Lab 3 Report

Palaskar Adarsh Mahesh

B20EE087

September 5, 2022

1 State feedback controller design

Aim: The objective of this experiment is to find the feedback matrix K such that the closed loop system

Consider a system with a state model:

$$\frac{dx}{du} = Ax + Bu$$
$$y = Cx$$

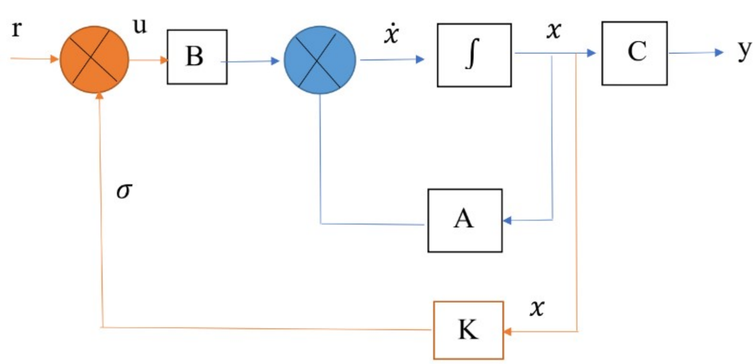

Figure 1: Block diagram with state feedback

$$U = r - \sigma$$
$$= r - Kx$$

$$\dot{x} = Ax + B(r - Kx)$$
$$= Ax + (Br - BKx)$$
$$= (A - BK)x + Br$$

Figure 2: Calculating dependencies

1. Checking the Controllability of the system

Controllability matrix Qc = [B AB] when A is 2X2 matrix and similar pattern is followed for square matrices of higher dimensions. If the determinant of Qc != 0, the system is completely controllable. If the determinant of Qc != 0, then the rank of matrix Qc = order of the system and the system is completely state controllable.

2. Determining the characteristic equation of the original system:
The characteristic equation for a system is obtained by:

$$|\lambda I - A| = 0$$

3. Determining the transformation matrix Pc:
The transformation matrix converts the given state model to Controllable Canonical form(CCF)

The transformation matrix, $P_c = \begin{bmatrix} P_1 \\ P_1 A \\ \vdots \\ P_1 A^{n-1} \end{bmatrix}$

And, $P_1 = [0\ 0 \ldots 0 1\ ]Q_C^{-1}$

4. Determining the state feedback gain matrix:

The state feedback matrix can be calculated using the following equation.

$$K = [b_n - a_n\ b_{n-1} - a_{n-1} \ldots \ldots \ldots .. b_2 - a_2\ b_1 - a_1]\ P_c$$

For the given question:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}u$$

where

$$\mathbf{x} = [\theta \quad \dot{\theta} \quad z \quad \dot{z}\,]^T$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 16.3106 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -1.0637 & 0 & 0 & 0 \end{bmatrix}; \mathbf{b} = \begin{bmatrix} 0 \\ -1.4458 \\ 0 \\ 0.9639 \end{bmatrix}$$

$z(t)$ = horizontal displacement of the pivot on the cart

$\theta(t)$ = rotational angle of the pendulum

Lab - 3

The controllability matrix can be calculated by :-

$$Q_c = \begin{bmatrix} B & AB & A^2B & A^3B \end{bmatrix}$$

$$\therefore Q_c = \begin{bmatrix} 0 & -1.45 & 0 & -23.58 \\ -1.45 & 0 & -23.58 & 0 \\ 0 & 0.96 & 0 & 1.54 \\ 0.96 & 0 & 1.54 & 0 \end{bmatrix}$$

$\det(Q_c) \neq 0$

$\therefore$ System is completely controllable.

For characteristic eq^n

$$|\lambda I - A| = 0$$

We get -

$$\begin{vmatrix} \lambda & 1 & 0 & 0 \\ 16.3106 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 1 \\ -1.0637 & 0 & 0 & \lambda \end{vmatrix} = 0$$

$$\boxed{\therefore \lambda^4 - (16.3106)\lambda^2 = 0}$$

The poles are given as

$$\begin{bmatrix} -1.3 & -1 & -2 & -4 \end{bmatrix}$$

For characteristic eq^n of poles

$$(\lambda+1.3)(\lambda+1)(\lambda+2)(\lambda+4) = 0$$

$$\boxed{\lambda^4 + (8.3)\lambda^3 + (23.1)\lambda^2 + (26.2)\lambda + 10.4 = 0}$$

For the transformation matrix P_c :-

$$P_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} Q_c^{-1}$$

$$\therefore P_1 = \begin{bmatrix} -0.047 & 0 & -0.0705 & 0 \end{bmatrix}$$

Now, $P_c = \begin{bmatrix} P_1 \\ P_1 A \\ P_1 A^2 \\ P_1 A^3 \end{bmatrix}$

$$\therefore P_c = \begin{bmatrix} -0.047 & 0 & -0.0705 & 0 \\ 0 & -0.0470 & 0 & -0.0705 \\ -0.6917 & 0 & 0 & 0 \\ 0 & -0.6917 & 0 & 0 \end{bmatrix}$$

For determining the state feedback gain matrix, we have

$$K = \begin{bmatrix} b_n - a_n & b_{n-1} \, a_{n-1} & \cdots & b_1 - a_1 \end{bmatrix} P_c$$

$$\therefore K = \begin{bmatrix} b_4 - a_4 & b_3 - a_3 & b_2 - a_2 & b_1 - a_1 \end{bmatrix} P_c$$

$$K = \begin{bmatrix} 10.4 & 26.2 & 39.4106 & 8.3 \end{bmatrix} P_c$$

Calculating, we get:-

$$\boxed{K = \begin{bmatrix} -27.7475 & -6.9723 & -0.7332 & -1.8472 \end{bmatrix}}$$

The Matlab code for the above calculations is as follows:

```
A = [0 1 0 0;
     16.3106 0 0 0;
     0 0 0 1;
     -1.0637 0 0 0]
B = [0; -1.4458; 0; 0.9639]

Qc = ctrb(A,B)
det(Qc)
%since Qc != 0 system is controllable

ch_eqn_coeff = charpoly(A)
ch_eqn_coeff_2_4 = ch_eqn_coeff(1,2:5)


poles = [-1.3 -1, -2 -4]
poles_eqn_coeff = poly(poles)
poles_eqn_coeff_2_4 = poles_eqn_coeff(1,2:5)

P1 = [0 0 0 1] / (Qc)

Pc = [P1; P1*A; P1*A^2; P1*A^3]

ch_eqn_coeff_2_4_rev = flip(ch_eqn_coeff_2_4)

poles_eqn_coeff_2_4_rev = flip(poles_eqn_coeff_2_4)

K = (poles_eqn_coeff_2_4_rev - ch_eqn_coeff_2_4_rev)*Pc
```

```
Qc = 4×4
            0   -1.4458         0  -23.5819
      -1.4458         0  -23.5819         0
            0    0.9639         0    1.5379
       0.9639         0    1.5379         0

ans = 420.5398
ch_eqn_coeff = 1×5
       1.0000         0  -16.3106         0         0

poles = 1×4
      -1.3000   -1.0000   -2.0000   -4.0000

poles_eqn_coeff = 1×5
       1.0000    8.3000   23.1000   26.2000   10.4000

P1 = 1×4
      -0.0470         0   -0.0705         0

Pc = 4×4
      -0.0470         0   -0.0705         0
            0   -0.0470         0   -0.0705
      -0.6917         0         0         0
            0   -0.6917         0         0

K = 1×4
     -27.7475   -6.9723   -0.7332   -1.8472
```

Figure 3: Output obtained on the Matlab terminal

Conclusion: The state feedback gain matrix was obtained using the above process. Also the calculations were verified using Matlab.