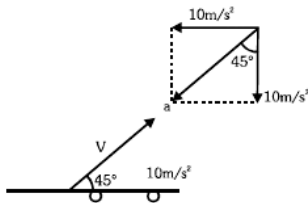
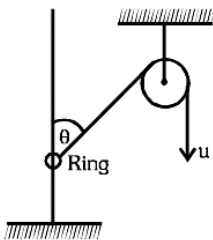
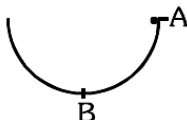



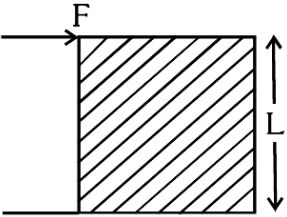
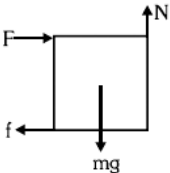
1.	A boy is standing on a flatcar that is moving with uniform acceleration on a level track. At the instant when flatcar is moving with velocity 10 m/s due east. The boy throws a ball with velocity 20 m/s in direction $45^\circ$ above the east with respect to the flatcar in plane of motion of flatcar. What should be the acceleration of the flatcar so that the boy can catch the ball without moving anywhere on the flatcar? Assume acceleration due to gravity $10 \text{ m/s}^2$ .	A
	10 $\text{m/s}^2$ eastwards	
	10 $\text{m/s}^2$ westwards	
	20 $\text{m/s}^2$ eastwards	
	20 $\text{m/s}^2$ westwards	
Sol.	<p>w.r.t. flat car, velocity of projection makes angle <math>45^\circ</math> with east. In order to catch the ball without moving trajectory has to be a straight line. So acceleration wrt flat car should be just opposite to projection velocity.</p> 	
2.	Find the speed of the ring as a function of ' $\theta$ ' if rope is pulled down with constant speed $u$ :-	B
		
	$\frac{u}{\sin \theta}$	
	$\frac{u}{\cos \theta}$	
	$\frac{u}{2 \sin \theta}$	
	$\frac{u}{2 \cos \theta}$	
Sol.	<p>Let speed of ring be <math>v</math> then <math>v \cos \theta = u</math></p> $\Rightarrow v = \frac{u}{\cos \theta}$	
3.	A small ball can move in a vertical plane along a semicircle of radius $r$ without friction. At what speed is the ball to launch from point A so that its acceleration is $3g$ at point B ?	C
		
	$(3gr)^{1/2}$	
	$(2gr)^{1/2}$	
	$(gr)^{1/2}$	

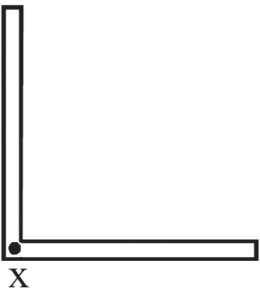
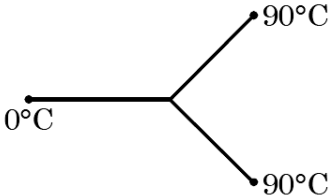
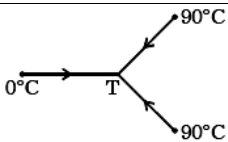
29- Correct

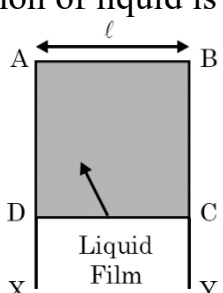
19- Incorrect

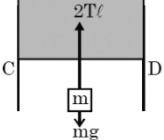
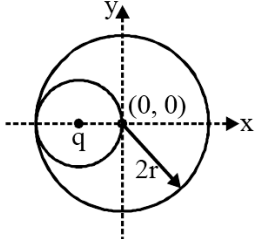
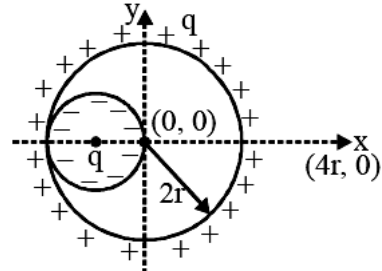
X

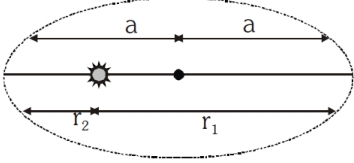
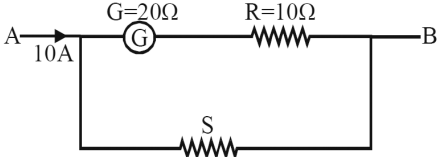
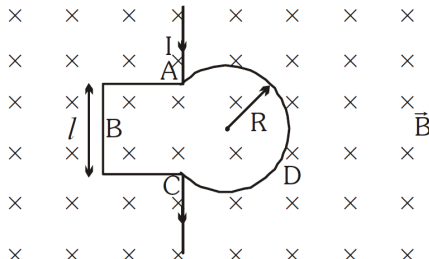
✓

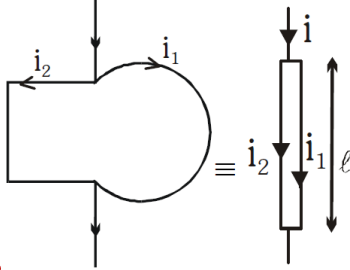
	$2(gr)^{1/2}$	
<b>Sol.</b>	$mgR + \frac{1}{2}mv'^2 = \frac{1}{2}m(v')^2$ $\Rightarrow \frac{mv'^2}{R} = 3g$	
<b>4.</b>	<p>A block of mass <math>m</math> sliding on a horizontal frictionless surface has kinetic energy <math>K</math>. It collides head on with another stationary identical block. The collision is perfectly inelastic. How much energy is lost in this collision?</p> 	<b>B</b>
	$K/4$	
	$K/2$	
	$K/3$	
	$3K/4$	
<b>Sol.</b>	<p>Velocity after collision <math>mu = 2mv_0</math>  <math>\Rightarrow v_0 = u/2</math></p> <p>So final kinetic energy <math>= \frac{1}{2}(2m)\left(\frac{u}{2}\right)^2 = \frac{K}{2}</math></p> <p>loss in kinetic energy <math>K - \frac{K}{2} = \frac{K}{2}</math></p>	
<b>5.</b>	<p>A cubical block of side <math>L</math> rests on a rough horizontal surface with coefficient of friction <math>\mu</math>. A horizontal force <math>F</math> is applied on the block as shown. If the coefficient of friction is sufficiently high so that the block does not slip before toppling, the minimum force required to topple the block is</p> 	<b>C</b>
	Infinitesimal	
	$mg/4$	
	$mg/2$	
	$Mg(1-\mu)$	
<b>Sol.</b>	<p><math>FL \geq mg\left(\frac{L}{2}\right) \Rightarrow F \geq \frac{mg}{2}</math></p> 	
<b>6.</b>	<p>Given structure is lying in vertical plane and is hinged at x. Structure is made up of 2 identical rods with mass <math>M</math> &amp; length <math>L</math> each. Initially it is at rest and released. Initially, one of the rods of the structure is vertical and the other horizontal. Initial angular acceleration about x is</p>	<b>C</b>

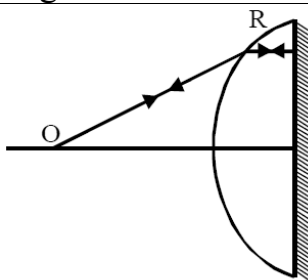
		
	$\left(\frac{3g}{10L}\right)$	
	$\left(\frac{g}{5L}\right)$	
	$\left(\frac{6g}{8L}\right)$	
	$\left(\frac{g}{2L}\right)$	
<b>Sol.</b>	$\frac{MgL}{2} = \frac{2ML^2\alpha}{3}$	
<b>7.</b>	<p>Three rods made of same material &amp; having the same cross section have been joined as shown in the figure. Each rod is of the same length. The left and right ends are kept at 0°C &amp; 90°C respectively. The temperature of the junction of the three rods will be :</p> 	<b>B</b>
	45°C	
	60°C	
	30°C	
	20°C	
<b>Sol.</b>	 <p>Let temperature of junction is T</p> $\frac{kA(90 - T)}{\ell} + \frac{kA(90 - T)}{\ell}$ $+ \frac{kA(0 - T)}{\ell} = 0$ <p>T = 60°C</p>	
<b>8.</b>	Assuming Newton's law of cooling to be valid. The temperature of body changes from 60°C to 40°C in 7 minutes. Temperature of surroundings being 10°C, its temperature after next 7 minutes, is :-	<b>D</b>
	7°C	
	14°C	
	21°C	
	28°C	

<b>Sol.</b>	<p style="text-align: center;"><b>OR</b></p> <p>According to Newton's law of cooling</p> $\frac{\theta_2 - \theta_1}{t} = K \left( \frac{\theta_1 + \theta_2}{2} - \theta_s \right)$ <p>Since the temperature decreases from <math>60^\circ\text{C}</math> to <math>40^\circ\text{C}</math> in 7 minutes</p> $\frac{60 - 40}{7} = K \left( \frac{60 + 40}{2} - 10 \right)$ $\Rightarrow \frac{20}{7} = K (50 - 10) \Rightarrow K = \frac{1}{14}$ <p>If the temp. of object becomes <math>\theta'</math> in next 7 minutes then</p> $\frac{40 - \theta'}{7} = \frac{1}{14} \left( \frac{40 + \theta'}{2} - 10 \right)$ $\Rightarrow 40 - \theta' = \frac{1}{4} (40 + \theta' - 20)$ $\Rightarrow 160 - 4\theta' = 20 + \theta'$ $\Rightarrow 5\theta' = 140 \Rightarrow \theta' = 28^\circ\text{C}$ <p style="text-align: center;"><b>OR</b></p> <p>According to Newton's law of cooling</p> $-\frac{d\theta}{dt} = K (\theta - \theta_0) \text{ or } dt = -\frac{1}{K} \frac{d\theta}{(\theta - \theta_0)}$ $\therefore \int_0^t dt = -\frac{1}{K} \int_{\theta_1}^{\theta_2} \frac{d\theta}{(\theta - \theta_0)}$ $\Rightarrow t = \frac{1}{K} \log_e \left\{ \frac{(\theta_1 - \theta_0)}{(\theta_2 - \theta_0)} \right\}$ <p>As per the question <math>7 = \frac{1}{K} \log_e \left\{ \frac{60 - 10}{40 - 10} \right\}</math></p> <p>Also <math>7 = \frac{1}{K} \log_e \left\{ \frac{40 - 10}{\theta - 10} \right\}</math></p> <p>from above equations we have</p> $\log_e \left( \frac{50}{30} \right) = \log_e \left( \frac{30}{\theta - 10} \right)$ $\therefore \frac{5}{3} = \frac{30}{\theta - 10} \Rightarrow 5 - 50 = 90$ $\Rightarrow q = \frac{100}{5} = 28^\circ\text{C}$	
<b>9.</b>	A carnot engine works between the temperature of 1092 K to 273 K is used as heat pump. It consumes 1260 watt of electrical energy and heat is extracted from the sink containing water at $0^\circ\text{C}$ . Find the rate of freezing of water :-	<b>A</b>
	1.25 gm/sec	
	2 gm/sec	
	3 gm/sec	
	5 gm/sec	
<b>Sol.</b>	$\eta = 1 - \frac{273}{1092} = \frac{3}{4}$ $\text{C.O.P.} = \frac{1 - \eta}{\eta} = \frac{1}{3} = \frac{Q_2}{w}$ $\Rightarrow Q_2 = \frac{1}{3} w = \frac{1}{3} \times 1260 \text{ watt}$ $= 420 \text{ watt}$ $\therefore \frac{dm}{dt} \times 80 \times 4.2 = 420$	
<b>10</b>	<p>A liquid film is formed over a frame ABCD as shown in figure. A massless wire CD can slide without friction. The mass to be hung from CD to keep it in equilibrium is (Surface tension of liquid is T)</p> 	<b>B</b>
	$T\ell / g$	
	$2T\ell / g$	
	$g / 2T\ell$	
	$T\ell / 2g$	

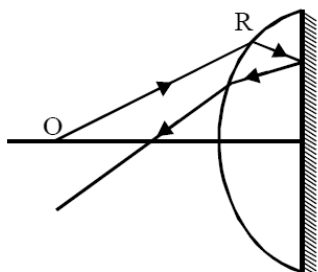
<b>Sol.</b>	 <p>For equilibrium, <math>2T\ell = mg</math></p> $m = \frac{2T\ell}{g}$	
<b>11.</b>	<p>A spherical cavity of radius <math>r</math> is made in a conducting sphere of radius <math>2r</math>. A charge <math>q</math> is kept at the centre of cavity as shown in the figure. Find the magnitude of the total electric field at <math>(4r, 0, 0)</math>.</p> 	<b>A</b>
	$\frac{Kq}{16r^2}$	
	$\frac{4Kq}{r^2}$	
	$\frac{Kq}{25r^2}$	
	$\frac{4Kq}{9r^2}$	
<b>Sol.</b>	$\vec{E} = \frac{kq}{(4r)^2} = \frac{kq}{16r^2}$ 	
<del><b>12.</b></del>	<p>A planet of mass <math>m</math> moves around the sun of mass <math>M</math> in an elliptical orbit. The maximum and minimum distances of the planet from the sun are <math>r_1</math> and <math>r_2</math> respectively. Therefore, the time period of the planet is proportional to</p>	<b>B</b>
	$(r_1 + r_2)^3$	
	$(r_1 + r_2)^{3/2}$	
	$(r_1 + r_2)^{2/3}$	
	$(r_1 + r_2)^4$	

<b>Sol.</b>	 $2a = r_1 + r_2 \Rightarrow a = \frac{r_1 + r_2}{2};$ $T \propto a^{3/2} \Rightarrow T \propto (r_1 + r_2)^{3/2}$	
<b>13.</b>	<p>Full scale deflection current for galvanometer is 1 mA. What should be the value of shunt resistance (approximately) so that galvanometer shows half scale deflection for the circuit shown.</p> 	<b>A</b>
	1.5 mΩ	
	3 mΩ	
	10 mΩ	
	15 mΩ	
<b>Sol.</b>	$(20 + 10) \frac{1 \times 10^{-3}}{2} = s \times \left( 10 - \frac{1 \times 10^{-3}}{2} \right)$ $\Rightarrow s \approx 1.5 \text{ m}\Omega$	
<b>14.</b>	<p>The figure shows a conducting loop ABCDA placed in a uniform magnetic field (strength B) perpendicular to its plane. The part ABC is the <math>(3/4)^{\text{th}}</math> portion of the square of side length <math>l</math>. The part ADC is a circular arc of radius R. The points A and C are connected to a battery which supply a current I to the circuit. The magnetic force on the loop due to the field B is</p> 	<b>B</b>
	Zero	
	BIl	
	2BIR	
	$\frac{BI/R}{l + R}$	

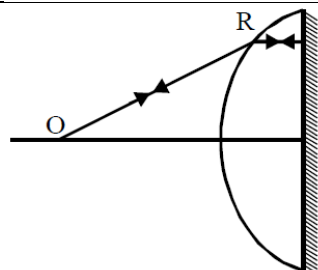
<b>Sol.</b>	 $F = Bi_1\ell + Bi_2\ell = B\ell(i_1 + i_2) = Bi\ell$	
<b>15.</b>	A time varying uniform magnetic field passes through a circular region of radius R. The magnetic field is directed outwards and it is a function of radial distance 'r' and time 't' according to relation $B = B_0rt$ . The induced electric field strength at a radial distance R/2 from the centre will be	<b>A</b>
	$\frac{B_0R^2}{12}$	
	$\frac{B_0R^2}{6}$	
	$\frac{2B_0R^2}{3}$	
	$\frac{B_0R^2}{16}$	
<b>Sol.</b>	$\phi = \int_0^{R/2} B(2\pi r)dr = B_0t2\pi\left(\frac{r^3}{3}\right)_0^{R/2}$ $= B_0(2\pi)\frac{1}{3}\left(\frac{R^3}{8}\right)t$ $\therefore E\left(2\pi\left(\frac{R}{2}\right)\right) = \frac{-d\phi}{dt} = -B_0(2\pi)\frac{R^3}{24}$ $\Rightarrow E = \frac{B_0R^2}{12}$	
<b>16.</b>	A sinusoidal voltage of peak value 283 V and frequency 50 Hz is applied to a series LCR circuit in which $R = 3\Omega$ , $L = 25.48\text{ mH}$ , and $C = 796\mu\text{F}$ . The power factor is :	<b>B</b>
	0.4	
	0.6	
	0.8	
	0.3	
<b>Sol.</b>	$\text{p.f.} = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$	
<b>17.</b>	In a hydrogen atom, electron moves from second excited state to first excited state and then from first excited state to ground state. The ratio of wavelengths is	<b>C</b>

	$\frac{20}{3}$	
	$\frac{15}{4}$	
	$\frac{27}{5}$	
	$\frac{5}{1}$	
<b>Sol.</b>	<p>From Bohr model <math>\frac{1}{\lambda} = R \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right)</math></p> <p><math>\frac{1}{\lambda_1} = R \left( \frac{1}{2^2} - \frac{1}{3^2} \right) = \frac{5}{36} R</math> ..... (i)</p> <p>and <math>\frac{1}{\lambda_2} = R \left( \frac{1}{1^2} - \frac{1}{2^2} \right) = \frac{3}{4} R</math> ..... (ii)</p> <p>Dividing eq. (i) and (ii), we get <math>\frac{\lambda_1}{\lambda_2} = \frac{27}{5}</math></p>	
<b>18.</b>	<p>A proton is bombarded on a stationary Lithium nucleus. As a result of collision two <math>\alpha</math>-particles are produced. The direction of motion of the <math>\alpha</math>-particles with the initial direction of motion makes an angle <math>\cos^{-1} \frac{1}{4}</math>. If B.E/Nucleon for <math>\text{Li}^7</math> and <math>\text{He}^4</math> are 5.60 MeV and 7.06 MeV respectively, then :</p> <p>Kinetic energy of striking proton is 17.28 MeV.</p> <p>Kinetic energy of striking proton is 8.64 MeV.</p> <p>Kinetic energy of striking proton is 4.32 MeV.</p> <p>Kinetic energy of striking proton is 2.16 MeV.</p>	<b>A</b>
<b>Sol.</b>	<p>Q value of the reaction</p> <p><math>Q = (2 \times 4 \times 7.06 - 7 \times 5.6) = 17.28 \text{ MeV}</math> ... (i)</p> <p><math>K_p + Q = 2K_\alpha</math> ..... (ii)</p> <p><math>\sqrt{2m_p K_p} = 2\sqrt{2m_\alpha K_\alpha} \cos \alpha</math></p> <p><math>K_p = K_\alpha</math> ..... (iii)</p> <p>So <math>K_p = 17.28 \text{ MeV}</math></p>	
<b>19.</b>	<p>A thin plano-convex glass lens (<math>\mu = 1.5</math>) has its plane surface reflecting and R is the radius of curvature of curved part, then which of the following ray diagram is true for an object placed at O ?</p>	<b>A</b>
	 <p>object is placed at distance 2R from lens</p>	

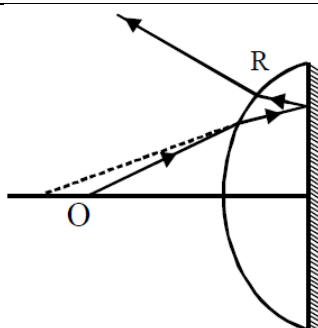




object is placed at distance  
2R from lens



object is placed at distance  
3R from lens



object is placed at distance  
2R from lens

**Sol.**

$$\frac{1}{f_{eq}} = \frac{1}{f_m} - \frac{2}{f_L}$$

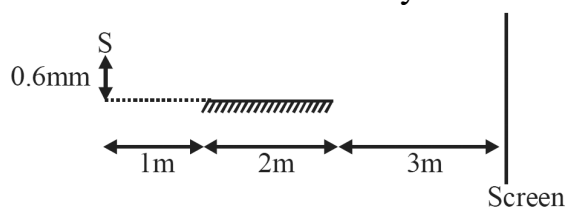
$$\frac{1}{f_L} = (\mu - 1) \left[ \frac{+1}{R} \right] = \frac{1}{2R}$$

$$\therefore \frac{1}{f_q} = -\frac{1}{R} \Rightarrow f_{eq} = -R \text{ [concave mirror]}$$

$\therefore$  Image of object kept at 2R will form at  
2R

**2.**

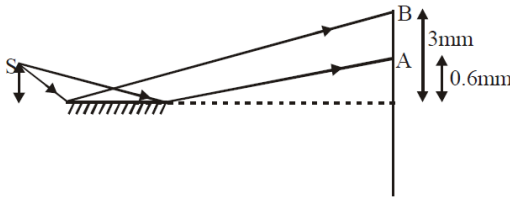
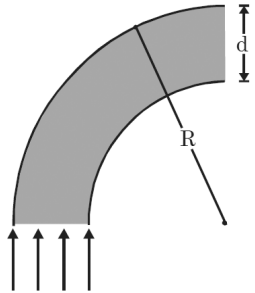
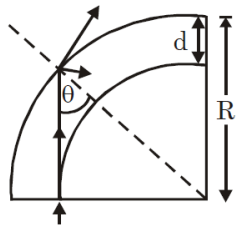
In a particular set up, interference is observed on an screen as shown. Wavelength of light emitted by source is  $4000 \text{ \AA}$ . Out of two coherent sources, one is obtained by reflection with a plane mirror of length 2m. The number of points on the screen where intensity is minimum, is :



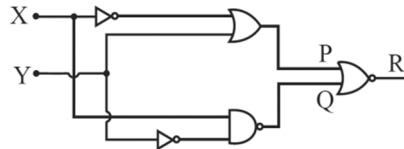
**C**

2

3

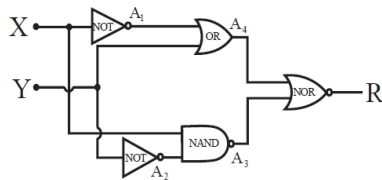
	1	
	4	
<b>Sol.</b>	 <p>At A</p> $\frac{(0.6)(1.2) \times 10^{-6}}{6} = n_1 (4 \times 10^{-7})$ $\Rightarrow n_1 = 0.3$ <p>At B</p> $\frac{(3)(1.2) \times 10^{-6}}{6} = n_2 (4 \times 10^{-7}) \Rightarrow n_2 = 1.5$ <p>So, there will be two maximas in the pattern.</p>	
<b>21.</b>	<p>An optical fiber has index of refraction <math>n = 1.40</math> and diameter <math>d = 100 \mu\text{m}</math>. It is surrounded by air. Light is sent into the fiber along the axis as shown in figure. Find smallest outside radius <math>R</math> (in <math>\mu\text{m}</math>) permitted for a bend in the fiber for no light to escape is.</p> 	<b>350</b>
<b>Sol.</b>	 <p>A ray along the inner edge will escape if any ray escapes. Its angle of incidence is described by <math>\sin \theta = \frac{R-d}{R}</math> and by <math>n \sin \theta &gt; 1 \sin 90^\circ</math>.</p> <p>Then <math>\frac{n(R-d)}{R} &gt; 1 \Rightarrow nR - nd &gt; R</math></p> $\Rightarrow nR - R > nd \quad R > \frac{nd}{n-1}$ $R_{\min} = \frac{1.40(100 \times 10^{-6} \text{ m})}{0.40} = 350 \times 10^{-6} \text{ m}$	
<b>22.</b>	For the given logic gate circuit, to get output as 1 at R, find the input of X,	<b>1</b>

if input value of Y is zero.



**Sol.**

The given circuit can be drawn as shown in the figure given below

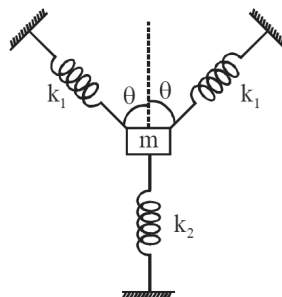


Truth table for this given logic gate is

Given inputs in the options						
X	Y	A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>	A <sub>4</sub>	R
0	0	1	1	1	1	0
1	0	0	1	0	0	1
1	1	0	0	1	1	0
0	1	1	0	1	1	0

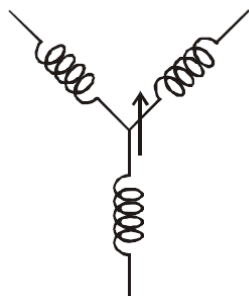
So to get output R = 1, inputs must be X = 1 and Y = 0

A block of mass  $m$  is connected to three springs as shown in the figure. The block is displaced down slightly and left free, it starts oscillating. Find the time period (in sec) of oscillations (neglecting gravity). ( $m = \pi^2 \text{ kg}$ ;  $\pi^2 = 10$ ;  $\theta = 60^\circ$ ;  $k_1 = 4 \text{ N/m}$ ;  $k_2 = 23 \text{ N/m}$ )



**Sol.**

at GP



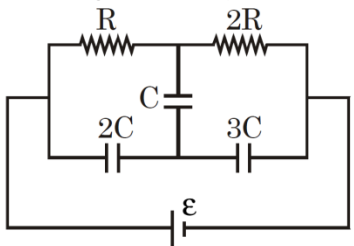
$$F_{\text{restoring}} = 2k_1 x \cos^2 \theta + k_2 x$$

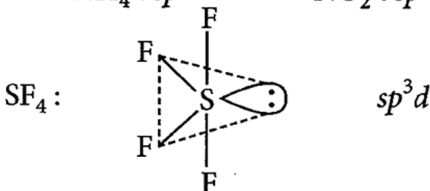
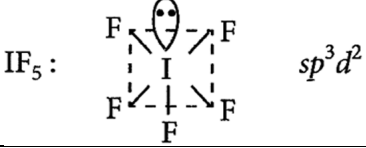
$$\text{So, TP} = 2\pi \sqrt{\frac{m}{2k_1 \cos^2 \theta + k_2}}$$

**24.**

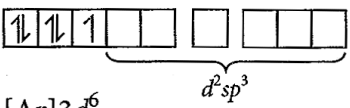
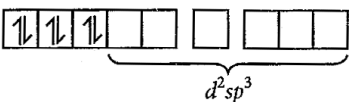
A piece of  $-10^\circ \text{C}$  ice is heated to  $-1^\circ \text{C}$  using a certain quantity of energy. Then another 19 times as much energy is necessary to finally convert entire ice to water at  $0^\circ \text{C}$ . Using that the specific heat of ice is half of the specific


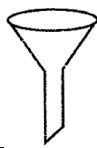

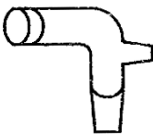
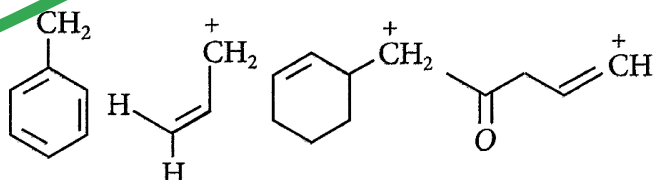
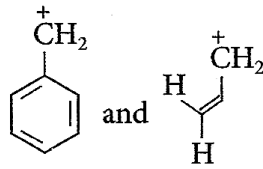
**357**

	heat of water [ $S_w = 4.2 \text{ kJ/(kg } ^\circ\text{C)}$ ], determine the heat of fusion of ice (in $\text{kJ/kg}$ ) from the above measurement data.																									
<b>Sol.</b>	$Q_1 = m (2.1 \text{ kJ/kg}^\circ\text{C}) (9^\circ\text{C})$ $Q_2 = 19Q_1 = m(2.1 \text{ kJ/kg}^\circ\text{C}) (1^\circ\text{C})$ $19[(18.9)m] = (2.1 + L_f)m$ $359.10 = 2.1 + L_f$ $L_f = 359.1 - 2.1 = 357. \text{ kJ/kg}$																									
<b>25.</b>	Find the charge (in $\mu\text{C}$ ) on the capacitor C in the figure shown. Internal resistance of a source is to be neglected. (Take : $\varepsilon = 9 \text{ V}$ , $C = 2\mu\text{F}$ ) 	<b>4</b>																								
<b>Sol.</b>	$V_R = 3\text{V}$ $V_{2R} = 6\text{V}$ $C(V_0 - 6) + 2C (V_0 - 9) + 3C (V_0 - 0) = 0$ $\Rightarrow V_0 = 4 \text{ V}$ $\therefore Q = 4 \mu\text{C}$																									
<b>26.</b>	The number of neutrons and electrons, respectively, present in the radioactive isotope of hydrogen is	<b>A</b>																								
	2 and 1																									
	2 and 2																									
	3 and 1																									
	1 and 1																									
<b>Sol.</b>	Radioactive isotope of hydrogen is tritium ${}^3_1\text{H}$ . Number of electrons = 1; Number of neutrons = $3 - 1 = 2$																									
<b>27.</b>	Match List-I with List-II. <table border="1" data-bbox="205 1364 876 1688"><thead><tr><th colspan="2">List-I (Species)</th><th colspan="2">List-II (Hybrid Orbitals)</th></tr></thead><tbody><tr><td>(A)</td><td><math>\text{SF}_4</math></td><td>(i)</td><td><math>sp^3d^2</math></td></tr><tr><td>(B)</td><td><math>\text{IF}_5</math></td><td>(ii)</td><td><math>d^2sp^3</math></td></tr><tr><td>(C)</td><td><math>\text{NO}_2^+</math></td><td>(iii)</td><td><math>sp^3d</math></td></tr><tr><td>(D)</td><td><math>\text{NH}_4^+</math></td><td>(iv)</td><td><math>sp^3</math></td></tr><tr><td></td><td></td><td>(v)</td><td><math>sp</math></td></tr></tbody></table> Choose the correct answer from the options given below:	List-I (Species)		List-II (Hybrid Orbitals)		(A)	$\text{SF}_4$	(i)	$sp^3d^2$	(B)	$\text{IF}_5$	(ii)	$d^2sp^3$	(C)	$\text{NO}_2^+$	(iii)	$sp^3d$	(D)	$\text{NH}_4^+$	(iv)	$sp^3$			(v)	$sp$	<b>A</b>
List-I (Species)		List-II (Hybrid Orbitals)																								
(A)	$\text{SF}_4$	(i)	$sp^3d^2$																							
(B)	$\text{IF}_5$	(ii)	$d^2sp^3$																							
(C)	$\text{NO}_2^+$	(iii)	$sp^3d$																							
(D)	$\text{NH}_4^+$	(iv)	$sp^3$																							
		(v)	$sp$																							
	(A) – (iii), (B) – (i), (C) – (v) and (D) – (iv)																									
	(A) – (ii), (B) – (i), (C) – (iv) and (D) – (v)																									
	(A) – (iv), (B) – (iii), (C) – (ii) and (D) – (v)																									
	(A) – (i), (B) – (ii), (C) – (v) and (D) – (iii)																									

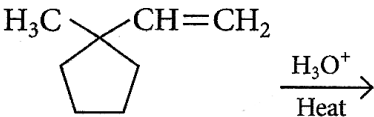
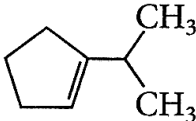
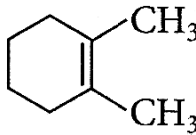
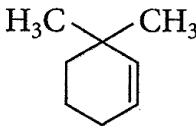
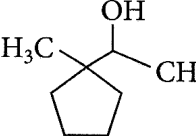
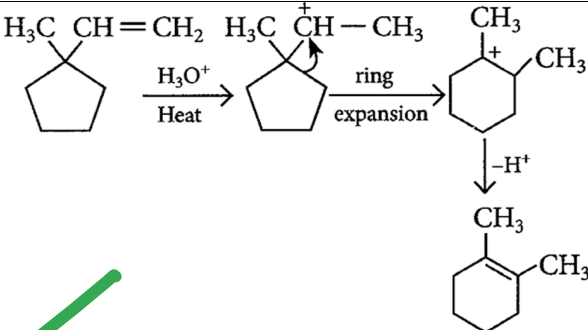
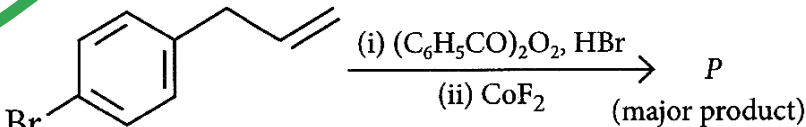
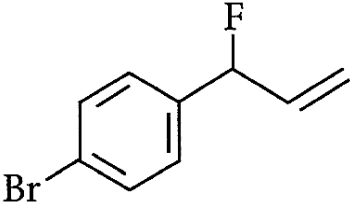
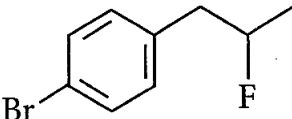
<b>Sol.</b>	$\text{NH}_4^+ : sp^3$ $\text{NO}_2^+ : sp$  $\text{SF}_4 :$ $sp^3d$  $\text{IF}_5 :$ $sp^3d^2$	
<b>28.</b>	<p>The <math>K_{sp}</math>, for the following dissociation is <math>1.6 \times 10^{-5}</math></p> $\text{PbCl}_2(s) \rightleftharpoons \text{Pb}^{2+}(aq) + 2\text{Cl}^-(aq)$ <p>Which of the following choices is correct for a mixture of 300 mL 0.134 M <math>\text{Pb}(\text{NO}_3)_2</math> and 100 mL 0.4 M <math>\text{NaCl}</math>?</p>	<b>B</b>
	$Q < K_{sp}$	
	$Q > K_{sp}$	
	$Q = K_{sp}$	
	Not enough data provided	
<b>Sol.</b>	$\text{PbCl}_{2(s)} \rightleftharpoons \text{Pb}_{(aq)}^{2+} + 2\text{Cl}_{(aq)}^-$ $K_{sp} = 1.6 \times 10^{-5}$ $[\text{Pb}^{2+}] = 0.134 \text{ M in } 300 \text{ mL}$ $[\text{Cl}^-] = 0.4 \text{ M in } 100 \text{ mL}$ $Q = [\text{Pb}^{2+}] [\text{Cl}^-]^2 \quad \dots(i)$ $[\text{Pb}^{2+}] = \frac{0.134 \times 300}{400} = 0.1005$ $[\text{Cl}^-] = \frac{0.4 \times 100}{400} = 0.1$ $\text{From eq. (i) } Q = (0.1005)(0.1)^2 = 1.0 \times 10^{-3} \therefore Q > K_{sp}$	
<b>29.</b>	<p><math>\Lambda_m^\circ</math> for <math>\text{NaCl}</math>, <math>\text{HCl}</math> and <math>\text{NaA}</math> are 126.4, 425.9 and 100.5 <math>\text{S cm}^2 \text{ mol}^{-1}</math>, respectively. If the conductivity of 0.001 M <math>\text{HA}</math> is <math>5 \times 10^{-5} \text{ S cm}^{-1}</math>, degree<sup>-1</sup> of <math>\text{HA}</math> is of dissociation</p>	<b>C</b>
	0.75	
	0.25	
	0.125	
	0.50	
<b>Sol.</b>	<p>Given, <math>\Lambda_m^\circ \text{NaCl} = 126.4 \text{ S cm}^2 \text{ mol}^{-1}</math></p> $\Lambda_m^\circ \text{HCl} = 425.9 \text{ S cm}^2 \text{ mol}^{-1}$ $\Lambda_m^\circ \text{NaA} = 100.5 \text{ S cm}^2 \text{ mol}^{-1}$ $\Lambda_m^\circ \text{HA} = \Lambda_m^\circ \text{HCl} + \Lambda_m^\circ \text{NaA} - \Lambda_m^\circ \text{NaCl}$ $= 425.9 + 100.5 - 126.4 = 400 \text{ S cm}^2 \text{ mol}^{-1}$ $\Lambda_m = \frac{1000 \kappa}{C} = 5 \times 10^{-5} \times \frac{1000}{0.001} = 50$ $\alpha = \frac{\Lambda_m}{\Lambda_m^\circ} = \frac{50}{400} = 0.125$	
<b>30.</b>	The correct reaction profile diagram for a positive catalyst reaction	<b>D</b>

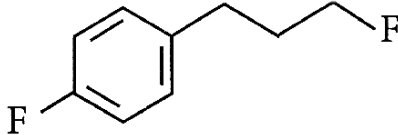
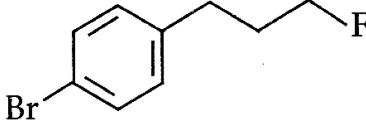
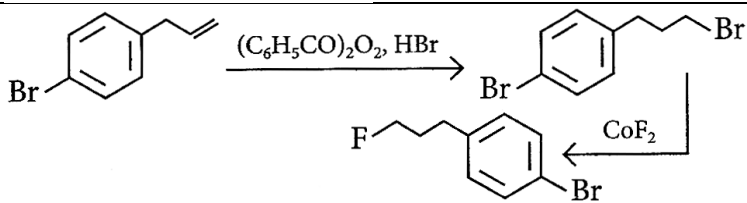
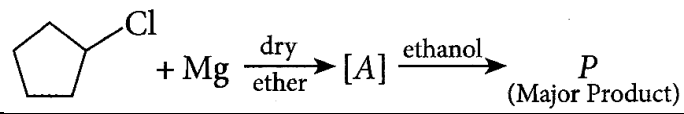
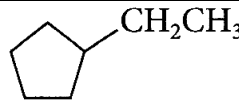
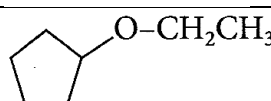
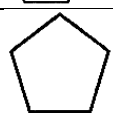
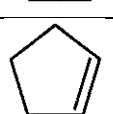
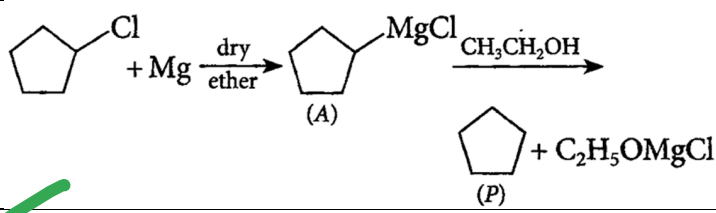
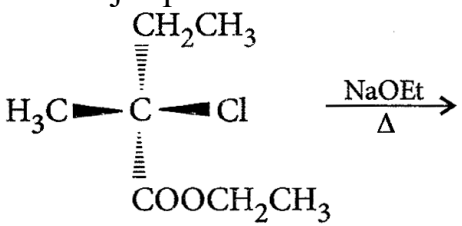
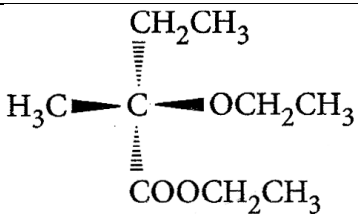
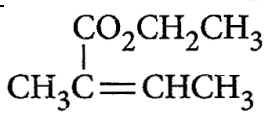
<b>Sol.</b>	Positive catalyst increases the rate of reaction and decreases the activation energy. So the correct reaction profile diagram for a positive catalyst is option (d).	
<b>31.</b>	<p>Given below are the oxides :</p> <p><math>\text{Na}_2\text{O}</math>, <math>\text{As}_2\text{O}_3</math>, <math>\text{N}_2\text{O}</math>, <math>\text{NO}</math> and <math>\text{Cl}_2\text{O}_7</math></p> <p>Number of amphoteric oxides is:</p>	<b>B</b>
	0	
	1	
	2	
	3	
<b>Sol.</b>	Amphoteric oxide : $\text{As}_2\text{O}_3$	
<b>32.</b>	<p>On heating, lead (II) nitrate gives a brown gas (A). The gas (A) on cooling changes to a colourless solid/liquid (B). (B) on heating with <math>\text{NO}</math> changes to a blue solid (C). The oxidation number of nitrogen in solid (C) is</p>	<b>C</b>
	+ 5	
	+ 4	
	+ 3	

	+ 1	
<b>Sol.</b>	<p>Lead nitrate on heating gives <math>\text{NO}_2</math> gas which is brown in colour, on cooling it dimerises to colourless <math>\text{N}_2\text{O}_4</math>. <math>\text{N}_2\text{O}_4</math> on heating with <math>\text{NO}</math> changes to <math>\text{N}_2\text{O}_3</math>. The reactions can be written as :</p> $\text{Pb}(\text{NO}_3)_2 \xrightarrow{\Delta} \text{PbO} + 2\text{NO}_2$ <p style="text-align: center;">Brown gas (A)</p> $2\text{NO}_2 \xrightarrow{\text{Cooling}} \text{N}_2\text{O}_4; \text{N}_2\text{O}_4 + 2\text{NO} \longrightarrow 2\text{N}_2\text{O}_3$ <p style="text-align: center;">Brown gas                      (B)                      Blue solid (C)</p> <p>Oxidation state of nitrogen in <math>\text{N}_2\text{O}_3</math> :</p> $2(x) + 3(-2) = 0 \Rightarrow 2x = +6 \Rightarrow x = +3$	
<b>33.</b>	Which of the following arrangements does not represent the correct order of the property stated against it?	<b>B</b>
	$\text{Sc} < \text{Ti} < \text{Cr} < \text{Mn}$ : Number of oxidation states	
	$\text{V}^{2+} < \text{Cr}^{2+} < \text{Mn}^{2+} < \text{Fe}^{2+}$ : Paramagnetic behaviour	
	$\text{Ni}^{2+} < \text{Co}^{2+} < \text{Fe}^{2+} < \text{Mn}^{2+}$ : Ionic size	
	$\text{Co}^{3+} < \text{Fe}^{3+} < \text{Cr}^{3+} < \text{Sc}^{3+}$ : Stability in aqueous solution	
<b>Sol.</b>	Number of unpaired electrons in $\text{Fe}^{2+}$ is less than $\text{Mn}^{2+}$ , so $\text{Fe}^{2+}$ is less paramagnetic than $\text{Mn}^{2+}$ .	
<b>34.</b>	<p>Given below are two statements:</p> <p><b>Statement I:</b> <math>[\text{Mn}(\text{CN})_6]^{3-}</math>, <math>[\text{Fe}(\text{CN})_6]^{3-}</math> and <math>[\text{Co}(\text{C}_2\text{O}_4)_3]^{3-}</math> are <math>d^2sp^3</math> hybridised.</p> <p><b>Statement II:</b> <math>[\text{MnCl}_6]^{3-}</math> and <math>[\text{FeF}_6]^{3-}</math> are paramagnetic and have 4 and 5 unpaired electrons respectively.</p> <p>In the light of the above statements, choose the correct answer from the options given below.</p>	<b>D</b>
	Statement I is correct but statement II is false.	
	Both statement I and statement II are false.	
	Statement I is incorrect but statement II is true.	
	Both statement I and statement II are true.	
<b>Sol.</b>	<p>The spin only magnetic moment is 4.90 B.M. which indicates that the number of unpaired electrons is</p> <p><math>\text{Fe}^{3+}</math> : <math>[\text{Ar}]3d^5</math></p> <p><math>[\text{Fe}(\text{CN})_6]^{3-}</math> : </p> <p><math>\text{Co}^{3+}</math> : <math>[\text{Ar}]3d^6</math></p> <p><math>[\text{Co}(\text{C}_2\text{O}_4)_3]^{3-}</math> : </p> <p>As <math>\text{CN}^-</math> and <math>\text{C}_2\text{O}_4^{2-}</math> are strong field ligands so pairing of electrons takes place and inner orbital complexes are formed. While in case of <math>[\text{MnCl}_6]^{3-}</math> and <math>[\text{FeF}_6]^{3-}</math>, due to the presence of weak field ligands, pairing does not take place.</p>	
<b>35.</b>	Which of the following is structure of a separating funnel?	<b>A</b>

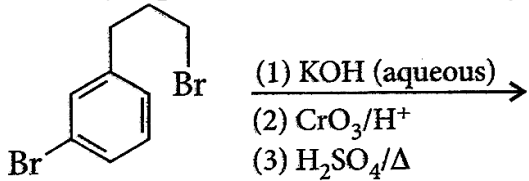
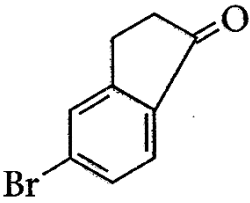
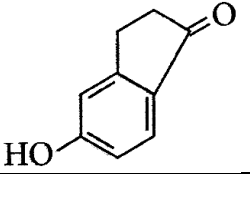
		
		
		
		
36	<p><b>List-I</b> [Test/Reagents/Observation(s)]</p> <p>(A) Lassaigne's test (B) Cu(II) oxide (C) Silver nitrate (D) The sodium fusion extract gives black precipitate with acetic acid and lead acetate</p> <p>The correct match is :</p>	<p><b>List-II</b> (Species detected)</p> <p>(i) Carbon (ii) Sulphur (iii) N, S, P and halogen (iv) Halogen specifically</p>
		A
	(A) – (iii), (B) – (i), (C) – (iv), (D) – (ii)	
	(A) – (i), (B) – (iv), (C) – (iii), (D) – (ii)	
	(A) – (iii), (B) – (i), (C) – (ii), (D) – (iv)	
	(A) – (i), (B) – (ii), (C) – (iv), (D) – (iii)	
37.	 <p>(A) (B) (C) (D)</p> <p>Among the given species the resonance stabilised carbocations are</p>	D
	(A), (B) and (C) only	
	(C) and (D) only	
	(A), (B) and (D) only	
	(A) and (B) only	
Sol.	 <p>are resonance stabilised carbocations because positive charge is in conjugation with double bond.</p>	
38.	<p>Identify the correct statement for the below given transformation.</p> $\text{CH}_3 - \text{CH}_2 - \text{CH}_2 - \underset{\text{N}^+(\text{CH}_3)_3}{\text{CH}} - \text{CH}_3 \xrightarrow{\text{C}_2\text{H}_5\text{ONa}} \text{A} + \text{B}$ <p>(Major) (Minor)</p>	C

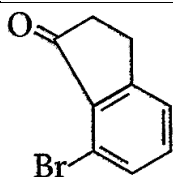
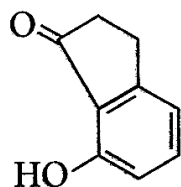


	A - $\text{CH}_3\text{CH}_2\text{CH}=\text{CH}-\text{CH}_3$ , B - $\text{CH}_3\text{CH}_2\text{CH}_2\text{CH}=\text{CH}_2$ , Saytzeff products	
	A - $\text{CH}_3\text{CH}_2\text{CH}=\text{CH}-\text{CH}_3$ , B - $\text{CH}_3\text{CH}_2\text{CH}_2\text{CH}=\text{CH}_2$ , Hofmann products	
	A - $\text{CH}_3\text{CH}_2\text{CH}_2\text{CH}=\text{CH}_2$ , B - $\text{CH}_3\text{CH}_2\text{CH}=\text{CHCH}_3$ , Hofmann products	
	A - $\text{CH}_3\text{CH}_2\text{CH}_2\text{CH}=\text{CH}_2$ , B - $\text{CH}_3\text{CH}_2\text{CH}=\text{CHCH}_3$ , Saytzeff products	
Sol.	$\text{CH}_3-\text{CH}_2-\text{CH}_2-\underset{\text{(CH}_3)_3\text{N}^+}{\text{CH}}-\overset{\text{H}}{\underset{\text{H}}{\text{C}}}-\text{H} \xrightarrow{\text{CH}_3\text{CH}_2\text{O}} \text{CH}_3\text{CH}_2\text{CH}_2\text{CH}=\text{CH}_2$ <p>(A) (Major) (Hofmann elimination) + <math>\text{CH}_3\text{CH}_2\text{CH}=\text{CHCH}_3</math> (B) (Minor product)</p>	
39.	<p>The major product in the following reaction is :</p> 	<b>B</b>
		
		
		
		
Sol.		
40.	 <p>Major product P of above reaction is</p>	<b>D</b>
		
		

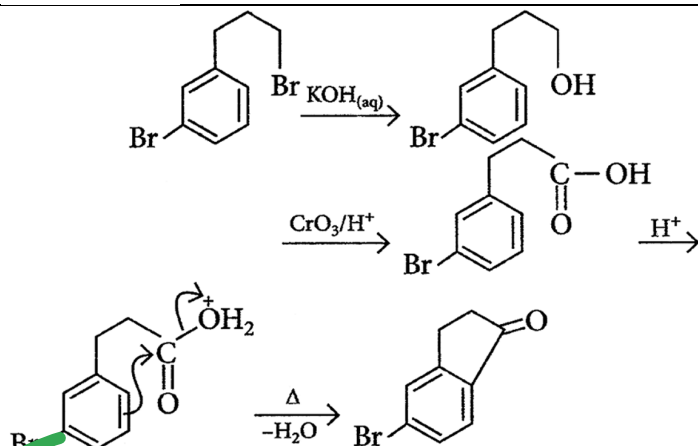
		
		
<b>Sol.</b>	 <p>HBr in presence of organic peroxide gives anti-Markovnikov addition product.</p>	
<b>41.</b>	<p>In the following sequence of reactions the P is</p> 	<b>C</b>
		
		
		
		
<b>Sol.</b>		
<b>42.</b>	<p>The major product of the following reaction is</p> 	<b>B</b>
		
		

	$\begin{array}{c} \text{OCH}_2\text{CH}_3 \\   \\ \text{H}_3\text{CH}_2\text{C} \blacktriangleleft \text{C} \blacktriangleright \text{CO}_2\text{CH}_2\text{CH}_3 \\   \\ \text{CH}_3 \end{array}$													
	$\begin{array}{c} \text{CH}_3\text{CH}_2\text{C}=\text{CH}_2 \\   \\ \text{CO}_2\text{CH}_2\text{CH}_3 \end{array}$													
<b>Sol.</b>	<p>As the given alkyl halide is tertiary, thus, elimination is preferred over substitution. According to Saytzeff rule, substituted alkene is formed as a major product.</p> $\begin{array}{c} \text{CH}_2\text{CH}_3 \\   \\ \text{CH}_3 \blacktriangleleft \text{C} \blacktriangleright \text{Cl} \\   \\ \text{COOCH}_2\text{CH}_3 \end{array} \xrightarrow[\text{-HCl}]{\text{NaOEt, } \Delta} \begin{array}{c} \text{CH}_3 \text{---} \text{C} = \text{CHCH}_3 \\   \\ \text{COOCH}_2\text{CH}_3 \end{array}$													
<b>43.</b>	<p>Two compounds A and B with same molecular formula (<math>\text{C}_3\text{H}_6\text{O}</math>) undergo Grignard's reaction with methylmagnesium bromide to give products C and D. Products C and D show following chemical tests.</p> <table border="1"> <thead> <tr> <th>Test</th><th>C</th><th>D</th></tr> </thead> <tbody> <tr> <td>Ceric ammonium nitrate test</td><td>Positive</td><td>Positive</td></tr> <tr> <td>Lucas test</td><td>Turbidity obtained after five minutes</td><td>Turbidity obtained immediately</td></tr> <tr> <td>Iodoform test</td><td>Positive</td><td>Negative</td></tr> </tbody> </table> <p>C and D respectively are</p>	Test	C	D	Ceric ammonium nitrate test	Positive	Positive	Lucas test	Turbidity obtained after five minutes	Turbidity obtained immediately	Iodoform test	Positive	Negative	<b>D</b>
Test	C	D												
Ceric ammonium nitrate test	Positive	Positive												
Lucas test	Turbidity obtained after five minutes	Turbidity obtained immediately												
Iodoform test	Positive	Negative												
	$\begin{array}{c} \text{CH}_3 \\   \\ \text{C} = \text{H}_3\text{C} \text{---} \text{C} \text{---} \text{OH} \\   \\ \text{CH}_3 \end{array}; \begin{array}{c} \text{D} = \text{H}_3\text{C} \text{---} \text{CH}_2 \text{---} \text{CH} \text{---} \text{CH}_3 \\   \\ \text{OH} \end{array}$													
	$\text{C} = \text{H}_3\text{C} \text{---} \text{CH}_2 \text{---} \text{CH}_2 \text{---} \text{CH}_2 \text{---} \text{OH};$ $\begin{array}{c} \text{CH}_3 \\   \\ \text{D} = \text{H}_3\text{C} \text{---} \text{C} \text{---} \text{OH} \\   \\ \text{CH}_3 \end{array}$													
	$\text{C} = \text{H}_3\text{C} \text{---} \text{CH}_2 \text{---} \text{CH}_2 \text{---} \text{CH}_2 \text{---} \text{OH};$ $\begin{array}{c} \text{D} = \text{H}_3\text{C} \text{---} \text{CH}_2 \text{---} \text{CH} \text{---} \text{CH}_3 \\   \\ \text{OH} \end{array}$													
	$\begin{array}{c} \text{OH} \\   \\ \text{C} = \text{H}_3\text{C} \text{---} \text{CH}_2 \text{---} \text{CH} \text{---} \text{CH}_3; \end{array} \begin{array}{c} \text{CH}_3 \\   \\ \text{D} = \text{H}_3\text{C} \text{---} \text{C} \text{---} \text{OH} \\   \\ \text{CH}_3 \end{array}$													

<p><b>Sol.</b></p>	<p>The possible isomers for <math>C_3H_6O</math> are</p> $CH_3-\overset{\overset{O}{\parallel}}{C}-CH_3 \text{ and } CH_3-CH_2-\overset{\overset{O}{\parallel}}{C}-H$ $CH_3-\overset{\overset{O}{\parallel}}{C}-CH_3 + CH_3MgBr \rightarrow CH_3-\overset{\overset{OMgBr}{\mid}}{C}-CH_3$ $\hspace{10em} \mid$ $\hspace{10em} CH_3$ $\hspace{10em} \swarrow \quad \searrow$ $\hspace{10em} H_3O^+$ $CH_3-\overset{\overset{OH}{\mid}}{C}-CH_3$ $\hspace{10em} \mid$ $\hspace{10em} CH_3$ <p>(D) (<math>3^\circ</math> alcohol)</p> <p>(Ceric ammonium nitrate : +ve test, Lucas test : Turbidity obtained immediately, Iodoform test : -ve test)</p> $CH_3-CH_2-\overset{\overset{O}{\parallel}}{C}-H + CH_3MgBr \rightarrow CH_3-CH_2-\overset{\overset{OMgBr}{\mid}}{C}-H$ $\hspace{10em} \mid$ $\hspace{10em} CH_3$ $\hspace{10em} \swarrow \quad \searrow$ $\hspace{10em} H_3O^+$ $CH_3-CH_2-\overset{\overset{OH}{\mid}}{C}-H$ $\hspace{10em} \mid$ $\hspace{10em} CH_3$ <p>(C) (<math>2^\circ</math> alcohol)</p> <p>Ceric ammonium nitrate : +ve test, Lucas test : Turbidity obtained after 5 minutes, Iodoform test : +ve test)</p> <p>Thus, C is <math>2^\circ</math> alcohol, <math>CH_3-CH_2-\overset{\overset{OH}{\mid}}{CH}-CH_3</math> and D is</p> <p><math>3^\circ</math> alcohol, <math>CH_3-\overset{\overset{OH}{\mid}}{C}-CH_3</math>.</p>	
<p><b>44.</b></p>	<p>The major product of the following reaction is</p> <div style="text-align: center;">  </div> <div style="display: flex; justify-content: space-around; margin-top: 20px;"> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> </div>	<p><b>A</b></p>



Sol.



45. Match List-I with List-II.

List I (Enzymatic reaction)		List II (Enzyme)	
A.	Sucrose $\rightarrow$ Glucose and Fructose	I.	Zymase
B.	Glucose $\rightarrow$ Ethylalcohol and $\text{CO}_2$	II.	Pepsin
C.	Starch $\rightarrow$ Maltose	III.	Invertase
D.	Proteins $\rightarrow$ Amino acids	IV.	Diastase

Choose the correct from the option given below.

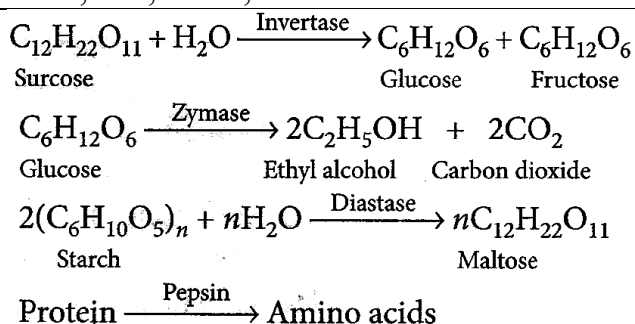
A-I, B-II, C-IV, D-III

A-I, B-IV, C-III, D-II

A-III, B-I, C-II, D-IV

A-III, B-I, C-IV, D-II

Sol.



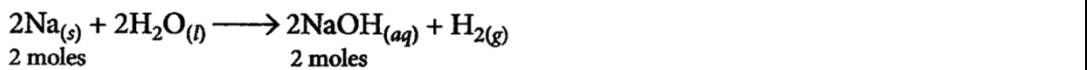
46. The volume of HCl, containing  $73 \text{ g L}^{-1}$ , required to completely neutralise NaOH obtained by reacting  $0.69 \text{ g}$  of metallic sodium with water, is \_\_\_\_\_ mL. (Nearest Integer)  
(Given: molar masses of Na, Cl, O, H, are 23, 35.5, 16 and  $1 \text{ g mol}^{-1}$ )

15

respectively)

**Sol.**

Reaction of sodium with water :



$$\text{Moles of Na} = \frac{0.69}{23} = 0.03 \text{ mole}$$

$$\text{Mole of NaOH produced} = 0.03 \text{ mole} = M_1 V_1$$

$$M_2(\text{HCl}) = \frac{73}{36.5} = 2 \text{ mol/L}$$

$$M_1 V_1 = M_2 V_2$$

$$0.03 = 2 \times V_2$$

$$V_2 = \frac{0.03}{2} \text{ L} = 0.015 \text{ L or } 15 \text{ mL}$$

~~47.~~

Data given for the following reactions as follows :

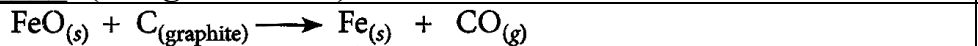


Substance	$\Delta H^\circ$ (kJ mol <sup>-1</sup> )	$\Delta S^\circ$ (J mol <sup>-1</sup> K <sup>-1</sup> )
FeO <sub>(s)</sub>	-266.3	57.49
C <sub>(graphite)</sub>	0	5.74
Fe <sub>(s)</sub>	0	27.28
CO <sub>(g)</sub>	-110.5	197.6

The minimum temperature is K at which the reaction becomes spontaneous is \_\_\_\_\_. (Integer answer)

964

**Sol.**



$\Delta H_f^\circ(\text{kJ/mol})$	-266.3	0	0	-110.5
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$\Delta S^\circ$ (Jmol <sup>-1</sup> K <sup>-1</sup> )	57.49	5.74	27.28	197.6
--	-------	------	-------	-------

$$\begin{aligned}\Delta H^\circ_{\text{reaction}} &= (\Delta H^\circ_{f(\text{Fe})} + \Delta H^\circ_{f(\text{CO})} - (\Delta H^\circ_{f(\text{FeO})} + \Delta H^\circ_{f(\text{C})}) \\ &= (0 - 110.5) - (-266.3 + 0) \\ &= -110.5 + 266.3 = 155.8 \text{ kJ/mol}\end{aligned}$$

$$\begin{aligned}\Delta S^\circ_{\text{reaction}} &= (\Delta S^\circ_{\text{Fe}} + \Delta S^\circ_{\text{CO}}) - (\Delta S^\circ_{\text{FeO}} + \Delta S^\circ_{\text{C}}) \\ &= (27.28 + 197.6) - (57.49 + 5.74) \\ &= 161.65 \text{ J/mol-K}\end{aligned}$$

$$T_{\min.} = \frac{\Delta H}{\Delta S} = \frac{155.8 \times 10^3 \text{ J/mol}}{161.65 \text{ J/mol-K}} \approx 964 \text{ K}$$

48. 

If the concentration of glucose ( $\text{C}_6\text{H}_{12}\text{O}_6$ ) in blood is  $0.72 \text{ g L}^{-1}$ , the molarity of glucose in blood is  $\times 10^{-3} \text{ M}$ . (Nearest Integer)  
(Given: Atomic mass of C = 12, H = 1, O = 16 u)

4

**Sol.**

$$M = \frac{W_{\text{solute}}}{M_{\text{solute}} \times V_{\text{soln}} (\text{in L})} = \frac{0.72}{180}$$
$$= 0.004 = 4 \times 10^{-3} \text{ M}$$

✓	10.	
---	-----	--

[Fe(CN)<sub>6</sub>]<sup>3-</sup> should be an inner orbital complex. Ignoring the pairing energy, the value of crystal field stabilization energy for this complex is (–) \_\_\_\_\_ Δ<sub>0</sub>. **(Nearest Integer)**

2

Sol.

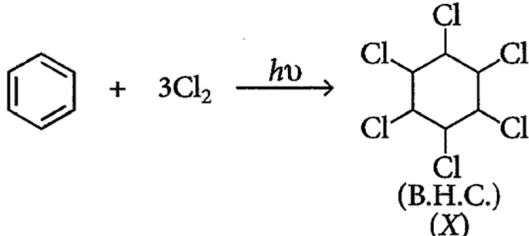
Fe<sup>3+</sup> ion configuration : [Ar]3d<sup>5</sup>
$$\text{Fe}^{3+} \text{ in } [\text{Fe}(\text{CN})_6]^{3-} : t_{2g}^5 e_g^0$$

$$\text{CFSE} = 5 \times (-0.4)\Delta_o = -2\Delta_o$$

**50.**

In the presence of sunlight, benzene reacts with  $\text{Cl}_2$  to give product, X. The

6

	number of hydrogen in X is _____.	
<b>Sol.</b>	 <p>Product 'X' contains 6 hydrogen atom.</p>	
<b>51.</b>	Let $S = \{z \in \mathbb{C} :  z - 1  = 1 \text{ and } (\sqrt{2} - 1)(z + \bar{z}) - i(z - \bar{z}) = 2\sqrt{2}\}$ . Let $z_1, z_2 \in S$ be such that $ z_1  = \max_{z \in S}  z $ and $ z_2  = \min_{z \in S}  z $ . Then, $ \sqrt{2}z_1 - z_2 ^2$ equals	<b>C</b>
	4	
	3	
	2	
	1	
<b>Sol.</b>	<p>(c) Given <math> z - 1  = 1</math>  <math>(x - 1)^2 + y^2 = 1</math>  and <math>(\sqrt{2} - 1)(z + \bar{z}) - i(z - \bar{z}) = 2\sqrt{2}</math>  <math>(\sqrt{2} - 1)(2x) - i(2iy) = 2\sqrt{2}</math>  <math>[\because z + \bar{z} = 2x, z - \bar{z} = 2iy]</math>  <math>2(\sqrt{2} - 1)x + 2y = 2\sqrt{2}</math>  <math>(\sqrt{2} - 1)x + y = \sqrt{2}</math>  On solving Eqs. (i) and (ii), we get  <math>x = 1, \frac{1}{2 - \sqrt{2}}</math>  When <math>x = 1</math>, then <math>y = 1</math>  <math>\therefore z_2 = 1 + i</math>  and when <math>x = \frac{1}{2 - \sqrt{2}}</math>, then  <math>y = \sqrt{2} - \frac{1}{\sqrt{2}}</math>  <math>\therefore z_1 = 1 + \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}</math>  <math>\therefore  \sqrt{2}z_1 - z_2 ^2</math>  <math>= \left  \sqrt{2} \left( 1 + \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right) - (1 + i) \right ^2</math>  <math>= (\sqrt{2})^2 = 2</math></p>	
<b>52.</b>	For a non-zero complex number $z$ , let $\arg(z)$ denote the principal argument of $z$ , with $-\pi < \arg(z) \leq \pi$ . Let $\omega$ be the cube root of the unity for which $0 < \arg(\omega) < \pi$ . Let $\alpha = \arg\left(\sum_{n=1}^{2025} (-\omega)^n\right)$ . Then the value of $\left \frac{3\alpha}{\pi}\right $ is _____.	<b>A</b>
	2	
	4	
	10	
	8	

<b>Sol.</b>	<p>We have, <math>\alpha = \arg\left(\sum_{n=1}^{2025} (-\omega)^n\right)</math></p> $= \arg(-\omega + \omega^2 - \omega^3 + \dots + (-\omega)^{2025})$ $= \arg\left(\frac{-\omega((- \omega)^{2025} - 1)}{-\omega - 1}\right)$ $= \arg\left(\frac{-\omega((- \omega)^{3 \times 675} - 1)}{-\omega - 1}\right)$ $= \arg\left(\frac{-\omega(-1 - 1)}{-\omega - 1}\right) = \arg\left(\frac{2\omega}{\omega^2}\right)$ $= \arg\left(\frac{2}{\omega}\right) = \arg(2\omega^2)$ $= \tan^{-1}\left(\frac{-\sqrt{3}}{-1}\right) = -\pi + \frac{\pi}{3} = \frac{-2\pi}{3}$ <p><math>[\because x &lt; 0 \text{ and } y &lt; 0, \text{ then } \arg(z) = -\pi + \theta]</math></p> $\Rightarrow \frac{3\alpha}{\pi} = -2$	
<b>53.</b>	All the letters of the word "GTWENTY" are written in all possible ways with or without meaning and these words are written as in a dictionary. The serial number of the word "GTWENTY" is _____.	<b>B</b>
	550	
	553	
	560	
	600	
<b>Sol.</b>	<p>(553) : Number of words starts with</p> $E = \frac{6!}{2!}$ <p>Number of words starts with <math>GE = \frac{5!}{2!}</math></p> <p>Number of words starts with <math>GN = \frac{5!}{2!}</math></p> <p>Number of words starts with <math>GTE = 4!</math></p> <p>Number of words starts with <math>GTN = 4!</math></p> <p>Number of words starts with <math>GTT = 4!</math></p> <p>Next word will be GTWENTY</p> <p><math>\therefore</math> Rank of GTWENTY</p> $= \frac{6!}{2!} + 2 \times \frac{5!}{2!} + 3 \times 4! + 1 = 553$	
<b>54.</b>	Let $\alpha = \sum_{k=0}^n \left(\frac{{}^nC_k^2}{k+1}\right)$ and $\beta = \sum_{k=0}^{n-1} \left(\frac{{}^nC_k {}^nC_{k+1}}{k+2}\right)$ . If $5\alpha = 6\beta$ , then $n$ equals _____.	<b>A</b>
	10	
	20	
	40	
	50	



<b>Sol.</b>	<p>(10) : We have, <math>\alpha = \sum_{k=0}^n \left( \frac{{}^nC_k}{k+1} \right)</math></p> <p>and</p> $\beta = \sum_{k=0}^{n-1} \left( \frac{{}^nC_k \cdot {}^nC_{k+1}}{k+2} \right)$ <p>Now, <math>\alpha = \sum_{k=0}^n \frac{{}^nC_k}{k+1} \cdot {}^nC_k</math></p> $= \sum_{k=0}^n \frac{{}^{n+1}C_{k+1}}{n+1} \cdot {}^nC_{n-k}$ $= \frac{1}{n+1} \sum_{k=0}^n {}^{n+1}C_{k+1} \cdot {}^nC_{n-k}$ $= \frac{1}{n+1} \cdot {}^{2n+1}C_{n+1}$ <p>Now, <math>\beta = \sum_{k=0}^{n-1} {}^nC_{n-k} \cdot \frac{n+1}{(k+2)(n+1)} \cdot {}^nC_{k+1}</math></p> $= \frac{1}{n+1} \sum_{k=0}^{n-1} {}^nC_{n-k} \cdot {}^{n+1}C_{k+2}$ $= \frac{1}{n+1} \cdot {}^{2n+1}C_{n+2}$ $\therefore \frac{\beta}{\alpha} = \frac{{}^{2n+1}C_{n+2}}{{}^{2n+1}C_{n+1}} = \frac{(2n+1-n-1)! \cdot (n+1)!}{(2n+1-n-2)! \cdot (n+2)!}$ $= \frac{n! \cdot (n+1)!}{(n-1)!(n+2)(n+1)!} = \frac{n}{n+2} = \frac{5}{6} \Rightarrow n = 10$	
<b>55.</b>	<p>A line passes through the origin and makes equal angles with the positive coordinate axes. It intersects the line <math>L_1: 2x + y + 6 = 0</math> and <math>L_2: 4x + 2y - p = 0, p &gt; 0</math>, at the points <math>A</math> and <math>B</math>, respectively. If <math>AB = \frac{9}{\sqrt{2}}</math> and the foot of the perpendicular from the point <math>A</math> on the line <math>L_2</math> is <math>M</math>, then <math>AM/BM</math> is equal to</p>	<b>B</b>
	5	
	3	
	4	
	2	
<b>Sol.</b>	<p>(b) The line passing through origin and making equal angles with the axes is</p> $y = x \Rightarrow 2x + x + 6 = 0$ $\Rightarrow x = -2, y = -2$ $\therefore A \equiv (-2, -2)$ <p>Also, <math>4x + 2y - p = 0 \Rightarrow x = p/6</math></p> $\Rightarrow B = \left( \frac{p}{6}, \frac{p}{6} \right)$ <p>Given, <math>AB = \frac{9}{\sqrt{2}}</math></p> $\Rightarrow \sqrt{\left( \frac{p}{6} + 2 \right)^2 + \left( \frac{p}{6} + 2 \right)^2} = \frac{9}{\sqrt{2}}$ $\Rightarrow \left( \frac{p}{6} + 2 \right) \sqrt{2} = \frac{9}{\sqrt{2}} \Rightarrow \frac{p+12}{6} = \frac{9}{2}$ $p = 15 \Rightarrow B = \left( \frac{5}{2}, \frac{5}{2} \right)$ $AM = \frac{ 4(-2) + 2(-2) - 15 }{\sqrt{4^2 + 2^2}} = \frac{27}{2\sqrt{5}}$ $\therefore BM^2 = AB^2 - AM^2$ $= \left( \frac{9\sqrt{2}}{2} \right)^2 - \left( \frac{27}{2\sqrt{5}} \right)^2 = \frac{81}{20}$ $\Rightarrow BM = \frac{9}{2\sqrt{5}}$ $\frac{AM}{BM} = \frac{27}{2\sqrt{5}} \times \frac{2\sqrt{5}}{9} = 3$	
<b>56.</b>	<p>Let the circle <math>C_1: x^2 + y^2 - 2(x + y) + 1 = 0</math> and <math>C_2</math> be a circle having centre at <math>(-1, 0)</math> and radius 2. If the line of the common chord of <math>C_1</math> and <math>C_2</math> intersects the</p>	<b>A</b>

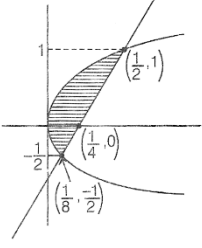
	y-axis at the point $P$ , then the square of the distance of $P$ from the centre of $C_1$ is:	
	2	
	6	
	4	
	1	
<b>Sol.</b>	<p>a) : We have <math>C_1 : x^2 + y^2 - 2(x + y) + 1 = 0</math></p> <p><math>C_2 : (x+1)^2 + y^2 - 4 = 0</math></p> <p>common chord, we have <math>C_1 - C_2 = 0</math></p> $x^2 - y^2 - 2x - 2y + 1 - x^2 - y^2 - 2x + 3 = 0$ $-4x - 2y + 4 = 0 \Rightarrow 2x + y - 2 = 0$ <p>since, common chord intersects <math>y</math>-axis</p> $= 0 \Rightarrow y = 2$ <p>So, point of intersection of common chord <math>y</math>-axis is <math>P(0, 2)</math>.</p> <p>required distance <math>= \left( \sqrt{(1-0)^2 + (1-2)^2} \right)^2 = 2</math></p>	
<b>57.</b>	Let $f: R - \{0\} \rightarrow R$ be a function such that $f(x) - 6f\left(\frac{1}{x}\right) = \frac{35}{3x} - \frac{5}{2}$ . If the $\lim_{x \rightarrow 0} \left( \frac{1}{\alpha x} + f(x) \right) = \beta$ ; $\alpha, \beta \in R$ , then $\alpha + 2\beta$ is equal to	<b>C</b>
	6	
	3	
	4	
	5	
<b>Sol.</b>	<p>(c) Given</p> $f(x) - 6f(1/x) = \frac{35}{3x} - \frac{5}{2} \quad \dots\dots(i)$ <p>Replace <math>x</math> by <math>1/x</math> in Eq. (i), we get</p> $f(1/x) - 6f(x) = \frac{35x}{3} - \frac{5}{2} \quad \dots\dots(ii)$ <p>On solving Eqs. (i) and (ii), we get</p> $f(x) = -2x - \frac{1}{3x} + \frac{1}{2}$ <p>Now, <math>\beta = \lim_{x \rightarrow 0} \left( \frac{1}{\alpha x} + f(x) \right)</math></p> $\Rightarrow \beta = \lim_{x \rightarrow 0} \left( \frac{1}{\alpha x} - 2x - \frac{1}{3x} + \frac{1}{2} \right)$ <p>limits exists finitely iff <math>\alpha = 3 \Rightarrow \beta = 1/2</math></p> <p>Hence, <math>\alpha + 2\beta = 3 + 2 \times \frac{1}{2} = 4</math></p>	
<b>58.</b>	In a set of $2n$ distinct observations, each of the observations below the median of all the observations is increased by 5 and each of the remaining observation is decreased by 3. Then, the mean of the new set of observations	<b>A</b>
	increases by 1	
	decreases by 1	
	decreases by 2	
	increases by 2	

<p><b>Sol.</b></p>	<p>(a) There are <math>2n</math> observations <math>x_1, x_2, \dots, x_{2n}</math>.</p> <p>So, mean = <math>\sum_{i=1}^{2n} \frac{x_i}{2n} = \bar{X}</math></p> $\Rightarrow \bar{X} = \frac{x_1 + x_2 + \dots + x_n + x_{n+1} + \dots + x_{2n}}{2n} \dots (i)$ <p>As, given in the question that total terms are <math>2n</math>, which is even.</p> <p>So, median of <math>2n</math> terms will be</p> $\frac{x_n + x_{n+1}}{2}$ <p>So, median will lie between <math>x_n</math> and <math>x_{n+1}</math>.</p> <p>As, we are given that each observation below median is increased by 5.</p> <p>So, for each <math>i</math> from <math>i = 1</math> to <math>i = n</math>, <math>x_i</math> becomes <math>x_i + 5</math>.</p> <p>As, we are given that each observation on above median is decreased by 3.</p> <p>So, for each <math>i</math> from <math>i = n + 1</math> to <math>i = 2n</math>, <math>x_i</math> becomes <math>x_i - 3</math>.</p> <p>So, new mean</p> $= \frac{(x_1 + 5) + (x_2 + 5) + \dots + (x_n + 5) + (x_{n+1} - 3) + \dots + (x_{2n} - 3)}{2n}$ $= \frac{x_1 + x_2 + \dots + x_n + 5n + x_{n+1} + \dots + x_{2n} - 3n}{2n}$ $= \frac{x_1 + x_2 + \dots + x_{2n}}{2n} + \frac{2n}{2n}$ $= \bar{X} + 1$ <p>So, it increase by 1.</p>	
<p><b>59.</b></p>	<p>Define a relation <math>R</math> on the interval <math>[0, \pi/2)</math> be <math>x R y</math> if and only if <math>\sec^2 x - \tan^2 y = 1</math>. Then, <math>R</math> is</p>	<p><b>D</b></p>
	<p>Both reflexive and transitive but not symmetric</p>	
	<p>Reflexive but neither symmetric not transitive</p>	
	<p>Both reflexive and symmetric but not transitive</p>	
	<p>An equivalence relation</p>	
<p><b>Sol.</b></p>	<p>(d) <math>xRy \Rightarrow \sec^2 x - \tan^2 y = 1</math></p> <p><b>To check Reflexivity</b></p> <p><math>xRx \Rightarrow \sec^2 x - \tan^2 x = 1</math> which is true.</p> <p>So, <math>R</math> is reflexive.</p> <p><b>To check symmetricity</b></p> <p><math>xRy \Rightarrow yRx</math></p> <p>i.e. <math>\sec^2 x - \tan^2 y = 1</math></p> $\Rightarrow \tan^2 x + 1 - \sec^2 y + 1 = 1$ $\Rightarrow \tan^2 x - \sec^2 y = -1$ $\Rightarrow \sec^2 y - \tan^2 x = 1 \Rightarrow yRx.$ <p>So, <math>R</math> is symmetric.</p> <p><b>To check Transitivity</b></p> <p><math>xRy, yRz \Rightarrow xRz</math></p> <p>i.e. <math>\sec^2 x - \tan^2 y = 1 \dots (i)</math></p> $\Rightarrow \sec^2 y - \tan^2 z = 1 \dots (ii)$ <p>On adding Eqs. (i) and (ii), we get</p> $\sec^2 x + \sec^2 y - \tan^2 y - \tan^2 z = 2$ $\Rightarrow \sec^2 x + 1 - \tan^2 z = 2$ $\Rightarrow \sec^2 x - \tan^2 z = 1 \Rightarrow xRz$ <p><math>\therefore R</math> is transitive.</p> <p>Hence, <math>R</math> is an equivalence relation.</p>	
<p><b>60.</b></p>	<p>Let <math>f(x) = \lim_{n \rightarrow \infty} \sum_{r=0}^n \left( \frac{\tan(x/2^{r+1}) + \tan^3(x/2^{r+1})}{1 - \tan^2(x/2^{r+1})} \right)</math>. Then <math>\lim_{x \rightarrow 0} \frac{e^x - e^{f(x)}}{(x - f(x))}</math> is equal to _____.</p>	<p><b>A</b></p>
	<p>1</p>	
	<p>4</p>	
	<p>5</p>	
	<p>6</p>	

<b>Sol.</b>	<p>: We have,</p> $= \lim_{n \rightarrow \infty} \sum_{r=0}^n \left[ \frac{2 \tan\left(\frac{x}{2^{r+1}}\right) - \tan\left(\frac{x}{2^{r+1}}\right) + \tan^3\left(\frac{x}{2^{r+1}}\right)}{1 - \tan^2\left(\frac{x}{2^{r+1}}\right)} \right]$ $= \lim_{n \rightarrow \infty} \sum_{r=0}^n \left[ \frac{2 \tan\left(\frac{x}{2^{r+1}}\right)}{1 - \tan^2\left(\frac{x}{2^{r+1}}\right)} - \frac{\tan\left(\frac{x}{2^{r+1}}\right) \left\{ 1 - \tan^2\left(\frac{x}{2^{r+1}}\right) \right\}}{\left\{ 1 - \tan^2\left(\frac{x}{2^{r+1}}\right) \right\}} \right]$ $= \lim_{n \rightarrow \infty} \sum_{r=0}^n \left( \tan \frac{x}{2^r} - \tan\left(\frac{x}{2^{r+1}}\right) \right) = \tan x$ <p>Now,</p> $\lim_{x \rightarrow 0} \left( \frac{e^x - e^{\tan x}}{x - \tan x} \right) = \lim_{x \rightarrow 0} e^{\tan x} \frac{(e^{x - \tan x} - 1)}{(x - \tan x)}$ $= 1 \quad \left[ \because \lim_{y \rightarrow 0} \frac{e^y - 1}{y} = 1 \right]$	
<b>61.</b>	Let A and B two square matrices of order 3 such that $ A  = 3$ and $ B  = 2$ . Then $ A^T A (\text{adj}(2A))^{-1} (\text{adj}(4B)) (\text{adj}(AB))^{-1} A A^T $ is equal to	<b>C</b>
	108	
	32	
	64	
	81	
<b>Sol.</b>	<p>(c) : We have</p> $ A^T A (\text{adj}(2A))^{-1} (\text{adj}(4B)) (\text{adj}(AB))^{-1} A A^T $ $=  A^T   A  \frac{1}{ \text{adj } 2A }  \text{adj } 4B  \frac{1}{ \text{adj } AB }  A   A^T $ $=  A ^2 \frac{1}{(2^2)^3  \text{adj } A } (4^2)^3  \text{adj } B  \frac{1}{ \text{adj } AB }  A ^2$ <p>[<math>\because  A  =  A^T </math> and <math>\text{adj } kA = k^{n-1} \text{adj } A</math> where <math>n</math> is order of <math>A</math>]</p> $=  A ^2 \frac{1}{2^6  A ^2} 4^6  B ^2 \frac{1}{ A ^2  B ^2}  A ^2$ <p>[<math>\because  \text{adj } A  =  A ^{n-1}</math> where <math>n</math> is order of <math>A</math>]</p> $= \frac{4^6}{2^6} = 64$	
<b>62.</b>	Let the system of equations $x + 5y - z = 1$ $4x + 3y - 3z = 7$ $24x + y + \lambda z = \mu$ $\lambda, \mu \in \mathbf{R}$ , having infinitely many solutions. Then the number of the solutions of this system, if $x, y, z$ are integers and satisfy $7 \leq x + y + z \leq 77$ , is:	<b>A</b>
	3	
	5	
	6	
	4	

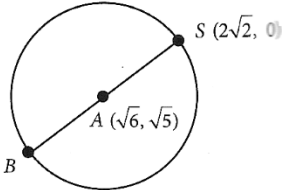
<b>Sol.</b>	<p>(a) : For infinitely many solutions, we have, <math>D = 0</math></p> $\Rightarrow \begin{vmatrix} 1 & 5 & -1 \\ 4 & 3 & -3 \\ 24 & 1 & \lambda \end{vmatrix} = 0$ $\Rightarrow 1(3\lambda + 3) - 5(4\lambda + 72) - 1(4 - 72) = 0$ $\Rightarrow 17\lambda = -289 \Rightarrow \lambda = -17 \quad \dots(i)$ <p>Now, <math>D_1 = 0</math></p> $\Rightarrow \begin{vmatrix} 1 & 5 & -1 \\ 7 & 3 & -3 \\ \mu & 1 & -17 \end{vmatrix} = 0$ $\Rightarrow 1(-51 + 3) - 5(-119 + 3\mu) - 1(7 - 3\mu) = 0$ $\Rightarrow -48 + 595 - 15\mu - 7 + 3\mu = 0$ $\Rightarrow 12\mu = 540$ $\Rightarrow \mu = 45 \quad \dots(ii)$ <p>Using (i) and (ii), we get</p> $x + 5y - z = 1, 4x + 3y - 3z = 7, 24x + y - 17z = 45$ $\Rightarrow z = x + 5y - 1$ $\therefore 4x + 3y - 3x - 15y + 3 = 7$ $\Rightarrow x - 12y = 4$ $\Rightarrow x = 4 + 12y \text{ and } z = 4 + 12y + 5y - 1 = 3 + 17y$ $\therefore (x, y, z) = (4 + 12k, k, 3 + 17k)$ <p style="text-align: right;">(<math>\because</math> Assume <math>y = k</math>)</p> <p>Also, <math>7 \leq 7 + 30k \leq 77</math></p> $\Rightarrow 0 \leq 30k < 70$ $\Rightarrow 0 \leq k \leq 2.3 \Rightarrow k = 0, 1, 2$ <p>Thus, there are three possible solutions.</p>	
<b>63.</b>	<p>Let <math>f</math> and <math>g</math> be two functions defined by <math>f(x) = \begin{cases} x + 1, &amp; x &lt; 0 \\  x - 1 , &amp; x \geq 0 \end{cases}</math> and <math>g(x) = \begin{cases} x + 1, &amp; x &lt; 0 \\ 1, &amp; x \geq 0 \end{cases}</math> Then <math>(gof)(x)</math> is</p>	<b>A</b>
	Continuous everywhere but not differentiable exactly at one point	
	Not continuous at $x = -1$	
	Continuous everywhere but not differentiable at $x = 1$	
	Differentiable everywhere	
<b>Sol.</b>	<p>(a) : We have,</p> $f(x) = \begin{cases} x + 1, & x < 0 \\  x - 1 , & x \geq 0 \end{cases}$ <p>and <math>g(x) = \begin{cases} x + 1, &amp; x &lt; 0 \\ 1, &amp; x \geq 0 \end{cases}</math></p> $f(x) = \begin{cases} x + 1, & x < 0 \\ x - 1, & x \geq 1 \\ 1 - x, & 0 \leq x < 1 \end{cases}$ $\therefore (gof)(x) = \begin{cases} x + 2, & x < -1 \\ 1, & x \geq -1 \end{cases}$ <p><math>gof(x)</math> is not differentiable at <math>x = -1</math> but it is continuous everywhere.</p>	
<b>64.</b>	<p>If the function <math>g: (\infty, \infty) \rightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)</math> is given by <math>g(u) = 2 \tan^{-1}(e^u) - \frac{\pi}{2}</math>. Then, <math>g</math> is</p>	<b>C</b>
	Even and is strictly increasing in $(0, \infty)$	
	Odd and is strictly decreasing in $(-\infty, \infty)$	

	Odd and is strictly increasing in $(-\infty, \infty)$	
	Neither even nor odd but is strictly increasing in $(-\infty, \infty)$	
<b>Sol.</b>	<p>(c) Given, <math>g(u) = 2\tan^{-1}(e^u) - \frac{\pi}{2}</math> for <math>u \in (-\infty, \infty)</math></p> $g(-u) = 2\tan^{-1}(e^{-u}) - \frac{\pi}{2}$ $= 2(\cot^{-1}(e^u)) - \frac{\pi}{2}$ $[\because \tan^{-1}(1/x) = \cot^{-1} x]$ $= 2\left(\frac{\pi}{2} - \tan^{-1}(e^u)\right) - \frac{\pi}{2}$ $= \pi/2 - 2\tan^{-1}(e^u) = -g(u)$ <p><math>\therefore g(-u) = -g(u)</math>  <math>\Rightarrow g(u)</math> is an odd function.          We have, <math>g(u) = 2\tan^{-1}(e^u) - \pi/2</math></p> $g'(u) = \frac{2e^u}{1+e^{2u}}$ $g'(u) > 0, \forall x \in R \quad [\because e^u > 0]$ <p>So, <math>g(u)</math> is strictly increasing for all <math>x \in R</math>.</p>	
<b>65.</b>	<p>Let the slope of the line <math>45x + 5y + 3 = 0</math> be <math>27r_1 + \frac{9r_2}{2}</math> for some <math>r_1, r_2 \in R</math>.</p> <p>Then <math>\lim_{x \rightarrow 3} \left( \int_3^x \frac{8t^2}{\frac{3r_2x}{2} - r_2x^2 - r_1x^3 - 3x} dt \right)</math> is equal to _____.</p>	<b>A</b>
	12	
	15	
	18	
	20	
<b>Sol.</b>	<p>(12) : Slope of line <math>45x + 5y + 3 = 0</math> is i.e., <math>-9</math></p> <p><math>-9 = 27(-2) + \frac{9 \times 10}{2}</math></p> <p><math>\therefore r_1 = -2</math> and <math>r_2 = 10</math>, we have</p> $\lim_{x \rightarrow 3} \int_3^x \frac{8t^2}{15x - 10x^2 + 2x^3 - 3x} dt$ $= \lim_{x \rightarrow 3} \frac{1}{15x - 10x^2 + 2x^3 - 3x} \left[ \frac{8t^3}{3} \right]_3^x$ $= \lim_{x \rightarrow 3} \frac{1}{15x - 10x^2 + 2x^3 - 3x} \left[ \frac{8x^3}{3} - 72 \right]$ $= \lim_{x \rightarrow 3} \frac{8x^2}{15 - 20x + 6x^2 - 3}$ $= \frac{72}{15 - 60 + 54 - 3} = 12$	
<b>66.</b>	The area (in sq. units) of the region described by $\{(x, y): y^2 \leq 2x \text{ and } y \geq 4x - 1\}$ is	<b>B</b>
	11/12	
	9/32	
	8/9	
	11/32	

<b>Sol.</b>	<p>(b) Given, <math>y^2 \leq 2x</math>, <math>y \geq 4x - 1</math></p> <p><b>Point of intersection</b></p>  <p> <math>y^2 = 2x</math> and <math>y = 4x - 1</math>  <math>y = 2y^2 - 1</math>  <math>\Rightarrow 2y^2 - y - 1 = 0</math>  <math>\Rightarrow 2y^2 - 2y + y - 1 = 0</math>  <math>\Rightarrow 2y(y - 1) + 1(y - 1) = 0</math>  <math>\Rightarrow (y - 1)(2y + 1) = 0</math>  <math>\Rightarrow y = 1, -\frac{1}{2} \Rightarrow x = \frac{1}{2}, \frac{1}{8}</math> </p> <p><math>\therefore</math> Point of intersection  <math>\left(\frac{1}{2}, 1\right), \left(\frac{1}{8}, -\frac{1}{2}\right)</math></p> <p><math>\therefore</math> Required area <math>= \int_{-1/2}^1 (x_1 - x_2) dy</math></p> $= \int_{-1/2}^1 \left( \frac{y+1}{4} - \left( \frac{y^2}{2} \right) \right) dy$ $= \left[ \frac{(y+1)^2}{8} - \frac{y^3}{6} \right]_{-1/2}^1$ $= \left( \frac{4}{8} - \frac{1}{6} \right) - \left( \frac{1}{32} + \frac{1}{48} \right) = \left( \frac{2}{6} \right) - \left( \frac{5}{96} \right)$ $= \frac{32 - 5}{96} = \frac{27}{96} = \frac{9}{32}$	
<b>67.</b>	<p>If <math>y = y(x)</math> satisfies the differential equation <math>8\sqrt{x}(\sqrt{9 + \sqrt{x}})dy = \left( \sqrt{4 + \sqrt{9 + \sqrt{x}}} \right)^{-1} dx, x &gt; 0</math> and <math>y(0) = \sqrt{7}</math>, then <math>y(256)</math> equals to</p>	<b>B</b>
	16	
	3	
	9	
	80	
<b>Sol.</b>	<p>(b) <math>\frac{dy}{dx} = \frac{1}{8\sqrt{x}\sqrt{9 + \sqrt{x}}\sqrt{4 + \sqrt{9 + \sqrt{x}}}}</math></p> <p style="text-align: right;">(given)</p> <p><math>\Rightarrow y = \sqrt{4 + \sqrt{9 + \sqrt{x}}} + c</math></p> <p>Now, <math>y(0) = \sqrt{7} + c</math></p> <p><math>\Rightarrow c = 0</math></p> <p><math>\therefore y(256) = \sqrt{4 + \sqrt{9 + 16}}</math></p> <p style="text-align: center;"><math>= \sqrt{4 + 5} = 3</math></p>	
<b>68.</b>	<p>If the shortest distance between the lines <math>\frac{x+2}{2} = \frac{y+3}{3} = \frac{z-5}{4}</math> and <math>\frac{x-3}{1} = \frac{y-2}{-3} = \frac{z+4}{2}</math> is <math>\frac{38}{3\sqrt{5}}k</math>, and <math>\int_0^k [x^2] dx = \alpha - \sqrt{\alpha}</math>, where <math>[x]</math> denotes the greatest integer function, then <math>6\alpha^3</math> is equal to</p>	<b>B</b>
	50	
	48	
	55	
	60	

<p><b>Sol.</b></p>	<p>(48) : <math>L_1: \frac{x+2}{2} = \frac{y+3}{3} = \frac{z-5}{4}</math></p> <p><math>L_2: \frac{x-3}{1} = \frac{y-2}{-3} = \frac{z+4}{2}</math></p> <p><math>\therefore \vec{a}_1 = -2\hat{i} - 3\hat{j} + 5\hat{k}, \vec{a}_2 = 3\hat{i} + 2\hat{j} - 4\hat{k},</math></p> <p><math>\vec{b}_1 = 2\hat{i} + 3\hat{j} + 4\hat{k}</math> and <math>\vec{b}_2 = \hat{i} - 3\hat{j} + 2\hat{k}</math></p> <p>Now, <math>\vec{a}_2 - \vec{a}_1 = 5\hat{i} + 5\hat{j} - 9\hat{k}</math></p> <p>Shortest distance between the lines</p> $= \frac{ (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) }{ \vec{b}_1 \times \vec{b}_2 }$ <p>and <math> \vec{b}_1 \times \vec{b}_2  = \begin{vmatrix} \hat{i} &amp; \hat{j} &amp; \hat{k} \\ 2 &amp; 3 &amp; 4 \\ 1 &amp; -3 &amp; 2 \end{vmatrix}</math></p> $= \hat{i}(6+12) - \hat{j}(4-4) + \hat{k}(-6-3) = 18\hat{i} - 9\hat{k}$ $d = \frac{ (5\hat{i} + 5\hat{j} - 9\hat{k}) \cdot (18\hat{i} - 9\hat{k}) }{\sqrt{324 + 81}}$ $= \frac{ 90 + 81 }{9\sqrt{5}} = \frac{171}{9\sqrt{5}}$ <p>From the question, we get</p> $\frac{38}{3\sqrt{5}} k = \frac{171}{9\sqrt{5}} \Rightarrow k = \frac{3}{2}$ $\therefore \int_0^k [x^2] dx = \int_0^{\frac{3}{2}} [x^2] dx$ $= \int_0^1 0 dx + \int_1^{\sqrt{2}} 1 dx + \int_{\sqrt{2}}^{\frac{3}{2}} 2 dx$ $= 0 + (\sqrt{2} - 1) + 2\left(\frac{3}{2} - \sqrt{2}\right)$ $= \sqrt{2} - 1 + 3 - 2\sqrt{2} = 2 - \sqrt{2}$ $= \alpha - \sqrt{\alpha} \quad \text{(Given)}$ <p>Hence, <math>\alpha = 2</math></p> $\therefore 6\alpha^3 = 6 \times 8 = 48$	
<p><b>69.</b></p>	<p>Let S be the focus of the hyperbola <math>\frac{x^2}{3} - \frac{y^2}{5} = 1</math>, on the positive x-axis. Let C be the circle with its centre at <math>A(\sqrt{6}, \sqrt{5})</math> and passing through the point S. If O is the origin and SAB is a diameter of C, then the square of the area of the triangle OSB is equal to _____.</p>	<p><b>D</b></p>
	<p>20</p>	
	<p>30</p>	
	<p>35</p>	
	<p>40</p>	



<b>Sol.</b>	<p>(40) :</p> $\frac{x^3}{3} - \frac{y^2}{5} = 1 \Rightarrow a = \sqrt{3}, b = \sqrt{5}$ $\therefore e = \sqrt{1 + \frac{5}{3}} \Rightarrow e = \sqrt{\frac{8}{3}}$ $\therefore ae = \sqrt{8} = 2\sqrt{2} \quad \therefore S = (2\sqrt{2}, 0)$  <p>A is mid-point of BS  <math>\Rightarrow B(2\sqrt{6} - 2\sqrt{2}, 2\sqrt{5})</math>          Area of <math>\Delta(OSB)</math>,  <math display="block">A = \frac{1}{2} \begin{vmatrix} 0 &amp; 0 &amp; 1 \\ 2\sqrt{2} &amp; 0 &amp; 1 \\ 2\sqrt{6} - 2\sqrt{2} &amp; 2\sqrt{5} &amp; 1 \end{vmatrix} = 2\sqrt{10}</math>  <math>\Rightarrow A^2 = 40</math></p>	
<b>70.</b>	<p>Let <math>A = \{1, 2, 3, 4, 5\}</math> and <math>B = \{1, 2, 3, 4, 5, 6\}</math>. Then the number of functions <math>f: A \rightarrow B</math> satisfying <math>f(1) + f(2) = f(4) - 1</math> is equal to _____.</p> <p>350 250 360 150</p>	<b>C</b>
<b>Sol.</b>	<p>(360) : We have, <math>A = \{1, 2, 3, 4, 5\}</math> ;  <math>B = \{1, 2, 3, 4, 5, 6\}</math>          Also, <math>f(1) + f(2) = f(4) - 1</math>  <math>\Rightarrow f(1) + f(2) + 1 = f(4)</math>          Here, <math>f(4) \leq 6</math>  <math>f(1) + f(2) + 1 \leq 6</math>  <math>f(1) + f(2) \leq 5</math>          At <math>f(1) = 1</math>, then, <math>f(2) = 1, 2, 3, 4</math>  <math>\Rightarrow</math> 4 mappings, <math>f(1) = 2</math>, then, <math>f(2) = 1, 2, 3</math>  <math>\Rightarrow</math> 3 mappings, <math>f(1) = 3</math>, then, <math>f(2) = 1, 2</math>  <math>\Rightarrow</math> 2 mappings, <math>f(1) = 4</math>, then, <math>f(2) = 1</math>  <math>\Rightarrow</math> 1 mapping  <math>f(3)</math> and <math>f(5)</math> have 6 mappings each.  <math>\therefore</math> Total number of functions  <math>= 10 \times 6 \times 6 = 360</math></p>	
<b>71.</b>	<p>Let <math>S_n = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \dots</math> upto <math>n</math> terms. If the sum of the first six terms of an A.P. with first term of an A.P. with first term <math>-p</math> and common difference <math>p</math> is <math>\sqrt{2026S_{2025}}</math>, then the absolute difference between 20<sup>th</sup> and 15<sup>th</sup> terms of the A.P. is</p>	<b>25</b>

<b>Sol.</b>	<p>(c) : <math>S_n = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \dots</math> u p</p> <p><math>n</math>-terms</p> $= \frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \dots + \frac{1}{n(n-1)}$ $= \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \dots + \left(\frac{1}{n} - \frac{1}{n+1}\right)$ $= 1 - \frac{1}{n+1} = \frac{n}{n+1}$ $\therefore \sqrt{2026 S_{2025}} = \sqrt{2026 \times \frac{2025}{2025+1}} = 45$ <p><math>S_6 = 45</math>, where <math>S_6</math> denotes the sum of first 6 terms of an A.P.</p> $\therefore \frac{6}{2}(-2p + 5p) = 45 \Rightarrow p = 5$ $ a_{20} - a_{15}  =  (-5 + 19 \times 5) - (-5 + 14 \times 5) $ $=  90 - 65  = 25$	
<b>72.</b>	<p>A bag contains six balls of different colours. Two balls are drawn in succession with replacement. The probability that both the balls are of the same colour is <math>p</math>. Next four balls are drawn in succession with replacement and the probability that exactly three balls are of the same colours is <math>q</math>. If <math>p:q = m:n</math>, where <math>m</math> and <math>n</math> are coprime, then <math>m + n</math> is equal to.....</p>	<b>14</b>
<b>Sol.</b>	<p>(14) <math>\therefore</math> Bag contains balls of different colours.</p> <p><math>p</math> = Probability of both drawn balls are of same colour <math>= \frac{{}^6C_1}{{}^6P_2} = \frac{1}{6 \times 6} = \frac{1}{6}</math></p> <p><math>q</math> = Probability that exactly three from four balls are of the same colour</p> $= \frac{{}^6C_1 \times {}^5C_1 \times {}^4C_1}{{}^6P_4}$ $= \frac{6 \times 5 \times 4}{6 \times 6 \times 6 \times 6} = \frac{5}{54}$ $\frac{p}{q} = \frac{(1/6)}{(5/54)} = \frac{1}{6} \times \frac{54}{5} = \frac{9}{5} = \frac{m}{n}$ <p>So, <math>m = 9</math> and <math>n = 5</math></p> <p>Therefore, <math>m + n = 9 + 5 = 14</math></p>	
<b>73.</b>	<p>Let <math>[t]</math> denote the greatest integer less than or equal to <math>t</math>. Let <math>f: [0, \infty] \rightarrow R</math> be a function defined by <math>f(x) = \left[\frac{x}{2} + 3\right] - [\sqrt{x}]</math>. Let <math>S</math> be the set of all points in the interval <math>[0, 8]</math> at which <math>f</math> is not continuous. Then, <math>\sum_{a \in S} a</math> is equal to.....</p>	<b>17</b>
<b>Sol.</b>	<p>(17) <math>\left[\frac{x}{2} + 3\right]</math> is discontinuous at <math>x = 2, 4, 6, 8</math></p> <p><math>\sqrt{x}</math> is discontinuous at <math>x = 1, 4</math></p> <p><math>F(x)</math> is discontinuous at <math>x = 1, 2, 6, 8</math></p> $\Sigma a = 1 + 2 + 6 + 8 = 17$	
<b>74.</b>	<p>Let <math>\lambda \in Z, \vec{a} = \lambda \hat{i} + \hat{j} - \hat{k}</math> and <math>\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}</math>. Let <math>\vec{c}</math> be a vector such that <math>(\vec{a} + \vec{b} + \vec{c}) \times \vec{c} = 0, \vec{a} \cdot \vec{c} = -17</math> and <math>\vec{b} \cdot \vec{c} = -20</math>. Then <math> \vec{c} \times (\lambda \hat{i} + \hat{j} + \hat{k}) ^2</math> is</p>	<b>46</b>

<p><b>Sol.</b></p>	<p>(a) : As <math>(\vec{a} + \vec{b} + \vec{c}) \times \vec{c} = 0</math></p> <p><math>\Rightarrow (\vec{a} + \vec{b}) \times \vec{c} = 0 \Rightarrow \vec{c} = m((\lambda + 3)\hat{i} - \hat{j})</math></p> <p>As <math>\vec{a} \cdot \vec{c} = -17</math></p> <p><math>m(\lambda(\lambda + 3) - 1) = -17</math></p> <p>and <math>\vec{b} \cdot \vec{c} = -20</math></p> <p><math>m(3(\lambda + 3) + 2) = -20</math></p> <p>From (1) and (2), <math>\frac{\lambda^2 + 3\lambda - 1}{3\lambda + 11} = \frac{17}{20}</math></p> <p><math>20\lambda^2 + 60\lambda - 20 = 51\lambda + 187</math></p> <p><math>\Rightarrow 20\lambda^2 + 9\lambda - 207 = 0</math></p> <p><math>\Rightarrow (20\lambda + 69)(\lambda - 3) = 0</math></p> <p><math>\Rightarrow \lambda = 3</math> (as <math>\lambda \in \mathbb{Z}</math>)</p> <p><math>m = -1</math> (from (1)) <math>\Rightarrow \vec{c} = -(6\hat{i} + \hat{j})</math></p> <p>Now, <math>\vec{c} \times (\lambda\hat{i} + \hat{j} + \hat{k}) = \begin{vmatrix} \hat{i} &amp; \hat{j} &amp; \hat{k} \\ -6 &amp; 0 &amp; -1 \\ 3 &amp; 1 &amp; 1 \end{vmatrix} = \hat{i} + 3\hat{j} - \hat{k}</math></p> <p>Now, <math> \vec{c} \times (\lambda\hat{i} + \hat{j} + \hat{k}) ^2 = 46</math></p>	
<p><b>5.</b></p>	<p>Let <math>\alpha = \frac{1}{\sin 60^\circ \sin 61^\circ} + \frac{1}{\sin 62^\circ \sin 63^\circ} + \dots + \frac{1}{\sin 118^\circ \sin 119^\circ}</math>.</p> <p>Then, the value of <math>\left(\frac{\operatorname{cosec} 1^\circ}{\alpha}\right)^2</math> is .....</p>	<p><b>3</b></p>
<p><b>Sol.</b></p>	<p>(3)</p> <p><math>\alpha = \frac{1}{\sin 60^\circ \cdot \sin 61^\circ} + \frac{1}{\sin 62^\circ \cdot \sin 63^\circ} + \dots + \frac{1}{\sin 118^\circ \cdot \sin 119^\circ}</math></p> <p><math>\Rightarrow \alpha = \frac{1}{\sin 1^\circ} \left( \frac{\sin(61^\circ - 60^\circ)}{\sin 60^\circ \cdot \sin 61^\circ} + \frac{\sin(63^\circ - 62^\circ)}{\sin 62^\circ \cdot \sin 63^\circ} + \dots + \frac{\sin(119^\circ - 118^\circ)}{\sin 118^\circ \cdot \sin 119^\circ} \right)</math></p> <p><math>\Rightarrow \alpha = \frac{1}{\sin 1^\circ} (\cot 60^\circ - \cot 61^\circ + \cot 62^\circ - \cot 63^\circ + \dots + \cot 118^\circ - \cot 119^\circ)</math></p> <p><math>\Rightarrow \alpha = \frac{1}{\sin 1^\circ} (\cot 60^\circ - \cot 61^\circ + \cot 62^\circ - \cot 63^\circ + \dots - \cot 62^\circ + \cot 61^\circ)</math></p> <p><math>\Rightarrow \alpha = \frac{1}{\sin 1^\circ} \times \frac{1}{\sqrt{3}}</math></p> <p><math>\Rightarrow 3 = \left(\frac{\operatorname{cosec} 1^\circ}{\alpha}\right)^2</math></p>	