

Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in Units & Dimensions - Part 1

1. Fundamental Units
2. Supplementary Units
3. Must Know / Practice Dimensional Formulae
4. Principle of Homogeneity
5. Conversion of Units
6. Dimension in Terms of other Physical Quantity
7. KEY Points (Dimensionless Qty)

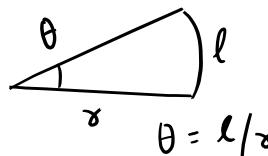
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1. FUNDAMENTAL UNITS

	Quantities	SI UNIT	CGS	SYMBOL
1.	LENGTH	m	cm	L
2.	MASS	kg	g	M
3.	TIME	s	s	T
4.	TEMP°	K		θ
5.	CURRENT	A		A
6.	INTENSITY	cd		cd
7.	AMOUNT	mol		mol

2. SUPPLEMENTARY UNITS

(i) ANGLE → Radian



$$\theta = l/r$$

(ii) SOLID ANGLE → Steradian



$$\sigma = A/r^2$$

3. MUST KNOW DIMENSIONAL FORMULAE

Units & Dimensions – Part 1

	Physical Qty	DIMENSIONAL FORMULAE
1	FORCE	MLT^{-2}
2	ENERGY	ML^2T^{-2}
3	ϵ_0	$M^{-1}L^{-3}T^4A^2$ EDUNI
4	μ_0	$MLT^{-2}A^{-2}$ EDUNI

NOTE: You may practice to find Dimension of some constants like:

Ex. Stefan boltzmann (const. σ)

$$\frac{dQ}{dt} = \sigma e A T^4 \Rightarrow \sigma = \frac{0}{t \times A \times T^4}$$

$$[\sigma] = \frac{ML^2T^{-2}}{T \times L^2 \times \theta^4} = MT^{-3}\theta^{-4}$$

$\hookrightarrow e$ is dimensionless.

(a) Gravitational const., $G = M^{-1}L^3T^{-2}$

(b) Gas const., $R = ML^2T^{-2}\theta^{-1}$

(c) Boltzmann's const., $K_B = ML^2T^{-2}\theta^{-1}$ EDUNI

(d) Planck const., $\epsilon_0 = ML^2T^{-1}$ EDUNI

(e) Rydberg's const., $R_y = L^{-1}$

(f) Magnetising field

$$H = \frac{B}{\mu_0} = \frac{\mu_0 i / 2\pi}{\mu_0}$$

$$= \frac{i}{8} \Rightarrow A L^{-1}$$

4 PRINCIPLE OF HOMOGENEITY

(i) If $a = b + c$ ex $s = ut + \frac{1}{2}at^2$
 $\Rightarrow [a] = [b] = [c]$ $[s] = [ut] = [at^2]$

5. CONVERSION OF UNITS

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 amount \rightarrow ηu \rightarrow units (can be SI, CGS
 of physical gty. numeric value or given in question)

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Ex: Find how many POISE (C.G.S unit of viscosity) is equal to 1 POISEUILLE (in SI)

Soln: $F = 6\pi\eta rV \Rightarrow [\eta] = \frac{[F]}{[r][V]} = \frac{MLT^{-2}}{L \times LT^{-1}} = ML^{-1}T^{-1}$

$$\begin{aligned} n_1 u_1 &= n_2 u_2 \\ \Rightarrow n_1 \times (g cm^{-1}s^{-1}) &= 1 \times (kg m^{-1}s^{-1}) \\ \Rightarrow n_1 &= \frac{kg m^{-1}s^{-1}}{g cm^{-1}s^{-1}} = \frac{10^3 g \times 10^{-2} cm^{-1} \times s^{-1}}{g cm^{-1}s^{-1}} \\ &= 10 \end{aligned}$$

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6. DIMENSION IN TERMS OF OTHER PHYSICAL QUANTITY

EX: Expression for time in terms of G (Gravitational const.), h (Planck const.) and c (speed of light) is proportional to?

Sol: $EDU\propto G^a h^b c^c$ $\Rightarrow [t] = [G]^a [h]^b [c]^c$

 $\Rightarrow [M^0 L^0 T^1] = [M^{-1} L^3 T^{-2}]^a [ML^2 T^{-1}]^b [LT^{-1}]^c$
 $\Rightarrow [M^0 L^0 T^1] = [M^{-a+b} L^{3a+2b+c} T^{-2a-b-c}]$
 $\therefore 0 = -a+b, 0 = 3a+2b+c, 1 = -2a-b-c$
 $\Rightarrow a = b = 1/2, c = -5/2$
 $\therefore t \propto G^{1/2} h^{1/2} c^{-5/2}$ EDU or $t \propto \sqrt{\frac{Gh}{c^5}}$ EDU

7 KEY POINTS (dimensionless & ty)

- (i) All trigonometric ratios ($\sin \theta, \cos \theta$ etc.)
- (ii) Angle, θ ($\sin \frac{ab}{c}$, here $[ab] = [c]$)
- (iii) Exponential functions, $e^x \rightarrow x$ must be dimensionless
- (iv) EDU $e^{-t/RC} \rightarrow [t] = [RC]$ EDU EDU
- (v) Reynolds number ($Re = \frac{\rho v D}{\eta}$), Dielectric const. (κ)
Refractive index and many more.

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Topics to cover in Error Analysis - Part 2

1. Absolute Error, Relative Error & Percentage Error
2. Combination of Errors
3. Vernier caliper and Screw Gauge

Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. Absolute Error, Relative Error & Percentage Error

(Error = True Value – Measured Value)

$$\text{STEP 1: } R_{\text{avg}} = \frac{R_1 + R_2 + \dots + R_n}{n} \quad (\text{we take } R_{\text{avg}} \text{ as True Value})$$

$$\text{STEP 2: Absolute Error } \Delta R_1 = R_1 - R_{\text{avg}}$$

$$\Delta R_2 = R_2 - R_{\text{avg}}$$

...

$$\Delta R_n = R_n - R_{\text{avg}}$$

(take only magnitude)

STEP 3: Mean Absolute Error

$$\Delta R_{\text{avg}} = \frac{|\Delta R_1| + |\Delta R_2| + \dots + |\Delta R_n|}{n}$$

$$\text{Final Reading} = \underbrace{R_{\text{avg}}}_{\substack{\downarrow \\ \text{Rel Error}}} \pm \underbrace{\Delta R_{\text{avg}}}_{\substack{\downarrow \\ \text{1. Error}}}$$

$$\frac{\Delta R_{\text{avg}}}{R_{\text{avg}}} \quad \frac{\Delta R_{\text{avg}} \times 100}{R_{\text{avg}}}$$

2 Combination of Errors

↳ error in measurement is very small

(b) Product or Division

(a) Sum or difference

$$l = 4.1 \pm 0.1 \text{ cm}, b = 3.3 \pm 0.1 \text{ cm}$$

$$S = l + b = 7.4 \pm 0.2 \text{ cm} \quad S = l - b = 0.8 \pm 0.2 \text{ cm}$$

$$P = d \frac{x^a y^b}{z^c} \quad \begin{cases} \text{Error is } \Delta x, \\ \Delta y \text{ & } \Delta z. \\ \text{Find Error in } P \end{cases}$$

$$\begin{aligned} \Rightarrow \ln P &= a \ln x + b \ln y - c \ln z + \ln d \\ \Rightarrow \frac{dP}{P} &= a \frac{dx}{x} + b \frac{dy}{y} - c \frac{dz}{z} + 0 \end{aligned}$$

To find Max Error in P

$$\frac{\Delta P}{P} = a \frac{\Delta x}{x} + b \frac{\Delta y}{y} + c \frac{\Delta z}{z}$$

$\left(\frac{\Delta x}{x} \text{ is Rel Error in } x \right)$

3. Vernier Caliper & Screw Gauge

↳ Must refer video for this from our youtube channel "EDUNITI".

Space to add concepts learnt from PYQs if any

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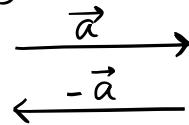
Topics to cover in Vectors - Part 3

1. Basic properties of vectors
2. Addition of vectors
3. Subtraction of vectors
4. Resolution of vectors in 2-D & 3-D
5. Dot product (scalar product)
6. Cross product (vector product)

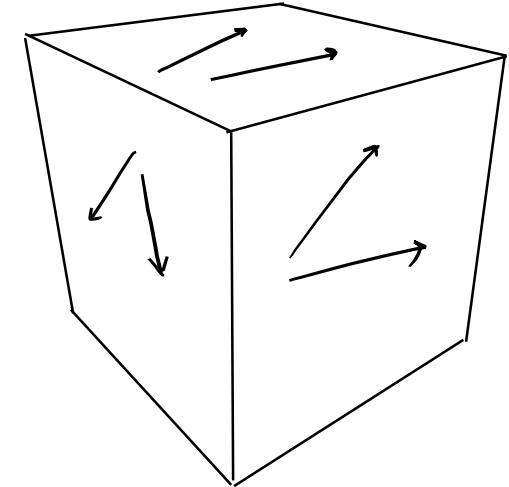
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1. Basic Properties of Vectors

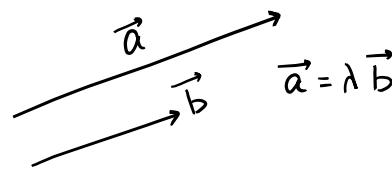
(a) Negative Vectors



(b) Coplanar Vectors

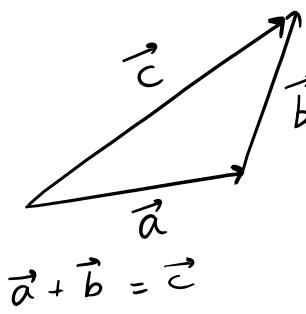


(c) Collinear Vectors

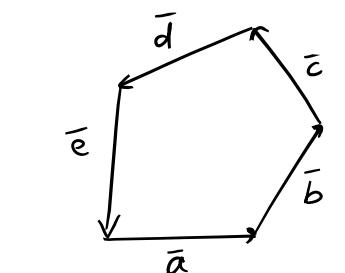


2. Addition of Vectors

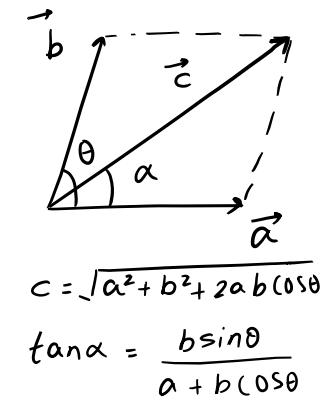
Triangle Law



Polygon Law (an extension of triangle law)



Parallelogram Law



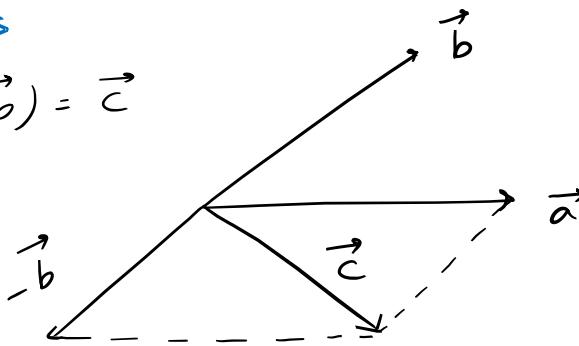
$$\bar{a} + \bar{b} + \bar{c} + \bar{d} + \bar{e} = 0$$

$$c = \sqrt{a^2 + b^2 + 2ab \cos \theta}$$

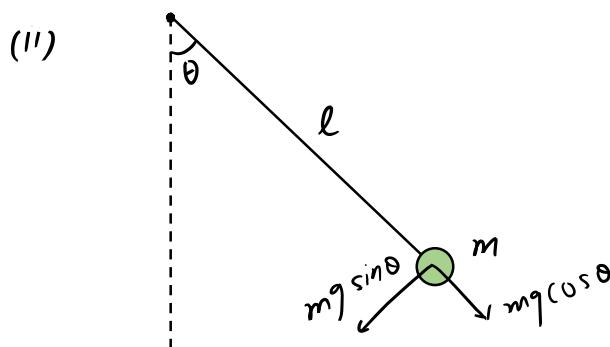
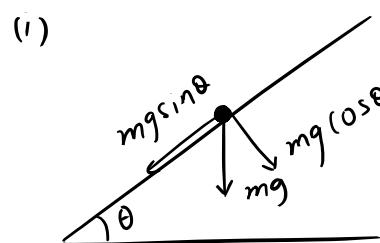
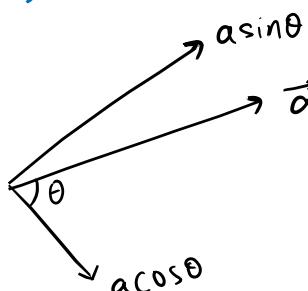
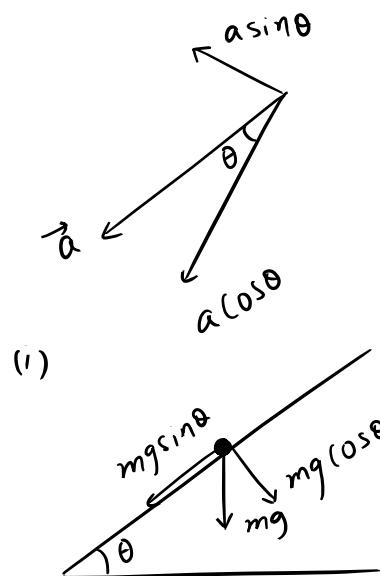
$$\tan \alpha = \frac{b \sin \theta}{a + b \cos \theta}$$

3. Subtraction of Vectors

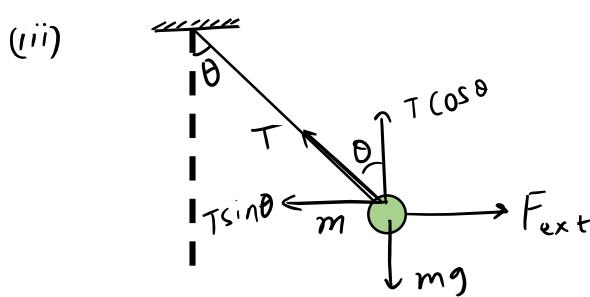
$$\vec{a} - \vec{b} = \vec{c} \Rightarrow \vec{a} + (-\vec{b}) = \vec{c}$$



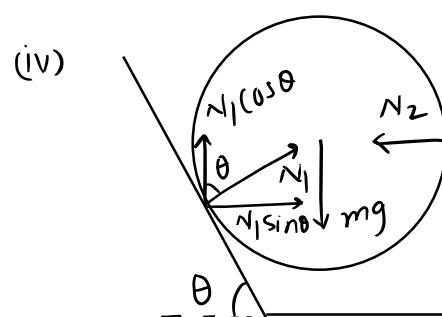
4. Resolution of Vectors (why resolve and how decide axis)



If particle is in accelerated state or tends to move.



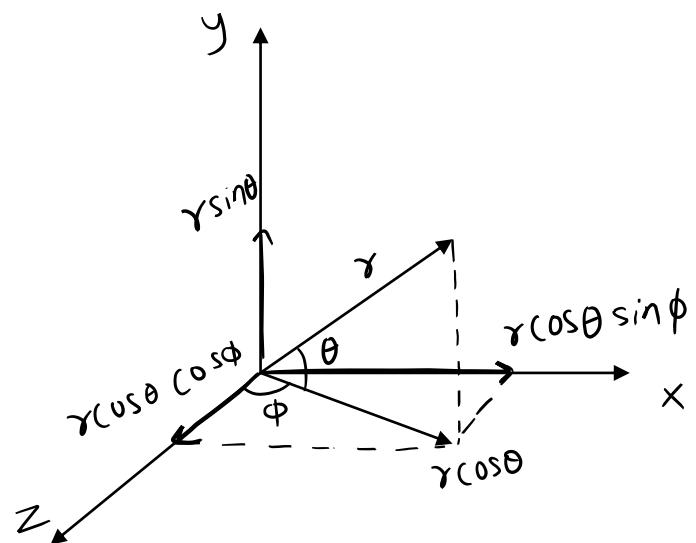
NOTE: These forces makes closed polygon



If particle is in equilibrium

Resolution of Vectors (3 dimension)

$$\vec{r} = r(\cos\theta \sin\phi \hat{i} + \sin\theta \hat{j} + r \cos\theta \cos\phi \hat{k})$$



5. Dot Product (Scalar Product)

$$\hookrightarrow \vec{a} \cdot \vec{b} = ab \cos \theta$$

$$\hookrightarrow \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a} \quad (\text{commutative law})$$

$$\hookrightarrow \vec{a} \cdot \vec{a} = a^2 \quad \# \text{ It's application.}$$

(i) If $\vec{a} \cdot \vec{b} = 0 \Rightarrow$ Vectors are orthogonal (90°)

(ii) Find angle betⁿ two Vectors, $\cos \theta = \frac{\vec{a} \cdot \vec{b}}{ab}$

(iii) Finding Projection of one Vector on another:

$$\vec{a} \cdot \vec{b} = ab \cos \theta$$

$$\therefore a \cos \theta = \frac{\vec{a} \cdot \vec{b}}{b}$$

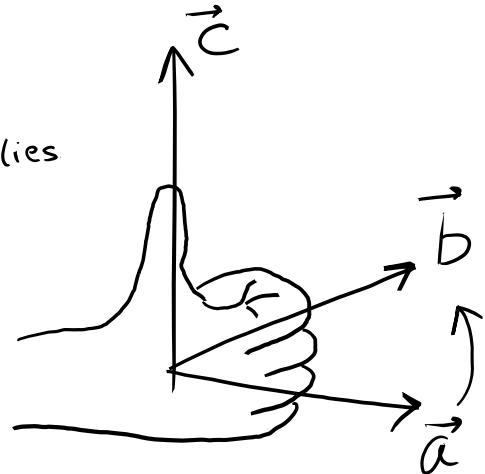
6. Cross Product (Vector Product)

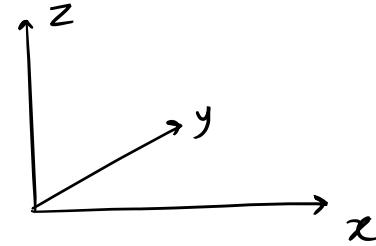
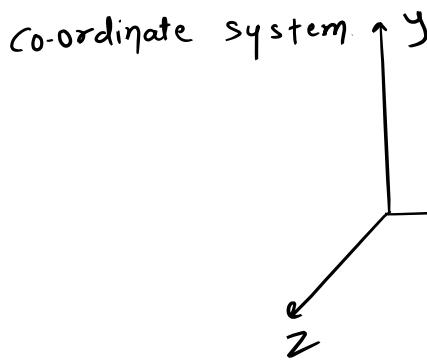
$$\hookrightarrow \vec{a} \times \vec{b} = \vec{c}, \vec{c} = ab \sin \theta \hat{n}$$

(i) \vec{c} is \perp to plane in which \vec{a} & \vec{b} lies

$$(ii) \vec{a} \times \vec{b} \neq \vec{b} \times \vec{a} \quad (\vec{a} \times \vec{b} = -\vec{b} \times \vec{a})$$

$$(iii) \vec{a} \times \vec{a} = 0$$





$$\begin{aligned}\hat{x} \times \hat{y} &= \hat{z} \\ \hat{y} \times \hat{z} &= \hat{x} \\ \hat{z} \times \hat{x} &= \hat{y}\end{aligned}$$

Cross Product (Vector Product)

$$\text{Ex: } \vec{a} = 2\hat{i} + 3\hat{j} + 5\hat{k}$$

$$\vec{b} = \hat{i} - 3\hat{j} + 4\hat{k}$$

find \vec{c}

$$\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 5 \\ 1 & -3 & 4 \end{vmatrix}$$

$$\begin{aligned}\vec{c} &= \hat{i}(12 + 15) - \hat{j}(8 - 5) + \hat{k}(-6 - 3) \\ &= 17\hat{i} - 3\hat{j} - 9\hat{k}\end{aligned}$$

Space to add concepts learnt from PYQs if any

Vectors – Part 3

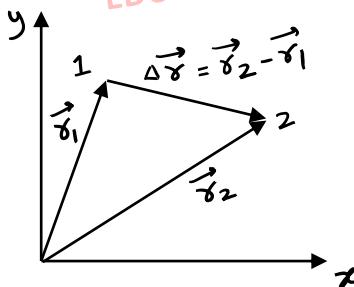
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Topics to cover in Motion in 1D – PART 1

- 1. Average velocity & acceleration
 - 2. Instantaneous velocity & acceleration (Calculus based)
 - 3. Uniform acceleration (equations of motion)
 - 4. Important points under motion under gravity
 - 5. Basics of graphs
 - 6. Types of graphs in 1D motion ($x-t$, $v-t$, $a-t$)

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1. Average Velocity & acceleration



$$(a) \overrightarrow{V_{av}} = \frac{\overrightarrow{r_2} - \overrightarrow{r_1}}{t_2 - t_1} = \frac{\Delta \overrightarrow{r}}{\Delta t} \quad \text{or} \quad \text{If } v = f(t) \text{ is given}$$

$$V_{av} = \frac{\int_{t_1}^{t_2} v dt}{t_2 - t_1}$$

$$(b) \vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta \vec{v}}{\Delta t} \quad \text{or} \quad \text{If } a = f(t) \text{ is given}$$

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$$a_{av} = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} a \, dt$$

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2. Instantaneous V & a (calculus based)

$$\begin{array}{l} x = f(t) \\ \downarrow \\ v = dx/dt \\ \curvearrowleft \\ a = \frac{dv}{dt} \end{array}$$

$$\begin{aligned}
 V &= f(t) \\
 &= f(x) \\
 a &= \frac{dV}{dt} \\
 &= V \frac{dV}{dx} \\
 &\# \frac{dx}{dt} = f(t) \\
 &\Rightarrow \int dx = \int f(t) dt
 \end{aligned}$$

$$\# \frac{dx}{dt} = f(x)$$

$$\Rightarrow \int \frac{dx}{f(x)} = \int dt$$

$$a = f(t) \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} = \frac{dy}{dt} \text{ or } v \frac{dv}{dx}$$

$$= f(v) \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} = f(x)$$

↓

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Ex: $v \frac{dv}{dx} = f(x)$

$$\Rightarrow \int v dv = \int f(x) dx$$

$$\Rightarrow \int v dv = \int f(x) dx$$

3. Uniform Acceleration ($a = \text{constant}$)

PART 1 – MOTION IN 1D

(i) $V = u + at$

(ii) $S = ut + \frac{1}{2}at^2$

(iii) $V^2 = u^2 + 2as$

(iv) $S = ut + \frac{1}{2}a(t^2 - 1)$ EDUNI

here $V, u, a \& S$
are vectors.
so be careful about signs.

If $u=0$, $t = \frac{V}{a}$, $S = \sqrt{\frac{2S}{a}}$, $S = \frac{V^2}{2a}$

$S = vt - \frac{1}{2}at^2$

4. Imp Points in Motion Under Gravity

(i) time of ascent = time of decent = $u/g = t$
Total time, $T = 2u/g$

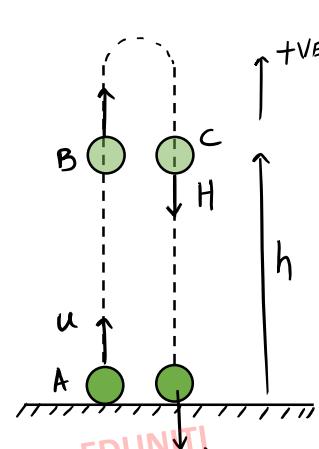
(ii) $H = \frac{u^2}{2g}$ (iii) $t = \sqrt{\frac{2H}{g}}$

(iv) Particle is at "h" twice in its journey EDUNI: EDUNI

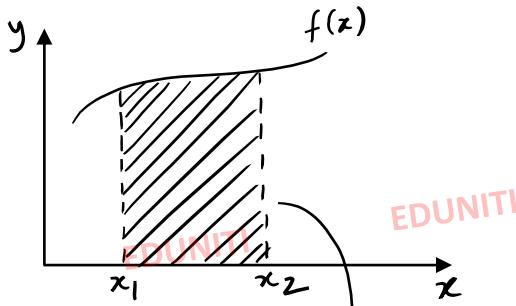
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$$h = ut - \frac{1}{2}gt^2 \xrightarrow{\text{Solving this}} t_{AB} \quad t_{AC}$$

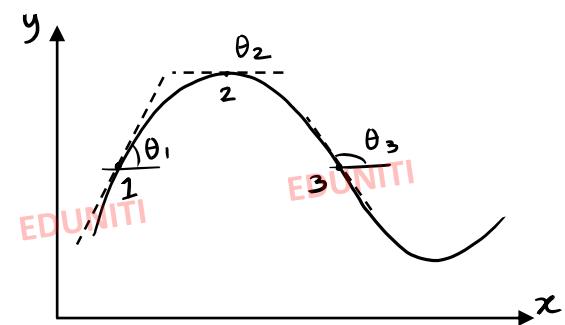
$(t_{AC} > t_{AB})$



5. Graphs Basics



Area under curve = $\int_{x_1}^{x_2} f(x) dx$

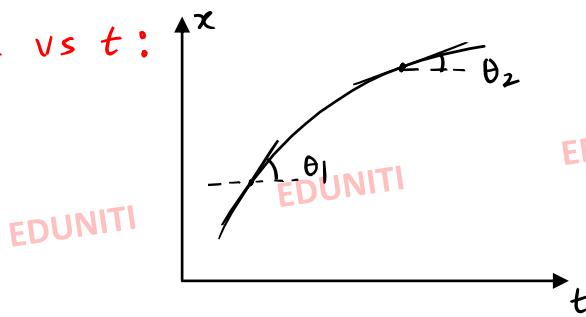


↳ Slope, $\frac{dy}{dx} = \tan \theta$

1 $\tan \theta_1 +ve$ 2 zero 3 $\tan \theta -ve$

6. Types of Graphs in 1-d Motion

PART 1 – MOTION IN 1D

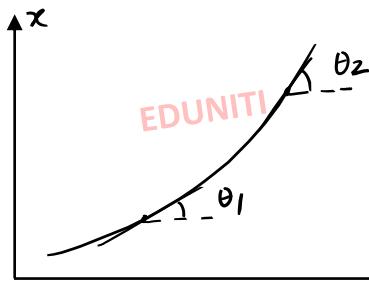
(i) x vs t :

$$\text{slope, } \frac{dx}{dt} = v$$

↳ with t slope $\uparrow \Rightarrow v \uparrow$

↳ with t slope $\downarrow \Rightarrow v \downarrow$

∴ Retarding Motion



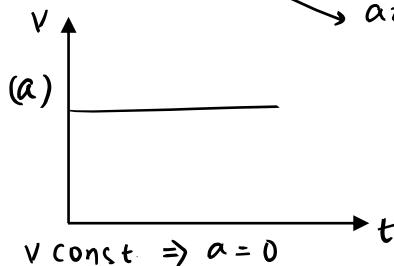
↳ with t slope $\uparrow \Rightarrow v \uparrow$

∴ Accelerated Motion

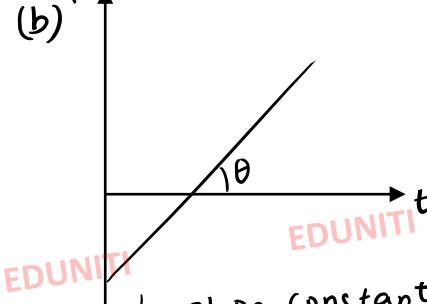
(ii) v vs t

↳ slope, $\frac{dv}{dt} = a$

area under curve $= \int v dt = \text{Displacement}$

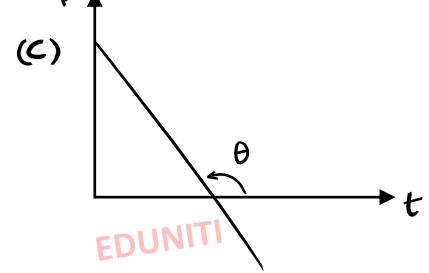


(b)

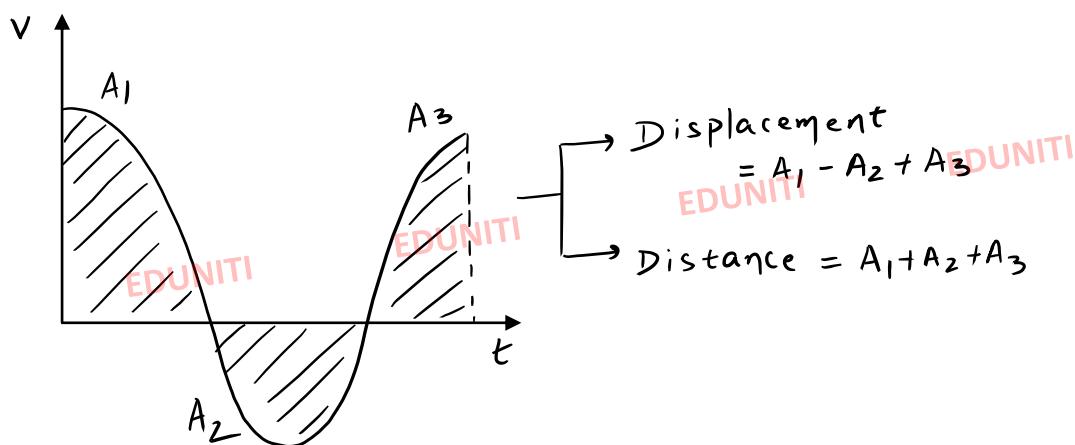


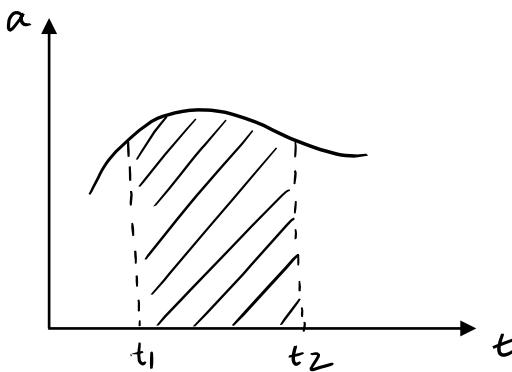
↳ Slope constant and $v \uparrow$
 $\Rightarrow a \text{ const} \& +ve$

(c)



↳ slope const.
 $\text{and } v \downarrow$
 $\Rightarrow a \text{ const}$
 $\& -ve$
 (retardation)



(iii) a vs t 

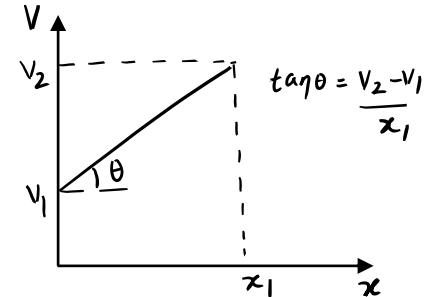
$$\text{Area} = \Delta v = v - u$$

(if area is below t axis,
take it as -ve)

(iv) v vs x

↳ slope, $\frac{dv}{dx} = \frac{a}{v}$

Ex7



$$y = mx + c$$

$$\Rightarrow v = x \tan \theta + v_1$$

do $v \frac{dv}{dx}$ to get $a = f(x)$

Space to add concepts learnt from PYQs if any

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Topics to cover in Motion in 2D – PART 2

1. Motion in a Plane
2. Projectile Motion (Ground to Ground)
 - a) Standard equation
 - b) Condition for same range and max range
 - c) Analysis of projectile at any time t
 - d) Equation of trajectory
3. Projectile fired from a tower
4. Projectile motion on a inclined plane
5. Projectile from a moving trolley

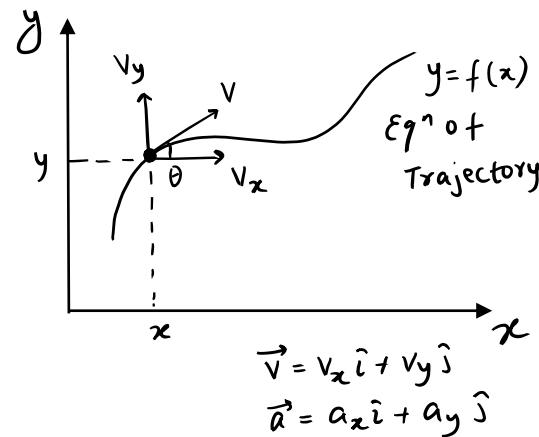
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1. Motion in a Plane

→ 2D motion is Vector Sum of two independent 1D motion (along x & y)

If \vec{a} is constant:

$$\begin{aligned} v_x &= u_x + a_x t \\ v_y &= u_y + a_y t \\ x &= u_x t + \frac{1}{2} a_x t^2 \\ y &= u_y t + \frac{1}{2} a_y t^2 \\ \vec{r} &= x \hat{i} + y \hat{j} \end{aligned}$$



2 Projectile Motion (Ground to Ground)

2a Standard eqn :

(i) Time of Flight

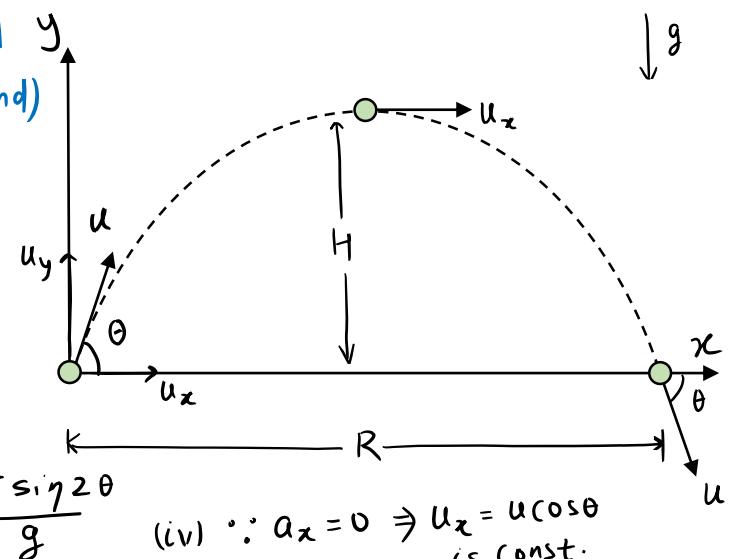
$$T = \frac{2u_y}{g} = \frac{2u \sin \theta}{g}$$

$$(ii) H = \frac{u^2 y}{2g} = \frac{u^2 \sin^2 \theta}{2g}$$

(iii) Range, $R = u_x \cdot T$

$$= \frac{2u_x u_y}{g} = \frac{u^2 \sin 2\theta}{g}$$

(iv) $\because a_x = 0 \Rightarrow u_x = u \cos \theta$
is const.



2b. Condition for R_{\max} & Same Range :

PART 2 – MOTION IN 2D

(i) $R = \frac{u^2 \sin 2\theta}{g}$

(ii) If $\theta_1 + \theta_2 = 90^\circ$



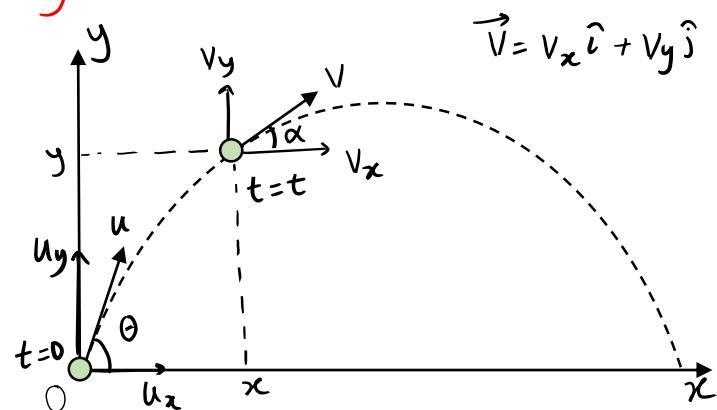
2c. Analysis of Projectile at any time t

(i) $v_x = u_x$, $v_y = u_y - gt$

$v = \sqrt{v_x^2 + v_y^2}$, $\alpha = \tan^{-1} \left(\frac{v_y}{v_x} \right)$

(ii) $x = u_x t$, $y = u_y t - \frac{1}{2} g t^2$

$\vec{r} = x \hat{i} + y \hat{j}$, $r = \sqrt{x^2 + y^2}$



$$\begin{aligned} u_x &= u \cos \theta, & u_y &= u \sin \theta \\ a_x &= 0, & a_y &= -g \end{aligned} \quad \left. \begin{array}{l} \text{Given} \end{array} \right\}$$

2d. Eqn of Trajectory

Using $x = u \cos \theta \cdot t$ and

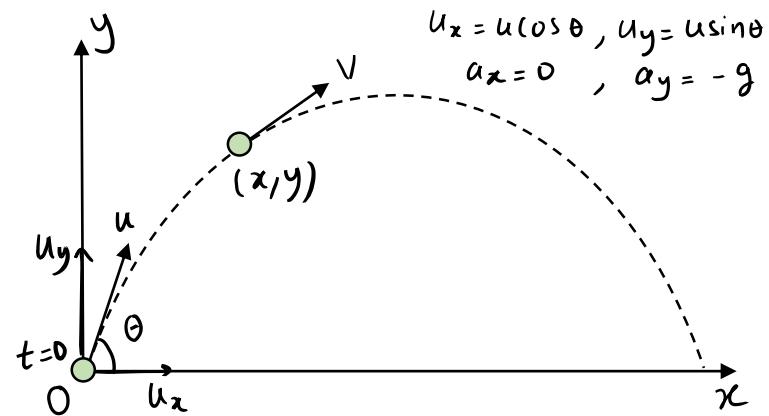
$y = u \sin \theta \cdot t - \frac{1}{2} g t^2$

(i) $y = x \tan \theta - \frac{g x^2}{2 u^2 \cos^2 \theta}$

 $(y = ax - bx^2) \rightarrow$ Quadratic Eqn

NOTE

Range, $R = \frac{a}{b}$



(ii) $y = x \tan \theta \left(1 - \frac{x}{R} \right)$

Range

3. Projectile Fired from a Tower

PART 2 – MOTION IN 2D

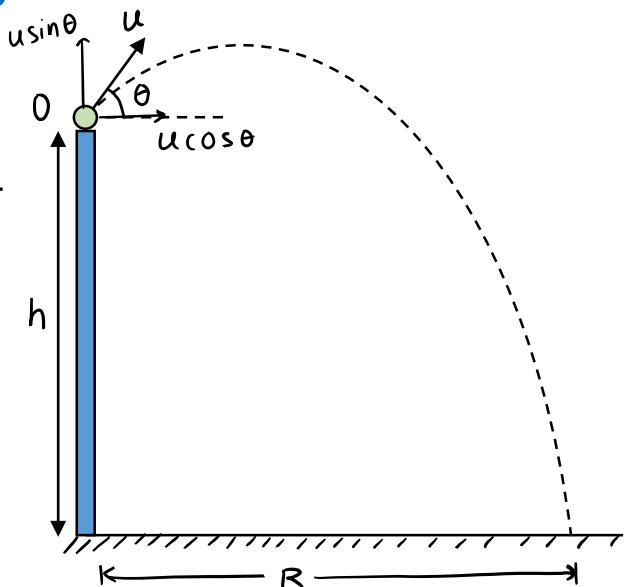
3a. At Some angle

(i) To find T (write eqn along y)

$$s_y = u_y t + \frac{1}{2} a_y t^2 \Rightarrow -h = u \sin \theta \cdot T - \frac{1}{2} g T^2$$

↓ Find T

$$(ii) R = u \cos \theta \cdot T$$



3b. Projected horizontally

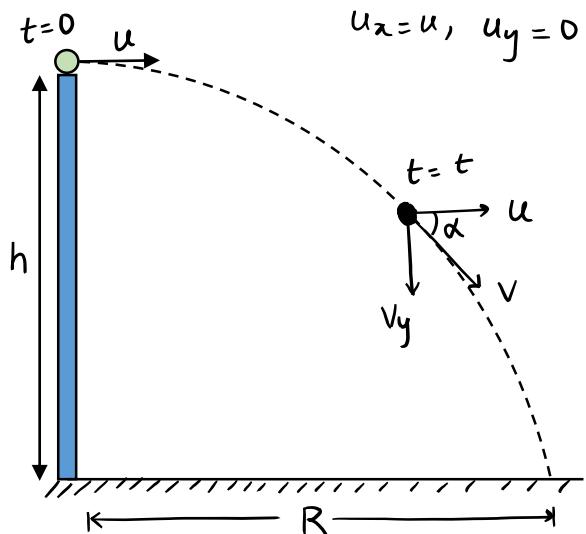
$$(i) \text{ Time of Flight, } T = \sqrt{\frac{2h}{g}}$$

$$(ii) \text{ Range, } R = u \cdot T = u \sqrt{\frac{2h}{g}}$$

$$(iii) v_y = gt \quad \therefore \alpha = \tan^{-1} \left(\frac{v_y}{u} \right)$$

and

$$V = \sqrt{u^2 + v_y^2}$$

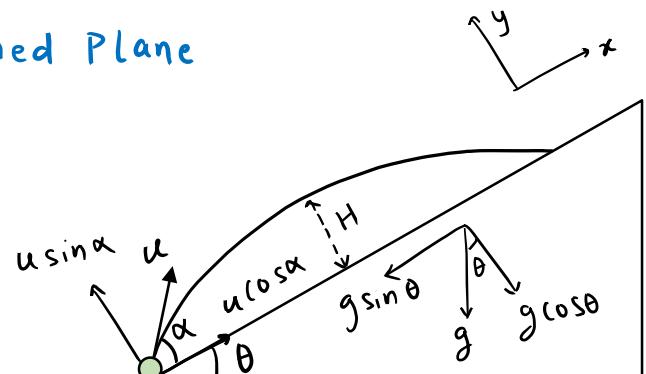


4. Projectile Motion on Inclined Plane

4a. UP the Incline

$$(i) T = \frac{2u_y}{a_y} = \frac{2u \sin \alpha}{g \cos \theta}$$

$$(ii) H = \frac{u^2 y}{2 a_y} = \frac{u^2 \sin^2 \alpha}{2 g \cos \theta}$$



$$(iii) \text{ For Range, } x = u_x t + \frac{1}{2} a_x t^2$$

$$\Rightarrow R = u \cos \alpha \cdot T - \frac{1}{2} g \sin \theta \cdot T^2$$

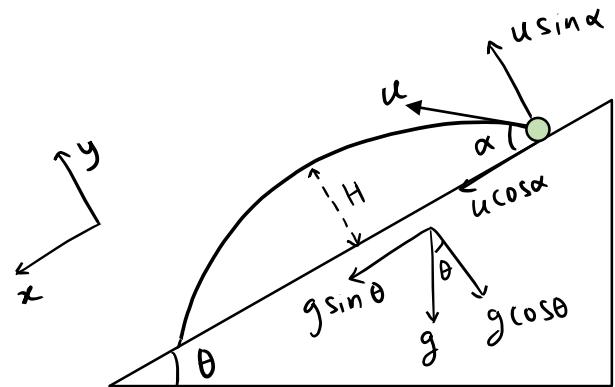
4b. Down the Incline

$$(i) T = \frac{2u_y}{a_y} = \frac{2u \sin \alpha}{g \cos \theta}$$

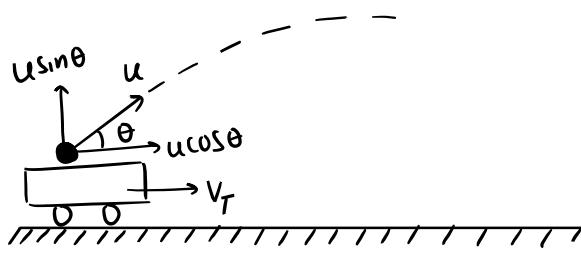
$$(ii) H = \frac{u^2 y}{2 a_y} = \frac{u^2 \sin^2 \alpha}{2 g \cos \theta}$$

$$(iii) \text{ for Range, } x = u_x t + \frac{1}{2} a_x t^2$$

$$\Rightarrow R = u \cos \alpha \cdot T + \frac{1}{2} g \sin \theta \cdot T^2$$



5. Projectile from moving Trolley



Concept: wrt ground

$$u_x = u \cos \theta + V_T$$

$$u_y = u \sin \theta$$

$$(i) T = \frac{2u \sin \theta}{g}$$

$$(ii) H = \frac{u^2 \sin^2 \theta}{2g}$$

$$(iii) R = u_x \cdot T = (u \cos \theta + V_T) \cdot \frac{2u \sin \theta}{g}$$

$$\hookrightarrow \text{To find } R_{\max}, \frac{dR}{d\theta} = 0$$

Space to add concepts learnt from PYQs if any

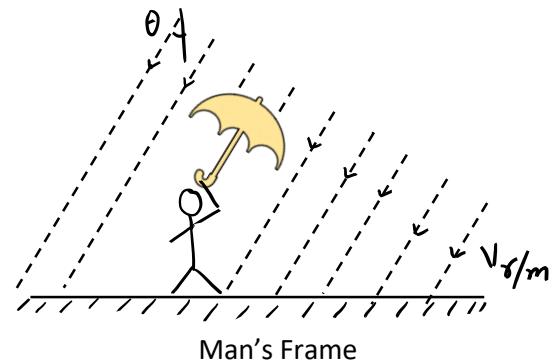
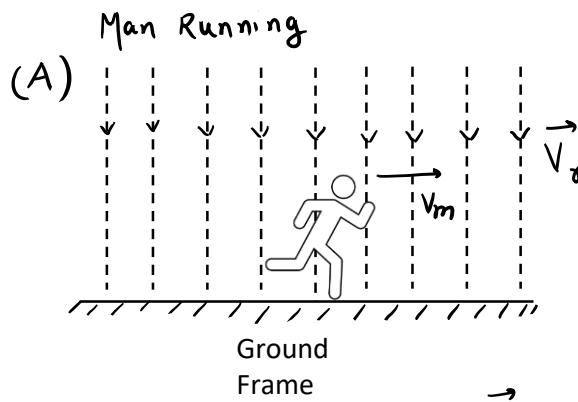
Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in Relative Motion – PART 3

1. Rain – Man
2. River – Swimmer (basic terms)
3. River Swimmer Case A – Least Time
4. River Swimmer Case B – Shortest Path (zero drift)
5. River Swimmer Case C – Shortest path (non zero drift)

Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. Rain - Man



Another way:

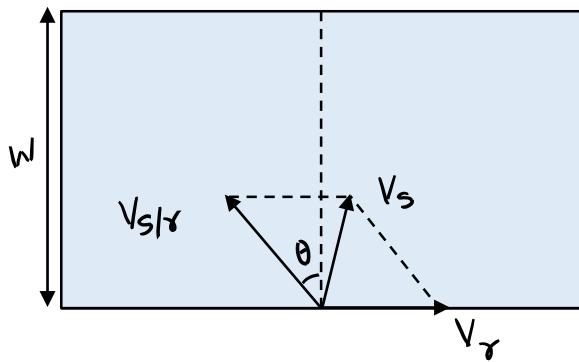
$$\vec{V}_r = \vec{V}_m + \vec{V}_{r/m}$$

$$(i) \tan \theta = V_m / V_{r/m}$$

$$(ii) V_{r/m} = \sqrt{V_m^2 + V_r^2}$$

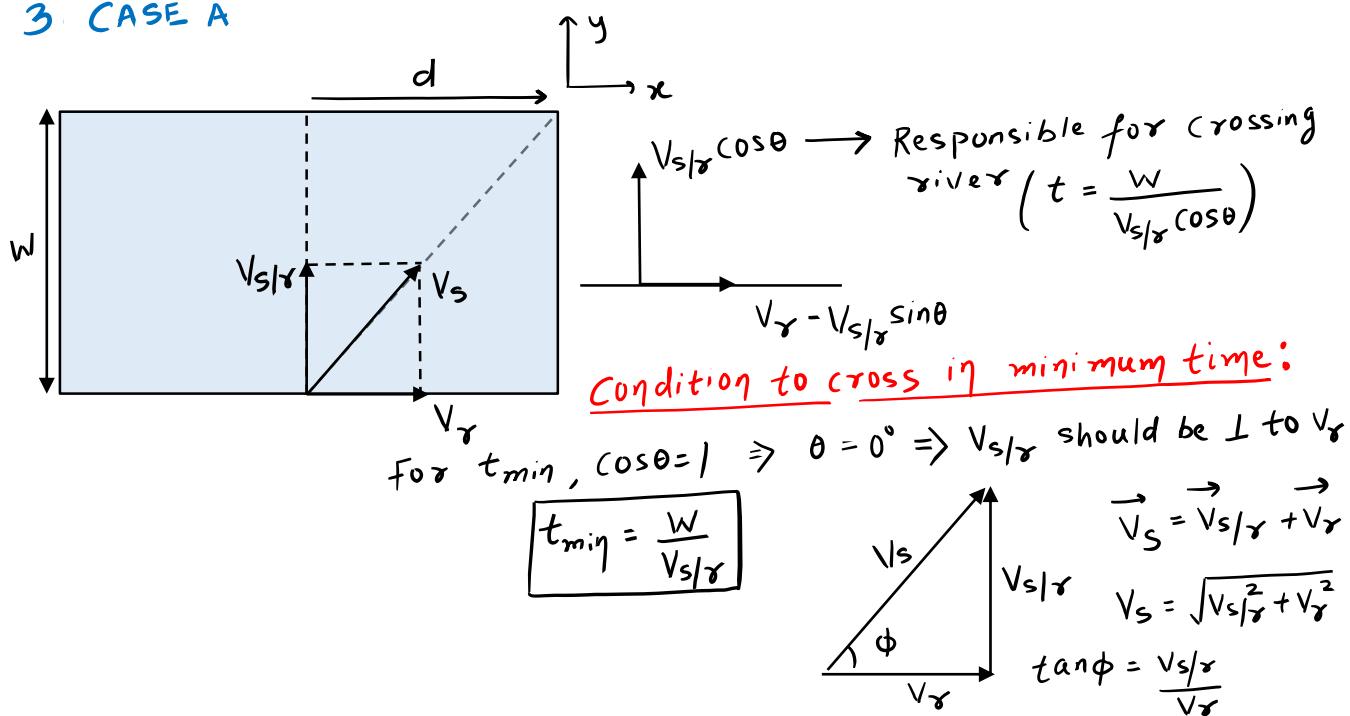
$$\vec{V}_{r/m} = \vec{V}_r - \vec{V}_m$$

2. River- Swimmer (basis terms)

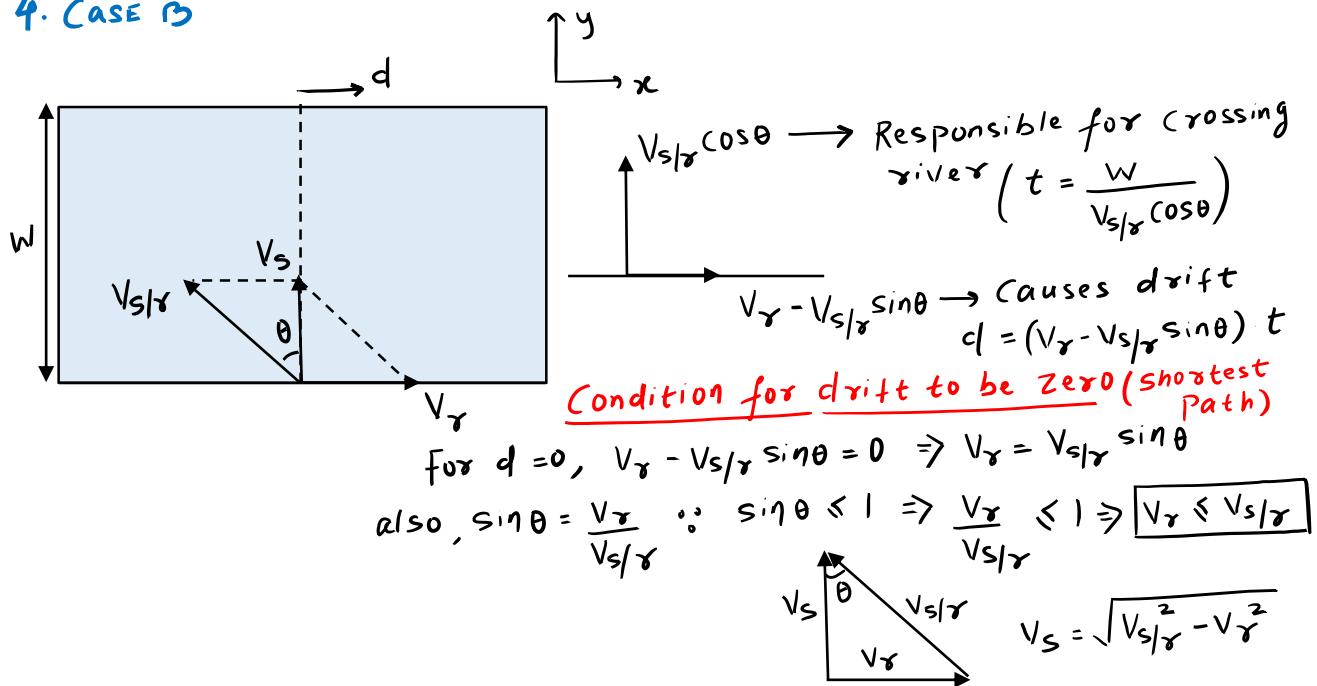


\vec{V}_r : velocity of river w.r.t Ground
 $\vec{V}_{s/r}$: velocity of swimmer w.r.t river (vel of swimmer in still river)
 \vec{V}_s : velocity of swimmer w.r.t Ground

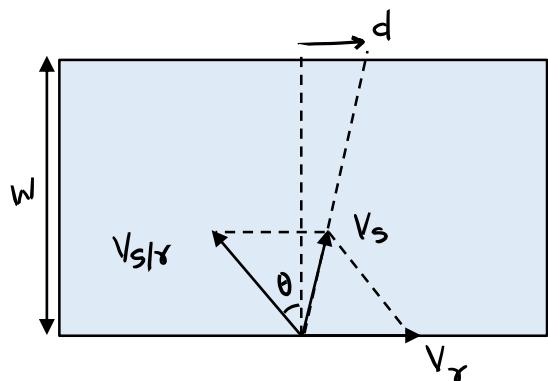
3. CASE A



4. CASE B



5. Case C



$V_{s/r} \cos \theta$ → Responsible for crossing river ($t = \frac{W}{V_{s/r} \cos \theta}$)

$V_r - V_{s/r} \sin \theta$ → Causes drift
 $d = (V_r - V_{s/r} \sin \theta) t$

If $V_r > V_{s/r}$ d can never be zero!

so, $d = (V_r - V_{s/r} \sin \theta) \frac{W}{V_{s/r} \cos \theta} \therefore$ Cond' for d_{\min} ?

Differentiate w.r.t θ and equate to zero.

We get : $\sin \theta = \frac{V_{s/r}}{V_r} \Rightarrow \boxed{\theta = \sin^{-1} \left(\frac{V_{s/r}}{V_r} \right)}$

Space to add concepts learnt from PYQs if any

Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

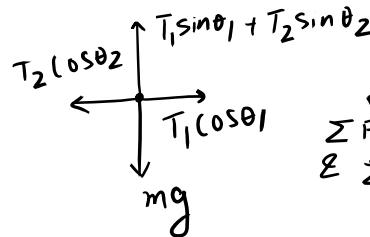
Topics to cover in NLM – PART 1

1. Static Equilibrium
2. Pulley Block Systems
3. Concept of Weighing Machine
4. Concept of Spring Balance
5. Spring Cutting & Combination
6. Pseudo Force
7. Variable Force

Note: For video refer Revision Series Playlist on EDUNI YouTube Channel

1. Static Equilibrium

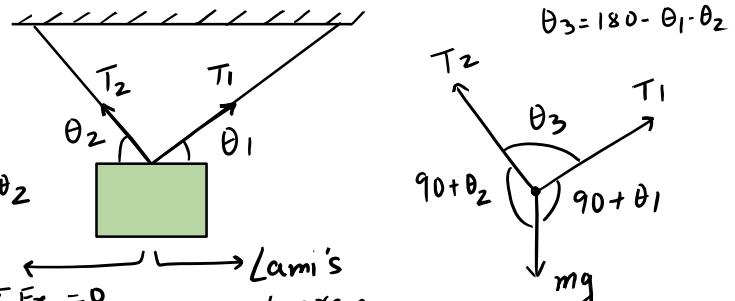
↪ $\sum \vec{F} = 0$ (body at rest)



$\sum F_x = 0$ & $\sum F_y = 0$

$$T_1 \sin \theta_1 + T_2 \sin \theta_2 = mg$$

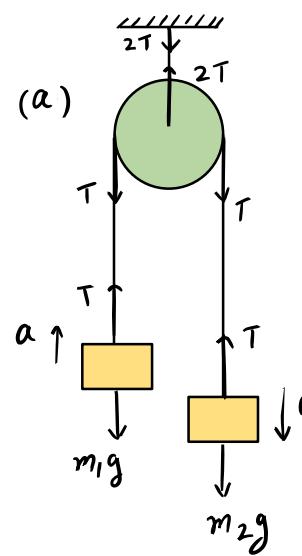
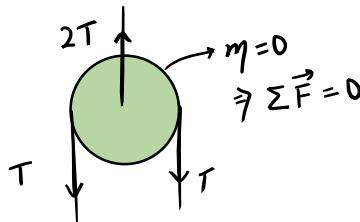
$$T_1 \cos \theta_1 = T_2 \cos \theta_2$$



$$\frac{mg}{\sin \theta_3} = \frac{T_1}{\sin(90 + \theta_2)} = \frac{T_2}{\sin(90 + \theta_1)}$$

2. Pulley-Block Systems

↪ Pulley is massless & smooth



$$\sum F = ma$$

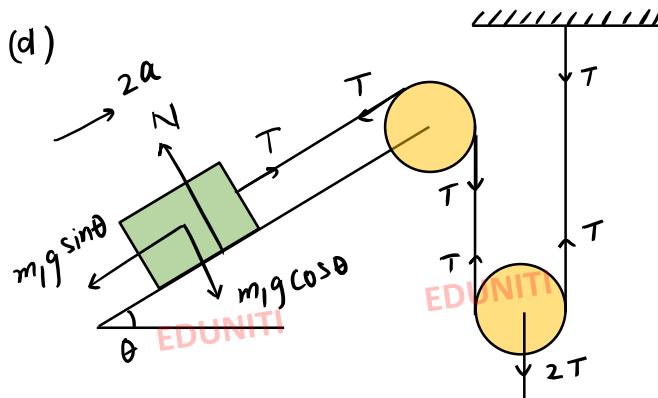
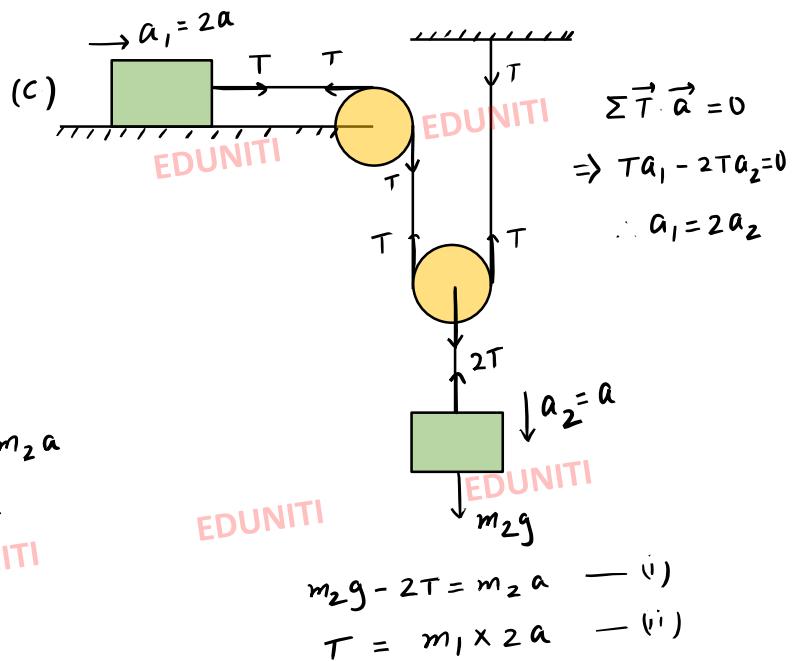
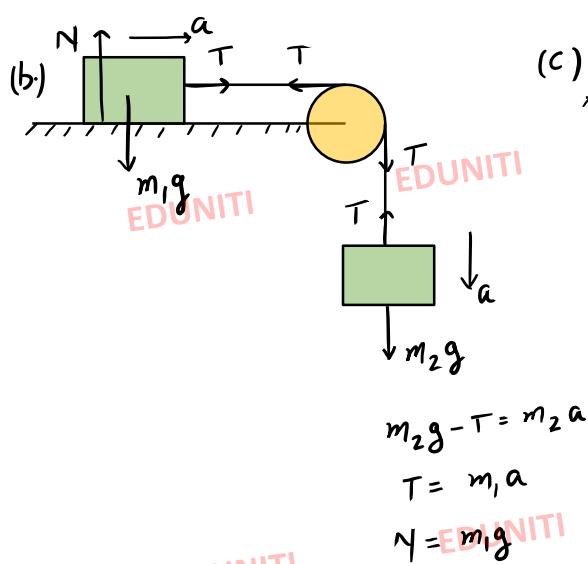
$$\downarrow$$

$$m_2 g - T = m_2 a \quad (i)$$

$$T - m_1 g = m_1 a \quad (ii)$$

$$\# T = \frac{2m_1 m_2 g}{m_1 + m_2}$$

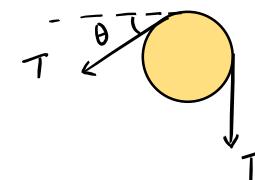
$$\# a = \left(\frac{m_2 - m_1}{m_1 + m_2} \right) g$$



$$m_2 g - 2T = m_2 a \quad \text{--- (i)}$$

$$T - m_1 g \sin \theta = m_1 \times 2a \quad \text{--- (ii)}$$

$$N = m_1 g \cos \theta \quad \text{--- (iii)}$$



clamp force

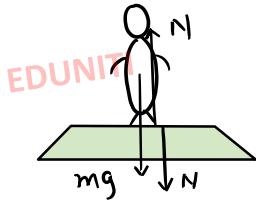
$$= \sqrt{T^2 + T^2 + 2T^2 \cos(\frac{\pi}{2} - \theta)}$$

EDUNITI

EDUNITI

3. Concept of Weighing Machine

Measures Normal Reaction acting on surface of m/c.



$$EDUNITI \quad N = mg$$

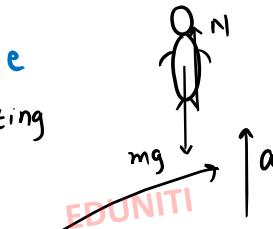
$$EDUNITI \quad \text{Reading} = \frac{N}{g} = m$$

If $N=0$

⇒ weightlessness

EDUNITI

ex: free fall



$$EDUNITI \quad N - mg = ma \Rightarrow N = m(g + a)$$

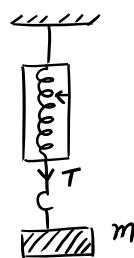
$$EDUNITI \quad \text{Reading} = \frac{N}{g} = m\left(1 + \frac{a}{g}\right)$$

$$EDUNITI \quad mg - N = ma \Rightarrow N = m(g - a)$$

$$EDUNITI \quad \text{Reading} = \frac{N}{g} = m\left(1 - \frac{a}{g}\right)$$

4. Concept of Spring Balance (massless spring)

Measures Tension in string attached to it.



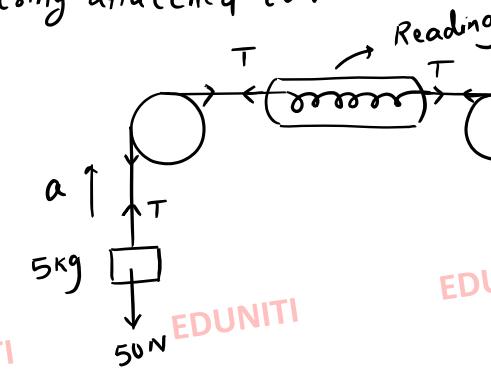
$$T = mg$$

$$EDUNITI \quad \text{Reading} = \frac{T}{g} = m$$

EDUNITI

$$EDUNITI \quad 100 - T = 10a \quad (i)$$

$$EDUNITI \quad T - 50 = 5a \quad (ii)$$

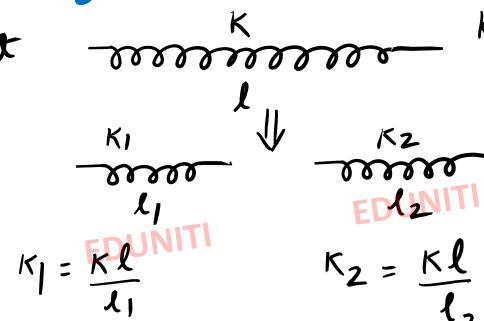


$$EDUNITI \quad \therefore T = \frac{200}{3} N$$

$$EDUNITI \quad \text{Reading} = \frac{20}{3} \text{ kg}$$

5. Spring cutting & Combination

Cut

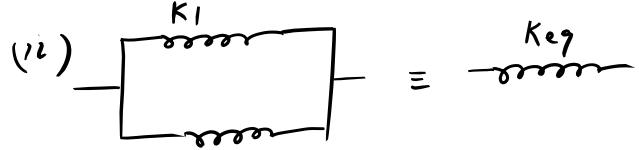


$$EDUNITI \quad K_1 = \frac{Kl}{l_1}$$

$$EDUNITI \quad K_2 = \frac{Kl}{l_2}$$

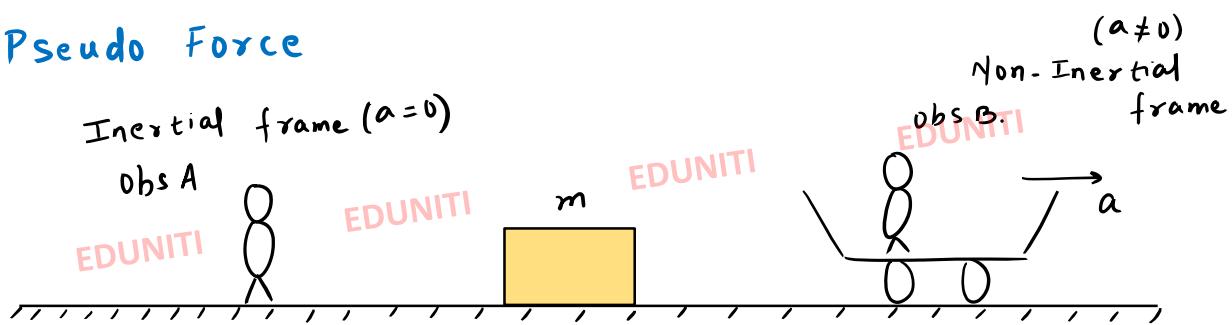
$K \cdot l = \text{const.}$ # Combination

$$(i) \quad EDUNITI \quad \frac{1}{K_{\text{eq}}} = \frac{1}{K_1} + \frac{1}{K_2}$$



$$EDUNITI \quad K_{\text{eq}} = K_1 + K_2$$

6. Pseudo Force



obs A. \rightarrow Body is at rest

obs B. \rightarrow $a \leftarrow$ m \therefore accelerated, thus force acting on it must be F_p $F_p = ma$
 (false force)

7. Variable Force

$F \rightarrow a$

$$F(t) \rightarrow F(t) = m \frac{dv}{dt} \Rightarrow \int_0^t F(t) dt = \int_u^v m dv$$

$$F(x) \rightarrow F(x) = m v \frac{dv}{dx} \Rightarrow \int_0^x F(x) dx = \int_u^v m v dv$$

$$F(v) \rightarrow F(v) = m v \frac{dv}{dx} \Rightarrow \int_0^x dx = \int_u^v \frac{m v}{F(v)} dv$$

$$F(v) = m \frac{dv}{dt} \Rightarrow \int_0^t dt = \int_u^v m \frac{dv}{F(v)}$$

Space to add concepts learnt from PYQs if any

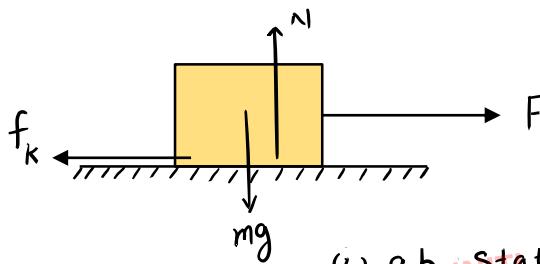
Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in Friction – PART 2

1. Variation of Friction with External Force
2. Direction of Friction (learn through FBD)
3. Pushing vs Pulling
4. Minimum force to start slipping
5. Contact Force
6. Angle of Repose
7. Two Block System
8. Friction on Chain

Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. Variation of Friction with External Force

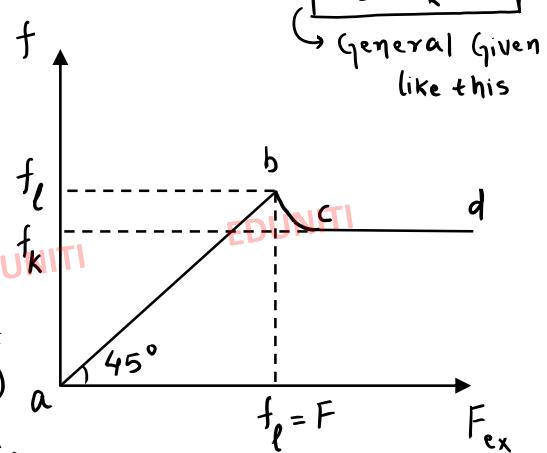


(i) ab : static friction
(block is at rest w.r.t ground)

(ii) At b : $f_{s\max}$ or f_l

(iii) cd : Kinetic friction ($f_k < f_{s\max}$)

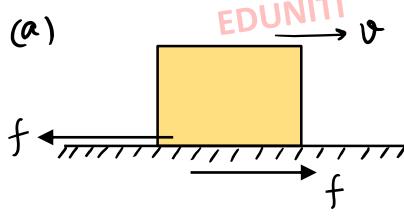
$$(iv) f_{s\max} = \mu_s N, f_k = \mu_k N$$



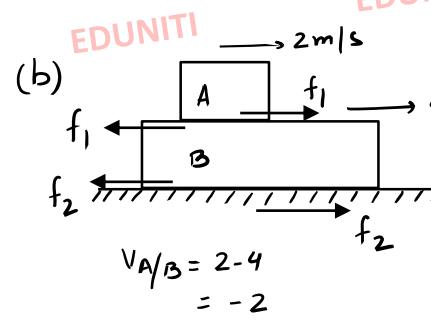
2. Direction of friction

↳ friction opposes the relative motion between two bodies

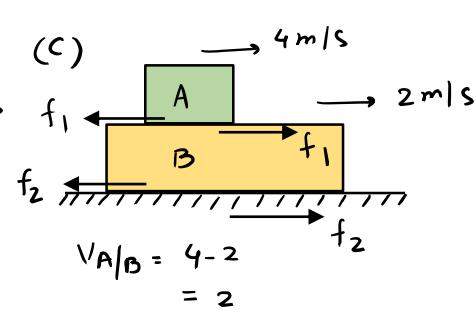
(a)

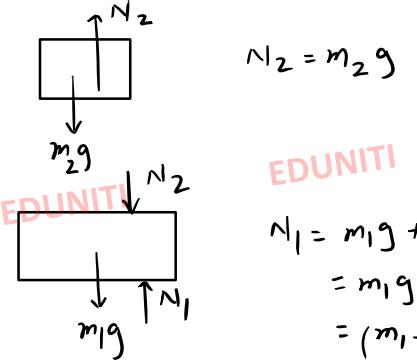
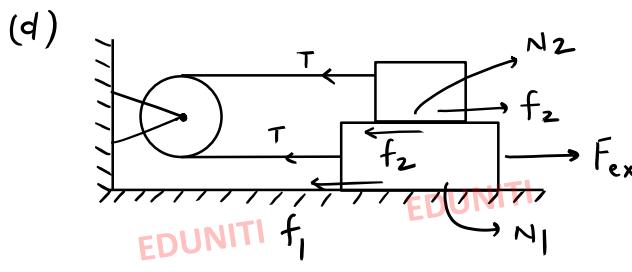


(b)



(c)





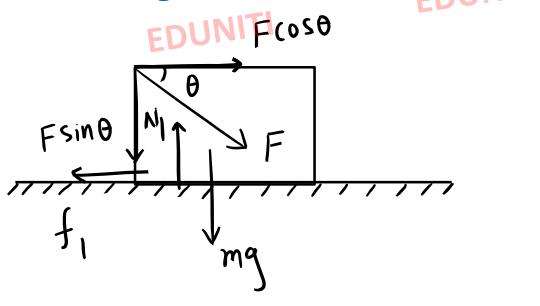
EDUNITI

$$\begin{aligned}N_1 &= m_1 g + N_2 \\&= m_1 g + m_2 g \\&= (m_1 + m_2) g\end{aligned}$$

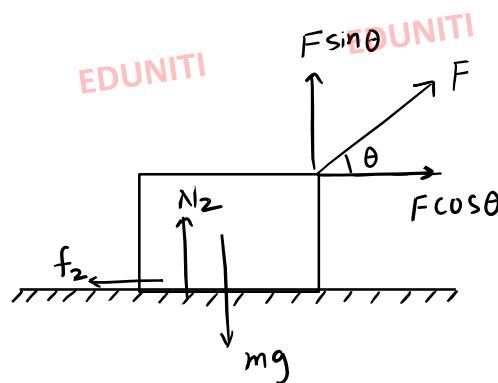
Assuming slipping at all places :

$$f_1 = \mu_1 N_1, \quad f_2 = \mu_2 N_2$$

3. Pushing vs Pulling



$$\begin{aligned}N_1 &= mg + F \sin \theta \\f_1 &= \mu N_1\end{aligned}$$

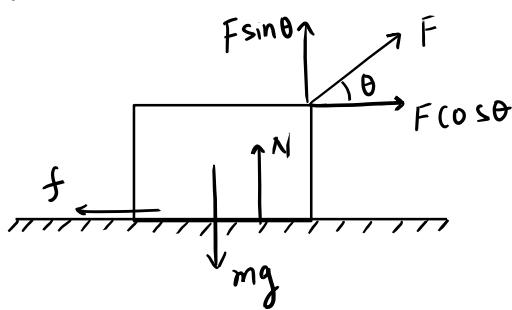


$$N_2 = mg - F \sin \theta$$

To slide, friction to overcome

$$f_2 = \mu N_2$$

4. Minimum Force to Start Slipping (condⁿ on θ)



To $\text{EDMINI}^{\text{NITI}}$ F, $\cos \theta + \mu \sin \theta \rightarrow \text{Maximized}$

To just start slipping :

$$F \cos \theta = f = \mu N$$

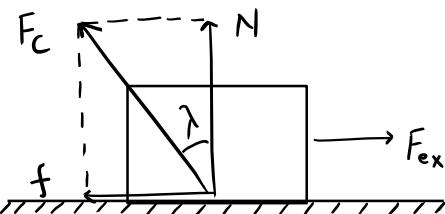
$$\Rightarrow F \cos \theta = \mu (mg - F \sin \theta)$$

$$\Rightarrow F = \frac{\mu mg}{\cos \theta + \mu \sin \theta}$$

EDUNITI

$$\Rightarrow \frac{d}{d\theta} (\cos \theta + \mu \sin \theta) = 0 \Rightarrow \tan \theta = 1$$

$$\therefore \cos \theta = \frac{1}{\sqrt{\mu^2 + 1}}, \quad \sin \theta = \frac{\mu}{\sqrt{\mu^2 + 1}} \Rightarrow F_{\min} = \frac{\mu mg}{\sqrt{\mu^2 + 1}}$$

5. Contact Force (F_c)

$$F_c = \sqrt{N^2 + f^2}$$

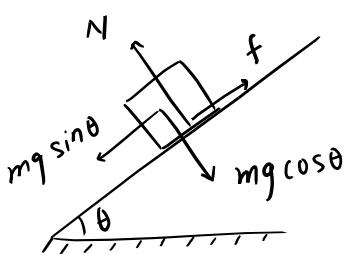
$$\begin{array}{l} \xrightarrow{1} \\ \downarrow F_{c,\min} \\ = N \quad \{ f=0 \} \end{array} \quad \begin{array}{l} \downarrow \\ F_{c,\max} \\ = \sqrt{N^2 + \mu N^2} \\ = N \sqrt{1+\mu^2} \quad \{ \mu = \mu_s \} \end{array}$$

$$\# N \leq F_c \leq N\sqrt{1+\mu^2}$$

6. Angle of Repose

↳ Minimum angle for which body slips due to its weight

Block starts to just slip if:



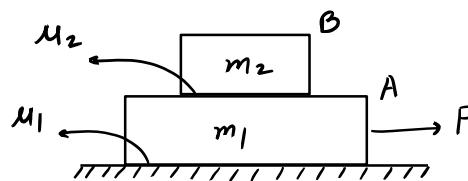
$$\begin{aligned} mgs \sin \theta &= f_{s\max} \\ \Rightarrow mgs \sin \theta &= \mu N \Rightarrow mgs \sin \theta = \mu mg \cos \theta \\ \therefore \tan \theta &= \mu \quad \text{or} \quad \boxed{\theta = \tan^{-1} \mu} \end{aligned}$$

(i) If angle is $< \tan^{-1} \mu \Rightarrow$ No slipping

(ii) If angle is $> \tan^{-1} \mu \Rightarrow$ slipping

$$a = g \sin \theta - \mu g \cos \theta$$

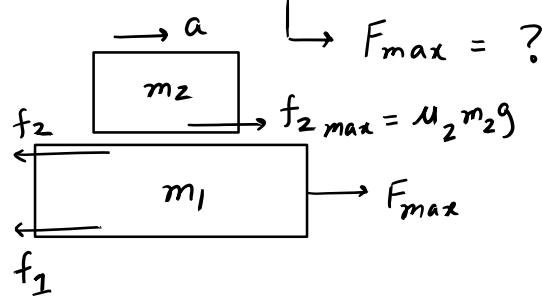
7. Two Block System (Block over block)



(i) Block B will never move before A

(ii) Min F to move A = $\mu_1(m_1+m_2)g = f_1$

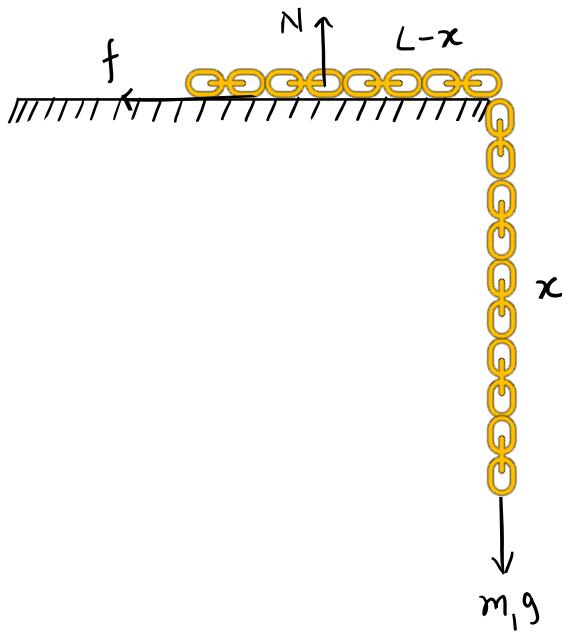
(iii) On further $\uparrow F$ upto a F_{\max}
 m_2 & m_1 moves together (m_2 not slipping on m_1)



$$\therefore a = \frac{\mu_2 m_2 g}{m_2} = \mu_2 g$$

$$\Rightarrow \boxed{F_{\max} - f_1 = (m_1 + m_2) \mu_2 g}$$

8. Friction on chain



$m_1 \rightarrow$ hanging mass

For Maximum hanging Length without slipping:

$$m_1 g = f_{\max}$$

$$\Rightarrow \frac{m}{L} x g = \mu \underbrace{\frac{m}{L} (L-x) g}_{N}$$

$$\Rightarrow x = \mu (L-x)$$

$$\therefore \boxed{x = \frac{\mu L}{1 + \mu}}$$

Space to add concepts learnt from PYQs if any

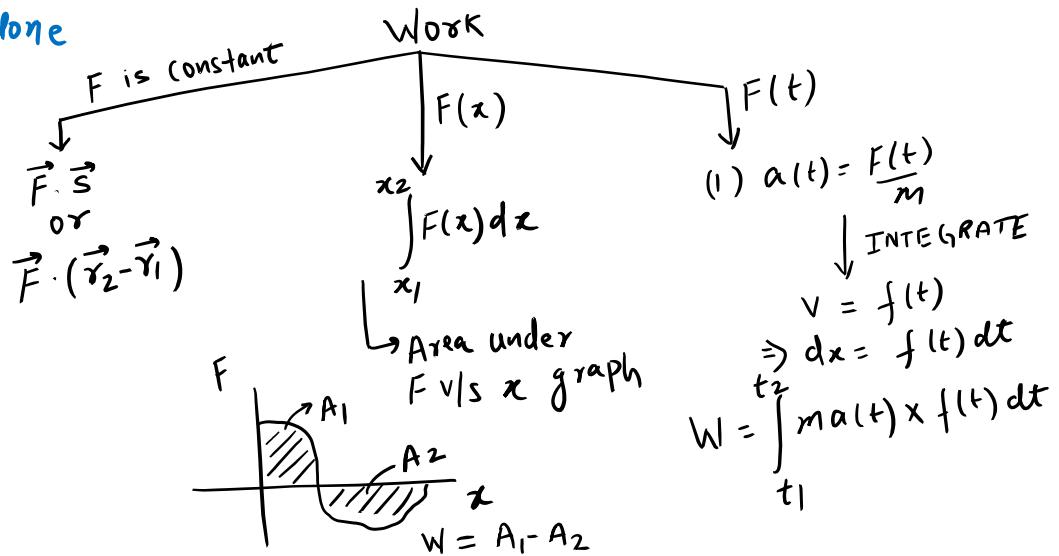
Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in Work Energy Power

1. Work done
2. Work Energy Theorem
3. Conservation of Mechanical Energy
4. Cases where Work done is zero
5. Potential Energy
6. How to write Change in P.E.
7. Conservative Force
8. Power

Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. Work done



2. Work-Energy Theorem

↪ Net work done = change in Kinetic energy

$$W_{\text{net}} = K_f - K_i \Rightarrow W_{\text{net}} = \Delta K$$

↪ Work can be due to Various forces:

↪ Conservative force (gravitational, electrostatic, spring force)

↪ Non-conservative (friction, viscous force)

↪ External agent (person applying forces)

↪ Tension, Normal

$$W_{\text{net}} = K_f - K_i \Rightarrow W_{\text{net}} = \Delta K$$

$$\hookrightarrow W_C + W_{NC} + W_{\text{ext}} = \Delta K$$

$$\Rightarrow -\Delta U + W_{NC} + W_{\text{ext}} = \Delta K$$

$$\Rightarrow W_{NC} + W_{\text{ext}} = \underbrace{\Delta K + \Delta U}_{\text{Total change in Mechanical Energy}}$$

3. Conservation of Energy

If on $W_{NC} = 0$ & $W_{\text{ext}} = 0$ (either they are absent or their work done is zero)

$$\Rightarrow 0 = \Delta K + \Delta U$$

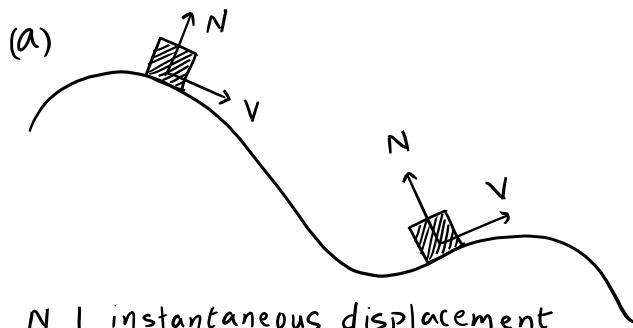
Total change in Mech Energy is zero

\Rightarrow Energy conservation

$$\text{or } 0 = K_f - K_i + U_f - U_i$$

$$\Rightarrow K_i + U_i = K_f + U_f$$

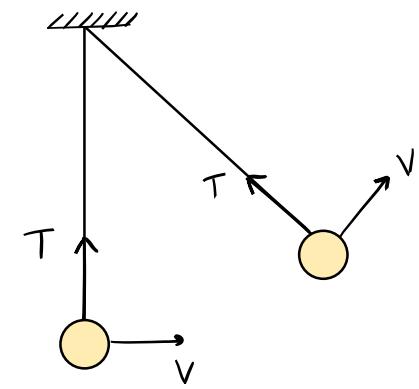
4. Situation where $W = 0$



$$N \perp \text{instantaneous displacement}$$

$$\Rightarrow W_N = 0$$

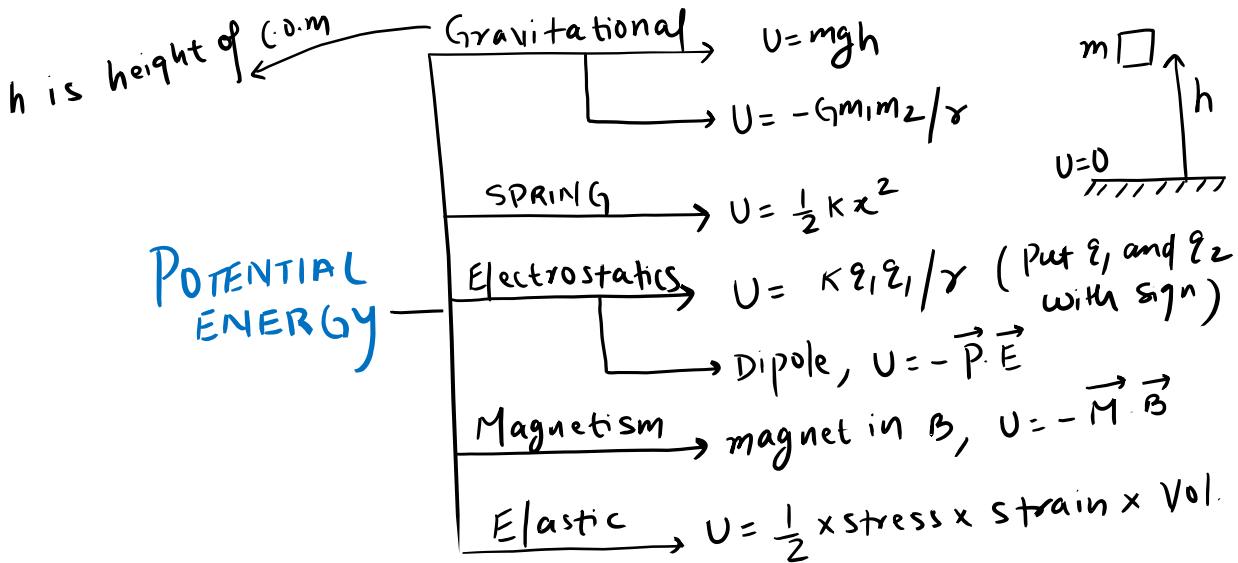
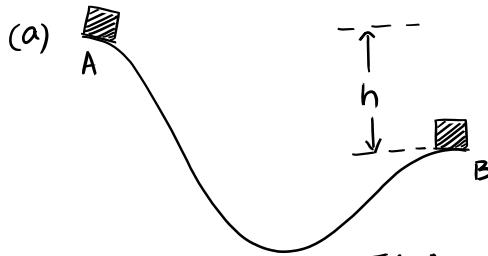
(b)



$$T \perp \text{to instantaneous displacement of bob}$$

$$\Rightarrow W_T = 0$$

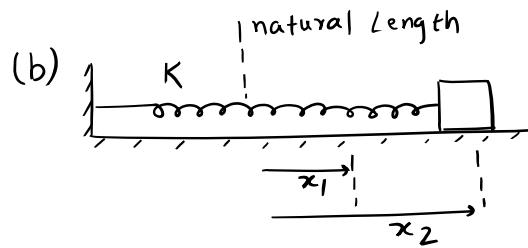
5.

6. How to write ΔU If $A \rightarrow B$

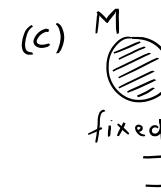
$$\Delta U = -mgh$$

If $B \rightarrow A$

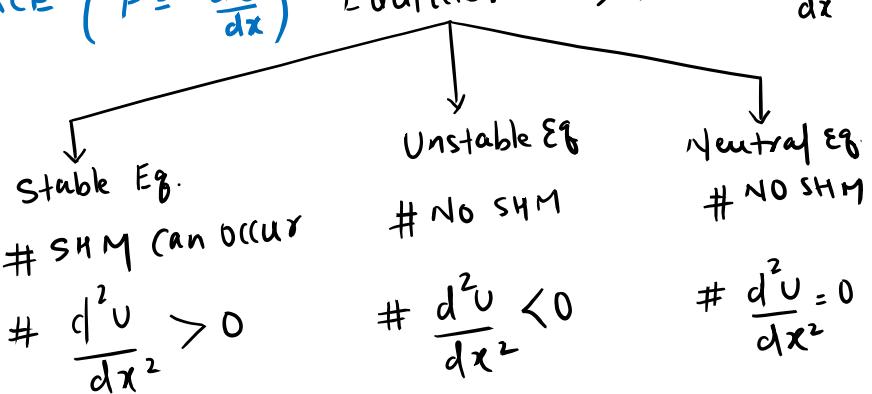
$$\Delta U = mgh$$

from $x_1 \rightarrow x_2$

$$\Delta U = \frac{1}{2}Kx_2^2 - \frac{1}{2}Kx_1^2$$



$$A \rightarrow B, \Delta U = -\frac{GMm}{r_2} - \left(-\frac{GMm}{r_1}\right)$$

7. CONSERVATIVE FORCE ($F = -\frac{dU}{dx}$) EQUILIBRIUM $\Rightarrow F=0$ OR $\frac{dU}{dx}=0$ 

8

POWER (unit: watt)

$$P_{av} = \frac{\Delta W}{\Delta t}$$

$$\begin{aligned} P_{inst} &= \frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} \\ &= \vec{F} \cdot \vec{V} \\ &= FV \cos \theta \end{aligned}$$

Space to add concepts learnt from PYQs if any

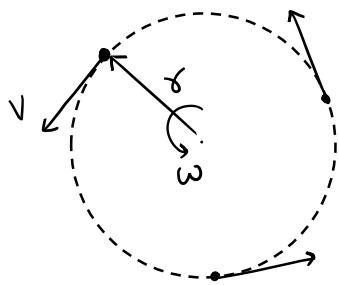
Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in Circular Motion

1. Kinematics of Circular Motion
2. Radius of Curvature
3. Relative Angular Velocity
4. Uniform vs Non-Uniform Circular Motion
5. Non-Uniform C.M. ($\alpha = \text{Const.}$)
6. Non-Uniform C.M. ($\alpha \neq \text{Const.}$)
7. Centripetal Force
8. Conical Pendulum
9. Centrifugal Force
10. Dynamics of Vertical C.M.
11. Vertical C.M. of a Pendulum Bob (string)
12. Vertical C.M of a Light Rod Pendulum
13. Vertical C.M. Inside a Circular Track
14. Vertical C.M. Outside a Circular Track
15. Vertical C.M. in a Circular Tube
16. Banked Roads

Note: For video refer Revision Series Playlist on EDUNI YouTube Channel

1 Kinematics of Circular Motion



(a) \rightarrow Angular displacement, $\Delta\theta$ (in rad)

\hookrightarrow Angular velocity, ω (rad/s)

$$\omega_{av} = \frac{\Delta\theta}{\Delta t} \quad \omega_{inst} = \frac{d\theta}{dt}$$

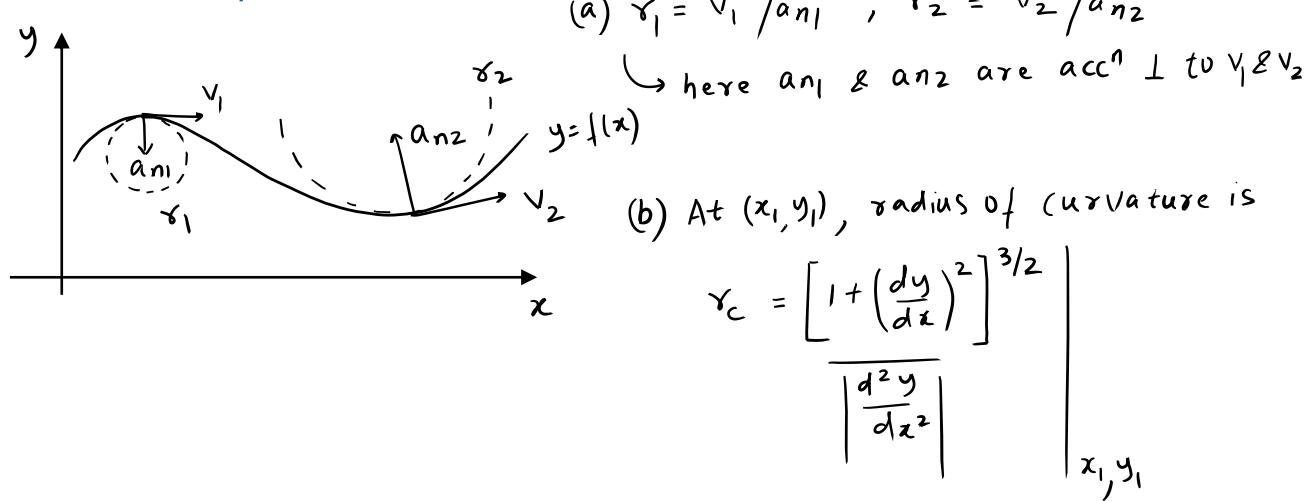
\hookrightarrow Angular accⁿ, α (rad/s²) = $\frac{d\omega}{dt} = \omega \frac{d\omega}{d\theta}$

(b) Relation between V and ω : $V = \omega r$

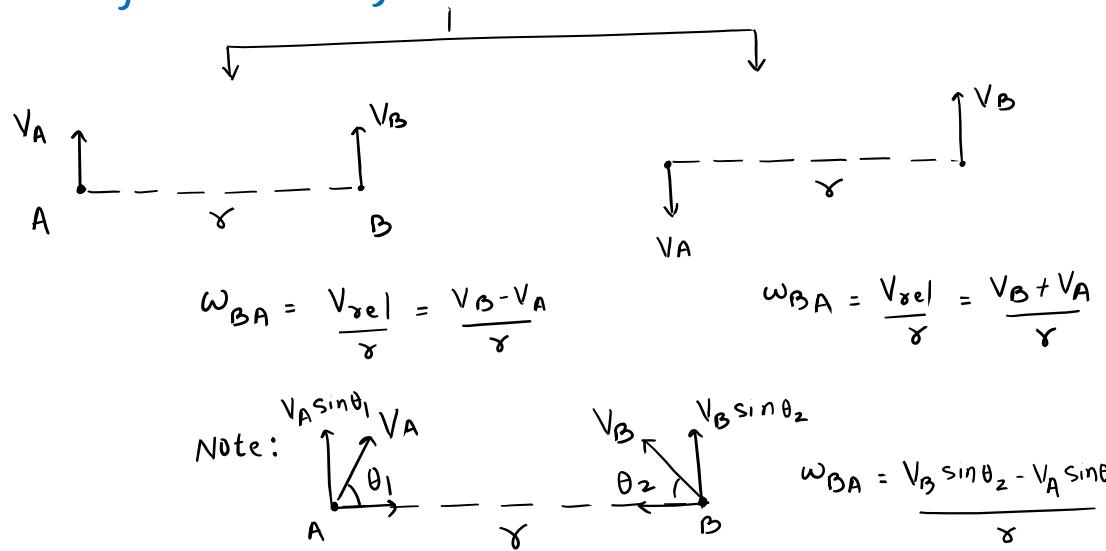
(c) $\frac{dV}{dt} = r \frac{d\omega}{dt} \Rightarrow a_t = r \alpha$ $\left\{ \begin{array}{l} a_t \text{ changes speed} \\ \text{of particle} \end{array} \right.$

(d) \because dirⁿ changing, $a_n = \frac{V^2}{r} = \omega^2 r$ $\left\{ \begin{array}{l} \text{normal or} \\ \text{centripetal acc}^n \end{array} \right.$

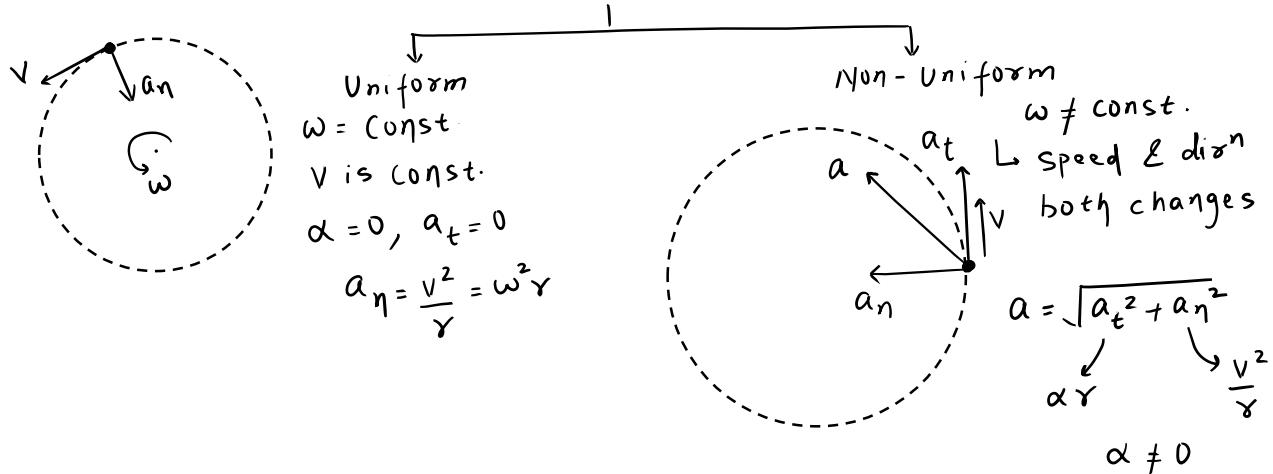
2. Radius of Curvature



3. Relative Angular Velocity



4. Uniform vs Non-uniform Circular Motion



5. Non-Uniform CM ($\alpha = \text{Const.}$)

$$\omega_f = \omega_i + \alpha t$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\theta$$

$$\theta = \omega_i t + \frac{1}{2} \alpha t^2$$

$$\Delta\theta\eta = \omega_i + \frac{\alpha}{2} (2n-1)$$

Note:

(i) If both ω_i and α are in same sense (CW or ACW), take α +ve.(ii) If α opp sense of ω_i , α -ve
(retardation, ex-fan switching off)

(iii) Be carefull about units

$$\omega \text{ in rpm} \rightarrow \omega \text{ in rad/s}$$

$$\text{Ex. } 50 \text{ rpm} \rightarrow \frac{50 \times 2\pi}{60} \text{ rad/s}$$

6. Non-Uniform CM ($\alpha \neq \text{Const.}$)

$$\omega = \frac{d\theta}{dt}, \alpha = \frac{d\omega}{dt} = \omega \frac{d\omega}{d\theta}$$

Ex. $\alpha = 2t$ and particle starts from rest. At $t = 3s$ find α ? ($\gamma = 2m$)Sol: At $t = 3s, \alpha = 2 \times 3 = 6 \text{ rad/s}^2$

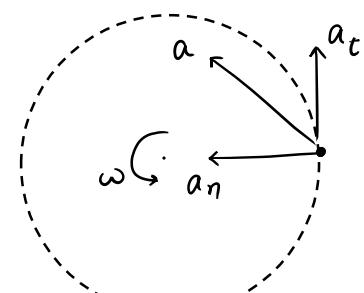
$$a_t = \alpha \gamma = 6 \times 2 = 12 \text{ m/s}^2$$

$$\alpha = 2t \Rightarrow \frac{d\omega}{dt} = 2t$$

$$\Rightarrow \int_0^3 d\omega = \int_0^3 2t dt \Rightarrow \omega = 9 \text{ rad/s}$$

$$a_\eta = \omega^2 \gamma = 81 \times 2 = 162 \text{ m/s}^2$$

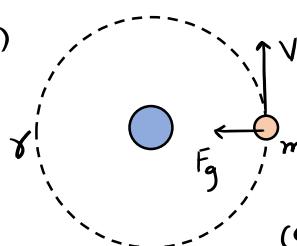
$$\therefore a = \sqrt{12^2 + 162^2} \approx 162.4 \text{ m/s}^2$$



7. Centripetal Force

Net force towards centre of circular path, $F_{\text{net}} = m a_\eta = \frac{m v^2}{r}$

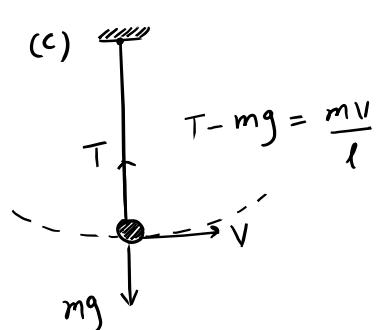
(a)



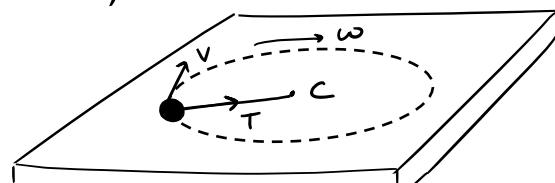
$$F_g = \frac{mv^2}{r}$$

$$\Rightarrow \frac{GMm}{r^2} = \frac{mv^2}{r}$$

(c)



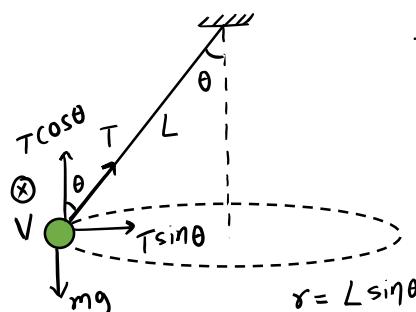
(b)



$$T - mg = \frac{mv^2}{l}$$

$$T = \frac{mv^2}{l} = m \omega^2 l$$

8. Conical Pendulum

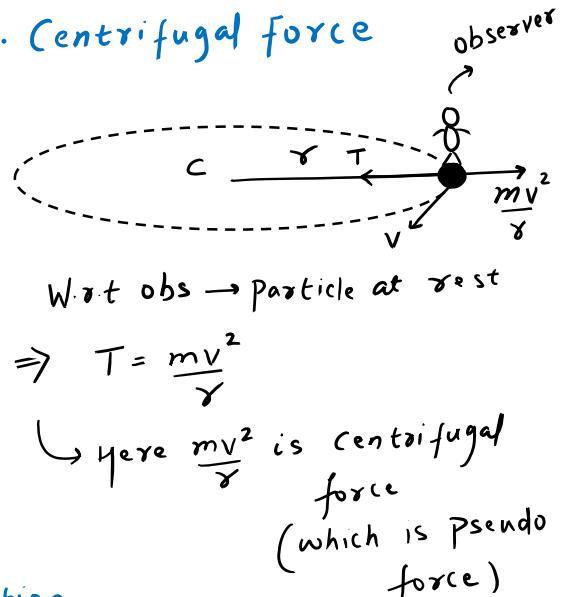


$$T \sin \theta = \frac{mv^2}{r}$$

$$T \cos \theta = mg$$

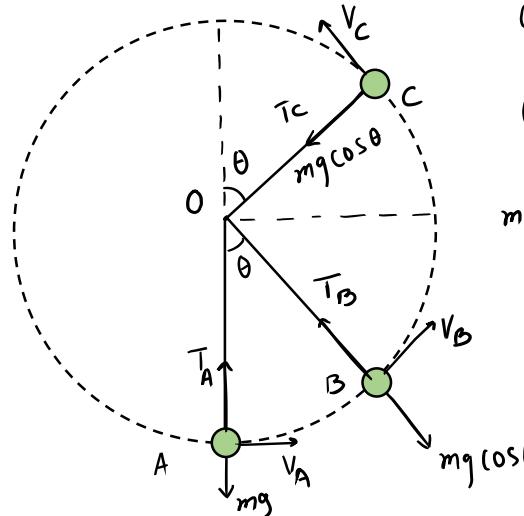
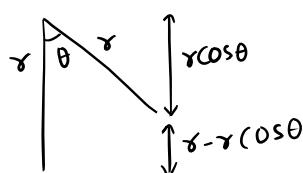
$$\Rightarrow \tan \theta = \frac{v^2}{rg}$$

9. Centrifugal force



10. Dynamics of Vertical Circular Motion

→ particle along circular path in vertical plane



$$(i) T_A - mg = \frac{mv_A^2}{r}$$

$$(ii) T_B - mg \cos \theta = \frac{mv_B^2}{r}$$

$$mg r (1 - \cos \theta) = \frac{1}{2} mv_A^2 - \frac{1}{2} mv_B^2$$

$$(iii) T_C + mg \cos \theta = \frac{mv_C^2}{r}$$

$$mg r (1 + \cos \theta) = \frac{1}{2} mv_A^2 - \frac{1}{2} mv_C^2$$

11. Vertical C.M. of a Pendulum Bob

$$(a) u \leq \sqrt{2gl}$$

$$\text{Gain in P.E.} = \text{Loss in K.E.}$$

$$\Rightarrow mg l (1 - \cos \theta) = \frac{1}{2} mv^2$$

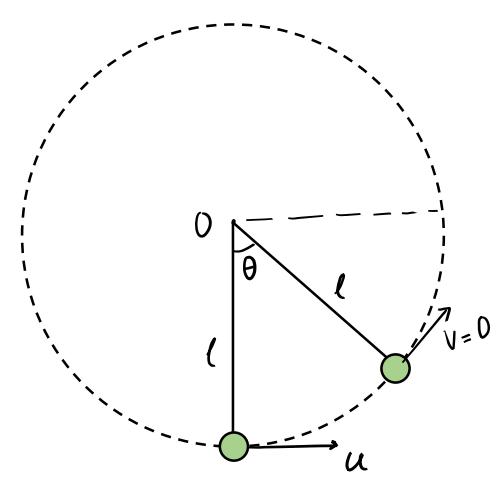
$$\therefore \cos \theta = \frac{2gl - u^2}{2gl} \quad \rightarrow (i) u = \sqrt{2gl}$$

$$\Rightarrow \theta = 90^\circ$$

$$(ii) \text{ use this if } u \leq \sqrt{2gl}$$

$$\text{ex. } u = \sqrt{gl}$$

$$\Rightarrow \theta = 60^\circ$$

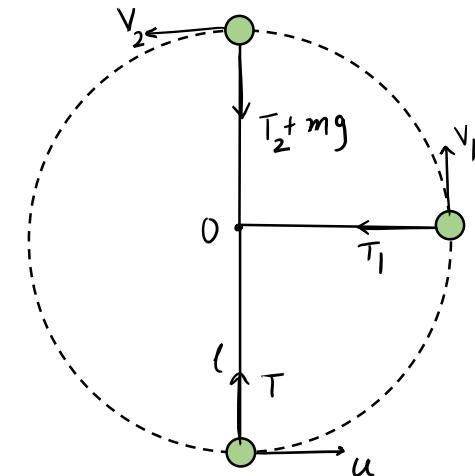


(b) $u > \sqrt{5gl}$

(i) bob does complete circle

(ii) If $u = \sqrt{5gl} \Rightarrow T_2 = 0$ but $v_2 \neq 0$ (iii) $T - T_1 = 3mg \quad \&$

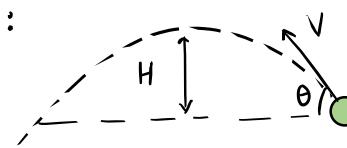
$$T - T_2 = 6mg$$

(c) $\sqrt{2gl} < u < \sqrt{5gl}$

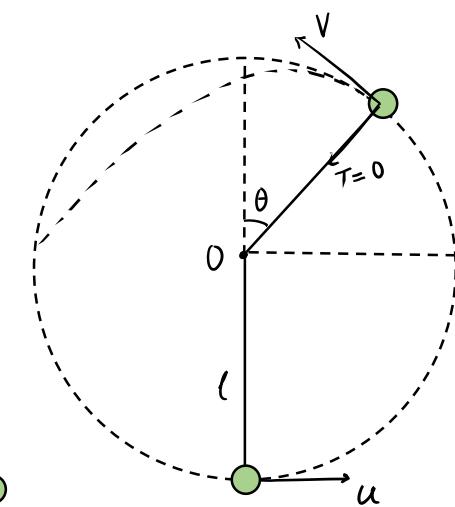
↳ rises above horizontal and somewhere $T=0$ (string slacks) and bob does projectile motion

$$(i) \cos\theta = \frac{u^2 - 2gl}{3gl} \quad \& \text{ from (conserv Energy)} \\ v = \sqrt{u^2 - 2gl(1 + \cos\theta)}$$

NOTE :



$$H = \frac{v^2 \sin^2 \theta}{2g}$$



12 Vertical CM. of a Light Rod Pendulum

↳ Here there is no issue of $T=0$ as rod can't slack (massless rod)

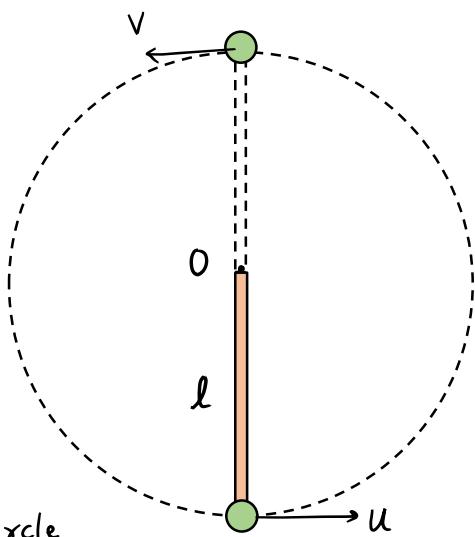
↳ for $u > \sqrt{4gl} \rightarrow$ complete circle

NOTE :

$$(2) \text{ for } u = \sqrt{4gl}$$

$$v = 0$$

↳ Due to inertia
it completes circle



13. Vertical CM. Inside a Circular Track

↪ All cases are same as pendulum Bob with string.
(instead of T its N here)

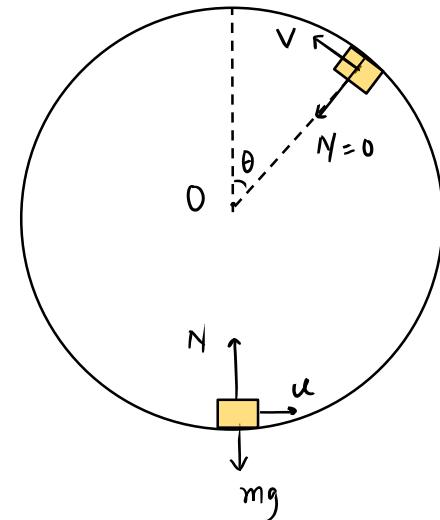
(i) $u > \sqrt{5g\gamma} \rightarrow$ complete circle

(ii) $\sqrt{2g\gamma} < u < \sqrt{5g\gamma} \rightarrow$ cross horizontal and at some θ with vertical $N=0$

$$\cos\theta = \frac{u^2 - 2g\gamma}{3g\gamma} \text{ and}$$

$$V = \sqrt{u^2 - 2g\gamma(1 + \cos\theta)}$$

↪ use this θ & V for projectile motion



14. Vertical CM. Outside a Circular Track

$A \rightarrow B : \text{Loss in PE} = \text{Gain in KE}$

$$\Rightarrow mg\gamma(1 - \cos\theta) = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$\# V = \sqrt{u^2 + 2g\gamma(1 - \cos\theta)}$$

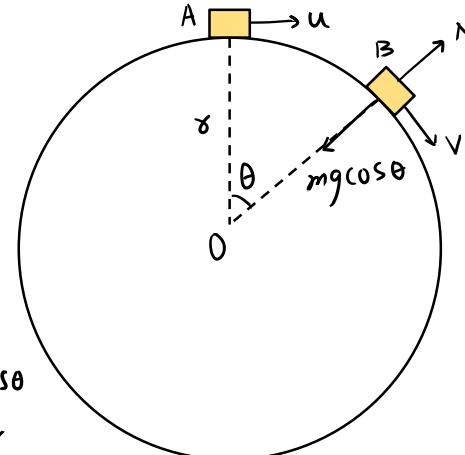
$$\text{and, } mg\cos\theta - N = \frac{mv^2}{r}$$

→ contact loose : ($N=0$)

$$v^2 = rg\cos\theta$$

$$\Rightarrow u^2 + 2g\gamma(1 - \cos\theta) = rg\cos\theta$$

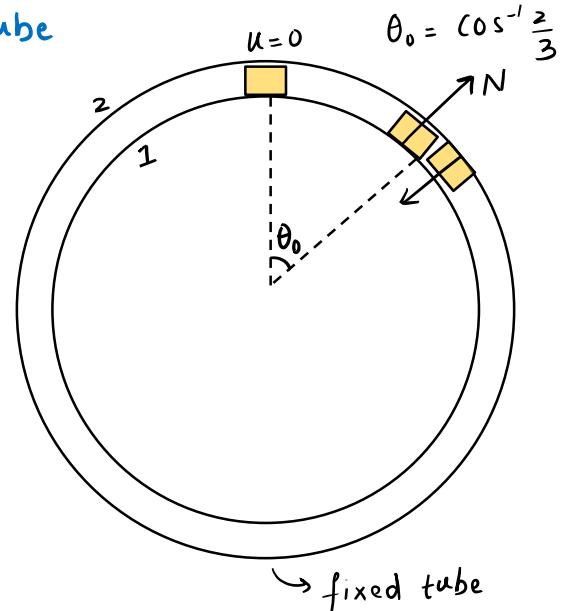
$$\# \cos\theta = \frac{u^2 + 2g\gamma}{3g\gamma}$$



15. Vertical CM. Inside a Circular Tube

↪ till $\theta < \cos^{-1} \frac{2}{3}$, block on 1
(N on block radially outwards)

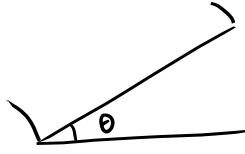
↪ for $\theta > \cos^{-1} \frac{2}{3}$, block on 2
(N on block radially inwards)



16. Banked and Horizontal Road

1. For no slipping

$$\sqrt{Rg \left(\frac{\tan \theta - \mu}{1 + \mu \tan \theta} \right)} \leq v \leq \sqrt{Rg \left(\frac{\tan \theta + \mu}{1 - \mu \tan \theta} \right)}$$



2. If $v = \sqrt{Rg \tan \theta}$, $f = 0$ (car can turn on slippery road + no wear & tear of tyres)

3. If $v > \sqrt{Rg \tan \theta}$, f acts down the slope
& $v < \sqrt{Rg \tan \theta}$, f acts up the slope

4. For horizontal circular turn,
 $v \leq \sqrt{\mu R g}$

Space to add concepts learnt from PYQs if any

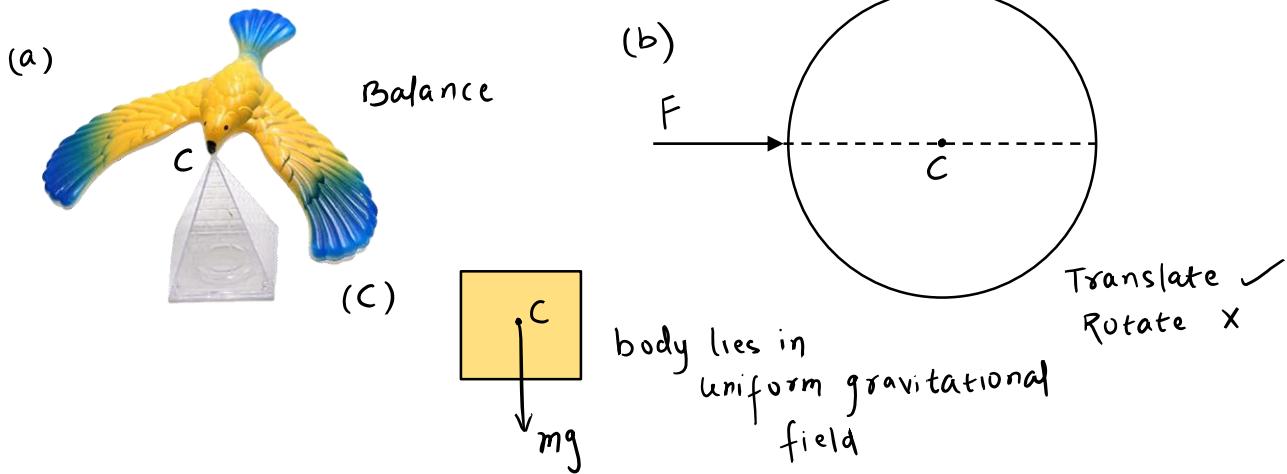
Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in COM – PART 1

1. Understanding of COM
2. COM of Multiparticle System
3. COM of Two Particle System
4. COM of Continuous Mass Distribution
5. COM of Standard Bodies (*uniform mass density*)
6. COM of Bodies (*Non-uniform mass density*)
7. COM of Combined Structure
8. COM in Cavity Problems (2 Methods)
9. Displacement of COM
10. Velocity & Acceleration of COM

Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. Understanding of COM



2 COM of Multiparticle System

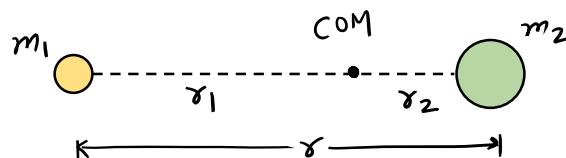
$$\vec{r}_C = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3}{m_1 + m_2 + m_3}$$

$$x_C = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$y_C = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

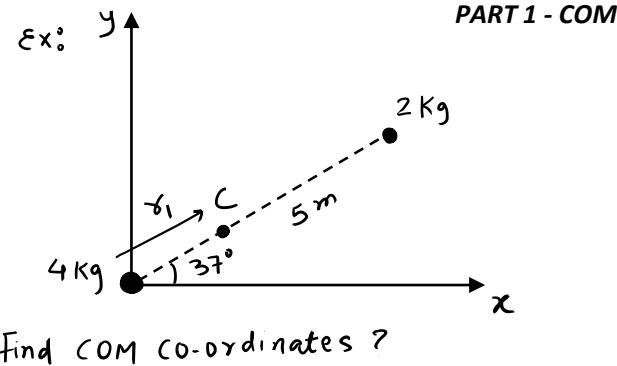
$$z_C = \frac{m_1 z_1 + m_2 z_2 + m_3 z_3}{m_1 + m_2 + m_3}$$

3. COM of 2-Particle System



$$(i) r_1 = \frac{m_2 r}{m_1 + m_2}, r_2 = \frac{m_1 r}{m_1 + m_2}$$

$$(ii) m_1 r_1 = m_2 r_2$$



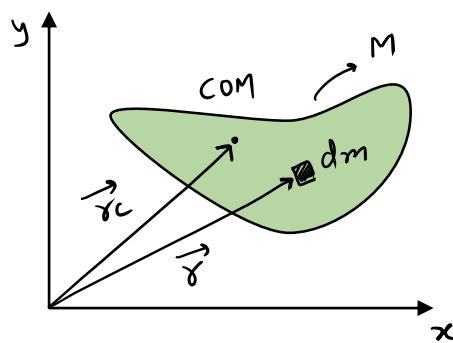
$$\text{Sol: } r_1 = \frac{2 \times 5}{2+4} = \frac{10}{6} = \frac{5}{3} \text{ m}$$

$$\therefore \frac{5}{3} \cos 37^\circ, \frac{5}{3} \sin 37^\circ$$

or

$$\boxed{\frac{4}{3}, 1}$$

4. COM of a continuous Mass Distribution

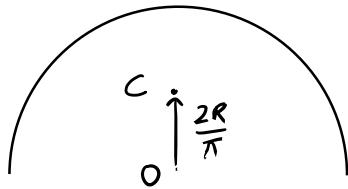


$$\vec{r}_c = \frac{\int \vec{r} dm}{\int dm} = \frac{1}{M} \int \vec{r} dm$$

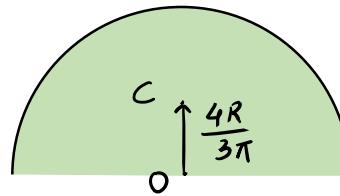
$$\left\{ \begin{array}{l} x_c = \frac{\int x dm}{\int dm} \\ y_c = \frac{\int y dm}{\int dm} \\ z_c = \frac{\int z dm}{\int dm} \end{array} \right.$$

5. COM of Standard bodies (uniform mass distribution)

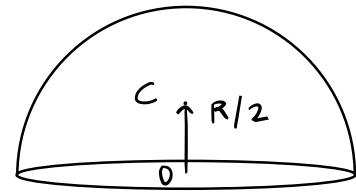
1. Semicircular Ring



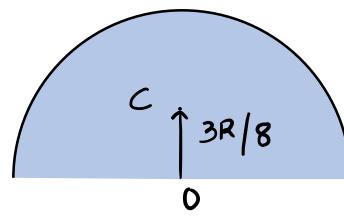
2. Semicircular Disc



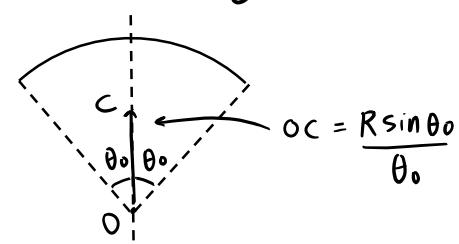
3. Hollow Hemisphere



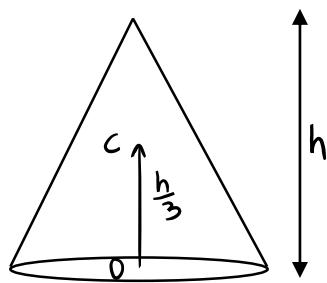
4. SOLID Hemisphere



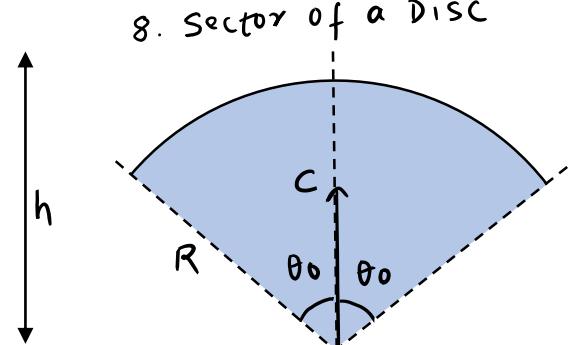
5. Arc of Ring



6. Hollow Cone



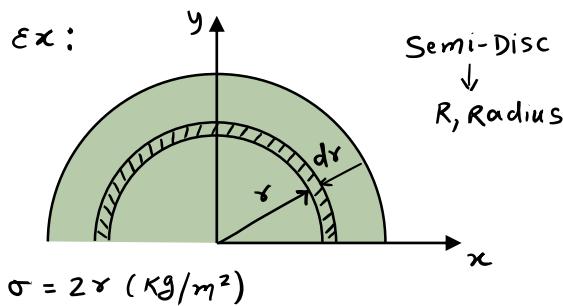
7. Solid Cone



8. Sector of a Disc

$$OC = \frac{2R}{3} \frac{\sin \theta_0}{\theta_0}$$

6. COM of body (non-uniform mass distribution)



Soln. COM lies on y axis

$$y_{cm} = \frac{\int y dm}{\int dm}$$

$$dm = \sigma(r) \times dA$$

$$= 2r \times \pi r dr$$

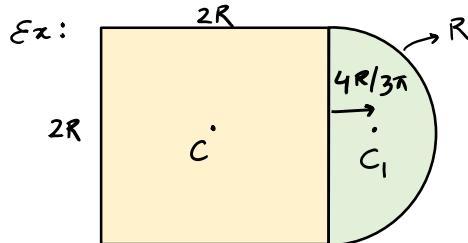
$$= 2\pi r^2 dr$$

$$\Rightarrow y_{cm} = \frac{\int \frac{2r}{\pi} \times 2\pi r^2 dr}{\int 2\pi r^2 dr}$$

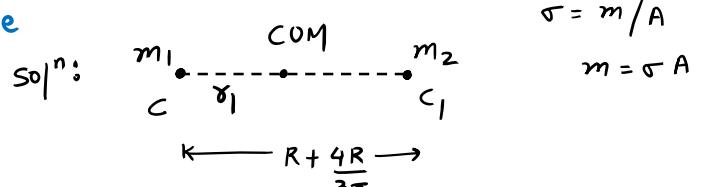
$$y = \frac{2r}{\pi} \left(\frac{\text{com of } dm}{dm} \right)$$

$$= \frac{4r^4/4}{2\pi \frac{r^3}{3}} \Big|_0^R = \boxed{\frac{3R}{2\pi}}$$

7. COM of Combined Structure



Both are same material. Find COM from C.

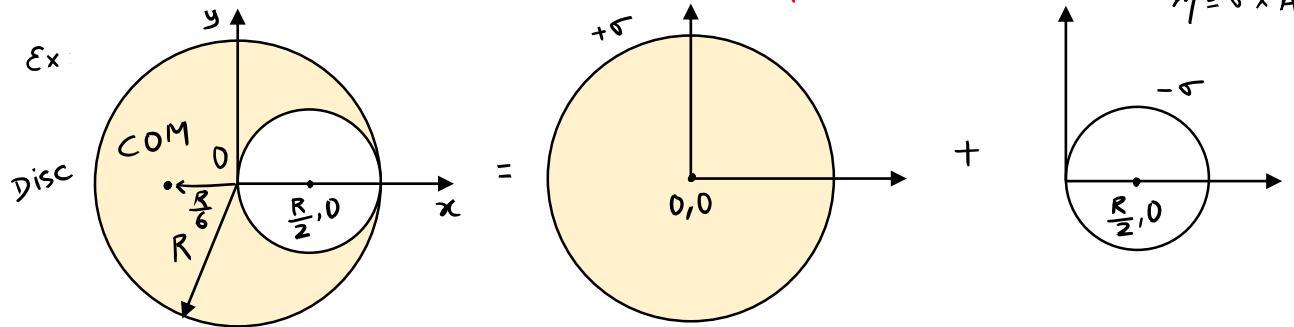


$$\gamma_1 = \frac{m_2 \left(R + \frac{4R}{3\pi} \right)}{m_1 + m_2} = \frac{\sigma \pi R^2 \left(R + \frac{4R}{3\pi} \right)}{\sigma 4R^2 + \sigma \frac{\pi R^2}{2}}$$

$$= \frac{R(3\pi + 4)}{3(8 + \pi)} \text{ Ans.}$$

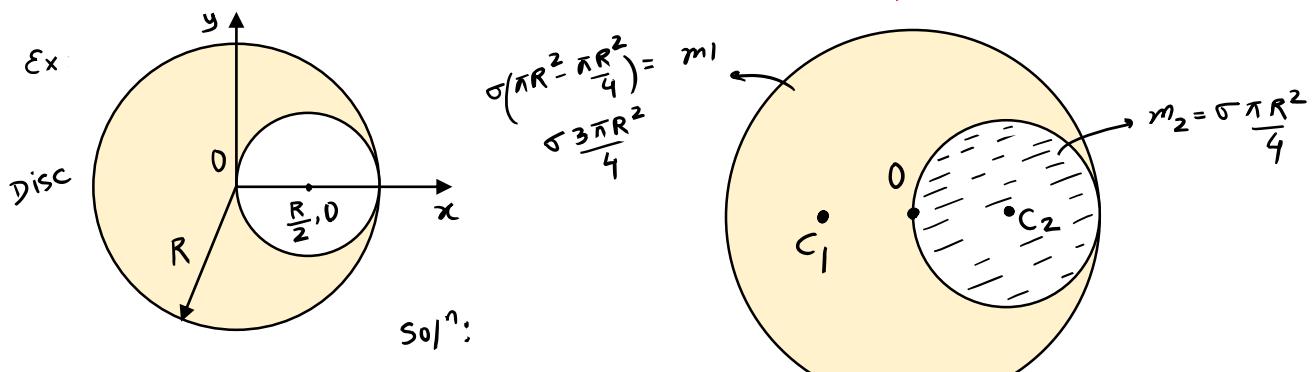
8. COM in Cavity Problems

PART 1 - COM



$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{(\sigma \pi R^2 \times 0) + (-\sigma \pi \frac{R^2}{4} \times \frac{R}{2})}{\sigma \pi R^2 - \sigma \pi \frac{R^2}{4}} = -\frac{R}{6}$$

2nd Method



Sol:

$$m_1 = \sigma (\pi R^2 - \pi \frac{R^2}{4}) = \sigma \frac{3\pi R^2}{4}$$

$$m_2 = \sigma \pi \frac{R^2}{4}$$

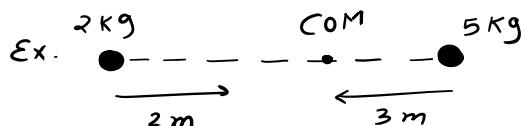
$$m_1 x = m_2 \frac{R}{2} \Rightarrow \frac{\sigma 3\pi R^2}{4} x = \frac{\sigma \pi R^2}{4} \times \frac{R}{2}$$

$$\Rightarrow x = \frac{R}{6}$$

9. Displacement of COM

$$\vec{r}_c = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots}{m_1 + m_2 + \dots}$$

$$\Rightarrow \Delta \vec{r}_c = \frac{m_1 \Delta \vec{r}_1 + m_2 \Delta \vec{r}_2 + \dots}{m_1 + m_2 + \dots}$$



Find displacement of COM

$$Sol: \Delta x_{cm} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2}$$

$$= \frac{(2 \times 2) + (5 \times -3)}{2 + 5} = 4 \frac{-15}{7} = \boxed{-\frac{11}{7} m}$$

10. Velocity & Acceleration of COM
 v_{cm} a_{cm}

PART 1 - COM

$$\vec{r}_c = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots}{m_1 + m_2 + \dots} \xrightarrow{d\vec{r}_c/dt} \vec{v}_{cm} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots}{m_1 + m_2 + \dots} \Rightarrow M \vec{v}_{cm} = \vec{P}_{net}$$

$$\downarrow \frac{d\vec{v}_{cm}/dt}{M}$$

$$\vec{a}_{cm} = \frac{m_1 \vec{a}_1 + m_2 \vec{a}_2 + \dots}{m_1 + m_2 + \dots} \Rightarrow M \vec{a}_{cm} = \vec{F}_{net}$$

NOTE:

$$(i) \vec{F}_{net} = \frac{d\vec{P}_{net}}{dt} \quad (ii) \vec{F}_{net} = 0 \Rightarrow \vec{P}_{net} = \text{const.} \\ (a_{cm} = 0) \quad (\vec{v}_{cm} = \text{const.})$$

Space to add concepts learnt from PYQs if any

Space to add concepts learnt from PYQs if any

Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in Cons of P & Collision – PART 2

1. Understanding of Momentum Conservation
2. COM at rest question
3. Boy-Trolley Situation
4. Gun-Bullet Problem (*muzzle velocity*)
5. Explosion Concept
6. Explosion of Projectile
7. Bullet hitting Block / Bob
8. Block moving over another block
9. Two Block Spring System
10. Understanding of Head-on vs Oblique Collision
11. Concept of Impulse
12. Coefficient of Restitution
13. Collision with Floor
14. Head-on Collision
15. Oblique Collision
16. Mass-variation Concept

Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. Understanding of Momentum Conservation

$$\vec{F}_{\text{net}} = M \vec{a}_{\text{cm}} \quad \& \quad \vec{P}_{\text{net}} = M \vec{v}_{\text{cm}}$$

$$\vec{F}_{\text{net}} = \frac{d\vec{P}_{\text{net}}}{dt} \quad \left\{ \begin{array}{l} M: \text{System} \\ \text{mass} \end{array} \right\}$$

If $\vec{F}_{\text{net}} = 0$

\downarrow
 $\vec{P}_{\text{net}} = \text{const.}$

\downarrow
 $\vec{v}_{\text{cm}} = \text{const.}$

\downarrow
If $v_{\text{cm}} = 0$, it
remain zero
 \Rightarrow CM at rest
although particles in
system may
move

\downarrow
If $v_{\text{cm}} \neq 0$
 \Rightarrow CM continues
to move with
const Velocity.

PART 2 – Cons of P & Collision

2. COM at rest ($\vec{F}_{\text{net}} = 0, V_{\text{cm}} = 0$)

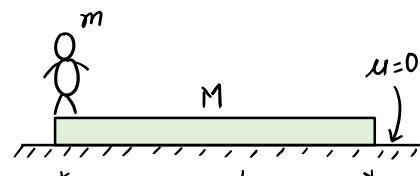
$$\vec{v}_{\text{cm}} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots}{m_1 + m_2 + \dots}$$

$$\Rightarrow 0 = m_1 \Delta \vec{v}_1 + m_2 \Delta \vec{v}_2 + \dots$$

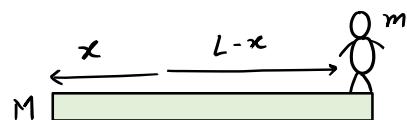
$$\{ \Delta \vec{v}_{\text{cm}} = 0 \}$$

here $\Delta \vec{v}_1, \Delta \vec{v}_2 \dots$ are w.r.t. ground

Ex 1. If boy walks to other end of plank, find distance moved by Plank.



Solⁿ:

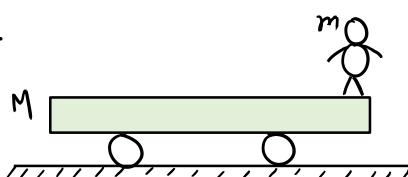


$$m_1 \Delta x_1 + m_2 \Delta x_2 = 0 \Rightarrow m(L-x) + M(-x) = 0$$

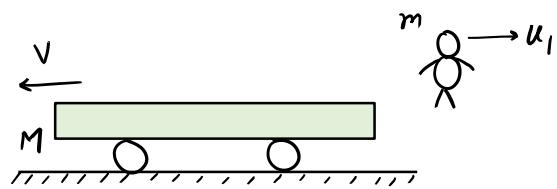
$$\therefore x = \frac{mL}{m+M} \text{ Ans}$$

3. Boy-Trolley Situation ($\vec{F}_{\text{net}} = 0, \vec{P}_{\text{net}} = \text{Const.}$)

Ex 2.



Solⁿ: NOTE - u is rel to trolley after jump.



$$V_b/T = V_b - V_T \Rightarrow u = u_1 + V \Rightarrow u_1 = u - V$$

$$\therefore \vec{P}_i = 0 \Rightarrow \vec{P}_f = 0$$

$$\Rightarrow mu_1 = MV$$

$$\Rightarrow m(u-V) = MV \Rightarrow V = \frac{mu}{m+M} \text{ Ans}$$

4. Gun-bullet (muzzle velocity)



Muzzle velocity is u, find recoil speed of gun

Solⁿ: Note - Muzzle vel. is vel of bullet w.r.t muzzle (end of Gun)



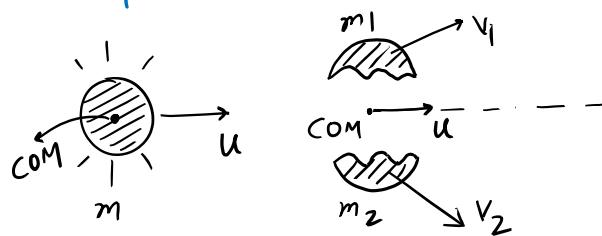
$$V_b/g = V_b - V_g$$

$$\Rightarrow u = u_1 + V \Rightarrow u_1 = u - V$$

$$\therefore \vec{P}_i = 0 \Rightarrow \vec{P}_f = 0 \Rightarrow MV = mu(u-V)$$

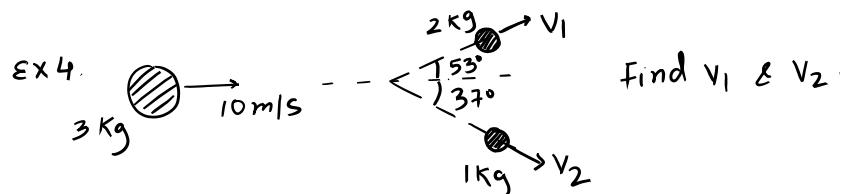
$$\therefore V = \frac{mu}{m+M} \text{ Ans}$$

5. Explosion Concept



(i) Explosion due to internal forces,
 $\sum \vec{F}_{\text{ext}} = 0 \Rightarrow \vec{P}_{\text{sys}} = \text{const}$
 $\Rightarrow \vec{V}_{\text{cm}} = \text{const}$

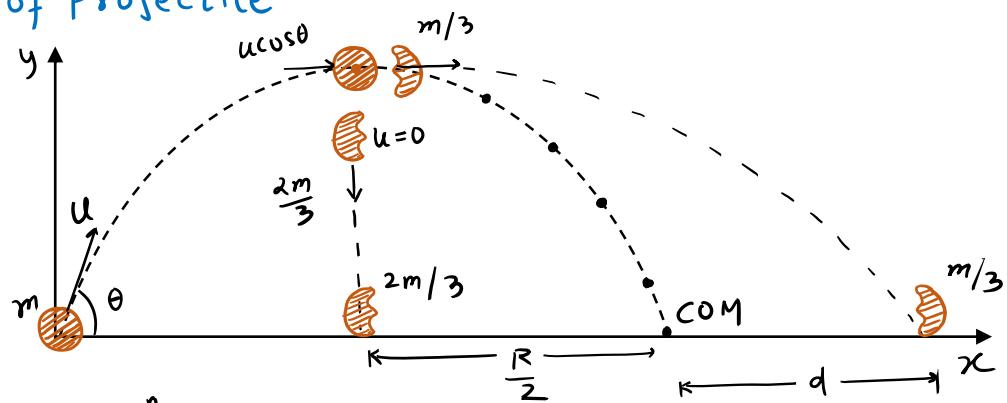
$$(ii) m\vec{u} = m_1\vec{v}_1 + m_2\vec{v}_2$$



Solⁿ: $\vec{P}_x = \text{const} \Rightarrow 30 = 2v_1 \cos 53^\circ + v_2 \cos 37^\circ \quad (i)$
 $\vec{P}_y = \text{const} \Rightarrow 0 = 2v_1 \sin 53^\circ - v_2 \sin 37^\circ \quad (ii)$
 $\therefore v_1 = 9 \text{ m/s}, v_2 = 24 \text{ m/s}$

6. Explosion of Projectile

Ex 5. Find distance of $\frac{m}{3}$ from origin.



Solⁿ: $\frac{2m}{3} \times \frac{R}{2} = \frac{m}{3} \times d$
 $\Rightarrow d = R \quad \therefore \text{Answer} = d + R = 2R = 2 \times \frac{u^2 \sin 2\theta}{g}$

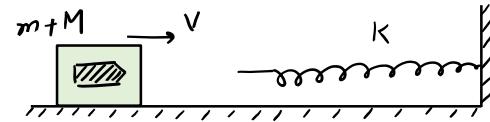
7. Bullet hitting Block/Bob

↳ many questions are formed

$$\vec{P} = \text{const}$$

$$KE + PE = \text{const}$$

Solⁿ:



$$mu = (m+M)v \Rightarrow v = \frac{mu}{m+M}$$

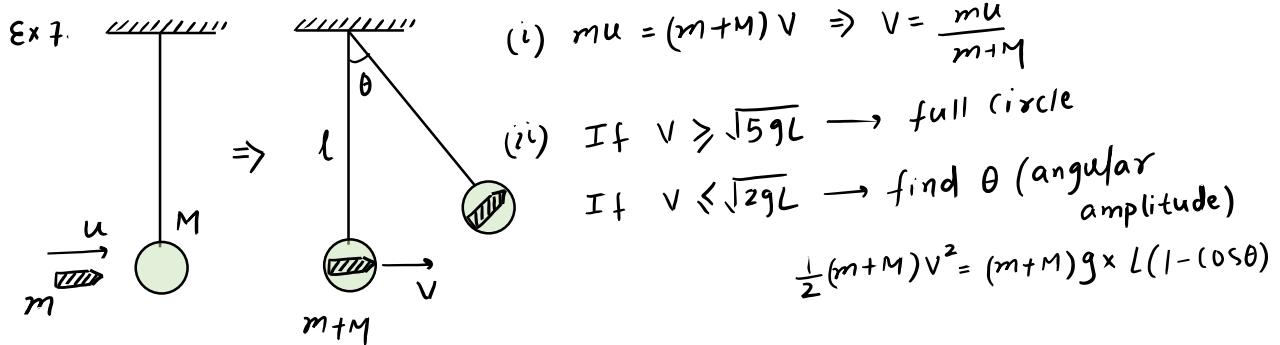
Loss in KE = Gain in PE

$$\Rightarrow \frac{1}{2}(m+M) \left(\frac{mu}{m+M} \right)^2 = \frac{1}{2} K x^2$$

$$\therefore x = \frac{mu}{\sqrt{K(m+M)}}$$

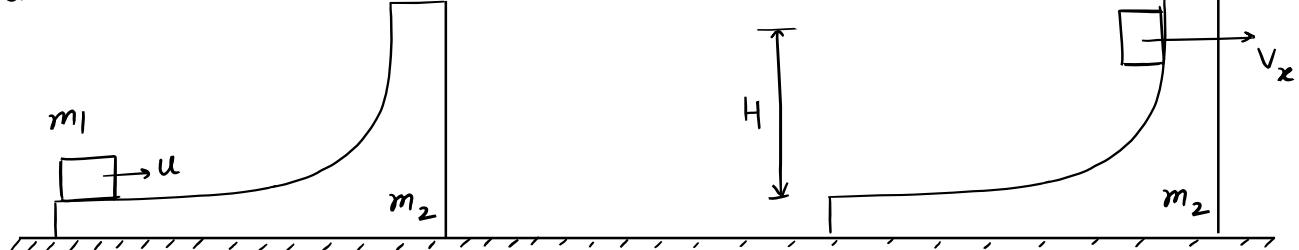


bullet gets embedded in block.
 Find max compression in spring.



8. Block moving over another block

Ex 8.

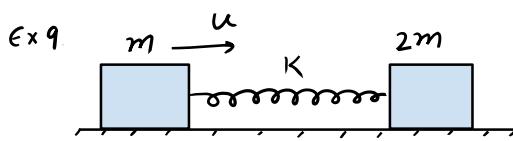


no friction
anywhere
Let us analyse.

$$(i) \vec{P}_x = \text{const} \Rightarrow m_1 u = (m_1 + m_2) v_x \\ \Rightarrow v_x = \frac{m_1 u}{m_1 + m_2}$$

$$(ii) \text{Gain in PE} = \text{Loss in KE} \\ \Rightarrow m_1 gh = \frac{1}{2} m_1 u^2 - \frac{1}{2} (m_1 + m_2) v_x^2$$

9. TWO BLOCK - SPRING SYSTEM



Spring in natural length

Find max compression in spring.

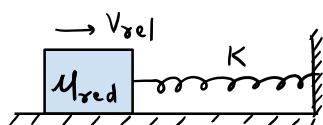
1st Method
Soln: At max comp. both block has same speed.



$$(i) \vec{P} = \text{const} \Rightarrow mu = mv + 2mv \\ \Rightarrow v = u/3$$

$$(ii) \text{TE} = \text{const} \Rightarrow \frac{1}{2} mu^2 = \frac{1}{2} mv^2 + \frac{1}{2} 2mv^2 + \frac{1}{2} Kx^2 \Rightarrow x = u \sqrt{\frac{2m}{3K}}$$

2nd Method



$$M_{\text{red}} = \frac{m_1 m_2}{m_1 + m_2}$$

$$\text{Loss in KE} = \text{Gain in PE} \Rightarrow \frac{1}{2} M_{\text{red}} v_{\text{rel}}^2 = \frac{1}{2} Kx^2$$

$$\Rightarrow \frac{1}{2} \left(\frac{m_1 m_2}{m_1 + m_2} \right) x^2 = \frac{1}{2} Kx^2 \therefore x = u \sqrt{\frac{2m}{3K}}$$

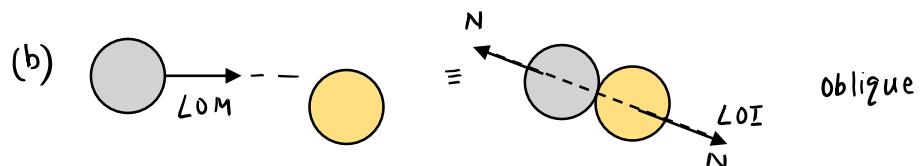
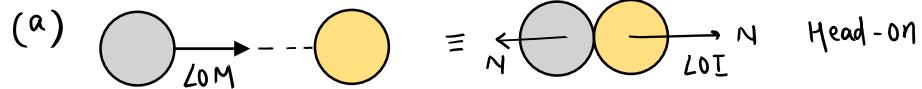
10. Understanding of Head-on vs Oblique Collision

LOM → Line of Motion

↓
LOM & LOI are
same

↓
LOM & LOI are different

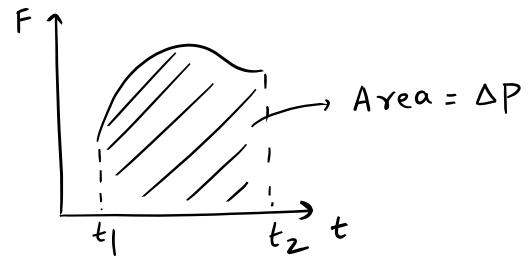
LOI → Line of Impact



11. Concept of Impulse (J) \rightarrow change in P due to F ($t_1 \rightarrow t_2$)

$$F = \frac{dp}{dt} \Rightarrow dp = F dt$$

↓ ↓
 $F = \text{const.}$ $F = f(t)$
 $\Delta P = F \Delta t$ $\Delta P = \int_{t_1}^{t_2} F dt$



(i) Impulsive force \rightarrow large force acting for short time
(they bring ΔP)

NOTE: During collision / impact mg is taken as non-impulsive (no ΔP due to mg)

12. Coefficient of Restitution (e)

$$c = \frac{\text{Vel of separation of Pt. of Contact}}{\text{Vel of approach of Pt of Contact}} \quad \{ \text{Vel along Line of impact} \}$$

```

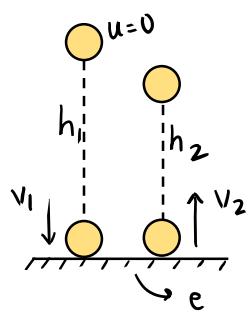
graph TD
    A[Collision on basis of KE loss] --> B[Elastic  
e=1]
    A --> C[Partial elastic  
0 < e < 1]
    A --> D[Inelastic  
e=0]
  
```

$$(a) \quad \text{Initial velocity} = 4 \text{ m/s} \quad \text{Final velocity} = 2 \text{ m/s} \Rightarrow \text{Final velocity} = 3 \text{ m/s} \quad \text{Final velocity} = 4 \text{ m/s} \quad e = \frac{4-3}{4-2} = \frac{1}{2}$$

$$(b) \quad \text{Initial state: } \text{Left circle } 4 \text{ m/s right, } \text{Right circle } 1 \text{ m/s left.} \quad \Rightarrow \quad \text{Final state: } \text{Left circle } 3 \text{ m/s right, } \text{Right circle } 5 \text{ m/s right.} \quad e = \frac{5-3}{4+1} = \frac{2}{5}$$

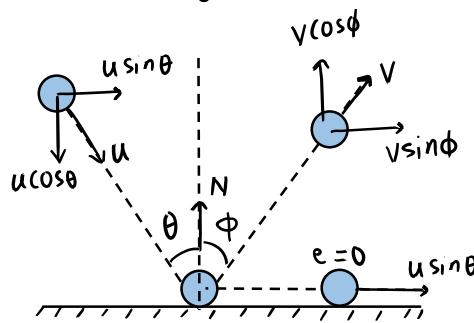
13. Collision with Floor

PART 2 – Cons of P & Collision



$$(i) e = \frac{v_2}{v_1} \Rightarrow v_2 = e v_1 \quad \left\{ v_1 = \sqrt{2gh_1} \right\}$$

$$(ii) h_2 = \frac{v_2^2}{2g} = \frac{e^2 v_1^2}{2g} = e^2 h_1$$



$$v \sin \phi = u \sin \theta \quad (1)$$

(as no Normal is \perp to it)

$$(i) \text{ If } e=1 \Rightarrow v=u \ \& \ \phi=\theta$$

$$(ii) \text{ If } 0 < e < 1 \Rightarrow v \cos \phi = e u \cos \theta \quad -(2)$$

$$(1)/(2) \Rightarrow \tan \phi = \frac{\tan \theta}{e}$$

$$(1)^2 + (2)^2 \Rightarrow v = u \sqrt{\sin^2 \theta + e^2 \cos^2 \theta}$$

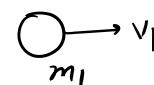
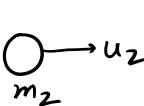
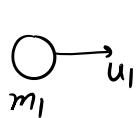
$$(iii) \text{ If } e=0 \Rightarrow v \cos \phi = 0$$

Ball moves horizontally

$$(iv) \text{ Impulse, } \Delta P = m(v \cos \phi + u \cos \theta)$$

$$\left\{ J = 2mu \cos \theta, e=1 \right\}$$

14. Head-on Collision



$$(i) P_i = P_f \Rightarrow m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 \quad \left\{ \begin{array}{l} \text{using these} \\ \text{two eqn} \end{array} \right.$$

$$(ii) e = \frac{v_2 - v_1}{u_1 - u_2}$$

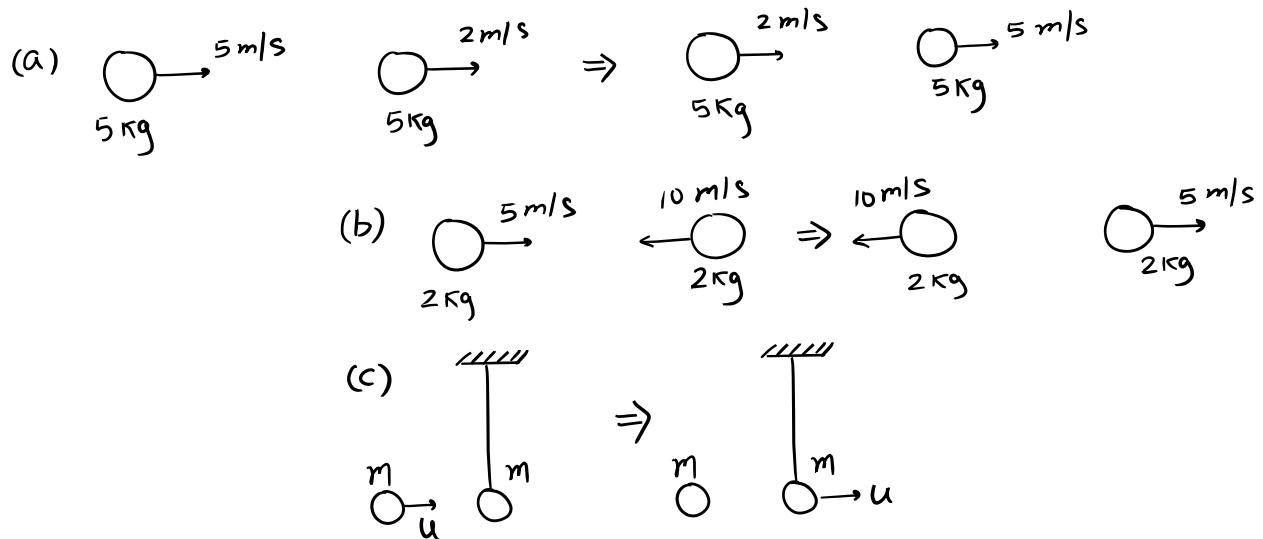
$\left\{ \begin{array}{l} \text{find } v_1 \& v_2 \end{array} \right.$

$$\begin{array}{c} | \\ e=1 \\ \downarrow \\ KE = \text{const} \\ K_i = K_f \end{array}$$

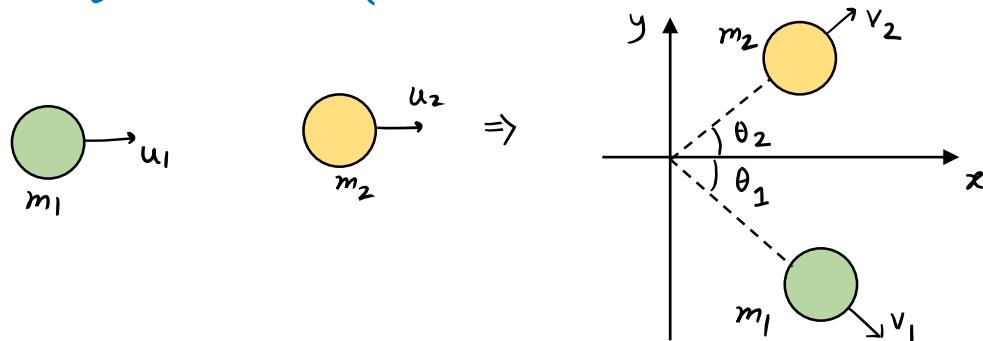
$$\begin{array}{c} | \\ 0 < e < 1 \\ \downarrow \\ KE \text{ loss} \\ K_i - K_f \end{array}$$

$$\begin{array}{c} | \\ e=0 \\ \downarrow \\ KE \text{ loss Max} \\ K_i - K_f \end{array}$$

Note: If $m_1 = m_2$ & $e=1$, then after collision bodies exchange velocities



15. Oblique Collision (elastic, $e=1$) → 2D Collision



$$(i) \vec{P}_x = \text{const} \Rightarrow m_1 u_1 + m_2 u_2 = m_1 v_1 \cos \theta_1 + m_2 v_2 \cos \theta_2$$

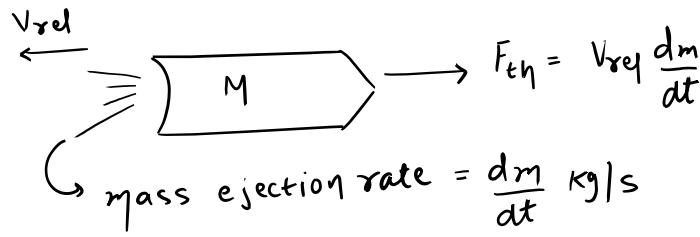
$$(ii) \vec{P}_y = \text{const} \Rightarrow 0 = m_2 v_2 \sin \theta_2 - m_1 v_1 \sin \theta_1$$

$$(iii) \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

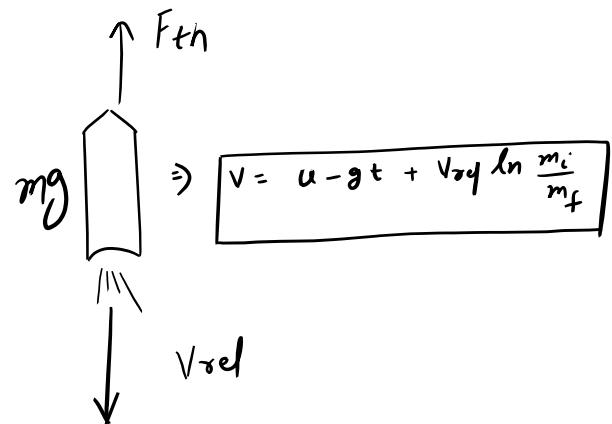
Note: If $m_1 = m_2$, $u_2 = 0$, elastic collision

$$\Rightarrow \boxed{\theta_1 + \theta_2 = 90^\circ}$$

16 Mass - Variation



Rocket Propulsion Equation

Space to add concepts learnt from PYQs if any

Space to add concepts learnt from PYQs if any

Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in ROTATIONAL MOTION – PART 1

1. Translatory vs Rotatory Motion
2. Moment of Inertia
3. Parallel & Perpendicular Axis Theorem
4. MOI of Standard Bodies (*uniform mass distribution*)
5. Addition or Subtraction of MOI
6. Radius of Gyration
7. Torque

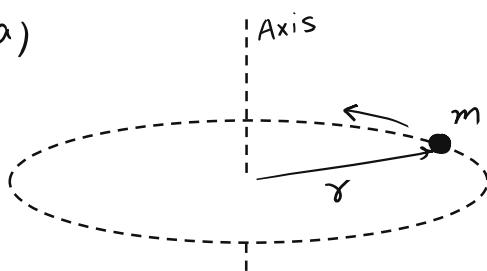
Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. Translatory vs Rotatory Motion

Refer animation shown in our revision video (Rotation-Part 1)

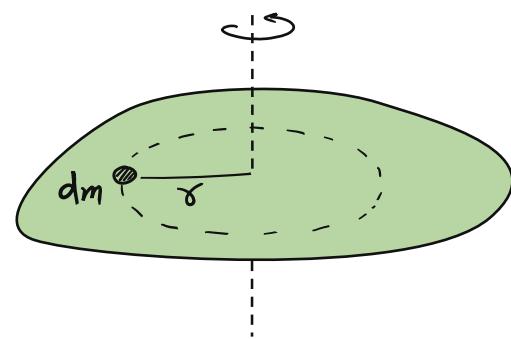
2 Moment of Inertia (I)

(a)



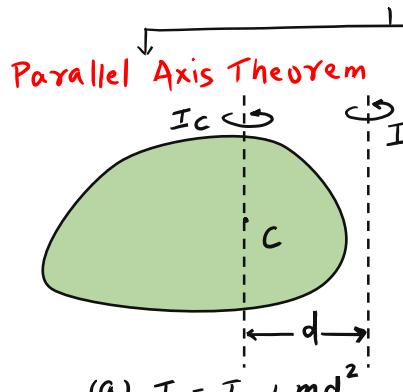
$$I = m r^2$$

(b)



$$dI = dm \cdot r^2 \Rightarrow I = \int dm \cdot r^2$$

3. PARALLEL AND PERPENDICULAR AXIS THEOREM

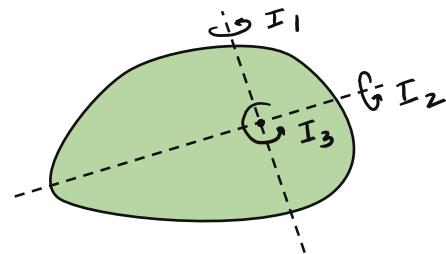


$$(a) I = I_c + md^2$$

(b) Applicable always

(c) I_c is MOI about an axis passing C.O.M.

Perpendicular Axis Theorem



$$(a) I_3 = I_1 + I_2$$

(b) Applicable for plane-Lamina

(c) I_1 and I_2 are about axes which are in the plane of body.

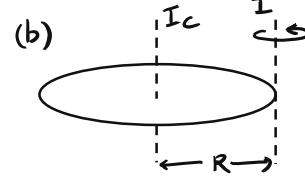
4. MOI of Standard bodies

PART 1 – ROTATION

(i) MOI OF RING



$$I = MR^2$$

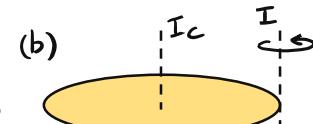


$$I = 2MR^2$$

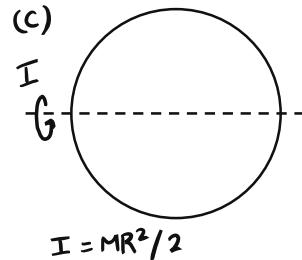
(ii) MOI OF DISC



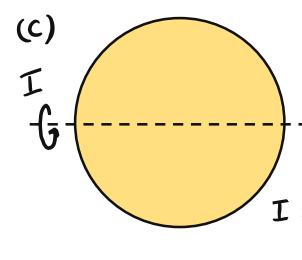
$$I = MR^2/2$$



$$I = \frac{3}{2}MR^2$$

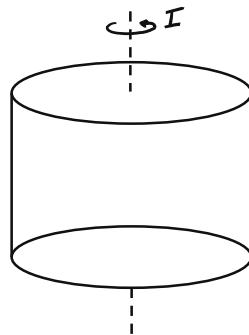


$$I = MR^2/2$$



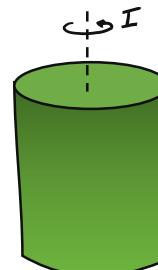
$$I = \frac{1}{4}MR^2$$

(iii) MOI OF HOLLOW CYLINDER

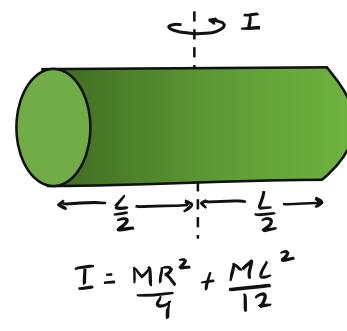


$$I = MR^2$$

(iv) MOI OF SOLID CYLINDER

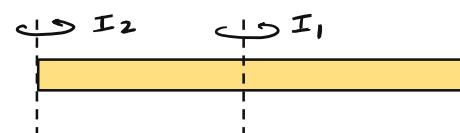


$$I = \frac{1}{2}MR^2$$



$$I = \frac{1}{4}MR^2 + \frac{ML^2}{12}$$

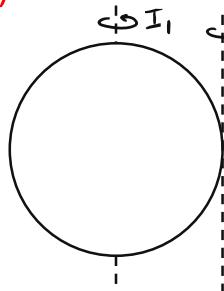
(v) MOI OF THIN ROD



$$I_1 = ML^2/12$$

$$I_2 = ML^2/3$$

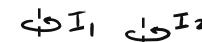
(vi) MOI OF HOLLOW SPHERE



$$I_1 = \frac{2}{3}MR^2$$

$$I_2 = \frac{5}{3}MR^2$$

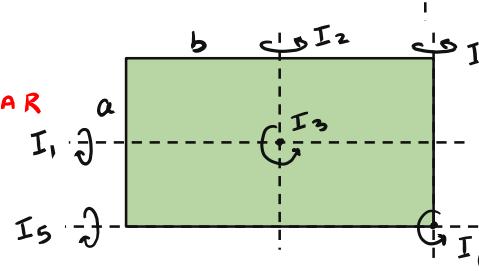
(vii) MOI OF SOLID SPHERE



$$I_1 = \frac{2}{5}MR^2$$

$$I_2 = \frac{7}{5}MR^2$$

(viii) MOI OF RECTANGULAR SHEET



$$I_1 = Ma^2/12$$

$$I_2 = Mb^2/12$$

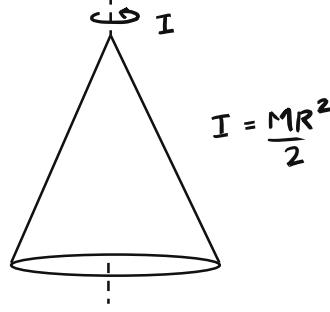
$$I_3 = I_1 + I_2$$

$$I_4 = Mb^2/3$$

$$I_5 = Ma^2/3$$

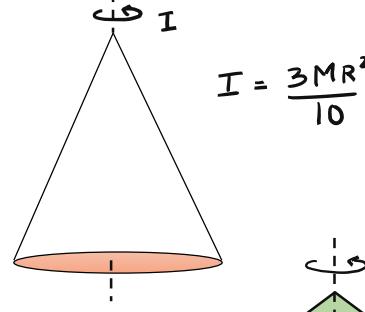
$$I_6 = I_4 + I_5$$

(ix) MOI OF HOLLOW CONE



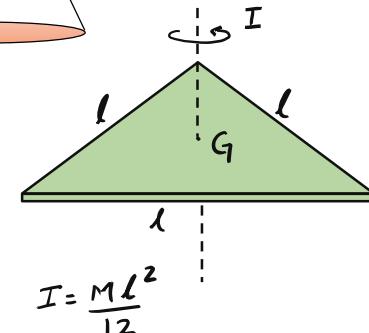
$$I = \frac{MR^2}{2}$$

(x) MOI OF SOLID CONE



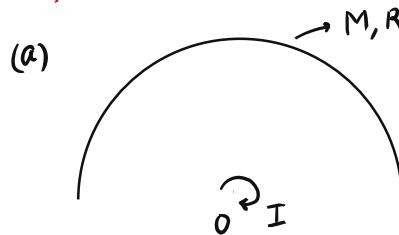
$$I = \frac{3MR^2}{10}$$

(xi) MOMENT OF INERTIA OF EQUILATERAL TRIANGLE SHEET (about Centroid)

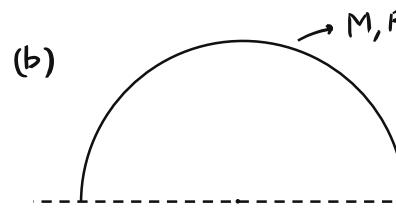


$$I = \frac{ML^2}{12}$$

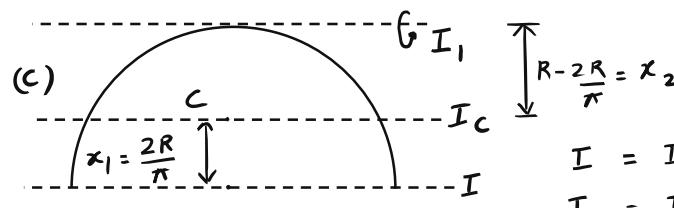
(xii) MOI OF SEMICIRCULAR RING



$$I = MR^2$$



$$I = \frac{MR^2}{2}$$

Find I_1 ?

$$I = I_c + Mx_1^2 \quad (1)$$

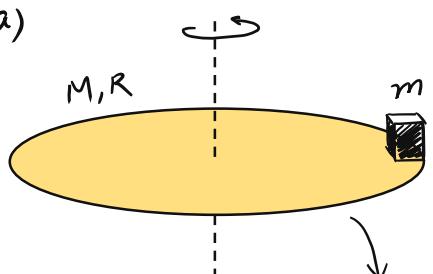
$$I_1 = I_c + Mx_2^2 \quad (2)$$

from (2) and (1) :

$$I_1 = I + M(x_2^2 - x_1^2)$$

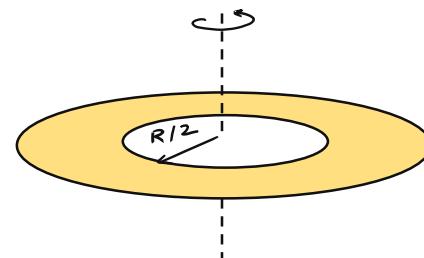
5. Addition or Subtraction of MOI

(a)



$$I = \frac{MR^2}{2} + MR^2$$

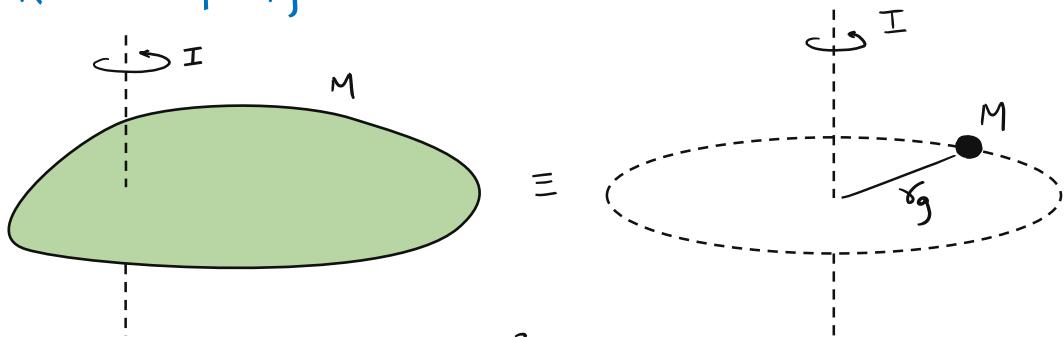
(b)



$$\begin{aligned}
 I &= \frac{MR^2}{2} - \text{MOI of removed Disc} \\
 &= \frac{MR^2}{2} - \left(\frac{M \times \pi R^2}{4} \right) \left(\frac{R}{2} \right)^2 \times \frac{1}{2} \\
 &= \frac{MR^2}{2} - \frac{MR^2}{32} = \frac{15}{32} MR^2
 \end{aligned}$$

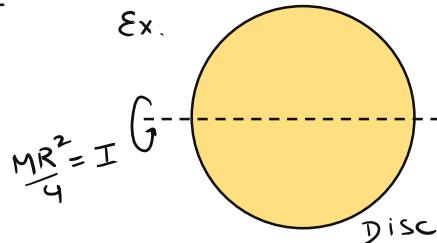
6. Radius of Gyration

PART 1 – ROTATION

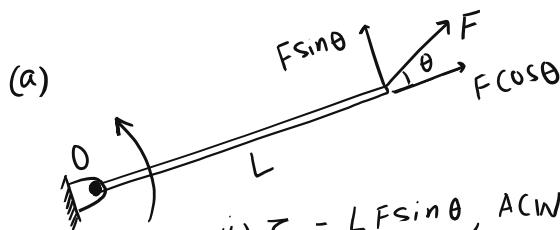


$$I = M r_g^2$$

$$\Rightarrow r_g = \sqrt{\frac{I}{M}}$$



$$r_g = \sqrt{\frac{MR^2/4}{M}} = \frac{R}{2}$$

7. Torque ($\vec{\tau} = \vec{\theta} \times \vec{F}$), N-m

$$(i) \tau = L F \sin \theta, \text{ ACW}$$

(ii) $F \cos \theta$ passes through O, can't have rotational effect

$$\sum \vec{\tau} = I \vec{\alpha}$$

angular accⁿ

static equilibrium problems

$$\sum \vec{\tau} = 0 \text{ and } \sum \vec{F} = 0$$

both ω and $V_{cm} = 0$

$$\text{Soln. } m_2 g - T_2 = m_2 a \quad \text{--- (i)}$$

$$T_1 - m_1 g = m_1 a \quad \text{--- (ii)}$$

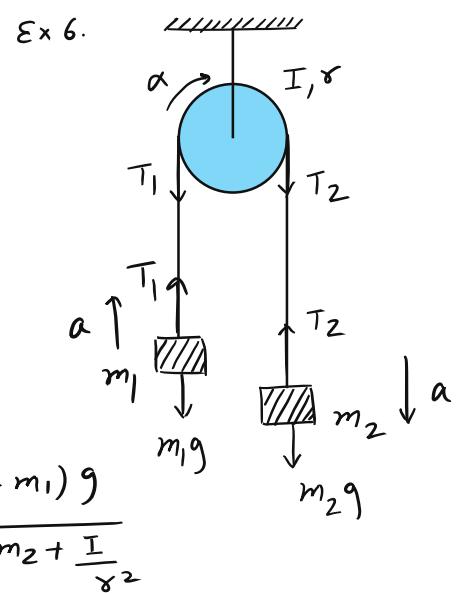
$$T_2 \gamma - T_1 \gamma = I \alpha$$

$$\Rightarrow (T_2 - T_1) \gamma = I \cdot \frac{a}{\gamma}$$

$$T_2 - T_1 = \frac{I a}{\gamma^2} \quad \text{--- (iii)}$$

$$(i) + (ii) + (iii)$$

$$(m_2 - m_1) g = a \left(m_1 + m_2 + \frac{I}{\gamma^2} \right) \therefore a = \frac{(m_2 - m_1) g}{m_1 + m_2 + \frac{I}{\gamma^2}}$$



Space to add concepts learnt from PYQs if any

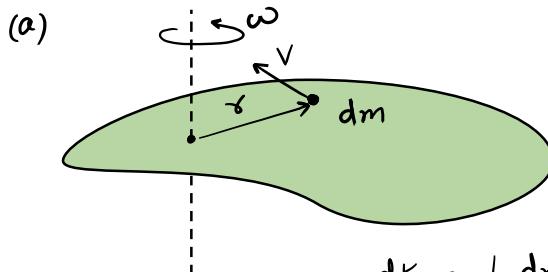
Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in ROTATIONAL MOTION – PART 2

1. K.E. in Rotational Motion
2. Angular Momentum
3. Conservation of Angular Momentum (*Impulse Equation*)
4. Standard Examples of $L = \text{Const.}$
5. Fixed & Free Collision
6. Hinge Reaction

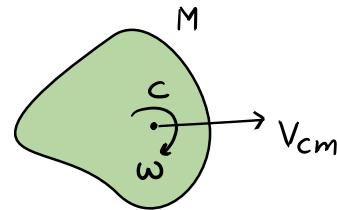
Note: For video refer Revision Series Playlist on EDUNI YouTube Channel

1. K.E. in Rotational Motion



$$\begin{aligned} dK &= \frac{1}{2} dm v^2 \\ &= \frac{1}{2} dm (\omega r)^2 \\ \Rightarrow K &= \frac{1}{2} \omega^2 \int dm r^2 \\ \boxed{K = \frac{1}{2} I \omega^2} \end{aligned}$$

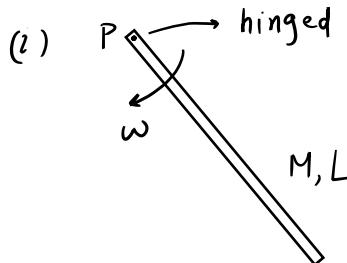
(b)



here Axis of rotation Passes COM

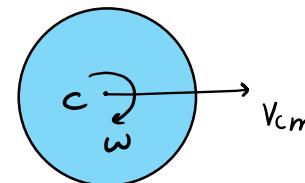
$$K = K_R + K_T$$

$$K = \frac{1}{2} I \omega^2 + \frac{1}{2} M v_{cm}^2$$



$$\begin{aligned} KE &= \frac{1}{2} I_P \omega^2 \\ &= \frac{1}{2} \cdot \frac{ML^2}{3} \cdot \omega^2 \end{aligned}$$

(ii)



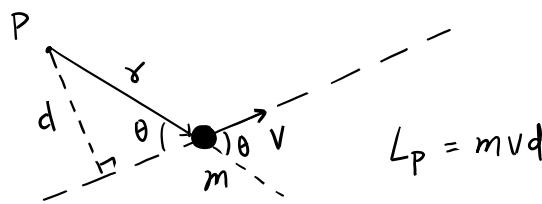
$$I_c = kmr^2$$

1 if Ring / Hollow cyl.
 $\frac{1}{2}$ if Disc / Solid cyl.
 $\frac{2}{3}$ if Hollow sphere
 $\frac{2}{5}$ if Solid sphere

$$KE = \frac{1}{2} I_c \omega^2 + \frac{1}{2} m v_{cm}^2$$

2. Angular Momentum (L)

(a) For Point mass

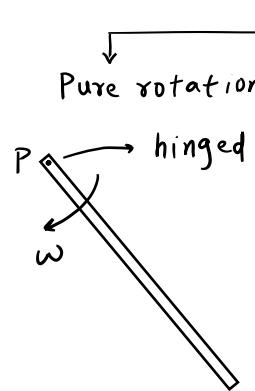


$$\# \vec{L} = \vec{r} \times \vec{p} \\ = \vec{r} \times m \vec{v} \\ \therefore L = m v \underbrace{r \sin \theta}_{d}$$

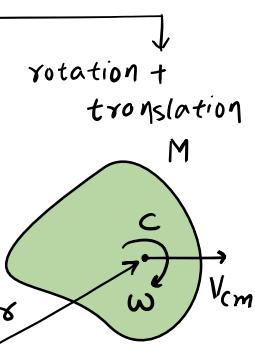
↳ for above fig: dir " ⊙ "

(b) Rigid body

PART 2 – ROTATION



$$L_p = I_p \omega$$



$$\vec{L}_p = I_{cm} \vec{\omega} + \vec{r}_{cm} \times \vec{p}_{cm}$$

3. Conservation of Angular Momentum

$$(a) \tau = \frac{dL}{dt}$$

If $\tau_{ext} = 0 \Rightarrow L$ is const or conserved

$$\left\{ \text{similar to } F = \frac{dp}{dt} \right\}$$

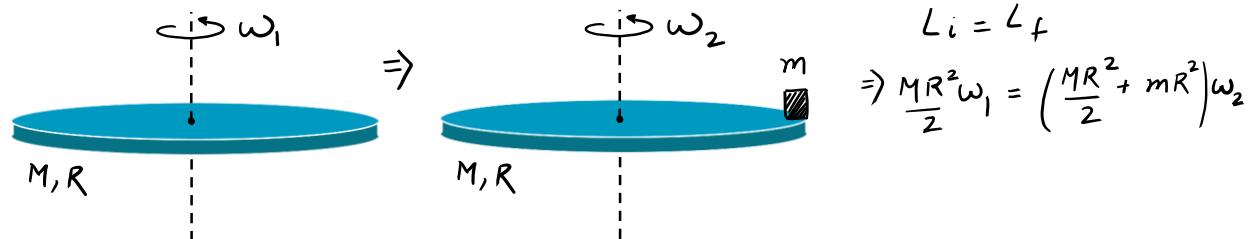
$$(b) \text{ If } \tau_{ext} \neq 0$$

$$\Rightarrow \tau dt = dL$$

$$\text{or } \boxed{\int \tau dt = \Delta L}$$

↳ Angular impulse = ΔL 4. Standard Examples of $L = \text{const.}$

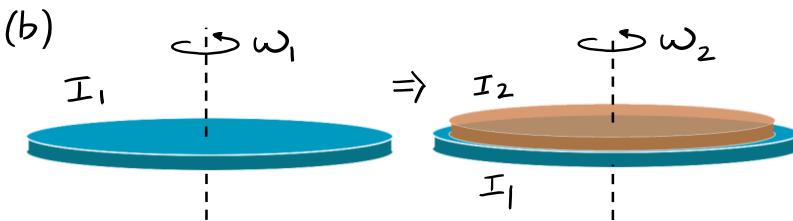
(a)



$$L_i = L_f$$

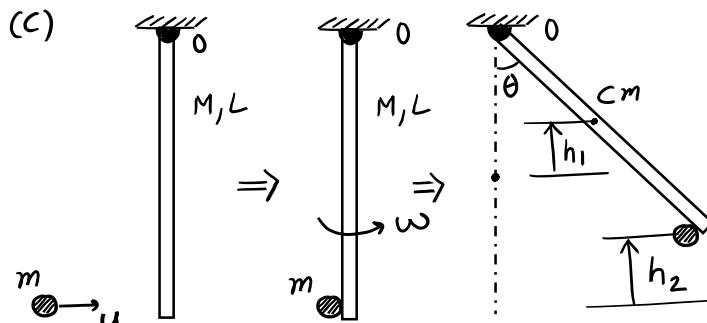
$$\Rightarrow \frac{MR^2}{2} \omega_1 = \left(\frac{MR^2}{2} + mR^2 \right) \omega_2$$

(b)



$$(i) I_1 \omega_1 = (I_1 + I_2) \omega_2$$

$$(ii) \text{Loss in KE} = \frac{1}{2} I_1 \omega_1^2 - \frac{1}{2} (I_1 + I_2) \omega_2^2$$



$\hookrightarrow \because \tau_{\text{net}} \text{ about } 0 \text{ is zero}$
during impact

$\Rightarrow L$ is same just before &
just after

$$muL = \left(\frac{ML^2}{3} + mL^2 \right) \omega \quad \text{--- (i)}$$

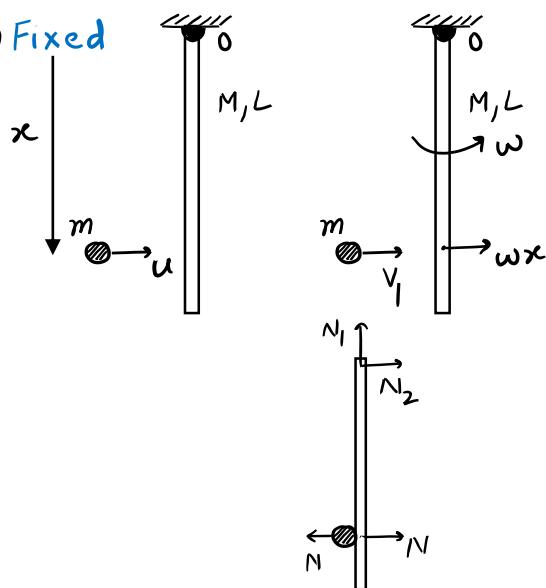
Gain in PE = Loss in KE

$$\Rightarrow Mg h_1 + mg h_2 = \frac{1}{2} \left(\frac{ML^2}{3} + mL^2 \right) \omega^2$$

$$\frac{L}{2} (1 - \cos \theta)$$

5. Fixed & Free Collision (e = coefficient of restitution)

(a) Fixed



$$L_i = L_f$$

$$\Rightarrow mu x = mv_1 x + \frac{ML^2}{3} \omega \quad \text{--- (i)}$$

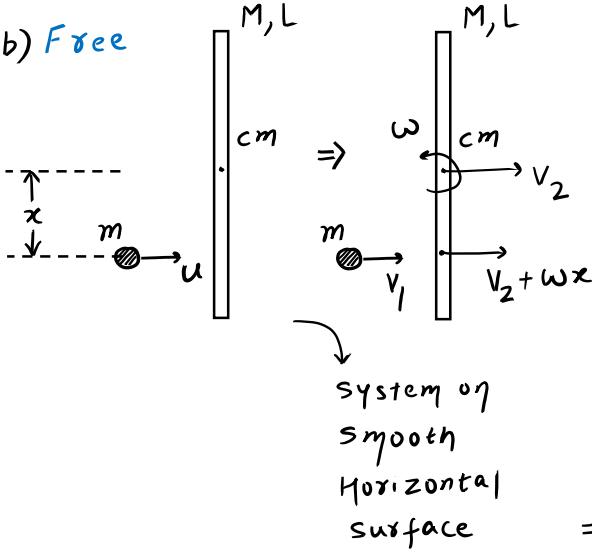
$$e = \frac{\text{Vel of separation}}{\text{Vel of approach}}$$

$$e = \frac{\omega x - v_1}{u} \quad \text{--- (ii)}$$

Hence solve (i) & (ii)
to get v_1 and ω

P is not conserved

(b) Free



(i) L is cons. about any point

About cm : $L_i = L_f$

$$\Rightarrow mu x = mv_1 x + \frac{ML^2}{12} \omega \quad \text{--- (1)}$$

(ii) P is cons. (As $F_{\text{net}} = 0$)

$$P_i = P_f \Rightarrow mu = mv_1 + Mv_2 \quad \text{--- (2)}$$

$$(iii) e = \frac{v_2 + \omega x - v_1}{u} \quad \text{--- (3)}$$

Solving (1), (2) & (3) we get

v_1, v_2 and ω

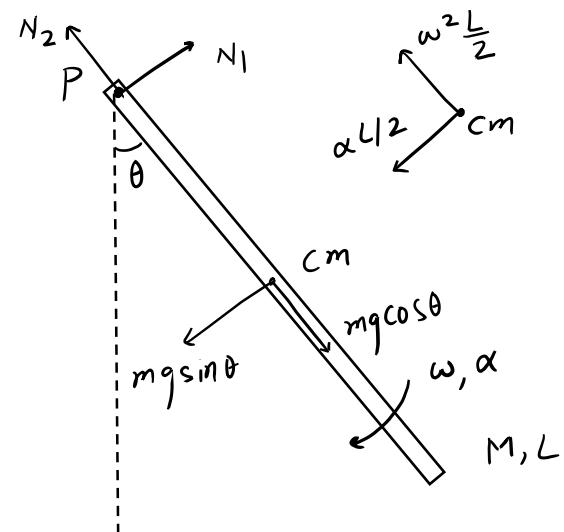
6. Hinge reaction

(i) Always take 2 \perp components of hinge γK^n

$$\hookrightarrow mg \sin \theta - N_1 = m \frac{\alpha L}{2}$$

$$\hookrightarrow N_2 - mg \cos \theta = m \frac{\omega^2 L}{2}$$

$$\therefore \gamma K^n = \sqrt{N_1^2 + N_2^2}$$

Space to add concepts learnt from PYQs if any

Space to add concepts learnt from PYQs if any

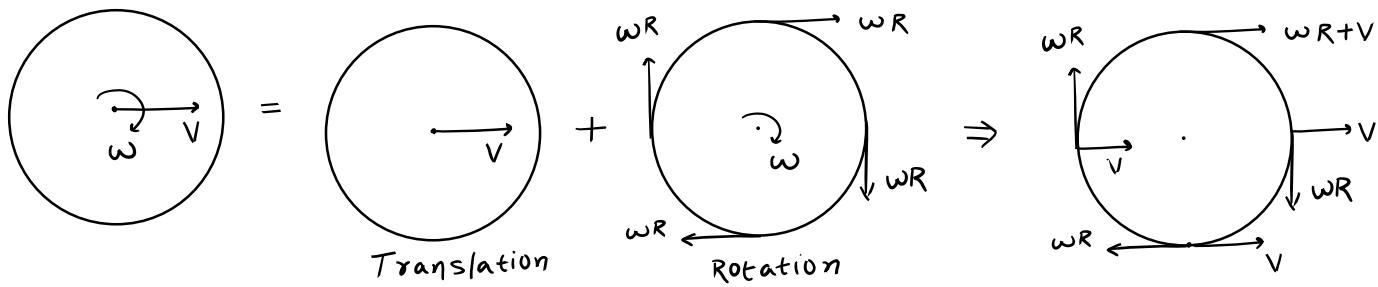
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Topics to cover in ROTATIONAL MOTION – PART 3

1. Velocity of a Point on Circumference (Trans + Rotational)
2. Acceleration of a Point of Circumference
3. Rolling Motion (*pure rolling & rolling with slipping*)
4. Total Kinetic Energy in Pure Rolling
5. External Force in Rolling
6. Energy Conservation (*pure rolling on Inclined*)
7. Acceleration & Friction (*Rolling on Inclined Plane*)
8. Toppling

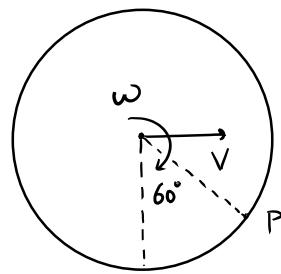
Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. Velocity of Point on Circumference (Trans + Rotational)

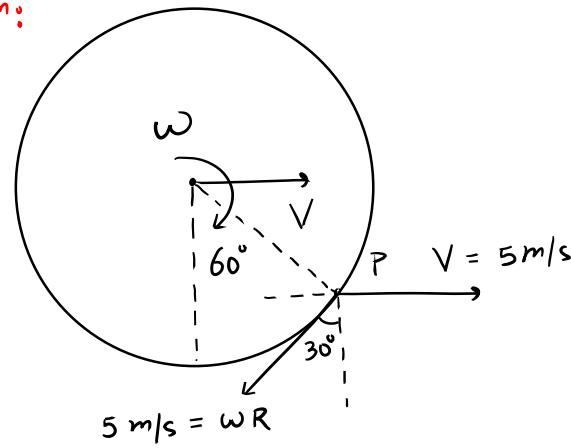


$$V = 5 \text{ m/s}, \omega = 2.5 \text{ rad/s}, R = 2 \text{ m} \text{ find } V_p ?$$

Ex 1.



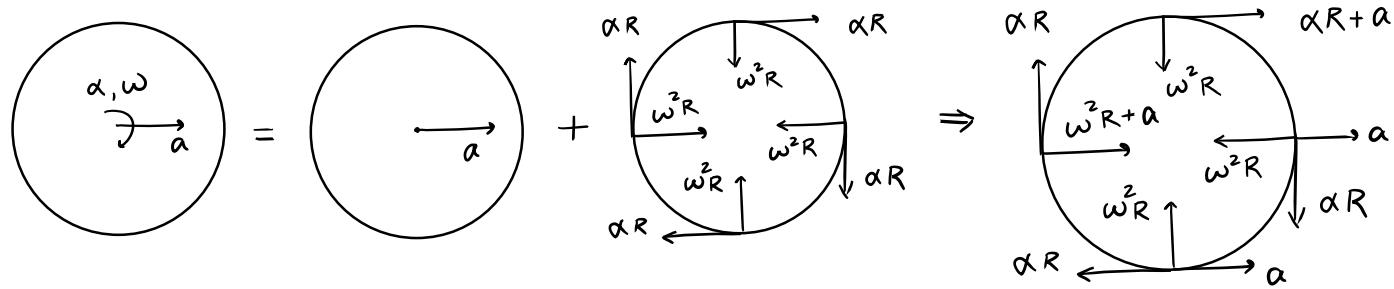
solⁿ:



$$V_p = \sqrt{5^2 + 5^2 + 2.5^2 \cos 120^\circ} = \boxed{5 \text{ m/s}}$$

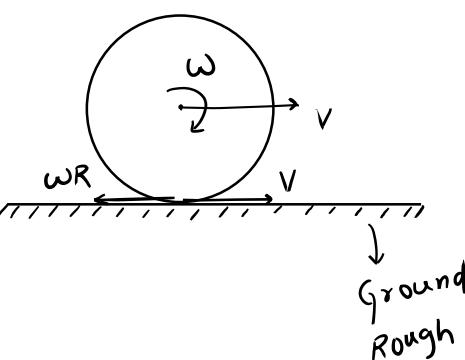
2. Acceleration of Point on circumference

PART 3 – ROTATION



3. Rolling Motion

(a) Pure rolling: No relative motion of Pt. of Contact or Pt. of Contact have same velocity.



$$\therefore V_{\text{Ground}} = 0 \Rightarrow V - \omega R = 0 \therefore V = \omega R$$

$$(i) V = \omega R \quad (\because a = \alpha R)$$

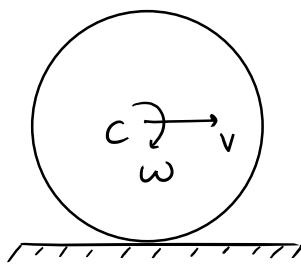
(ii) No friction force acting

(b) Rolling with Slipping: $V \neq \omega R$

↳ If $V > \omega R \Rightarrow f_K$ acts backward

↳ If $\omega R > V \Rightarrow f_K$ acts forward

NOTE: f_K until Pure rolling starts

4. KE in Pure Rolling ($V = \omega R$)

$$I_C = k m R^2$$

↳ 1 if Ring or Hollow Cylinder

$\frac{1}{2}$ if Disc or Solid Cylinder

$\frac{2}{3}$ if Hollow Sphere

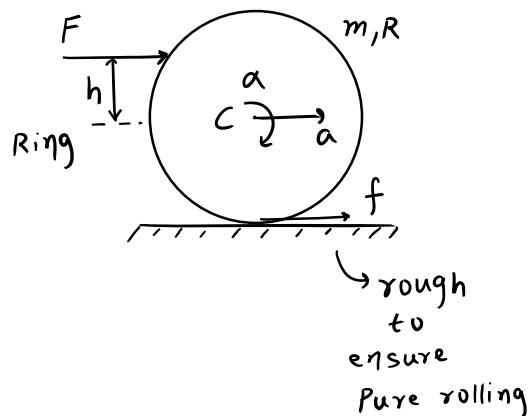
$\frac{2}{5}$ if Solid Sphere

$$KE = K_T + K_R = \frac{1}{2} m V^2 + \frac{1}{2} \cdot k m R^2 \cdot \left(\frac{V}{R}\right)^2$$

$$\therefore K.E. = \frac{1}{2} m V^2 (1 + k)$$

5. External Force in Rolling

PART 3 – ROTATION



NOTE: (i) : Pure rolling $\Rightarrow f$ is static
 (ii) : Pt. of contact is at instantaneous rest $\Rightarrow W_f = 0$

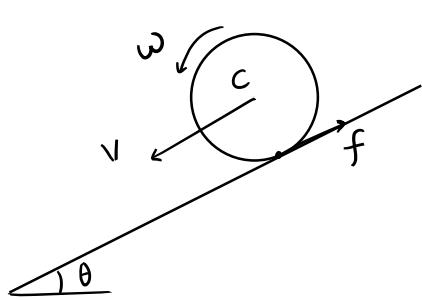
$$F + f = ma \quad \text{--- (1)}$$

$$Fh - fR = mR^2 \cdot \alpha \quad \left\{ \alpha = \frac{a}{R} \right\}$$

$$\Rightarrow Fh - fR = mR^2 \frac{a}{R} \quad \text{--- (2)}$$

Solve (1) & (2) to get
 a and f .

6. Energy Conservation (pure rolling on Inclined)



$$v = \omega R \quad \& \quad a = \alpha R$$

NOTE: (i) $v \uparrow$ due to gravity

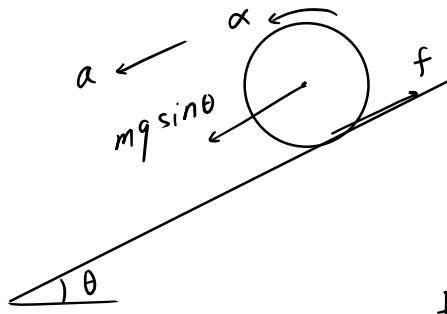
(ii) Thus ω must \uparrow so that $v = \omega R$

(iii) Thus f acts (Creating Torque about C)

$$\text{Static} \Rightarrow W_f = 0$$

↓
 Energy Cons. is applicable

7. Acceleration & Friction – Rolling on Inclined Plane



$$mg \sin \theta - f = ma \quad \text{--- (1)}$$

$$fR = kmR^2 \cdot \alpha \quad \left\{ \alpha = \frac{a}{R} \right\}$$

$$\Rightarrow fR = kmR^2 \frac{a}{R} \Rightarrow f = kma \quad \text{--- (2)}$$

From (1) and (2) :

$$a = \frac{g \sin \theta}{1 + k}$$

$$f = \frac{km g \sin \theta}{1 + k}$$

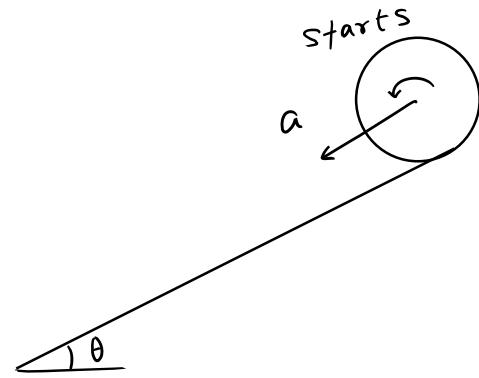
$$a = \frac{gs \sin \theta}{1+k} \quad \left\{ \text{lower } k \Rightarrow \text{higher } a \right.$$

PART 3 – ROTATION

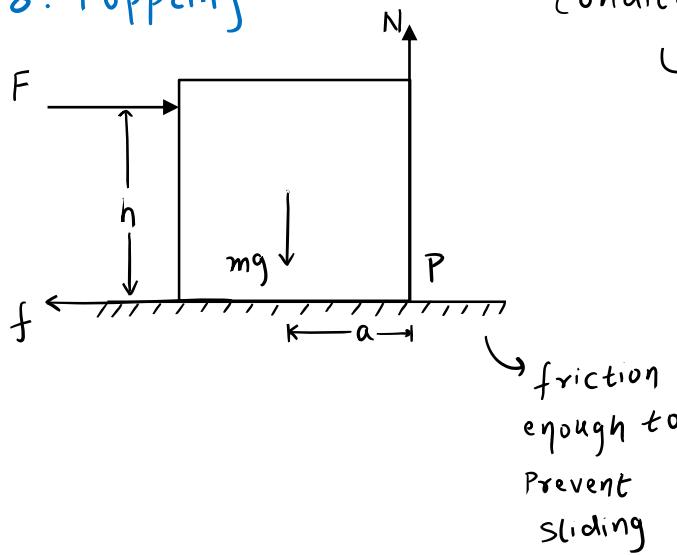
$$\begin{array}{cccc}
 & \downarrow & \downarrow & \downarrow \\
 \text{Ring} & \text{Disc} & \text{H. sphere} & \text{S. sphere} \\
 k=1 & k=\frac{1}{2} & k=\frac{2}{3} & k=\frac{2}{5}
 \end{array}$$

$$\Rightarrow a_{\text{S.S.}} > a_{\text{disc}} > a_{\text{H.S.}} > a_{\text{ring}}$$

↓
Solid sphere reaches 1st if
all starts together

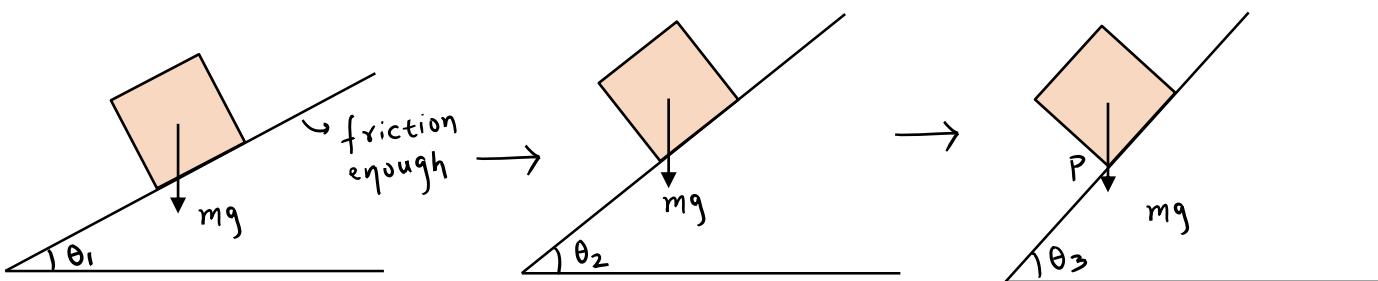


8. Toppling



Increase θ (Toppling on incline)

θ_3 is limiting angle.



Space to add concepts learnt from PYQs if any

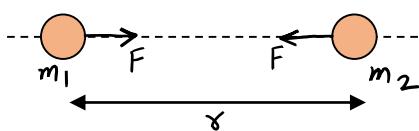
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Topics to cover in GRAVITATION

1. Law of Gravitation
2. Gravitational Field
3. g due to a Ring
4. g due to a long wire
5. g due to a thin spherical shell
6. g due to a solid sphere
7. Potential Energy
8. Gravitational Potential (point mass & ring)
9. Potential due to Spherical Shell
10. Potential due to Solid Sphere
11. Variation of g with height
12. Variation of g with depth
13. Variation of g with rotation
14. Escape Velocity
15. Orbital Velocity, Time Period, KE, PE & TE
16. Geostationary Satellite
17. Discussion on Elliptical Path
18. Speed at Perigee and Apogee
19. Projected velocity vs path
20. Kepler's Law
21. Double Mass System

Note: For video refer Revision Series Playlist on EDUNI YouTube Channel

1. LAW OF GRAVITATION

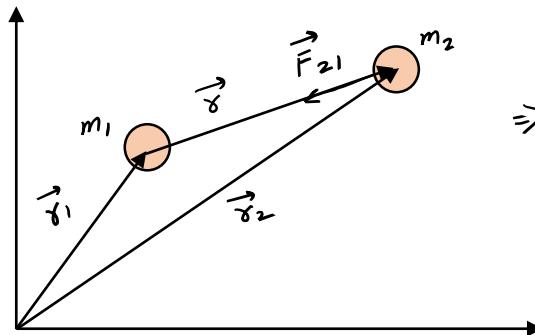


$$F = G \frac{m_1 m_2}{r^2}$$

G : universal gravitational constant
 $6.67 \times 10^{-11} \text{ N m}^2 / \text{kg}^2$

NOTE:
1. F acts along line joining masses.
2. Here m_1 and m_2 are point masses.

IN VECTOR FORM



$$\Rightarrow \vec{F}_{21} = - \frac{G m_1 m_2}{|\vec{r}_2 - \vec{r}_1|^3} (\vec{r}_2 - \vec{r}_1)$$

2. GRAVITATIONAL FIELD

→ A way of expressing influence of a mass

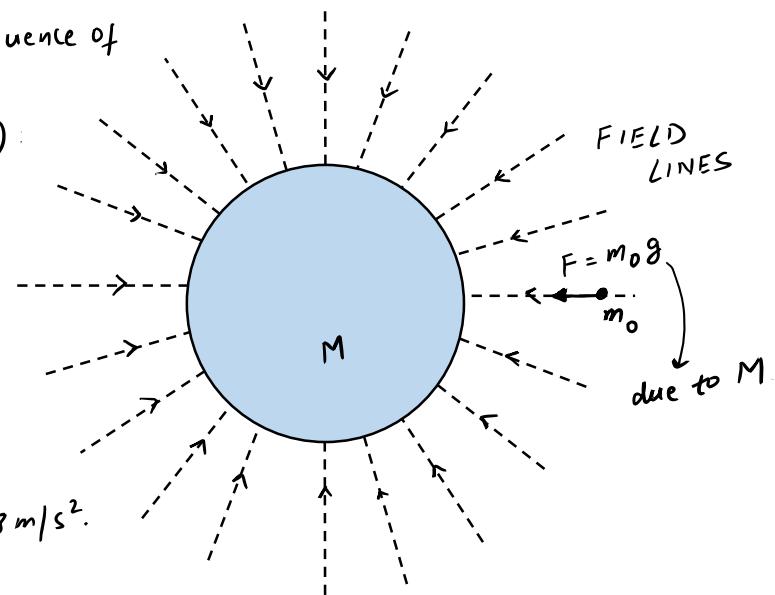
Gravitational Field strength (g):

$$m \cdot \gamma \cdot g = \frac{G m}{\gamma^2}$$

NOTE:

1. g is also called "acceleration due to gravity"

2. At earth's surface its close to $g_0 = 9.8 \text{ m/s}^2$.



3. FOR A RING

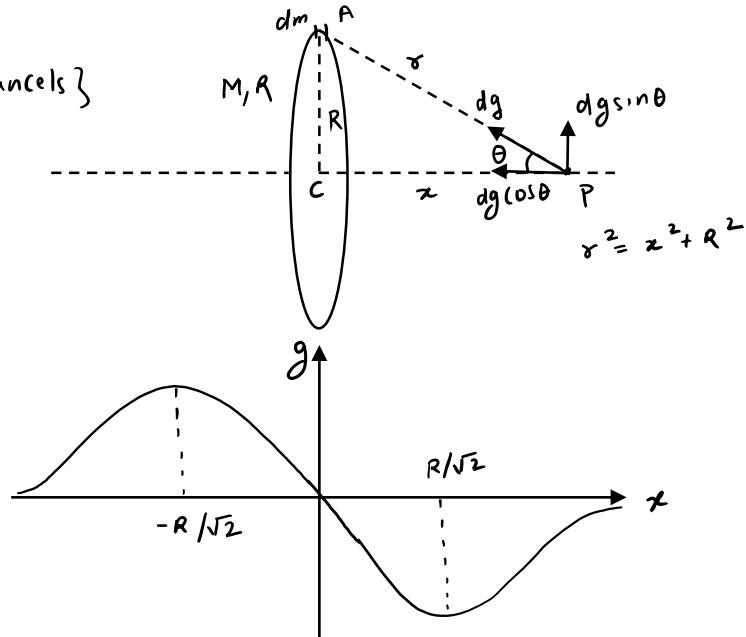
$$\int dg \sin \theta = 0 \quad \{ \text{vertical comp. cancels} \}$$

$$g(x) = \int dg \cos \theta$$

$$= \int \frac{G dm \cos \theta}{\gamma^2} = \int \frac{G dm}{\gamma^2} \times \frac{x}{\gamma}$$

$$= \frac{G x}{(x^2 + R^2)^{3/2}} \int dm$$

$$= \boxed{\frac{G M x}{(x^2 + R^2)^{3/2}}}$$



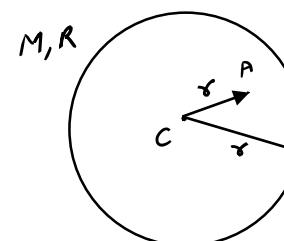
4. FOR A LONG WIRE

$$g = \frac{2 G \lambda}{\gamma}$$

$$\lambda (\text{kg/m})$$

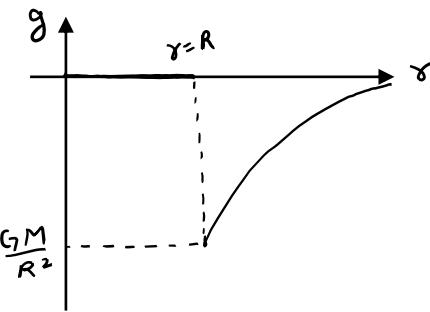
$$\gamma \quad g$$

5. FOR THIN SPHERICAL SHELL

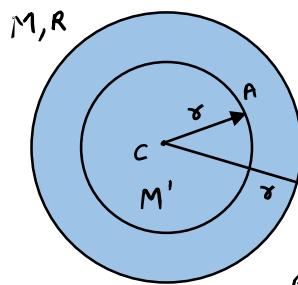


A: For $r < R$, $g = 0$

B: For $r > R$, $g = -\frac{GM}{r^2}$



6 FOR SOLID SPHERE



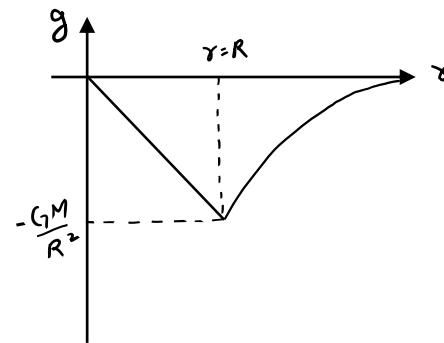
A: For $\gamma < R$

$$M' = M r^3 / R^3$$

$$\Rightarrow g = -\frac{GM'}{r^2} = -\frac{GMr}{R^3}$$

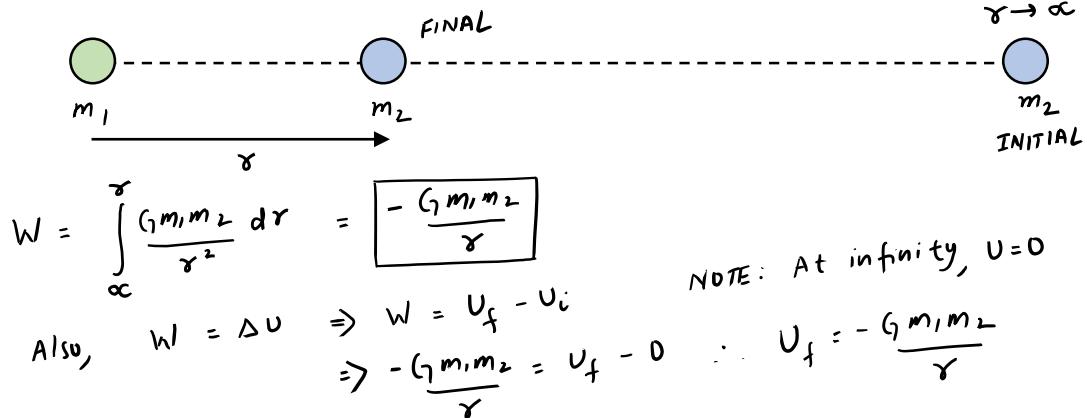
β : FOR $r > R$

$$g = -\frac{GM}{r^2}$$



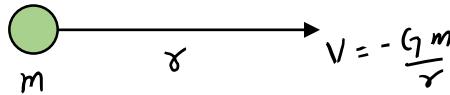
NOTE : Taking radially inwards
as -ve direction

7 POTENTIAL ENERGY U : Work done to bring m_2 very slowly from ∞ to a separation s from m_1 .



8. GRAVITATIONAL POTENTIAL V

POINT Mass:

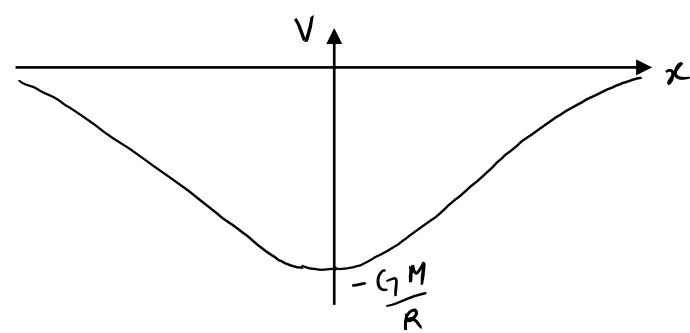
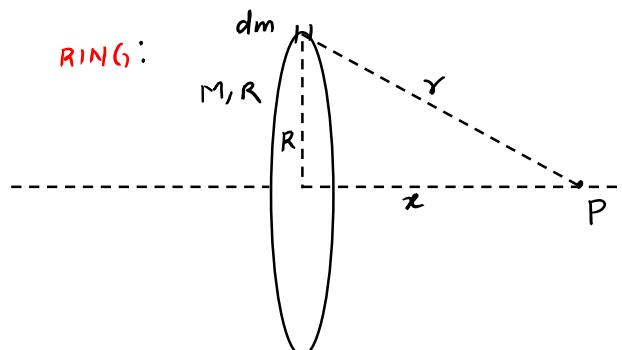


$$\text{RING: } dV = -G \frac{dm}{r}$$

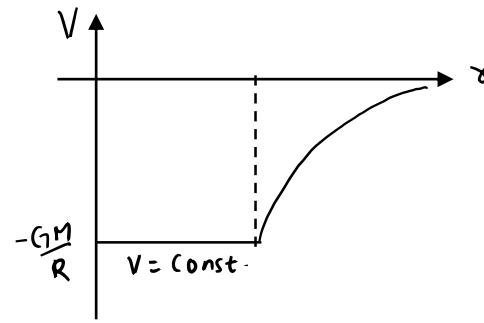
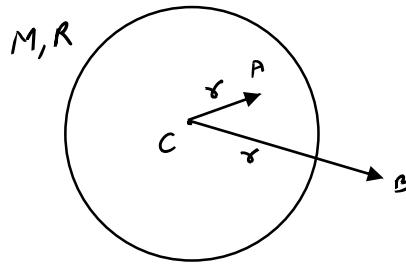
$$\Rightarrow V = -\frac{G}{r} \int dm$$

$$V = -\frac{GM}{(R^2 + x^2)^{1/2}}$$

RING



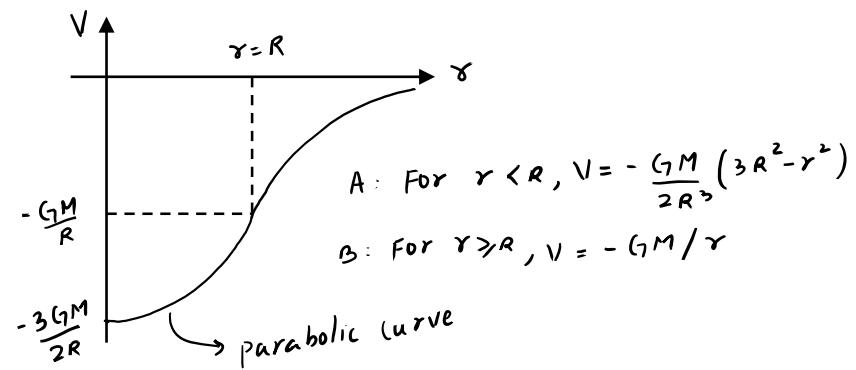
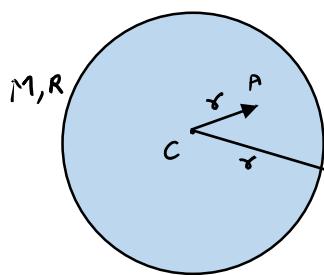
9. V DUE TO SPHERICAL SHELL



A: For $r < R$, $V = -\frac{GM}{R}$ B: For $r > R$, $V = -\frac{GM}{r}$

→ **NOTE:** For an outside point, assume all mass to be situated at centre.

10. V DUE TO SOLID SPHERE



→ **NOTE:** For an outside point, assume all mass to be situated at centre.

11. VARIATION OF g WITH HEIGHT

At height "h":

$$g = \frac{GM}{(R+h)^2} = \frac{GM}{R^2} \left(1 + \frac{h}{R}\right)^{-2}, \quad g_0 = \frac{GM}{R^2}$$

If $\frac{h}{R} \ll 1$ or $h \ll R$

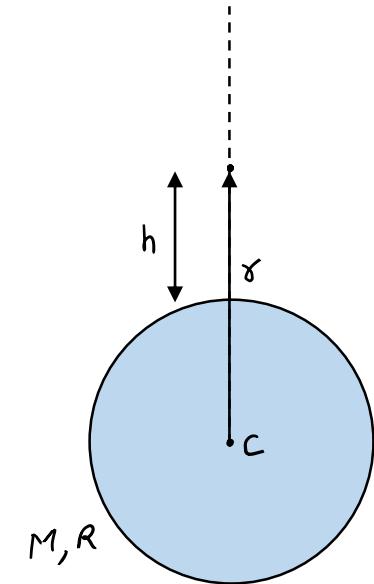
$$\Rightarrow g = g_0 \left(1 - \frac{2h}{R}\right)$$

NOTE: only if $h \ll R$ (for earth till $h \sim 400$ Km)

1. use $g = g_0 \left(1 - \frac{2h}{R}\right)$

2. If h is comparable with R , use

$$g = \frac{GM}{(R+h)^2} = \frac{g_0 R^2}{(R+h)^2}$$



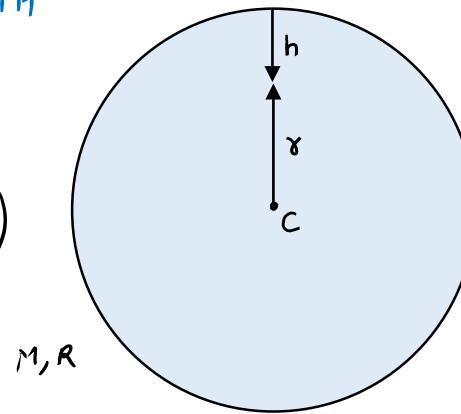
12. VARIATION OF g WITH DEPTH

At a depth "h":

$$g = \frac{GM\gamma}{R^3}, \quad \gamma = R-h$$

$$\Rightarrow g = \frac{GM(R-h)}{R^3} = \frac{GM}{R^2} \left(\frac{R-h}{R} \right)$$

$$\therefore g = g_0 \left(1 - \frac{h}{R} \right)$$

13. VARIATION OF g WITH ROTATION

$$\gamma = R \cos \theta$$

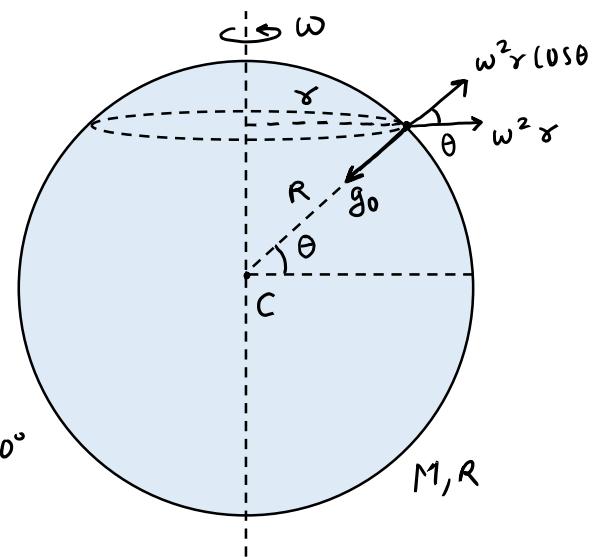
$g_{\text{effective}}$ towards centre is

$$g_{\text{eff}} = g_0 - \omega^2 \gamma \cos \theta$$

$$= g_0 - \omega^2 R \cos^2 \theta$$

NOTE:

- $\omega^2 R = 0.034 \text{ rad/s}$ (for Earth)
- At equator, $g_{\text{eff}} = g_0 - \omega^2 R$, $\theta = 0^\circ$
- At poles, $g_{\text{eff}} = g_0$, $\theta = 90^\circ$.

14. ESCAPE VELOCITY v_e

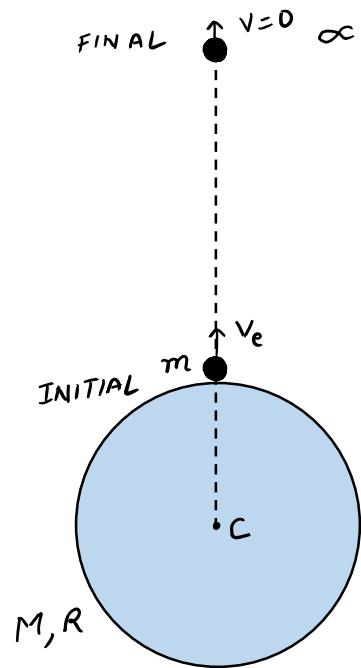
Minimum velocity v_e , so that particle escapes planet's gravitational pull.
(the distance where g of planet is zero is taken is ∞)

$$K_i + U_i = K_f + U_f$$

$$\Rightarrow \frac{1}{2}mv_e^2 - \frac{GMm}{R} = 0 + 0$$

$$\therefore v_e = \sqrt{\frac{2GM}{R}}$$

$\approx 11.2 \text{ km/s}$ at EARTH'S Surface.



15. ORBITAL VELOCITY V_0 , T, K, U, T.E., B.E.

$$(a) F = m V_0^2 / r \Rightarrow \frac{GMm}{r^2} = m \frac{V_0^2}{r} \Rightarrow V_0 = \sqrt{\frac{GM}{r}}$$

$$(b) T = \frac{2\pi r}{V_0} = \frac{2\pi r}{\sqrt{GM/r}} = \frac{2\pi}{\sqrt{GM}} r^{3/2}$$

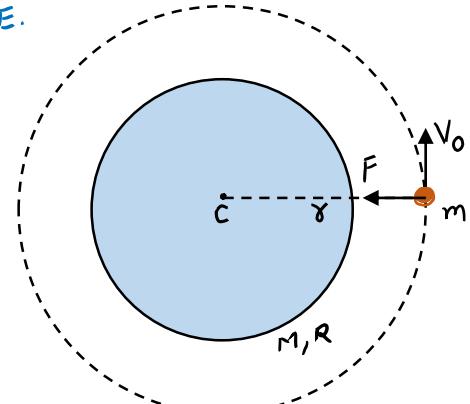
$$(c) K = \frac{1}{2} m V_0^2 = \frac{GMm}{2r}$$

$$U = -\frac{GMm}{r}, \quad T.E. = K + U = -\frac{GMm}{2r}$$

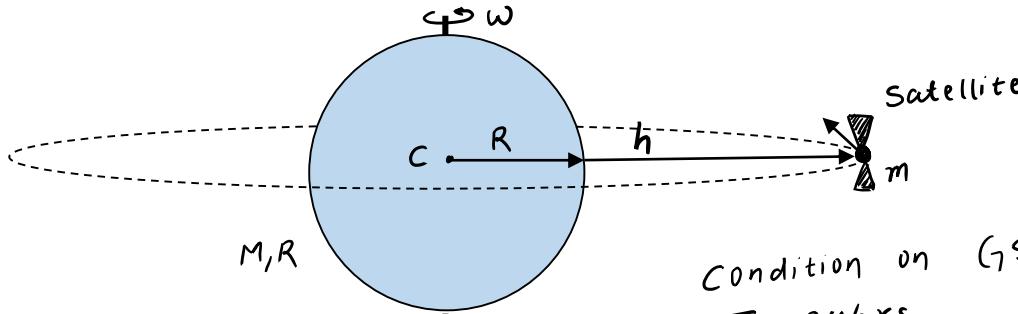
$$\therefore K = \left| \frac{U}{2} \right| = |T.E|$$

$$(d) \text{ Binding Energy, } \boxed{B.E. = -T.E.} = \frac{GMm}{2r}$$

Energy required to be given so that "m" escapes.



16. GEOSTATIONARY SATELLITE



$$T = \frac{2\pi}{\sqrt{GM}} r^{3/2}, \quad r = R + h$$

$$T = 24 \text{ hrs}, \quad R = 6400 \text{ km},$$

$$M = 6 \times 10^{24} \text{ kg}$$

we get $\boxed{h \approx 36000 \text{ km}}$

Condition on G.SAT :

1. $T = 24 \text{ hrs}$
2. Must lie on equatorial plane
3. must revolve along the direction of rotation of earth.

17. DISCUSSION ON ELLIPTICAL PATH

About τ , L , $U+K$

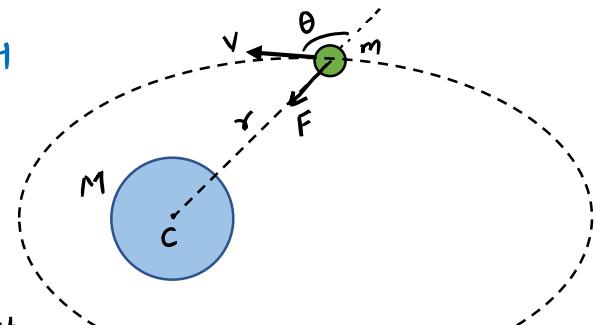
(1) As "m" revolves, F always passes through C. Thus torque τ about C is zero.

$\therefore \tau = 0 \Rightarrow L$ is constant.

$$\text{or } mv r \sin \theta = \text{const.}$$

(2) $U + K = \text{constant}$

$$\Rightarrow -\frac{GMm}{r} + \frac{1}{2} mv^2 = \text{const.}$$



18. SPEED AT PERIGEE / APOGEE

$$L_p = L_A \Rightarrow m V_p a(1-e) = m V_A a(1+e) \quad \dots \text{--- (1)}$$

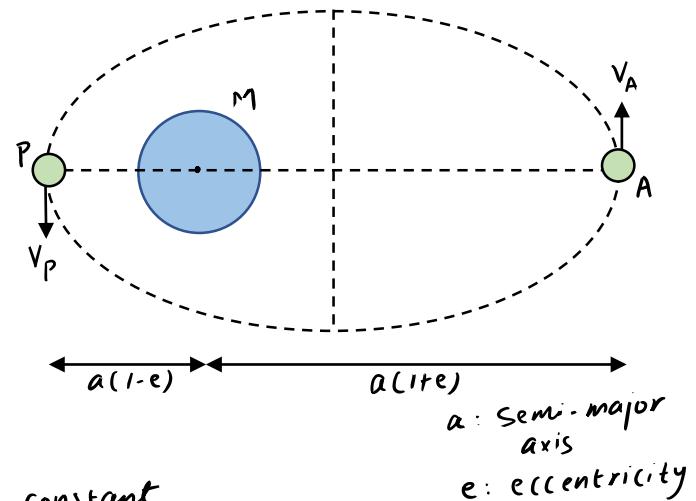
$$U_p + K_p = U_A + K_A$$

$$\Rightarrow -\frac{G M m}{a(1-e)} + \frac{1}{2} m V_p^2 = -\frac{G M m}{a(1+e)} + \frac{1}{2} m V_A^2 \quad \dots \text{--- (2)}$$

Solving (1) and (2), we get

$$V_p = \sqrt{\frac{G M (1+e)}{a(1-e)}}, \quad V_A = \sqrt{\frac{G M (1-e)}{a(1+e)}}$$

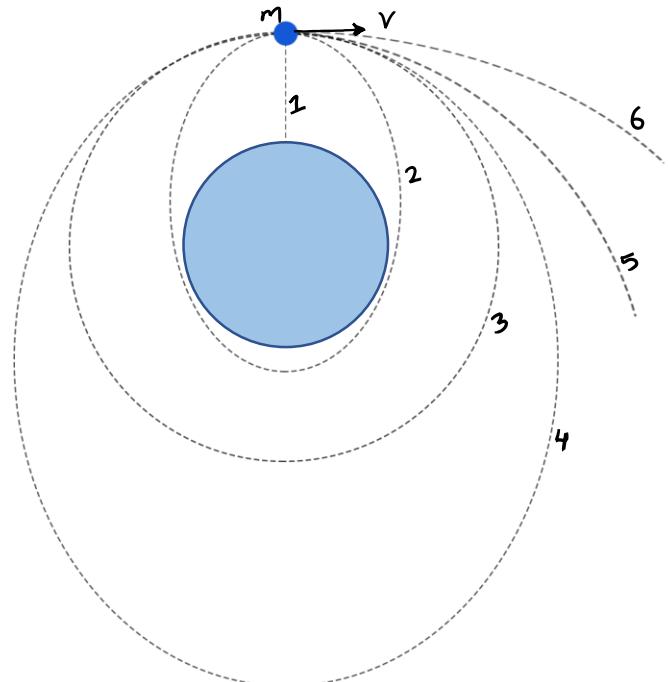
NOTE: Total energy of system is constant
i.e. $-\frac{G M m}{2a}$



19. PROJECTED VELOCITY V/S PATH

If:

- (a) $V=0 \Rightarrow$ path 1, straight line
- (b) $0 < V < V_0 \Rightarrow$ path 2, Elliptical
- (c) $V = V_0 \Rightarrow$ path 3, Circular
- (d) $V_0 < V < V_e \Rightarrow$ path 4, Elliptical
- (e) $V = V_e \Rightarrow$ Path 5, Parabola
- (f) $V > V_e \Rightarrow$ Path 6, Hyperbola



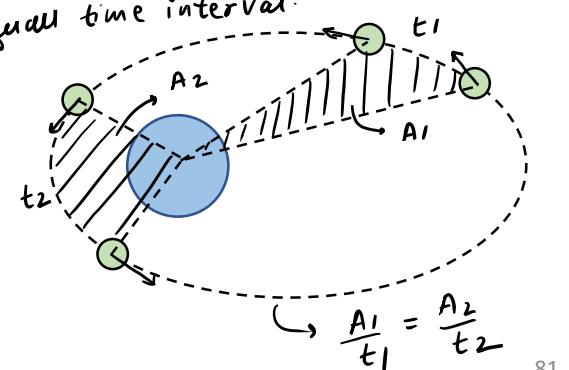
20. KEPLER'S LAW

1. Law of Orbit: Planet revolves around sun in an elliptical path with sun at one of the foci.

2. Law of Area: Line joining planet and sun sweeps equal area in equal time interval.

$$\frac{dA}{dt} = \frac{L}{2m} = \text{constant.}$$

3. Law of Periods: $T^2 \propto a^3$



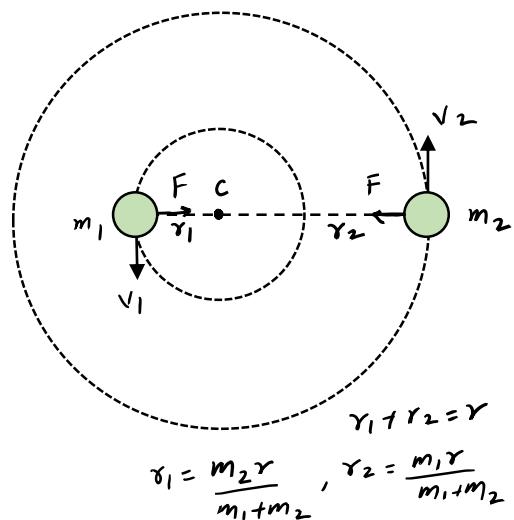
21. DOUBLE MASS SYSTEM

- Considering 2 Mass System, they revolve about their common centre of mass, 'C'.
- That point is called "BARYCENTER"

NOTE:

1. Both m_1 and m_2 has same Time Period T and ω , $\omega = \sqrt{\frac{G(m_1+m_2)}{r^3}}$

$$2. \frac{v_1}{v_2} = \frac{\omega r_1}{\omega r_2} \Rightarrow \frac{v_1}{v_2} = \frac{r_1}{r_2}$$



Space to add concepts learnt from PYQs if any

Space to add concepts learnt from PYQs if any

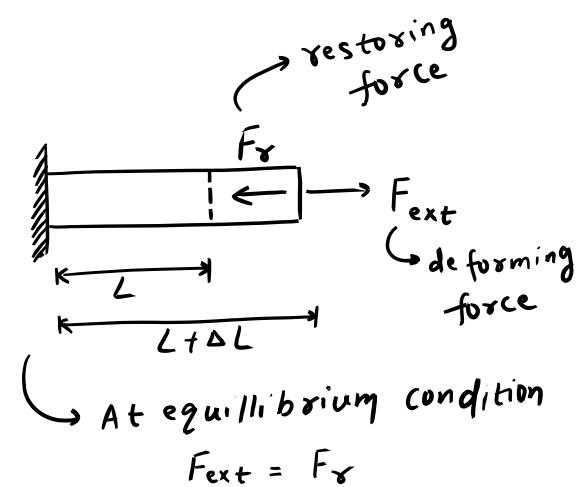
Topics to cover in Properties of Solids | Elasticity

1. Elastic body vs Plastic body
2. Stress & Strain in elastic material (example)
3. Relation between Stress & Strain (Hooke's Law)
4. Modulus of Elasticity (standard example)
5. Analogy with springs
6. Potential Energy
7. Elongation due to Self weight
8. Poisson Ratio
9. Relation among Modulus of elasticity

1. Elastic vs Plastic body

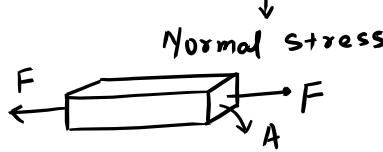
↳ body that regains original shape after removal of deforming force
Ex: Steel

↳ body doesn't regain shape
Ex: dough, clay

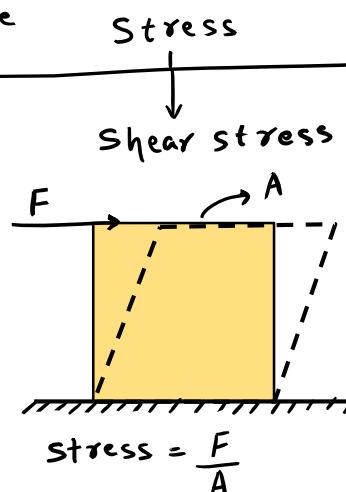


2. Stress & strain (elastic material)

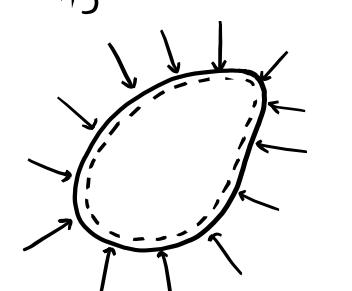
↳ due to restoring force



$$\text{Stress} = \frac{F}{A}$$

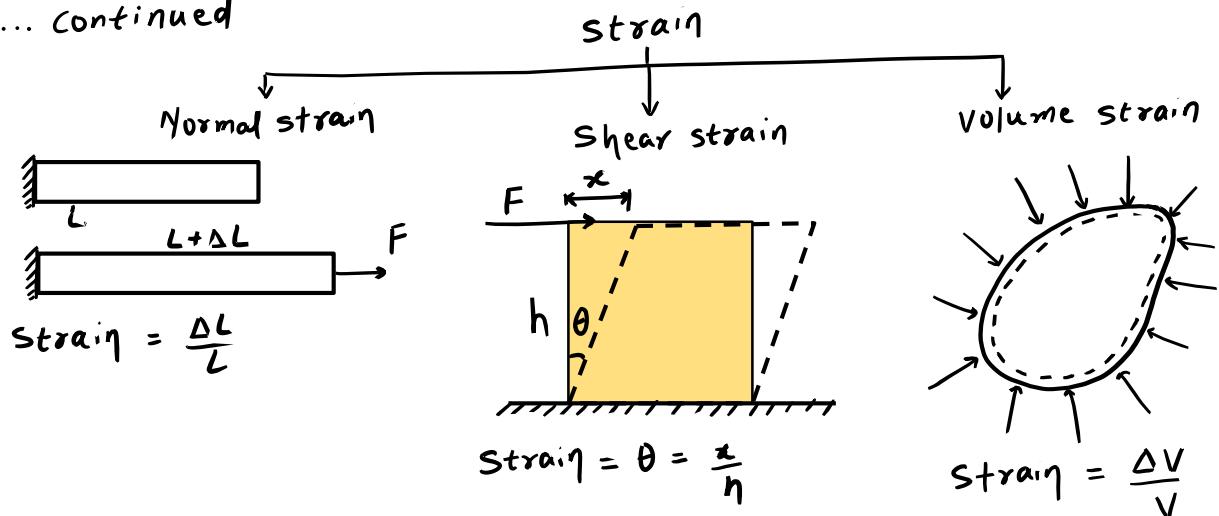


$$\text{Stress} = \frac{F}{A}$$

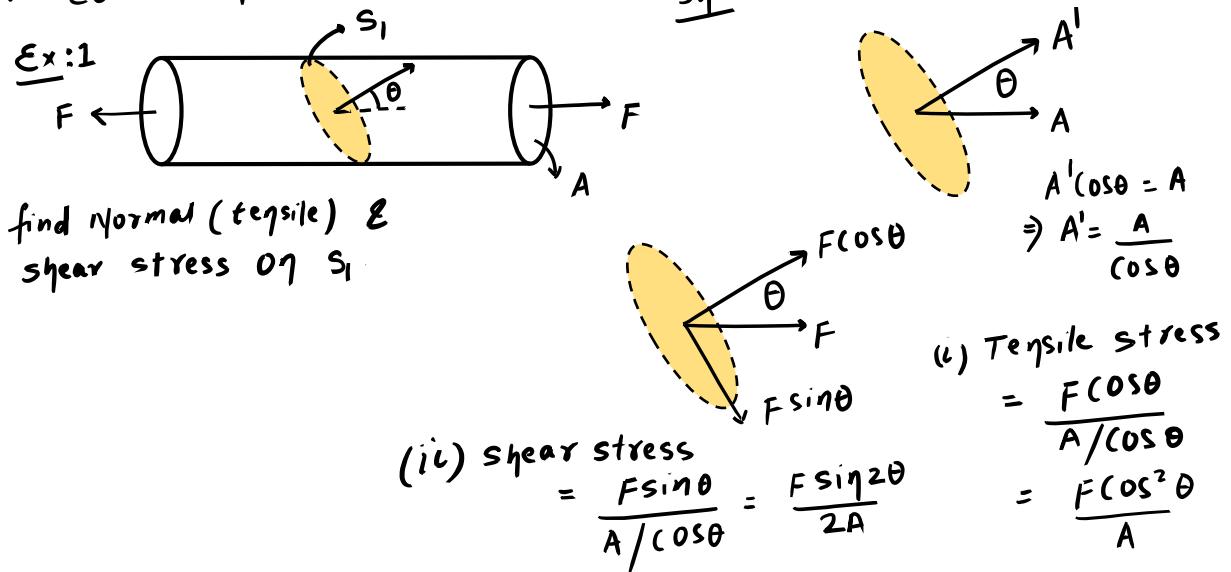


Here it is ΔP or P_{excess}

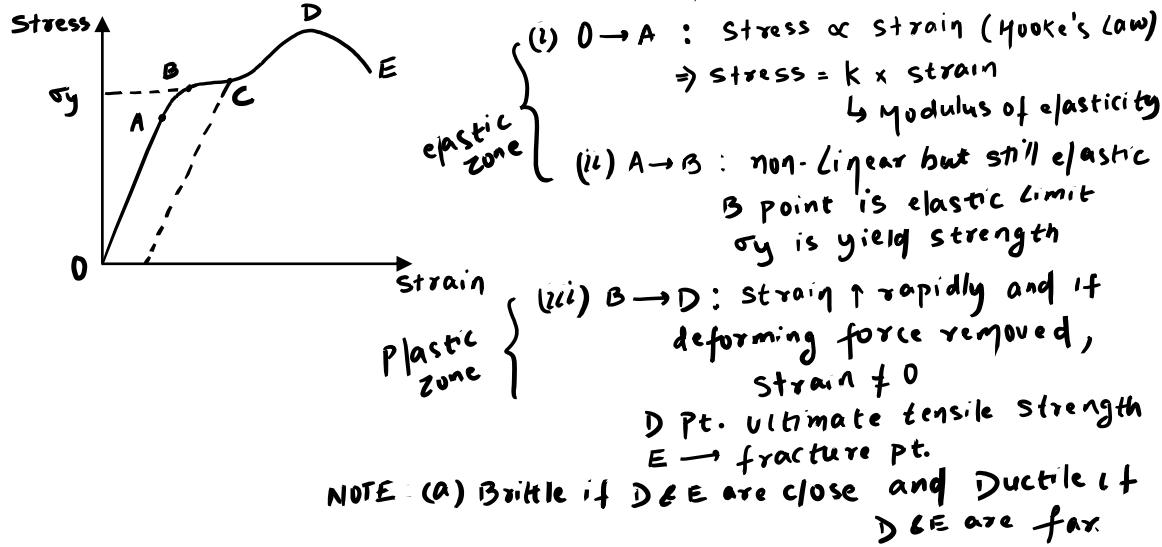
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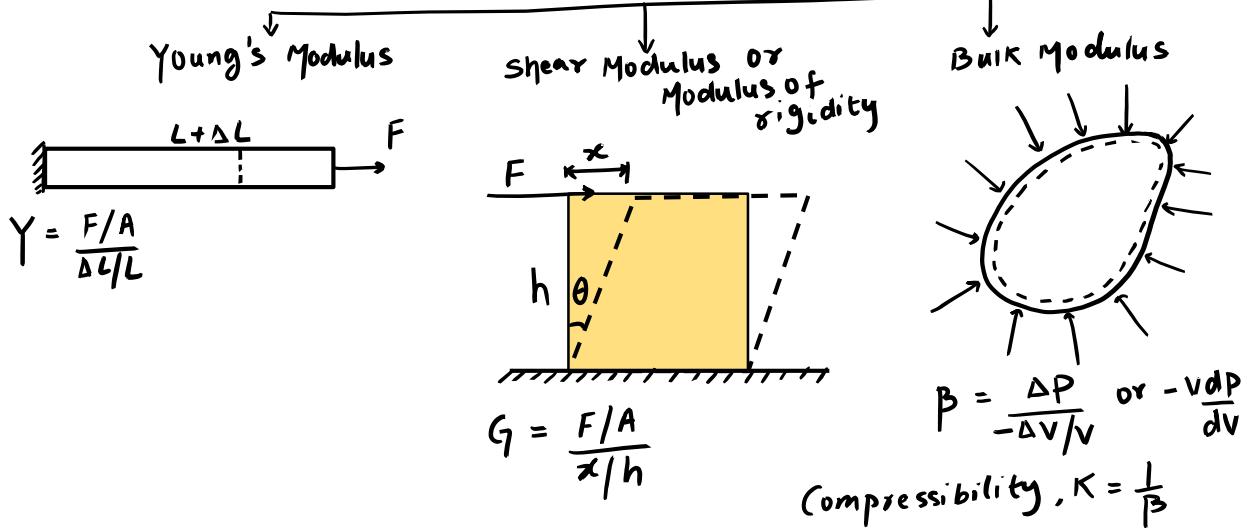
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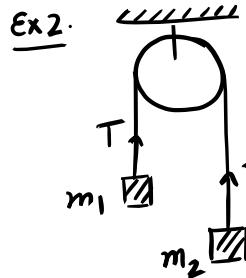
3. Relation between Stress & Strain (Hooke's Law)



4 Modulus of Elasticity

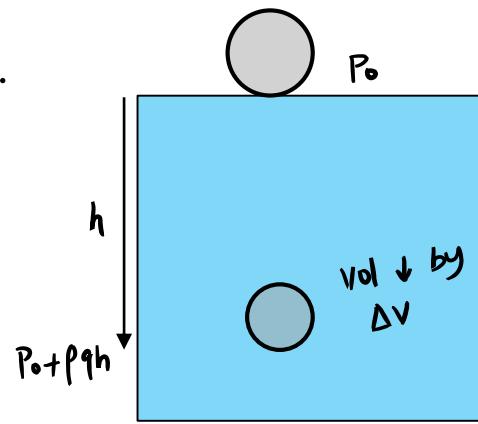


... continued



$$\therefore \frac{T}{A} = Y \cdot \frac{\Delta L}{L}$$

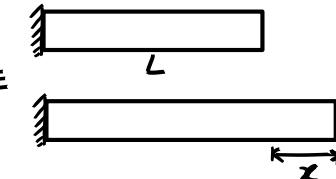
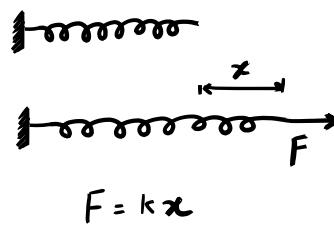
$$T = \frac{2m_1 m_2 g}{m_1 + m_2}$$

Ex 3.

$$\Delta P = B \times \frac{\Delta V}{V} \Rightarrow \Delta V = \frac{V}{B} \cdot \Delta P$$

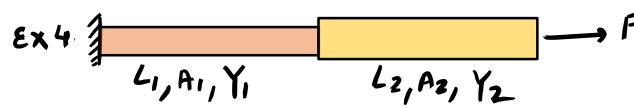
$$\Rightarrow \Delta V = \frac{V}{B} \cdot \rho g h$$

5. Analogy with springs



$$\frac{F}{A} = Y \cdot \frac{x}{L} \Rightarrow F = \boxed{\frac{YA}{L}} x$$

equivalent force constant
 $k = \frac{YA}{L}$



$$K_1 = \frac{Y_1 A_1}{L_1}, \quad K_2 = \frac{Y_2 A_2}{L_2} \quad \therefore K_{eq} = \frac{K_1 K_2}{K_1 + K_2}$$

$$\Rightarrow F = K_{eq} x$$

6. Potential Energy

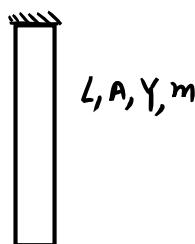
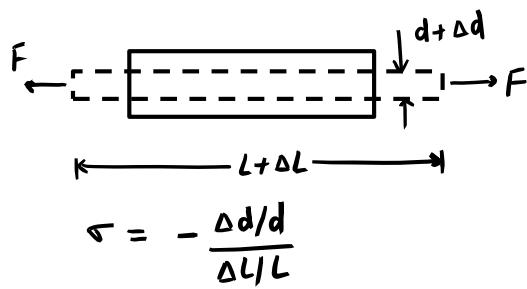
(a) Energy density, $u = \frac{1}{2} \times \text{stress} \times \text{strain}$
 J/m^3

\Rightarrow Energy, $U = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{Volume}$

Note: $U = \frac{1}{2} Kx^2 = \frac{1}{2} \frac{YA}{L} \cdot x^2$

7. Elongation due to Self weight

$\hookrightarrow x = \frac{MgL}{2YA}$

8. Poisson ratio, σ 

$$\sigma = -\frac{\Delta d/d}{\Delta L/L}$$

9. Relation among Y , G & B

(a) $B = \frac{Y}{3(1-2\sigma)}$

(b) $G = \frac{Y}{2(1+\sigma)}$

(c) $B = \frac{YG}{9G-3Y}$ { this Question
asked in
JEE 2021, Feb }

from
(a) & (b)

Space to add concepts learnt from PYQs if any

Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in FLUID STATICS – PART 1

1. Introductory Points (density, pressure)
2. Pressure Variation (vessel at rest)
3. Free Surface (vessel at rest, vessel accelerated linearly, vessel rotating)
4. Pressure Variation (vessel linearly accelerated, rotating)
5. Pascal's Law (hydraulic lift)
6. Barometer
7. Force on side walls due to liquid
8. Torque on side walls
9. Archimedes' Principle (floatation, Apparent weight, center of buoyancy)

Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. Introductory Points

(a) Density, $\rho = m/V$ (kg/m^3 or g/cm^3)

(b) $\rho_{\text{mix}} = \frac{\text{total mass}}{\text{total volume}}$

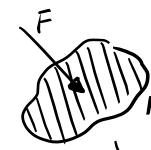
(c) Relative density or specific gravity = ρ/ρ_w at 4°C

Ex: Rel density of Hg = 13.6 $\Rightarrow \rho_{\text{Hg}} = 13.6 \times 10^3 \text{ kg/m}^3$
 $= 13600 \text{ kg/m}^3$
 $(\text{or } 13.6 \text{ g/cc})$

(d) Pressure = F_{\perp}/A

→ Pascal or Pa (N/m^2)

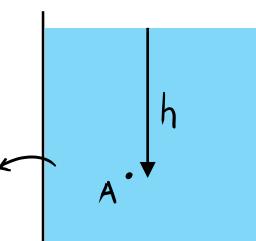
→ $P_0 = 1 \text{ atm} = 1013 \times 10^3 \text{ Pa}$
 $\approx 10^5 \text{ Pa}$



Pressure acts \perp to Surface

2. Pressure Variation (vessel at rest)

(a)



$$P_A = \rho g h + P_0$$

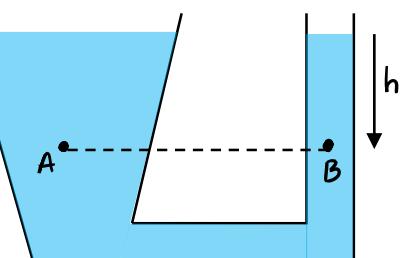
↓
Gauge pressure (P_g)

Absolute Pressure

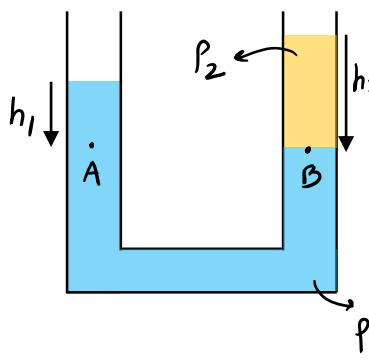
$$(i) P_A = P_B = \rho g h + P_0$$

(ii) Pressure at depth h is independent of shape of vessel

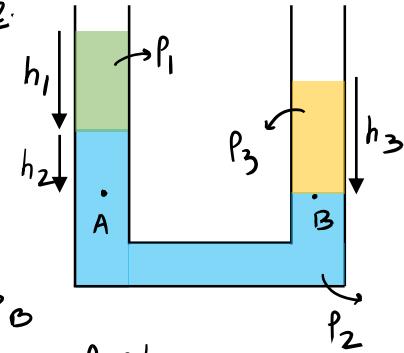
(b)



(iii) At Same Level, Pressure is Same (Same Liquid)

Ex 1.

$$P_A = P_B \Rightarrow P_1 g h_1 + P_0 = P_2 g h_2 + P_0$$

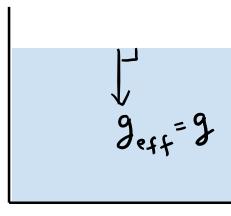
Ex 2.

$$P_A = P_B \Rightarrow P_0 + P_1 g h_1 + P_2 g h_2 = P_0 + P_3 g h_3$$

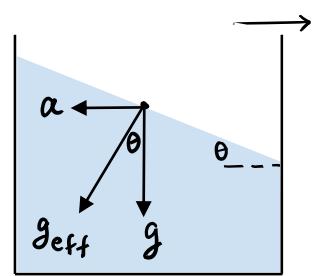
3. Free Surface

↳ free surface is always \perp to g_{eff}

(a)

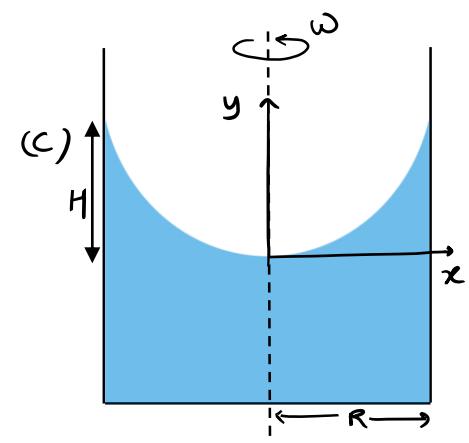


(b)



$$g_{eff} = \sqrt{a^2 + g^2}$$

$$\tan \theta = \frac{a}{g}$$



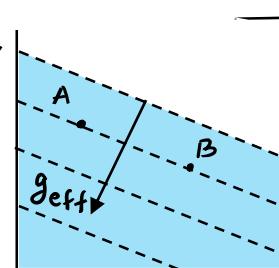
$$y = \frac{\omega^2 x^2}{2g}$$

$$\text{If } x = R, y = H$$

$$\therefore H = \frac{\omega^2 R^2}{2g}$$

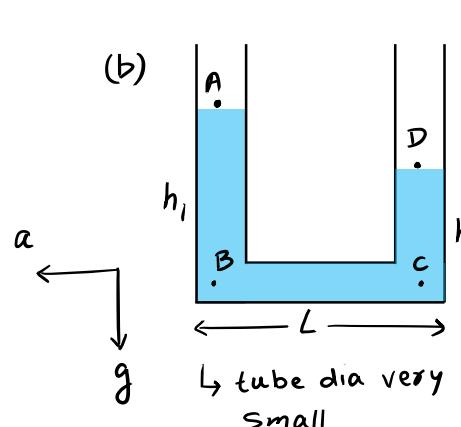
4. Pressure Variation (accelerated system)

(a)



$$P_A = P_B = P_0 + \rho g_{eff} h$$

(b)



↳ tube dia very small

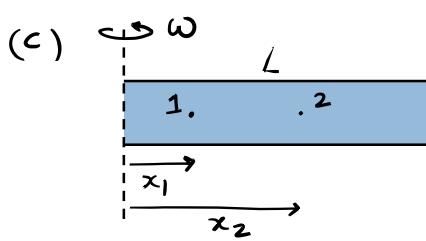
Write eqn from A \rightarrow D \therefore

$$P_A + \rho g h_1 - \rho a L - \rho g h_2 = P_D$$

$$P_A = P_D = P_0$$

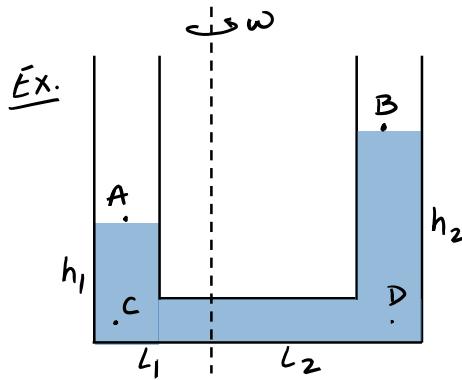
$$\Rightarrow h_1 - h_2 = \frac{a L}{g}$$

PART 1 – FLUID STATICS



$$P_2 - P_1 = \frac{1}{2} \rho \omega^2 (x_2^2 - x_1^2) \quad \left\{ \begin{array}{l} \text{If } x_1 = 0 \text{ i.e. at axis} \\ P_2 - P_1 = \frac{1}{2} \rho \omega^2 x_2^2 \end{array} \right.$$

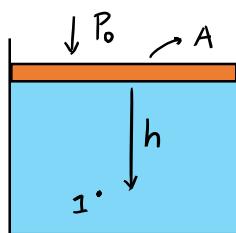
↳ As you move from axis to location 2, Pressure increases by $\frac{1}{2} \rho \omega^2 x_2^2$



$$\begin{aligned} P_A + \rho g h_1 - \frac{1}{2} \rho \omega^2 L_1^2 \\ + \frac{1}{2} \rho \omega^2 L_2^2 - \rho g h_2 = P_B \\ (P_A = P_B = P_0) \\ h_2 - h_1 = \frac{\omega^2}{2g} (L_2^2 - L_1^2) \end{aligned}$$

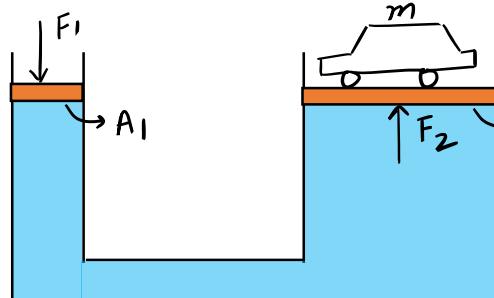
5. Pascal's Law

↳ External pressure gets distributed evenly in all directions



$$P_1 = \rho g h + \frac{mg}{A} + P_0$$

Hydraulic Lift



$$\begin{aligned} F_2 &= P A_2 \\ &= F_1 \cdot \frac{A_2}{A_1} \end{aligned}$$

$$\Rightarrow F_2 = F_1 \left(\frac{A_2}{A_1} \right)$$

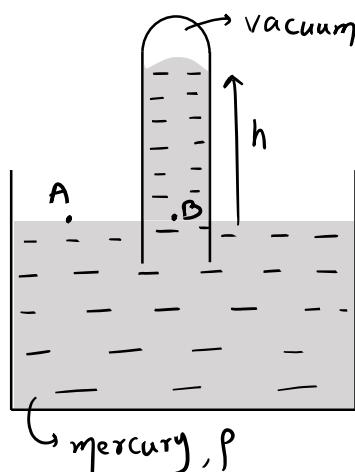
∴ $A_2 \gg A_1 \Rightarrow F_2 \gg F_1$

$$\text{Ex. } mg = F_2 = F_1 \frac{A_2}{A_1}$$

$$\Rightarrow F_1 = \frac{A_1}{A_2} \times mg$$

6. Barometer

↳ setup to measure atmospheric pressure



$$P_A = P_B$$

$$\Rightarrow P_0 = \rho g h$$

$$\text{If } P_0 = 1.013 \times 10^5 \text{ Pa}$$

$$h = \frac{P_0}{\rho g} = \frac{1.013 \times 10^5}{13600 \times 9.8}$$

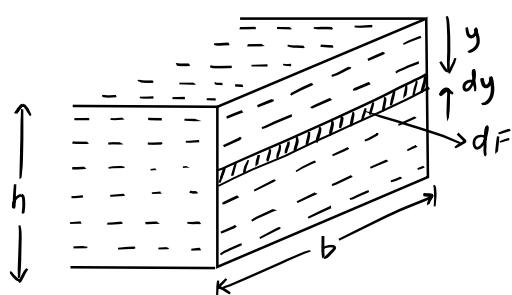
$$= 0.76 \text{ m or } 760 \text{ mm}$$

∴ 1 atm is also written as 760 mm of Hg.

NOTE: We don't use H_2O because $h \approx 10 \text{ m}$ (not practical)

7. Force on sidewalls (due to liquid)

PART 1 – FLUID STATICS



$$dF = P(y) \times dA = \rho g y \times b dy$$

$$\Rightarrow F = \rho g b \int_0^h y dy = \rho g b \times \frac{h^2}{2} = \frac{\rho g h \times b h}{2}$$

$$\therefore F = P_{av} \times \text{Area of wall}$$

$$\hookrightarrow P_{av} = \rho g \frac{h}{2} \text{ (Pressure at center)}$$

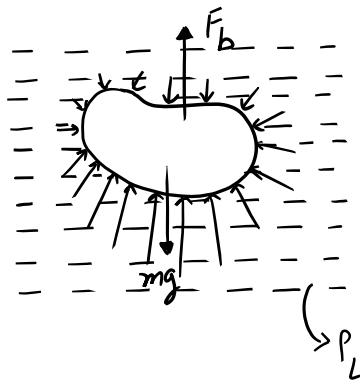
8. Torque on side walls

$$dc = dF \times (h-y) \Rightarrow dc = \rho g y \cdot b dy \cdot (h-y)$$

$$\Rightarrow c = \rho g b \int_0^h y(h-y) dy = \boxed{\frac{\rho g b h^3}{6}}$$

9. Archimedes' Principle / Floatation

(a)



Buoyant force, $F_b = \rho_L V g$ { V is volume of displaced liquid }

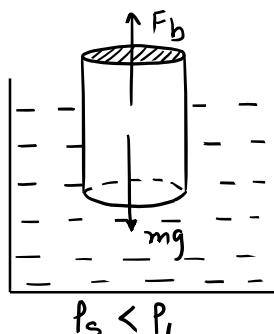
Body is solid (not hollow from inside) :

(i) Body SINKS : $\rho_s > \rho_L$

(ii) Body Floats : $\rho_s \leq \rho_L$

$\rho_s < \rho_L$ \swarrow $\rho_s = \rho_L$
 Some portion is outside \searrow Completely Submerged

(b)



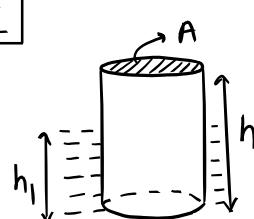
$$F_b = mg \Rightarrow \rho_L V_1 g = \rho_s V g \quad \left\{ \begin{array}{l} V_1 : \text{displaced liquid vol.} \\ V : \text{vol of body} \end{array} \right.$$

$$\Rightarrow \boxed{\frac{V_1}{V} = \frac{\rho_s}{\rho_L}}$$

→ Ex: Ice floating in water

$$\frac{V_1}{V} = \frac{900}{1000} \Rightarrow \frac{V_1}{V} = 0.9$$

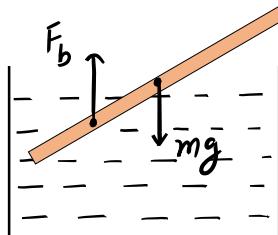
∴ 90% of ice is inside water



$$\frac{A h_1}{A h} = \frac{\rho_s}{\rho_L} \Rightarrow \boxed{\frac{h_1}{h} = \frac{\rho_s}{\rho_L}}$$

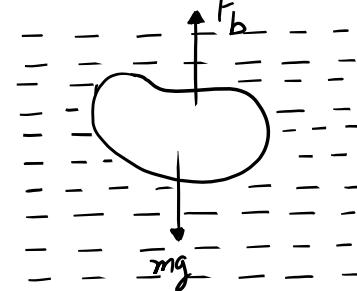
↪ Valid for uniform cross-section

(C) Center of Buoyancy



- (i) C.O.B. is C.O.M of displaced liquid
- (ii) F_b passes through C.O.B.

(d) Apparent weight



$$(i) W_{app} = mg - F_b \quad \left\{ V = \frac{m}{\rho_s} \right.$$

$$\Rightarrow W_{app} = mg - \rho_L V g$$

$$= mg - \rho_L \frac{m}{\rho_s} g = mg \left(1 - \frac{\rho_L}{\rho_s} \right)$$

$$\therefore \text{Reading} = m \left(1 - \frac{\rho_L}{\rho_s} \right)$$

Space to add concepts learnt from PYQs if any

Space to add concepts learnt from PYQs if any

Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in FLUID DYNAMICS – PART 2

1. Types of Flow (Streamline, Turbulent)
2. Reynolds Number
3. Ideal Fluid (4 assumptions)
4. Equation of Continuity
5. Bernoulli's Equation
6. Pressure-speed Relation (horizontal flow)
7. Speed of Efflux – Torricelli Law
8. Speed of Efflux (important other case)
9. Vessel emptying time (y as a function of time)
10. Thrust on Vessel (liquid coming out)
11. Venturimeter

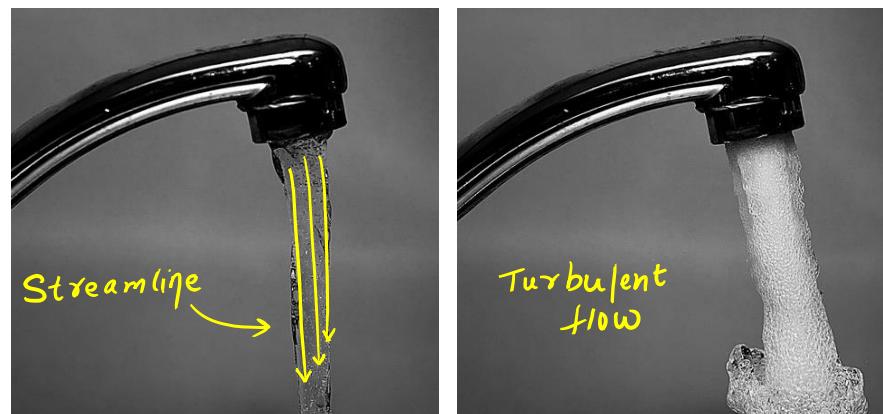
Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. Streamline & Turbulent flow

(Laminar or steady flow)

↳ Velocity of fluid at a point is always same

↳ random flow / mixing of flow
↳ vel of fluid at any point changes



2 Reynolds Number (Re)

$$Re = \frac{\rho v d}{\eta} \rightarrow \text{Dimensionless qty}$$

$Re < 1000 \rightarrow$ streamline flow

$\rho \rightarrow$ density of liquid
 $v \rightarrow$ speed
 $d \rightarrow$ Pipe cross secⁿ dia
 $\eta \rightarrow$ coeff of viscosity

$Re > 2000 \rightarrow$ Turbulent flow

$1000 < Re < 2000 \rightarrow$ Unsteady flow

↳ As mentioned in NCERT

3. Ideal Fluid

PART 2 – FLUID DYNAMICS

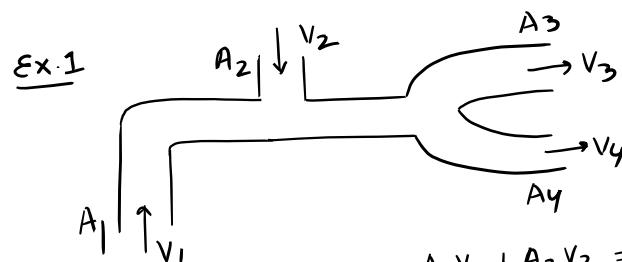
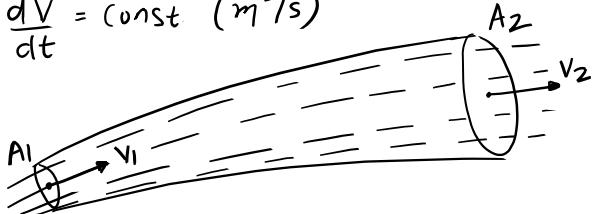
- (i) fluid is incompressible (ρ is const.) (iii) streamline flow
 (ii) non-viscous (no fluid friction) (iv) no rotational effect

4. Eqⁿ of continuity (incompressible)

↪ Volume flow rate is constant, $\frac{dV}{dt} = \text{const.}$ (m^3/s)

$$\Rightarrow \frac{A dx}{dt} = \text{const.} \Rightarrow A v = \text{const.}$$

$$A_1 v_1 = A_2 v_2$$



$$A_1 v_1 + A_2 v_2 = A_3 v_3 + A_4 v_4$$

5. Bernoulli's Eqⁿ

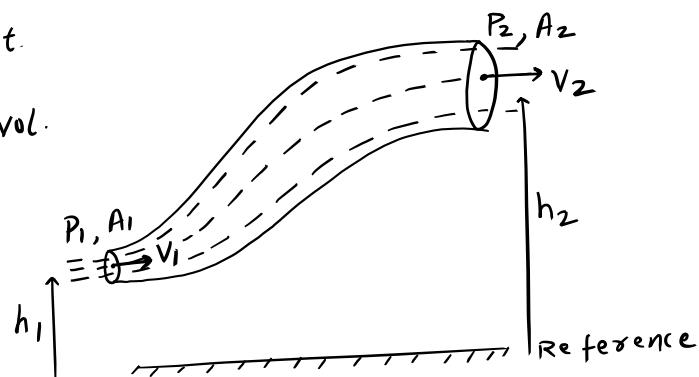
↪ based on conservation of mechanical energy to flow of Ideal fluid

$$\# P + \frac{1}{2} \rho v^2 + \rho g h = \text{const.}$$

Pressure
Energy/vol

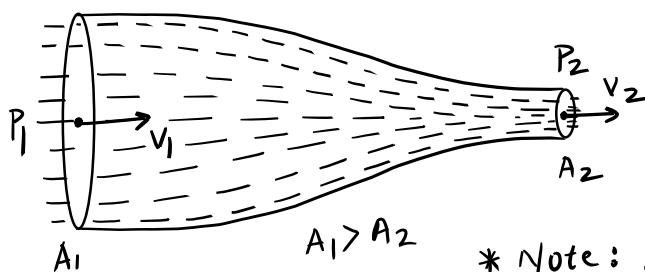
K.E./vol

PE/vol.



$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$$

6. P - v Relation (horizontal flow)



∴ flow is horizontal

$$\Rightarrow P + \frac{1}{2} \rho v^2 = \text{const}$$

Also $A_1 v_1 = A_2 v_2$, so $v_1 < v_2$

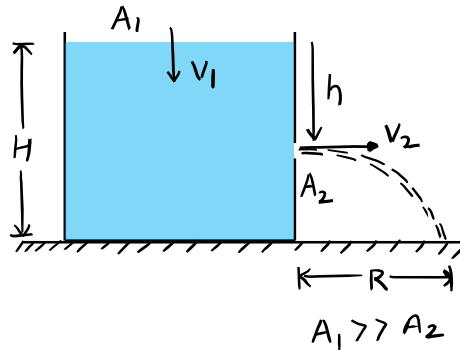
$$\therefore P_1 > P_2$$

* Note: Loss in pressure energy/vol = gain in KE/vol.
 (for horizontal flow)

7. Speed of Efflux: Torricelli's Law

PART 2 – FLUID DYNAMICS

↳ speed of efflux from an open tank is identical to freely falling body.



$$\Rightarrow V_2 >> V_1$$

↳ speed of efflux

→ Just to left and right of hole:

$$\text{Loss in Press. Energy / vol} = \text{Gain of KE / Vol.}$$

$$\Rightarrow (P_0 + \rho gh) - (P_0) = \frac{1}{2} \rho V_2^2 - \frac{1}{2} \rho V_1^2 \approx 0$$

$$\Rightarrow V_2 = \sqrt{2gh}$$

$$(i) \text{ Range, } R = V_2 \times \sqrt{\frac{2(H-h)}{g}} = 2\sqrt{h(H-h)}$$

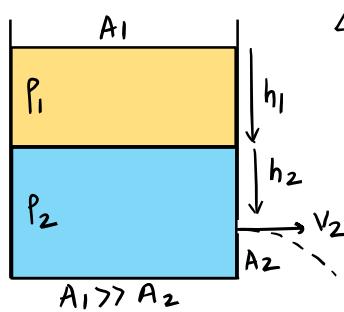
(ii) for R to be max:

$$\frac{dR}{dh} = 0 \Rightarrow h = H/2$$

$$\text{and } R_{\max} = H$$

8. Speed of Efflux (other cases)

(a)

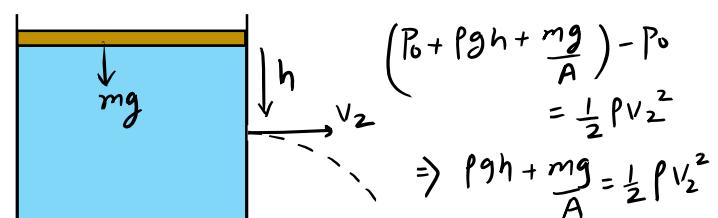


$$\text{Loss in Press. Energy / vol} = \text{Gain of KE / Vol.}$$

$$(P_0 + P_1 gh_1 + P_2 gh_2) - P_0 = \frac{1}{2} \rho_2 V_2^2 \quad \{ V_1 \text{ neglected}$$

$$\Rightarrow P_1 gh_1 + P_2 gh_2 = \frac{1}{2} \rho_2 V_2^2$$

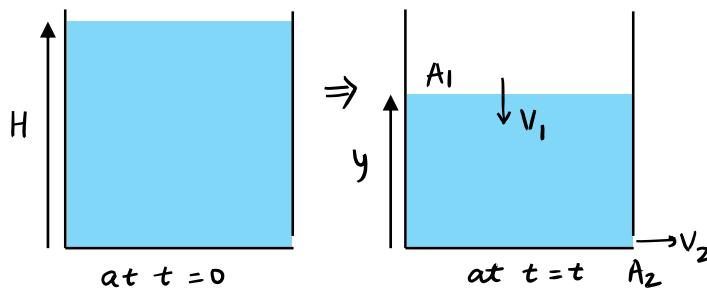
(b)



$$(P_0 + \rho gh + \frac{mg}{A}) - P_0 = \frac{1}{2} \rho V_2^2$$

$$\Rightarrow \rho gh + \frac{mg}{A} = \frac{1}{2} \rho V_2^2$$

9. Vessel Emptying time



$$A_1 V_1 = A_2 V_2 \Rightarrow A_1 \left(-\frac{dy}{dt} \right) = A_2 \sqrt{2gy}$$

$$\Rightarrow \int_{H}^{y} \frac{dy}{\sqrt{y}} = - \frac{A_2}{A_1} \sqrt{2g} \int_0^t dt$$

$$\Rightarrow 2(\sqrt{y} - \sqrt{H}) = - \frac{A_2}{A_1} \sqrt{2g} \cdot t$$

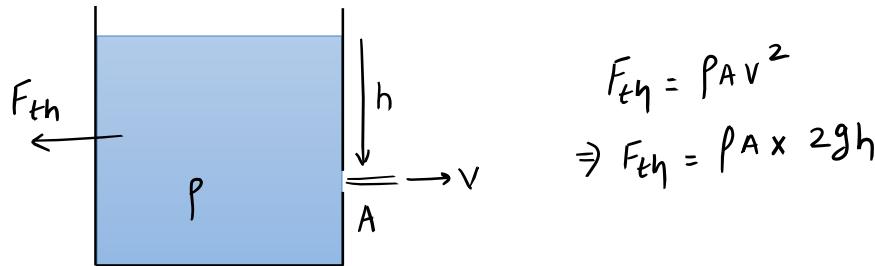
$$\therefore t = \frac{A_1}{A_2} \sqrt{\frac{2}{g}} (\sqrt{H} - \sqrt{y})$$

↳ To empty $y=0$ at $t=t_0$

$$\Rightarrow t_0 = \frac{A_1}{A_2} \sqrt{\frac{2H}{g}}$$

10. Thrust on Vessel (Liquid coming out)

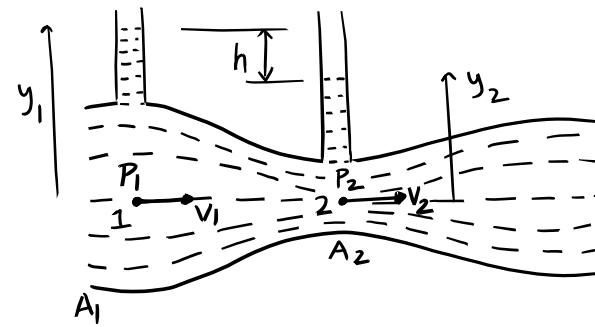
PART 2 – FLUID DYNAMICS



11. Venturiometer

↪ A device / setup to measure flow speed

(a) Setup 1:



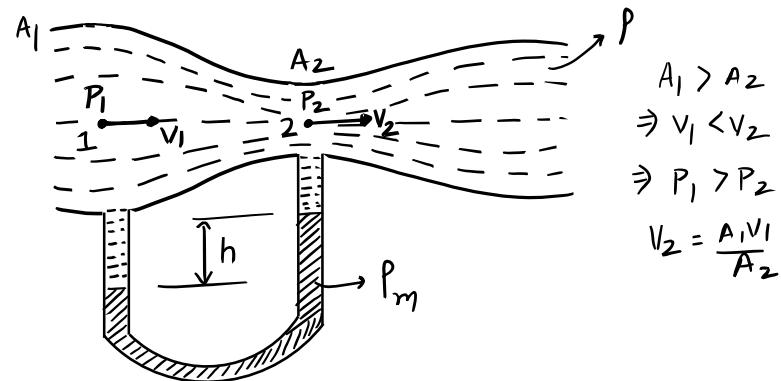
$$A_1 > A_2 \Rightarrow v_1 < v_2 \Rightarrow P_1 > P_2$$

$$v_2 = \frac{A_1 v_1}{A_2}$$

$$P_1 - P_2 = \frac{1}{2} \rho v_2^2 - \frac{1}{2} \rho v_1^2 \Rightarrow \rho g y_1 - \rho g y_2 = \frac{1}{2} \rho \left(\frac{A_1 v_1}{A_2} \right)^2 - \frac{1}{2} \rho v_1^2$$

$$\Rightarrow \rho g h = \frac{1}{2} \rho v_1^2 \left[\left(\frac{A_1}{A_2} \right)^2 - 1 \right] \therefore v_1 = \sqrt{\frac{2gh}{\left(\frac{A_1}{A_2} \right)^2 - 1}}$$

(b) Setup 2:



$$A_1 > A_2 \Rightarrow v_1 < v_2 \Rightarrow P_1 > P_2$$

$$v_2 = \frac{A_1 v_1}{A_2}$$

$$P_1 - P_2 = \frac{1}{2} \rho v_2^2 - \frac{1}{2} \rho v_1^2 \Rightarrow P_m g h = \frac{1}{2} \rho \left(\frac{A_1 v_1}{A_2} \right)^2 - \frac{1}{2} \rho v_1^2$$

$$\therefore v_1 = \sqrt{\frac{2P_m gh}{\rho \left[\left(\frac{A_1}{A_2} \right)^2 - 1 \right]}}$$

Space to add concepts learnt from PYQs if any

Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in FLUID PROPERTIES – PART 3

1. Cohesive vs Adhesive Force
2. Understanding Surface Tension ($S = F/L$)
3. Lifting bodies (ring, disc, plate etc.)
4. Surface Energy ($S = E/A$)
5. Drop Coalesce & Bubble collapse
6. Excess Pressure (Drop & Bubble)
7. 2 Soap bubbles in contact (Externally / Internally)
8. Angle of contact
9. Capillary Action (in tube as well as parallel plates)
10. Insufficient tube length
11. Force to separate glass plates
12. Understanding Viscous Force
13. Poiseuille's Equation – Flow through narrow tube
14. Stokes' Law & Terminal velocity

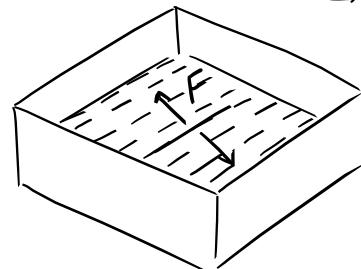
Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. Cohesive vs Adhesive Force

↓
Force of attraction
between molecules
of same substance

→ Force of attraction between molecules of
different substance.

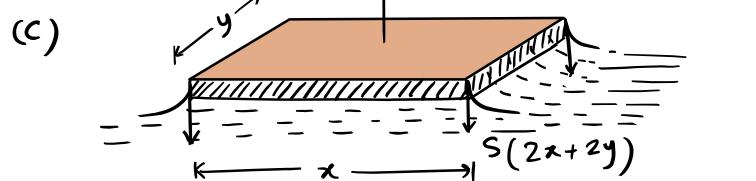
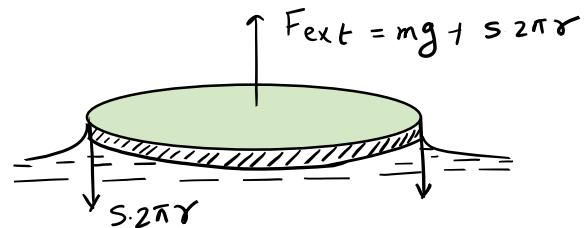
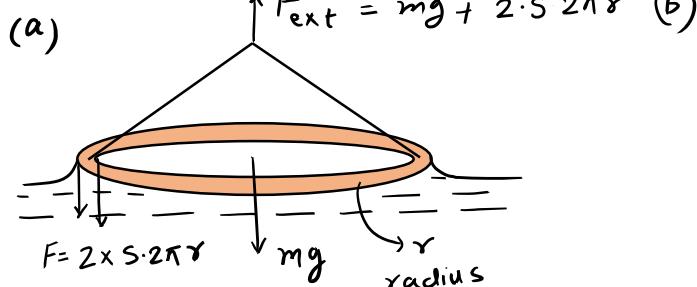
2. Understanding Surface Tension



→ property of liquid where Surface
(i) tries have minimum Area
(ii) Acts as stretched membrane
→ $S = \frac{F}{L}$ or $F = SL$

NOTE:
F acts perpendicular to Line
and tangential to free Surface.

3. Lifting of bodies



4. Surface Energy

↳ Due to intermolecular interaction, free surface of liquid has energy called "Surface Energy"

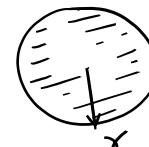
$$S = \frac{E}{A} \quad \text{or} \quad E = S \cdot A$$

A : Surface Area

S : Surface Tension

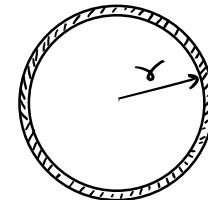
E : Surface energy

Ex 1. Energy of drop



$$E = s \cdot 4\pi r^2$$

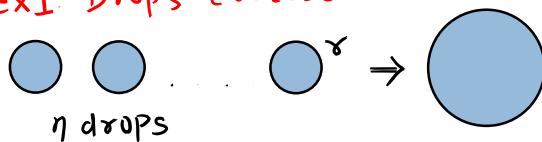
Ex 2. Energy of Bubble



$$E = 2 \times s \cdot 4\pi r^2$$

5. Standard Questions on Surface Energy

Ex 1. Drops Coalesce



Here Area $\downarrow \Rightarrow$ Energy \downarrow

$$E_i = n \cdot s \cdot 4\pi r^2$$

$$E_f = s \cdot 4\pi R^2 = s \cdot 4\pi r^2 \cdot n^{2/3}$$

$\therefore E_i - E_f$ increases Temp of Bigger drop by $\Delta\theta$

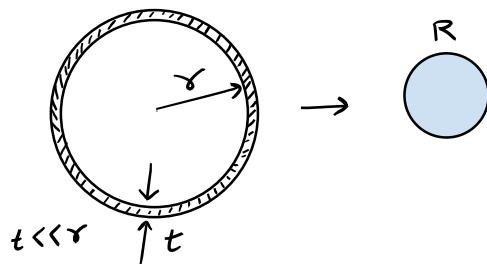
$$\Rightarrow E_i - E_f = m s \Delta\theta$$

\therefore Volume is same

$$\Rightarrow \frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3$$

$$\text{or, } R = r n^{1/3}$$

Ex 2. Bubble Collapse



$$E_i = 2 \times S \cdot 4\pi r^2$$

$$E_f = S \cdot 4\pi R^2$$

$$= S \cdot 4\pi (3r^2 t)^{2/3}$$

\therefore Vol of liquid is same

$$4\pi r^2 \times t = \frac{4}{3}\pi R^3$$

$$R = (3r^2 t)^{1/3}$$

$$\therefore E_i - E_f = m s \Delta \theta$$

6. Excess Pressure in Liquid Drop and Bubble

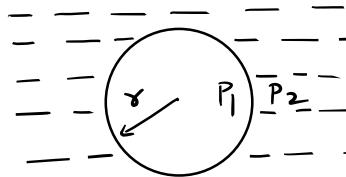
Drop 

$$P_{ex} = P_1 - P_2 = \frac{2S}{r}$$

Bubble 

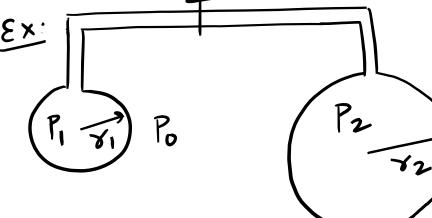
$$P_{ex} = P_1 - P_2 = \frac{4S}{r}$$

#NOTE: If the bubble is in liquid (ex. Aquarium)



$$P_1 = P_2 + \rho g h$$

$$P_1 - P_2 = \frac{2S}{r}$$

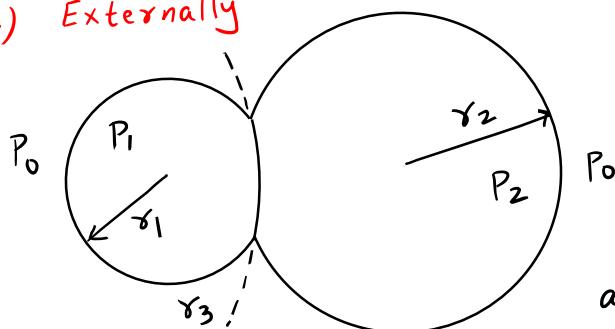
Ex: 

$$P_1 = \frac{4S}{r_1} + P_0$$

$$P_2 = \frac{4S}{r_2} + P_0$$

7. TWO SOAP bubbles in Contact

(1) Externally



$$P_2 - P_0 = \frac{4S}{r_2} \quad \dots (1)$$

$$P_1 - P_0 = \frac{4S}{r_1} \quad \dots (2)$$

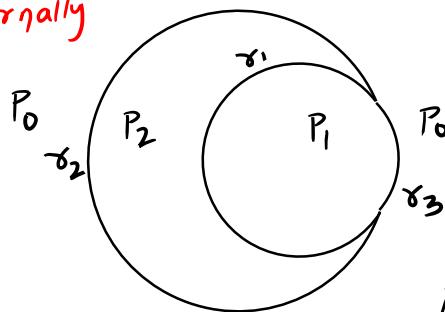
$$\text{Subtract (2) & (1)} \Rightarrow P_1 - P_2 = \frac{4S}{r_1} - \frac{4S}{r_2}$$

$$\text{also, } P_1 - P_2 = \frac{4S}{r_3}$$

$$\therefore \frac{4S}{r_3} = \frac{4S}{r_1} - \frac{4S}{r_2} \Rightarrow$$

$$r_3 = \frac{r_1 r_2}{r_2 - r_1}$$

(ii) Internally



$$P_1 - P_2 = \frac{4s}{r_1} \quad \text{--- (1)}$$

$$P_2 - P_0 = \frac{4s}{r_2} \quad \text{--- (2)}$$

$$\text{Adding (1) \& (2)} \Rightarrow P_1 - P_0 = \frac{4s}{r_1} + \frac{4s}{r_2}$$

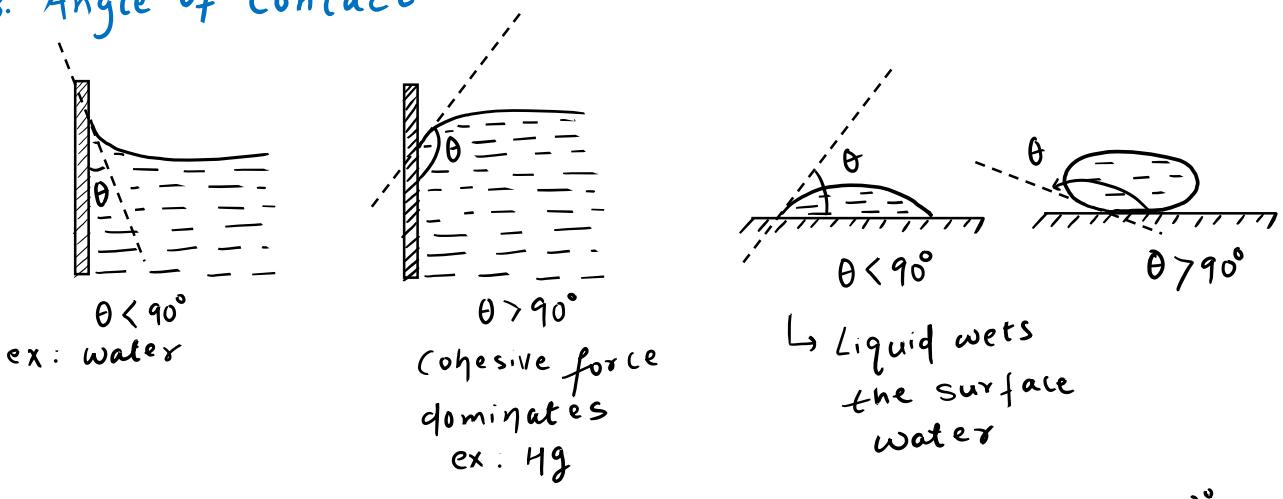
$$\text{Also, } P_1 - P_0 = \frac{4s}{r_3}$$

$$\therefore \frac{4s}{r_3} = \frac{4s}{r_1} + \frac{4s}{r_2} \Rightarrow$$

$$r_3 = \frac{r_1 r_2}{r_1 + r_2}$$

r_3 will be smallest

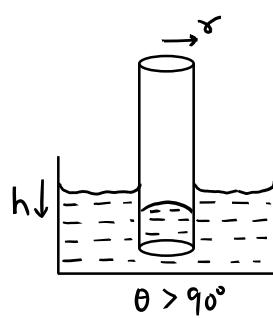
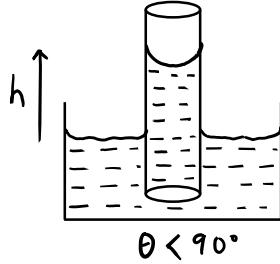
8. Angle of Contact



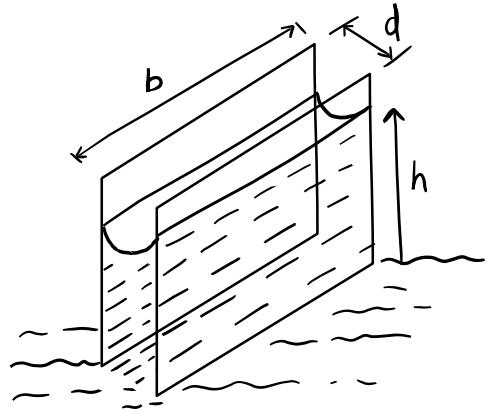
Note: Generally θ for distilled H_2O is 0° .

9. Capillary Action

(i) Thin tube



(ii)



$$\# h = \frac{2s \cos \theta}{\gamma \rho g} \text{ or } \frac{2s}{R \rho g}$$

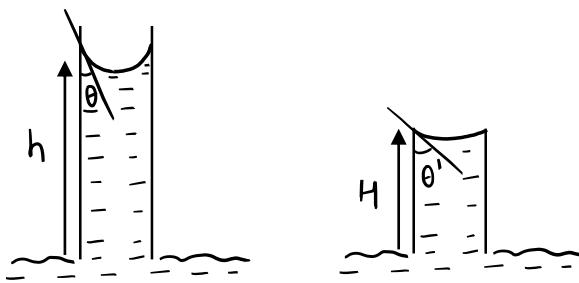
$$h = \frac{2s \cos \theta}{d \rho g}$$

r is tube radius, R is meniscus radius

↳ Generally if its H_2O $\cos \theta = 1$.

10 Insufficient Tube Length ($H < h$)

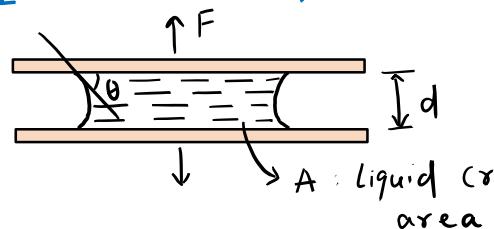
PART 3 – FLUID PROPERTIES



$$h = \frac{2s \cos \theta}{\gamma g} \quad \text{but } H < h$$

→ In this case liquid won't overflow rather it reaches top and adjusts contact angle to θ' ($\theta' > \theta$)

11. Force to separate Glass Plates

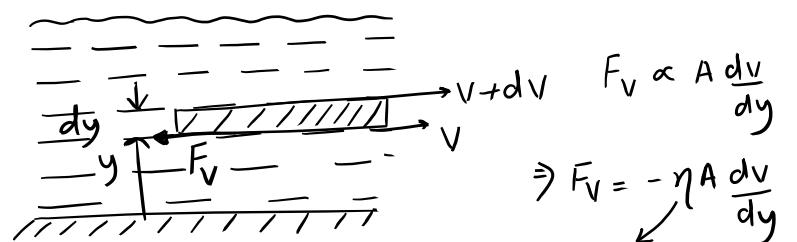
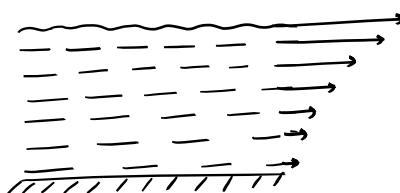


$$F = \frac{2sA \cos \theta}{d}$$

{ If it's H₂O
 $\cos \theta = 1$

12. Understanding Viscous Force

→ Fluid opposes the relative motion between its different layers.



$$F_V \propto A \frac{dv}{dy}$$

$$\Rightarrow F_V = -\eta A \frac{dv}{dy}$$

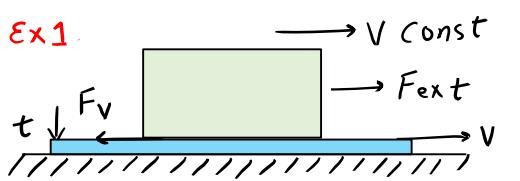
Coef of Viscosity

$$(\eta) = ML^{-1}T^{-1}$$

SI unit: Poiseuille, PI

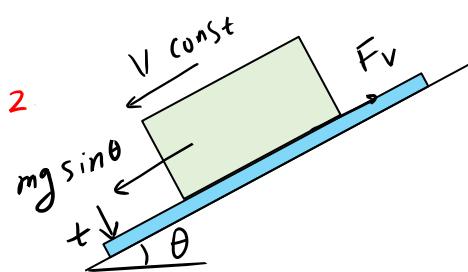
C.G.S: Poise 1 PI = 10 Poise

Ex 1



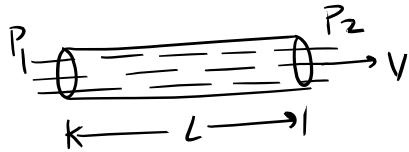
$$F_{ext} = F_V = \eta A \frac{dv}{dy} = \eta A \frac{V}{t}$$

Ex 2



$$mg \sin \theta = \eta A \frac{V}{t}$$

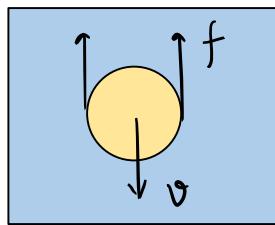
13. Poiseuille's eqn - Flow through Narrow Tube



$$\text{Flow rate, } Q = \frac{\pi r^4}{8\eta L} (P_1 - P_2)$$

(m³/s)

14. Stokes' Law & Terminal Velocity



$$f = 6\pi\eta rV$$

→ As $V \uparrow$, $F_v \uparrow$

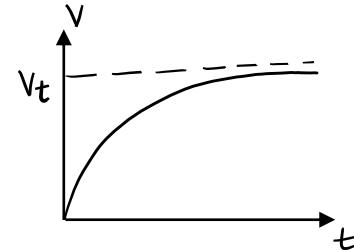
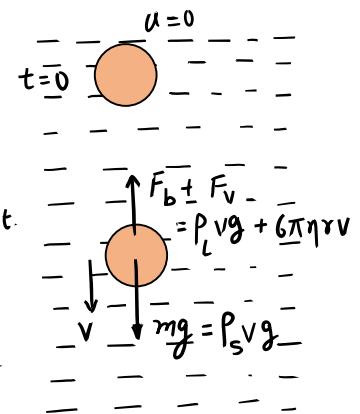
and at certain speed

$$F_b + F_v = mg \quad \left\{ \Rightarrow a = 0, V = \text{const.} \right.$$

Terminal
velocity

$$\Rightarrow P_L \frac{4}{3}\pi r^3 g + 6\pi\eta rV_t = P_s \frac{4}{3}\pi r^3 g$$

$$V_t = \frac{2r^2 g}{9\eta} (P_s - P_L)$$



Space to add concepts learnt from PYQs if any

Space to add concepts learnt from PYQs if any

Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in SIMPLE HARMONIC MOTION

1. Kinematics and Energy of SHM
2. Steps to Find Time Period (Linear/Angular SHM)
3. Spring Block and Simple Pendulum
4. T of Spring Block (under constant force)
5. Combination of Spring
6. Spring Cut
7. Two Block System (Reduced Mass)
8. SHM of Block in a Liquid
9. SHM of Piston in Cylinder
10. SHM in Tunnel of Planet
11. SHM of Charge
12. Physical Pendulum
13. Torsional Pendulum
14. SHM of Dipole in Field (Electric/Magnetic Dipole)
15. Time Period of Simple Pendulum (Different Conditions)
16. Superposition of SHM
17. Damped Oscillation

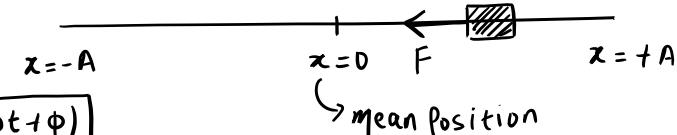
Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. KINEMATICS/ENERGY OF SHM

$$\begin{aligned} F &\propto -x \\ \Rightarrow F &= -kx \\ a &= -\left(\frac{k}{m}\right)x \end{aligned}$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \Rightarrow x = A \sin(\omega t + \phi)$$

$$\omega^2 = \frac{k}{m} \quad \text{or} \quad T = 2\pi \sqrt{\frac{m}{k}}$$



mean position

$$\begin{aligned} v_{\max} &= \pm \omega A, \quad x = 0 \\ v_{\min} &= 0, \quad x = \pm A \end{aligned}$$

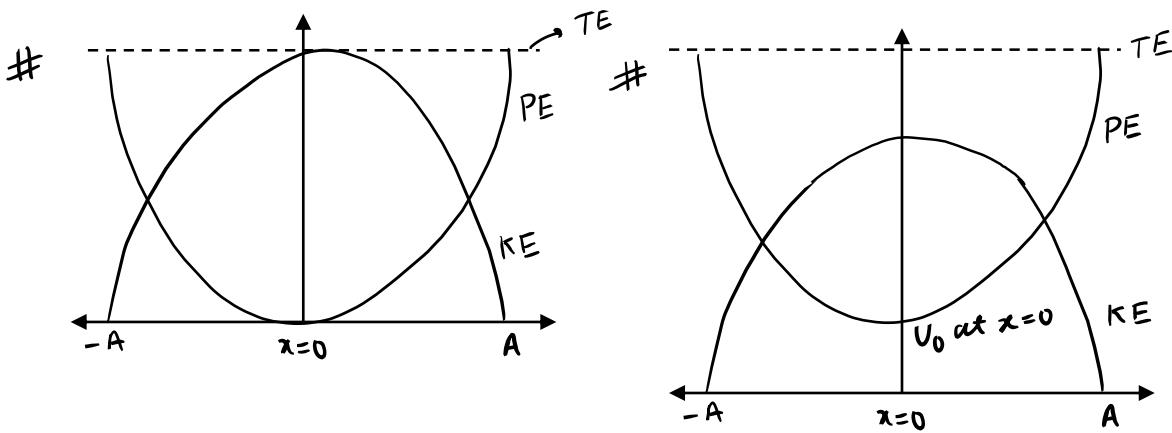
$$\begin{aligned} (a) \quad v(x) &= \omega \sqrt{A^2 - x^2} \\ v(t) &= A \omega \cos \omega t \end{aligned}$$

$$\begin{aligned} (b) \quad a(x) &= -\omega^2 x \\ a(t) &= -\omega^2 A \sin \omega t \end{aligned}$$

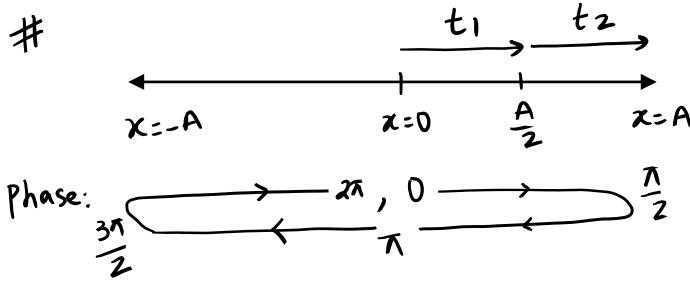
$$\begin{aligned} a_{\max} &= \pm \omega^2 A, \quad x = \mp A \\ a_{\min} &= 0, \quad x = 0 \end{aligned}$$

$$(c) \quad KE(x) = \frac{1}{2} m \omega^2 (A^2 - x^2), \quad PE(x) = \frac{1}{2} m \omega^2 x^2$$

$$TE = KE + PE = \frac{1}{2} m \omega^2 A^2 \quad \text{constant.}$$



At mean position
system has PE.



$$\begin{aligned}
 \text{(i) } t_{x=0 \rightarrow x=A} &= T/4 \\
 \text{(ii) } x = A \sin \omega t \\
 &\Rightarrow \frac{A}{2} = A \sin \omega t_1 \\
 &\Rightarrow \frac{1}{2} = \sin \omega t_1 \Rightarrow \frac{\pi}{6} = \frac{2\pi}{T} t_1
 \end{aligned}$$

$$t_1 = \frac{T}{12}$$

$$\text{(iii) } t_2 = \frac{T}{4} - t_1 = \boxed{\frac{1}{6}}$$

2. STEPS TO FIND TIME PERIOD

LINEAR SHM

(1) Give linear displacement of x from mean position

(2) Find linear acceleration $F=ma$

(3) you will get

$$a = - \underbrace{(\text{some constant}) \times x}_{\omega^2}$$

$$\therefore T = 2\pi/\omega$$

ANGULAR SHM

(1) Give angular displacement of θ (small) from mean position.

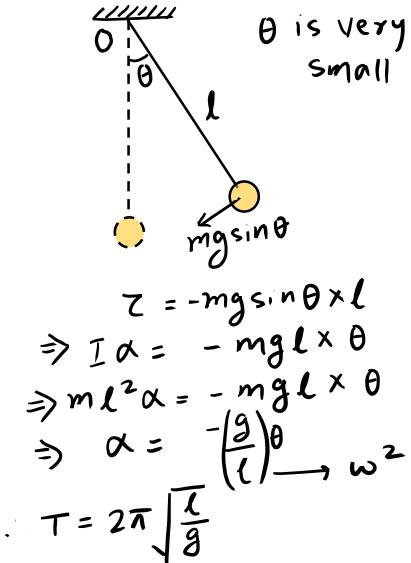
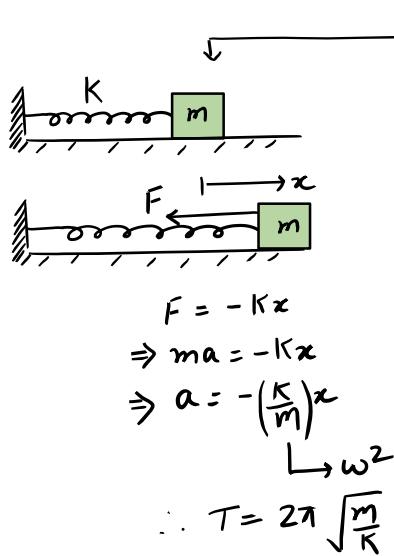
(2) Find angular acceleration $\tau = I\alpha$

(3) you will get

$$\alpha = - \underbrace{(\text{some const.}) \times \theta}_{\omega^2}$$

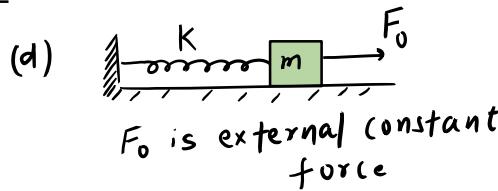
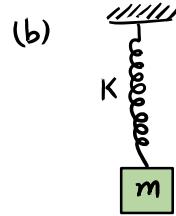
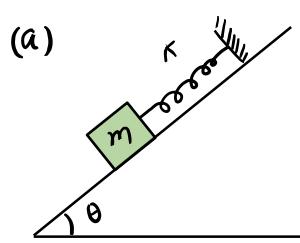
$$\therefore T = 2\pi/\omega$$

3. SPRING BLOCK AND SIMPLE PENDULUM



4. SPRING BLOCK (under presence of Constant force)

NOTE: Result remains same (Only mean position changes)



(i) In all cases

$$T = 2\pi \sqrt{\frac{m}{K}}$$

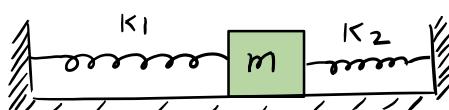
(ii) Mean position changes

5. COMBINATION OF SPRING

(a) \equiv $K_{eq} = \frac{K_1 K_2}{K_1 + K_2}$

(b) \equiv $K_{eq} = K_1 + K_2$

Ex: 1



Here still K_1 and K_2 are in parallel

$$T = 2\pi \sqrt{\frac{m}{K_1 + K_2}}$$

Ex: 2 $T = 2\pi \sqrt{\frac{m}{3K}}$

$$K_{eq} = \frac{2K}{2} + 2K = 3K$$

6. SPRING CUT ($K \cdot l = \text{const.}$)

$$\text{Original} = \text{Cut} \rightarrow K, l = K_1, l_1 + K_2, l_2$$

$$Kl = K_1 l_1 = K_2 l_2$$

$$K_1 = \frac{Kl}{l_1}, \quad K_2 = \frac{Kl}{l_2}$$

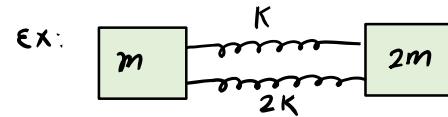
Ex: $\frac{K, l}{K_1, l_1 + K_2, l_2}$

$$\frac{K, l}{\frac{K}{l/3} + \frac{K}{l/3}} = \frac{K, l}{\frac{2K}{l}} = \frac{l}{2}$$

$$\therefore \text{Each } K' = 3K$$

7. TWO BLOCK SYSTEM

$$(\text{Reduced mass, } M_{\text{red}} = \frac{m_1 m_2}{m_1 + m_2})$$

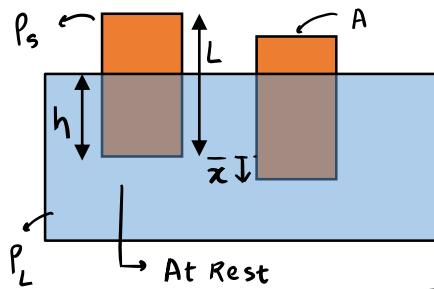


$$T = 2\pi \sqrt{\frac{M_{\text{red}}}{K_{\text{eq}}}}$$

$$\# M_{\text{red}} = \frac{m \times 2m}{m + 2m} = \frac{2m}{3}$$

$$K_{\text{eq}} = K + 2K = 3K$$

$$T = 2\pi \sqrt{\frac{2m}{9K}}$$

8. BLOCK IN LIQUID ($P_s < P_L$)

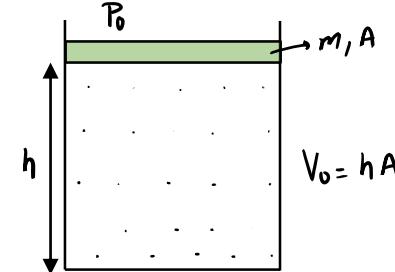
$$T = 2\pi \sqrt{\frac{h}{g}} \quad \text{or} \quad T = 2\pi \sqrt{\frac{m}{AP_L g}}$$

A. cross-section area

At equilibrium:

$$\frac{P_s}{P_L} = \frac{h}{L}$$

9. SHM OF PISTON IN CYLINDER



Piston given small displacement

Isothermal

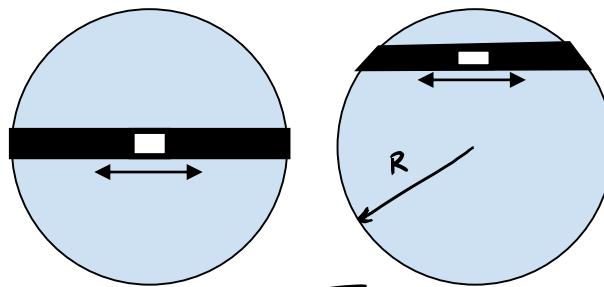
$$T = 2\pi \sqrt{\frac{mV_0}{P_0 A^2}}$$

Adiabatic

$$T = 2\pi \sqrt{\frac{mV_0}{\gamma P_0 A^2}}$$

$$C_P/C_V$$

10. SHM IN TUNNEL IN A PLANET

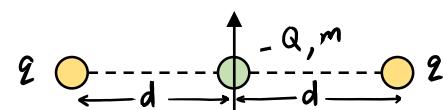


$$T = 2\pi \sqrt{\frac{R}{g}}$$

$$\text{where } g = \frac{GM}{R^2}$$

11. SHM OF CHARGE

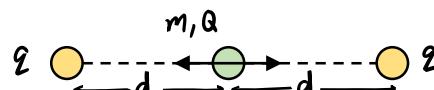
(a)



For small disturbance

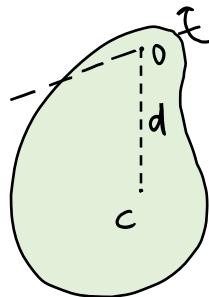
$$T = 2\pi \sqrt{\frac{md^3}{2KQ^2}}, \quad K = \frac{1}{4\pi\epsilon_0}$$

(b)



$$T = 2\pi \sqrt{\frac{md^3}{4KQ^2}}, \quad K = \frac{1}{4\pi\epsilon_0}$$

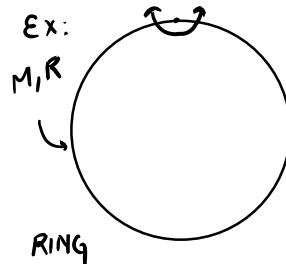
12. PHYSICAL PENDULUM



$$T = 2\pi \sqrt{\frac{I}{mgd}}$$

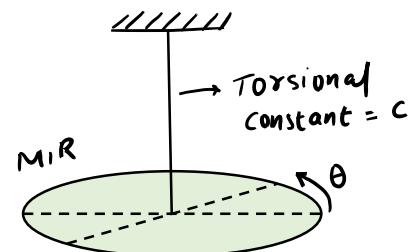
I : MOI of body about O (Point of suspension)

d : Distance between O and C (Centre of mass)



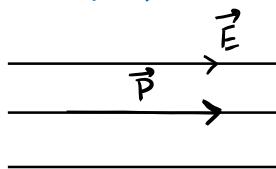
$$\begin{aligned} I &= MR^2 + MR^2 \\ &= 2MR^2 \\ d &= R \\ \therefore T &= 2\pi \sqrt{\frac{2MR^2}{MgR}} \\ \Rightarrow T &= 2\pi \sqrt{\frac{2R}{g}} \end{aligned}$$

13. TORSIONAL PENDULUM



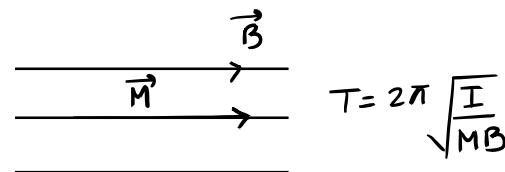
$$\begin{aligned} \tau &= -C\theta \\ \Rightarrow I\alpha &= -C\theta \\ \therefore \alpha &= -\left(\frac{C}{I}\right)\theta \\ \therefore T &= 2\pi \sqrt{\frac{I}{C}} \end{aligned}$$

14. SHM OF DIPOLE IN FIELD (small angular displacement)



$$T = 2\pi \sqrt{\frac{I}{PE}}$$

I : Moment of Inertia

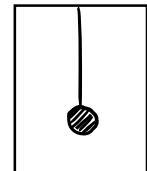


$$T = 2\pi \sqrt{\frac{I}{MB}}$$

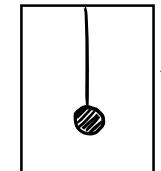
I : Moment of Inertia

15. T OF SIMPLE PENDULUM (Different Condition)

$$T = 2\pi \sqrt{\frac{L}{g_{\text{eff}}}}$$

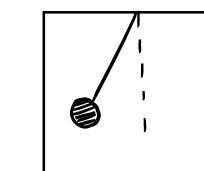


$$g_{\text{eff}} = a + g$$



$$g_{\text{eff}} = g - a$$

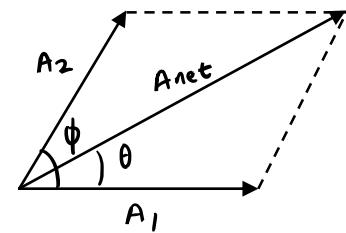
$$a$$



$$g_{\text{eff}} = \sqrt{a^2 + g^2}$$

16. SUPERPOSITION OF SHM

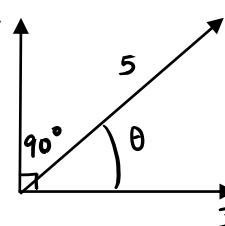
$$\begin{aligned} Y &= A_1 \sin \omega t + A_2 \sin(\omega t + \phi) \\ &= A_{\text{net}} \sin(\omega t + \theta) \\ \downarrow & \quad \tan^{-1} \left(\frac{A_2 \sin \phi}{A_1 + A_2 \cos \phi} \right) \\ \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos \phi} & \end{aligned}$$



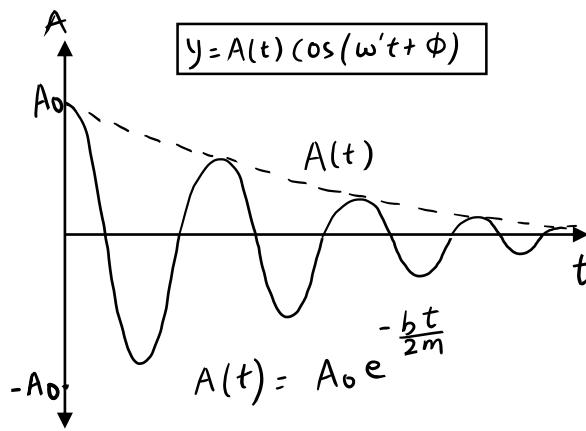
$$\text{Ex: } Y = 3 \sin(\omega t + 30^\circ) + 4 \sin(\omega t + 120^\circ)$$

$$\Rightarrow Y = 5 \sin(\omega t + 30^\circ + \theta)$$

$$\text{where } \theta = \tan^{-1} \frac{4}{3}$$



17. Damped Oscillation



b: Damping Constant (depends on medium)

$$y = A(t) \cos(\omega't + \phi)$$

$$\rightarrow A(t) = A_0 e^{-\frac{bt}{2m}}$$

$$\rightarrow \omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

Space to add concepts learnt from PYQs if any

Space to add concepts learnt from PYQs if any

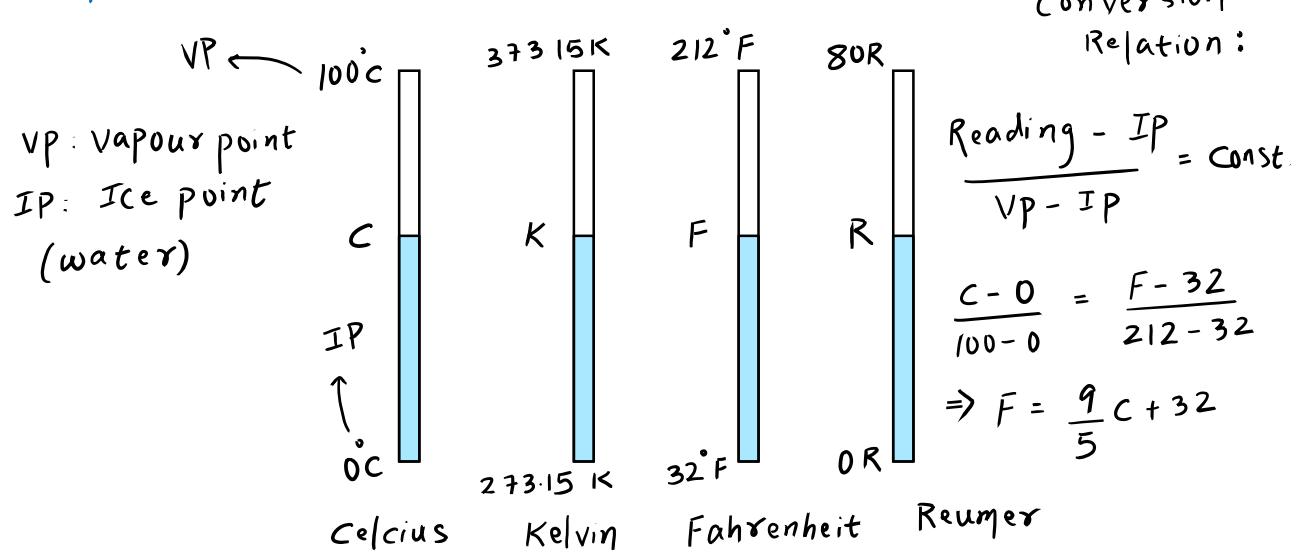
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Topics to cover in Thermal Properties – Part 1

1. Temperature Scales
2. Thermal Expansion
3. Effect of Temperature on density
4. Anomalous behavior of Water
5. Expansion in Cavities / Gaps etc.
6. Apparent Expansion of Liquid (*Hg rise in Thermometer*)
7. Effect of Temperature on Pendulum Clocks (*Second's Pendulum*)
8. Thermal Stress
9. Bimetallic Strip
10. Calorimetry – Heat Exchange
11. Phase Transformation (*Graph Understanding*)
12. Water Equivalent

Note: For video refer Revision Series Playlist on EDUNI YouTube Channel

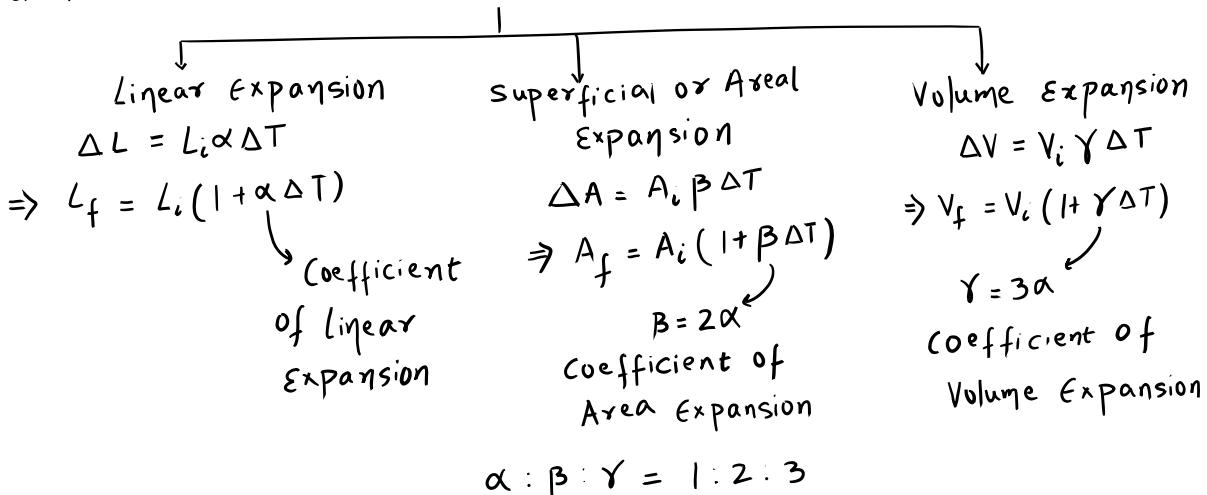
1. Temperature Scales



2. Thermal Expansion

↳ distance between any two points ↑ due to rise in temp.

Part 1 – Thermal Properties



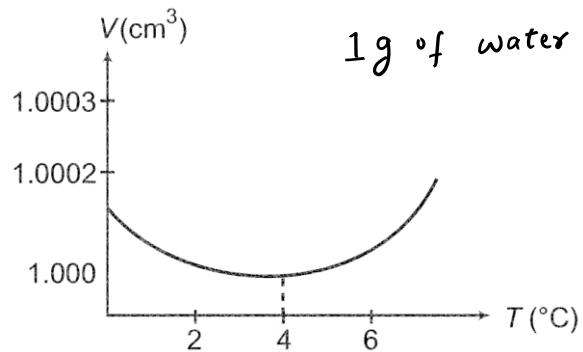
3. Effect of temperature on density

$$\rho_i = \frac{m}{V_i}, \quad \rho_f = \frac{m}{V_f} = \frac{m}{V_i(1 + \gamma \Delta T)}$$

$$\Rightarrow \rho_f = \frac{\rho_i}{(1 + \gamma \Delta T)}$$

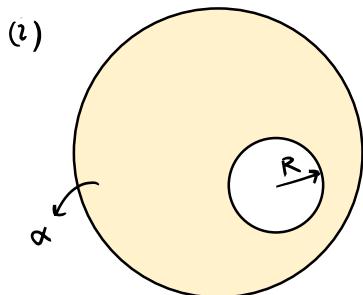
$$\text{If } \gamma \Delta T \ll 1 \Rightarrow \boxed{\rho_f = \rho_i (1 - \gamma \Delta T)}$$

4. Anomalous Expansion of Water



↳ This anomalous behaviour causes ice to form first at the surface of Lake in cold weather

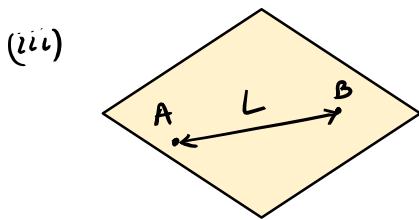
5. Expansion in Cavities/ Gaps etc.



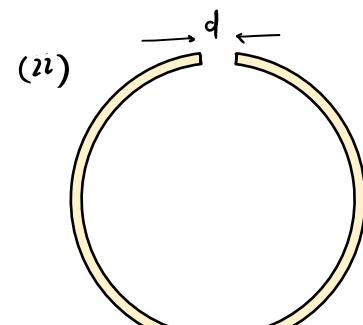
Disc, $\Delta T \rightarrow R_f = R(1 + \alpha \Delta T)$
 $A_f = \pi R^2 (1 + 2\alpha \Delta T)$

Sphere, $\Delta T \rightarrow R_f = R(1 + \alpha \Delta T)$

$$V_f = \frac{4}{3} \pi R^3 (1 + 3\alpha \Delta T)$$



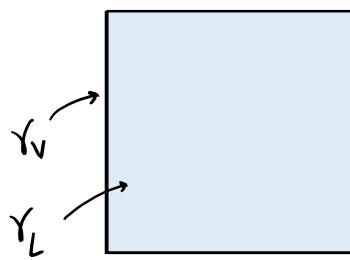
$$L_f = L(1 + \alpha \Delta T)$$



$$d_f = d(1 + \alpha \Delta T)$$

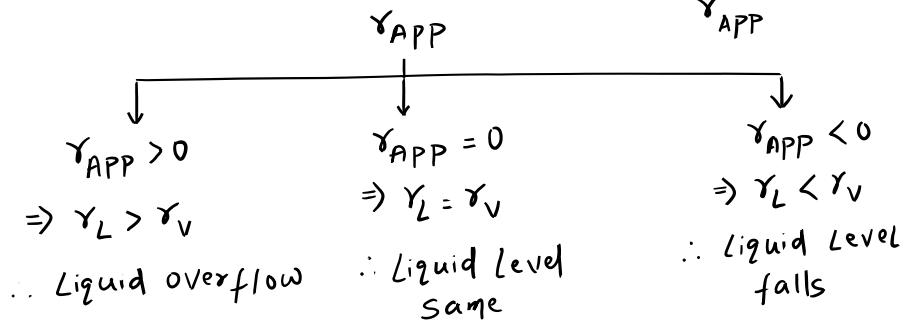
6 Apparent Expansion of Liquid

Part 1 – Thermal Properties



on increasing temp^o (ΔT) $\Rightarrow \Delta V_V = V_0 \gamma_V \Delta T$, $\Delta V_L = V_0 \gamma_L \Delta T$

\therefore Apparent Expansion, $\Delta V_{app} = V_0 (\gamma_L - \gamma_V) \Delta T$

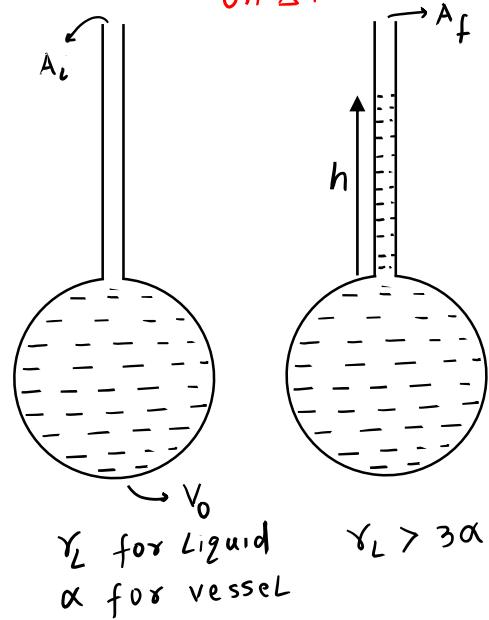


Solⁿ: App \uparrow in vol of liquid,
 $\Delta V = V_0 (\gamma_L - 3\alpha) \Delta T$

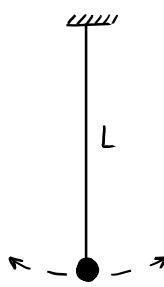
$$\therefore \Delta V = h \times A_f$$

$$\Rightarrow h = \frac{\Delta V}{A_f} = \frac{V_0 (\gamma_L - 3\alpha) \Delta T}{A_i (1 + 2\alpha \Delta T)}$$

Ex 2. Mercury rise in thermometer on ΔT .



7. Effect of Temperature on Pendulum Clocks



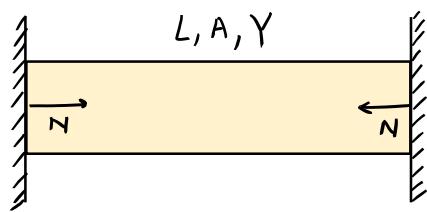
Second's pendulum \rightarrow for 1 oscillation of Pendulum, clock shows 2 sec

$$\Rightarrow T \text{ of Pendulum is also } 2 \text{ sec} \quad \left\{ T = 2\pi \sqrt{\frac{L}{g}} \right.$$

\downarrow

Temp \uparrow , $\Delta\theta$
 $\Rightarrow L \uparrow \Rightarrow T \uparrow$
 \therefore clock becomes slow/Looses time

Temp \downarrow , $\Delta\theta$
 $\Rightarrow L \downarrow \Rightarrow T \downarrow$
 \therefore fast/Gains time

8. Effect of ΔT on Rod between walls (thermal stress)

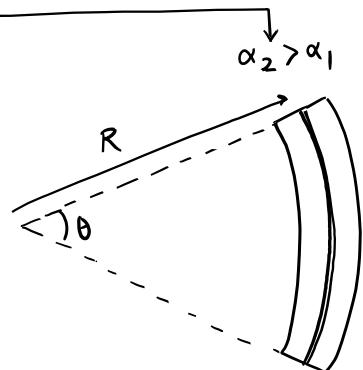
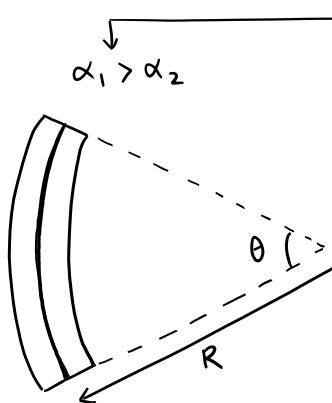
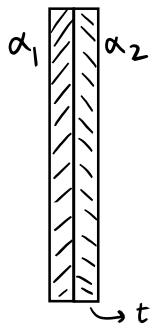
On ↑ temp° by ΔT , $\Delta L = L\alpha \Delta T$
(ΔL is also compression by N)

$$\text{Thermal stress} = Y \times \frac{\Delta L}{L}$$

$$\text{and, } N = A Y \alpha \Delta T = Y \alpha \Delta T$$

9. Bimetallic strip

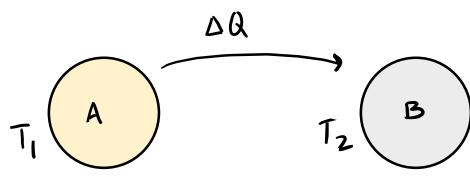
on heating by ΔT



$$R = \frac{t}{(\alpha_1 - \alpha_2) \Delta T}$$

$$R = \frac{t}{(\alpha_2 - \alpha_1) \Delta T}$$

10. Calorimetry - Heat Exchange



$$T_1 > T_2$$

Assuming no heat lost to Surrounding.

(i) Heat loss by A = Heat Gain by B

$$\Rightarrow m_A s_A (T_1 - T) = m_B s_B (T - T_2)$$

↳ T is final temp°

↳ $s = \frac{\Delta Q}{m \Delta T}$, specific heat capacity

J/kg°C or cal/g°C

NOTE :

Heat Capacity, $C = ms$

$$\therefore \Delta Q = C \Delta T$$

$$\hookrightarrow 1 \text{ cal} = 4.2 \text{ J}$$

11 Phase Transformation

↳ heat required to change phase, $\Delta Q = m L \rightarrow$ Latent heat, J/kg or cal/g

Latent heat of fusion

$$L_f = \frac{Q}{m}$$

Latent heat of vaporisation

$$L_v = Q/m$$

(i) L_f for ice = 80 cal/g { By losing 80 cal , 1g of water changes to ice at 0°C & Vice-Versa

(ii) L_v for steam = 540 cal/g

(iii) During phase change T is const.

(i) AB \rightarrow solid

(ii) T_2 \rightarrow Melting Pt.

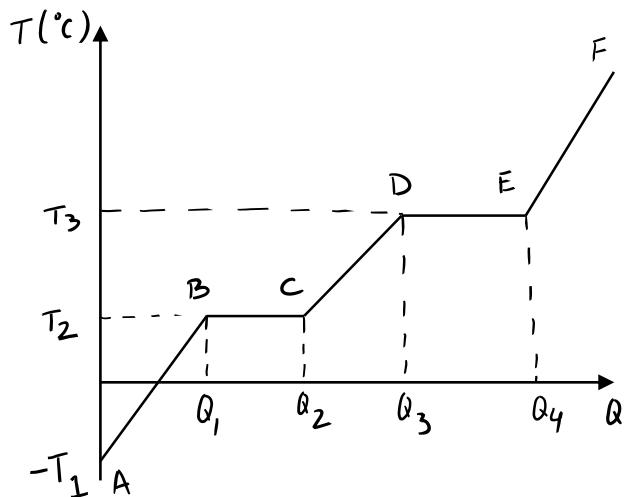
(iii) CD \rightarrow Liquid

(iv) T_3 \rightarrow Boiling Pt.

(v) EF \rightarrow Vapour

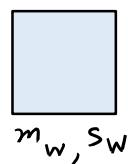
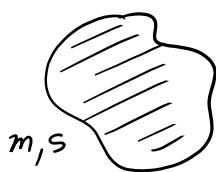
$$Q_1 = m s_s (T_2 + T_1) \quad Q_2 - Q_1 = m L_f$$

$$Q_3 - Q_2 = m s_L (T_3 - T_2) \quad Q_4 - Q_3 = m L_v$$



12. Water Equivalent

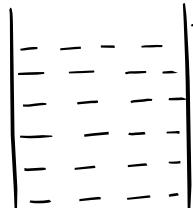
↳ Amount of water which requires the same amount of heat for same ΔT as that of object.



$$m s \Delta T = m_w s_w \Delta T \Rightarrow m_w = \frac{m s}{s_w}$$

W (Water Equivalent)

Ex:



→ Suppose water equivalent of container is 50g

⇒ Instead of container, take 50g water

Space to add concepts learnt from PYQs if any

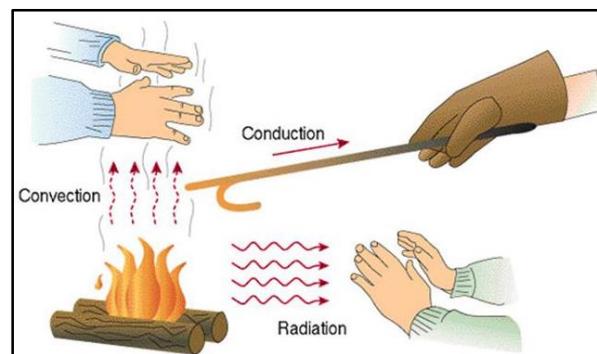
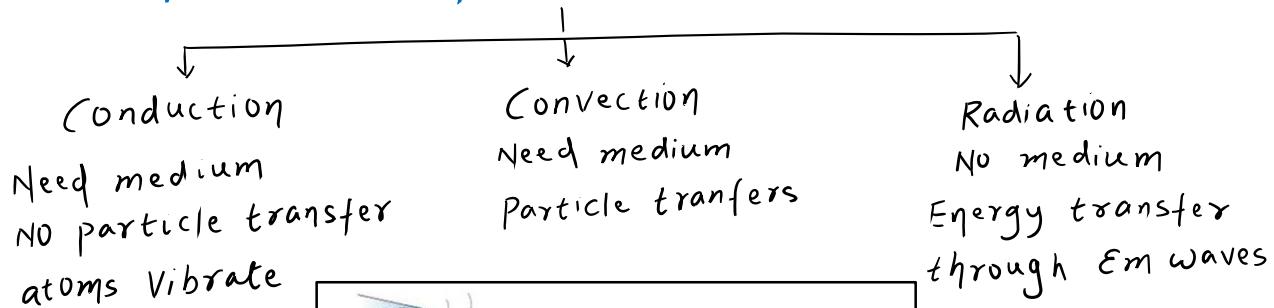
Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in Heat Transfer – Part 2

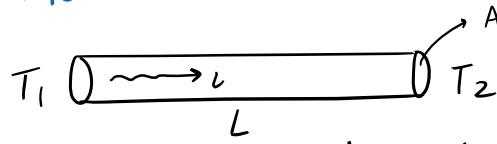
1. Modes of Heat Transfer
2. Thermal Conduction
3. Analogy with Ohm's Law
4. Interface & Junction Temperature
5. Temperature variation at Steady State
6. Equivalent Thermal Conductivity
7. Radial Heat Conduction
8. Freezing of Lake
9. Thermal Radiation
10. Stefan – Boltzmann Law
11. Newton's Law of Cooling
12. Variation of body temperature as per Newton's Law
13. Average form of Newton's Law of Cooling
14. Wien's Displacement Law

Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. Modes of heat transfer



2. Thermal Conduction



If $T_1 > T_2$, at steady state

$$\text{Heat current, } i = \frac{dQ}{dt} = \frac{KA(T_1 - T_2)}{L} \quad \{ \text{J/s} \}$$

(i) $\frac{dQ}{dt}$ is constant

(ii) K is thermal conductivity
 $\{ \text{W/m-K} \}$

3. Analogy with Ohm's Law

$$V_1 \xrightarrow{i} V_2 \quad I = \frac{V_1 - V_2}{R}$$

where $R = \frac{\rho L}{A}$

$$\text{Similarly, } i = \frac{T_1 - T_2}{L/KA} = \frac{T_1 - T_2}{R_{th}}$$

$$\therefore R_{th} = \frac{L}{KA}$$

Thermal Resistance

Note:

Resistance in series & parallel concept valid

4. Interface & Junction Temperature



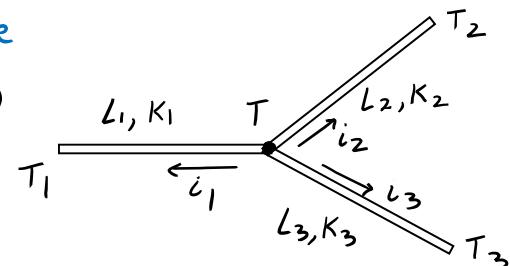
$\therefore i$ is same both rods

$$\Rightarrow \frac{T_1 - T}{L_1/K_1 A} = \frac{T - T_2}{L_2/K_2 A}$$

Solve for T .

$$R_1 = \frac{L_1}{K_1 A}, R_2 = \frac{L_2}{K_2 A}$$

(b)

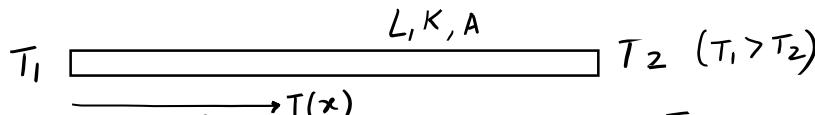


$$i_1 + i_2 + i_3 = 0$$

$$\Rightarrow \frac{T - T_1}{R_1} + \frac{T - T_2}{R_2} + \frac{T - T_3}{R_3} = 0$$

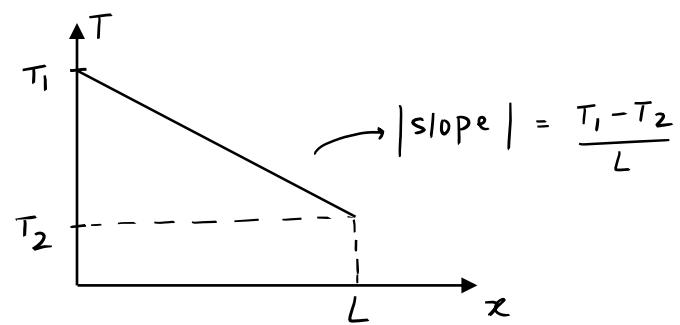
$$\text{where, } R_1 = \frac{L_1}{K_1 A_1}, R_2 = \frac{L_2}{K_2 A_2}, R_3 = \frac{L_3}{K_3 A_3}$$

5. Temperature Variation at Steady state



$$\frac{T_1 - T(x)}{x/KA} = \frac{T_1 - T_2}{L/KA}$$

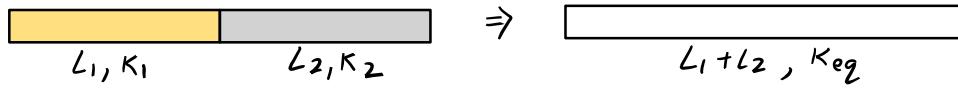
$$\Rightarrow T(x) = -\left(\frac{T_1 - T_2}{L}\right)x + T_1$$



6. Equivalent Thermal Conductivity (K_{eq})

Part 2 – Heat Transfer

(a) In Series



$$R_1 + R_2 = R_{eq} \Rightarrow \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} = \frac{L_1 + L_2}{K_{eq} A}$$

$$\Rightarrow K_{eq} = \frac{K_1 K_2 (L_1 + L_2)}{L_1 K_2 + L_2 K_1}$$

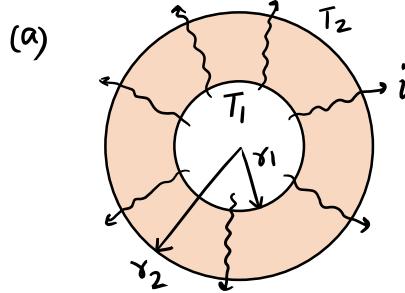
(b) In Parallel



$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R_{eq}}$$

$$\Rightarrow \frac{K_1 A_1}{L} + \frac{K_2 A_2}{L} = \frac{K_{eq} (A_1 + A_2)}{L} \therefore K_{eq} = \frac{K_1 A_1 + K_2 A_2}{A_1 + A_2}$$

7. Radial Heat Conduction

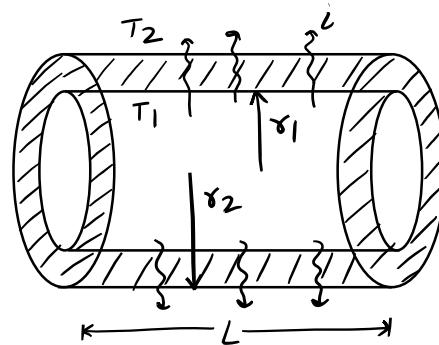


Spherical shell

$$i = \frac{T_1 - T_2}{R_{th}}$$

$$\rightarrow \frac{r_2 - r_1}{4\pi K r_1 r_2}$$

(b)

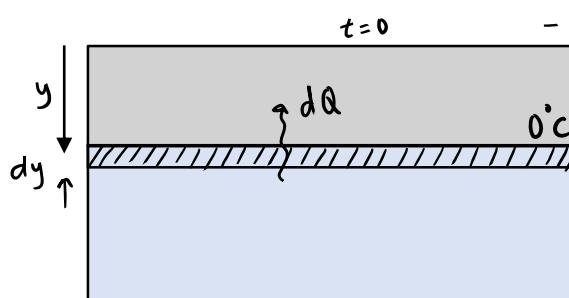


Cylindrical shell

$$i = \frac{T_1 - T_2}{R_{th}}$$

$$\rightarrow \frac{\ln(r_2/r_1)}{2\pi K L}$$

8. Freezing of Lake (time taken to freeze depth y)



- (i) at $t=0$ Lake starts to freeze from top
- (ii) at time t 'y' depth freezes
- (iii) In next dt time dy freezes by losing dQ heat.

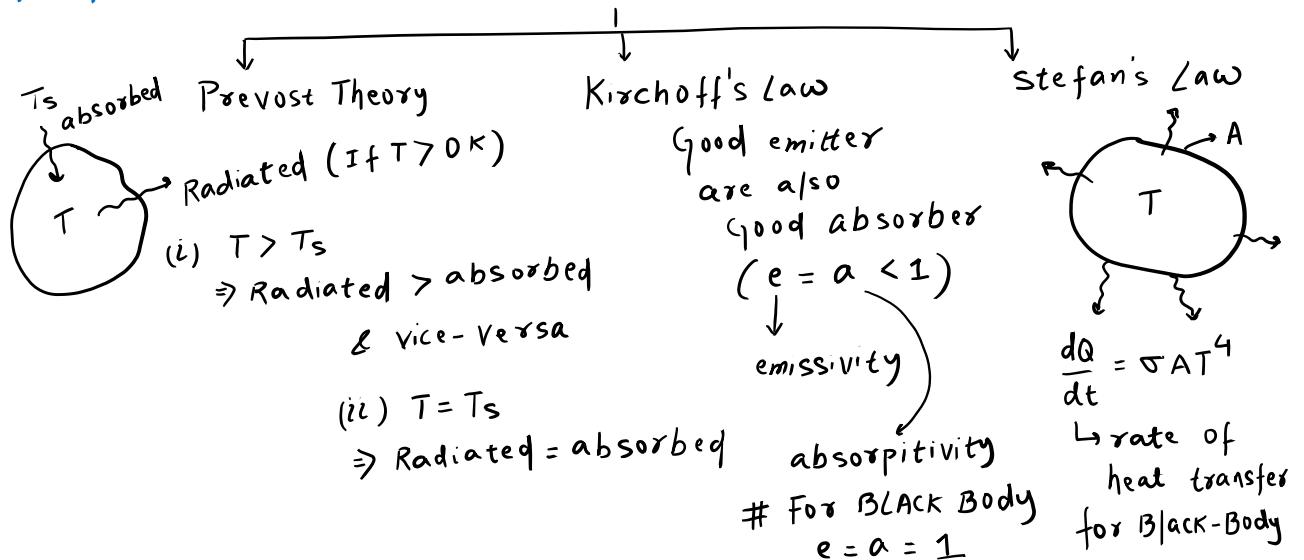
$$\frac{dQ}{dt} = \frac{KA}{y} T \Rightarrow dm \cdot L_f = \frac{KA}{y} T \cdot dt$$

$$\Rightarrow PA dy \cdot L_f = \frac{KA}{y} T dt \Rightarrow \frac{PL_f}{KT} \int_0^y dy = \int_0^t dt$$

$$\therefore t = \frac{\rho L_f \cdot y^2}{2KT}$$

9. Thermal Radiation

Part 2 – Heat Transfer



10. Stefan-Boltzmann Law

Net rate of heat loss by body

$$\frac{dQ}{dt} = \sigma e A (T^4 - T_s^4), \quad T > T_s$$

stefan-Boltzmann const

Rate of cooling

$$\Delta Q = m s \Delta T \Rightarrow \frac{dQ}{dt} = -m s \frac{dT}{dt}$$

as $T \downarrow$

$$\Rightarrow \frac{dT}{dt} = -\frac{\sigma e A}{m s} (T^4 - T_s^4)$$

11. Newton's Law of Cooling

Applicable if T is close to T_s ($T - T_s = \Delta T$ or $T = T_s + \Delta T$)

$$\frac{dT}{dt} = -\frac{\sigma e A}{m s} \left[(T_s + \Delta T)^4 - T_s^4 \right] = -\frac{\sigma e A}{m s} \cdot T_s^4 \left[\left(1 + \frac{\Delta T}{T_s}\right)^4 - 1 \right] \quad \because \Delta T \text{ is small}$$

$$(1 + x)^n \approx 1 + nx$$

$$\Rightarrow \frac{dT}{dt} = -\frac{\sigma e A}{m s} \cdot T_s^4 \left(1 + 4 \frac{\Delta T}{T_s} - 1 \right)$$

$$\Rightarrow \frac{dT}{dt} = -\frac{4 \sigma e A}{m s} T_s^3 \Delta T \Rightarrow \frac{dT}{dt} = -\frac{4 \sigma e A T_s^3}{m s} (T - T_s)$$

↓

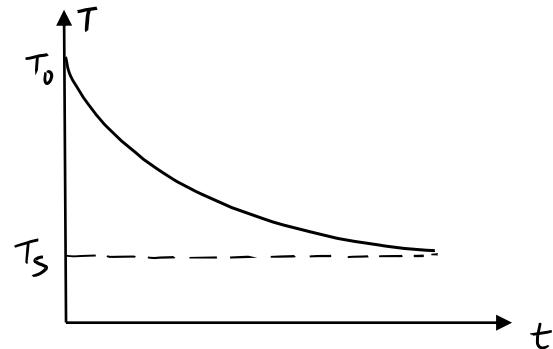
$$\frac{dT}{dt} \propto (T - T_s)$$

12. Variation of body temp as per Newton's Law

$$\frac{dT}{dt} = -\frac{4\sigma e A T_s^3}{ms} (T - T_s) \text{ or } \frac{dT}{dt} = -C(T - T_s) \quad \left\{ C = \frac{4\sigma e A T_s^3}{ms} \right.$$

$$\Rightarrow \int_{T_0}^T \frac{dT}{T - T_s} = -C \int_0^t dt \quad \Rightarrow \boxed{T = T_s + (T_0 - T_s) e^{-Ct}}$$

T_0 is bodies temp' at $t=0$



13. Average form of Newton's Law of Cooling

↳ If body cools from T_2 to T_1 in time t ,

$$\frac{T_2 - T_1}{t} = C \left(\frac{T_2 + T_1 - T_s}{2} \right)$$

Ex7. In 5 minutes, a body cools from 75°C to 65°C at room temperature of 25°C . The temperature of body at the end of next 5 minutes is _____ $^\circ\text{C}$.

Solⁿ: $\frac{75 - 65}{5} = C \left(\frac{75 + 65 - 25}{2} \right) \Rightarrow C = \frac{2}{45}$

$$\frac{65 - T}{5} = \frac{2}{45} \left(\frac{65 + T - 25}{2} \right) \Rightarrow \boxed{T = 57^\circ\text{C}} \text{ Ans.}$$

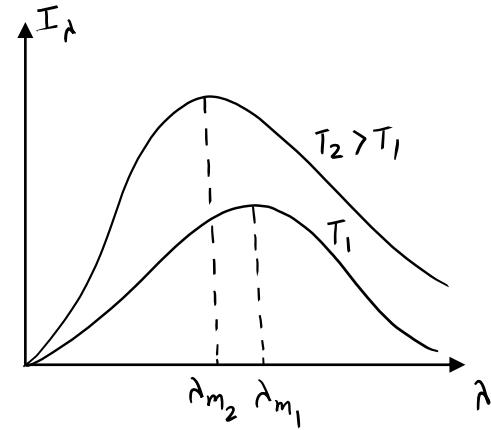
14. Wien's Displacement Law

↳ Energy distribution curve of black body radiation

(i) For a Given T of body, Various wavelength is emitted.

$$(ii) \lambda_m \propto \frac{1}{T} \Rightarrow \lambda_m T = b$$

↳ Wien's const.
 $2.8 \times 10^{-3} \text{ m-K}$



Space to add concepts learnt from PYQs if any

Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in KINETIC THEORY OF GASES– PART 1

- 1.Postulates of KTG
- 2.Boyle's Law
- 3.Charles Law
- 4.Dalton's Law of Partial Pressures
- 5.Different forms of Ideal Gas Law
- 6.Pressure exerted by a Gas
- 7.Maxwell's distribution of velocities
- 8.Degree of Freedom
- 9.Equipartition of energy and Internal energy
- 10.Mixing Gases
- 11.Mean Free Path
- 12.Mean Free Time

Note: For video refer Revision Series Playlist on EDUNI YouTube Channel

1. Postulates of KTG:

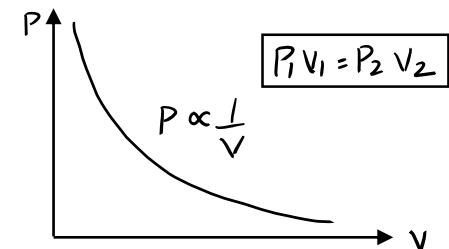
1. Molecule size negligible
2. Molecule is assumed to be hard sphere and all collision elastic
3. No interaction among molecules => Total Energy = K.E + P.E = K.E
4. Collision time is negligible
5. Effect of Gravity on molecule is neglected
6. Real gases obeys Ideal gas law $PV=nRT$ at very high T and very low P

$$\text{At STP, } [P=1\text{ atm}, T=273\text{ K}] \quad PV=nRT$$

$$\Rightarrow V = \frac{1 \times 8.314 \times 273}{1.013 \times 10^5} = 2.24 \times 10^{-2} \text{ m}^3 = 22.4 \text{ L}$$

2. BOYLE'S LAW, $T = \text{const}$

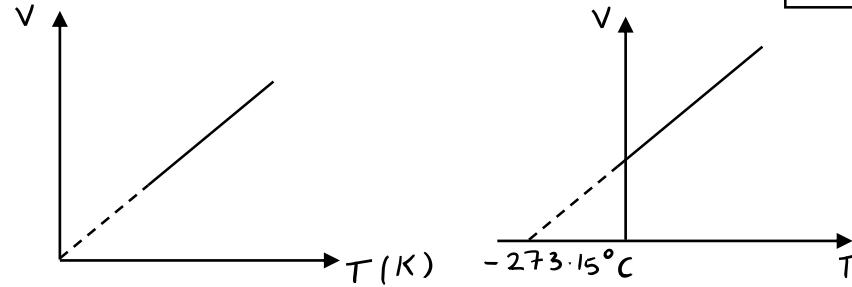
Boyle's law states that the pressure of a fixed amount of a gas varies inversely with the volume if the temperature is maintained constant.



3. CHARLES LAW, $P = \text{const.}$

Charles law states that the pressure remaining constant, the volume of a fixed amount of a gas varies directly with its absolute temperature.

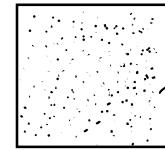
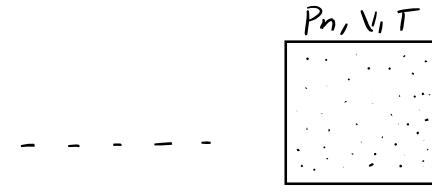
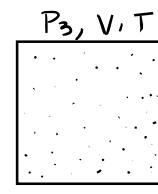
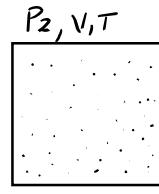
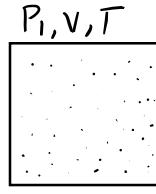
$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$



4. DALTON'S LAW OF PARTIAL PRESSURES

Pressure exerted by a mixture of non interacting gases is equal to the sum of their partial pressures. Hence, for a mixture of n gases, the total pressure of the gas is given by

$$P = P_1 + P_2 + P_3 + \dots + P_n$$



$$P_1 + P_2 + \dots + P_n$$

V, T

5. DIFFERENT FORMS OF IDEAL GAS LAW

N : Total number of molecules of gas.

N_A : Avogadro No, 6.023×10^{23}

m : mass of gas

M : molecular mass

$$PV = nRT$$

$$PV = \frac{N}{N_A} RT$$

$$PV = \frac{m}{M} RT$$

$$\Rightarrow PV = NkT$$

where, $k = \frac{R}{N_A}$ is

Boltzmann's constant

$$k = 1.38 \times 10^{-23} \text{ J/K}$$

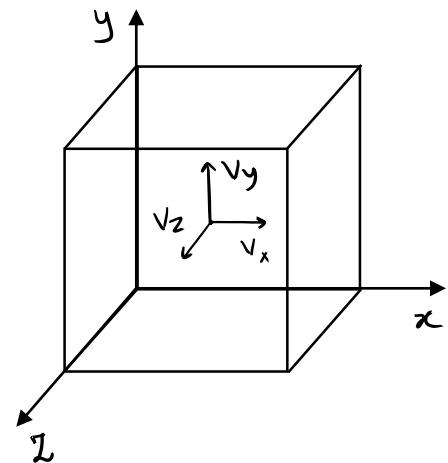
$$\Rightarrow P = \frac{m}{V} \frac{RT}{M}$$

$$\Rightarrow P = \frac{RT}{M}$$

6. PRESSURE EXERTED BY GAS:

Momentum transferred to a container wall per second per unit of its surface area is

avg pressure , $P = \frac{1}{3} \rho v_{rms}^2$



7. MAXWELL'S DISTRIBUTION OF VELOCITIES

A particle speed probability distribution shows how the speeds of molecules are distributed for an ideal gas.

n_v : no. of molecules per unit range of speeds.

$$V_{mp} = \sqrt{\frac{2RT}{M}}$$

$$V_{av} = \sqrt{\frac{8RT}{\pi M}}$$

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

NOTE :

(1) T : Temp° in K

M : Molecular mass (kg)

v : speed in m/s

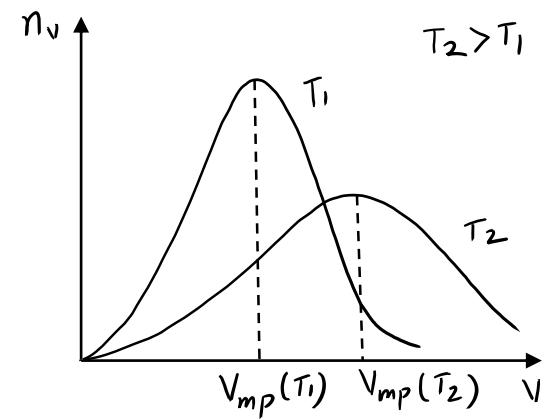
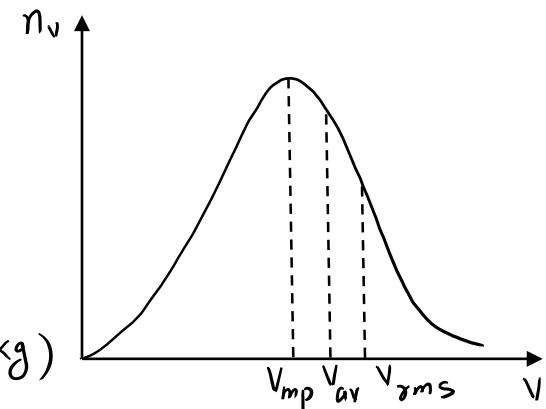
(2) $V_{mp} < V_{av} < V_{rms}$ for a given gas at T temp.

(3) Area under curve gives total no. of molecules

ON Increasing T :

(1) V_{mp} increases

(2) More molecules have high speed



8. DEGREE OF FREEDOM, f

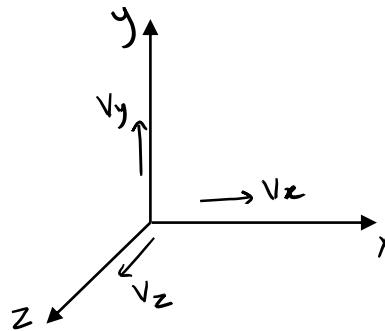
Number of ways a molecule can Participate in contributing to the total mechanical energy of that molecule

MODES

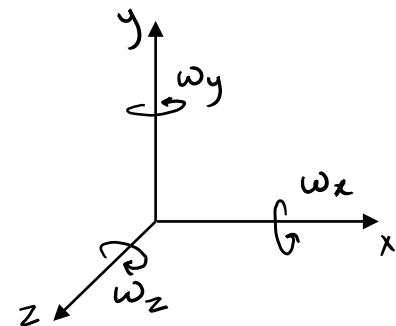
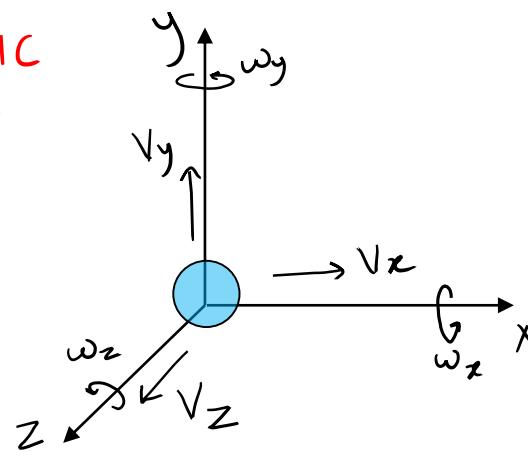
VIBRATIONAL

- (1) 2 DOF for each mode
- (2) Occurs at very temp°

TRANSLATIONAL



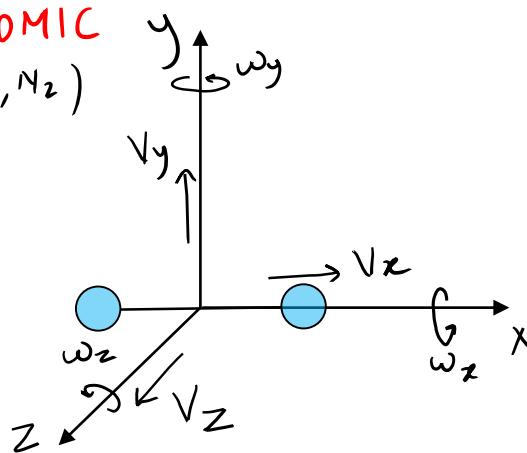
ROTATIONAL

(A) MONOATOMIC
(e.g. He, Ne, Ar)

NOTE:

- (1) Since molecules size is assumed to be zero implies $\frac{1}{2}I\omega_y^2, \frac{1}{2}I\omega_x^2, \frac{1}{2}I\omega_z^2$ is zero as $I = 0$

$$f = 3 \quad \text{Due to translational}$$

(B) DIATOMIC
(e.g. H₂, O₂, N₂)

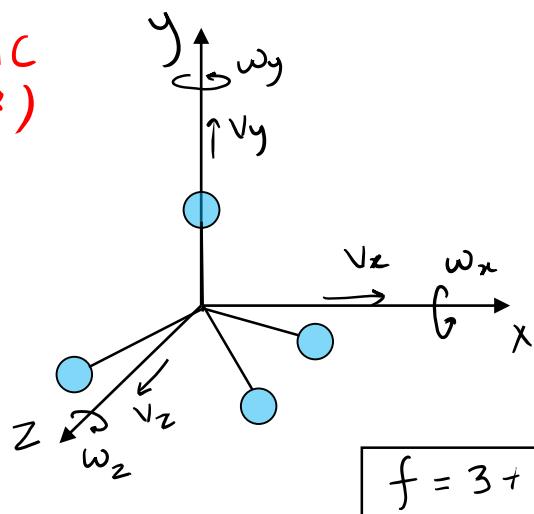
NOTE:

- (1) Since molecules are along x-axis, implies $\frac{1}{2}I\omega_x^2 = 0$

$$f = 3 + 2 = 5 \quad \text{At low temp°}$$

$$f = 3 + 2 + 2 = 7 \quad \text{At high temp°}$$

(C) **POLYATOMIC (NON-LINEAR)**
Eg: $\text{H}_2\text{O}, \text{CH}_4$



NOTE:
 (1) LINEAR Polyatomic
Eg: $\text{CO}_2, f = 3 + 2 = 5$
 (2) Vibrational DOF
will be mentioned
in problems.

$$f = 3 + 3 = 6$$

9. EQUIPARTITION OF ENERGY AND INTERNAL ENERGY, U

According to this law, for any system in thermal equilibrium, the total energy is distributed equally amongst all the degrees of freedom with the average energy associated with each degree of freedom equal to $kT/2$ per molecule $RT/2$ or per mole.

Implies If 1 mole of gas has f DOF,

$$U = \frac{fRT}{2}$$

∴ For n mole of gas

$$U = \frac{f n R T}{2} \quad \text{Total Kinetic energy.}$$

10. MIXING GASES

(1.) Equivalent f for a gaseous mixture (Gases mixed at same temp°)

$$f_{\text{mix}} = \frac{n_1 f_1 + n_2 f_2 + \dots + n_N f_N}{n_1 + n_2 + \dots + n_N}$$

(2.) FINAL temp° of gaseous mixture
(Gases at different temp° mixed at const. Volume
in thermally insulated vessel, V remains const)

$$T_f = \frac{f_1 n_1 T_1 + f_2 n_2 T_2 + \dots + f_N n_N T_N}{f_1 n_1 + f_2 n_2 + \dots + f_N n_N}$$

11 MEAN FREE PATH, λ

The free path travelled by a molecule between two successive collisions

$$\lambda = \frac{1}{\sqrt{2\pi} d^2 n}$$

d : diameter of molecule
 n : no. of molecules per unit volume, $\frac{N}{V}$

NOTE: using $PV = NkT$ or $\frac{P}{kT} = \frac{N}{V} = n$

$$\lambda = \frac{kT}{\sqrt{2\pi} d^2 P}$$

Replacing n by P/kT

12. MEAN FREE TIME, τ

Time between two successive collisions

$$\tau = \frac{\lambda}{V_{av}}$$

NOTE: For τ calculation

Depending on problem we need to find dependence of τ with variables.

e.g.: If T and P are changing

$$\text{where, } \lambda = \frac{kT}{\sqrt{2\pi} d^2 P} \text{ and } V_{av} = \sqrt{\frac{8RT}{M}}$$

$$\lambda \propto \frac{1}{P} \text{ and } V_{av} \propto \sqrt{T}$$

$$\Rightarrow \tau \propto \frac{1}{P} \times \frac{1}{\sqrt{T}} \Rightarrow \tau \propto \frac{\sqrt{T}}{P}$$

Space to add concepts learnt from PYQs if any

Space to add concepts learnt from PYQs if any

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Topics to cover in THERMODYNAMICS – PART 2

- 1.Thermal Equilibrium
- 2.Zeroth Law of Thermodynamics
- 3.First Law of Thermodynamics (sign conventions)
- 4.Thermodynamics Process
- 5.Molar Heat Capacity (C_p & C_v)
- 6.Adiabatic Constant
7. C_p , C_v & Adiabatic Constant for Mixture of Gases
- 8.Work done in various Processes
- 9.Indicator Diagram (cyclic process)
- 10.Work done using PV indicator diagram
- 11.Adiabatic Process (equations and PV graph)
- 12.Isothermal vs Adiabatic : PV Slope
- 13.Reversible Polytropic Process
- 14.Free Expansion
- 15.Second Law of Thermodynamics
- 16.Thermal Efficiency of a Heat Engine
- 17.Coefficient of Performance of a Refrigerator

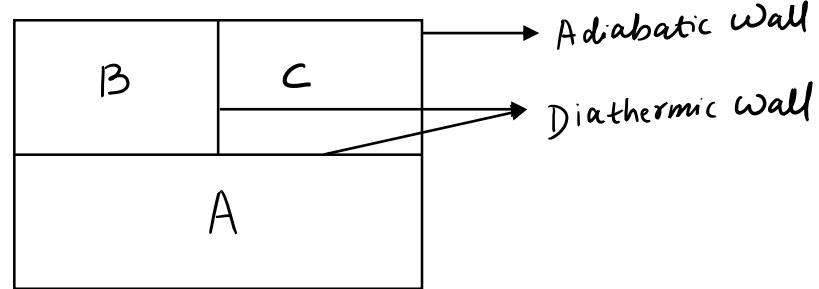
Note: For video refer Revision Series Playlist on EDUNI YouTube Channel

1 THERMAL EQUILIBRIUM

Two different temperature body in contact exchange heat until common temperature is reached. This is Called state of Thermal Equilibrium

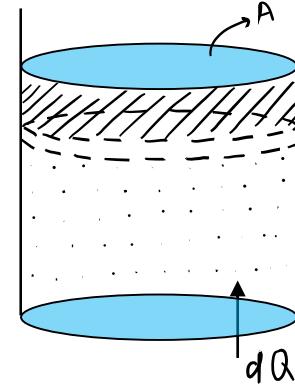
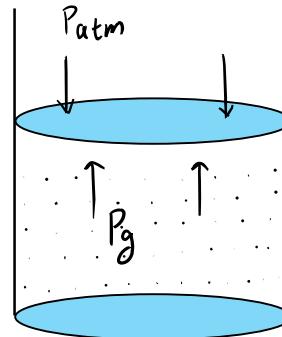
2. ZEROTH LAW OF THERMODYNAMICS

If two systems (B and C) are separately in thermal equilibrium with a third one (A), then they themselves are in thermal equilibrium with each other.



3. FIRST LAW OF THERMODYNAMICS

$$dQ = dU + dW$$



$$\begin{aligned} \textcircled{1} \quad dQ &= \dots \\ \textcircled{2} \quad dW &= P_g A dx \\ &= P_g dV \end{aligned}$$

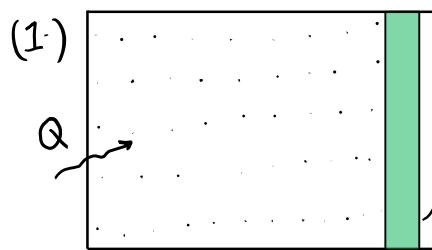
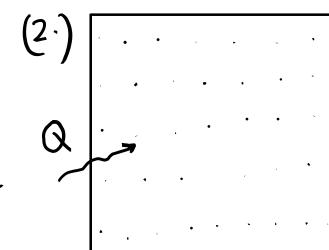
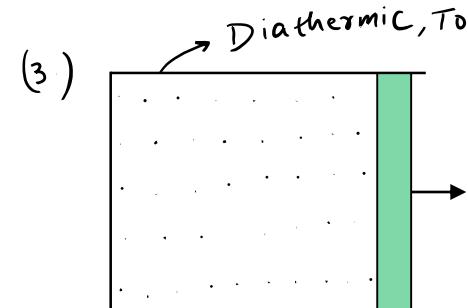
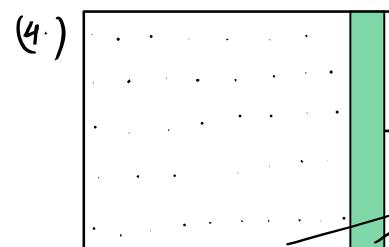
$$\textcircled{3} \quad V = f \frac{R n T}{2}$$

$$\Rightarrow dU = f \frac{R n dT}{2}$$

"SIGN CONVENTION"

- (1) If heat is supplied, ΔQ is +VE
 If heat is rejected, ΔQ is -VE
- (2) If gas does work (volume of gas increases), ΔW is +VE
 If work is done on the gas (volume of gas decreases), ΔW is -VE
- (3) $T \uparrow \Rightarrow \Delta U$ is +VE
 $T \downarrow \Rightarrow \Delta U$ is -VE

4. THERMODYNAMIC PROCESS

Isobaric Process $V \propto T$ Isochoric Process $P \propto T$ Isothermal Process $P \propto \frac{1}{V}$ Adiabatic Process $P V^{\gamma} = C$

5. MOLAR HEAT CAPACITY, C

Molar heat capacity of a substance is defined as the amount of heat required to raise the temperature of one mole of a substance by a unit degree.

$$C = \frac{dQ}{n dT} \Rightarrow \int dQ = \int_{T_1}^{T_2} n C dT$$

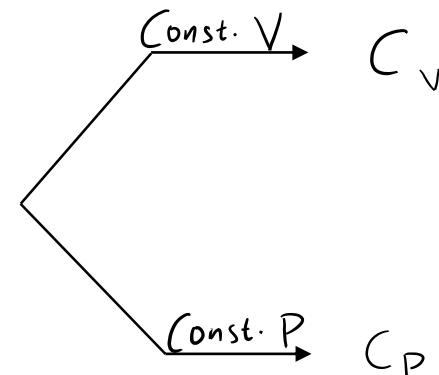
UNIT: $J \text{ mol}^{-1} \text{ K}^{-1}$

$$\Delta Q = n C (T_2 - T_1)$$

$$\Rightarrow \Delta Q = n C \Delta T$$

NOTE:

C depends on the way of heating / process.



C_P, C_V

$$(1.) C_V = \frac{dQ}{n dT} = \frac{dU + dW}{n dT}^0$$

$$= \frac{(fR n dT)/2}{n dT} \Rightarrow C_V = \frac{fR}{2}$$

$$\text{ALSO, } U = \frac{fR n T}{2}$$

$$\Rightarrow U = n C_V T \quad \text{or}$$

$$dU = n C_V dT$$

$$(2.) C_P = \frac{dQ}{n dT} = \frac{dU + dW}{n dT} = \frac{n C_V dT + P dV}{n dT}$$

$$\Rightarrow C_P = C_V + \frac{P dV}{n dT} \left(\begin{array}{l} PV = nRT \\ \Rightarrow P dV = nR dT \end{array} \right)$$

$$\Rightarrow C_P = C_V + R$$

$$C_P - C_V = R$$

MAYER'S
RELATION

$$C_P = \left(\frac{f+2}{2} \right) R$$

6. ADIABATIC CONSTANT, γ

$$\gamma = \frac{C_P}{C_V} \Rightarrow \gamma = 1 + \frac{2}{f} \Rightarrow f = \frac{2}{\gamma - 1}$$

C_P, C_V in terms of γ and R

$$C_V = \frac{fR}{2} = \frac{R}{\gamma - 1}$$

$$C_P = \gamma C_V = \frac{\gamma R}{\gamma - 1}$$

GAS	f	C_P	C_V	γ
Monoatomic	3	$\frac{5R}{2}$	$\frac{3R}{2}$	$5/3$
Diatomeric	5	$\frac{7R}{2}$	$\frac{5R}{2}$	$7/5$
NON-LINEAR Polyatomic	6	4R	3R	$4/3$

* Vibration mode only if mentioned

7. C_p , C_v and γ FOR MIXTURE OF GASESGases at same temp^o T,

n_1	C_{V1}	C_{P1}	f_1
n_2	C_{V2}	C_{P2}	f_2
n_3	C_{V3}	C_{P3}	f_3
⋮	⋮	⋮	⋮

$$C_{V_{mix}} = \frac{n_1 C_{V1} + n_2 C_{V2} + \dots}{n_1 + n_2 + \dots}$$

$$C_{P_{mix}} = \frac{n_1 C_{P1} + n_2 C_{P2} + \dots}{n_1 + n_2 + \dots}$$

$$f_{mix} = \frac{n_1 f_1 + n_2 f_2 + \dots}{n_1 + n_2 + \dots}$$

$$\gamma_{mix} = \frac{C_{P_{mix}}}{C_{V_{mix}}} \quad \text{OR} \quad 1 + \frac{2}{f_{mix}}$$

8. WORK DONE IN VARIOUS PROCESSES $\Delta W = \int_{V_1}^{V_2} P dV$, $PV = nRT$ ISOCHORIC
 $V = \text{const.}$
 $\Delta W = 0$ ISOBARIC
 $P = \text{const.}$
 $\Delta W = P(V_2 - V_1)$
OR
 $\Delta W = nR(T_2 - T_1)$ ISOTHERMAL
 $T = \text{const.}$

$$\Delta W = nRT \ln\left(\frac{V_2}{V_1}\right)$$

OR

$$\Delta W = nRT \ln\left(\frac{P_1}{P_2}\right)$$

ADIASTATIC
 $\Delta Q = 0$
 $\Rightarrow \Delta W = -\Delta U$
 $\Rightarrow \Delta W = -nC_V \Delta T$

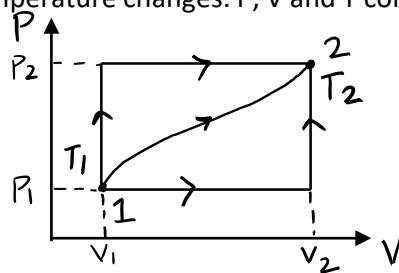
$$\Delta W = \frac{nR(T_2 - T_1)}{1 - \gamma}$$

OR

$$\Delta W = \frac{P_2 V_2 - P_1 V_1}{1 - \gamma}$$

9. INDICATOR DIAGRAM

When a gas is heated/cooled/compressed etc. thermodynamic parameters Pressure, Volume and Temperature changes. P, V and T combinedly defines the state of a Gas.

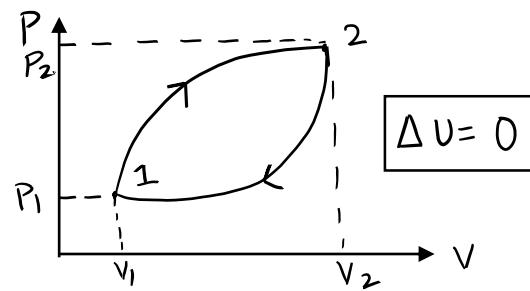
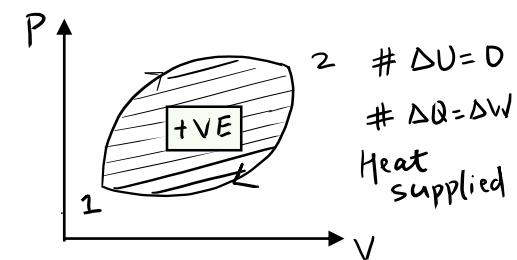
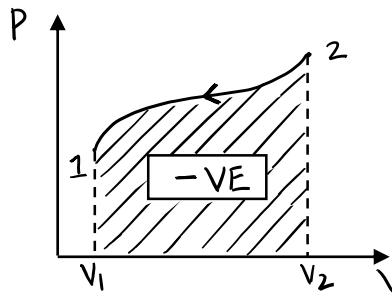
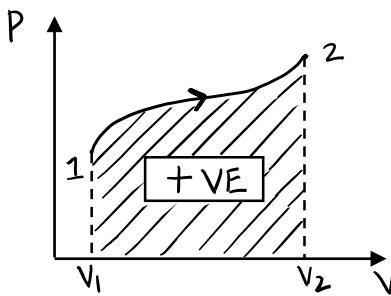


NOTE:

- (1) ΔU is state variable, only depends on T_1 and T_2 and not path dependent.
- (2) ΔQ and ΔW both depends on path or how P and V are related.

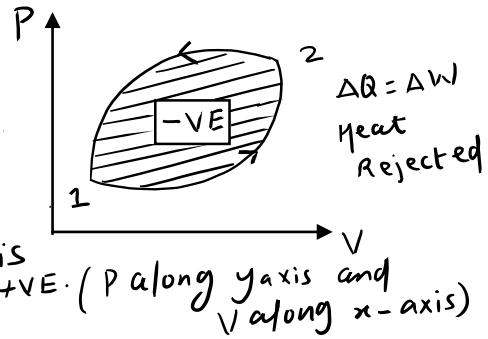
CYCLIC PROCESSES

A process in which initial and final state are same (state means P, V and T are same)

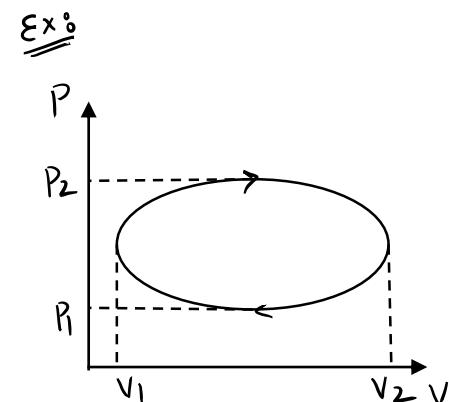
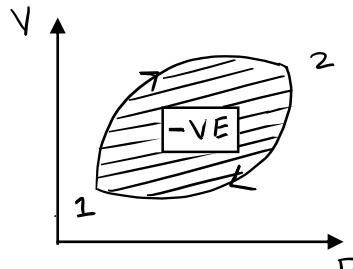
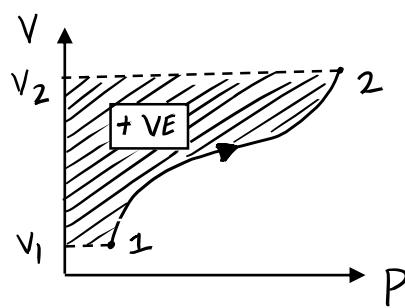
10. ΔW USING P-V INDICATOR DIAGRAM

NOTE:

- (1.) For non-cyclic, area is between curve and V-axis.
 $V \uparrow \Rightarrow \Delta W$ is +VE
- (2.) For cyclic, area is of the loop. If clockwise $\Rightarrow \Delta W$ is +VE. (P along y-axis and V along x-axis)



... CONTINUED WITH Ex.



→ Ellipse ($A = \pi ab$)

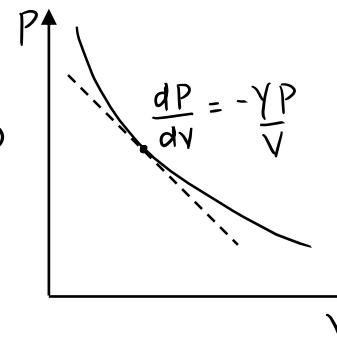
$$\Delta W = \pi \left(\frac{V_2 - V_1}{2} \right) \left(\frac{P_2 - P_1}{2} \right)$$

11. ADIABATIC PROCESS, $\Delta Q=0$

(1.) $PV^\gamma = C$

$$\frac{dP}{dV} \cdot V^\gamma + P \gamma V^{\gamma-1} = 0$$

$$\Rightarrow \frac{dP}{dV} = -\frac{\gamma P}{V}$$

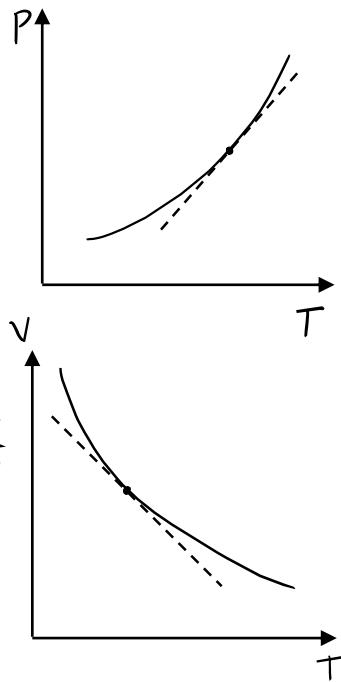


(2.) $T^\gamma P^{1-\gamma} = C$

$$\frac{dP}{dT} = \frac{\gamma}{(\gamma-1)} \frac{P}{T}$$

(3.) $T V^{\gamma-1} = C$

$$\frac{dV}{dT} = -\frac{1}{\gamma-1} \frac{V}{T}$$



12. ISOTHERMAL V/S ADIABATIC : PV SLOPE

$$\hookrightarrow PV = \text{const.}$$

$$\Rightarrow \frac{dP}{dV} \cdot V + P = 0$$

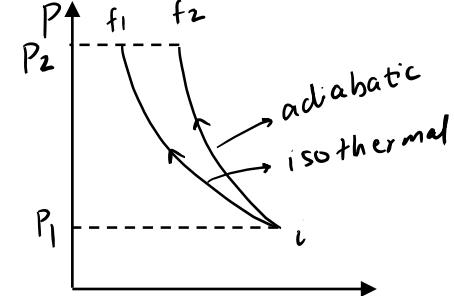
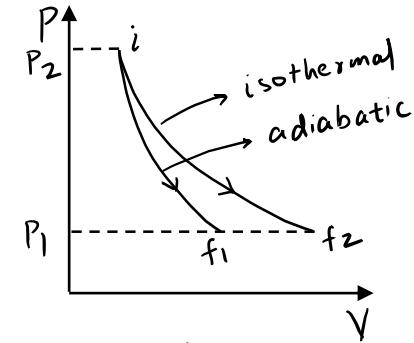
$$\Rightarrow \frac{dP}{dV} = -\frac{P}{V}$$

$$\hookrightarrow PV^\gamma = \text{const.}$$

$$\Rightarrow \frac{dP}{dV} = -\frac{\gamma P}{V}$$

Thus,

$$\left| \frac{dP}{dV} \right|_{\text{ad}} > \left| \frac{dP}{dV} \right|_{\text{iso}}$$



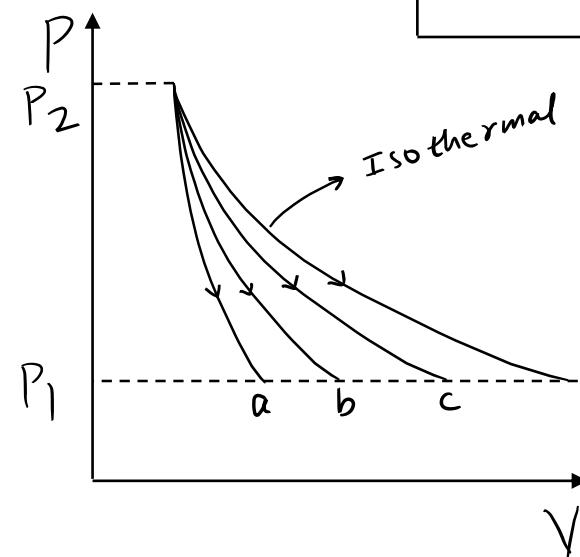
PV SLOPE AMONG GASES

$$\left| \frac{dP}{dV} \right| = \frac{\gamma P}{V} \text{ (adiabatic)}$$

a: monoatomic, $\gamma = \frac{5}{3} = 1.67$

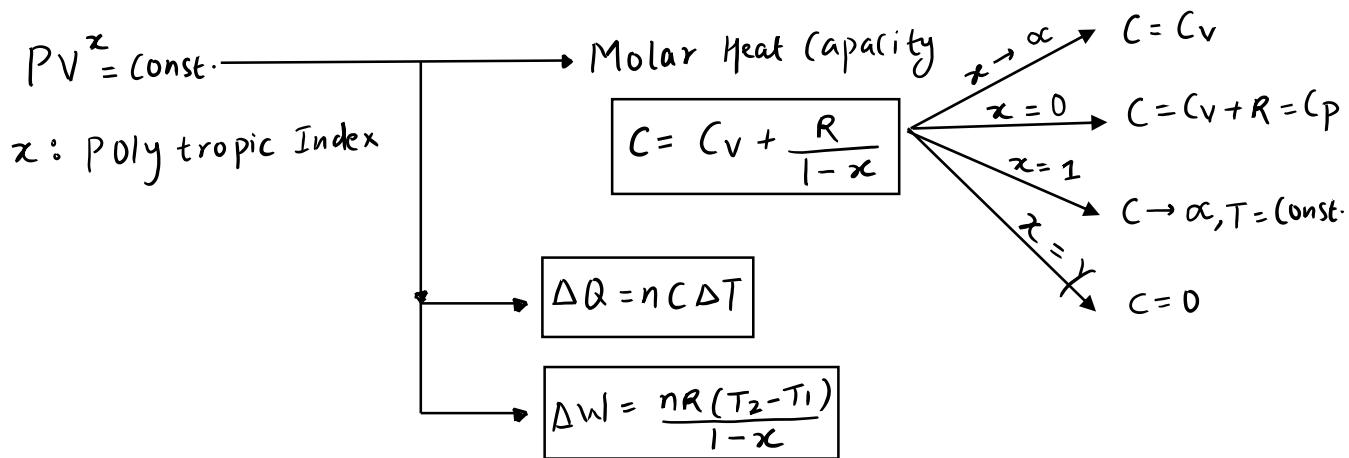
b: diatomic, $\gamma = \frac{7}{5} = 1.4$

c: polyatomic, $\gamma = \frac{4}{3} = 1.3$

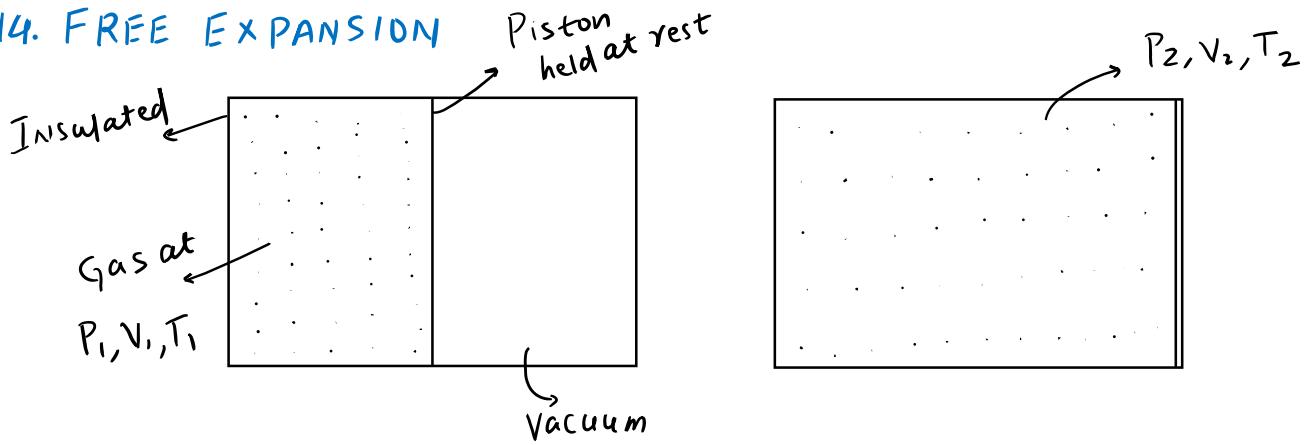


13. REVERSIBLE POLYTROPIC PROCESS

Reversible Processes are Quasi-Static (very slow process such that system is always in thermodynamic Equilibrium) and Non-dissipative (no loss in energy due to friction, viscous force etc.)



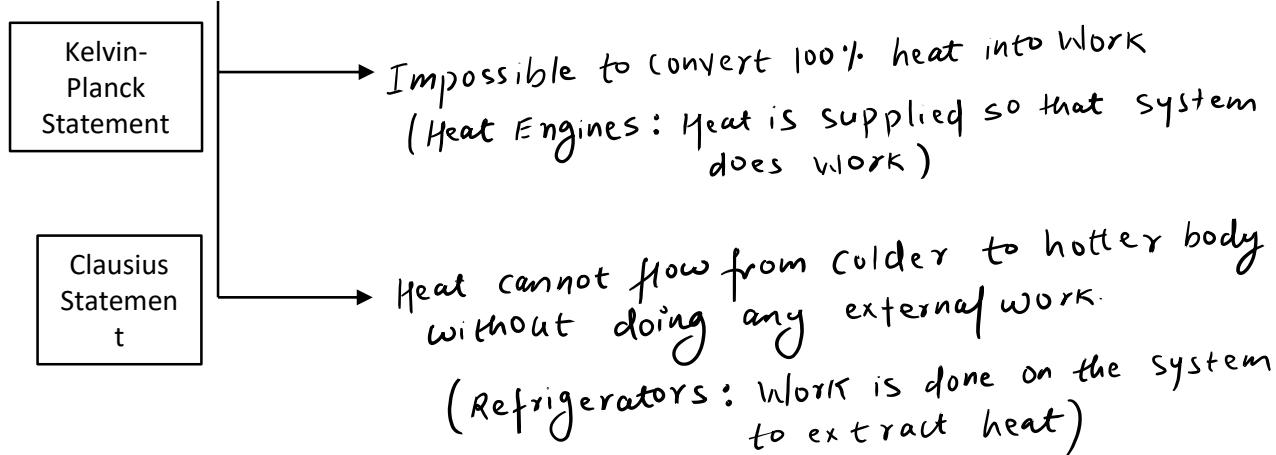
14. FREE EXPANSION



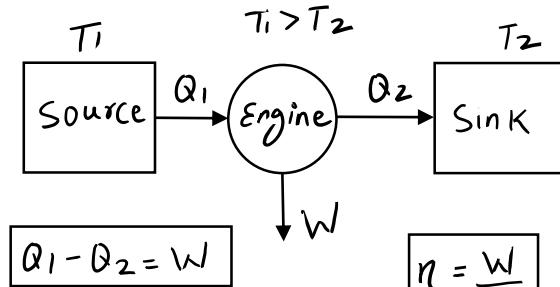
NOTE:

- (1) If no opposition $\Rightarrow \Delta W = 0$
- (2) $\Delta Q = 0 \Rightarrow \Delta U = 0$ or $T = \text{const.} \Rightarrow T_1 = T_2$
- (3) $P_1 V_1 = P_2 V_2$ * Imp for problem solving

15. SECOND LAW OF THERMODYNAMICS

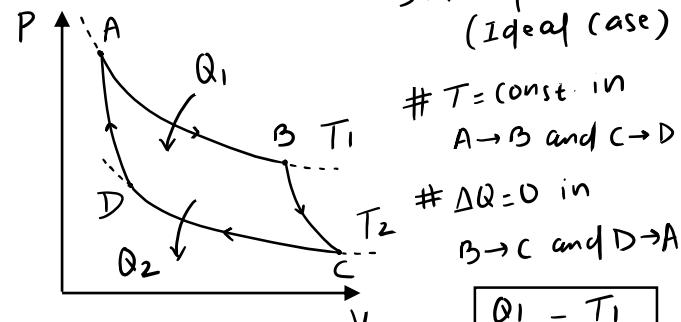


16. THERMAL EFFICIENCY OF A HEAT ENGINE



Carnot Engine:

Based on Carnot cycle



$$\eta = \frac{W}{Q_1}$$

or

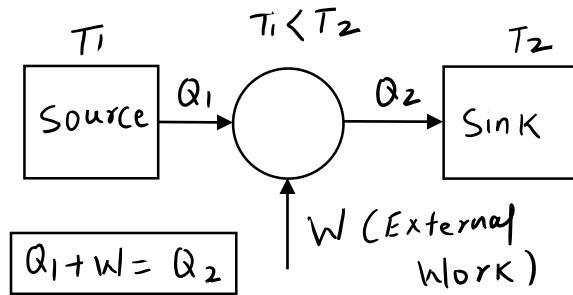
$$\eta = 1 - \frac{Q_2}{Q_1}$$

* Q_1 and Q_2 are magnitudes

$$\therefore \eta = 1 - \frac{T_2}{T_1}$$

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

17. COEFFICIENT OF PERFORMANCE OF A REFRIGERATOR



A best refrigeration cycle is one that removes the greatest amount of heat Q_1 from inside the refrigerator for least amount of external work W .

$$COP = \frac{Q_1}{W} = \frac{Q_1}{Q_2 - Q_1}$$

If it follows Carnot cycle i.e. $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$

$$\Rightarrow COP = \frac{T_1}{T_2 - T_1}$$

Space to add concepts learnt from PYQs if any

Space to add concepts learnt from PYQs if any

Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in String Waves – PART 1

1. Waves (Transverse & Longitudinal)
2. Wave Equation
3. Path & Phase Difference
4. Direction of Wave through equation
5. Particle velocity & acceleration
6. Direction of particle velocity
7. Wave speed on strings
8. Average Power & Intensity Transmitted

Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

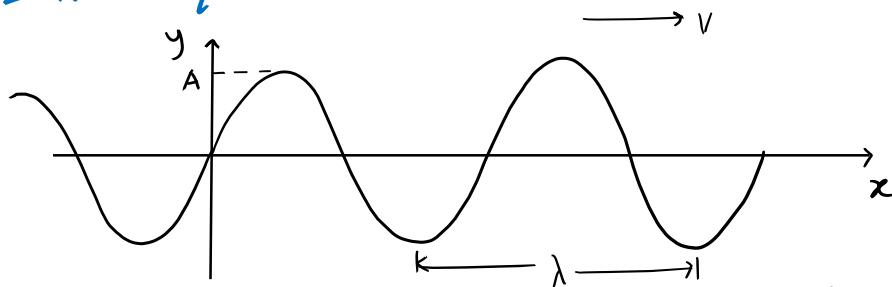
1. Waves (Transverse & Longitudinal)

Energy is transmitted from one point to another
↳ also called Travelling waves

Transverse Disturbance travels in x-dirⁿ but particle moves in dirⁿ \perp to x-axis
↳ string waves

Longitudinal Particle moves along the same dirⁿ as disturbance
↳ Sound waves

2. Wave Equation (string waves)



Wave Speed

$$V = \frac{\text{coeff of } t}{\text{coeff of } x} = \frac{\omega}{K} = f\lambda$$

$$y = A \sin(\omega t - Kx + \phi)$$

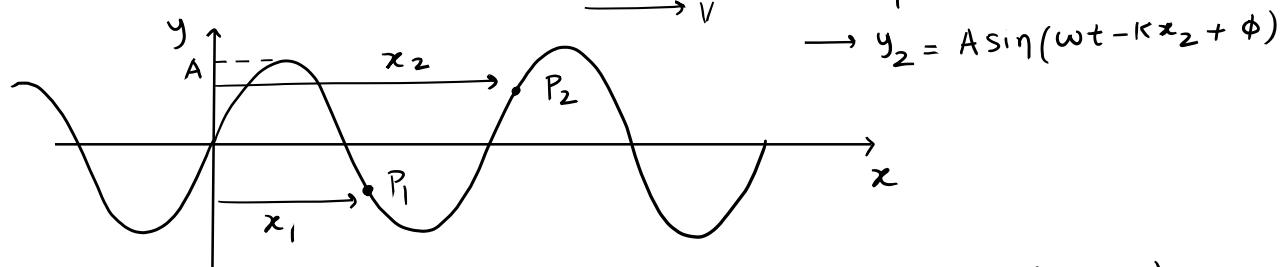
↳ Amplitude

↳ Angular freq, $\omega = 2\pi f = \frac{2\pi}{T}$

↳ Initial phase const.

↳ Angular wave no, $K = \frac{2\pi}{\lambda}$

3. Path & Phase difference

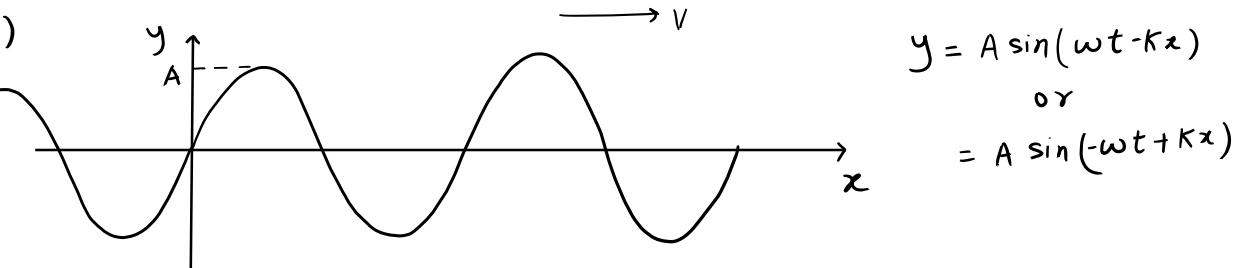


Phase difference, $\Delta\phi = \phi_1 - \phi_2 = k(x_2 - x_1)$

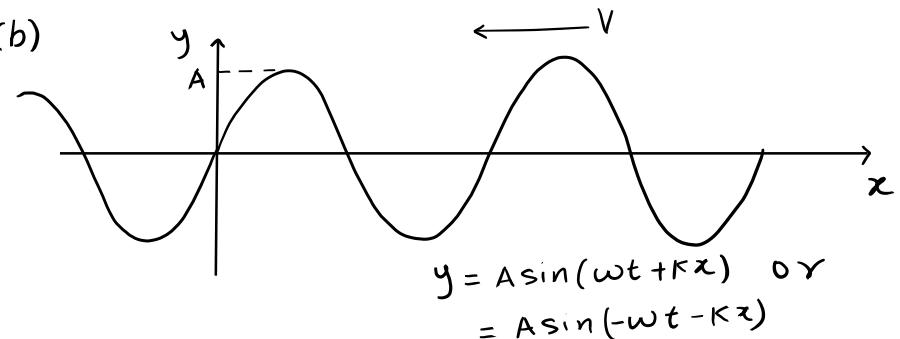
$$\Rightarrow \boxed{\Delta\phi = \frac{2\pi}{\lambda} \cdot \Delta x} \quad \Delta x \text{ is path difference}$$

4. Direction of Wave Motion in Eqⁿ

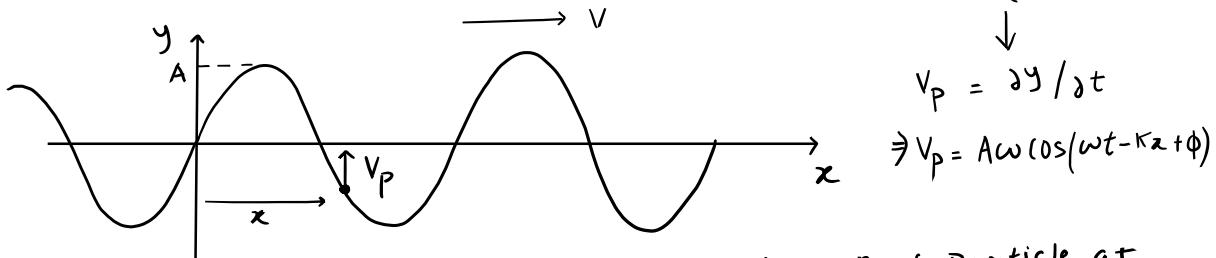
(a)



(b)



5. Particle Velocity & Acceleration

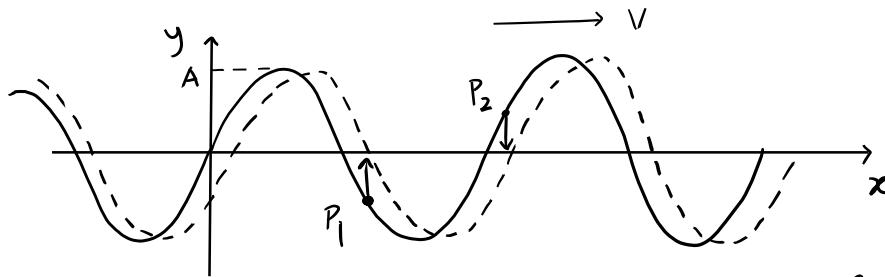


or, $\boxed{V_p = \omega \sqrt{A^2 - y^2}}$ { y is posⁿ of particle at x distance at time t

and, $a_p = \frac{dV_p}{dt} \Rightarrow \boxed{a_p = -\omega^2 y}$

6. Particle Direction at time t

String Waves

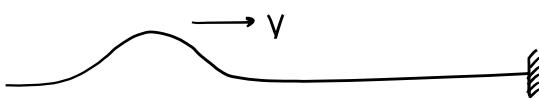


Method 1 : $v_p = -v \times \text{slope}$ at P_1 slope is $-ve$
 v_p is $+ve \Rightarrow \uparrow$

Method 2. Draw wave after dt time
Same result.

7. Wave Speed of strings

Ex. 5.



$$v = \sqrt{\frac{T}{\mu}}$$

Tension in string
mass/length

 $M \gg m$

Find time to reach top.

$$\text{soln} : M \gg m \Rightarrow T \text{ in string} \approx Mg$$

$$\therefore v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Mg}{m/L}} = \sqrt{\frac{MgL}{m}} \Rightarrow t = \frac{L}{v} = \sqrt{\frac{mL}{Mg}}$$

8. Avg. Power & Intensity Transmitted

$$\hookrightarrow P_{av} = \frac{1}{2} \rho s v \omega^2 A^2$$

 ρ : density of string s : cross-secⁿ area of string v : wave speed, $\sqrt{T/\mu}$

$$P_{av} = 2\pi^2 f^2 A^2 \rho v s$$

 ω : $2\pi f$ A : Amplitude

$$\hookrightarrow I = P_{av}/s = 2\pi^2 f^2 A^2 \rho v$$

Space to add concepts learnt from PYQs if any

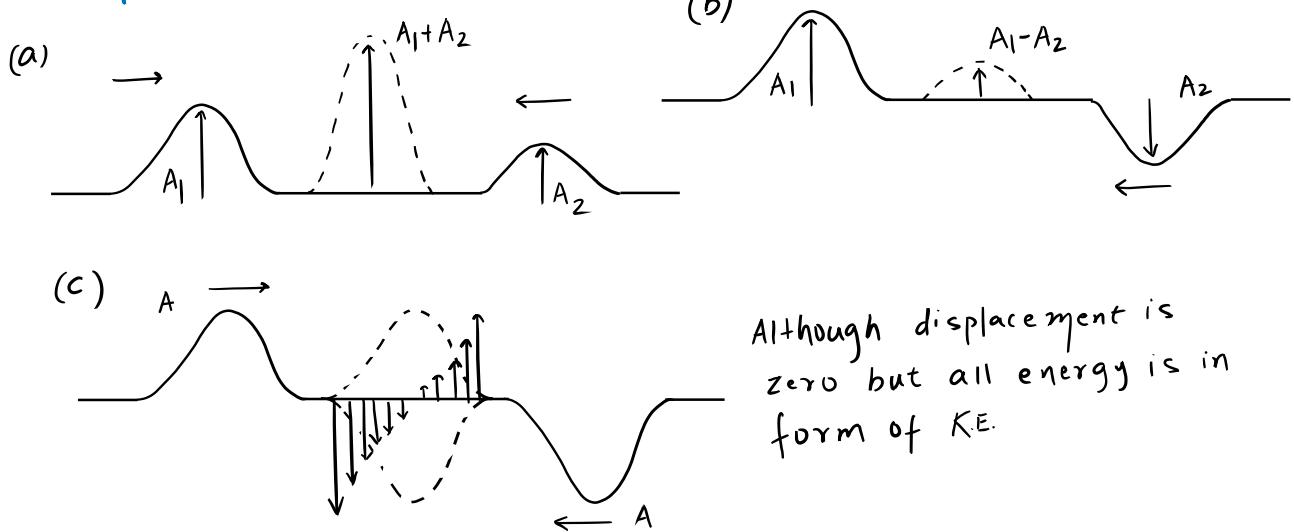
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Topics to cover in String Waves – PART 2

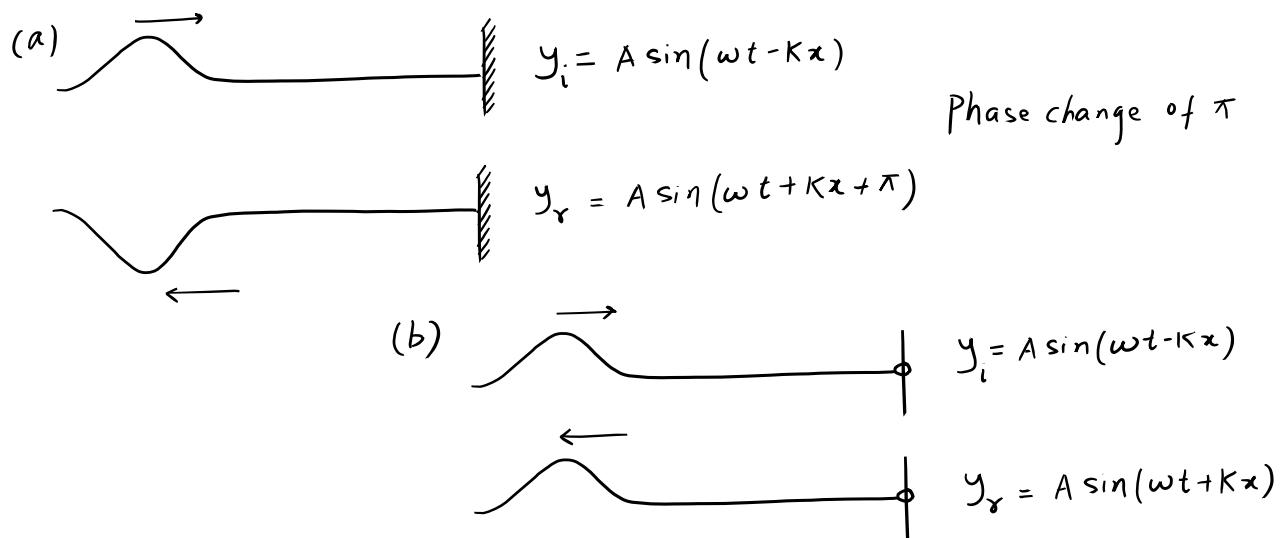
1. Superposition of Pulse
2. Reflection from fixed & free ends
3. Reflection & Transmission between two strings
4. Standing or Stationary Waves
5. Key Points in Standing Waves
6. Equation of Standing Waves
7. Standing wave in Clamped string (between 2 fixed ends)
8. Standing wave in Clamped String (1 fixed & 1 free end)
9. Standing wave in Composite String
10. Sonometer

Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

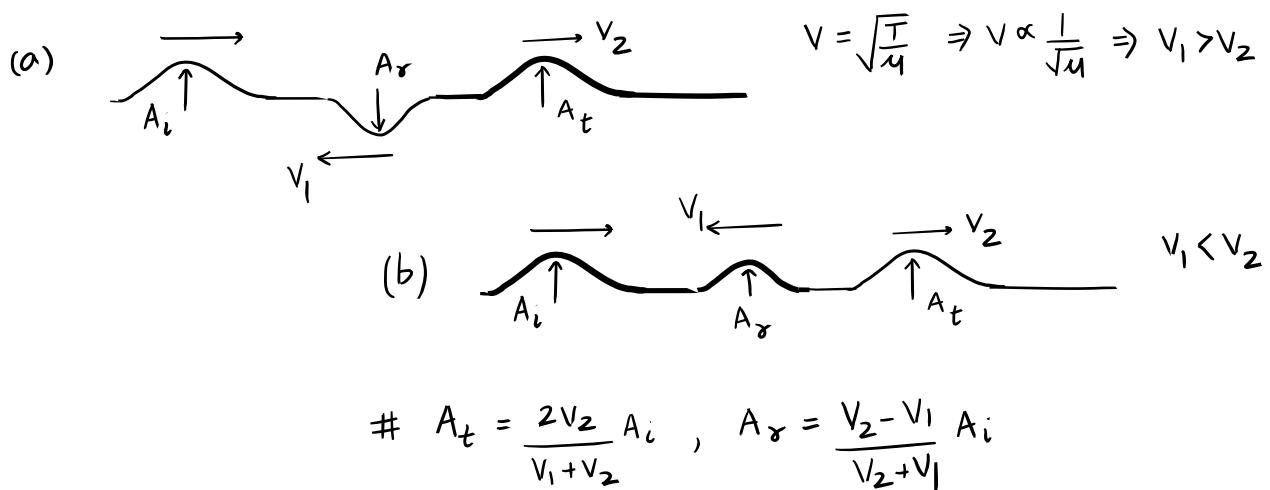
1. Superposition of Pulse



2. Reflection from fixed & free ends



3. Reflection & transmission between two strings

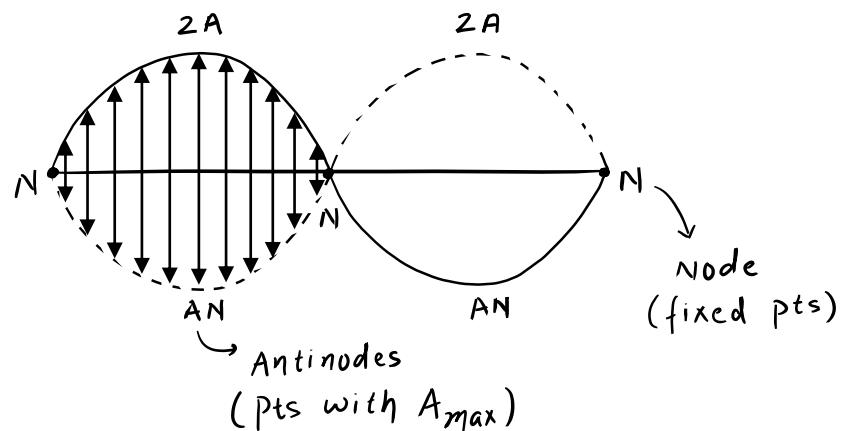


4. Standing waves or stationary waves

↳ formed due to superpos'n of two coherent waves travelling in opp direction

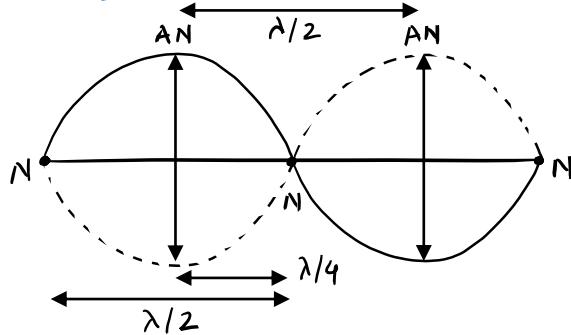
↳ both waves

had 'A' Amp.



5. Key Points in standing waves

String Waves



- (i) All Points between two successive NODES are in same phase
- (ii) All points across a NODE are in opp phase ($\Delta\phi = \pi$)
- (iii) separation betⁿ two consecutive NODES OR ANTINODES is $\lambda/2$.
- (iv) and betⁿ consecutive NODES and ANTINODE is $\lambda/4$.

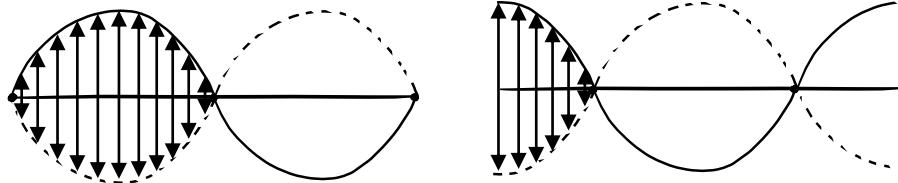
6. Equation of standing waves

$$y = 2A \sin(Kx + \phi_1) \sin(\omega t + \phi_2)$$

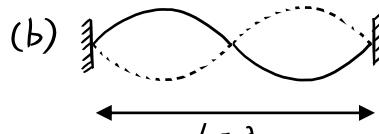
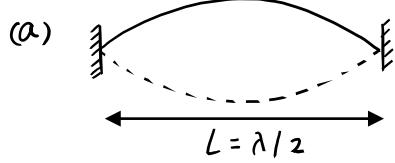
Amplitude of a point at distance x

$$\Rightarrow y = 2A \sin Kx \sin(\omega t + \phi_2) \quad (at x=0, \text{ NODE})$$

$$y = 2A \cos Kx \sin(\omega t + \phi_2) \quad (at x=0, \text{ AN})$$



7. Standing Wave in clamped string (fixed at both ends)



$$f_1 = \frac{V}{\lambda} \Rightarrow f_1 = \frac{1}{2L} \sqrt{\frac{T}{\mu}}$$

fundamental freq
or
1st Harmonic

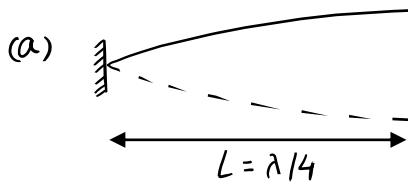
$$f_2 = \frac{1}{L} \sqrt{\frac{T}{\mu}} = \frac{2}{2L} \sqrt{\frac{T}{\mu}} \rightarrow 2^{\text{nd}} \text{ harmonic}$$

$$\begin{aligned} & \text{n}^{\text{th}} \text{ Harmonic} \\ & (n-1)^{\text{th}} \text{ overtone} \Rightarrow f = \frac{n}{2L} \sqrt{\frac{T}{\mu}} \\ & = nf_1 \end{aligned}$$

NOTE: If $f_1 = 100 \text{ Hz} \Rightarrow$ string will be in resonance with tuning fork of 100, 200, 300 Hz...

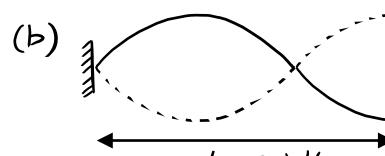
8. Standing Wave in Clamped String (fixed at one end)

String Waves



$$f_1 = \frac{V}{\lambda} = \frac{1}{4L} \sqrt{\frac{T}{\mu}}$$

↳ fundamental freq or
1st harmonic



$$L = 3\lambda/4$$

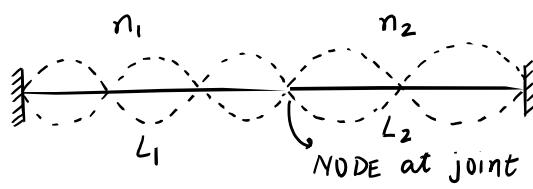
$$f_3 = \frac{3}{4L} \sqrt{\frac{T}{\mu}}$$

3rd harmonic
1st overtone

$$f = \frac{(2n+1)}{4L} \sqrt{\frac{T}{\mu}} = (2n+1) f_1$$

↳ (2n+1)th harmonic
↳ nth overtone

NOTE: If $f_1 = 100\text{ Hz} \Rightarrow$ String will be in resonance with tuning fork of 100, 300, 500 Hz...

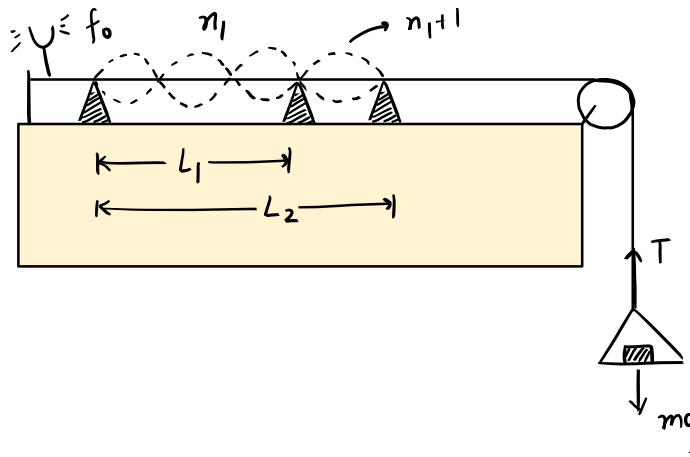
9. Standing Wave in Composite String $\text{freq,1} = \text{freq,2}$ 

$$\Rightarrow \frac{n_1}{2L_1} \sqrt{\frac{T}{\mu_1}} = \frac{n_2}{2L_2} \sqrt{\frac{T}{\mu_2}}$$

$$\Rightarrow \frac{n_1}{n_2} = \frac{L_1}{L_2} \times \sqrt{\frac{\mu_2}{\mu_1}}$$

↳ will learn its application
in example ahead

10. Sonometer (to find wave speed in strings)



$$\therefore L_2 - L_1 = \frac{\lambda}{2}$$

$$\Rightarrow \lambda = 2(L_2 - L_1)$$

$$\therefore V = f_0 \lambda = 2f_0(L_2 - L_1)$$

↳ Simply saying, it is
standing waves betw
2 fixed ends.

Space to add concepts learnt from PYQs if any

Note: To best use these 1st watch the video from "Revision Series Playlist" on Eduniti YouTube Channel (PYQs are also there for practice)

Topics to cover in Sound Waves – PART 3

1. Displacement & Pressure waves
2. Speed of sound waves
3. Beat Frequency
4. Doppler's Effect
5. Different cases (how to write sign)
6. Effect of Wind Speed
7. Sound reflected from fixed wall/object
8. Source and observer not in same line

Note: For video refer Revision Series Playlist on EDUNITI YouTube Channel

1. Displacement and Pressure Wave

$$\hookrightarrow s = s_0 \sin \omega \left(t - \frac{x}{v} \right) \quad \hookrightarrow P = P_0 \cos \omega \left(t - \frac{x}{v} \right), \quad P_0 = \frac{B \omega s_0}{v}$$

Here P_0 is amplitude Pressure

2. Speed of sound waves

$$V_{\text{liquid}} = \sqrt{\frac{B}{P}}, \quad V_{\text{solid}} = \sqrt{\frac{Y}{P}}, \quad V_{\text{gas}} = \sqrt{\frac{Y P}{M}} = \sqrt{\frac{Y R T}{M}}$$

B : Bulk Modulus

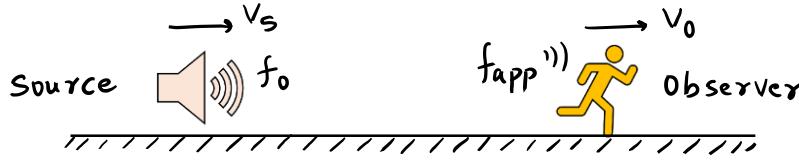
Y : Young's Modulus, γ : Adiabatic Constant

3. Beat frequency: Superposition of two waves of almost equal frequencies.

$$f_b = |f_1 - f_2|$$

4. DOPPLER'S EFFECT

When a source of sound and a listener are in motion relative to each other, the frequency of the sound heard by the listener is not the same as the source frequency.



MOST GENERAL

$$f_{app} = f_0 \left(\frac{V \pm V_w \pm V_o}{V \pm V_w \pm V_s} \right)$$

v_s : Speed of source

v_o : Speed of observer

f_0 : Source Sound frequency

f_{app} : Apparent frequency heard by observer

V : Speed of sound in still air

5. CASES (v_s and v_o in same direction)

NOTE:

If any speed tries to decrease separation between source and observer, sign is taken so as to $\uparrow f_{app}$ and vice-versa.

$f_{app} = f_0 \left(\frac{V - V_o}{V} \right)$ $f_{app} = f_0 \left(\frac{V}{V - V_s} \right)$ $f_{app} = f_0 \left(\frac{V - V_o}{V - V_s} \right)$

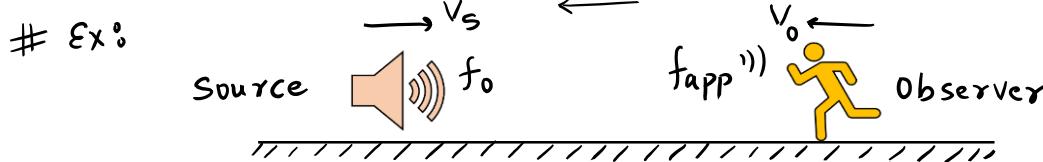
$f_{app} = f_0 \left(\frac{V + V_o}{V} \right)$ $f_{app} = f_0 \left(\frac{V}{V + V_s} \right)$ $f_{app} = f_0 \left(\frac{V + V_o}{V + V_s} \right)$

5(ii) CASES (v_s and v_o in opposite direction)6. EFFECT OF WIND SPEED (v_w)

↪ If "v" is along $v_w \Rightarrow v + v_w$

↪ If "v" is opposite $v_w \Rightarrow v - v_w$

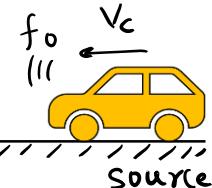
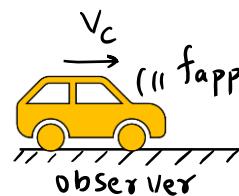
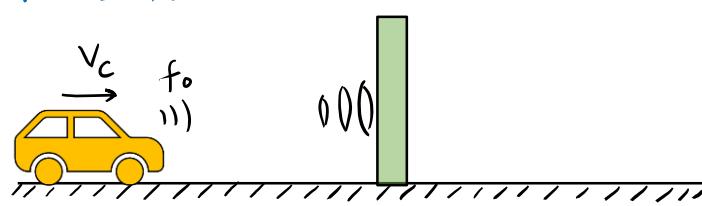
$$f_{app} = f_0 \left(\frac{v - v_w + v_o}{v - v_w - v_s} \right)$$



7. SOUND REFLECTED FROM "FIXED WALL"

Car honk freq = f_0

fapp heard by driver
after reflection.

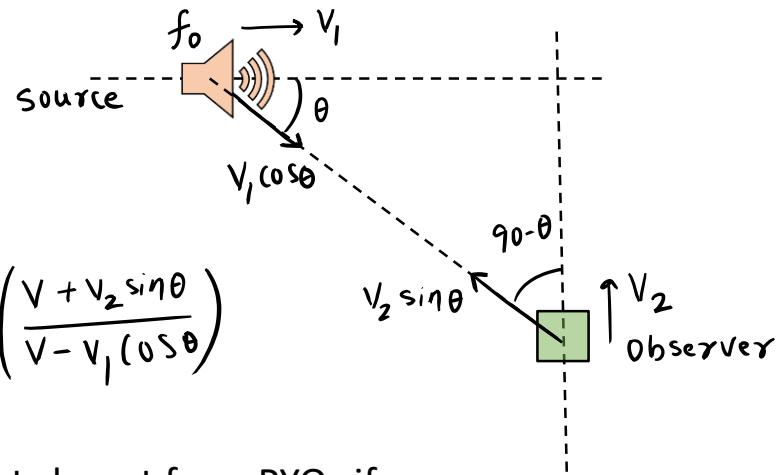


$$f_{app} = f_0 \left(\frac{v + v_c}{v - v_c} \right)$$

Beat frequency, $f_b = f_{app} - f_0$

8. SOURCE AND OBSERVER NOT IN SAME LINE

Take components of Velocities along line joining them.



Space to add concepts learnt from PYQs if any

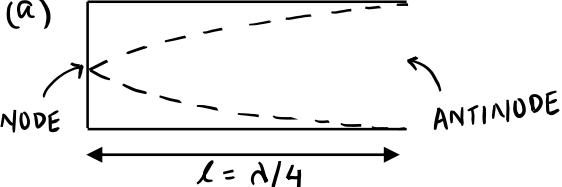
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Topics to cover in Sound Waves – PART 4

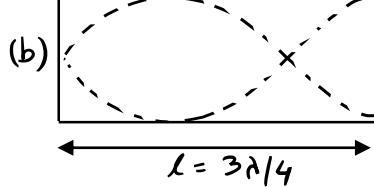
1. Closed Organ Pipe
2. Open Organ Pipe
3. End Correction
4. Resonance Tube

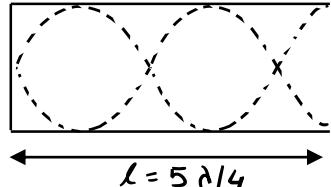
Note: For video refer Revision Series Playlist on EDUNI YouTube Channel

1. CLOSED ORGAN PIPE (STATIONARY WAVES)

(a)  NODE ANTIODE $\lambda = \lambda/4$

$\therefore \lambda = 4l, f_0 = \frac{v}{4l}$ → fundamental frequency or 1st harmonic

(b)  $\Rightarrow \lambda = 4l/3$
 $\therefore f_3 = \frac{3v}{4l} \rightarrow 3^{\text{rd}} \text{ Harmonic or 1}^{\text{st}} \text{ overtone}$

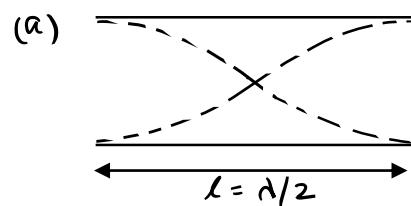
(c)  $\Rightarrow \lambda = 4l/5$
 $\therefore f_5 = \frac{5v}{4l} \rightarrow 5^{\text{th}} \text{ Harmonic or 2}^{\text{nd}} \text{ overtone}$

$v = \sqrt{\frac{YRT}{M}}$ or $\sqrt{\frac{YP}{\rho}}$
 $\gamma = C_p/C_v$

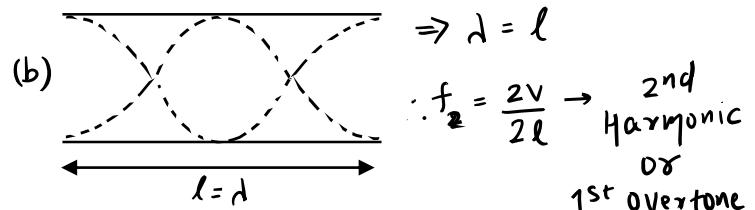
If frequency of tuning fork matches with odd Multiple of fundamental frequency, Resonance occurs

$f_n = (2n+1)f_0$
 $\hookrightarrow n^{\text{th}} \text{ overtone}$

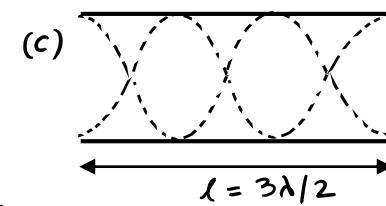
2. OPEN ORGAN PIPE (STATIONARY WAVES)



$$\therefore \lambda = 2l, \boxed{f_0 = \frac{V}{2l}} \rightarrow \text{fundamental frequency or 1st Harmonic}$$



$$\Rightarrow \lambda = l \quad \therefore f_2 = \frac{2V}{2l} \rightarrow \text{2nd Harmonic or 1st overtone}$$



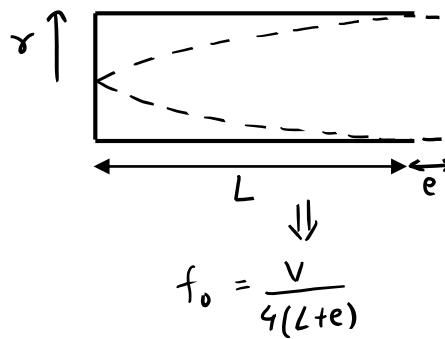
$$\Rightarrow \lambda = 2l/3 \quad \therefore f_3 = \frac{3V}{2l} \rightarrow \text{3rd Harmonic or 2nd overtone}$$

If frequency of tuning fork matches with integer Multiple of fundamental frequency, Resonance occurs

$$f_n = (n+1) f_0 \quad \hookrightarrow n^{\text{th}} \text{ overtone}$$

3 END CORRECTION (e)

At open side, ANTI NODE is formed a little outside.



$$\boxed{e = 0.6r} \quad r = \text{Pipe radius.} \quad L = \text{Pipe true length}$$

$$\downarrow \quad f_0 = \frac{V}{2(L+2e)}$$

4 RESONANCE TUBE

Used to find Speed of sound in air.

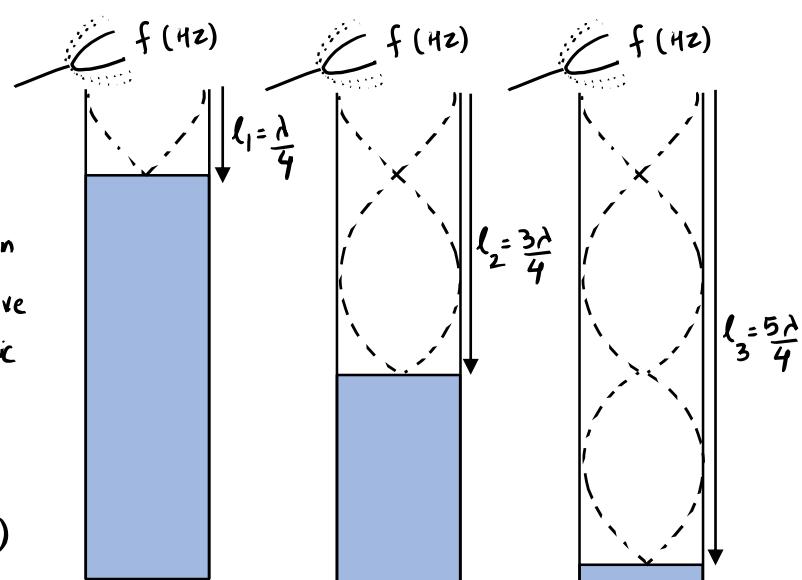
(1) Difference betⁿ any two consecutive Resonance/harmonic is $\lambda/2$

$$l_2 - l_1 = \lambda/2$$

$$\Rightarrow \lambda = 2(l_2 - l_1)$$

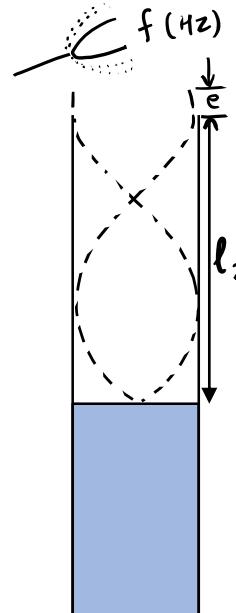
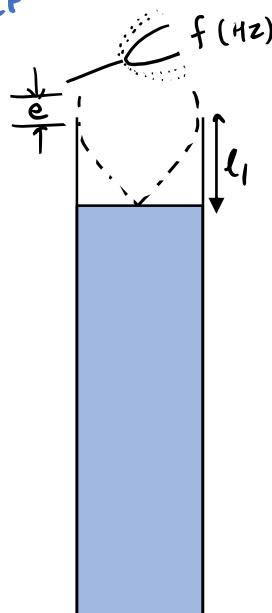
$$V = \lambda f = 2f(l_2 - l_1)$$

NOTE: ONCE f is fixed, λ is also fixed.



end correction effect

l_1 and l_2
are length
measured on
scale.



$$(l_2 + e) - (l_1 + e) = \frac{\lambda}{2}$$

$$\Rightarrow l_2 - l_1 = \frac{\lambda}{2}$$

$$v = f \times 2(l_2 - l_1)$$

Hence end correction
has no effect on
final result of
experiment.

Space to add concepts learnt from PYQs if any