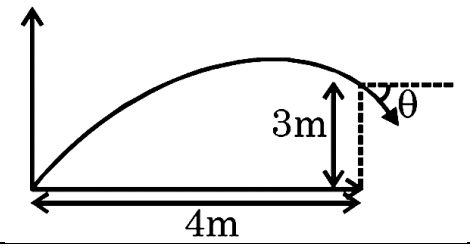
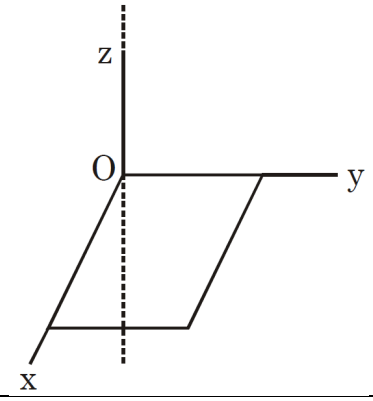
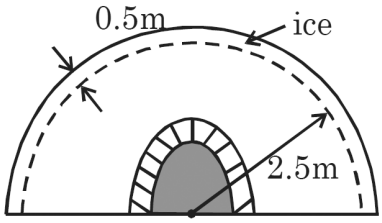
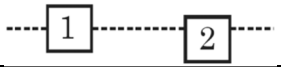
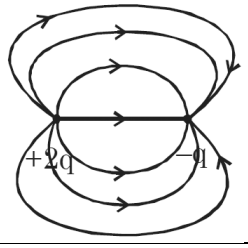
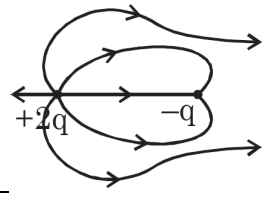
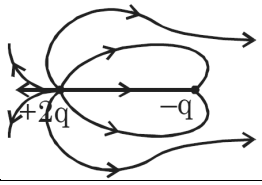
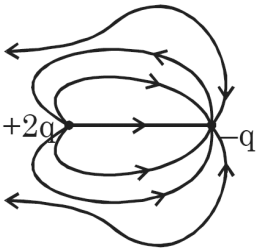
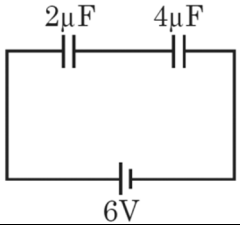
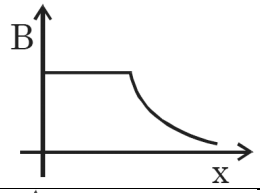
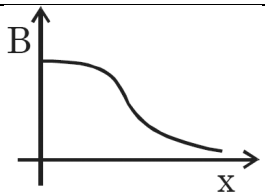
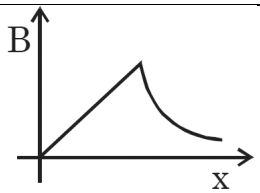
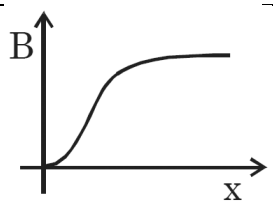
1.	Which of the following is a CORRECT statement?	**C**
	$2.3056 + 10.138 - 7.4671 = 4.9765$	
	$2.38 \times 1.0 = 2.38$	
	$\frac{8.05}{3.1} = 2.6$	
	$(1.11 - 0.1) \times 9.0 = 9.0$	
Sol.	(1) $2.3056 \rightarrow$ 4 decimals $10.138 \rightarrow$ 3 decimals $-7.4671 \rightarrow$ 4 decimals ——————— 4.9765 Answer should have 3 decimals. (2) $2.38 \times 1.0 = 2.38 \rightarrow$ answer should have 2 significant digits (3) $\frac{8.05}{3.1} = 2.59 \approx 2.6$ answer should have 2SD (4) $1.11 - 0.1 = 1.01 \approx 1.0$ but is is an intermediate step so we keep 1 digit extra. $1.01 \times 9.0 = 9.09 \approx 9.1 \rightarrow$ both 1.01 and 9.0 have 2SD	
2.	A projectile travels at θ below horizontal 1 sec after projection as shown. What is the value of θ?  	**B**
	$\tan^{-1}(2)$	
	$\tan^{-1}\left(\frac{1}{2}\right)$	
	37^{o}	
	53^{o}	

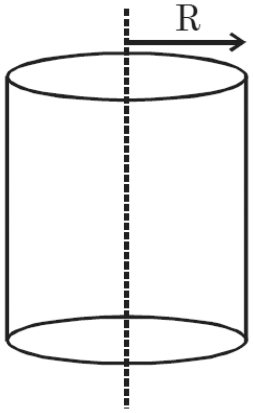
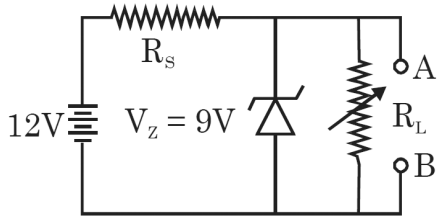
Sol.	$\tan\theta = \frac{v_y}{v_x} = \frac{u_y - gt}{u_x}$ $u_x = \frac{4}{1} m/s$ $3 = u_y \times 1 - \frac{1}{2} \times 10 \times 1^2$ $u_y = 8$ m/s $\Rightarrow \tan\theta = \left\|\frac{8-10}{4}\right\| = \left\|-\frac{1}{2}\right\|$	
3.	An electron in a research apparatus follows a circular path. On the electron's first circuit of the apparatus, its speed is v_0 and the radius of its circular path is R. Each time it makes one circuit, it passes a short region where it receives a "kick" and gains an additional speed of $\frac{v_0}{100}$. The electron follows a circular path such that the magnitude of its acceleration is always the same. What is the radius of the circular path after the electron has received 10 kicks ?	**C**
	$\frac{R}{1.1}$	
	1.1 R	
	1.2 R	
	$\frac{R}{1.21}$	
Sol.	$a = \frac{v_0^2}{R}$ after 10 kicks, $v = v_0 + \frac{10v_0}{100} = 1.1v_0$ $a = \frac{(1.1v_0)^2}{R_1} = \frac{v_0^2}{R}$ $\Rightarrow R_1 = 1.21$ R	
4.	A square of mass m is made of thin wire with sides of length a. Calculate the rotational inertia about the z-axis.  	**B**

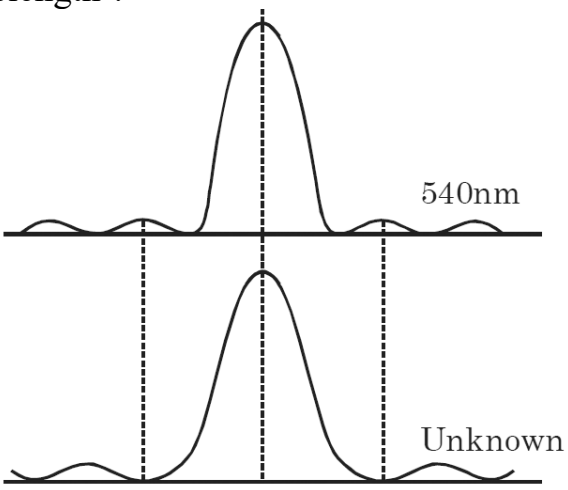
	$\frac{7ma^2}{6}$	
	$\frac{5ma^2}{6}$	
	$\frac{7ma^2}{12}$	
	$\frac{5ma^2}{12}$	
Sol.	$I = \frac{m}{4}\times\frac{a^2}{3}\times 2 + \frac{m}{4}\times\left(\frac{a^2}{12}+\left(\frac{a}{2}\right)^2+a^2\right)\times 2$ $= \frac{ma^2}{6}+\frac{m}{2}\left(\frac{a^2+3a^2+12a^2}{12}\right)$ $= \frac{ma^2}{6}+\frac{2}{3}ma^2 = \frac{ma^2+4ma^2}{6} = \frac{5ma^2}{6}$	
5.	If we want that a simple pendulum oscillates with same time period in summer as it did in winter, what changes to the simple pendulum should we do during summer ?	**B**
	Increase the length	
	Decrease the length	
	Increase mass of bob	
	Decrease mass of bob	
Sol.	$T = 2\pi\sqrt{\frac{\ell}{g}}$ In summer, length will increase due to expansion, hence for same time period we have to decrease ℓ in summer.	
6.	An igloo, a hemispherical enclosure built of ice (k = 1.67 J/m-s°C), has an inner radius of 2.50 m. The thickness of the ice is 0.5 m. At what rate must thermal energy be generated to maintain the air inside the igloo at 5°C when the outside temperature is –40°C ? Ignore all thermal energy losses through the ground or by air currents. 	**A**
	2.25 πkW	
	4.5 πkW	
	1.25 πkW	
	5 πkW	

Sol.	$\frac{dH}{dt} = -2\pi x^2 k \frac{dT}{dx}$ $\Rightarrow \frac{dH}{dt} \times \int_{2.5}^{3} \frac{dx}{x^2} = 2\pi k \int_{-40}^{5} dT$ $\frac{dH}{dt} = \frac{2\pi \times k \times 45}{\frac{1}{2.5} - \frac{1}{3}} = 2.25\pi kW$	
7.	Two blocks of same mass but different density ρ_1 & ρ_2 are floating in water. Block–2 has higher fraction of its volume submerged in water. If buoyant force on them are B_1 & B_2 	**D**
	$B_1 > B_2, \rho_1 > \rho_2$	
	$B_1 < B_2, \rho_1 < \rho_2$	
	$B_1 = B_2, \rho_1 > \rho_2$	
	$B_1 = B_2, \rho_1 < \rho_2$	
Sol.	$B = mg \Rightarrow B_1 = B_2$ $\rho_1 V_1 g = B = \rho_w v_{LD} g$ $\frac{\rho_1}{\rho_W} = \frac{V_{LD}}{V_1} = f_1$ $\frac{\rho_2}{\rho_W} = f_2$ $\Rightarrow \rho_2 > \rho_1$	
8.	A plastic ball is rising in water with terminal speed v. If we use a different plastic whose density is double that of the original ball but still less than density of water, keeping the radius same, the terminal speed would be :	**D**
	Doubled	
	Half	
	More than original value	
	Less than original value	
Sol.	$v = \frac{2}{9} \frac{r^2 g(\rho - \sigma)}{\eta}$ $v' = \frac{2}{9} r^2 g \frac{(\rho - 2\sigma)}{\eta}$	
9.	The electric field line pattern for charges +2q & – q are shown. Select the most appropriate diagram :	**B**
		

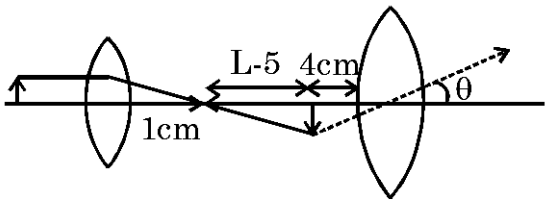
		
		
		
Sol.	Field lines are proportional to quantity of charge & they originate at positive charge & terminates at –ve charge or at ∞	
10.	In half deflection method, the resistance of resistance box in series with the galvanometer is 10,000 Ω. When the galvanometer is shunted with 10 Ω resistance, its reading drops to half the initial value. The resistance of galvanometer is	**B**
	5 Ω	
	10 Ω	
	20 Ω	
	25 Ω	
Sol.	$i = \frac{v}{R+G}$ $\frac{i}{2} = \frac{v}{\frac{SG}{S+G}+R} \times \frac{S}{S+G}$ $\Rightarrow \frac{2S}{SG+RG+RS} = \frac{1}{R+G}$ $SG + SR = RG$ $\frac{SR}{R-S} = G$ $R >> S$ $\Rightarrow G \approx S$	
11.	Two air filled capacitors are connected as shown. The left capacitor is suddenly filled with a dielectric of constant K. The work done by battery after that is 24μJ. Dielectric constant K is 	**A**

	2	
	3	
	4	
	6	
Sol.	$WD = \Delta QV$ $\Rightarrow \Delta Q = 4\mu C$ $Q_i = \frac{2\times 4}{2+4}\times 6 = 8\mu C$ $Q_f = 12\mu C = C_{eq} \times 6$ $C_{eq} = 2\ \mu\ F \Rightarrow C_1 = C_2 = 4\ \mu F$ $k = \frac{C_1}{C_0} = 2$	
12.	An infinite cylindrical current carrying wire carries a current that is uniformly distributed over its cross section. The magnetic field as a function of distance from it's axis is	**C**
		
		
		
		

Sol.	 Applying Ampere's Law For inside $B2\pi r = \mu_0 J\pi r^2$ $\Rightarrow B \propto r$ For outside $B2\pi r = \mu_0 J\pi R^2$ $\Rightarrow B \propto \frac{1}{r}$	
13.	Mass of Bromine, Selenium and Krypton are given as : $M^{80}_{Br} = 79.918528$ u, $M(^{80}Kr) = 79.916376$ u, $M(^{80}Se) = 79.916521$u Which of the following β decays are possible? (i) ${}^{80}_{35}Br \rightarrow {}^{80}_{36}Kr + {}^{0}_{-1}e + \bar{\nu}$ (ii) ${}^{80}_{35}Br \rightarrow {}^{80}_{34}Se + {}^{0}_{-1}e + \nu$ (iii) ${}^{80}_{35}Br + {}^{0}_{-1}e \rightarrow {}^{80}_{34}Se + \nu$	**A**
	all 3	
	(i) & (iii) only	
	(ii) & (iii) only	
	(i) & (ii) only	
Sol.	$Q_{(i)} = (M_{Br} - M_{Kr})\, c^2 > 0$ $Q_{(ii)} = (M_{Br} - M_{Se} - 2M_e)\, c^2 > 0$ $Q_{(ii)} = (M_{Br} - M_{Se})\, c^2 > 0$	
14.	Drawing power from a 12V car battery, a 9V stabilized DC voltage is required to power a car stereo system, attached to the terminals A and B, as shown in the figure. If a Zener diode with ratings, V_z = 9V and P_{max} = 0.27 W, is connected as shown in the figure, for the above purpose, the minimum series resistance R_S must be 	**C**
	111Ω	

	103Ω	
	100Ω	
	97Ω	
Sol.	$i_{max} = \frac{P_{max}}{V_z} = \frac{3}{100}$ $\therefore\ R_{s_{min}} = \frac{V_{R_S}}{i_{max}}$ $= \frac{3}{3/100} = 100\Omega$	
15.	When light of wavelength 540 nm passes through a single slit of unknown width, the diffraction pattern displays a second maximum where the first minimum of light of an unknown wavelength had been observed to fall. What is the unknown wavelength ?  	**D**
	360 nm	
	270 nm	
	180 nm	
	810 nm	
Sol.	$\frac{\lambda_1}{w} = \frac{3\times 540}{2w}$ $\lambda_1 = 810\ \text{nm}$	
16.	You are trying to construct a compound microscope given two lenses with focal lengths f_1 = 1.0 cm and f_2 = 4.0 cm. How far apart (in cm) should you place the lenses in order to obtain an angular magnification of 60 ?	**B**
	10.60	
	14.60	
	7.30	
	3.60	

Sol.



for better resolution, smaller focal length should be objective.

$h_0 = 1$ cm $f_e = 4$cm.

$$Q_0 = \frac{h_0}{25}$$

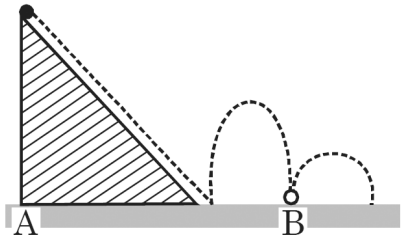
$$Q = \frac{60h_0}{25} = \frac{h_I}{4}$$

$$\frac{L-5}{1} = \frac{h_I}{h_0} = \frac{240}{25}$$

$L = 14.6$ cm

17. A small elastic ball of mass m is placed at the apex of a 45° inclined plane as shown in the figure below. The ball is allowed to slip without friction down the plane (along the dotted line), hit the ground (as shown) and bounce along it. If the height of the inclined plane is h and the coefficient of restitution between the ball and the ground is 0.5, then the distance AB, as marked on the figure, will be N × h, where N is

B



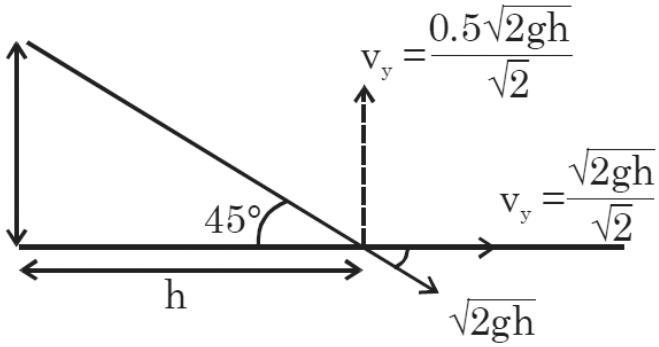
5

2

3

1

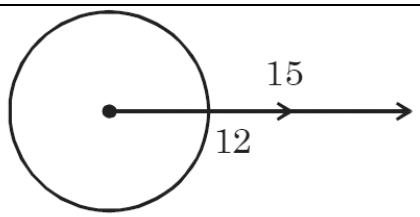
Sol.

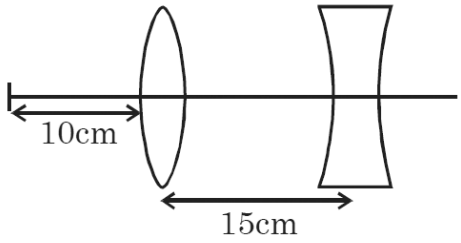
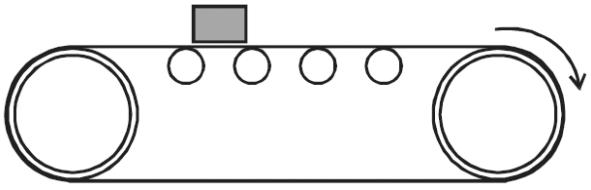


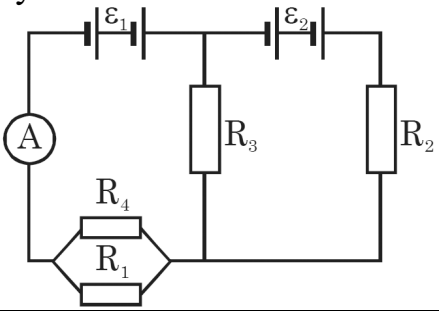
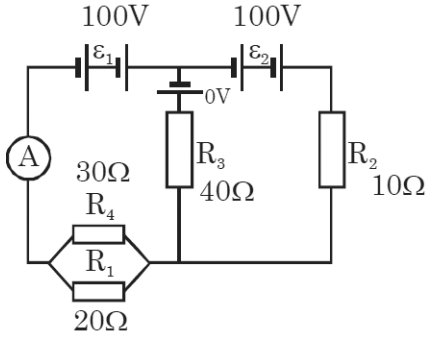
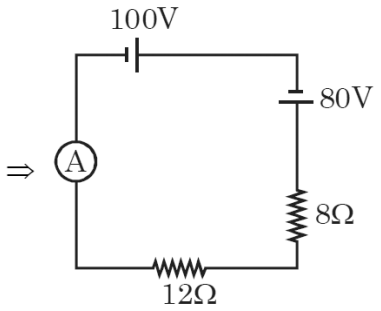
$$R = u_x \times T$$

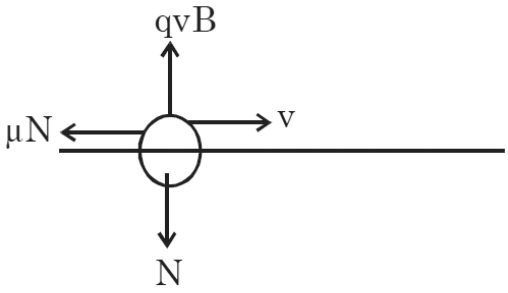
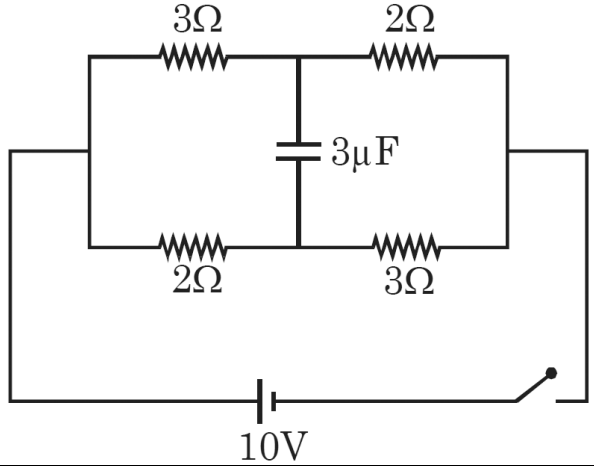
$$= \sqrt{gh} \times \frac{0.5\sqrt{gh} \times 2}{g} = h$$

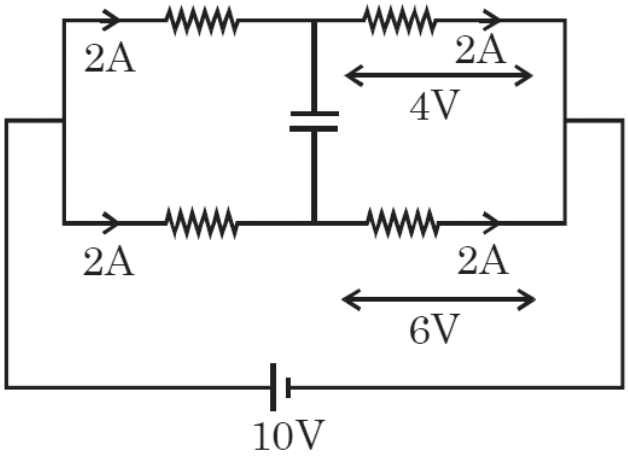
$$\Rightarrow AB = h + h = 2h$$

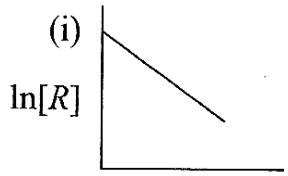
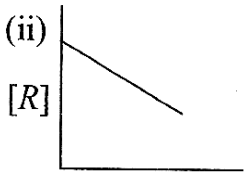
18.	An electron moves in the field produced by a charged sphere of radius 10 cm along the radius connecting the points separated by 12 and 15 cm from the centre of the sphere. The velocity of the electron changes thereby from 2×10^5 to 2×10^6 m/s. Determine the surface charge (in nC/m^2) density of the sphere. (Mass of electron = 9.1×10^{-31} kg)	**B**
	5.91	
	5.97	
	6.13	
	11.94	
Sol.	 $\frac{kQe}{0.12}+\frac{1}{2}mu^2=\frac{kQe}{0.15}+\frac{1}{2}mu^2$ $\frac{kQe\times 0.03}{0.12\times 0.15}=\frac{1}{2}m\times\left(v^2-u^2\right)$ $\frac{9\times10^9\times Q\times1.6\times10^{-19}}{0.12\times0.15}\times0.03=\frac{1}{2}\times4\times10^{10}\times99$ $Q=\frac{9.1\times99\times10^{-11}}{12}$ $=\frac{75.075\times10^{-12}}{4\pi\times(0.1)^2}=5.97\ \text{nC/m}^2$	
19.	You are given a converging lens with equal radii of curvature and a diverging lens with the same radii of curvature as those of the converging lens. The lenses are made of material with n = 1.50, and the radii of curvature are all 35 cm. They are placed at opposite ends of a tube 15 cm long, and the nearer lens which is converging is 10 cm from an object. What is the location of the image that results from the two refractions? Write the final distance (in cm) from diverging lens.	**A**
	15.85	
	3.75	
	31.70	
	20.35	

Sol.	 $\frac{1}{u}+\frac{1}{-10}=\frac{1}{35}$ $\frac{1}{u}=\frac{1}{35}-\frac{1}{10}=\frac{2-7}{70}=\frac{-1}{14}$ $\frac{1}{u}+\frac{1}{+29}=\frac{-1}{35}$ $\frac{1}{u}=\frac{-1}{35}-\frac{1}{29}=\frac{-64}{35\times29}$ $u=\frac{35\times29}{64}=15.85 \text{ to } 15.86$	
20.	The new white belt of a long horizontal conveyor is moving with a constant speed v = 3.0 m/s. A small block of carbon is placed on the belt with zero initial velocity relative to the ground. The block will slip a bit before moving with the belt, leaving a black mark on the belt (figure). How long is that mark (in m) if the coefficient of kinetic friction between the carbon block and the belt is 0.20 and the coefficient of static friction is 0.30? 	**C**
	5.50	
	3.25	
	2.25	
	7.25	
Sol.	$0^2=v^2-2\mu_k g S_{rel}$ $\frac{9}{2\times2}=S_{rel}=2.25m$	
21.	A wave given by equation y = 1mm sin $\left(\frac{nx}{30}-5nt\right)$ is produced in a string 100m long of mass 1kg. What is the tension (in N) in string ? x is in m & t in sec.	**225**
Sol.	$v=\frac{\omega}{k}=\frac{5\pi\times30}{\pi}=150=\sqrt{\frac{T}{\frac{1}{100}}}$ $\Rightarrow T=\frac{2250000}{100}=225N$	

22.	In the circuit in figure we have $\varepsilon_1 = \varepsilon_2 = 100V$, $R_1 = 20\Omega$, $R_2 = 10\Omega$, $R_3 = 40\Omega$ and $R_4 = 30\Omega$. Find the reading of the ammeter (in A). Disregard the resistance of the battery and the ammeter. 	**9**
Sol.	 $\varepsilon_{eq} = \dfrac{\frac{0}{40}+\frac{100}{10}}{\frac{1}{40}+\frac{1}{10}} = \dfrac{\frac{10}{5}}{\frac{}{40}} = 80V$  $R_{eq} = \dfrac{400}{50} = 8\Omega$ $R'_{eq} = \dfrac{30\times 20}{30+20} = \dfrac{600}{50} = 12$ $i = \dfrac{180}{12+8} = 9A$	
23.	A calorimeter consists of 400 g of water at 24°C. A 500 g piece of copper at 100°C is thrown into the water and the equilibrium temperature is found to be 36.5°C. What is the molar heat capacity of copper in cal/mol k, given that 1 mol of copper has a mass of 63.5g ? Neglect the heat capacity of the container.	**10**
Sol.	$400 \times 1 \times 12.5 = 500 \times 5 \times (100 - 36.5)$ $S = \dfrac{10}{63.5}$ cal/gm °C	

	$1\text{gm} = \frac{1}{63.5}\text{mol}$ $S = 10\ \text{cal/mol}\ ^\circ\text{C}$	
24.	In gravity free space, a bead of charge 1μC and mass 3 mg is threaded on a rough rod of friction coefficient μ = 0.3. A magnetic field of magnitude 0.2 T exists perpendicular to the rod. The bead is projected along the rod with a speed of 4m/s. How much distance (in m) will the bead cover before coming to rest?	**200**
	 $N = qvB$ $-\mu qvB = \frac{mdv}{dt} = mv\frac{dv}{dS}$ $\mu qBS = mv$ $S = \frac{3\times10^{-6}\times4}{0.3\times10^{-6}\times0.2} = 200\text{m}$	
25.	Initially the capacitor is uncharged. What is the steady state charge on it (in μC) after the switch is closed? 	**6**

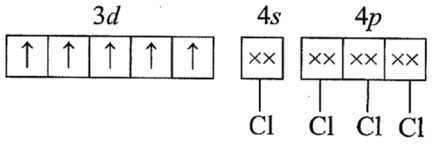
Sol.	 $Pd = 6 - 4 = 2\ V \Rightarrow Q = CV = 6\mu C$	
26.	Consider the following reaction. Black coloured solution (A) $\xrightarrow[\Delta]{KNO_3+KOH}$ Green colour solution (B) $\xrightarrow{CO_2}$ (C) + (A) Purple Pink compound(C) is decolorised by Fe^{2+}. The compounds A, B and C are	**A**
	MnO_2, K_2MnO_4, $KMnO_4$	
	MnO_2, $KMnO_4$, K_2MnO_4	
	$KMnO_4$, MnO_2, K_2MnO_4	
	K_2MnO_4, MnO_2, $KMnO_4$	
Sol.	Black colour compound is MnO_2(A). When it is fused with KOH and KNO_3, potassium manganate (K_2MnO_4) is formed. It is indicated by its green colour. In acidic medium, K_2MnO_4 is unstable and disproportionates. $\underset{(A)}{MnO_2} + KOH + KNO_3 \xrightarrow{\Delta} \underset{\text{Green colour } (B)}{K_2MnO_4}$ $CO_2 + H_2O \rightleftharpoons H_2CO_3 \rightleftharpoons H^+ + HCO_3^-$ $3MnO_4^{2-} + 4H^+ \longrightarrow \underset{(A)}{MnO_2} + \underset{(C)\,\text{Purple}}{2MnO_4^-} + 2H_2O$	
27.	A solution of $Ni(NO_3)_2$ is electrolysed between platinum electrodes using 0.1 faraday electricity. How many moles of Ni will be deposited at cathode?	**A**
	0.05	
	0.10	
	0.20	
	0.40	
Sol.	$Ni^{2+} + 2e^- \longrightarrow Ni$ $2 \times 96500\,C$ or 2F deposits 1 mole of Ni $\therefore$ 0.1F will deposit $= \frac{1}{2} \times 0.1$ mole of Ni $= \frac{0.1}{2} = 0.05$ mole	
28.	The given plots represents the variation of the concentration of a reactant R with time for two different reactions (i) and (ii). The respective orders of the reactions are:	**A**

	(i)  (ii) 	
	1, 0	
	2, 0	
	1, 2	
	2, 2	
Sol.	In graph (i), In [R] *vs* time is linear with positive intercept and negative slope. Hence, it is Ist order. In graph (ii), [Reactant] *vs* time is linear with positive intercept and negative slope. Hence, it is zero order.	
29.	An organic compound contains N, O, C and H. Which of the following methods can not be used for quantitative estimation of elements in the given organic compound?	**B**
	Duma's method	
	Carius method	
	Kjeldahl's method	
	Liebig method	
Sol.	Quantitative detection of elements The list of elements and their methods for quantitative analysis are given below as **Element** — **Methods of quantitative analysis** C — Liebig method H — Liebig method N — Duma's method, Kjeldahl's method Halogen, P — Carius method Since, the above compound does not contain any halogen or phosphorus. So, Carius method cannot be used for the given organic compound.	
30.	Given below are two statements. **Statement-I:** $[FeCl_4]^-$ has higher spin – only magnetic moment than $[Co(en)(NH_3)_2Cl_2]^+$. **Statement-II:** The cobalt ion in $[Co(en)(NH_3)_2Cl_2]^+$ has $sp^3\ d^2$ – hybridisation. In the light of the above statements choose the correct answer from the options given below.	**C**
	Both Statement I and Statement II are correct.	
	Both Statement I and Statement II are incorrect.	
	Statement I is correct but Statement II is incorrect.	
	Statement I is incorrect but Statement II is correct.	

Sol.

In $[FeCl_4]^-$: Cl^- is a weak ligand. So, it will not cause pairing

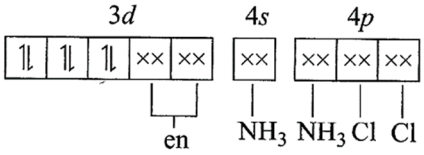
$Fe^{3+} = [Ar]3d^5$

$[FeCl_4]^- =$ 

Hybridisation : sp No. of unpaired electrons $(n) = 5$

Spin only magnetic moment $= \sqrt{n(n+2)} = \sqrt{5(5+2)} = \sqrt{35}$ BM

$[Co(en)(NH_3)_2Cl_2]^+$ $Co^{3+} = [Ar]3d^6$ $[Co(en)(NH_3)_2Cl_2]^+ =$



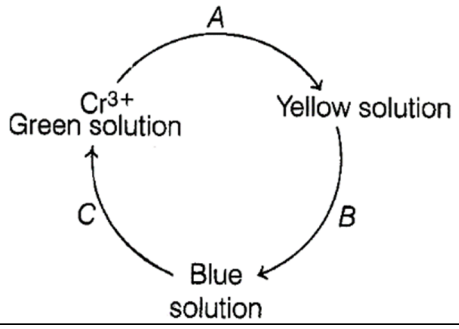
Hybridisation : d^2sp^3

Number of unpaired electron $(n) = 0$

Spin-only magnetic moment $(\mu) = \sqrt{n(n+2)} = 0$

Hence, $[FeCl_4]^-$ has higher spin only magnetic moment than $[Co(en)(NH_3)_2Cl_2]^+$ and Co^{3+} in $[Co(en)(NH_3)_2Cl_2]^+$ has d^2sp^3-hybridisation.

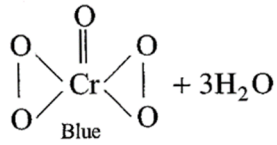
31. Identify A, B and C in the following sequence. **A**



	Alkaline H_2O_2, acidified H_2O_2, on standing
	Alkaline O_3, acidified O_3, Zn/HCl
	Acidified H_2O_2, alkaline H_2O_2, heat
	Alkaline O_3, heat, NH_4OH

Sol.

(a) (A) $\underset{\text{Green}}{2Cr^{3+}} + \underbrace{10OH^- + 3H_2O_2}_{A} \longrightarrow \underset{\text{Yellow}}{2CrO_4^{2-}} + 8H_2O$

(B) $CrO_4^{2-} + \underbrace{2H_2O_2 + 2H^+}_{B} \longrightarrow$  $+ 3H_2O$

In aqueous solution, CrO_5 is unstable and it further decomposes.

(C) $2\,CrO_5 \longrightarrow \underset{\text{(Amphoteric)}}{Cr_2O_3} + \frac{7}{2}O_2$

$Cr_2O_3 + 3\,H_2SO_4 \longrightarrow Cr_2(SO_4)_3 + 3\,H_2O$

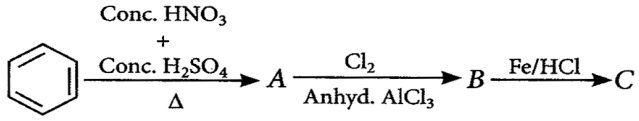
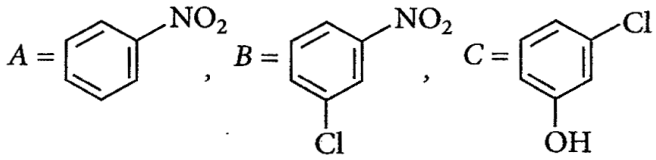
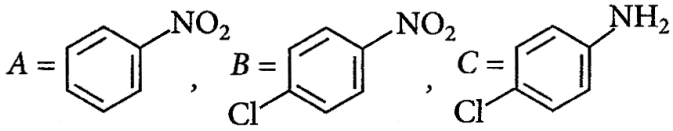
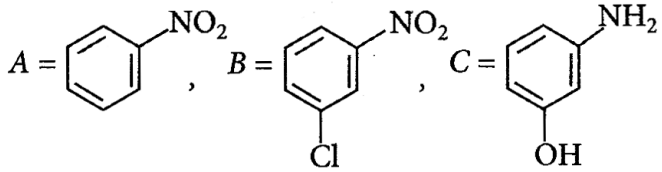
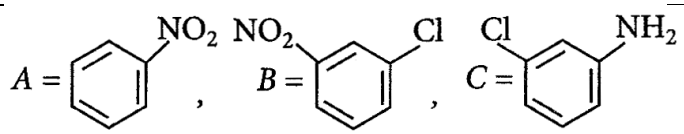
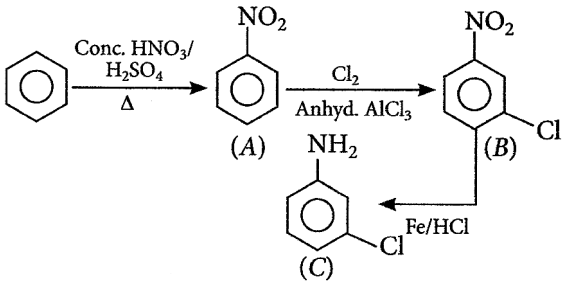
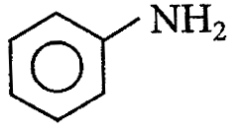
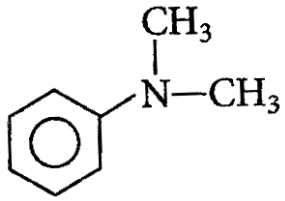
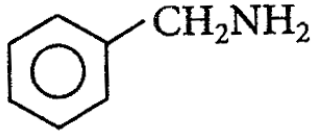
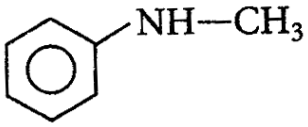
32. Calculate $\Delta S_{universe}$ for following chemical reaction **A**

C (graphite) + $2H_2(g) \longrightarrow CH_4(g)$;

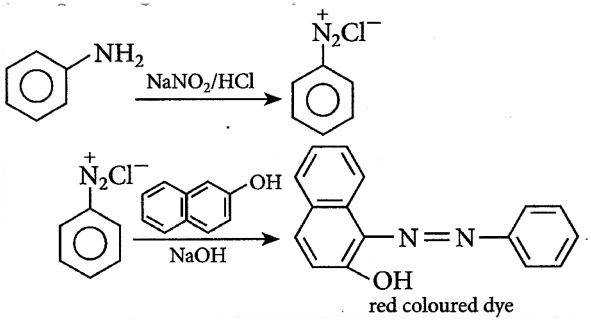
$\Delta_f H° = -74.81$ kJ at 298 K.

The standard entropies of C (graphite), $H_2(g)$ and $CH_4(g)$ are 5.740, 130.684

	and 186.264 J/K-mol, respectively.	
	$170.1\ \frac{J}{K}$	
	$125\ \frac{J}{K}$	
	$212\ \frac{J}{K}$	
	$138.2\ \frac{J}{K}$	
Sol.	$\Delta S_{Reaction} = [186.264] - [5.740 + 2 \times 130.684)]$ or $\Delta S_{System} = -80.844$ J/K $\Delta S_{Surrounding} = +\frac{\Delta H}{T} = \frac{74.81 \times 10^3}{298} = 251$ J/K $\Delta S_{Universe} = -80.844 + 251 = 170.1$ J/K	
33.	The minimum mass of NaBr which should be added in 200 ml of 0.0004M – $AgNO_3$ solution just to start precipitation of AgBr.K_{sp} of AgBr = 4×10^{-13}. (Br = 80)	**C**
	1.0×10^{-9} g	
	2×10^{-10} g	
	2.06×10^{-8} g	
	1.03×10^{-7} g	
Sol.	$\left[Br^-\right] = \frac{4 \times 10^{-13}}{4 \times 10^{-4}} = 10^{-9} M$ Mass of $NaBr$ added of $200\,ml = 103 \times \frac{10^{-9}}{1000} \times 200$ $= 2.06 \times 10^{-8} g$	
34.	Match the organic compounds in column- I with the Lassaigne's test results in column-II appropriately **Column-I** / **Column-II** (A) Aniline — (i) Red colour with $FeCl_3$ (B) Benzenesulphonic acid — (ii) Violet colour with sodium nitroprusside (C) Thiourea — (iii) Blue colour with hot and acidic solution of $FeSO_4$	**B**
	(A)-(ii); (B)-(i); (C)-(iii)	
	(A)-(iii); (B)-(ii); (C)-(i)	
	(B)-(ii); (B)-(iii); (C)-(i)	
	(C)-(iii); (B)-(i); (C)-(ii)	
35.	Number of cyclic tripeptide formed with 2 amino acids A and B is	**B**
	2	
	4	
	3	
	5	

Sol.	AAA, BBB, ABA, BAB are 4 possible cyclic structures.	
36.	Identify correct A, B and C in the reaction sequence given below. 	**D**
		
		
		
		
Sol.		
37.	The diazonium salt of which of the following compounds will form a coloured dye on reaction with β-naphthol in NaOH?	**A**
		
		
		
		

Sol. Benzene diazonium salt, when coupled with β-naphthol gives an azodye of red colour.



38. Match List-I with List-II. **B**

	List I (Reaction)		List II (Reagents)
A.	Hoffmann Degradation	I.	Conc. KOH, Δ
B.	Clemmensen Reduction	II.	$CHCl_3$, $NaOH/H_3O^+$
C.	Cannizzaro Reaction	III.	Br_2, NaOH
D.	Reimer-Tiemann Reaction	IV.	Zn-Hg/HCl

Choose the correct answer from the options given below.

(A) – III, (B) – IV, (C) – II, (D) – I

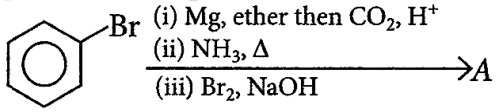
(A) – III, (B) – IV, (C) – I, (D) – II

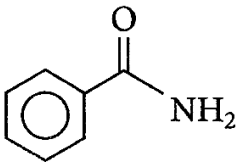
(A) – II, (B) – IV, (C) – I, (D) – III

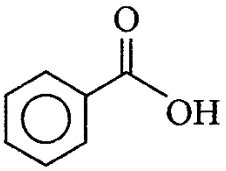
(A) – II, (B) – I, (C) – III, (D) – IV

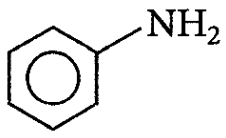
Sol. Hoffmann degradation – Br_2, NaOH Clemmensen reduction – Zn-Hg/HCl Cannizzaro reaction – conc. KOH, Δ Reimer-Tiemann reaction – $CHCl_3$, $NaOH/H_3O^+$

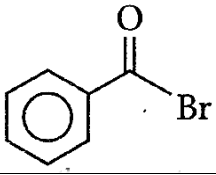
39. The final product A, formed in the following multistep reaction sequence is **C**



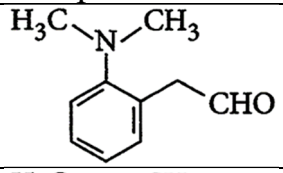
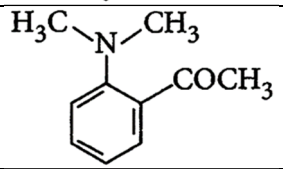
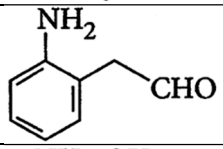
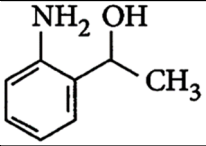


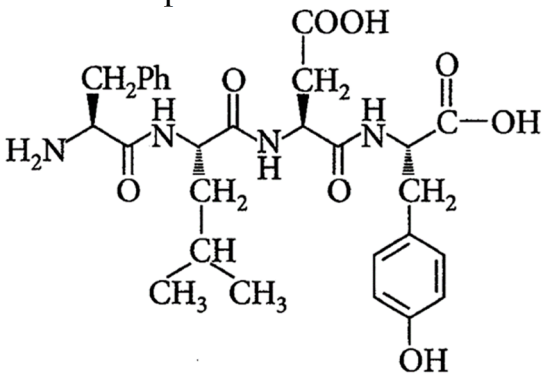
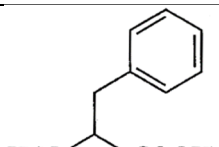
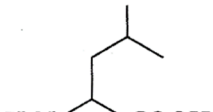
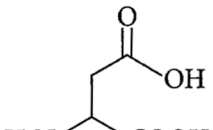
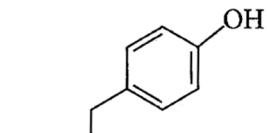
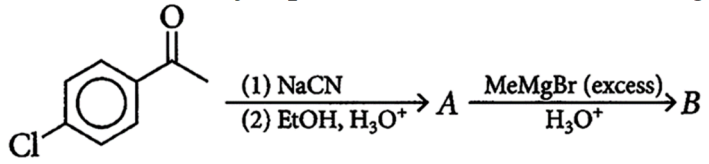
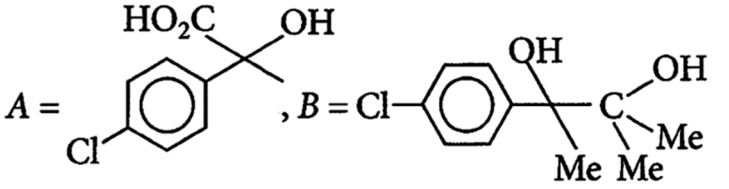
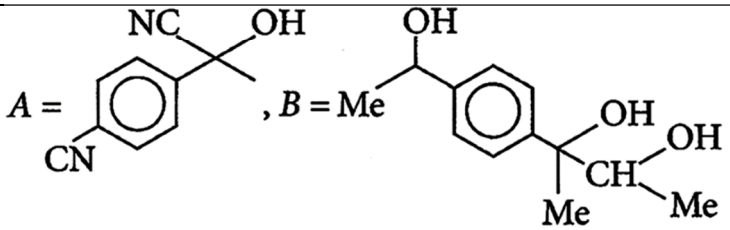
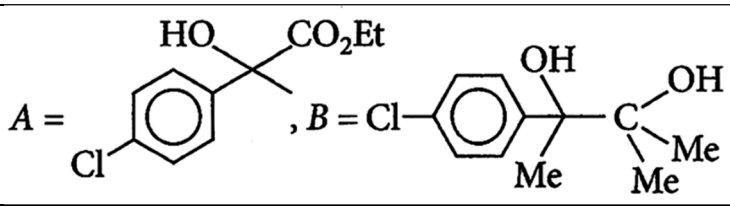


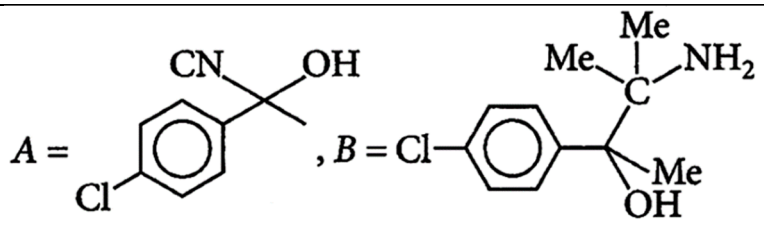
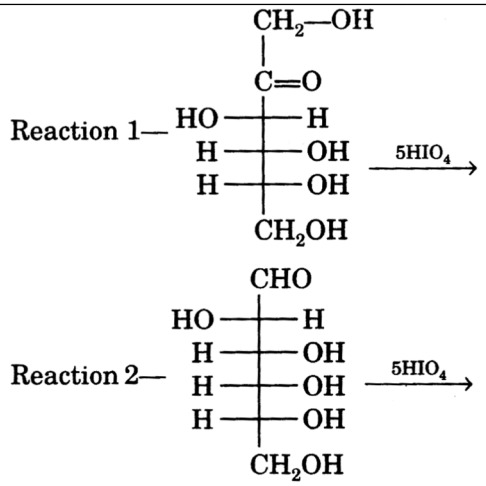


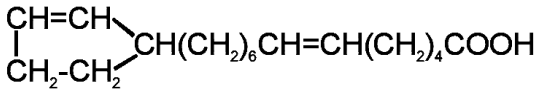
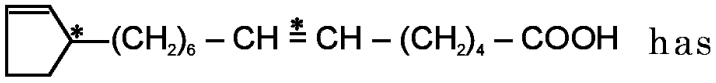
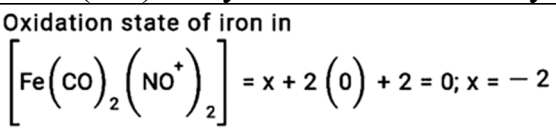
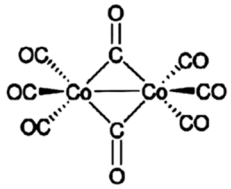
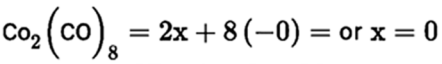


40. Resistance of a 0.1 M KCl solution in a conductance cell is 300 ohm and **A**

	specific conductance of 0.1 M KCl is 1.33×10^{-2} ohm^{-1} cm^{-1}. The resistance of 0.1 M NaCl solution in the same cell is 400 ohm. The equivalent conductance of the 0.1 M NaCl solution (in ohm^{-1} cm^2/g meq.) is :	
	100	
	400	
	300	
	150	
Sol.	$\kappa = \frac{1}{R}\frac{l}{A}$ i.e. $\frac{l}{A} = R \times \kappa = 300 \times 1.33 \times 10^{-2}$ $\simeq 4.0 cm^{-1}$ $\kappa_{NaCl} = \frac{1}{R}\frac{l}{A} = \frac{1}{400} \times 4.0$ $\Lambda_{m(NaCl)} = \kappa \times \frac{1000}{M} = \frac{4.0}{400} \times \frac{1000}{0.1}$ $= 100$	
41.	A chloride salt solution acidified with dil.HNO_3 gives a curdy white precipitate, [A] on addition of $AgNO_3$. [A] on treatment with NH_4OH gives a clear solution, B. A and B are respectively	**D**
	AgCl and $(NH_4)[Ag(OH)_2]$	
	$H[AgCl_3]$ and $(NH_4)[Ag(OH)_2]$	
	$H[AgCl_3]$ and $[Ag(NH_3)_2]Cl$	
	AgCl and $[Ag(NH_3)_2]Cl$	
Sol.	$\text{Chloride salt} + AgNO_3 + \text{dil. } HNO_3 \longrightarrow \underset{(A)}{AgCl}\downarrow$ $AgCl + 2NH_4OH \longrightarrow \underset{(B)}{[Ag(NH_3)_2]Cl} + 2H_2O$	
42.	The tests performed on compound X and their inferences are (i) 2,4-DNP test — Coloured precipitate (ii) Iodoform test — Yellow precipitate (iii) Azo-dye test — No dye formation Compound 'X' is	**B**
		
		
		
		
Sol.	Compound 'X' should be (b). Due to presence of —$COCH_3$ group, it undergoes iodoform test.	

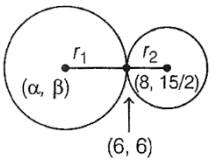
	Due to presence of 3°-amine group it does not participate in azo-dye test. On the other hand, presence of $PhCOCH_3$ group, it readily undergoes 2, 4-DNP test.	
43.	Following tetrapeptide can be represented as  (F, L, D, Y, I, Q, P are one letter codes for amino acids.)	**D**
	FIQY	
	YQLF	
	PIDY	
	FLDY	
Sol.	 Phenylalanine (F)  Leucine (L)  Aspartic acid (D)  Tyrosine (Y) Arranging these four peptides we will get the given tetrapeptide, thus the tetrapeptide is FLDY.	
44.	Find out the major products from the following reaction sequence. 	**C**
		
		
		

		
45.	For the adsorption of hydrogen on platinum the activation energy is 30 kJ mol^{-1} and for the adsorption of hydrogen on nickel, the activation energy is 41.4 kJ mol^{-1}. The logarithm of the ratio of the rates of chemisorption on equal areas of the metals at 300 K is _____. (Nearest integer) Given : In 10 = 2.3, R = 8.3 J K^{-1} mol^{-1}	**2**
	2	
	3	
	4	
	6	
Sol.	Given : E_{a_1} = 30 kJ mol^{-1}, E_{a_2} = 41.4 kJ mol^{-1}, and T = 300 K According to Arrhenius equation, $k = Ae^{-E_a/RT}$ $\frac{k_1}{k_2} = \frac{e^{-E_{a_1}/RT}}{e^{-E_{a_2}/RT}} = e^{(E_{a_2}/RT - E_{a_1}/RT)}$ or $2.3 \log \frac{k_1}{k_2} = \frac{E_{a_2}}{RT} - \frac{E_{a_1}}{RT}$ or $\log \frac{k_1}{k_2} = \frac{1}{2.3RT}(E_{a_2} - E_{a_1}) = \frac{(41.4-30)\times 1000}{2.3\times 8.3\times 300}$ $= 1.99 \approx 2$	
46.	 Ratio of moles of formaldehyde obtained in the reaction (1) and reaction (2).	**2**
47.	When chromite ore is fused in strong alkali in presence of air, the number of equivalent bond lengths in anionic part of Cr-containing product is x. Find the value of $\frac{1}{x}$. **(Multiply your answer by 100)**	**25**
Sol.	$FeCr_2O_4 + NaOH + O_2 \rightarrow Na_2CrO_4 + Fe_2O_3 + H_2O$ CrO_4^{2-} has 4 equal Cr–O bond length	
48.	In alkaline medium, thiosulphate ions are oxidised to sulpahte ions by MnO_4^- ions. The oxidation number of Mn decreases by.....units	**3**
Sol.	In alkaline medium $MnO_4^- \rightarrow MnO_2$ (+7) (+4) The oxidation number of Mn decreases by 3 units.	

49.	How many stereoisomers are possible for Gorlic acid ($C_{18}H_{30}O_2$) 	**4**
Sol.	The given compound  has two stereocentres therefore it has four stereo isomers.	
50.	The EAN of metal atoms in $[Fe(CO)_2(NO^+)_2]$ is 'x' and EAN of metal atoms in $Co_2(CO)_8$ is 'y'. The value of x + y is	**72**
Sol.	Oxidation state of iron in  So, EAN $= 26 - (-2) + 8 = 36 = x$  Oxidation state of cobalt in  So, EAN $= 27 + 1 + 8 = 36 = y$ So, $x + y = 72$	
51.	If the centre and radius of the circle $\left\|\frac{z-2}{z-3}\right\| = 2$ are respectively, (α, β) *and* γ, then $3(\alpha + \beta + \gamma)$ is equal to	**D**
	9	
	10	
	11	
	12	

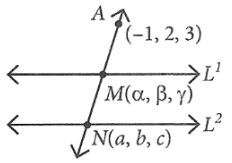
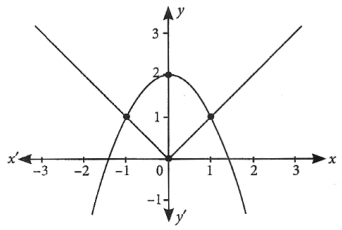
Sol.	*(d)* Let $z = x + iy$ We have, $\left\|\frac{z-2}{z-3}\right\| = 2$ $\Rightarrow \left\|\frac{x+iy-2}{x+iy-3}\right\| = 2$ $\Rightarrow \frac{\sqrt{(x-2)^2+y^2}}{\sqrt{(x-3)^2+y^2}} = 2$ $\Rightarrow (x-2)^2 + y^2 = 4[(x-3)^2 + y^2]$ $\Rightarrow x^2 - 4x + 4 + y^2$ $= 4[x^2 - 6x + 9 + y^2]$ $\Rightarrow x^2 + y^2 - \frac{20}{3}x + \frac{32}{3} = 0$ $\therefore$ Centre of circle is $\left(-\frac{1}{2}\times\left(\frac{-20}{3}\right), -\frac{1}{2}\times(0)\right)$ i.e. $\left(\frac{10}{3}, 0\right)$ And radius of circle $= \sqrt{\left(\frac{10}{3}\right)^2 + (0)^2 - \frac{32}{3}}$ $= \sqrt{\frac{100}{9} - \frac{32}{3}} = \sqrt{\frac{100-96}{9}} = \frac{2}{3}$ $\therefore \alpha = \frac{10}{3}, \beta = 0, \gamma = \frac{2}{3}$ So, $3(\alpha+\beta+\gamma) = 3\left(\frac{10}{3} + 0 + \frac{2}{3}\right)$ $= 10 + 2 = 12$	
52.	Let α, β be the roots of the equation $x^2 + 2\sqrt{2}x - 1 = 0$. The quadratic equation, whose roots are $\alpha^4 + \beta^4$ and $\frac{1}{10}(\alpha^6 + \beta^6)$, is	**A**
	$x^2 - 195x + 9056 = 0$	
	$x^2 - 190x + 9466 = 0$	
	$x^2 - 180x + 9506 = 0$	
	$x^2 - 195x + 9466 = 0$	
Sol.	Given α and β are the roots of the given quadratic equation. $x^2 + 2\sqrt{2}x - 1 = 0$ $\alpha + \beta = -2\sqrt{2}, \alpha\beta = -1$ Now, $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$ $= \{(\alpha+\beta)^2 - 2\alpha\beta\}^2 - 2(\alpha\beta)^2$ $= (8+2)^2 - 2(-1)^2 = 100 - 2 = 98$ and $\alpha^6 + \beta^6 = (\alpha^3 + \beta^3)^2 - 2\alpha^3\beta^3$ $= [(\alpha+\beta)\{(\alpha+\beta)^2 - 3\alpha\beta\}]^2 - 2(\alpha\beta)^3$ $= [(-2\sqrt{2})(8+3)]^2 - 2(-1)^3$ $= (8)(121) + 2 = 970$ $\therefore \frac{\alpha^6 + \beta^6}{10} = 97$ Hence, required quadratic equation is $x^2 - (98+97)x + (98)(97) = 0$ $\Rightarrow x^2 - 195x + 9506 = 0$	
53.	If the 1011^{th} term from the end in the binomial expansion of $\left(\frac{4x}{5} - \frac{5}{2x}\right)^{2022}$ is 1024 times 1011^{th} term from the beginning, then $\|x\|$ is equal to	**A**
	5/16	

	12	
	8	
	15	
Sol.	*(a)* Given expression $\left(\frac{4x}{5}-\frac{5}{2x}\right)^{2022}$ So, 1011th term from the end $=T_{1010+1}={}^{2022}C_{1010}\left(\frac{-5}{2x}\right)^{1012}\left(\frac{4x}{5}\right)^{1010}$ and 1011th term from the beginning $=T_{1010+1}={}^{2022}C_{1010}\left(\frac{4x}{5}\right)^{1012}\left(\frac{-5}{2x}\right)^{1010}$ Given that, 1011th term from the end in above binomial expression is 1024 times 1011th term from the beginning. Thus, ${}^{2022}C_{1010}\left(\frac{-5}{2x}\right)^{1012}\left(\frac{4x}{5}\right)^{1010}$ $=1024\ {}^{2022}C_{1010}\left(\frac{4x}{5}\right)^{1012}\left(\frac{-5}{2x}\right)^{1010}$ $\Rightarrow \left(\frac{-5}{2x}\right)^2=1024\left(\frac{4x}{5}\right)^2$ $\Rightarrow x^4=\frac{5^4}{64\times1024}\Rightarrow x^4=\frac{5^4}{2^{16}}$ $\Rightarrow \lvert x\rvert=\frac{5}{2^4}=\frac{5}{16}$	
54.	Let $s_1, s_2, s_3, \ldots., s_{10}$ respectively be the sum to 12 terms of 10 AP's whose first terms are 1, 2, 3, …,10 and the common differences are 1, 3, 5, …,19 respectively. Then, $\sum_{i=1}^{10} S_i$ is equal to	**D**
	7380	
	7360	
	7220	
	7260	
Sol.	*(d)* Given that let $s_1, s_2, \ldots, s_{10}$ respectively be the sum of 12 terms of 10 APs, whose first terms are 1, 2, 3, …, 10 and the common difference are 1, 3, 5, … 19, respectively. The first terms are $a_i = i$ and the common differences are $d_i = 2i-1$. Thus, $S_i=\frac{12}{2}[2i+11\times(2i-1)]$ $=6[24i-11]$ $\sum_{i=1}^{10}S_i=6\sum_{i=1}^{10}(24i-11)$ $=6[24\sum_{i=1}^{10}i-110]$ $=6\left[24\frac{(10)(10+1)}{2}-110\right]$ $=6[12\times10\times11-110]$ $=6[1320-110]$ $=6\times1210=7260$	
55.	Let P be a point on the parabola $y^2 = 4ax$, where $a > 0$. The normal to the parabola at P meets the x-axis at a point Q. The area of the triangle PFQ, where F is the focus of the parabola, is 120. If the slope of m of the normal and a are both positive integers, then the pair (a, m) is	**A**
	(2, 3)	
	(1, 3)	
	(2, 4)	

	(3, 4)	
Sol.	(a) : $2yy' = 4a$ Slope of normal to the parabola at point $P(at^2, 2at)$ is $m = -\frac{2at \times 2}{4a} = -t$ Now equation of normal at $P(at^2, 2at)$is $y - 2at = -t(x - at^2)$ So $Q(2a + at^2, 0)$ Area of $\Delta PFQ = \frac{1}{2}\begin{vmatrix} at^2 & 2at & 1 \\ 2a+at^2 & 0 & 1 \\ a & 0 & 1 \end{vmatrix} = 120$ $\Rightarrow \left\lvert \frac{2at}{2}(a+at^2) \right\rvert = 120$ $\Rightarrow \quad a^2t(1 + t^2)\rvert = 120 \Rightarrow \lvert 2^2 \times 3(1 + 3^2)\rvert = 120$ So, $m = 3, a = 2$	
56.	Let the circles $C_1: (x-\alpha)^2 + (y-\beta)^2 = r_1^2$ and $C_2: (x-8)^2 + \left(y - \frac{15}{2}\right)^2 = r_2^2$ touch each other externally at the point (6, 6). If the point (6, 6) divides the line segment joining the centres of the circles C_1 *and* C_2 internally in the ratio 2 : 1, then $(\alpha + \beta) + 4(r_1^2 + r_2^2)$ equals	**D**
	125	
	145	
	110	
	130	
Sol.	(d) $C_1 : (x-\alpha)^2 + (y-\beta)^2 = r_1^2$ $C_2 : (x-8)^2 + \left(y - \frac{15}{2}\right)^2 = r_2^2$  $r_1 : r_2 = 2 : 1$ So, $\frac{2 \times 8 + \alpha}{2+1} = 6 \Rightarrow \alpha = 2$ and $\frac{2 \times \frac{15}{2} + \beta}{2+1} = 6 \Rightarrow \beta = 3$ So, $(\alpha, \beta) = (2, 3)$ Now, $r_1 = \sqrt{(6-\alpha)^2 + (6-\beta)^2}$ $= \sqrt{(4)^2 + (3)^2} = 5$ $\Rightarrow \quad r_1^2 = 25$ and $r_2^2 = \left(\sqrt{(8-6)^2 + \left(\frac{15}{2} - 6\right)^2}\right)^2$ $= 4 + \frac{9}{4} = \frac{25}{4}$ Therefore, $(\alpha + \beta) + 4(r_1^2 + r_2^2)$ $= (2 + 3) + 4\left(25 + \frac{25}{4}\right)$ $= 5 + 4 \times \frac{125}{4} = 130$	
57.	The mean and standard deviation of 100 observations are 40 and 5.1,	**A**

	respectively. By mistake one observation is taken as 50 instead of 40. If the correct mean and the correct standard deviation are μ and σ respectively, then $10(\mu+\sigma)$ is equal to	
	449	
	451	
	447	
	445	
Sol.	*(a)* Let the observations be $x_1 x_2,\ldots,x_{99}$, 50 Mean $=\frac{x_1+x_2+x_3+\ldots..+x_{99}+50}{100}=40$ $\Rightarrow x_1+x_2+x_3+\ldots.+x_{99}=3950$ Correct Mean $=\frac{3950+40}{100}$ $\mu=\frac{399}{10}=39.9$ $(SD)^2=\sum_{i=1}^{99}\frac{x_i^2+2500}{100}-(40)^2$ $\sum_{i=1}^{99}x_i^2=160101$ $(\text{Correct SD})^2=\frac{160101+1600}{100}-\left(\frac{399}{10}\right)^2$ $SD=5$ $10(\mu+\sigma)=10(39.9+5)$ $=10(44.9)=449$	
58.	If the mean of the following probability distribution of a random variable X: $\begin{array}{\|c\|c\|c\|c\|c\|c\|}\hline X & 0 & 2 & 4 & 6 & 8\\ \hline P(X) & a & 2a & a+b & 2b & 3b\\ \hline\end{array}$ is $\frac{46}{9}$, then the variance of the distribution is	**C**
	$\frac{151}{27}$	
	$\frac{173}{27}$	
	$\frac{566}{81}$	
	$\frac{581}{81}$	
Sol.	(c) : $\begin{array}{\|c\|c\|c\|c\|c\|c\|}\hline X & 0 & 2 & 4 & 6 & 8\\ \hline P(X) & a & 2a & a+b^3 & 2b & 3b\\ \hline\end{array}$ Mean $=\sum x_i P(x_i)$ $\Rightarrow \frac{46}{9}=0+4a+4a+4b+12b+24b$ $\Rightarrow \frac{46}{9}=8a+40b$ $\Rightarrow 36a+180b=23$ ...(i) Also, $\sum_{i-1}^{n}P_i=1$ $\Rightarrow 4a+6b=1$ ...(ii) On solving (i) and (ii), we get $a=\frac{1}{12},\ b=\frac{1}{9}$ Now, $\sigma^2=\sum x_i^2P(x_i)-\left(\sum x_iP(x_i)\right)^2$ $=0+4\times2a+16(a+b)+36(2b)$ $+64(3b)-\left(\frac{46}{9}\right)^2$ $=8\left(a+2(a+b)+9b+24b\right)-\left(\frac{46}{9}\right)^2$ $=8(3a+35b)-\left(\frac{46}{9}\right)^2=8\left(\frac{3}{12}+\frac{35}{9}\right)-\left(\frac{46}{9}\right)^2$ $=8\left(\frac{149}{36}\right)-\left(\frac{46}{9}\right)^2=\frac{566}{81}$	

59.	Let $f: R \to R$ be a function defined by $f(x) = \left(2\left(1-\frac{x^{25}}{2}\right)(2+x^{25})\right)^{1/50}$. If the function $g(x) = f(f(f(x))) + f(f(x))$, then the greatest integer less than or equal to $g(1)$ is _____.	**A**
	2	
	8	
	10	
	20	
Sol.	**(2) :** We have, $f(x) = \left(2\left(1-\frac{x^{25}}{2}\right)(2+x^{25})\right)^{1/50}$ $= \left((2-x^{25})(2+x^{25})\right)^{1/50} = (4-x^{50})^{1/50}$ $f(f(x)) = x, f(f(f(x)) = (4-x^{50})^{1/50}$ $g(x) = (4-x^{50})^{1/50} + x,$ $g(1) = (4-1)^{1/50} + 1 = 3^{1/50} + 1$ Greatest integer of $g(1)$ is $[g(1)] = 2$	
60.	Let $A = \{1, 2, 3, \ldots\ldots, 100\}$, Let R be a relation on A defined by $(x, y)\ \epsilon\ R$ if and only if $2x = 3y$. Let R_1 be a symmetric relation on A such that $R \subset R_1$ and the number of elements in R_1 is n. Then, the minimum value of n is _____.	**B**
	70	
	66	
	64	
	75	
Sol.	**(66) :** We have , $A = \{1, 2, 3, ..., 100\}$ $R = \{(3, 2), (6, 4), (9, 6), (12, 8), ... (99, 66)\}$ $n(R) = 33$ Since, R_1 is symmetric and $R \subset R_1$ $\therefore \quad n(R_1) = 66$	
61.	Let a line passing through the point $(-1, 2, 3)$ intersect the lines $L_1: \frac{x-1}{3} = \frac{y-2}{2} = \frac{z+1}{-2}$ at $M(\alpha, \beta, \gamma)$ and $L_2: \frac{x+2}{-3} = \frac{y-2}{-2} = \frac{z-1}{4}$ at $N(a, b, c)$. Then, the value of $\frac{(\alpha+\beta+\gamma)^2}{(a+b+c)^2}$ equals_____.	**A**
	196	
	200	
	210	
	220	

Sol.	**(196) :** We have, $L_1 : \frac{x-1}{3} = \frac{y-2}{2} = \frac{z+1}{-2} = \lambda \in R$ $\therefore \quad M \equiv (3\lambda+1, 2\lambda+2, -2\lambda-1)$ $L_2 : \frac{x+2}{-3} = \frac{y-2}{-2} = \frac{z-1}{4} = \mu \in R$ $\therefore \quad N \equiv (-3\mu - 2, -2\mu + 2, 4\mu + 1)$ $\alpha + \beta + \gamma = 3\lambda + 2 \Rightarrow a + b + c = -\mu + 1$ Direction ratio's of line $AM = < 3\lambda + 2, 2\lambda, -2\lambda - 4 >$ Direction ratio's of line $AN = < -3\mu - 1, -2\mu, 4\mu - 2 >$  $\Rightarrow \quad \frac{3\lambda+2}{-3\mu-1} = \frac{2\lambda}{-2\mu} = \frac{-2\lambda-4}{4\mu-2}$ $\Rightarrow \quad \frac{3\lambda+2}{-3\mu-1} = \frac{\lambda}{-\mu}$ and $\frac{\lambda}{-\mu} = \frac{-2\lambda-4}{4\mu-2}$ $\Rightarrow \quad -3\lambda\mu - 2\mu = -3\mu\lambda - \lambda$ and $\quad 4\mu\lambda - 2\lambda = 2\mu\lambda + 4\mu$ $\Rightarrow \quad 2\mu = \lambda$ and $2\mu\lambda = 2\lambda + 4\mu$ $\Rightarrow \quad \lambda^2 = 2\lambda + 2\lambda \Rightarrow \lambda^2 = 4\lambda$ $\Rightarrow \quad \lambda(\lambda - 4) = 0 \Rightarrow \lambda = 0, \lambda = 4$ $\therefore \quad \frac{(\alpha+\beta+\gamma)^2}{(a+b+c)^2} = \frac{(3\times4+2)^2}{(-2+1)^2} = \frac{(14)^2}{1} = 196$	
62.	A function f is defined on $[-3, 3]$ as $f(x) = \begin{cases} \min\{\lvert x\rvert, 2 - x^2\}, & -2 \le x \le 2 \\ [\lvert x\rvert], & 2 < \lvert x\rvert \le 3 \end{cases}$ where $[x]$ denotes the greatest integer $\le x$. The number of points, where f is not differentiable in $(-3, 3)$ is _____.	**D**
	10	
	20	
	15	
	5	
Sol.	$f(x) = \begin{cases} \min\{\lvert x\rvert, 2-x^2\}, & -2 \le x \le 2 \\ [\lvert x\rvert], & 2 < \lvert x\rvert \le 3 \end{cases}$  From the graph, we see that number of points of non-differentiability in (−3, 3) = 5.	
63.	Let (2, 3) be the largest open interval in which the function $f(x) = 2\log_e(x-2) - x^2 + ax + 1$ is strictly increasing and (b, c) be the largest open interval, in strictly decreasing. Then, $100(a + b - c)$ is equal to	**C**
	420	
	160	
	360	
	280	

Sol.	*(c)* Given, $f(x)=2\log_e(x-2)-x^2+ax+1$ is increasing in (2, 3) $\therefore \quad f'(x)=\frac{2}{x-2}-2x+a\geq 0$ and $\quad f''(x)=\frac{-2}{(x-2)^2}-2<0$ then we can say that $f'(x)$ is strictly decreasing. and $\quad f'(3)\geq 0$ $\Rightarrow \quad 2-6+a\geq 0\Rightarrow a\geq 4$ $\therefore \quad a_{\min}=4$ Also given, $g(x)=(x-1)^3(x+2-a)^2$ is strictly decreasing. $\Rightarrow \quad g(x)=(x-1)^3(x+2-4)^2$ $=(x-1)^3(x-2)^2$ $\Rightarrow g'(x)=(x-1)^3 2(x-2)+(x-2)^2\cdot 3(x-1)^2$ $=(x-1)^2(x-2)(2x-2+3x-6)$ $=(x-1)^2(x-2)(5x-8)<0$ in $x\in\left(\frac{8}{5},2\right)$ $\therefore \quad b=8/5$ and $c=2$. $-\infty \leftarrow$ + 1 + 8/5 − 2 + $\rightarrow \infty$ $\therefore 100(a+b-c)=100(4+8/5-2)$ $=360$	
64.	Let $f(x)=\int\frac{2x}{(x^2+1)(x^2+3)}dx$. If $f(3)=\frac{1}{2}(\log_e 5-\log_e 6)$, then $f(4)$ is equal to	**B**
	$\frac{1}{2}(\log_e 19-\log_e 17)$	
	$\frac{1}{2}(\log_e 17-\log_e 19)$	
	$\log_e 19-\log_e 20$	
	$\log_e 17-\log_e 18$	
Sol.	*(b)* Given, $f(x)=\int\frac{2x}{(x^2+1)(x^2+3)}dx$ Put $\quad x^2=t\Rightarrow 2x\cdot dx=dt$ $f(x)=\int\frac{dt}{(t+1)(t+3)}$ $=\frac{1}{2}\int\frac{1}{t+1}-\frac{1}{t+3}dt$ $f(x)=\frac{1}{2}(\ln\lvert t+1\rvert-\ln\lvert t+3\rvert)+c$ $f(x)=\frac{1}{2}\ln\left\lvert\frac{t+1}{t+3}\right\rvert+c=\frac{1}{2}\ln\left\lvert\frac{x^2+1}{x^2+3}\right\rvert+c$ $f(3)=\frac{1}{2}\ln\left\lvert\frac{9+1}{9+3}\right\rvert+c=\frac{1}{2}\ln\frac{5}{6}+c$ $\frac{1}{2}(\ln 5-\ln 6)=\frac{1}{2}(\ln 5-\ln 6)+c$ $c=0$ So, $f(x)$ becomes $f(x)=\frac{1}{2}\ln\left\lvert\frac{x^2+1}{x^2+3}\right\rvert$ $f(4)=\frac{1}{2}\ln\left(\frac{17}{19}\right)=\frac{1}{2}(\log_e 17-\log_e 19)$	
65.	Let $f:(0,\infty)\to \boldsymbol{R}$ be a twice differentiable function. If for some $a\neq 0, \int_0^1 f(\lambda x)d\lambda=af(x), f(1)=1$ and $f(16)=\frac{1}{8}$, then $16-f'\left(\frac{1}{16}\right)$ is equal to_____.	**C**
	120	
	150	
	112	
	180	

Sol.	**(112) :** We have, $\int_0^1 f(\lambda x)d\lambda = af(x)$ $\Rightarrow \frac{1}{x}\int_0^x f(t)dt = af(x)$ Put $\lambda x = t \Rightarrow d\lambda = \frac{1}{x}dt$ $\Rightarrow \int_0^x f(t)dt = axf(x)$ $\Rightarrow f(x) = a(xf'(x) + f(x))$ $\Rightarrow (1-a)f(x) = axf'(x)$ $\Rightarrow \frac{f'(x)}{f(x)} = \frac{(1-a)}{a}\frac{1}{x}$ $\Rightarrow \ln f(x) = \frac{1-a}{a}\ln x + c$ (On integrating) Now, $x = 1, f(1) = 1 \Rightarrow c = 0$ Again $x = 16, \quad f(16) = \frac{1}{8}$ $\Rightarrow \frac{1}{8} = (16)^{\frac{1-a}{a}} \Rightarrow -3 = \frac{4-4a}{a} \Rightarrow a = 4$ Therefore, $f(x) = x^{-3/4}$ $\Rightarrow f'(x) = -\frac{3}{4}x^{-\frac{7}{4}}$ Hence, $16 - f'\left(\frac{1}{16}\right) = 16 - \left(-\frac{3}{4}(2^{-4})^{-7/4}\right)$ $= 16 + 96 = 112$	
66.	Consider the system of linear equations $x + y + z = 4\mu, x + 2y + 2\lambda z = 10\mu,$ $x + 3y + 4\lambda^2 z = \mu^2 + 15$, where $\lambda, \mu \in R$. Which one of the following statements is **NOT** correct?	**A**
	The system is inconsistent if $\lambda = \frac{1}{2}$ and $\mu \neq 1$	
	The system has unique solution if $\lambda \neq \frac{1}{2}$ and $\mu = 1,15$	
	The system has infinite number of solutions if $\lambda = \frac{1}{2}$ and $\mu = 15$	
	The system is consistent if $\lambda \neq \frac{1}{2}$	

Sol.

(a) : We have,

$$\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & 2\lambda\\ 1 & 3 & 4\lambda^2\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}4\mu\\ 10\mu\\ \mu^2+15\end{bmatrix}$$

For $\lambda=\frac{1}{2}$ and $\mu = 15$, we get

$$\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & 1\\ 1 & 3 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}60\\ 150\\ 240\end{bmatrix}$$

System is consistent but does not have unique solution as matrix have zero determinant because two columns are same.

Also, let $\lambda\neq\frac{1}{2}$ and $\lambda = \mu = 1$, then we have

$$\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & 2\\ 1 & 3 & 4\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}4\\ 10\\ 16\end{bmatrix}$$

$\Rightarrow \quad x+y+z=4; x+2y+2z$
$= 10; x+3y+4z=16$

On solving these equations, we get $(-2, 6, 0)$ as unique solution.

Clearly, for $\lambda=\frac{1}{2}$ and $\mu \neq 1$ *i.e.*, $\mu = 15$ the system is consistent and have infinite solution.

So, statement (a) is not correct.

67. Let $\boldsymbol{a}=\hat{\boldsymbol{i}}+2\hat{\boldsymbol{j}}+\hat{\boldsymbol{k}}$ and $\hat{\boldsymbol{b}}=2\hat{\boldsymbol{i}}+7\hat{\boldsymbol{j}}+3\hat{\boldsymbol{k}}$. Let $L_1: \boldsymbol{r}=\left(-\hat{\boldsymbol{i}}+2\hat{\boldsymbol{j}}+\hat{\boldsymbol{k}}\right)+\lambda\boldsymbol{a}, \lambda \in R$ and $L_2: \boldsymbol{r}=\left(\hat{\boldsymbol{j}}+\hat{\boldsymbol{k}}\right)+\mu\boldsymbol{b}, \mu \in R$ be two lines. If the line L_3 passes through the point of intersection of L_1 *and* L_2 and is parallel to $\boldsymbol{a}+\boldsymbol{b}$, then L_3 passes through the point

B

(5, 17, 4)

(8, 26, 12)

(2, 8, 5)

(–1, –1, 1)

Sol.

(b)

$L_1 : \mathbf{r} = (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}) + \lambda(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}})$

$\Rightarrow \mathbf{r} = (\lambda - 1)\hat{\mathbf{i}} + 2(\lambda + 1)\hat{\mathbf{j}} + (\lambda + 1)\hat{\mathbf{k}}$

$L_2 : \mathbf{r} = (\hat{\mathbf{j}} + \hat{\mathbf{k}}) + \mu(2\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$

$\Rightarrow \mathbf{r} = (2\mu)\hat{\mathbf{i}} + (1 + 7\mu)\hat{\mathbf{j}} + (1 + 3\mu)\hat{\mathbf{k}}$

For point of intersection, we equating respective components

i.e. $\lambda - 1 = 2\mu$ …(i)

$2(\lambda + 1) = 1 + 7\mu$ …(ii)

$\lambda + 1 = 1 + 3\mu$ …(iii)

On solving these equation simultaneously, we get

$\lambda = 3$ and $\mu = 1$

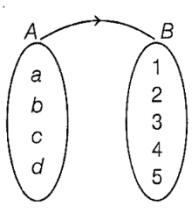
$\Rightarrow$ Point of intersection $= (2, 8, 4)$

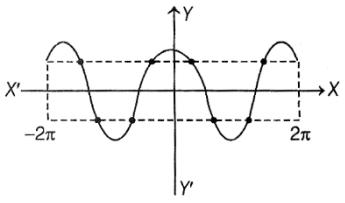
Given, $\mathbf{a} + \mathbf{b} = 3\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$

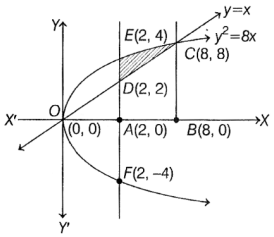
$L_3 : \mathbf{r} = 2\hat{\mathbf{i}} + 8\hat{\mathbf{j}} + 4\hat{\mathbf{k}} + \alpha(3\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$

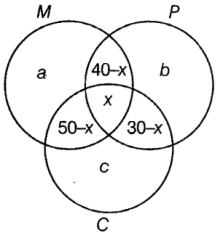
For $\alpha = 2$, $\mathbf{r} = 8\hat{\mathbf{i}} + 26\hat{\mathbf{j}} + 12\hat{\mathbf{k}}$

Hence, L_3 passes through (8, 26, 12)

68.	The probability that a randomly chosen one-one function from the set $\{a, b, c, d\}$ to the set $\{1, 2, 3, 4, 5\}$ satisfied $f(a) + 2f(b) - f(c) = f(d)$ is	**D**
	1/24	
	1/40	
	1/30	
	1/20	
Sol.	*(d)*  Here number of elements in sample $n(S) = 5 \times 4 \times 3 \times 2 = 120$ $f(a) + 2f(b) = f(c) + f(d)$ $5 \quad 2 \times 1 \quad 3 \quad 4$ $4 \quad 2 \times 2 \quad 3 \quad 5$ $1 \quad 2 \times 3 \quad 2 \quad 5$ $\therefore$ Number of favourable cases $n(A) = 2! \times 3 = 6$ $\therefore P(A) = \frac{\text{Number of favourable cases}}{\text{Total number of cases}}$ $\therefore P(A) = \frac{n(A)}{n(S)} = \frac{6}{120} = \frac{1}{20}$	
69.	Let $f: R \to R$ be a function defined by $f(x) = \frac{4^x}{4^x+2}$ and $M = \int_{f(a)}^{f(1-a)} xsin^4(x(1-x))dx, N = \int_{f(a)}^{f(1-a)} \sin^4\big(x(1-x)\big)dx. a \neq \frac{1}{2}$. If $\alpha M = \beta N, \alpha, \beta \in N$, then the least value of $\alpha^2 + \beta^2$ is equal to_____.	**A**
	5	
	10	
	25	
	40	

Sol.	(5) : We have, $f(a)=\frac{4^a}{4^a+2}$ and $f(1-a)=\frac{4^{(1-a)}}{4^{1-a}+2}=\frac{4^1\cdot 4^{-a}}{4^1\cdot 4^{-a}+2}=\frac{4^1\cdot 4^{-a}}{4^{-a}(4+2\cdot 4^a)}$ $=\frac{4}{(4+2\cdot 4^a)}=\frac{2}{2+4^a}$ $\Rightarrow f(a)+f(1-a)=\frac{4^a}{4^a+2}+\frac{2}{2+4^a}=\frac{4^a+2}{4^a+2}=1$ $M=\int_{f(a)}^{f(1-a)} x\sin^4[x(1-x)]\,dx,$ $N=\int_{f(a)}^{f(1-a)} \sin^4[x(1-x)]\,dx$ $\Rightarrow M=\int_{f(a)}^{f(1-a)} (1-x)\sin^4[(1-x)(1-1+x)]\,dx$ $\Rightarrow \quad M=\int_{f(a)}^{f(1-a)} (1-x)\sin^4[x(1-x)]\,dx$ $=\int_{f(a)}^{f(1-a)} \sin^4[x(1-x)]\,dx-\int_{f(a)}^{f(1-a)} x\sin^4[x(1-x)]\,dx$ $\Rightarrow \quad M=N-M \Rightarrow 2M=N \Rightarrow \frac{M}{N}=\frac{1}{2}$ $\Rightarrow \quad \frac{M}{N}=\frac{\beta}{\alpha}=\frac{1}{2} \Rightarrow \beta=1, \alpha=2$ Then, the least value of $\alpha^2+\beta^2=4+1=5$	
70.	If $\theta \in [-2\pi, 2\pi]$, then the number of solutions of $2\sqrt{2}\cos^2\theta+(2-\sqrt{6})\cos\theta-\sqrt{3}=0$, is equal to	**B**
	12	
	8	
	6	
	10	
Sol.	(b) $\cos\theta=\frac{\sqrt{6}-2\pm\sqrt{(\sqrt{6}-2)^2+8\sqrt{6}}}{4\sqrt{2}}$ $=\frac{\sqrt{3}}{2},-\frac{1}{\sqrt{2}}$  ∴Number of solutions = 8 i.e. $\frac{-11\pi}{6},\frac{-5\pi}{4},\frac{-3\pi}{4},\frac{-\pi}{6},\frac{\pi}{6},\frac{3\pi}{4},\frac{5\pi}{4},\frac{11\pi}{6}$	
71.	Let P be the set of seven-digit numbers with sum of their digits equal to 11. If the numbers in P are formed by using the digits 1, 2 and 3 only, then the number of elements in the set P is	**161**

Sol.	**Case (I)** Number of numbers created using 3221111 is $n_1 = \frac{7!}{4!2!} = \frac{7\times 6\times 5\times 4!}{4!\times 2\times 1} = 105$ **Case (II)** Number of numbers created using 2222111 is $n_2 = \frac{7!}{4!3!} = \frac{7\times 6\times 5\times 4!}{4!\times 3\times 2} = 35$ **Case (III)** Number of numbers created using 3311111 is $n_3 = \frac{7!}{5!2!} = \frac{7\times 6\times 5!}{5!\times 2\times 1} = 21$ Hence, total number of elements in the set $P = n_1 + n_2 + n_3 = 105 + 35 + 21 = 161$	
72.	If $\lim\limits_{x\to 1} \frac{(5x+1)^{1/3} - (x+5)^{1/3}}{(2x+3)^{1/2} - (x+4)^{1/2}} = \frac{m\sqrt{5}}{n(2n)^{\frac{2}{3}}}$, where $\gcd(m,n) = 1$, then $8m + 12n$ is equal to…….	**100**
Sol.	*(100)* Let $L = \lim\limits_{x\to 1} \frac{(5x+1)^{1/3} - (x+5)^{1/3}}{(2x+3)^{1/2} - (x+4)^{1/2}}$ $\left(\text{form}\frac{0}{0}\right)$ $\therefore L = \lim\limits_{x\to 1} \frac{\frac{1}{3}(5x+1)^{\frac{-2}{3}}\cdot 5 - \frac{1}{3}(x+5)^{\frac{-2}{3}}}{\frac{1}{2}(2x+3)^{\frac{-1}{2}}\cdot 2 - \frac{1}{2}(x+4)^{\frac{-1}{2}}}$ (Using L' Hospital Rule) $= \frac{8\sqrt{5}}{3\cdot 6^{2/3}} = \frac{m\sqrt{5}}{n(2n)^{2/3}}$ (given) On comparing, we get $m = 8$ and $n = 3$ Thus, $8m + 12n = 100$	
73.	Let α be the area of the largest region bounded by the curve $y^2 = 8x$ and the lines $y = x$ and $x = 2$, which lies in the first quadrant. Then, the value of 3α is equal to…….	**22**
Sol.	*(22)* Given equation of curves are $y^2 = 8x$ …(i) $y = x$ …(ii) and $x = 2$ …(iii)  Hence, area of shaded portion is $\int_2^8 (\sqrt{8x} - x)dx = \int_2^8 2\sqrt{2}\sqrt{x}\,dx - \int_2^8 x\,dx$ $= 2\sqrt{2}\left[\frac{x^{3/2}}{3/2}\right]_2^8 - \left[\frac{x^2}{2}\right]_2^8$ $= \frac{4\sqrt{2}}{3}[(8)^{3/2} - (2)^{3/2}] - \frac{1}{2}[(8)^2 - (2)^2]$ $= \frac{22}{3} = \alpha$ [given] Hence, $3\alpha = 22$	
74.	Let $A = \begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix}$ *and* $B = \begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix}, \alpha, \beta \in R$. Let α_1 be the value of α which satisfies $(A+B)^2 = A^2 + \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ and α_2 be the value of α which satisfies $(A+B)^2 = B^2$. Then, $\lvert\alpha_1 - \alpha_2\rvert$ is equal to……...	**2**

Sol.	(2) Now, $A+B=\begin{bmatrix}\beta+1 & 0\\ 3 & \alpha\end{bmatrix}$ $\Rightarrow (A+B)^2=\begin{bmatrix}\beta+1 & 0\\ 3 & \alpha\end{bmatrix}\begin{bmatrix}\beta+1 & 0\\ 3 & \alpha\end{bmatrix}$ $=\begin{bmatrix}(\beta+1)^2 & 0\\ 3(\beta+1)+3\alpha & \alpha^2\end{bmatrix}$ Now, $A^2=\begin{bmatrix}1 & -1\\ 2 & \alpha\end{bmatrix}\begin{bmatrix}1 & -1\\ 2 & \alpha\end{bmatrix}$ $=\begin{bmatrix}-1 & -1-\alpha\\ 2+2\alpha & \alpha^2-2\end{bmatrix}$ $\therefore A^2+\begin{bmatrix}2 & 2\\ 2 & 2\end{bmatrix}=\begin{bmatrix}1 & -\alpha+1\\ 2\alpha+4 & \alpha^2\end{bmatrix}$ $=\begin{bmatrix}(\beta+1)^2 & 0\\ 3(\alpha+\beta+1) & \alpha^2\end{bmatrix}$ $\Rightarrow \quad \alpha=1=\alpha_1$ (say) Now $B^2=\begin{bmatrix}\beta & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}\beta & 1\\ 1 & 0\end{bmatrix}=\begin{bmatrix}\beta^2+1 & \beta\\ \beta & 1\end{bmatrix}$ $=\begin{bmatrix}(\beta+1)^2 & 0\\ 3(\beta+1)+3\alpha & \alpha^2\end{bmatrix}$ $\therefore \beta=0, \alpha=-1=\alpha_2$ (say) $\therefore \lvert\alpha_1-\alpha_2\rvert=\lvert 1-(-1)\rvert=2$	
75.	In a survey of 220 students of a higher secondary school, it was found that atleast 125 and atmost 130 students studied Mathematics; atleast 85 and atmost 95 studied Physics; alteast 75 and atmost 90 studied Chemistry; 30 studied both Physics and Chemistry; 50 studied both Chemistry and Mathematics; 40 studied both Mathematics and Physics and 10 studied none of these subjects. Let m and n respectively be the least and the most number of students who studied all the three subjects. Then, $m+n$ is equal to…….	**45**
Sol.	(45) According to the question, $125 \le a+90-x \le 130$  $85 \le b+70-x \le 95$ $75 \le c+80-x \le 90$ $a+b+c+120-2x=210$ $\Rightarrow 15 \le x \le 45$ and $30-x \ge 0$ $\Rightarrow \quad 15 \le x \le 30$ $\Rightarrow m=15$ and $n=30$ Hence, $30+15=45$	