QUANTUM CONTROL

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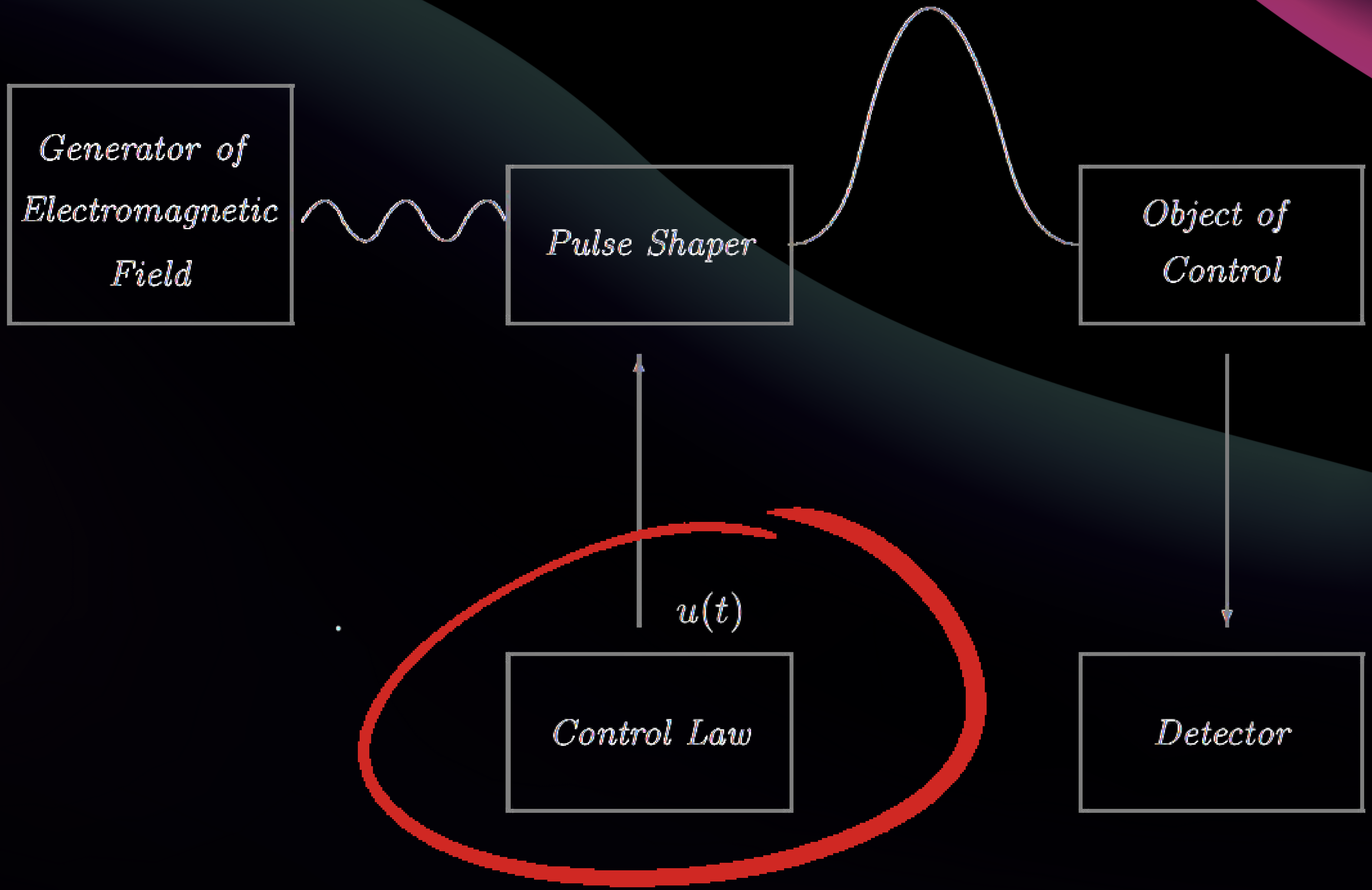
Need for Quantum Control Systems

- Quantum systems are very sensitive to the presence of environment which often destroys the main features of quantum dynamics. This is known as **de-coherence**.

- One method to prevent de-coherence is to obtain the desired state transfer in the least possible time so that the interaction with the environment becomes negligible.

What is Optimal Control?



Formulation of the Optimal Control Problem

Given a set X of (state) functions x: R → R^n
A set U of (control) functions u : R → R^m

Find x ∈ X and u ∈ U which minimize a cost functional J : X × U→ R

$$J(u) := ||x(T) - x_f||^2$$

- Incorporating a term which takes into consideration the energy used during the given interval of time:

$$J(u) = \lambda ||x(T) - x_f||^2 + \int_0^T ||u(t)||^2 dt$$

- The equation shows the euclidean distance between the final and desired state values. We can also incorporate a term that considers the energy used during a given interval of time. For example, in the form below λ > 0, will introduce a penalty on the final state and and energy like term for the control. We can thus manipulate the value of λ to control the amount of energy used to obtain a more accurate final state

Different formulations of optimal control problems

1. Problem of Mayer

$$J(u) := \phi(x(T), T),$$

- Minimizes cost function of the above form where phi is a smooth function.
- Mayer problems arise when there is a particular emphasis on the final state and/or time.

2. Problem of Lagrange:

$$J(u) := \int_0^T L(x(t), u(t), t)dt,$$

- Here, L is a smooth function.
- This problem occurs when the cost accumulates with time. For example, it can be used to minimize the energy used in the time duration required to reach the final state.

3. Problem of Bolza:

$$J(u) := \phi(x(T), T) + \int_0^T L(x(t), u(t), t)dt,$$

- The problem of Bolza is a combination of the problems of Mayer and Lagrange.
- Bolza problem arises when when there is a cumulative cost which increases during the control action but special emphasis is placed on the situation at the final time.

Optimal control problems for quantum systems

Assuming state (x) to be a pure state, and using the Schr¨odinger equation and Bolza's cost function we can prove that the differential equation describing the dynamics involving only real quantities is given by:

$$\dot{x} = \tilde{H}(u)x.$$

And the Cost function can be written as:

$$J(u) = \tilde{\phi}(x(T), T) + \int_0^T \tilde{L}(x(t), u(t), t)\, dt,$$

Conditions For Optimal Control

Pontryagrian Maximum Principle

- The Pontryagrian Maximum principle states that assuming u is optimal control and x the required state. Then, there exists a nonzero vector **λ** solution of the adjoint equations:

Optimal Control

- In order to get the optimal function we define a hamiltion operator such that :-
- We solve this equation using the boundary conditions from the previously stated theorem.

For Quantum Systems

- The hamiltonian takes the form:

$$h(\lambda, x, u) = \lambda^T \tilde{H}(u)x - \tilde{L}(x, u).$$

- If L is independent of x, lambda and x satisfy same equations.

Example

For a 2 level Quantum System

Suppose we can to control the spin of a +1/2 spin particle via an electromagnetic field:-

$$\dot{\phi} = (\sigma_z u_z + \sigma_x u_x + \sigma_y u_y)\phi$$

Equations Recap

- Cost Function:-

$$\frac{1}{2}\vec{\psi}^{\dagger} O \vec{\psi} + \frac{k}{2}\int_0^T u^2(t)dt$$

- Optimal Control Hamiltonian:-

$$\dot{\lambda} = \tilde{H}(u)\lambda + \tilde{L}_x^T$$

Summary

- We looked over one of the biggest problems in quantum information theory and the need for quantum control.
- We learned about the different formulations of the optimal control problem for different situations.
- We learned how to obtain the optimal control function.
- We learned the necessary conditions to obtain an optimal control function.

Thank you !!