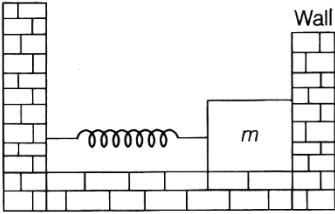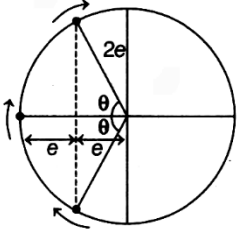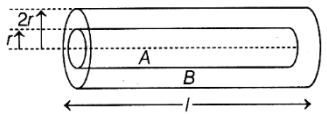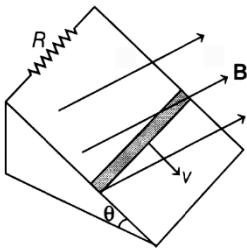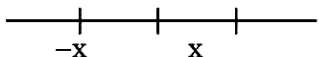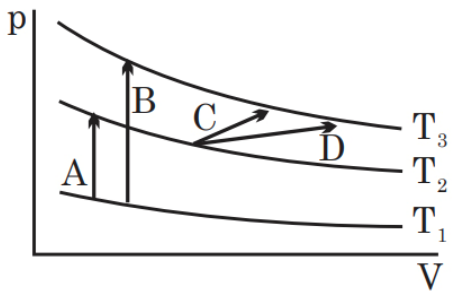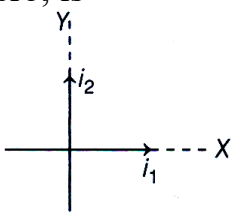| 1.	A conducting wire frame is placed in a magnetic field which is directed into the paper. The magnetic field is increasing at a constant rate. The directions of induced currents in wires AB and CD are	A
	B to A and D to C	
	A to B and C to D	
	A to B and D to C	
	B to A and C to D	
Sol.	The emf will be induced in each loop in such a way that induced current will be anti-clockwise in each loop to oppose the flux increasing in $\otimes$ direction : $\varepsilon_2 > \varepsilon_1$ $\because$ {Induced emf will be greater in the bigger loop} So, current in bigger loop will be anti-clockwise and in the smaller loop clockwise.	
2.	Figure shows a square loop of side 0.5m and resistance 10Ω. The magnetic field has a magnitude B = 1.0 T. The work done in pulling the loop out of the field uniformly in 2.0 is	A
	3.125×10^{-3} J	
	6.25×10^{-4} J	
	1.25×10^{-2} J	
	5.00×10^{-4} J	
Sol.	$v = \dfrac{0.5}{2} = 0.25$ m/s $i = \dfrac{Blv}{R} = \dfrac{1 \times 0.5 \times 0.25}{10} = 1.25 \times 10^{-2} A$ $F_{ext} = Bil = 1 \times 1.25 \times 10^{-2} \times 0.5$ $= 6.25 \times 10^{-3} N$ $W = F_{ext}l = 6.25 \times 10^{-3} \times 0.5 = 3.125 \times 10^{-3}$ J	
3.	A resistance of 20Ω is connected to a source of an alternating potential $V = 220 \sin(100\pi t)$. The time taken by the current to change from the peak value to rms value, is	D
	0.2 s	
	0.25 s	
	2.5×10^{-2} s	
	2.5×10^{-3} s	

Sol.	$i_{rms} = \dfrac{i_m}{\sqrt{2}} = i_m \sin \omega t_2 \Rightarrow \omega t_2 = \dfrac{\pi}{4}$ and $i_m = i_m \sin \omega t_1 \Rightarrow \omega t_1 = \dfrac{\pi}{2}$ Hence, $\omega(t_1 - t_2) = \dfrac{\pi}{2} - \dfrac{\pi}{4} \Rightarrow t_2 - t_1 = \dfrac{\pi}{4\omega}$ $= 2.5 \times 10^{-3}$ s	
4.	In a circuit an AC current and a DC current are supplied together. The expression of the instantaneous current is given as : $i = 3 + 4\sin\left(\omega t + \dfrac{\pi}{3}\right)$. The rms value of current is	**B**
	5 A	
	$\sqrt{17}$ A	
	$\dfrac{5}{\sqrt{2}}$ A	
	$\dfrac{7}{\sqrt{2}}$ A	
Sol.	$i^2 = 9 + 16 \sin^2\left(\omega t + \dfrac{\pi}{3}\right) + 24 \sin\left(\omega t + \dfrac{\pi}{3}\right)$ $i_{rms} = \sqrt{<i^2>} = \sqrt{9 + 16\left(\dfrac{1}{2}\right) + 24(0)} = \sqrt{17}$ A	
5.	An AC circuit has R = 100 Ω, C = 2 μF and L = 80 mH connected in series. The quality factor of the circuit is	**A**
	2	
	0.5	
	20	
	400	
Sol.	Given that, $R = 100\ \Omega, C = 2\ \mu F, L = 80\ mH$ For a series L - C - R AC circuit, Quality factor, $\phi = \dfrac{1}{R}\sqrt{\dfrac{L}{C}} = \dfrac{1}{100}\sqrt{\dfrac{80 \times 10^{-3}}{2 \times 10^{-6}}}$ $\phi = 2$	
6.	A circuit connected to an AC source of emf $e = e_0 \sin(100t)$ with t in seconds, gives a phase difference of $\dfrac{\pi}{4}$ between the emf e and current i. Which of the following circuits will exhibit this?	**C**
	RC circuit with $R = 1k\Omega$ and $C = 1\mu F$	
	RL circuit with $R = 1k\Omega$ and $L = 1mH$	
	RC circuit with $R = 1k\Omega$ and $C = 10\mu F$	
	RL circuit with $R = 1k\Omega$ and $L = 10mH$	
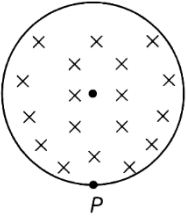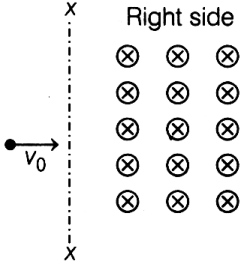
Sol.	Since, $\tan \dfrac{\pi}{4} = 1 = \dfrac{X_C \text{ or } X_L}{R}$ $\therefore$ For R-C circuit, we have $\qquad 1 = \dfrac{1}{C\omega R}$ or $\omega = \dfrac{1}{CR}$ ...(i) Similarly, for R-L circuit, we have $\qquad 1 = \dfrac{\omega L}{R} \Rightarrow \omega = \dfrac{R}{L}$ ...(ii) It is given in the question that, $\qquad \omega = 100$ rad/s Thus, again by substituting the given values of R, C or L option wise in the respective Eqs. (i) and (ii), we get that only for option (c), $\qquad \omega = \dfrac{1}{CR} = \dfrac{1}{10 \times 10^{-6} \times 10^3}$ or $\qquad \omega = 100$ rad/s		
7.	A particle of mass 1 kg is undergoing SHM, for which graph between force and displacement (from mean position) as shown. Its time period, in seconds, is 	**B**	
	$\pi / 3$		
	$2\pi / 3$		
	$\pi / 6$		
	$3 / \pi$		
Sol.	$F = -kx$ $\Rightarrow \qquad -13.5 = -k(1.5) \Rightarrow k = 9$ $\qquad T = 2\pi \sqrt{\dfrac{m}{k}} = 2\pi \sqrt{\dfrac{1}{9}} = \dfrac{2\pi}{3}$ s		
8.	A heavy particle of mass 8 kg is executing SHM on the X-axis with time period 6.28 s. If its maximum and minimum potential energy during motion is 80 J and 16 J respectively, then find out amplitude of the motion of particle	**D**	
	16 m		
	32 m		
	8 m		
	4 m		
Sol.	$K_{max} + U_{min} = K_{min} + U_{max}$ $\Rightarrow \qquad K_{max} + 16 = 0 + 80$ $\Rightarrow \qquad K_{max} = 64$ J $\qquad K_{max} = \dfrac{1}{2} m\omega^2 A^2$ $\Rightarrow \qquad 64 = \dfrac{1}{2} \times 8 \times \left(\dfrac{2\pi}{6.28}\right)^2 A^2 \Rightarrow A = 4$ m		
9.	In the figure, the block of mass m attached to the spring of stiffness k is in contact with the completely elastic wall and the compression in the spring is	**A**	
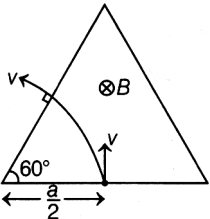
'e'. The spring is compressed further by 'e' by displacing the block towards left and is then released. If the collision between the block and the wall is completely elastic, then the time period of oscillations of the block will be

Wall

$\dfrac{2\pi}{3}\sqrt{\dfrac{m}{k}}$

$2\pi\sqrt{\dfrac{m}{k}}$

$\dfrac{\pi}{3}\sqrt{\dfrac{m}{k}}$

$\dfrac{\pi}{6}\sqrt{\dfrac{m}{k}}$

| Sol. | $\cos\theta = \dfrac{e}{2e} = \dfrac{1}{2}$

 $\Rightarrow \quad \theta = \dfrac{\pi}{3}$

 $\theta = \omega t$

 $\Rightarrow \quad 2\left(\dfrac{\pi}{3}\right) = \left(\sqrt{\dfrac{k}{m}}\right)t$

 $\Rightarrow \quad t = \dfrac{2\pi}{3}\sqrt{\dfrac{m}{k}}$ | |

| 10. | A composite cylinder is made of two different materials A and B of thermal conductivities K_A and K_B. The dimensions of the cylinder are as shown in the figure. The thermal resistance of the cylinder between the two end faces is

  | **A** |

| | $\dfrac{1}{\pi r^2 (K_A + 3K_B)}$ | |

| | $\dfrac{1}{\pi r^2}\left(\dfrac{1}{K_A} + \dfrac{3}{K_B}\right)$ | |

| | $\dfrac{1}{\pi r^2}\left(\dfrac{1}{K_A} + \dfrac{4}{K_B}\right)$ | |

| | $\dfrac{1}{\pi r^2 (K_A + 4K_B)}$ | |

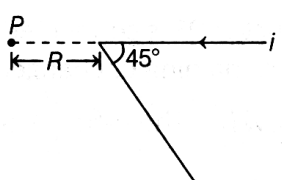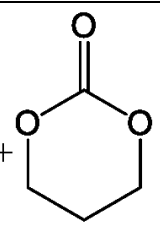|		
Sol.	$R_A = \dfrac{l}{K_A(\pi r^2)}$, $R_B = \dfrac{l}{K_B(\pi(2r)^2 - \pi r^2)} = \dfrac{l}{3K_B \pi r^2}$ $\dfrac{1}{R_{eq}} = \dfrac{1}{R_A} + \dfrac{1}{R_B}$ $\Rightarrow \quad \dfrac{1}{R_{eq}} = \dfrac{K_A(\pi r^2)}{l} + \dfrac{3K_B(\pi r^2)}{l}$ $\Rightarrow \quad R_{eq} = \dfrac{l}{\pi r^2(K_A + 3K_B)}$	
11.	One mole of an ideal gas undergoes a process in which $T = T_0 + aV^3$, where T_0 and a are positive constants and V is volume. The volume for which pressure will be minimum is	**A**
	$\left(\dfrac{T_0}{2a}\right)^{1/3}$	
	$\left(\dfrac{T_0}{3a}\right)^{1/3}$	
	$\left(\dfrac{a}{2T_0}\right)^{2/3}$	
	$\left(\dfrac{a}{3T_0}\right)^{2/3}$	
Sol.	$T = T_0 + aV^3$ $\Rightarrow \dfrac{pV}{nR} = T_0 + aV^3 \quad \Rightarrow \quad p = R\left(\dfrac{T_0}{V} + aV^2\right)$ **For minimum pressure,** $\dfrac{dp}{dV} = R\left(-\dfrac{T_0}{V^2} + 2aV\right) = 0$ $\Rightarrow V = \left(\dfrac{T_0}{2a}\right)^{1/3}$	
12.	A conducting wire of mass m slides down two smooth conducting bars, set at an angle θ to the horizontal as shown in figure. The separation between the bars is l. The system is located in the magnetic field B, perpendicular to the plane of the sliding wire and bars. The constant velocity of the wire is 	**A**
	$\dfrac{mgR \sin\theta}{B^2 l^2}$	
	$\dfrac{mgR}{B^2 l^2}$	
	$\dfrac{mgR \tan\theta}{B^2 l^2}$	
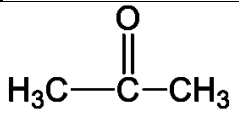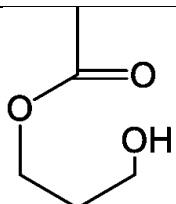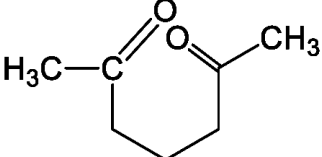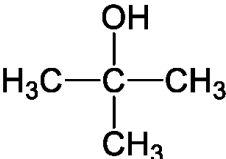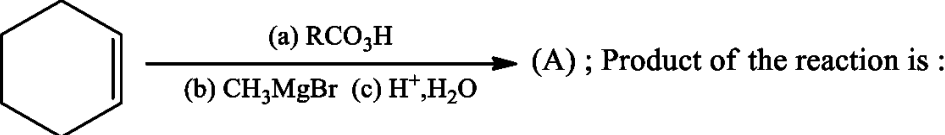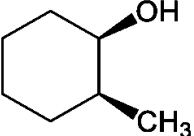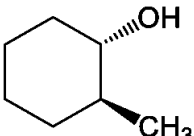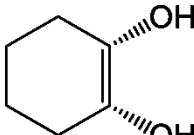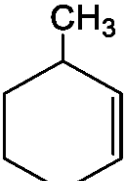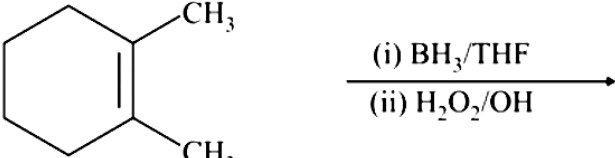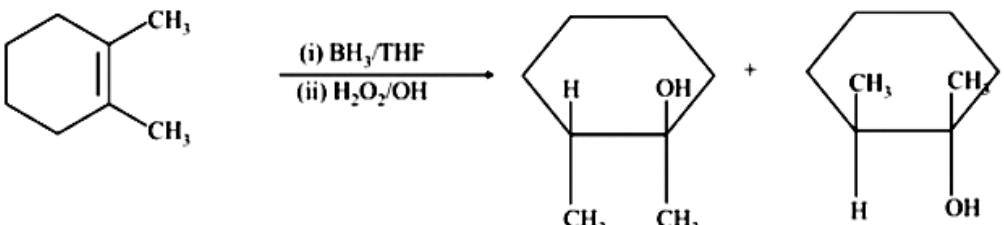
	$$\dfrac{mgR\sin\theta}{B^2 l^2 \cos^2\theta}$$	
Sol.	$i = \dfrac{Blv}{R}$; $\Sigma F = 0$ $\Rightarrow \qquad mg\sin\theta - Bil = 0$ $\Rightarrow \quad mg\sin\theta - B\left(\dfrac{Blv}{R}\right)l = 0$ $\Rightarrow \qquad\qquad v = \dfrac{mgR\sin\theta}{B^2 l^2}$	
13.	A circular hole in an aluminium plate is 2.54 cm in diameter at 0°C. What is the diameter when the temperature of the plate is raised to 100°C ? (Given, $\alpha_{Al} = 2.3 \times 10^{-5} / °C$)	**B**
	2.4558 cm	
	2.5458 cm	
	1.4558 cm	
	1.5458 cm	
Sol.	$d = d_0(1 + \alpha\Delta T) = 2.54(1 + 2.3 \times 10^{-5} \times 100)$ $\qquad\qquad = 2.5458$ cm	
14.	A 100 g of iron nail is hit by a 1.5 kg hammer striking at a velocity of 60 ms^{-1}. What will be the rise in the temperature of the nail if one-fourth of energy of the hammer goes into heating the nail? [specific heat capacity of iron = 0.42 Jg^{-1} °C^{-1}]	**C**
	675°C	
	1600°C	
	16.07°C	
	6.75°C	
Sol.	Mass of iron ball, $\qquad m_1 = 100$ g $= 0.1$ kg Mass of hammer, $m_2 = 1.5$ kg $\qquad v = 60$ m/s, $s = 0.42$Jg^{-1}°C^{-1} $\qquad\qquad = 420$ J kg^{-1}°C^{-1} According to question, $\dfrac{\text{Total kinetic energy of hammer}}{4} = $ Heat energy gain by nail $\Rightarrow \quad \dfrac{1}{4}\left[\dfrac{1}{2}m_2 v^2\right] = m_1 s\Delta T$ $\Rightarrow \dfrac{1}{4}\left[\dfrac{1}{2} \times 1.5 \times 60^2\right] = 0.1 \times 420 \times \Delta T$ $\Rightarrow \qquad\qquad \Delta T = \dfrac{675}{42} = 16.07\ °C$	
15.	A particle is in simple harmonic motion along the x axis. The amplitude of the motion is x_m. When it is at $x = x_1$ its kinetic energy is K = 5J and its potential energy (measured with U = 0 at x = 0) is U = 3J. When it is at $x = -x_1$, the kinetic and potential energies are:	**A**
	K = 5 J and U = 3J	
	K = 5 J and U = –3J	
	K = 8 J and U = 0	
	K = 0 and U = 8J	
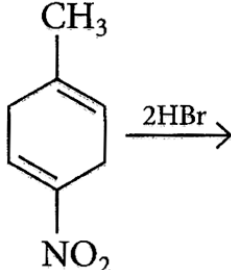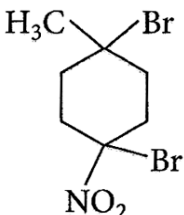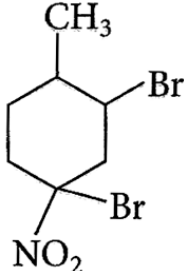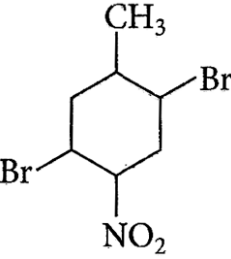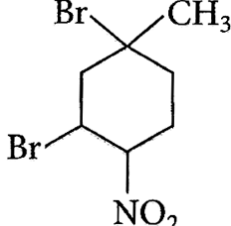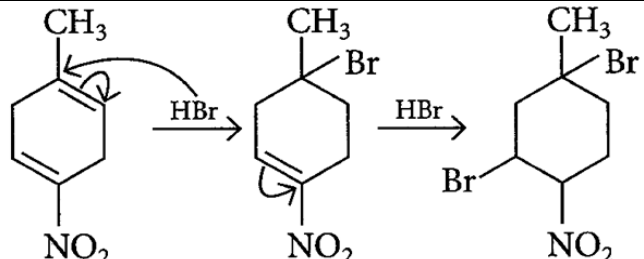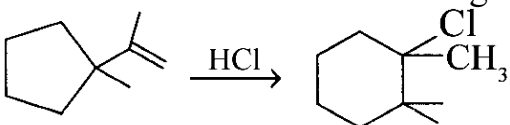
Sol.	$v = \omega\sqrt{A^2 - X^2}$ Speed is same at same separation from M.P., So KE at x = KE at –x So PE at x = PE at –x KE = 5J u = 3J	
16.	The diagram shows four thermodynamic processes carried out on an ideal gas. For which of these processes is the change in the internal energy of the gas the greatest? 	**B**
	A	
	B	
	C	
	D	
Sol.	Here $T_3 > T_2 > T_1$ $\Delta U = nC_v\Delta T$ Here ΔT is maximum in process B.	
17.	Two mutually perpendicular conductors carrying current i_1 and i_2 lie in one plane. If i_1 is taken along X-axis and i_2 along y-axis, the focus of points, where magnetic induction is zero, is 	**A**
	A straight line passing through origin and having slope $\dfrac{i_1}{i_2}$	
	A straight line passing through origin and having slope $\dfrac{i_2}{i_1}$	
	A circle	
	A hyperbola	
Sol.	At point (X, Y), $\mathbf{B} = \left(\dfrac{\mu_0 i_1}{2\pi Y} - \dfrac{\mu_0 i_2}{2\pi X}\right)\hat{\mathbf{k}} = 0$ $\Rightarrow \qquad Y = \left(\dfrac{i_1}{i_2}\right)X$	
18.	Figure shows a uniform magnetic field $B = kt$ confined to a cylindrical region of radius R. The instantaneous acceleration experienced by an electron placed at point P on the periphery of the field is	**B**

	Zero	
	$\dfrac{keR}{2m}$ towards left	
	$\dfrac{keR}{2m}$ towards right	
	$\dfrac{keR}{m}$ towards left	
Sol.	$\varepsilon = \dfrac{d\phi}{dt} \Rightarrow E(2\pi r) = \pi r^2 \dfrac{dB}{dt}$ $\Rightarrow \quad E = \dfrac{r}{2}\dfrac{dB}{dt}$ At point P, $\mathbf{E} = \dfrac{R}{2}\dfrac{dB}{dt}$ towards right, $\mathbf{a} = \dfrac{q\mathbf{E}}{m} = \dfrac{keR}{2m}$ towards left.	
19.	A charge particle having charge q_0 and mass m is entered into a magnetic field with velocity v_0 as shown. Uniform magnetic field B_0 is present only on right side of dotted line. Average force exerted on charge particle during time is inside magnetic field is	**A**
	$\dfrac{2qB_0 v_0}{\pi}$	
	$qB_0 v_0$	
	Zero	
	$2qB_0 v_0$	
Sol.	$F_{avg} = \dfrac{\Delta p}{\Delta t} = \dfrac{mv_0 - (-mv_0)}{\dfrac{\pi m}{qB}} = \dfrac{2qB_0 v_0}{\pi}$	
20.	A uniform magnetic field exists in region which forms an equilateral triangle of side a. The magnetic field is perpendicular to the plane of the triangle. A charge q enters into this magnetic field perpendicular with speed v along perpendicular bisector of one side and comes out along perpendicular bisector of other side. The magnetic induction in the triangle is	**B**
	$\dfrac{mv}{qa}$	

	$\dfrac{2mv}{qa}$	
	$\dfrac{mv}{2qa}$	
	$\dfrac{mv}{4qa}$	
Sol.	$R = \dfrac{mv}{qB} \quad \Rightarrow \quad \dfrac{a}{2} = \dfrac{mv}{qB} \quad \Rightarrow B = \dfrac{2mv}{qa}$	
21.	The magnetic flux ϕ (in weber) linked with a closed circuit of resistance $8\,\Omega$ varies with time (in seconds) as $\phi = 5t^2 - 36t + 1$. The induced current in the circuit at t = 2 s is……..A.	**2**
Sol.	Induced current, $$I = -\dfrac{e}{R} = -\left(\dfrac{\frac{d}{dt}\phi_B}{R}\right) = -\left(\dfrac{10t - 36}{8}\right)$$ Induced current at $t = 2s$ is, $$I = -\left(\dfrac{10 \times 2 - 36}{8}\right) = 2\,A$$	
22.	A body cools in 7 min from 60ºC to 45ºC in 5 min and to 40ºC in another 7 min. Find the temperature of the surrounding.	**28**
Sol.	$\theta_0 = 30°\,C$ $60°C \xrightarrow{7\,min} 40°C$ $\left\langle \dfrac{d\theta}{dt} \right\rangle = -K(\langle\theta\rangle - \theta_0)$ $\Rightarrow \dfrac{40 - 60}{7} = -K\left(\dfrac{60 + 40}{2} - 10\right)$ $40°C \xrightarrow{7\,min} \theta$ $\Rightarrow \dfrac{\theta - 40}{7} = -K\left(\dfrac{40 + \theta}{2} - 10\right)$ Solving Eqs. (i) and (ii), we get $\theta = 28°C$	
23.	A wall has two layers A and B each made of different material, both the layers have the same thickness. The thermal conductivity of the material A is thrice that of B. Under thermal equilibrium, the temperature difference across the wall B is 36ºC. The temperature difference across the wall A is	**12**
Sol.	$i_A = i_B$ $\Rightarrow \dfrac{(\Delta T)_A}{\left(\dfrac{d}{(3K)A}\right)} = \dfrac{36}{\left(\dfrac{d}{KA}\right)}$ $\Rightarrow (\Delta T)_A = 12°\,C$	
24.	Starting at temperature 300 K, one mole of an ideal diatomic gas ($\gamma = 1.4$) is first compressed adiabatically from volume V_1 to $V_2 = \dfrac{V_1}{16}$. It is then allowed to expand isobarically to volume $2V_2$. If all the processes are the	**1819**

	quasi-static, then the final temperature of the gas (in K) is (to the nearest integer)....... .	
Sol.	Initially gas is compressed adiabatically. Initial temperature, $T_1 = 300\,K$ Number of moles $= 1$, $\gamma = 1.4$ Initial volume $= V_1$ Final volume $= \dfrac{V_1}{16}$ Process equation is, $\quad T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ $\Rightarrow 300 V_1^{1.4-1} = T_2 \left(\dfrac{V_1}{16}\right)^{1.4-1}$ $\Rightarrow \qquad T_2 = 300 \times 2^{\frac{8}{5}}$ ...(i) In next process, gas is expanded isobarically. Initial volume, $\quad V_2 = \dfrac{V_1}{16}$ Final volume, $V_3 = 2\left(\dfrac{V_1}{16}\right) = \dfrac{V_1}{8}$ Now, for isobaric expansion, $\quad \dfrac{V_2}{T_2} = \dfrac{V_3}{T_3}$ $\Rightarrow \quad \dfrac{V_2}{300 \times 2^{\frac{8}{5}}} = \dfrac{2V_2}{T_3}$ $\Rightarrow \quad T_3 = 2 \times 300 \times 2^{\frac{8}{5}} = 300 \times 2^{\frac{13}{5}} = 1818.85$ or $\qquad T_3 = 1819\,K$	
25.	A long straight wire, carrying current I, is bent at its mid-point to form an angle of 45°. Induction of magnetic field at point P, distant R from point of bending is equal to $\dfrac{(\sqrt{x}-1)\mu_0 i}{4\pi R}$, then find value of x. 	**2**
26.	 $H_3C\!\!-\!\!MgBr +$ (excess) $\xrightarrow{\ H_3O^+\ }$ (X); Product(X) is :	**D**

	$H_3C-\overset{\displaystyle O}{\overset{\|}{C}}\overset{\displaystyle O}{\underset{\|}{\cdots}}CH_3$ (cyclohexane ring with two acetyl groups)	
	$\overset{\displaystyle OH}{H_3C-\underset{\underset{\displaystyle CH_3}{\|}}{\overset{\|}{C}}-CH_3}$	
27.	(cyclohexene) $\xrightarrow[\text{(b) } CH_3MgBr \text{ (c) } H^+,H_2O]{\text{(a) } RCO_3H}$ (A) ; Product of the reaction is :	**B**
	(cyclohexane with OH wedge and CH₃ wedge)	
	(cyclohexane with OH dashed and CH₃ wedge)	
	(cyclohexene with OH dashed and OH dashed)	
	CH_3 (methylcyclohexene)	
28.	(cyclohexene with two CH₃ groups) $\xrightarrow[\text{(ii) } H_2O_2/OH]{\text{(i) } BH_3/THF}$ Among the following correct statement is:	**D**
	Number of product in given reaction is one.	
	Reaction is regioselective	
	Number of products in given reaction is four.	
	Number of products in given reaction is two.	
Sol.	(cyclohexene with two CH₃ groups) $\xrightarrow[\text{(ii) } H_2O_2/OH]{\text{(i) } BH_3/THF}$ (product with H, OH, CH₃, CH₃) + (product with CH₃, CH₃, H, OH)	
	Number of products = 2.	
29.	The major product of the following reaction is	**D**
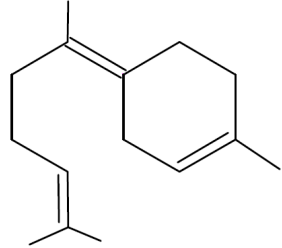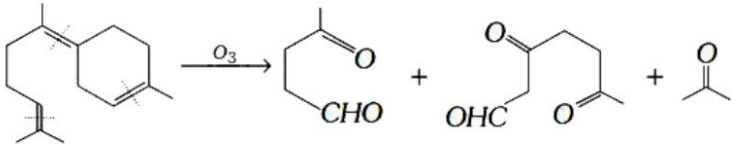
CH₃ ... 2HBr → ... NO₂ (as drawn)		
Sol.		
30.	The main reason for the rearrangement is	**B**
	The initially formed carbocation is secondary and it rearranges to the more stable tertiary one by shift of alkyl group	
	The reaction converts one tertiary carbocation into another to relieve tortional strain	
	The tertiary chloride is much more stable than a secondary one because of hyperconjugation	
	This is a radical chain reaction controlled by the relative bond strengths	

Sol.	 3° More stable	
31.	Which of the following compounds will lose optical activity after the reaction?	**B**
	$+ Pd/BaSO_4 \xrightarrow{H_2}$	
	$C\equiv C-CH_3 \xrightarrow[NH_3(\ell)]{Na}$	
	$C\equiv C-CH_3 + Pd/BaSO_3 \xrightarrow{H_2}$	
	$C\equiv C-CH_3 \xrightarrow[NH_3(\ell)]{Na}$	
Sol.	 *trans* (optically active) *trans-trans* (optically inactive)	
32.	Consider the following reaction. $+ Br_2 - H_2O \xrightarrow{NaCl}$ Which of the following would not be formed in the above reaction?	**D**
	2, 3-dibromo-2-methyl butane	
	2-chloro-3-bromo-2-methyl butane	
	3-bromo-2-methyl-2-butanol	
	2, 3-dichloro-2-methyl butane	
Sol.	 Dichloride is not formed in this reaction.	
33.	Which of the following reaction would not give at least one aldehyde product?	**C**
	1-pentyne $+ B_2H_6 \xrightarrow[NaOH]{H_2O_2}$	
	2-Butyne $+ Na \xrightarrow{NH_3\,(l)} \xrightarrow[Zn-H_2O]{O_3}$	
	1-Pentyne $+ KMnO_4 \xrightarrow{NaOH}$	

	Cyclopentene + O_3 $\xrightarrow[H_2O]{Zn}$	
Sol.	Alkyne, with $KMnO_4$ in alkaline medium gives carboxylic acid.	
34.	Four samples of acids and bases are taken for an experiment. **A.** 100 mL of 1 M NaOH and 100 mL of 1 M HCl. **B.** 100 mL of 2 M KOH and 100 mL of 1 M H_2SO_4. **C.** 100 mL of 1 M CH_3COOH and 100 mL of 1 M NaOH. **D.** 100 mL of 0.5 M KOH and 100 mL of 0.5 M HNO_3. Now, for each sample enthalpy of neutralization is calculated. Which of the following represents the correct result?	C
	Enthalpy of neutralization calculated in each case is found same.	
	In cases (A) and (D), the value of enthalpy of neutralization is same.	
	In cases (A), (B) and (D) the value of enthalpy of neutralization is same.	
	The value of enthalpy calculated is different for each sample.	
Sol.	As in cases (A), (B) and (D), the acids and bases taken are strong and the enthalpy of neutralisation of all strong acids with strong bases is same, i.e. – 57.3 kJ.	
35.	For complete combustion of ethanol, $$C_2H_5OH(l) + 3O_2(g) \rightarrow 2CO_2(g) + 3H_2O(l),$$ the amount of heat produced as measured in bomb calorimeter, is 1364.47 kJ mol^{-1} at 25ºC. Assuming ideality the enthalpy of combustion, ΔH, for the reaction will be (R = 8.314 Jk^{-1} mol^{-2})	B
	-1350.50 kJ mol^{-1}	
	$- 1366.95$ kJ mol^{-1}	
	$- 1361.95$ kJ mol^{-1}	
	-1460.50 kJ mol^{-1}	
Sol.	(b) : Given; $\Delta U = -1364.47$ kJ mol^{-1}, $\Delta n_g = 2 - 3 = -1$, $T = 25 + 273 = 298$ K $\Delta H = \Delta U + \Delta n_g RT$ $= -1364.47 + (-1) \times 8.314 \times 10^{-3} \times 298$ $= -1364.47 - 1 \times 8.314 \times 10^{-3} \times 298$ $= -1364.47 - 2477.57 \times 10^{-3} = -1366.95$ kJ mol^{-1} (Note : Given value of R is doubtful, it should be 8.314 J K^{-1} mol^{-1} or 8.314×10^{-3} kJ K^{-1} mol^{-1}.)	
36.	Which one of the following statements is not correct in respect of lyophilic sols?	C
	There is a considerable interaction between the dispersed phase and dispersion medium	
	These are quite stable and are not easily coagulated	
	They always carry significant characteristic charge	
	The particles are hydrated	
37.	Measuring Zeta potential is useful in determining which property of colloidal solution?	B
	Solubility	
	Stability of the colloidal particles	
	Size of the colloidal particles	
	Viscosity	

38.	Identify the statement (s) which is not correct with respect to surface phenomenon.	C
	If on adding electrolyte in an emulsion, the conductivity increases then it will be oil in water type emulsion.	
	Tyndall effect is observed when refractive indices of dispersed phase and dispersion medium differ largely.	
	Macromolecular colloids are generally lyophobic in nature	
	Gases which can react with adsorbents are generally chemisorbed	
Sol.	Macromolecular colloids like starch, gelatin are generally lyophilic.	
39.	Which of the following statements is incorrect?	D
	The electronic configuration of Cr is [Ar] $3d^5 4s^1$ [Atomic number of Cr = 24]	
	The magnetic quantum number may have a negative value	
	In silver atom, 23 electrons have a spin of one type and 24 of the opposite type. (Atomic number of Ag = 47)	
	The oxidation state of nitrogen in HN_3 is –3.	
Sol.	• $_{24}Cr = 1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 3d^5, 4s^1 = [Ar]3d^5, 4s^1$ • For magnetic quantum number (*m*), negative values are possible. For, *s*-subshell, $l = 0$, hence $m = 0$. For *p*-subshell, $l = 1$, hence $m = -1, 0, +1$ • $_{47}Ag = 1s^2, 2s^2 2p^6 3s^2 3p^6 3d^{10}, 4s^2 4p^6 4d^{10} 5s^1$ Hence, 23 electrons have a spin of one type and 24 of the opposite type. • oxidation state of N in N_3H is $-1/3$.	
40.	The wave function (Ψ) of 2s is given by $$\Psi_{2s} = \frac{1}{2\sqrt{2\pi}}\left(\frac{1}{a_0}\right)^{1/2}\left(2 - \frac{r}{a_0}\right)e^{-r/2a_0}$$ At $r = r_0$, radial node is formed. Thus, r_0 is terms of a_0	B
	• $r_0 = \dfrac{a_0}{2}$	
	• $r_0 = 2a_0$	
	• $r_0 = 4a_0$	
	• $r_0 = a_0$	
Sol.	When $r = r_0$, $\Psi_{2s} = 0$, then from the given equation $2 - \dfrac{r_0}{a_0} = 0$ or $r_0 = 2a_0$	
41.	Given below are two statements : **Statement I :** Bohr's theory accounts for the stability and line spectrum of Li^+ ion. **Statement II :** Bohr's theory was unable to explain the splitting of spectral lines in the presence of a magnetic field. • In the light of the above statements, choose the most appropriate answer from the options given below.	D
	• Both statement I and statement II are true.	

	• Statement I is true but statement II is false.	
	• Both statement I and statement II are false.	
	• Statement I is false but statement II is true.	
42.	• The de Broglie wavelength (λ) associated with a photoelectron varies with the frequency (υ) of the incident radiation as, [υ_0 is threshold frequency]	B
	• $\lambda \propto \dfrac{1}{(\upsilon - \upsilon_0)}$	
	• $\lambda \propto \dfrac{1}{(\upsilon - \upsilon_0)^{1/2}}$	
	• $\lambda \propto \dfrac{1}{(\upsilon - \upsilon_0)^{3/2}}$	
	• $\lambda \propto \dfrac{1}{(\upsilon - \upsilon_0)^{1/4}}$	
Sol.	• For electron, $\lambda = \dfrac{h}{\sqrt{2m\,K.E.}}$ $h\upsilon = h\upsilon_0 + K.E. \Rightarrow K.E. = h\upsilon - h\upsilon_0$ $\lambda = \dfrac{h}{\sqrt{2m(h\upsilon - h\upsilon_0)}} = \dfrac{h}{\sqrt{2mh(\upsilon - \upsilon_0)}} ;\ \lambda \propto \dfrac{1}{\sqrt{(\upsilon - \upsilon_0)}}$	
43.	Consider the following pairs of electrons (A) (i) $n = 3, l = 1, m_l = 1, m_s = +\dfrac{1}{2}$ (ii) $n = 3, l = 2, m_l = 1, m_s = +\dfrac{1}{2}$ (B) (i) $n = 3, l = 2, m_l = -2, m_s = -\dfrac{1}{2}$ (ii) $n = 3, l = 2, m_l = -1, m_s = -\dfrac{1}{2}$ (C) (i) $n = 4, l = 2, m_l = 2, m_s = +\dfrac{1}{2}$ (ii) $n = 3, l = 2, m_l = 2, m_s = +\dfrac{1}{2}$ • The pairs of electrons present in degenerate orbitals is/are	B
	only (A)	
	only (B)	
	only (C)	
	(B) and (C)	
Sol.	A has 3p and 3d electrons. B has both 3d electrons. C has 4d and 3d electrons. As B has both electrons from 3d-orbital so these are from degenerate orbitals.	
44.	The major product of the following reaction is $CH_3C \equiv CH \xrightarrow[\text{(ii) DI}]{\text{(i) DCl (1 equiv.)}}$	A

	CH$_3$C(I)(Cl)CHD$_2$	
	CH$_3$CD$_2$CH(Cl)(I)	
	CH$_3$CD(I)CHD(Cl)	
	CH$_3$CD(Cl)CHD(I)	
Sol.	CH$_3$C $\equiv$ CH $\xrightarrow{\text{(i)DCl}}$ CH$_3$—C=CHD Cl $\xrightarrow{\text{(ii) DI}}$ CH$_3$—C—CHD$_2$ (with I and Cl substituents on central C)	
45.	Which of the following molecule/species is most stable?	A
Sol.	As cyclopentenyl carbocation is aromatic in nature, it is most stable. $(4n + 2)$ no. of πe^- are present, where $n = 0$.	
46.	Reductive ozonolysis of a terpenoid of following structure gives how many different products	3
Sol.	Three Products	
47.	How many dichloride cyclopentanes (including stereoisomers) are obtained when cyclopentane reacts with excess chlorine at high temperature?	7

Sol.	Total isomers = 7	
48.	3 g of acetic acid is added to 250 mL of 0.1 M HCl and the solution made up to 500 mL. To 20 mL of this solution 1/2 mL of 5 M NaOH is added. The pH of the solution is _____. [**Given :** pK_a of acetic acid = 4.75, molar mass of acetic acid 60 g/mol, log 3 = 0.4771] Neglect any changes in volume. **(Multiply your answer by 100)**	**523**
Sol.	$$\text{mmoles of CH}_3\text{COOH} = \frac{3 \times 1000}{60} = 50$$ (molar mass of CH_3COOH = 60) mmoles of HCl = 250 × 0.1 = 25 $$\text{mmoles of CH}_3\text{COOH in 20 mL} = \frac{50}{500} \times 20 = 2$$ $$\text{mmoles of HCl} = \frac{25}{500} \times 20 = 1$$ $$NaOH + HCl \rightarrow NaCl + H_2O$$ Initial (mmoles) : 2.5 1 After reaction (mmoles) : 1.5 0 1 $$CH_3COOH + NaOH \rightarrow CH_3COONa + H_2O$$ Initial (mmoles) : 2 1.5 After reaction (mmoles) : 0.5 0 1.5 $$pH = pK_a + \log \frac{[CH_3COONa]}{[CH_3COOH]}$$ $$pH = pK_a + \log\left(\frac{1.5}{0.5}\right) = 4.75 + \log 3$$ $$= 4.75 + 0.4771 = 5.227 \approx 5.23$$	
49.	A piston filled with 0.04 mole of an ideal gas expands reversibly from 50 mL to 350 mL at a constant temperature of 310 K. It absorbs 208 J of heat during this process. The division of q and W for this process will be – x. The value of x is…….. .	**1**

Sol.	As we know, $\Delta U = q + W$, and the process is reversibly isothermal and expansion occurs, here. Therefore, $$q = -W$$ and $\Delta U = 0$ $$W = -2.303 \, nRT \log \frac{V_2}{V_1}$$ $$= -2.303 \times 0.04 \times 8.314 \times 310 \times \log \frac{350}{50}$$ $$= -237.42 \times \log 7$$ $$= -200.64$$ So, $q = +200.64$ The division of q and $W = \dfrac{-200.64}{200.64} = -1$	
50.	When the heat of a reaction at constant pressure is -2.5×10^3 cal and at 25°C temperature, entropy change for the reaction is 7.4 cal deg^{-1}, the Gibbs free energy for this reaction will be(–)……cal.	**4705**
Sol.	Heat at constant pressure means enthalpy, i.e. $$\Delta H = -2.5 \times 10^3 \text{ cal}$$ $$\Delta S = 7.4 \text{ cal deg}^{-1}$$ $$T = 298 \text{ K}$$ $$\Delta G = \Delta H - T\Delta S$$ $$\Delta G = -2.5 \times 10^3 - (298 \times 7.4)$$ $$\approx -4705 \text{ cal}$$	
51.	The number of numbers, strictly between 5000 and 10000 can be formed using the digits 1, 3, 5, 7, 9 without repetition, is	**D**
	6	
	12	
	120	
	72	
Sol.	**(d)** Number lies between 5000 & 10000 if numbers start from 5, 7, 9 _ _ _ _ For first position only 3 possibilities then, 4, 3, 2 Total numbers = $3 \times 4 \times 3 \times 2 = 72$	
52.	If the four letter words (need not be meaningful) are to be formed using the letters from the word "MEDITERRANEAN" such that the first letter is R and the fourth letter is E, then the total number of all such words is:	**B**
	110	
	59	
	$\dfrac{11!}{(2!)^3}$	
	56	
Sol.	**(b)** M, EEE, D. I, T, RR, AA, NN R _ _ E Two empty places can be filled with identical letters [EE, AA, NN] $\Rightarrow$ 3 ways Two empty places, can be filled with distinct letters [M, E, D, I, T, R, A, N] $\Rightarrow {}^8P_2$ $\therefore$ Number of words $3 + {}^8P_2 = 59$	
53.	A scientific committee is to be formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as	**A**

	Indians. Then the number of ways, the committee can be formed, is:		
	1625		
	575		
	560		
	1050		
Sol.	(a)		

Indians	Foreigners	Number of ways
2	4	$^6C_2 \times {}^8C_4 = 1050$
3	6	$^6C_3 \times {}^8C_6 = 560$
4	8	$^6C_4 \times {}^8C_8 = 15$

Total number of ways = 1625

54.	**Statement1:** The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is $^9C_3$. **Statement2:** The number of ways of choosing any 3 places from 9 different places is $^9C_3$.	**A**
	Statement–1 is true, Statement–2 is true; Statement–2 is not a correct explanation for Statement–1.	
	Statement–1 is true, Statement–2 is false.	
	Statement–1 is false, Statement–2 is true.	
	Statement–1 is true, Statement–2 is true; Statement–2 is a correct explanation for Statement–1.	
Sol.	(a) The number of ways of distributing 10 identical balls in 4 distinct boxes $= {}^{10-1}C_{4-1} = {}^9C_3$	
55.	If A denotes the sum of all the coefficients in the expansion of $(1 - 3x + 10x^2)^n$ and B denotes the sum of all the coefficients in the expansion of $(1 + x^2)^n$, then	**D**
	$B = A^3$	
	$3A = B$	
	$A = 3B$	
	$A = B^3$	
Sol.	**(d):** Putting $x = 1$ in $(1 - 3x + 10x^2)^n$, we get $A = (1 - 3 + 10)^n = 8^n$ ...(i) Putting $x = 1$ in $(1 + x^2)^n$, we get $B = (1 + 1)^n = 2^n$ ...(ii) From (i) and (ii), we get $A = B^3$	
56.	In an examination of mathematics paper, there are 20 questions of equal marks and the question paper is divided into three sections : A, B and C. A student is required to attempt total 15 questions taking at least 4 questions from each section. If section A has 8 questions, section B has 6 questions and section C has 6 questions, then the total number of ways a student can select 15 questions is	**B**
	10376	
	11376	
	11176	
	None of these	

Sol.	(11376) :				

Sol. (11376) :

A	B	C	Number of ways
5	6	4	$^8C_5 \times {}^6C_6 \times {}^6C_4 = 56 \times 1 \times 15 = 840$
6	5	4	$^8C_6 \times {}^6C_5 \times {}^6C_4 = 28 \times 6 \times 15 = 2520$
6	4	5	$^8C_6 \times {}^6C_4 \times {}^6C_5 = 28 \times 15 \times 6 = 2520$
5	5	5	$^8C_5 \times {}^6C_5 \times {}^6C_5 = 56 \times 6 \times 6 = 2016$
4	6	5	$^8C_4 \times {}^6C_6 \times {}^6C_5 = 70 \times 1 \times 6 = 420$
4	5	6	$^8C_4 \times {}^6C_5 \times {}^6C_6 = 70 \times 6 \times 1 = 420$
5	4	6	$^8C_5 \times {}^6C_4 \times {}^6C_6 = 56 \times 15 \times 1 = 840$
7	4	4	$^8C_7 \times {}^6C_4 \times {}^6C_4 = 8 \times 15 \times 15 = 1800$

$\therefore$ Required number of ways
= 840 + 2520 + 2520 + 2016 + 420 + 420 + 840 = 11376

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57. $^{n-1}C_r = (k^2 - 8) \, {}^nC_{r+1}$ if and only if | **A**

$2\sqrt{2} < k \le 3$

$2\sqrt{3} < k \le 3\sqrt{2}$

$2\sqrt{2} < k < 2\sqrt{3}$

$2\sqrt{3} < k < 3\sqrt{3}$

Sol. **(a) :** We have, $^{n-1}C_r = (k^2 - 8) \, {}^nC_{r+1}$

$\Rightarrow \dfrac{(n-1)!}{(n-(1+r))! \, r!} = (k^2 - 8) \dfrac{n!}{(n-(r+1))!(r+1)!}$

$\Rightarrow 1 = \dfrac{(k^2-8)n}{r+1} \Rightarrow k^2 - 8 = \dfrac{r+1}{n} \le 1 \qquad [\because n \ge r+1]$

$\Rightarrow k^2 - 8 \le 1 \Rightarrow k^2 - 9 \le 0 \Rightarrow (k-3)(k+3) \le 0$

$\Rightarrow -3 \le k \le 3 \qquad\qquad\qquad ...(i)$

But $k^2 > 8$, as $^{n-1}C_r$ can't be negative and 0.

$\Rightarrow k^2 - 8 > 0 \Rightarrow (k - 2\sqrt{2})(k + 2\sqrt{2}) > 0$

$\Rightarrow k < -2\sqrt{2}$ or $k > 2\sqrt{2} \qquad\qquad ...(ii)$

From (i) and (ii), $2\sqrt{2} < k \le 3$ or $-3 \le k < -2\sqrt{2}$

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58. Let $\alpha = \dfrac{(4!)!}{(4!)^{3!}}$ and $\beta = \dfrac{(5!)!}{(5!)^{4!}}$. Then | **C**

$\alpha \notin N$ and $\beta \in N$

$\alpha \in N$ and $\beta \notin N$

$\alpha \in N$ and $\beta \in N$

$\alpha \notin N$ and $\beta \notin N$

Sol. Not available

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59. If $\dfrac{^{11}C_1}{2} + \dfrac{^{11}C_2}{3} + \cdots + \dfrac{^{11}C_9}{10} = \dfrac{n}{m}$ with $\gcd(n, m) = 1$, then $n + m$ is equal to | **C**

2045

2080

2041

2036

Sol.	(2041) : $\dfrac{^{11}C_1}{2}+\dfrac{^{11}C_2}{3}+\ldots\ldots+\dfrac{^{11}C_9}{10}$	
	$=\left(^{11}C_0+\dfrac{^{11}C_1}{2}+\dfrac{^{11}C_2}{3}+\ldots\ldots+\dfrac{^{11}C_9}{10}+\dfrac{^{11}C_{10}}{11}+\dfrac{^{11}C_{11}}{12}\right)-1-1-\dfrac{1}{12}$	
	$=\dfrac{2^{11+1}-1}{12}-2-\dfrac{1}{12}=\dfrac{2^{12}-26}{12}=\dfrac{4070}{12}=\dfrac{2035}{6}$	
	$\therefore \quad n+m = 2035+6 = 2041$	
60.	Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is:	**C**
	500	
	200	
	300	
	350	
Sol	(c) Required no of ways = (total of ways) – (A and B included) = $^5C_2 . ^7C_3 - ^5C_1 \times ^5C_2 = 300$	
61.	Among the statements: **(S1)**: $2023^{2022} - 1999^{2022}$ is divisible by 8. **(S2)**: $13(13)^n - 11n - 13$ is divisible by 144 for infinitely many $n \in N$.	**C**
	Both (S1) and (S2) are incorrect	
	Only (S2) is correct	
	Both (S1) and (S2) are correct	
	Only (S1) is correct	
Sol	(c) Let $S_1 = (1999+24)^{2022} - (1999)^{2022}$ $= {}^{2022}C_1 (1999)^{2021}(24) + {}^{2022}C_2 (1999)^{2020}(24)^2 + \ldots$ so on $\Rightarrow$ S_1 is divisible by 8 Let $S_2 = 13(13^n) - 11n - 13$ $13^n = (1+12)^n = 1+12n+ {}^nC_2 12^2 + {}^nC_3 12^3 \ldots\ldots$ $13(13^n) - 11n - 13 = 145n + {}^nC_2 12^2 + {}^nC_3 12^3 \ldots\ldots$ If $(n = 144m, m \in N)$, then it is divisible by 144 for infinite value of n	
62.	If $(1+ax)^n = 1+8x+24x^2+\cdots$, then the value of a and n are	**A**
	2 and 4	
	2 and 3	
	3 and 6	
	1 and 2	
Sol.	(a) Given, $(1+ax)^n = 1+8x+24x^2+\ldots$ $\Rightarrow 1+n\cdot ax + \dfrac{n(n-1)}{1\cdot 2}a^2 x^2 + \ldots = 1+8x+24x^2+\ldots$ On comparing the coefficients of x and x^2, we get $na = 8$ and $\dfrac{n(n-1)}{1\cdot 2}a^2 = 24$ $\Rightarrow \qquad na(n-1)a = 48$ $\Rightarrow \qquad 8(8-a) = 48 \Rightarrow 8-a = 6$ $\Rightarrow \qquad a=2 \Rightarrow n=4$	
63.	Two packs of 52 cards are shuffled by together. The number of wys in which a man can be dealt 26 cards so that he does not get two cards of the	**A**

	same suit and same denomination, is	
	$\dfrac{^{52}C_{26} \times 2^{26}}{^{104}C_{26}}$	
	$2 \times {^{52}C_{26}}$	
	None of these	
Sol.	(a) 26 cards can be chosen out of 52 cards in $^{52}C_{26}$ ways, there are two ways in which each card can be dealt, because a card can be either from the first pack or from the second. Hence, the total number of ways is $^{52}C_{26} \times 2^{26}$.	
64.	The number of ways to distribute 30 identical candies among four children C_1, C_2, C_3 and C_4 so that C_2 receives atleast 4 and atmost 7 candies, C_3 receives atleast 2 and atmost 6 candies, is equal to	D
	205	
	615	
	510	
	430	
Sol	(d) We are given that $t_1 + t_2 + t_3 + t_4 = 30$ Now coefficient of x^{30} in $(1 + x + x^2 + ... + x^{30})^2$ $(x^4 + x^5 + x^6 + x^7)(x^2 + x^3 + x^4 + x^5 + x^6)$ $\Rightarrow x^6\left(\dfrac{1-x^{31}}{1-x}\right)^2 (1 + x + x^2 + x^3)(1 + x + x^2 + x^3 + x^4)$ $\Rightarrow x^6(1-x^{31})^2(1-x^4)(1-x^5)(1-x)^4$ $\Rightarrow x^6(1-x^4-x^5+x^9)(1+x^{62}-2x^{31})(1-x)^{-4}$ $\Rightarrow x^6(1-x^4-x^5+x^9)(1-x)^{-4}$ Coefficient of x^n in $(1-x)^{-r}$ is $^{n+r-1}C_{r-1}$ $\Rightarrow {^{27}C_3} - {^{23}C_3} - {^{22}C_3} - {^{18}C_3}$ $2925 - 1771 - 1540 + 816 = 430$ **OR** $x_2 \in [4, 7], x_3 \in [2, 6] \Rightarrow t_1 + t_2 + t_3 + t_4 = 24$ total ways = $^{24+4-1}C_{4-1} - {^{20+4-1}C_{4-1}} - {^{19+4-1}C_{4-1}} + {^{15+4-1}C_{4-1}}$ $= {^{27}C_3} - {^{23}C_3} - {^{22}C_3} + {^{18}C_3} = 430$	
65.	The least value of n for which the number of integral terms in the Binomial expansion of $\left(\sqrt[3]{7} + \sqrt[12]{11}\right)^n$ is 183, is:	A
	2184	
	2148	
	2172	
	2196	
Sol.	(a) General term $= {^nC_r}(7^{1/3})^{n-r}(11^{1/12})^r$ $= {^nC_r}(7)^{\frac{n-r}{3}}(11)^{r/12}$ For integral terms, r must be multiple of 12 $\therefore r = 12k, k \in W$ Total values of $r = 183$ Hence max. $r = 12(182)$ (Except $r = 0$) $= 2184$ Min. value of $n = 2184$ ($\because n \geq r$)	
66.	If $\sum_{r=0}^{10}\left(\dfrac{10^{r+1}-1}{10^r}\right) \cdot {^{11}C_{r+1}} = \dfrac{\alpha^{11}-11^{11}}{10^{10}}$, then α is equal to :	D
	15	
	11	

	24	
	20	
Sol.	(d) $\displaystyle\sum_{r=0}^{10}\left(\frac{10^{r+1}-1}{10^r}\right){}^{11}C_{r+1} = \sum_{r=0}^{10}\left(10-\frac{1}{10^r}\right){}^{11}C_{r+1}$	

$$=10\sum_{r=0}^{10}{}^{11}C_{r+1}-10\sum_{r=0}^{10}\left({}^{11}C_{r+1}\left(\frac{1}{10}\right)^{r+1}\right)$$

$$=10\left[{}^{11}C_1+{}^{11}C_2+....+{}^{11}C_{11}\right]$$

$$-10\left[{}^{11}C_1\left(\frac{1}{10}\right)^1+{}^{11}C_2\left(\frac{1}{10}\right)^2+....+{}^{11}C_{11}\left(\frac{1}{10}\right)^{11}\right]$$

$$=10[2^{11}-1]-10\left[\left(1+\frac{1}{10}\right)^{11}-1\right]$$

$$=10(2)^{11}-10-\frac{11^{11}}{10^{10}}+10$$

$$=\frac{(20)^{11}-11^{11}}{10^{10}}\Rightarrow \alpha=20.$$

67.	If $\displaystyle\sum_{k=1}^{31}({}^{31}C_k)({}^{31}C_{k-1})-\sum_{k=1}^{30}({}^{30}C_k)({}^{30}C_{k-1})=\frac{\alpha(60!)}{(30!)(31!)}, \alpha\in R,$ then the value of 16α is equal to	A
	1411	
	1320	
	1615	
	1855	

Sol.	(a) Given expression is $\displaystyle\sum_{K=1}^{31}{}^{31}C_K\cdot{}^{31}C_{K-1}$	

$$={}^{31}C_1\cdot{}^{31}C_0+{}^{31}C_2\cdot{}^{31}C_1+....+{}^{31}C_{31}\cdot{}^{31}C_{30}$$
Here, ${}^nL_r={}^nL_{n-r}$
$$={}^{31}C_0\cdot{}^{31}C_{30}+{}^{31}C_1\cdot{}^{31}C_{29}+....+{}^{31}C_{30}\cdot{}^{31}C_0$$
$$={}^{62}C_{30}.$$
Similarly

$$\sum_{K=1}^{30}({}^{30}C_K\cdot{}^{30}C_{K-1})={}^{60}C_{29}$$

$$=1\,{}^{62}C_{30}-{}^{60}C_{29}=\frac{62!}{30!32!}-\frac{60!}{29!31!}$$

$$=\frac{60!}{29!31!}\left\{\frac{62.61}{30.32}-1\right\}=\frac{60!}{30!31!}\left(\frac{2822}{32}\right)$$

compare above equation with $\dfrac{\alpha(60!)}{(30!)(31!)}$

So, $\alpha=\dfrac{2822}{32}$ $\therefore 16\alpha=16\times\dfrac{2822}{32}=1411$

68.	Fifteen identical balls have to be put in five different boxes. Each box can contain any number of balls, the total number of ways of putting the balls into the boxes so that each box contains atleast two balls, is equal to	A
	${}^9C_5$	
	${}^{10}C_5$	
	${}^6C_5$	
	None of these	

Sol.	(a) Let the balls put in , x_2, x_3, x_4 and x_5,
	we have
	
	            $x_3 + x_4 + x_5 = 15$, $x_i \geq 2$
	or $(x_1 - 2) + (x_2 - 2) + (x_3 - 2) + (x_4 - 2) + (x_5 - 2) = 5$
	$\Rightarrow$ $y_1 + y_2 + y_3 + y_4 + y_5 = 5$, $y_i = x_i - 2 \geq 0$
	The total number of ways is equal to the number of
	non-negative integral of the last equation, which is
	equal to
	    $^{5+5-1}C_5 = {}^9C_5$	

69.	The term independent of a in the expansion of $\left(1 + \sqrt{a} + \dfrac{1}{\sqrt{a}-1}\right)^{-30}$ is	**B**
	$^{30}C_{20}$	
	0	
	$^{30}C_{10}$	
	None of these	

Sol.	(b) $\left(1 + \sqrt{a} + \dfrac{1}{\sqrt{a}-1}\right)^{-30}$
	
	    $= \left(\dfrac{a}{\sqrt{a}-1}\right)^{-30} = \left(\dfrac{\sqrt{a}-1}{a}\right)^{30}$
	
	    $= \dfrac{1}{a^{30}}(1-\sqrt{a})^{30}$
	
	    $= \dfrac{1}{a^{30}}\{{}^{30}C_0 - {}^{30}C_1\sqrt{a} + \ldots + {}^{30}C_{30}(\sqrt{a})^{30}\}$
	
	There is no term independent of a.	

70.	If $n > \left(8 + 3\sqrt{7}\right)^{10}, n \in N$, then the least value of n is	**B**
	$\left(8 + 3\sqrt{7}\right)^{10} - \left(8 - 3\sqrt{7}\right)^{10}$	
	$\left(8 + 3\sqrt{7}\right)^{10} + \left(8 - 3\sqrt{7}\right)^{10}$	
	$\left(8 + 3\sqrt{7}\right)^{10} - \left(8 - 3\sqrt{7}\right)^{10} + 1$	
	$\left(8 + 3\sqrt{7}\right)^{10} - \left(8 - 3\sqrt{7}\right)^{10} - 1$	
Sol.	(b) Let $f = (8 - 3\sqrt{7})^{10}$, here $0 < f < 1$ $\therefore (8 + 3\sqrt{7})^{10} + (8 - 3\sqrt{7})^{10}$ is an integer, hence this is the value of n.	
71.	The number of 5–digit natural numbers, such that the product of their digits is 36, is	**180**
Sol.	**(180)** Given a 5 digit number product is 36. Multiple of $36 = 2^2 \times 3^2$ $= 2 \times 2 \times 3 \times 3 \times 1$ We need to make five digit number by using the digits $2, 2,$ 3 & 3 and the fifth digit could be 1. 5 digit possibles are, $\{2, 2, 3, 3, 1\}, \{4, 3, 3, 1, 1\}$ $\{6, 2, 3, 1, 1\}, \{9, 2, 2, 1, 1\}, \{4, 9, 1, 1, 1\}, \{6, 6, 1, 1, 1\}$ Total ways of all possibles are $\dfrac{\lfloor 5}{\lfloor 2 \lfloor 2} + \dfrac{\lfloor 5}{\lfloor 2 \lfloor 2} + \dfrac{\lfloor 5}{\lfloor 2} + \dfrac{\lfloor 5}{\lfloor 2 \lfloor 2} + \dfrac{\lfloor 5}{\lfloor 3} + \dfrac{\lfloor 5}{\lfloor 3 \lfloor 2}$ $\Rightarrow 30 + 30 + 60 + 30 + 20 + 10 = 180$	
72.	Let $^nC_{r-1} = 28, \ ^nC_r = 56 \ and \ ^nC_{r+1} = 70$. Let A (4cos t, 4 sin t),	**20**

B(2 sin t, – 2 cos t) and C(3r – n, r² – n – 1) be the vertices of a triangle ABC, where t is a parameter. If $(3x - 1)^2 + (3y)^2 = \alpha$, is the locus of the centroid of triangle ABC, then α equals:

Sol.

$$\because \frac{^nC_{r-1}}{^nC_r} = \frac{28}{56} \Rightarrow \frac{\dfrac{n!}{(r-1)!\,(n-r+1)!}}{\dfrac{n!}{r!\,(n-r)!}} = \frac{1}{2}$$

$$\Rightarrow \frac{r}{(n-r+1)} = \frac{1}{2} \Rightarrow 3r = n+1 \qquad \ldots\text{(i)}$$

$$\frac{^nC_r}{^nC_{r+1}} = \frac{56}{70} \Rightarrow \frac{n!}{(n-r)!\,r!} \times \frac{(n-r-1)!\,(r+1)!}{n!} = \frac{56}{70}$$

$$\Rightarrow \frac{(r+1)}{(n-r)} = \frac{4}{5} \Rightarrow 9r = 4n - 5 \qquad \ldots\text{(ii)}$$

On solving (i) and (ii) we get $r = 3, n = 8$

$\therefore C(3r - n, r^2 - n - 1) = C(1, 0)$

$\therefore (3x-1)^2 + (3y)^2 = (4\cos t + 2\sin t)^2 + (4\sin t - 2\cos t)^2$

$(3x-1)^2 + (3y)^2 = 20$

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73. If $\sum_{k=1}^{10} K^2 \left(10_{C_k}\right)^2 = 22000L,$ then L is equal to_____.   **221**

Sol.

(221) Given expression is $\displaystyle\sum_{K=1}^{10} K^2 \left(^{10}C_K\right)^2$

$$\Rightarrow \sum_{K=1}^{10} \left(K \cdot {}^{10}C_K\right)^2 = \sum_{K=1}^{10} \left(10 \cdot {}^9C_{K-1}\right)^2$$

$$= 100 \sum_{K=1}^{10} {}^9C_{K-1} \cdot {}^9C_{10-K} = 100\left(^{18}C_9\right) = 100\left(\frac{18!}{9!\,9!}\right)$$

$\Rightarrow 4862000 = 22000L.$ Therefore, $L = 221$

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74. If the 2nd term of the expansion $\left[a^{\frac{1}{13}} + \dfrac{a}{\sqrt{a^{-1}}}\right]^n$ is $14a^{5/2}$, then the value of $\dfrac{^nC_3}{^nC_2}$ is   **4**

Sol.

(4) $T_2 = {}^nC_1 (a^{1/13})^{n-1} \cdot a\sqrt{a} = 14a^{5/2}$

$$\Rightarrow n \cdot a^{\frac{n-1}{13}} = 14a$$

$$\Rightarrow n \cdot a^{\frac{n-14}{13}} = 14 \Rightarrow \frac{n-14}{13} = 0 \Rightarrow n = 14$$

$$\Rightarrow \frac{^{14}C_3}{^{14}C_2} = \frac{14!}{3!\cdot 11!} \cdot \frac{2!\cdot 12!}{14!} = \frac{12}{3} = 4$$

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75. If the sum of the coefficients of all even powers of x in the product $(1 + x + x^2 + \cdots + x^{2n})(1 - x + x^2 - x^3 + \cdots + x^{2n})$ is 61, then n is equal to   **30**

Sol.

(30) Let $(1 - x + x^2 \ldots x^{2n})(1 + x + x^2 \ldots x^{2n})$

$= a_0 + a_1 x + a_2 x^2 + \ldots$

put $x = 1$

$\quad 1(2n + 1) = a_0 + a_1 + a_2 + \ldots a_{2n} \qquad \ldots\text{(i)}$

put $x = -1$

$\quad (2n + 1) \times 1 = a_0 - a_1 + a_2 + \ldots a_{2n} \qquad \ldots\text{(ii)}$

Adding (i) and (ii), we get,

$\quad 4n + 2 = 2(a_0 + a_2 + \ldots) = 2 \times 61$

$\Rightarrow 2n + 1 = 61 \Rightarrow n = 30.$