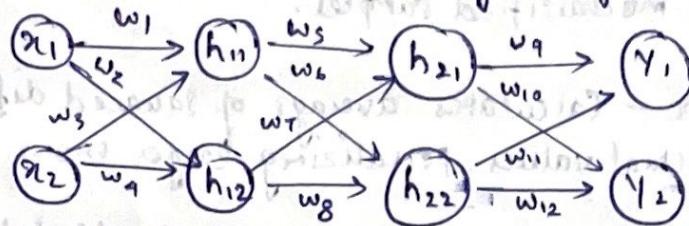


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ANN Numerical Assignment 3

Q1] find mean squared error for given feed forward N/w.



Given, $n_1 = 0.3, n_2 = 0.5, n_3 = 0.7, n_4 = 0.8$

$w_1 = 0.5, w_2 = 0.6, w_3 = 0.7, w_4 = 0.8$

$w_5 = 0.9, w_6 = 0.1, w_7 = 0.2, w_8 = 0.3$

$w_9 = 0.4, w_{10} = 0.5, w_{11} = 0.6, w_{12} = 0.7$

$y_0 = 0, y_1 = 1, b_1 = b_2 = b_3 = 0.2, \lambda = 1$

$$\Rightarrow \text{i)} \text{Net}_{11} = \sum w_{ini} + b_1 \\ = w_1 n_1 + w_2 n_2 + b_1 = (0.5 \times 0.3) + (0.7 \times 0.5) + 0.2 \\ = \underline{\underline{0.7}}$$

$$h_{11} = f(\text{Net}) = \frac{1}{1 + e^{-\lambda \text{Net}}} = \frac{1}{1 + e^{-0.7}} = \underline{\underline{0.668}}$$

$$\text{ii)} \text{Net}_{12} = \sum w_{ini} + b_1 \\ = w_2 n_1 + w_4 n_2 + b_1 = (0.6 \times 0.3) + (0.8 \times 0.5) + 0.2 \\ = \underline{\underline{0.78}}$$

$$h_{12} = \underline{\underline{0.686}}$$

$$\text{iii)} \text{Net}_{21} = \sum w_{ini} + b_2 \\ = w_5 h_{11} + w_9 h_{12} + b_2 \\ = 0.9 \times 0.668 + 0.2 \times 0.686 + 0.2 \\ = \underline{\underline{0.938}}$$

$$h_{21} = \underline{\underline{0.719}}$$

$$\text{iv)} \text{Net}_{22} = \sum w_{ini} + b_2 \\ = w_6 h_{11} + w_8 h_{12} + b_2 \\ = 0.1 \times 0.668 + 0.3 \times 0.686 + 0.2 = \underline{\underline{0.473}}$$

$$h_{22} = f(\text{Net}_{22}) = \frac{1}{1 + e^{-0.473}} = \underline{\underline{0.616}}$$

$$v) \gamma_1 = w_{10}h_{21} + w_{11}h_{22} = 0.4 \times 0.719 + 0.6 \times 0.616 = 0.657$$

$$\gamma_1 = f(\text{net}_1) = \frac{1}{1+e^{-0.657}} = \underline{\underline{0.659}}$$

$$vi) \text{Net}_2 = w_{10}h_{21} + w_{12}h_{22} = 0.5 \times 0.719 + 0.7 \times 0.616 = 0.791$$

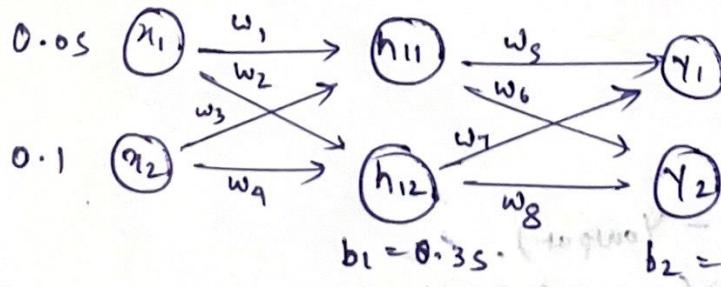
$$\gamma_2 = f(\text{net}_2) = \frac{1}{1+e^{-0.791}} = \underline{\underline{0.688}}$$

vii) Mean Squared error \rightarrow

$$\begin{aligned} \text{MSE} &= \frac{1}{2} \left[\frac{\text{actual}}{\text{Op}_1} - \frac{\text{target}}{\text{Op}_1} \right]^2 + \frac{1}{2} \left[\frac{\text{actual}}{\text{Op}_2} - \frac{\text{target}}{\text{Op}_2} \right]^2 \\ &= \frac{1}{2} (0.659 - 0)^2 + \frac{1}{2} (0.688 - 1)^2 \\ &= \underline{\underline{0.266}} \end{aligned}$$

Q2] Train network using backward propagation using forward & backward pass. Assume $\alpha = 0.5$, $\gamma_1 = 0.01$, $\gamma_2 = 0.99$.

$$\begin{aligned} \Rightarrow \gamma_1 &= 0.05, \gamma_2 = 0.01, b_1 = 0.35, b_2 = 0.6, \\ w_1 &= 0.15, w_2 = 0.2, w_3 = 0.25, w_4 = 0.36, \\ w_5 &= 0.4, w_6 = 0.95, w_7 = 0.5, w_8 = 0.55. \end{aligned}$$



— forward pass:

$$\begin{aligned} i) \text{net}_1 &= w_1 x_1 + w_3 x_2 + b_1 \\ &= 0.15 \times 0.05 + 0.25 \times 0.01 + 0.35 = 0.38 \end{aligned}$$

$$ii) h_{11} = f(\text{net}_1) = \frac{1}{1+e^{-0.38}} = \underline{\underline{0.593}}$$

$$\text{ii) } \text{Net}_2 = w_2 n_1 + w_1 n_2 + b_1$$

$$= 0.2 \times 0.05 + 0.36 \times 0.1 + 0.35$$

$$= \underline{\underline{0.396}}$$

$$h_{22} = \frac{1}{1 + e^{-0.396}} = \underline{\underline{0.398}}$$

$$\text{iii) } \text{Net}_3 = w_5 h_{14} + w_7 h_2 + b_2$$

$$= 0.4 \times 0.595 + 0.5 \times 0.398 + 0.6$$

$$= \underline{\underline{1.137}}$$

$$y_1 = \frac{1}{1 + e^{-1.137}} = \underline{\underline{0.757}}$$

$$\text{iv) } \text{Net} = w_6 h_1 + w_8 h_2 + b_2$$

$$= 0.95 \times 0.598 + 0.55 \times 0.398 + 0.6$$

$$= \underline{\underline{1.196}}$$

$$y_2 = f(\text{Net}) = \frac{1}{1 + e^{-1.196}} = \underline{\underline{0.767}}$$

Mean Squared error (MSE)

$$= \frac{1}{2} \left(\frac{\text{Actual}}{\text{Op}_1} - \frac{\text{target}}{\text{Op}_1} \right)^2 + \frac{1}{2} \left(\frac{\text{Actual}}{\text{Op}_2} - \frac{\text{target}}{\text{Op}_2} \right)^2$$

$$= \frac{1}{2} (0.757 - 0.01)^2 + \frac{1}{2} (0.767 - 0.99)^2$$

$$= \underline{\underline{0.3}}$$

Backward pass

$$\delta_5 = y_1 (1 - y_1) (\text{target} - \text{yourput})$$

$$= 0.757 (1 - 0.757) (0.01 - 0.757)$$

$$= \underline{\underline{-0.138}}$$

$$\begin{aligned}\delta_6 &= y_2 (1-y_2) (y_{\text{target}} - y_{\text{actual}}) \\ &= 0.767 (1-0.767) (0.99 - 0.767) \\ &= 0.041\end{aligned}$$

computing new wt value.

$$\begin{aligned}\Delta w_5 &= \eta \delta_5 y_1 \\ &= (-0.138)(0.75) \\ &= -0.1035\end{aligned}$$

$$\begin{aligned}\Delta w_5^{\text{new}} &= \Delta w_5 + w_5^{\text{old}} \\ &= 0.4 + (-0.1035)\end{aligned}$$

$$w_5^{\text{new}} = 0.296$$

$$\begin{aligned}\Delta w_7^{\text{new}} &= w_7^{\text{old}} + \Delta w_5 \\ &= 0.5 + (-0.103) \\ \Delta w_7^{\text{new}} &= 0.397\end{aligned}$$

$$\begin{aligned}\Delta w_6 &= \eta \delta_6 y_2 \\ &= 1(0.041)(0.76) \\ &= 0.031\end{aligned}$$

$$\begin{aligned}w_6^{\text{new}} &= w_6^{\text{old}} + \Delta w_6 \\ &= 0.45 + 0.031 \\ &= 0.481\end{aligned}$$

$$\begin{aligned}w_8^{\text{new}} &= w_8^{\text{old}} + \Delta w_6 \\ &= 0.55 + 0.031\end{aligned}$$

Hidden layer

$$\begin{aligned}\delta_4 &= 4(1-y_4) w_5 \delta_5 \\ &= h_{22} (1-h_{22}) w_5^{\text{new}} \delta_6 \\ &= 0.598 (1-0.598) \times 0.58 \times 0.041 \\ &= 0.0057\end{aligned}$$

$$\begin{aligned}
 \delta_3 &= h_{11} (1 - h_{11}) w_6^{\text{new}} \delta_6 \\
 &= 0.595 (1 - 0.595) 0.981 \times 0.04 \\
 &= 0.0047
 \end{aligned}$$

$$\begin{aligned}
 w_1^{\text{new}} &= w_1^{\text{old}} + \Delta w_1 \\
 &= 0.15 + 0.0027 \\
 &= 0.1527
 \end{aligned}$$

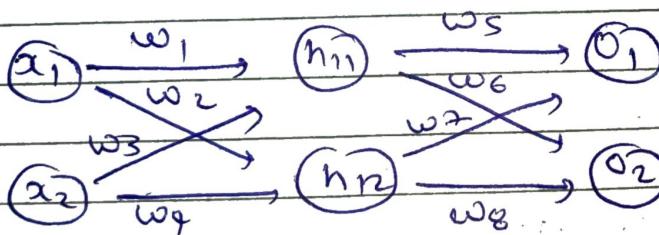
$$\begin{aligned}
 w_3^{\text{new}} &= w_3^{\text{old}} + \Delta w_3 \\
 &= 0.25 + 0.0027 \\
 &= 0.2527
 \end{aligned}$$

$$\begin{aligned}
 w_2^{\text{new}} &= w_2^{\text{old}} + \Delta w_2 \\
 &= 0.2 + 0.0034
 \end{aligned}$$

$$w_2^{\text{new}} = 0.2034$$

$$\begin{aligned}
 w_4^{\text{new}} &= w_4^{\text{old}} + \Delta w_4 \\
 &= 0.35 + 0.0034 \\
 &= 0.3634
 \end{aligned}$$

Q3)



$w_1 = 0.1$

$w_5 = 0.5$

$w_2 = 0.2$

$w_6 = 0.6$

$w_3 = 0.3$

$w_7 = 0.7$

$w_4 = 0.4$

$w_8 = 0.8$

$n = 0.25 \quad \lambda = 1 \quad x_1 = 0.1 \quad x_2 = 0.5$



forward pass.

$$\begin{aligned}
 \textcircled{a} \quad \text{net}_{11} &= w_1 x_1 + w_3 x_2 + b \\
 &= 0.1 \times 0.1 + 0.3 \times 0.5 + 0.025 \\
 &= 0.185 \\
 h_{11} &= f(\text{net}_{11}) = \frac{1}{1 + e^{-\lambda \text{net}_{11}}} = \frac{1}{1 + e^{-0.185}} \\
 &= 0.54
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{b} \quad \text{net}_{22} &= w_2 x_1 + w_4 x_2 + b \\
 &= 0.2 \times 0.1 + 0.4 \times 0.5 + 0.025 \\
 &= 0.245 \\
 h_{22} &= f(\text{net}_{22}) = \frac{1}{1 + e^{-0.245}} \\
 &= 0.56
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{c} \quad \text{net}_1 &= w_5 h_{11} + w_6 h_{22} + b_2 \\
 &= 0.65 \times 0.54 + 0.6 \times 0.56 + 0.35 \\
 &= 0.956 \\
 o_1 &= F(\text{net}_1) = \frac{1}{1 + e^{-0.956}} \\
 &= 0.722
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{d} \quad \text{net}_2 &= w_7 h_{11} + w_8 h_{22} + b_2 \\
 &= 0.7 \times 0.54 + 0.8 \times 0.56 + 0.35 \\
 &= 1.176 \\
 o_2 &= F(\text{net}_2) = \frac{1}{1 + e^{-1.176}} \\
 &= 0.75
 \end{aligned}$$

$$\begin{aligned}
 \text{MSE} &= \frac{1}{2} [(0.722 - 0.05)^2 + (0.75 - 0.95)^2] \\
 &= 0.243 \approx 0.25
 \end{aligned}$$

Backward pass.

off layer

$$\delta_5 = 0_1 (1-0_1) (T_1 - 0_1)$$

$$= 0.722 (1 - 0.722) (0.05 - 0.722)$$

$$= -0.134$$

$$\Delta w_5 = \eta \delta_5 0_1$$

$$= 0.25 \times (-0.134) 0.722$$

$$= 0.024$$

$$\Delta w_5 = \Delta w_6$$

$$w_5^{\text{new}} = w_5^{\text{old}} + \Delta w_5$$

$$= 0.5 - 0.024$$

$$= 0.476$$

$$w_6^{\text{new}} = w_6^{\text{old}} + \Delta w_6$$

$$= 0.6 - 0.024$$

$$= 0.576$$

$$\delta_6 = 0_2 (1-0_2) (T_2 - 0_2)$$

$$= 0.75 (1 - 0.754) (0.95 - 0.754)$$

$$= 0.037$$

$$\Delta w_7 = \eta \delta_6 0_2$$

$$= 0.25 \times 0.037 \times 0.75$$

$$= 0.0069$$

$$\Delta w_8 = \Delta w_7$$

$$w_7^{\text{new}} = w_7^{\text{old}} + \Delta w_7$$

$$= 0.7 + 0.0069$$

$$w_7^{\text{new}} = 0.7069$$

$$w_8^{\text{new}} = w_8^{\text{old}} + 0.0069$$

$$= 0.8069$$

Hidden layer

$$\delta_3 = h_{11} (1 - h_{11}) w_5^{\text{new}} \delta_5$$

$$= 0.54 (1 - 0.54) 0.47 (-0.13)$$

$$= 0.015$$

$$\Delta w_1 = \eta \delta_3 h_{11}$$

$$= 0.25 \times (-0.015 \times 0.54) = -0.002$$

$$\Delta w_3 = \Delta w_1$$

$$\Delta w_3 = \Delta w_3 = -0.002$$

$$w_3^{\text{new}} = w_3^{\text{old}} + \Delta w_3$$

$$= 0.1 + (-0.002)$$

$$= 0.098$$

$$w_3^{\text{new}} = w_3^{\text{old}} + \Delta w_3$$

$$= 0.3 + (-0.002)$$

$$= 0.298$$

$$\delta_4 = h_{22} (1 - h_{22}) w_8^{\text{new}} \delta_8$$

$$= 0.56 (1 - 0.56) 0.80 \times 0.037$$

$$= 0.007$$

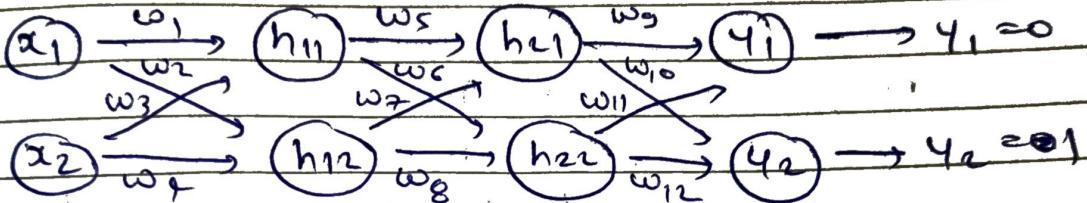
$$\Delta w_2 = \eta \delta_4 h_{22}$$

$$= 0.25 \times 0.007 \times 0.56 = 0.0009$$

$$\Delta w_4 = \Delta w_2 = 0.0009$$

$$\begin{aligned}
 w_2^{\text{new}} &= w_2^{\text{old}} + \Delta w_2 \\
 &= 0.2 + 0.0009 \\
 &= 0.2009
 \end{aligned}$$

$$\begin{aligned}
 w_4^{\text{new}} &= w_4^{\text{old}} + \Delta w_4 \\
 &= 0.4 + 0.009 \\
 w_4^{\text{new}} &= 0.4009
 \end{aligned}$$



$$b_1 = 0.5 \quad b_2 = 0.5$$

Find the error for given feed forward network

$$w_1 = 0.1$$

$$w_6 = 0.6$$

$$w_{11} = 0.2$$

$$w_2 = 0.3$$

$$w_7 = 0.7$$

$$b_1 = 0.5$$

$$w_3 = 0.3$$

$$w_8 = 0.8$$

$$b_2 = 0.5$$

$$w_4 = 0.4$$

$$w_9 = 0.9$$

$$y_1 = 0$$

$$w_5 = 0.5$$

$$w_{10} = 0.1$$

$$y_2 = 1$$

$$\begin{aligned}
 \text{net}_{11} &= \sum w_i x_i + b_1 \\
 &= w_1 x_1 + w_3 x_2 + b_1 \\
 &= 0.1 \times 0.3 + 0.3 \times 0.5 + 0.5 \\
 &= 0.68
 \end{aligned}$$

$$\begin{aligned}
 \text{net}_{12} &= \sum w_i x_i + b_1 \\
 &= w_2 x_1 + w_4 x_2 + b_1 \\
 &= 0.3 \times 0.3 + 0.4 \times 0.5 + 0.5
 \end{aligned}$$

$$\text{net}_{12} = 0.79$$

$$h_{11} = F(\text{net}_{11}) = \frac{1}{1+e^{-\lambda \text{net}_{11}}} = \frac{1}{1+e^{-0.68}}$$

$$h_{11} = 0.664$$

$$h_{12} = \frac{1}{1+e^{-0.79}}$$

$$h_{12} = 0.68$$

$$\begin{aligned} \text{net}_{21} &= \sum w_i x_i + b_2 \\ &= h_{11} w_5 + h_{12} w_7 + b_2 \\ &= 0.664 \times 0.5 + 0.68 \times 0.7 + 0.5 \\ &= 0.332 + 0.476 + 0.5 \end{aligned}$$

$$\text{net}_{21} = 1.308$$

$$h_{21} = F(\text{net}_{21})$$

$$= \frac{1}{1+e^{-1.308}} = 0.78$$

$$\begin{aligned} \text{net}_{22} &= \sum w_i x_i + b_2 \\ &= (h_{11} \times w_6) + (h_{12} \times w_8) + b_2 \\ &= 0.664 \times 0.6 + 0.68 \times 0.8 + 0.5 \\ &= 0.398 + 0.544 + 0.5 \\ &= 1.442 \end{aligned}$$

$$\text{net}_{22} = F(\text{net}_{22})$$

$$= 1$$

$$\begin{aligned}
 \text{net}_1 &= h_{21} \times w_9 + h_{22} \times w_{11} \\
 &= 0.78 \times 0.9 + 0.809 \times 0.2 \\
 &= 0.864 \\
 y_1 &= f(\text{net}_1) \\
 &= 0.703
 \end{aligned}$$

$$\begin{aligned}
 \text{net}_2 &= h_{21} \times w_{10} + h_{22} \times w_{12} \\
 &= 0.78 \times 0.1 + 0.809 \times 0.3 \\
 &= 0.321
 \end{aligned}$$

$$\begin{aligned}
 y_2 &= f(\text{net}_2) \\
 y_2 &\approx 0.58
 \end{aligned}$$

$$\begin{aligned}
 \text{MSE} &= \frac{1}{2} \left[\left[\frac{\text{Actual}}{\text{Op}_1} - \frac{\text{Target}}{\text{Op}_1} \right]^2 + \left[\frac{\text{Actual}}{\text{Op}_2} - \frac{\text{Target}}{\text{Op}_2} \right]^2 \right] \\
 &= \frac{1}{2} \left[(0.703 - 0)^2 + (0.58 - 1)^2 \right] \\
 &= \frac{1}{2} \left[0.494 + 0.176 \right] \\
 &= 0.335
 \end{aligned}$$