
Answer 5: For parabolic input, system having type 0 and type 1 will produce infinite steady state error.
Answers for questions 6, 7 and 8 are as follows:

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For input $5\,u(t)$ (unit step function)

$$G(s) = \frac{k\,(s+6)}{(s+4)(s+7)(s+9)(s+12)}$$

$$H(s) = 1$$

For unit step i/p $R(s) = 1/s$

$$K_p = \lim_{s \to 0} G(s)$$

$$= \lim_{s \to 0} \left(\frac{k\,(s+6)}{(s+4)(s+7)(s+9)(s+12)} \right)$$

$$K_p = \frac{6k}{3024}$$

$$K_p = 0.00198\,k = \frac{k}{504}$$

$$e_{ss} = 5 \left(\frac{1}{1+K_p} \right)$$

$$e_{ss} = \frac{5}{1+\frac{k}{504}}$$

$R_s = 5/s^2$, $i/p = 5\,t\,u(t)$:

$K_v = \lim_{s \to 0} s\,G(s)$

$= \lim_{s \to 0} s \times \dfrac{k\,(s+6)}{(s+4)(s+7)(s+9)(s+12)}$

$\boxed{K_v = 0}$

$\boxed{e_{ss} \to \infty}$

$R_s = 1/s^3$, $i/p = 5\,t^2\,u(t)$

$K_a = \lim_{s \to 0} s^2 \times \dfrac{k\,(s+6)}{(s+4)(s+7)(s+9)(s+12)}$

$\boxed{K_a = 0}$

$\boxed{e_{ss} \Rightarrow \infty}$

For $G(s) = \dfrac{k\,(s+6)\,(s+8)}{s\,(s+4)\,(s+7)\,(s+8)\,(s+12)}$

For input $5\,u(t)$

$R(s) = 1/s \quad , \quad H(s) = 1$

$K_p = \lim\limits_{s \to 0} G(s)$

$\boxed{K_p \longrightarrow \infty}$

$e_{ss} = 5\left(\dfrac{1}{1+K_p}\right)$

$\boxed{e_{ss} \cancel{\longrightarrow} = 0}$

For i/p $5\,t\,u(t)$

$R(s) = 1/s^2 \quad , \quad H(s) = 1$

$K_v = \lim\limits_{s \to 0} s\,G(s)$

$= \dfrac{k\,(48)}{3024}$

$\boxed{K_v = \dfrac{k}{63}}$

$\boxed{e_{ss} = \dfrac{5}{K_v} = \dfrac{63 \times 5}{k}}$

For i/p $5\ t^2\ u(t)$

$R(s) = 1/s^3$, $H(s) = 1$

$$K_a = \lim_{s \to 0} s^2 \left(G(s)\right)$$

$$\boxed{K_a = 0}$$

$$e_{ss} = \frac{1}{K_a}$$

$$\boxed{e_{ss} \longrightarrow \infty}$$

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For $G(s) = \dfrac{k(s+1)(s+6)(s+8)}{s^2(s+4)(s+7)(s+9)(s+12)}$

For input $5\,u(t)$

$R(s) = 1/s$, $H(s) = 1$

$K_p = \lim_{s \to 0} G(s)$

$\boxed{K_p \longrightarrow \infty}$

$e_{ss} = 5\left(\dfrac{1}{1+K_p}\right)$

$\boxed{e_{ss} = 0}$

For input $5\,t\,u(t)$

$R(s) = 1/s^2$, $H(s) = 1$

$K_v = \lim_{s \to 0} s\,G(s)$

$\boxed{K_v = \cancel{0}\ \infty}$

$\boxed{\&\ e_{ss} = 5\left(\dfrac{1}{K_v}\right) = 0}$

For input $5t^2u(t)$

$R(s) = 1/s^3$, $H(s) = 1$

$$K_a = \lim_{s \to 0} s^2 G(s)$$

$$K_a = \frac{k(48)}{3024}$$

$$\boxed{K_a = \frac{k}{63}}$$

$$e_{ss} = 5\left(\frac{1}{K_a}\right)$$

$$\boxed{e_{ss} = \frac{315}{k}}$$

4 Simulink implementation and Graphs:

4.1 No pole at origin of G(s):

$$G(s) = \frac{k(s+6)}{(s+4)(s+7)(s+9)(s+12)}$$

4.1.1 Step Input:

The simulink implementation is as follows:

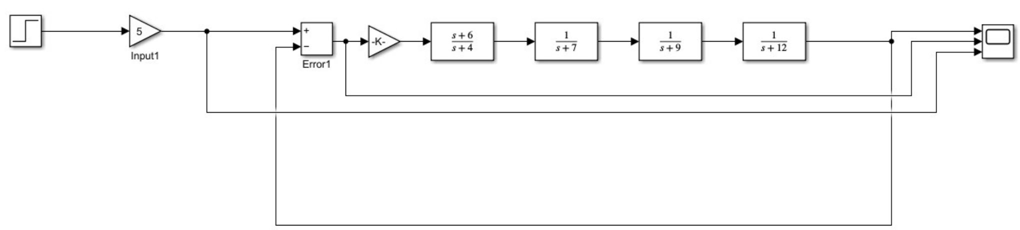

Figure 2: Simulink Implementation

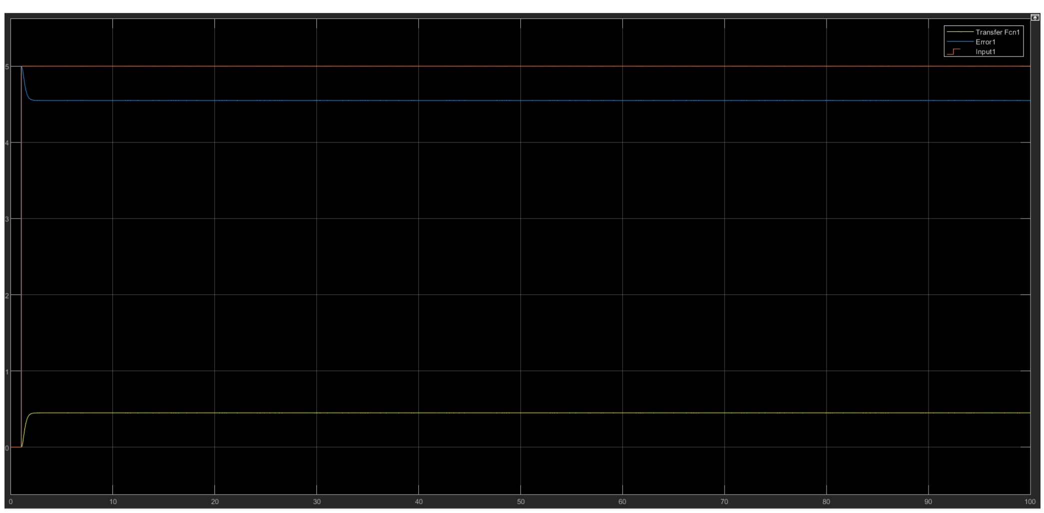

Figure 3: K = 50

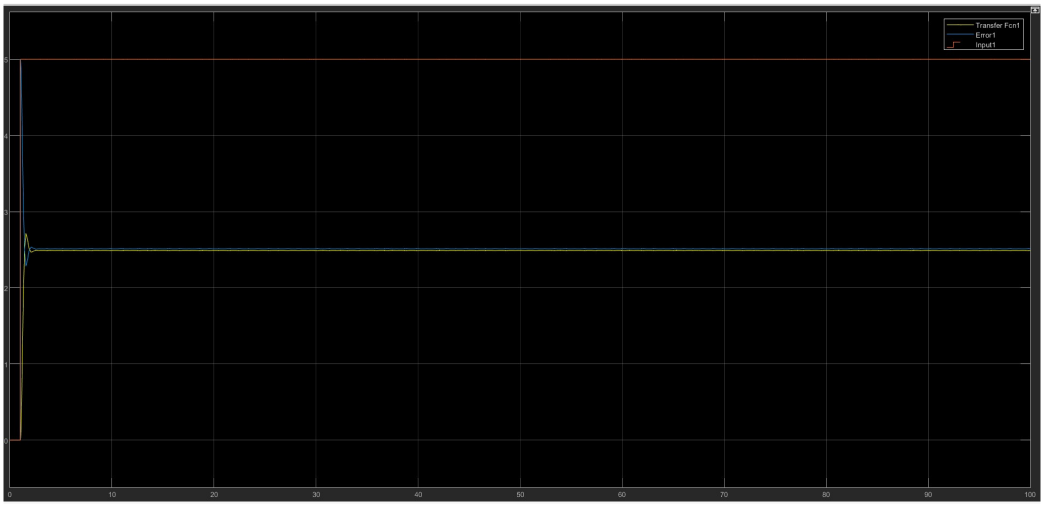

Figure 4: K = 500

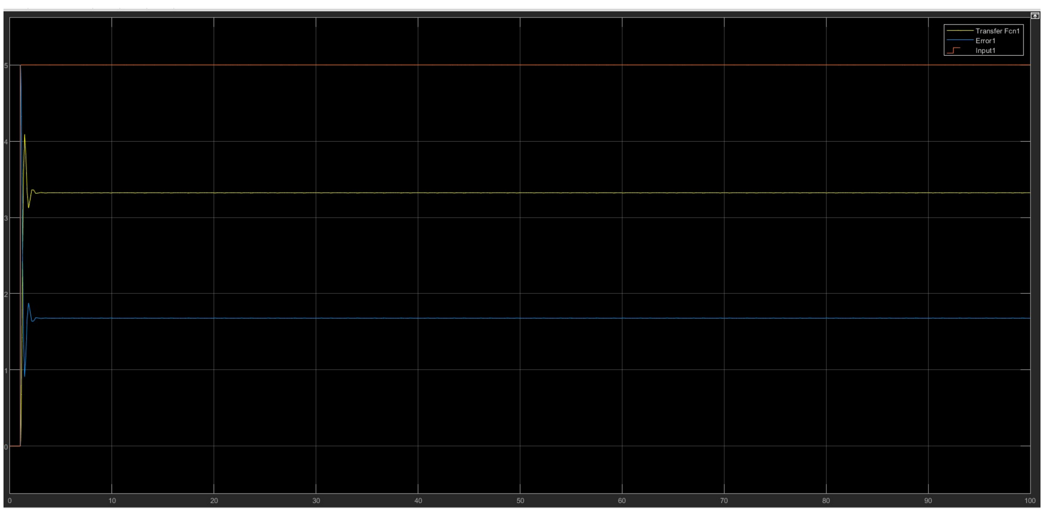

Figure 5: K = 1000

4.1.2 Ramp Input:

The simulink implementation is as follows:

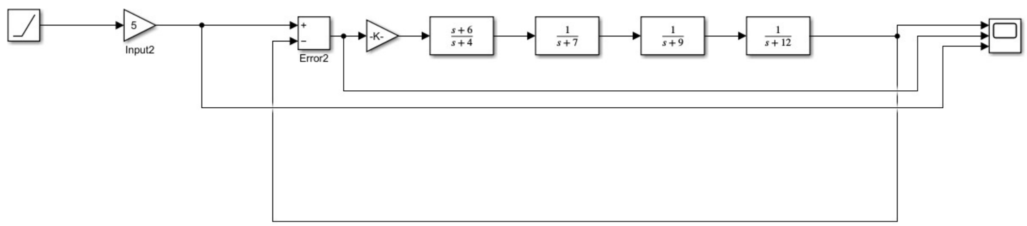

Figure 6: Simulink Implementation

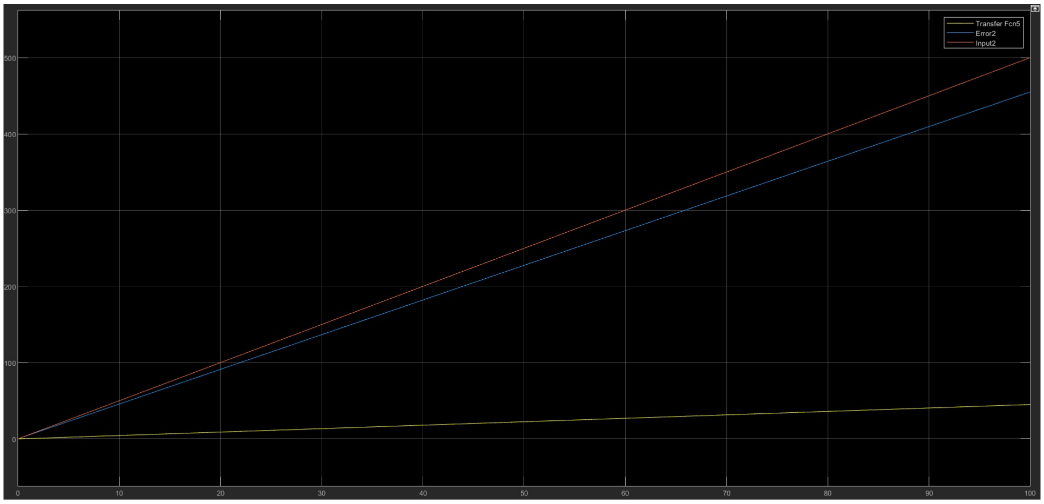

Figure 7: K = 50

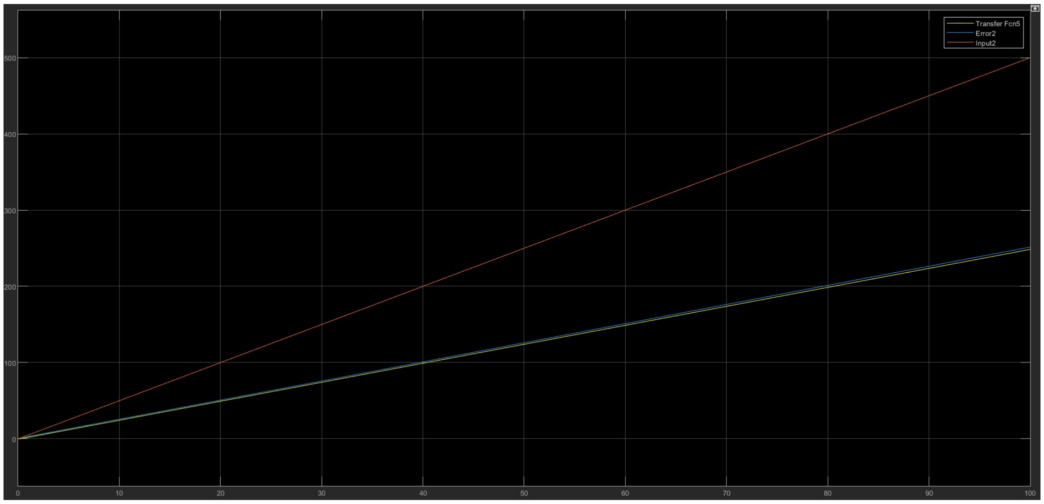

Figure 8: K = 500

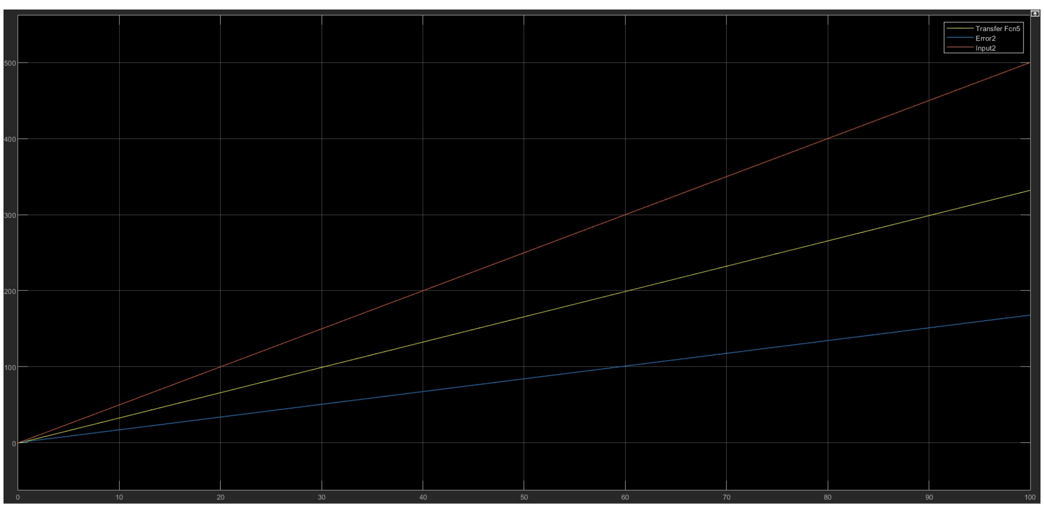

Figure 9: K = 1000

4.1.3 Parabolic Input:

The simulink implementation is as follows:

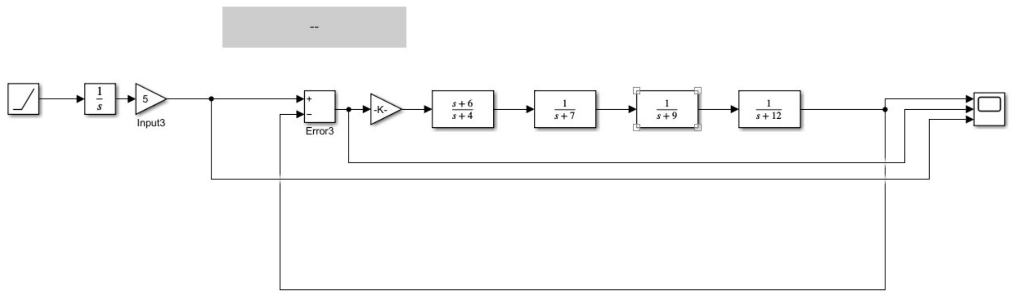

Figure 10: Simulink Implementation

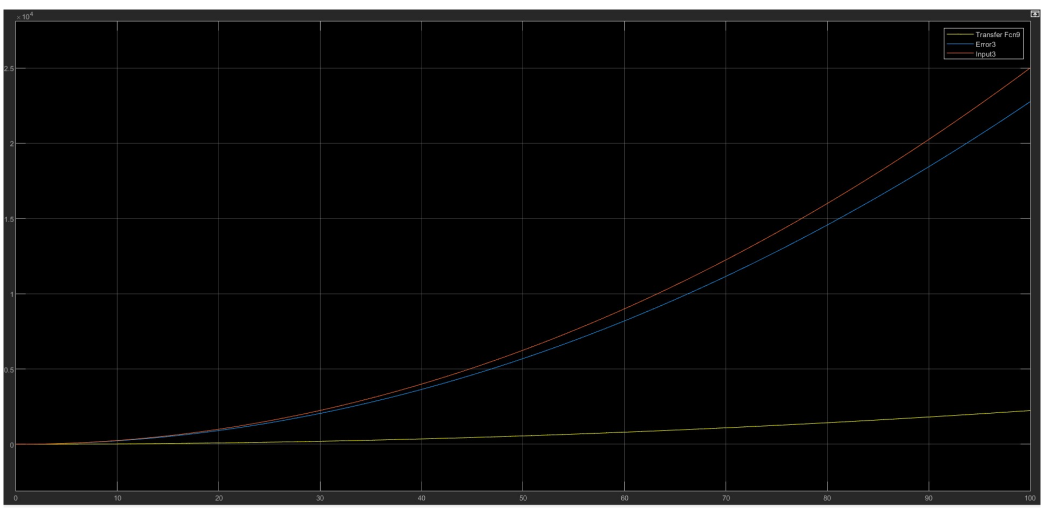

Figure 11: K = 50

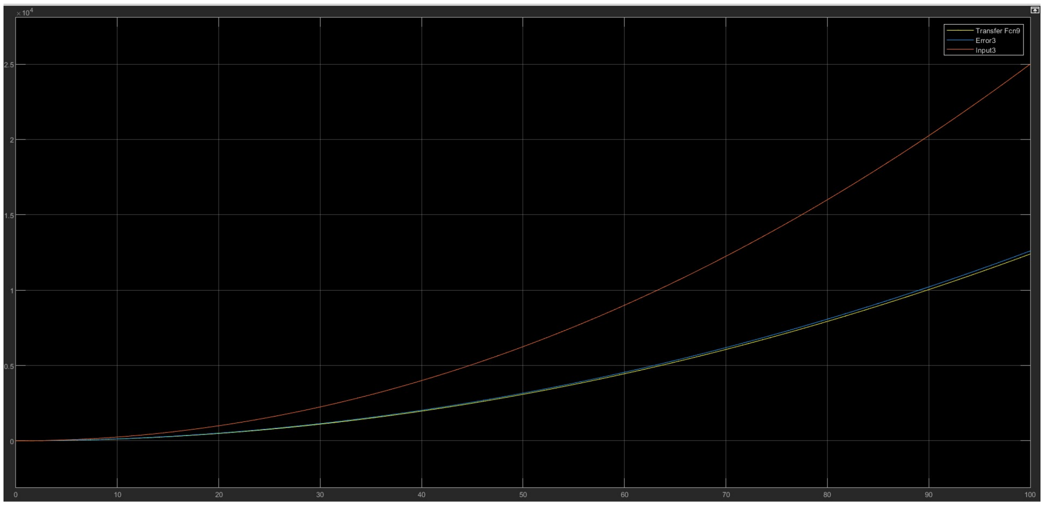

Figure 12: K = 500

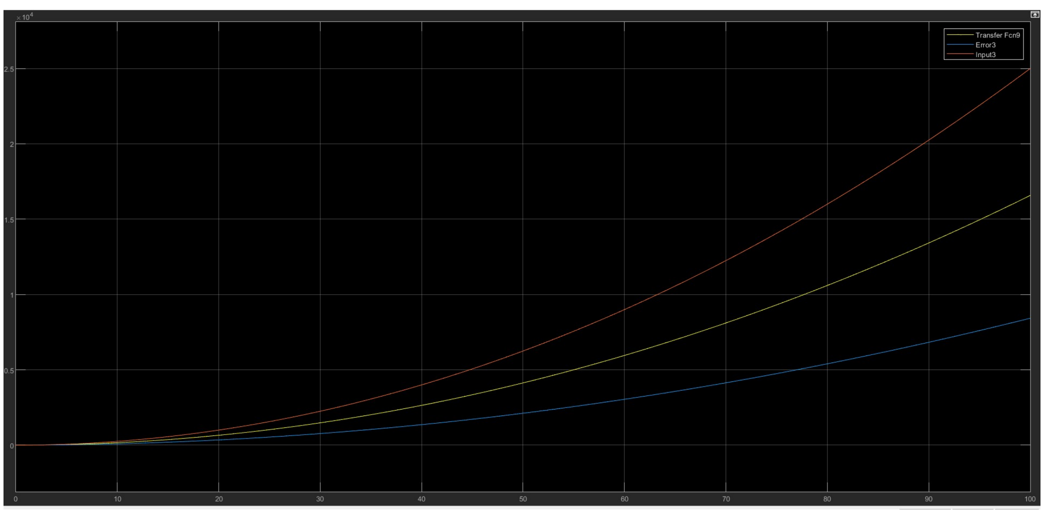

Figure 13: K = 1000

4.2 1 pole at origin of G(s):

$$G(s) = \frac{k(s+6)(s+8)}{s(s+4)(s+7)(s+9)(s+12)}$$

4.2.1 Step Input:

The simulink implementation is as follows:

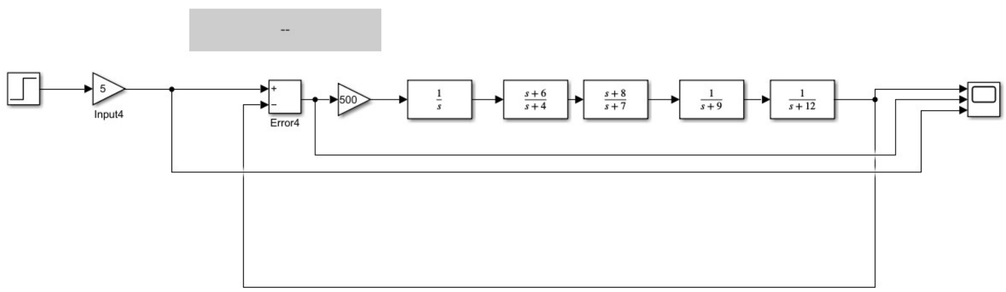

Figure 14: Simulink Implementation

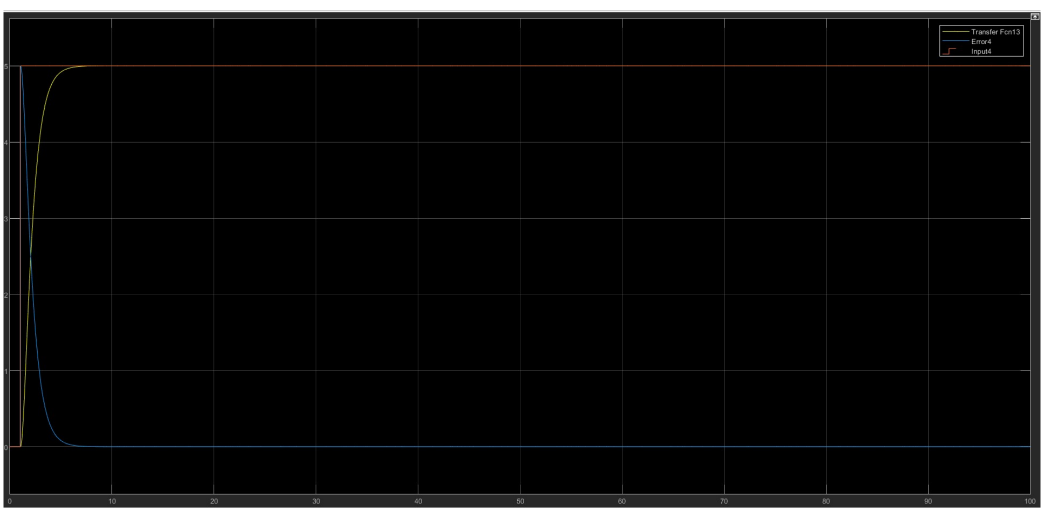

Figure 15: K = 50

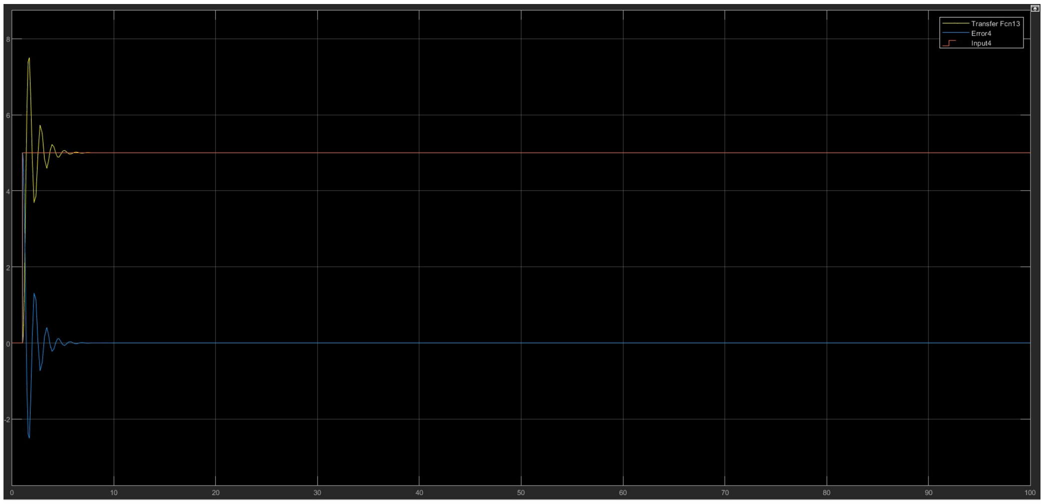

Figure 16: K = 500

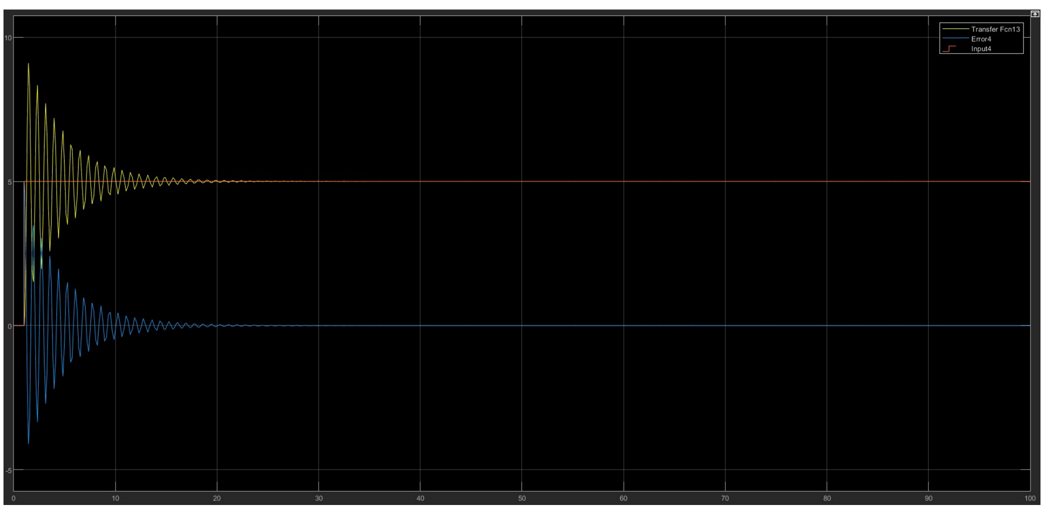

Figure 17: K = 1000

4.2.2 Ramp Input:

The simulink implementation is as follows:

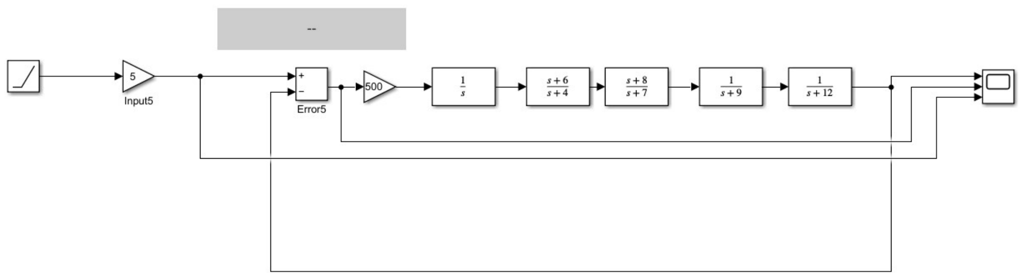

Figure 18: Simulink Implementation

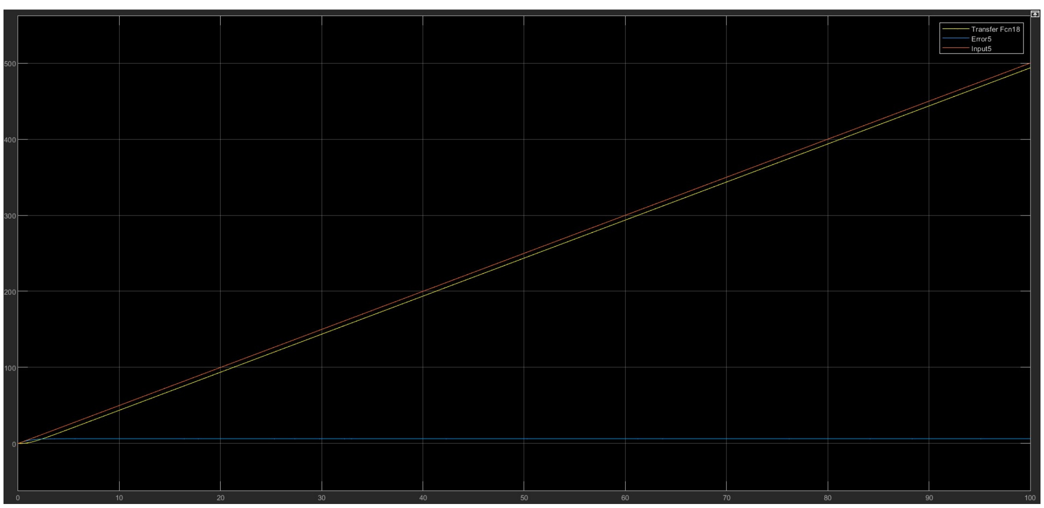

Figure 19: K = 50

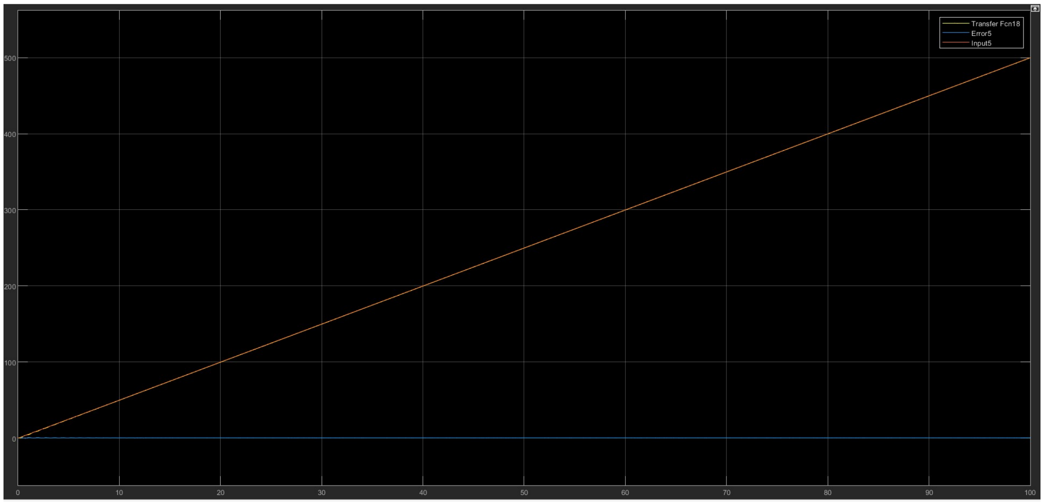

Figure 20: K = 500

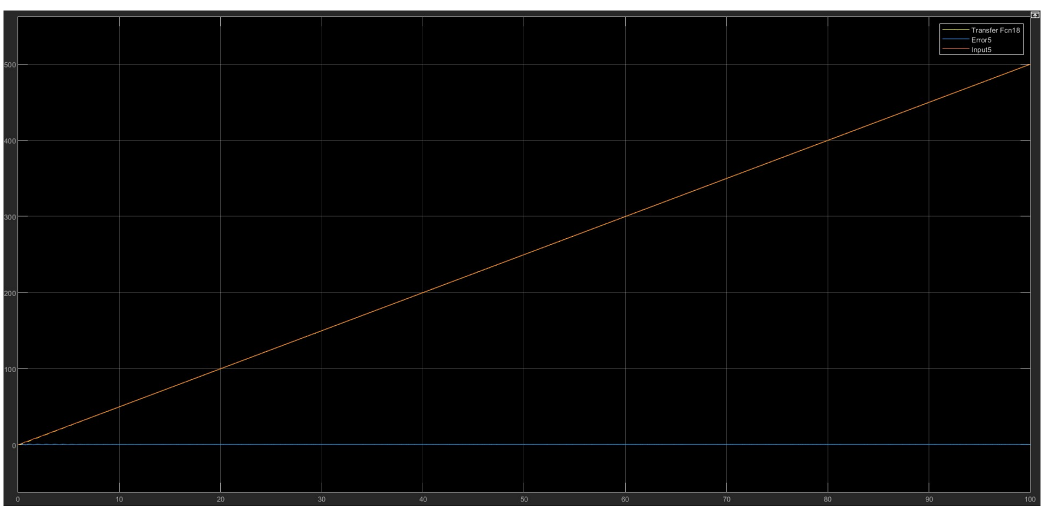

Figure 21: K = 1000

4.2.3 Parabolic Input:

The simulink implementation is as follows:

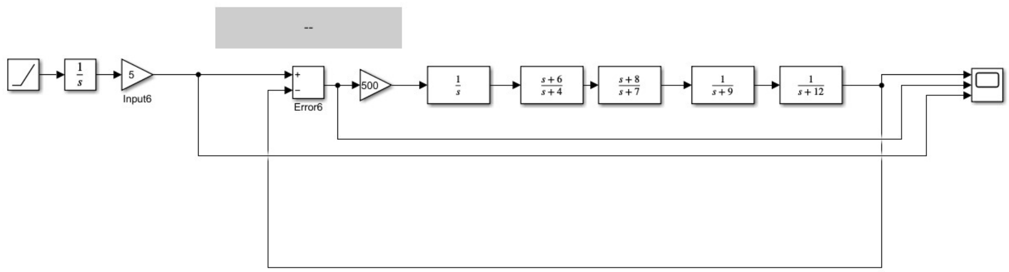

Figure 22: Simulink Implementation

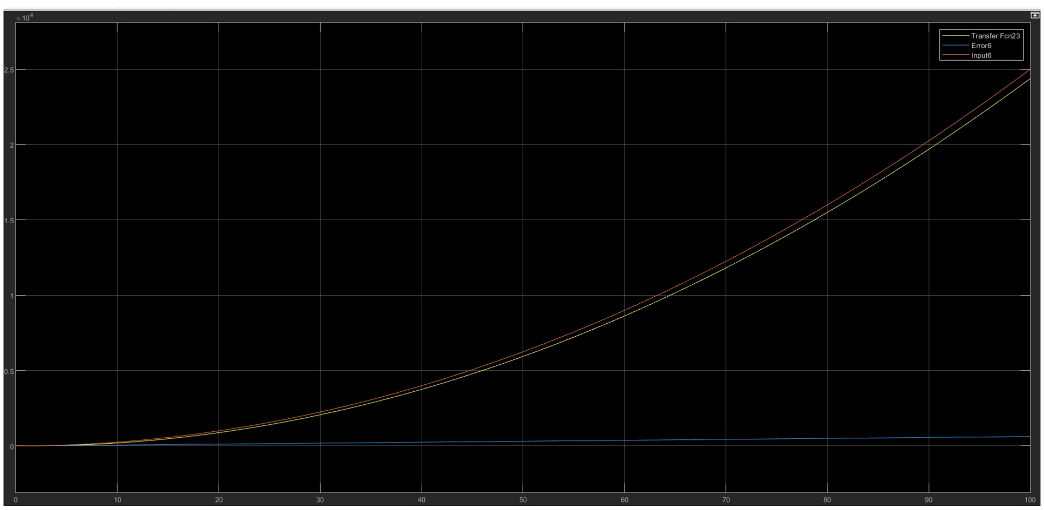

Figure 23: K = 50

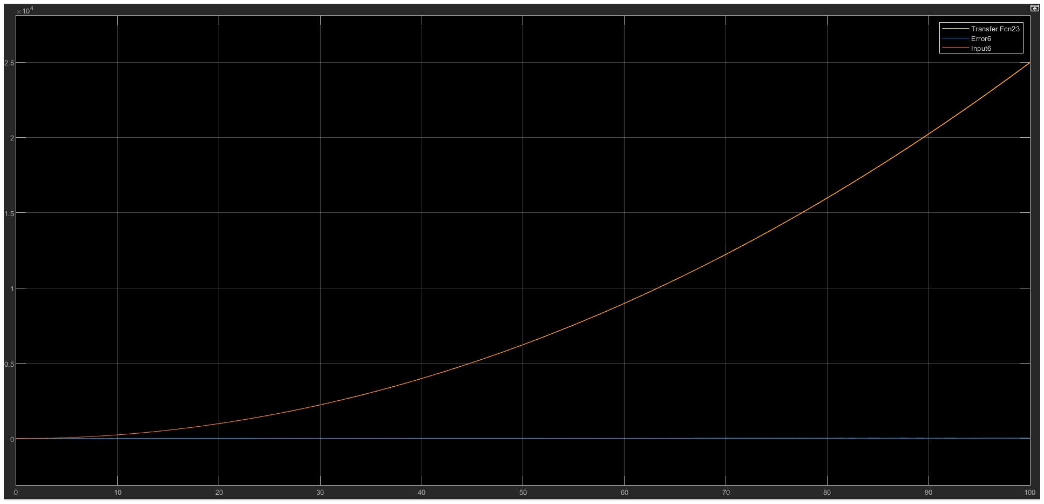

Figure 24: K = 500

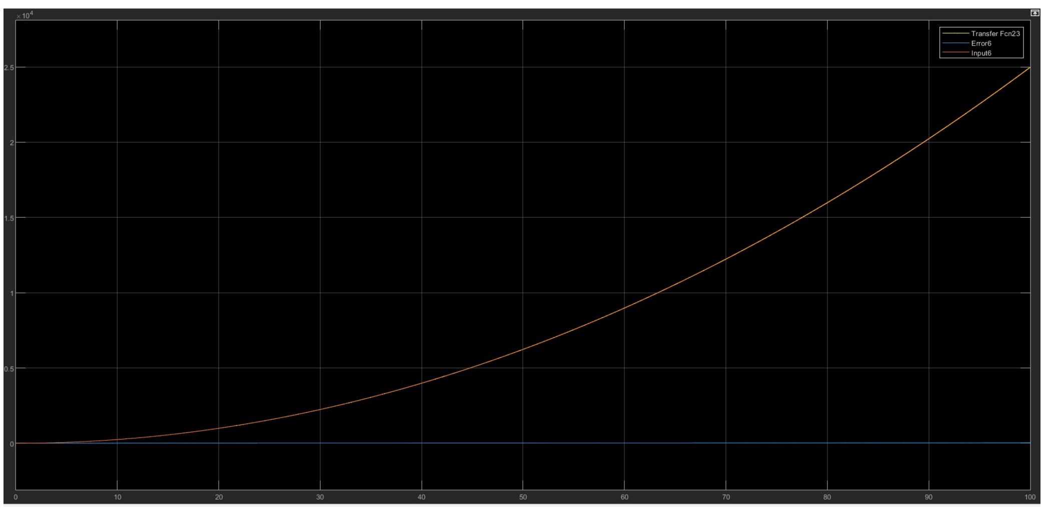

Figure 25: K = 1000

4.3 Two poles at origin of G(s):

$$G(s) = \frac{k(s+6)(s+8)(s+1)}{s^2(s+4)(s+7)(s+9)(s+12)}$$

4.3.1 Step Input:

The simulink implementation is as follows:

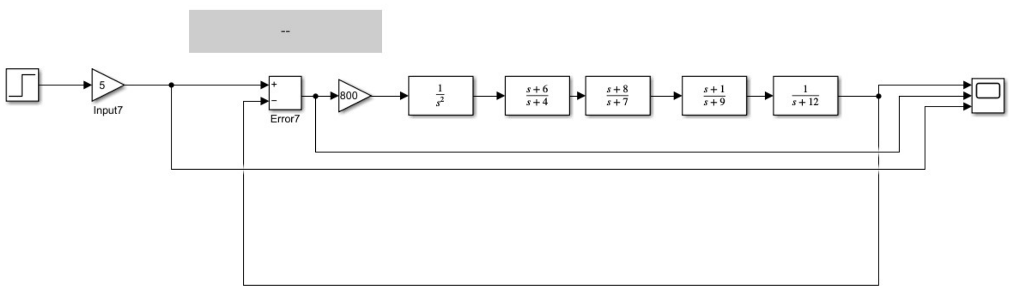

Figure 26: Simulink Implementation

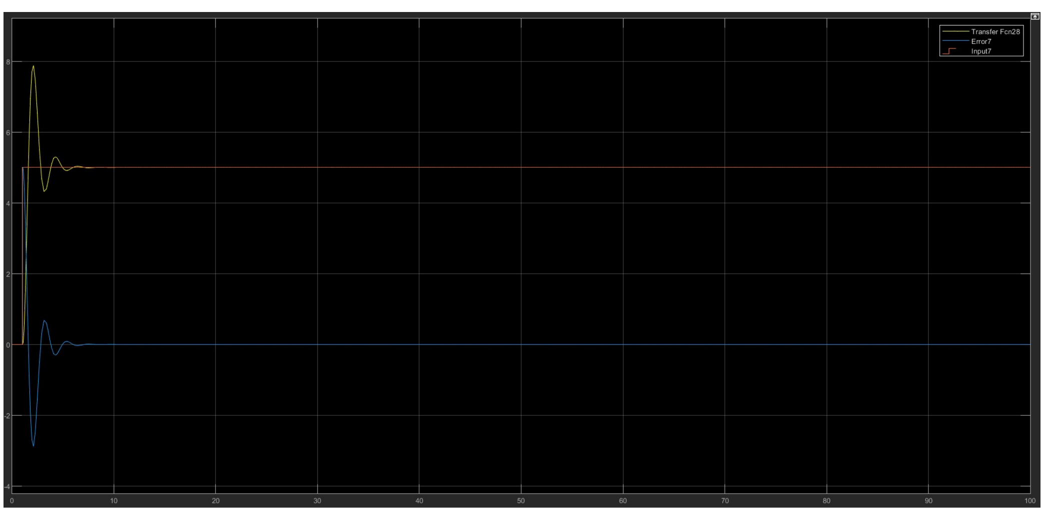

Figure 27: K = 200

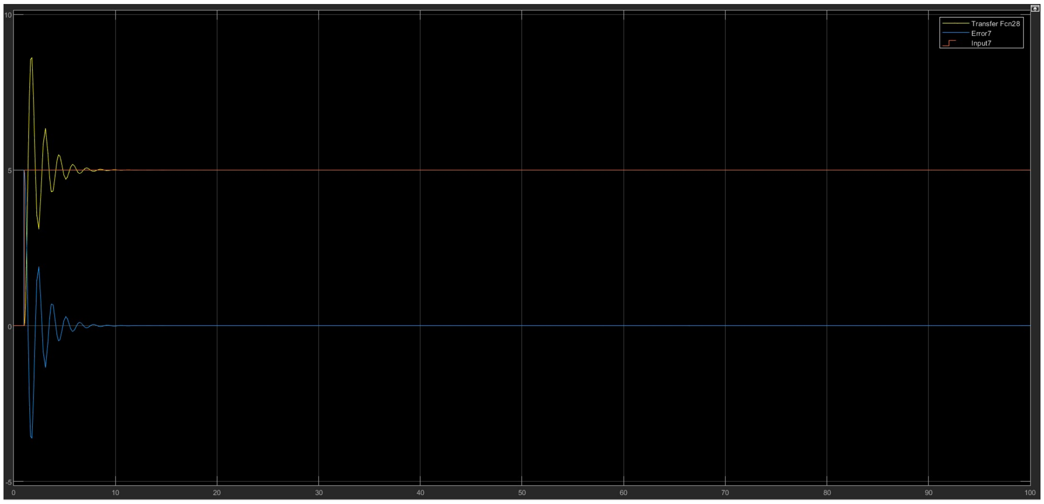

Figure 28: K = 400

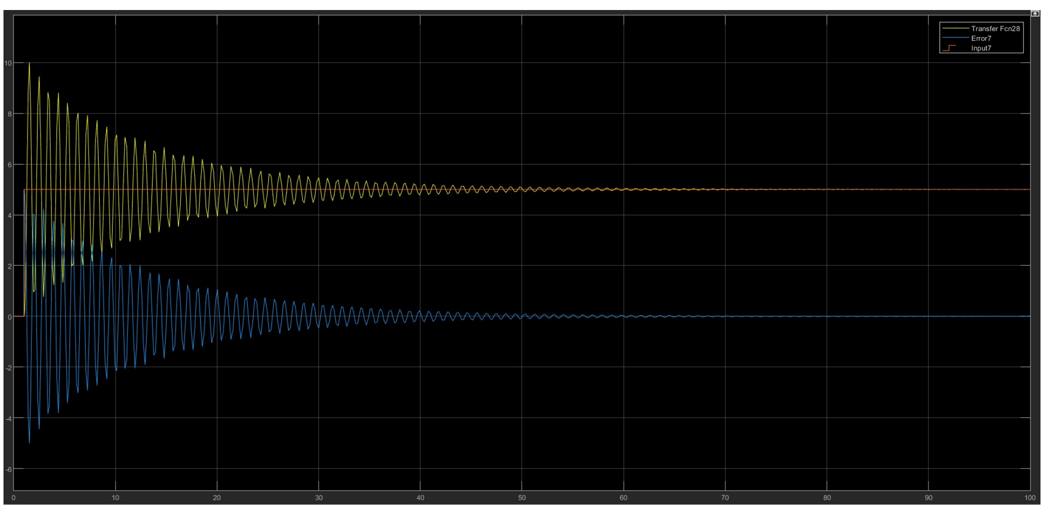

Figure 29: K = 800

4.3.2 Ramp Input:

The simulink implementation is as follows:

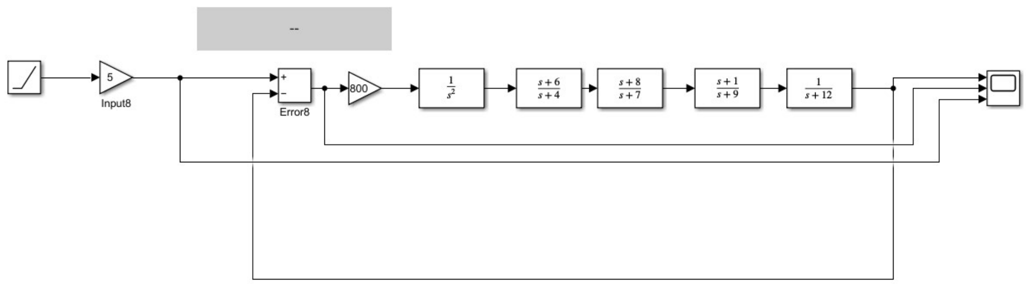

Figure 30: Simulink Implementation

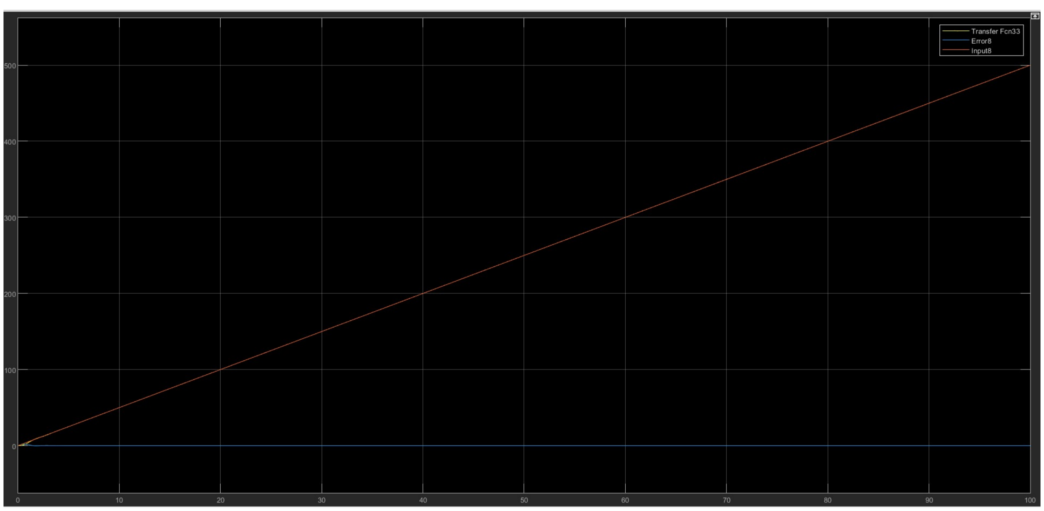

Figure 31: K = 200

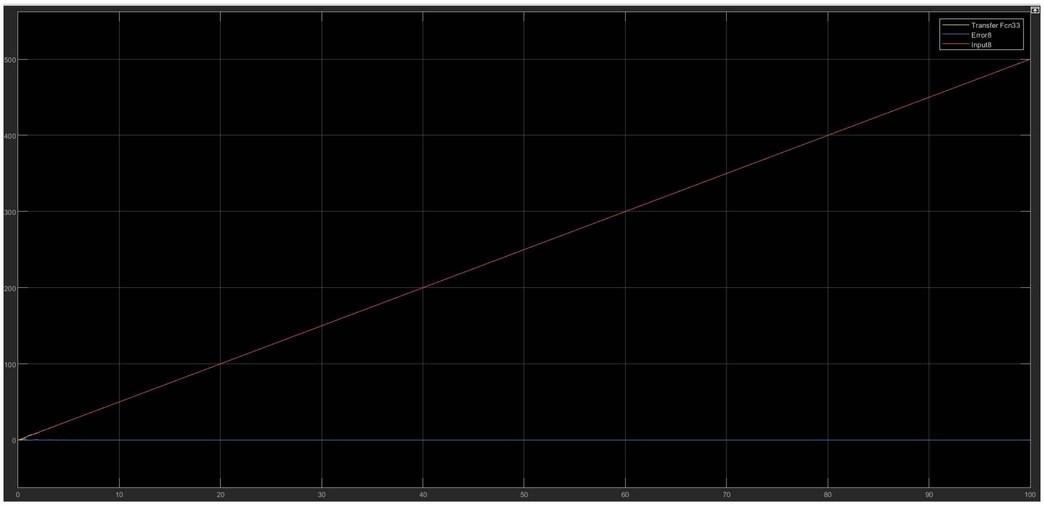

Figure 32: K = 400

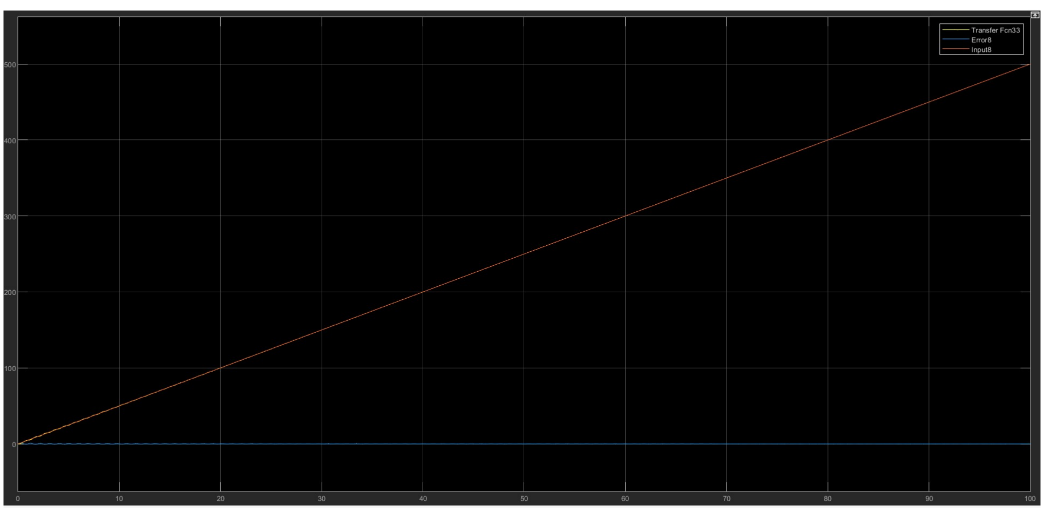

Figure 33: K = 800

4.3.3 Parabolic Input:

The simulink implementation is as follows:

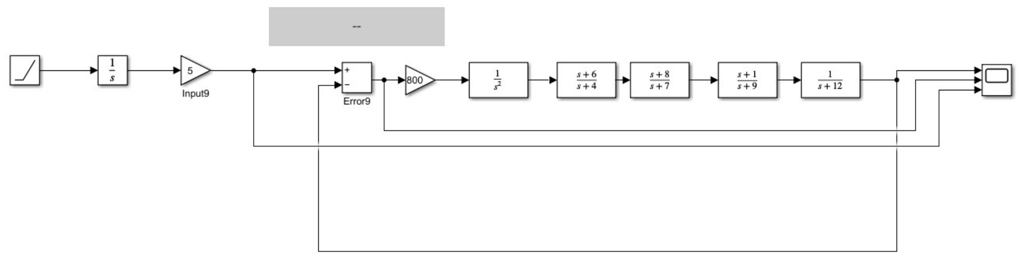

Figure 34: Simulink Implementation

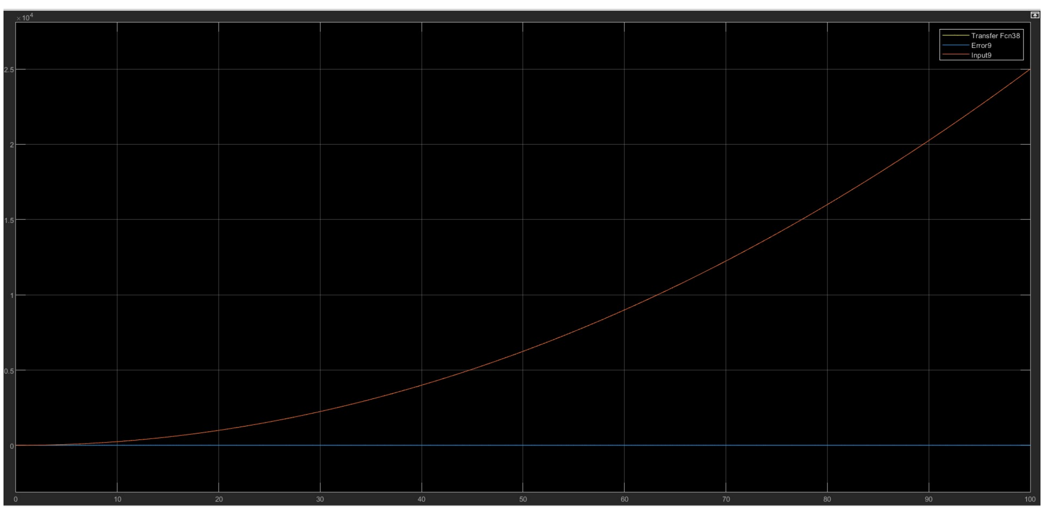

Figure 35: K = 200

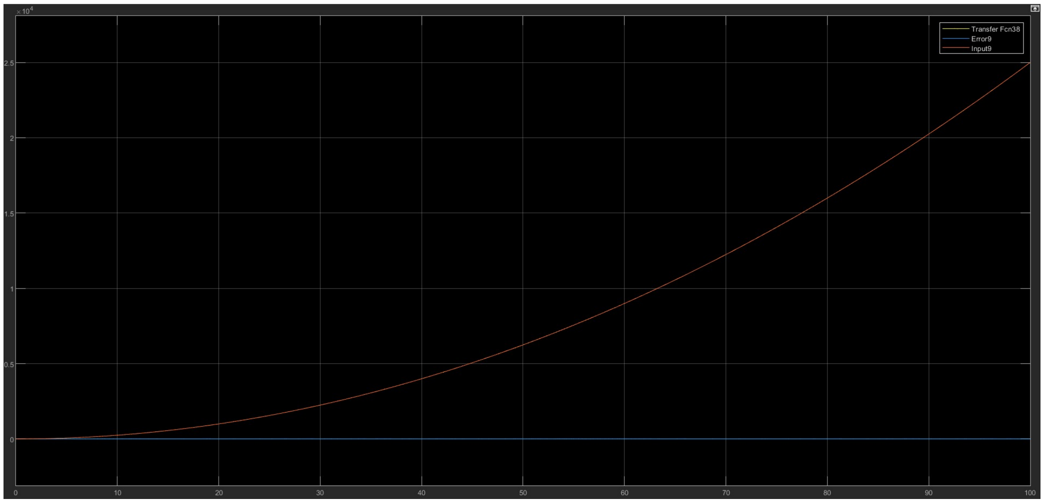

Figure 36: K = 400

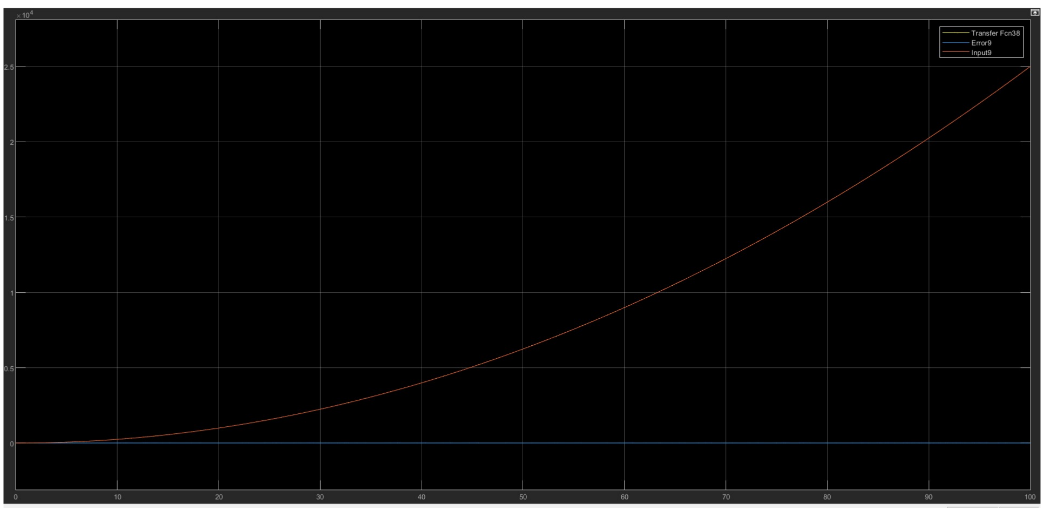

Figure 37: K = 800

5 Conclusion:

We observed and verified the effect of input waveform, loop gain, and system type upon steady state errors.