

Instructions: Use Matlab/Simulink to solve all problems. Save your work in an m-file and an slx-file. Project write-up should include the following:

1. a project write-up with clearly defined answers and any required hand calculations.
2. a print out of the Matlab command window showing your Matlab solution answers, if applicable. Clearly define each solution and the part to which it pertains.
3. a print out copy of your m-file program and any Simulink diagrams.
4. a print out copy of clearly labeled Figures/Plots
5. a soft copy of your program required to run all the problem parts (place it in the mycourses dropbox shell)

Design a sliding mode controller for the following system:

$$\ddot{x} + f_1(t, x, \dot{x}) + f_2(t, x) = b(t)u$$

with $f_1(t, x, \dot{x}) = \alpha_1(t)|x|\dot{x}^2$ and $f_2(t, x) = \alpha_2(t)x^3 \cos 2x$ where $\alpha_1(t)$, $\alpha_2(t)$, and $b(t)$ are unknown time-varying functions with known bounds:

$$\forall t \geq 0 \quad 1 \leq \alpha_1(t) \leq 2 \quad -1 \leq \alpha_2(t) \leq 5 \quad 4 \leq b(t) \leq 7$$

with $x_d = \sin(\pi t/2)$.

First derive the sliding mode control law form that verifies the sliding condition. Once you prove the sliding condition is verified derive the actual sliding mode control law that guarantees tracking stability for the system shown above. **You must treat each functionality, i.e., f_1 and f_2 separately in your formulation of the sliding mode control law and in particular the sliding mode gain, K !** Do not combine them into one function.

Simulate your closed-loop system for variations in $\alpha_1(t)$, $\alpha_2(t)$, and $b(t)$ for their extreme values shown above and evaluate the performance of your proposed control law. First, using a signum function verify the sliding condition is satisfied with a time history plot. Next, assume the control law is to be implemented in a real-world system (i.e., smooth the control effort in a thick boundary layer) and perform another simulation to show the tracking performance.

For your computer project write-up you must include the following both for the “perfect” tracking case using a signum function and the control law using a boundary layer (saturation function):

1. all hand calculations and derivations of your control law
2. Plot of position tracking performance: x overlaid with x_d versus time
3. Plot of velocity tracking performance: \dot{x} overlaid with \dot{x}_d versus time
4. Plot of position tracking error performance: \tilde{x} versus time
5. Plot of velocity tracking error performance: $\dot{\tilde{x}}$ versus time
6. Plot of the sliding condition: $s\dot{s}$ overlaid with $-\eta|s|$ versus time
7. Plot of your switching gain, K versus time
8. Plot of the control effort, u versus time
9. Plot of the variations of the system parameters, α_1 , α_2 , and b versus time
10. clearly labeled plots showing your control law performance including tracking error and control law input
11. copies of your m-files and Simulink diagrams
12. discussion on the performance of your control law