Quantum Control and Its Application: A Brief Introduction

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Abstract. The development of quantum information science has brought great changes to the fields of computer and communication, and deepened people's understanding of basic physical problems. The core problems of overcoming decoherence, preparing and manipulating quantum states are quantum control problems in essence, so the research on quantum control will attract more and more attention and be important in the development of quantum information. This paper studies several applications of quantum Lyapunov control and quantum feedback control in quantum information processing by using Schrodinger equation, and some principal equations as tools for finite-dimensional systems. In this paper, a brief introduction of quantum control and its applications is presented to review the recent development and its prospect.

1. Introduction

Quantum mechanics [1]–[3] is the science that describes the laws of microscopic material movement. It was established by many physicists such as Planck and Bohr in the early 20th century. The birth of quantum mechanics has changed human cognition of the microcosm, and its correctness can be accurately tested by experiments. Today's quantum mechanics has been widely used in the fields of chemistry, materials, information, nanotechnology, etc., which has greatly promoted the development of related disciplines and changed the lives of human beings. For example, quantum energy-based semiconductor band theory has promoted the development of semiconductor technology, enabling humans to enter the computer age. At the same time, quantum mechanics also provides a theoretical basis for modern technologies such as atomic energy, laser, and nuclear magnetic resonance.

Quantum informatics [4]–[7] is a combination of quantum mechanics and information science. The content of research is how to use quantum mechanics to calculate, store, encode and transmit information. Quantum computing and quantum communication are the two main research directions of quantum information science. The specific research contents of quantum information science include quantum algorithm, quantum entanglement theory, quantum cryptography, quantum teleportation, quantum dense coding and corresponding physical realization [4]–[7]. The idea of using quantum systems for information processing was first proposed by Feynman. In 1981, Feynman pointed out at the first physics conference of the Massachusetts Institute of Technology that classical computers could not effectively simulate the evolution of quantum systems, and quantum computers could accomplish such tasks. In 1985, Deutsch proposed the concept of a general quantum computer and a simple quantum algorithm. In 1994, Shor of Bell Labs proposed a quantum algorithm for prime decomposition. Since the current RSA encryption algorithm used by banks and networks relies on the complexity of prime decomposition, and quantum computers can easily crack this encryption system, quantum information science has attracted extensive attention from the scientific community and the government. Interestingly, quantum information technology can destroy the security of existing communications, but it also provides a completely secure communication method in principle. In 1996, Grover proposed a quantum search algorithm that only needs $\sqrt{n}$ steps to find targets from n unordered entries, which can significantly reduce resource consumption compared to classical search algorithms.

In the past three decades, quantum information science has achieved remarkable results in both theory and experiment, and it has further promoted the development of quantum physics itself [8]–[12]. Today's quantum information science has become an emerging and emerging discipline and is closely linked to various disciplines such as optics, atomic physics, mathematics, and control.

When considering the quantum properties of physical devices, each of these fields can be expressed as a problem of finding the best (i.e., optimum) information processing program. Among them, quantum error correction and cryptography are the key [12]. This implicit and complex problem involves the dynamics of quantum systems, the optimization of quantum systems, and real-time controllers. This kind of controller is usually used as a form of feedback control to realize conventional quantum control design. In addition, quantum computing is another important part of quantum control. It is more complicated, so we will not introduce it for a limited page.

In this paper, we mainly review two quantum control methods for quantum systems. Section II is the preliminaries for quantum control. Section III is the first control method, called *measurement-based quantum control*. Section IV is the second control method, named as *coherent feedback quantum control*. Section V is the conclusion.

2. Preliminaries for Quantum Control

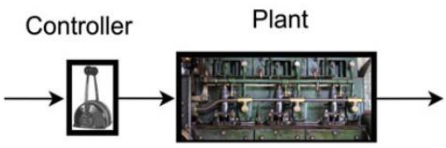

Figure 1. Open-loop controller and its signal diagram: this kind of controller directly actuates the modelled plants with a desired input, which has no feedback from the actual output of the system or other states

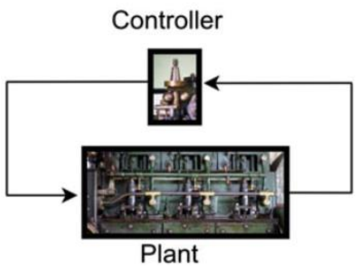

Figure 2. Closed-loop controller (measurement-based) and its signal diagram: this kind of controller actuates the modelled plants, and has a (or some) feedback from the actual output (i.e., measurement) of the system or other states

Usually, we need to model the quantum system, including the controller, the plant, the dynamics of the system, and etc. After that, we need to design a controller. And based on whether or not the controller uses the feedback from the output or other states, we can get two different kinds of controller forms: 1) open-loop controller and 2) closed-loop controller.

The open-loop controller is usually concerned with Hamiltonian system, which can be described by a Hamiltonian form:

$$\mathcal{H}(t) = \mathcal{H}_0 + \sum_{j=1}^{n} \mathcal{H}_j u_j(t),$$

Where $\mathcal{H}_0, \mathcal{H}_1, \ldots, \mathcal{H}_n$ are fixed self-adjoint operators and $u_1, u_2, \ldots, u_n$ are the predefined control variables, or functions. There are many definitions of controllability, that is, the degree to which an appropriate control strategy can be used to guide one state to another. This is naturally a bilinear control problem. Characterization inevitably involves checking the Lie algebra generated by the biased adjoint operator $-i\mathcal{H}_\alpha$, which uses the commutator as the Lie bracket, as shown in Figure 1.

The open-loop and closed-loop quantum systems are similar to the traditional systems, thus the analysis is similar, too. For the open-loop controller and system, geometric control theory is usually adopted to fulfill the desire, which has a wide application in nuclear magnetic resonance and chemistry. For the open-loop controller and system, the feedback is used to design a controller, which is dynamical (or sometimes static), usually from the output of the plant or transient states. This is measurement-based quantum control, as illustrated in Fig. 2. Another feedback that interacts the system and the controller each other is also considered, which is called as coherent control, as illustrated in Fig. 3. For more details, refer to Section III for measurement-based quantum control and Section IV for coherent feedback quantum control.

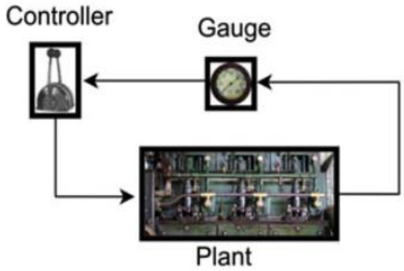

Figure 3. Closed-loop controller (coherent feedback) and its signal diagram: this kind of controller actuates the plant, and has a (or some) feedback from the actual output (i.e., measurement) of the system or other states, which interacts with the plant

From the standard bilinear model, note that the control variable directly acts as a controller output to actuate the plant as the carriers with important information of dynamic influence between the plant and the designed controller, namely a open-loop control process. However, to deal with this kind of system, we need to know more about a mathematical model which is necessary to deal with the problem, named Markov models. In the following context, we focus on the Markov models.

A simple Markov component is described by a triple ($\mathcal{S}, \mathcal{L}, \mathcal{H}$), where $\mathcal{S} = [s_{ij}] \in \mathbb{C}^{n\times n}$ represent the scattering operators, which is unitary, $\mathcal{L} = [l_i] \in \mathbb{C}^n$ represents the coupling operators, and $\mathcal{H}$ is the system Hamiltonian.

In the case of a single input and output, the unitary adapted quantum stochastic evolution is described as:

$$d\mathcal{U}_t = (\mathcal{S} - I)\mathcal{U}_t d\Gamma + \mathcal{L}\mathcal{U}_t d\mathcal{B}^\dagger - \mathcal{L}^\dagger \mathcal{S}\mathcal{U}_t d\mathcal{B} - \left(\frac{\mathcal{L}^\dagger\mathcal{L}}{2} + i\mathcal{H}\right)\mathcal{U}_t dt,$$

Where $\mathcal{B}$, $\mathcal{B}^\dagger$, and Γ are three fundamental denoted processes: creation, annihilation, and conservation, respectively. The following conditions are satisfied:

$$d\mathcal{B}d\mathcal{B}^\dagger = dt,\ d\mathcal{B}d\Gamma = d\mathcal{B}, d\Gamma d\mathcal{B}^\dagger = d\mathcal{B}^\dagger,\ d\Gamma d\Gamma = d\Gamma.$$

The above state dynamics are explicitly relevant to the normal associated Heisenberg equations (for more details, refer to [13]).

There is another kind networks of quantum control, i.e., the cascaded system. Here we consider the simplest form of a network, consisting of two cascaded systems. Actually, the cascaded system in the instantaneous feed forward limit is equivalent to the single component, i.e.,

$$(\mathcal{S}_2, \mathcal{L}_2, \mathcal{H}_2) \triangleleft (\mathcal{S}_1, \mathcal{L}_1, \mathcal{H}_1) \\ = \left(\mathcal{S}_2\mathcal{S}_1, \mathcal{L}_2 + \mathcal{S}_2\mathcal{L}_1, \mathcal{H}_1 + \mathcal{H}_2 + \mathrm{Im}\left\{\mathcal{L}_2^\dagger \mathcal{S}_2 \mathcal{L}_1\right\}\right),$$

Which is presented in [13] as the series product of the two models.

In reality, the open-loop controller is usually not adopted, unless the quantum system is really easy to deal with. From the engineering perspective, the close-loop control method is valuable to consider, for the real problems are usually sophisticated and strongly coupled. In this situation, the open-loop controller seems hard to fulfill a satisfactory desire. Thus, in this paper, we mainly focus on the measurement-based quantum control and coherent feedback quantum control, both of which belong to the feedback control method.

3. Measurement-based Quantum Control

In this part, we focus on the measurement-based quantum control, which includes the following parts:

the dynamics of the plant and its mathematical description, with any external or internal noise;

the dynamics of the measurement apparatus and its mathematical description, including: 1) measurement, 2) transition, and 3) additional noise;

the ways of the controller implemented to the system;

the desires (or the objectives) of the quantum system.

Among these, the control objective is most important, because in the quantum system the system variables are usually implicit. Also, according to the normal control methods, the stability of the feedback control for specific purposes and specific system have some limits, such as the negative eigenvalue of the system matrix. To fulfill the control structure, we need to model the specific cost to measure the output or the state performance. Usually, the cost is random (because the reading is random, so it is inevitable here), so we need to find the minimum value of the average cost. A controller designed for a specific system is to find the best controller among all possible controllers, that is, a physical system that continuously processes measurement readings and drives the system to minimize costs. Without the control of the target, there is no control problem.

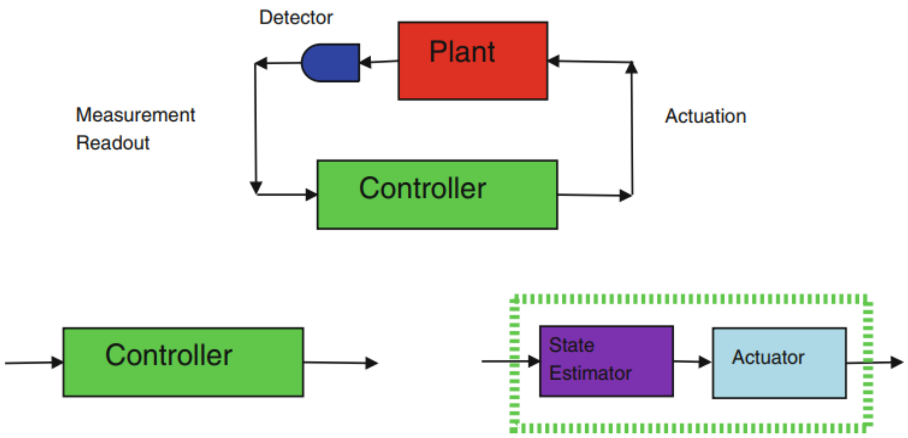

Figure 4. Separation principle: the controller in a quantum system can be separated into two parts, including a state estimator and an actuator

Notice that the separation principle in Fig. 4. The controller is separated into two parts: 1) a state estimate of a system to produce an optimum state based on measurements from the output or other states read out at a given time and 2) a given controller applied to the system based on the optimum state calculated in step 1. Thus, the controller design is simplified into two parts, and improve the efficiency of the controller design. A sketch of a system with separation is described in Fig. 4.

Although there has been a lot of discussion and researches on state estimation and its relevant application, especially considering that it is related to the basis of quantum measurement and quantum theory, the remaining control problems have become less popular, and we hope someone can make a progress. When the goal is to minimize the average cost mentioned above, such as the expected weighted average of the observed values at a given time interval, then the state estimator is any machine that can deal with the data from the filtered state that is the appropriate solution to the stochastic equation. But this situation is usually not the case. There is also a penalty implementation to make a compromise where the performance cost is too high. Another problem is the risk sensitive cost, which Matthew James had solved in the quantum case and has some variants for applications. The separation principal presented in Fig. 4 somewhat depends on the results of risk sensitivity cost and the measurement from the output or states.

In General, from the perspective of a pure operation, quantum theory and its relevant lemmas tells us what and how to calculate the probabilities of physical (or actual) events. If done correctly, this is consistent with the overall approach adopted by engineers. However, although the optimum estimated state of the output or the transient states at a given time obviously relies on the measurement settings and readings, there are more work needs to be done. It also relies on the control goals and the usage of the estimated states to guide the quantum system.

In a sense, quantum control based on measurement is more complicated than traditional quantum measurement. Traditional control has introduced unexpected fundamental problems because our best estimates of quantum control problems based on plant measurements are now in a way more complex than traditional quantum measurement problems. Traditional controls have brought unexpected fundamental problems. It is worth emphasizing that the quantum operating method is a natural fit with standard traditional engineering operating procedures. Therefore, Mermin's famous maxim on quantum feedback control is interesting.

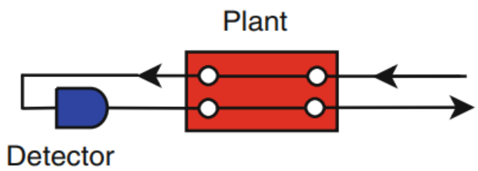

Figure 5. Direct measurement feedback method of Wiseman and Milburn

An example is given out here. There is a double-pass of a quantum light field through a plant, which can be modelled as the following series product: $(\mathrm{I,L,H_0})$, $\mathcal{U}(t+dt,t)=e^{-iFdJ}$. Here, the detector measures the component J (quadrature or photon count) of the output field from the first pass, and then inputs it as a direct proportional Hamiltonian for the second time, J is singular so we can describe it as a stochastic unitary process $\mathcal{U}$, as illustrated in Fig. 5.

Thus the homodyne detection and photon counting can be separated as following:

Homodyne detection: $J=\mathcal{B}+\mathcal{B}^{\dagger}$, $(dJ)^2=dt$. Thus, the closed-loop model is

$$(I,-iF,0)\triangleleft(I,\mathcal{L},\mathcal{H}_0)=\left(I,\mathcal{L}-iF,\mathcal{H}_0+\frac{F\mathcal{L}+\mathcal{L}^{\dagger}F}{2}\right);$$

Photon counting: $J=\Gamma$, $(dJ)^2=dJ$. Thus, the closed-loop model is

$$(S,0,0)\triangleleft(I,\mathcal{L},\mathcal{H}_0)=(S,S\mathcal{L},\mathcal{H}_0).$$

4. Coherent Feedback Quantum Control

In this part, we briefly introduce the coherent feedback quantum control, which is similar to the measurement-based quantum control. This coherent feedback is influenced by the plant or other transient states, which can be really sophisticated and strongly coupled.

The main disadvantage of the first control method (i.e., measurement-based control) is that it is limited by the time it takes to process the data from the measurement, which may be highly influenced by the noise externally and internally. In reality, we usually have to connect the device and the computer (or controller) using some interfaces which induces noise for example. Although some simple applications of quantum Kalman filtering have been implemented in real-time and offline variance calculation, there are still problems in implementing more computationally intensive controllers. In order to meet the high-performance standards required for quantum information processing, it is essential to go beyond traditional measurement-based controllers to consider fast Nan-level devices. This brings us into the field of quantum coherent feedback control. The above quantum feedback networks (some coupled measurement-based control system) is naturally suitable for describing the Nan photon control system driven by the continuous wave laser input field.

For general stability, considering the stability conditions, there are passivity and L_2 gain of general quantum systems proposed by James and Gough. Mathematically, we can replace state variables with operators. Linear models occur in systems with regular coordinates, where the triple $(\mathcal{S}, \mathcal{L}, \mathcal{H})$ implicitly represents a linear system, which mainly described by the Heisenberg-Langevin equation. This situation only occurs if $\mathcal{H}$ is quadratic, $\mathcal{L}$ is linear, and $\mathcal{S}$ does not depend on regular coordinates. Based on these ideas, the value of performance can be deal with easily, and one can generalize some known results on LQG problem, H^∞ control method, and their applications with those linear systems. And one thing needs to clarify that some theoretical results might be not implemented to the physical devices, e.g., although the robust control problem can be dealt with theoretically, the optimal controller is, unfortunately, not physically realized for the quantum system.

The Mabuchi Lab team has proposed physical applications, including the quantum error correction in the continuous time, using Nan photonic circuits with coherent feedback from the output or the states, rather than measurement-based control (less coupled). In recent time, they also developed a hardware description language to implement the previously described product series, connection and feedback reduction operations. This allows arbitrary quantum optical networks to be built under the Markov hypothesis. Through these technologies, opportunities for designing and synthesizing quantum control devices are expected to become more powerful. The realization of measurement-based nonlinearity in quantum optics also opens up the fields of quantum optics and quantum networks for future information processing applications.

Here, we also present the simplest implementation of coherent control, which only consists of a plant and a controller as an idealistic system in the form of Hamiltonian:

$$\mathcal{H} = \mathcal{H}_P \otimes I_C + I_P \otimes \mathcal{H}_C + \mathcal{V}_{PC},$$

Where $\mathcal{V}_{PC}$ gives the nontrivial coupling of the plant and controller. Notice that both the plant and the controller are quantized.

5. Conclusion

The development of quantum information science has brought great changes to the field of computer and communication and deepened people's understanding of basic physical problems. The core problems of overcoming decoherence, preparing and manipulating quantum states are essentially quantum control problems, so the research of quantum control will be more and more important in the development of quantum techniques. Some applications of quantum Lyapunov control and quantum feedback control in quantum information processing are studied by using Schrodinger equation and some principal equations as tools of finite-dimensional systems. This paper reviews some latest progress of quantum control and its applications.

This paper focuses on the application of several quantum control methods in quantum information processing, which will play an active role in the presentation of quantum states, the preparation of quantum gates, and the improvement of quantum control methods. In the subsequent research, quantum inversion can be studied by combining specific physical models and experimental parameters. In fact, the effect of acceleration control also depends on the design method of Lyapunov function (physical meaning), the intensity of control field, control precision and other factors. How to make this control method more widely applicable remains to be studied. In addition, how to choose the appropriate control Hamiltonian for accelerating evolution is also a meaningful problem. How the control with Lyapunov function can achieve high-precision quantum gates in other types of Hamiltonian and higher-dimensional systems remains to be studied. Meanwhile, the implementation of continuous local equivalent operators of CNOT gate also depends on specific models, and whether other models can implement equivalent operators of CNOT gate or other meaningful operator sets remains to be studied. The application of this memory effect in quantum information and its correspondence with classical non-Markov properties need to be further studied. The exact meaning of the dynamic mapping in quantum non-Markov processes also needs to be further analyzed and clarified.

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