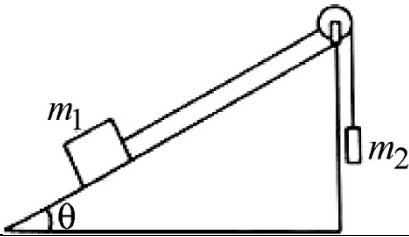
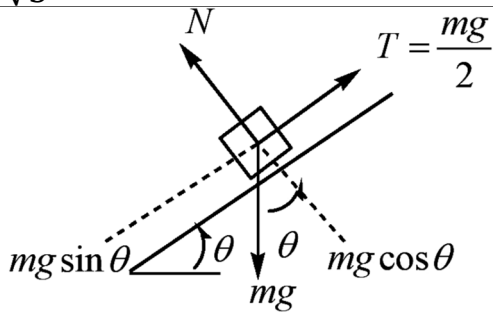
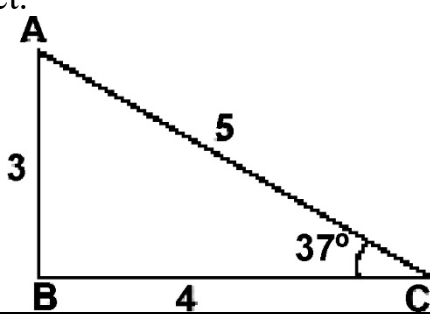
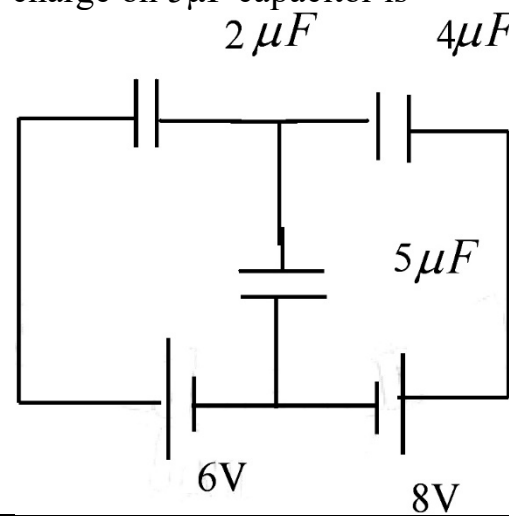
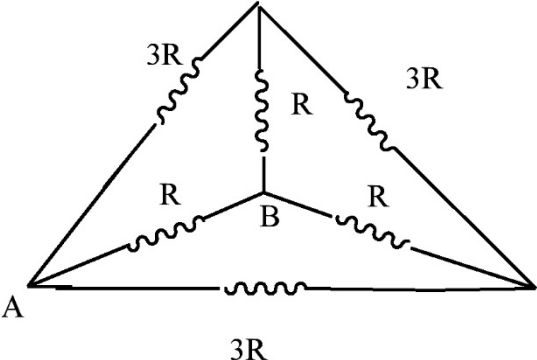
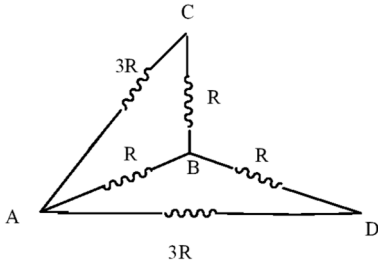


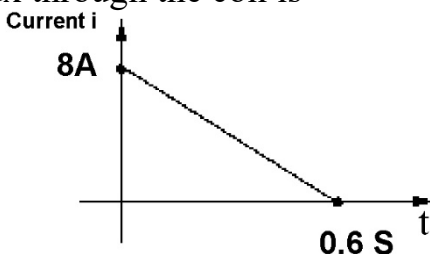
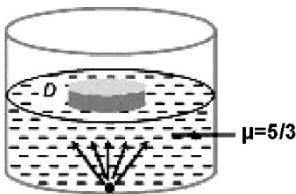
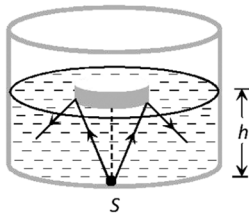
1.	The dependency of speed of waves (ripples) on the surface of water (v) on the density of waver (ρ), their wave length (λ) and surface tension (S) is given by	C
	$\frac{S}{\rho\lambda}$	
	$\sqrt{\frac{S\lambda}{\rho}}$	
	$\sqrt{\frac{S}{\rho\lambda}}$	
	$\left(\frac{S}{\rho\lambda}\right)^{\frac{1}{3}}$	
Sol.	$V = \rho^a \lambda^b S^c = \left(\frac{M}{L^3}\right)^a (L)^b (MT^{-2})^c$ $\Rightarrow a + c = 0; \quad -3a + b = 1 \quad \& \quad -1 = -2c$ $\text{SOLVING} \quad c = \frac{1}{2}; \quad a = -\frac{1}{2} \quad \& \quad b = -\frac{1}{2} \quad \therefore v = \sqrt{\frac{S}{\rho\lambda}}$	
2.	If a ball is thrown vertically upwards with speed u, the distance covered during the last t seconds of its ascent is	A
	$\frac{1}{2}gt^2$	
	$ut - \frac{1}{2}gt^2$	
	$(u - gt)t$	
	ut	
Sol.	<p>The distance covered by the ball during the last t seconds of its upward motion = Distance covered by it in first t seconds of its downward motion</p> <p>From $h = ut + \frac{1}{2}gt^2$</p> <p>$h = \frac{1}{2}gt^2$ [As u = 0 for it downward motion]</p>	
3.	<p>Two bodies of masses $m_1 = m$ kg and $m_2 = \frac{m}{2}$ kg are connected by a light string going over a smooth light pulley on a smooth inclined plane as shown in the figure. The system is at rest. The force exerted by the inclined plane on the body of mass m_1 will be</p> 	A
	$\frac{\sqrt{3}mg}{2}$	
	$\frac{mg}{2}$	
	2mg	

	$\frac{mg}{\sqrt{3}}$	
Sol.	 <p>Force exerted by inclined plane on m_1 is N.</p> $mg \sin \theta = \frac{mg}{2} \Rightarrow \theta = 30^\circ$ $mg \cos \theta = N \Rightarrow N = \frac{\sqrt{3}}{2} mg$	
4.	<p>Consider a series LCR circuit with an ac source then</p> <p>Statement I: Kirchhoff's current and voltage laws are applicable for instantaneous values of currents and voltages in the circuit.</p> <p>Statement II: Kirchhoff's current and voltage laws application for root mean square values of currents and voltages in the circuit.</p>	C
	Both the statements are TRUE	
	Both the statements are FALSE	
	Statement I is TRUE and Statement II is FALSE	
	Statement I is FALSE and Statement II is TRUE	
Sol.	Kirchoff's law are true for instantaneous values of currents and voltages only.	
5.	A ball of mass m makes a head on elastic collision with a second ball (at rest) and rebounds with a speed equal to one – fourth of its original speed. What is the mass of the second ball?	D
	$(3/5) m$	
	$(1/3) m$	
	$(7/3) m$	
	$(5/3) m$	
Sol.	<p>Conservation of the momentum $m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \therefore v_1' = -\frac{v_1}{4}$</p> $mv_1 = m\left(\frac{-v_1}{4}\right) + m_2 v_2'; \quad m_2 v_2' = 5m \frac{v_1}{4}$ $v_1 - v_2 = v_2' - v_1'$ $v_2' = v_1 + v_1' = \frac{3v_1}{4} \quad \therefore m_2 = \frac{5m}{3}$	
6.	The bulk modulus of a liquid is $3 \times 10^8 \text{ Nm}^{-2}$. The pressure required to reduce the volume of the liquid by 5 % is	C
	$5 \times 10^6 \text{ N/m}^2$	
	$10 \times 10^6 \text{ N/m}^2$	
	$15 \times 10^6 \text{ N/m}^2$	
	$20 \times 10^6 \text{ N/m}^2$	

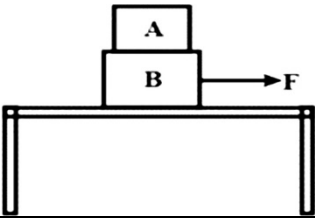
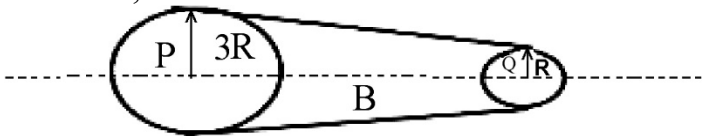
Sol.	$B = \frac{\Delta p}{\frac{-\Delta v}{v}} \Rightarrow \Delta p = -B \frac{\Delta v}{v} = 3 \times 10^8 \times \frac{5}{100} = 15 \times 10^6 \text{ Pa.}$	
7.	<p>Given below are two statements one is labeled as Assertion(A) and the other is labelled Reason(R).</p> <p>Assertion(A): Some insects are able to walk on water.</p> <p>Reason(R): The surface of water acts as a stretched membrane due to surface tension</p>	A
	(A) & (R) are true and (R) is the correct explanation of (A)	
	(A) & (R) are true but (R) is not correct explanation of (A)	
	(A) is true but (R) is false	
	Both (A) and (R) are false	
Sol.	INSECTS WALK ON WATER DUE TO SURFACE TENSION.	
8.	<p>Two different metal bodies A and B; mass of A being twice that of B, are heated at a uniform rate under similar condition. The variation of the temperature of the bodies is graphically represented as below. The ratio of the specific heat capacities of A and B is</p>	B
	$\frac{8}{3}$	
	$\frac{9}{40}$	
	$\frac{12}{35}$	
	$\frac{3}{8}$	
Sol.	$dQ = msdt$ $\Rightarrow S = \frac{dQ}{mdt} \times \frac{dt}{dt} a \frac{dt}{mT}$ $\therefore \frac{S_A}{S_B} = \frac{3}{5} \times \frac{1}{2} \times \frac{90}{120} = \frac{9}{40}$	
9.	A particle executes simple harmonic motion (amplitude = A) between $x = -A$ and $x = +A$. The time taken for it to go from 0 to $A/2$ is T_1 and to go from $A/2$ to A is T_2 . Then	A
	$T_1 < T_2$	
	$T_1 > T_2$	

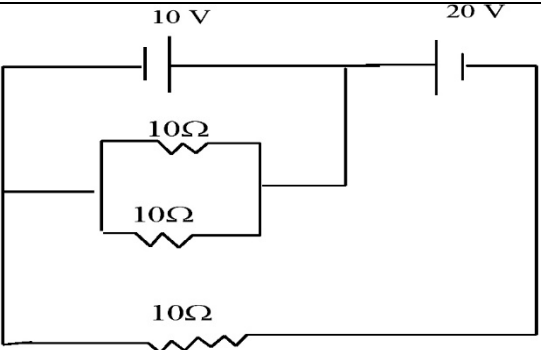
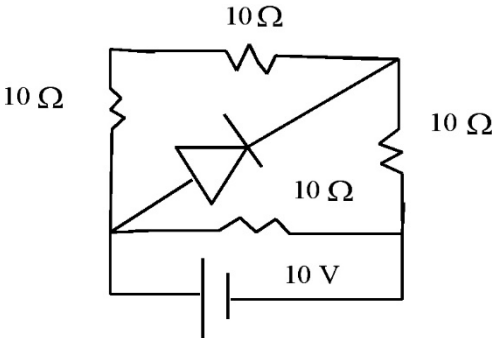
	$T_1 = T_2$	
	$T_1 = 2T_2$	
Sol.	<p>Using $x = A \sin \omega t$</p> <p>For $x = A/2$, $\sin \omega T_1 = 1/2 \Rightarrow T_1 = \frac{\pi}{6\omega}$</p> <p>For $x = A$, $\sin \omega(T_1 + T_2) = 1 \Rightarrow T_1 + T_2 = \frac{\pi}{2\omega}$</p> <p>$\Rightarrow T_2 = \frac{\pi}{2\omega} - T_1 = \frac{\pi}{2\omega} - \frac{\pi}{6\omega} = \frac{\pi}{3\omega}$ i.e. $T_1 < T_2$</p>	
10.	<p>ABC is a right angled triangular plate of uniform thickness. I_1, I_2 and I_3 are moments of inertia about AB, BC and AC respectively. Then which of the following relation is correct.</p> 	C
	$I_1 = I_2 = I_3$	
	$I_2 > I_1 > I_3$	
	$I_3 < I_2 < I_1$	
	$I_3 > I_1 > I_2$	
Sol.	<p>$AC > BC > AB$</p> <p>Moment of inertia of a body depends on distribution of mass from the axis of rotation. The mass is farthest from the axis AB. So, I_1 is maximum and nearest from the axis AC. So, I_3 is minimum or $I_3 < I_2 < I_1$.</p>	
11.	<p>In the circuit shown the charge on $5\mu F$ capacitor is</p> 	B
	$10 \mu C$	
	$20 \mu C$	
	$30 \mu C$	
	$40 \mu C$	

Sol.	<p>The p.d across $5\mu F$ capacitor is</p> $= \frac{C_1 V_1 + C_2 V_2 + C_3 V_3}{C_1 + C_2 + C_3} = \frac{2\mu \times 6 + 4\mu \times 8}{2\mu + 4\mu + 5\mu} = \frac{44}{11} = 4V$ $q = CV = 5 \times 4 = 20\mu C$	
12.	<p>Find the equivalent resistance between A and B in the adjacent circuit</p> 	D
	$\frac{R}{3}$	
	R	
	3R	
	$\frac{2R}{3}$	
Sol.	<p>C & D are equipotential</p> $R_{AB} = (3R \sim 4R) \parallel (3R \sim 4R) \parallel R$ $= 4R \parallel 4R \parallel R = 2R \parallel R = \frac{2R}{3}$ 	
13.	<p>A point particle 'P' of mass m and charge q and another point particle 'Q' of mass 2 m and charge q are attached at the ends of a massless rod of length 'l'. The system is now rotated with uniform angular velocity 'w' about an axis passing through the centre of mass & perpendicular to the length of the rod. The magnetic moment of the system is</p>	A
	$\frac{5ql^2 w}{18}$	
	$\frac{4ql^2 w}{9}$	
	$\frac{8ql^2 w}{9}$	
	$\frac{ql^2 w}{9}$	
Sol.	$M = iA = q \frac{w}{2\pi} \pi r^2 = \frac{q\omega r^2}{2} = \frac{q\omega}{2} \left(\left(\frac{l}{3} \right)^2 + \left(\frac{2l}{3} \right)^2 \right) = \frac{5ql^2 w}{18}$	
14.	In a coil of resistance 10Ω a current is induced by changing the magnetic flux	B

	<p>through it. The current induced as a function of time is shown in given figure. The magnitude of change in flux through the coil is</p> 	
	25 Wb	
	24 Wb	
	20 Wb	
	15 Wb	
Sol.	$q = \frac{\Delta\phi}{R} \Rightarrow \Delta\phi = Rq = 10 \times \text{Area Under graph}$ $= 10 \times \frac{1}{2} \times 0.6 \times 8 = 24 \text{ Wb}$	
15.	<p>A point source of light S is placed at the bottom of a vessel containing a liquid of refractive index $5/3$. A person is viewing the source from above the surface. There is an opaque disc D of radius 1 cm floating on the surface of the liquid. The centre of the disc lies vertically above the source S. The liquid from the vessel is gradually drained out through a tap. The maximum height of the liquid for which the source cannot be seen at all from above is</p> 	C
	1.50 cm	
	1.64 cm	
	1.33 cm	
	1.86 cm	
Sol.	<p>Suppose the maximum height of the liquid is h for which the source is not visible. Hence radius of the disc</p> $r = \frac{h}{\sqrt{\mu^2 - 1}}$ $1 = \frac{h}{\sqrt{\left(\frac{5}{3}\right)^2 - 1}} \Rightarrow h = 1.33 \text{ cm}$ 	
16.	<p>In a Fraunhofer single slit diffraction experiment, the slit is illuminated by two different wavelengths λ_1 and λ_2. If the first principal minima of λ_1 coincides with the first principal maxima of λ_2 then $\frac{\lambda_1}{\lambda_2} =$</p>	B
	$\frac{2}{1}$	
	$\frac{3}{2}$	

	$\frac{5}{3}$	
	$\frac{8}{5}$	
Sol.	<p>for 1st principle minima: $\frac{d}{2} \sin \theta = \frac{\lambda_1}{2}$ ----- (1)</p> <p>for 1st principle maxima $\frac{d}{3} \sin \theta = \frac{\lambda_2}{2}$ ----- (2)</p> <p>$\frac{(1)}{(2)} = \frac{\lambda_1}{\lambda_2} = \frac{3}{2}$</p>	
17.	Consider the following statements	
	<p>1. The de-broglie wavelength of an electron accelerated from rest by 150 V is about 1 Å</p> <p>2. There is no de-broglie wave associated with a car moving at 10 m/s</p> <p>3. When a bomb at rest explodes into two pieces, the de-broglie wavelength's of the two pieces are equal</p> <p>4. For Oxygen molecule of mass 'm' the de-broglie wavelength at T K is $\frac{h}{\sqrt{3mkT}}$</p> <p>The correct statements are</p>	D
	All are correct	
	1, 2, 3 only	
	1 and 3 only	
	1, 3 and 4 only	
Sol.	$\lambda = \sqrt{\frac{150}{V}} \text{ Å}$	
18.	Imagine that the electron in a hydrogen atom is replaced by a muon (μ). The mass of muon particle is 207 times that of an electron and charge is equal to the charge of an electron. The ionization potential of this hydrogen atom will be:	D
	27.2 eV	
	331.2 eV	
	13.6 eV	
	2815.2 eV	
Sol.	<p>So, $E \propto m$</p> <p>Ionization potential = $13.6 \times \frac{(mass_\mu)}{(mass_e)} \text{ eV} = 13.6 \times 207 \text{ eV}$</p> <p>= 2815.2 eV</p>	
19.	A nucleus A of mass number 220 and binding fraction 5.6 MeV per nucleon splits into two fragments of mass numbers 100 and 120 each of average binding energy 6.4 MeV per nucleon. The energy released per fission will be	D
	0.8 MeV	
	275 MeV	
	220 MeV	
	176 MeV	
Sol.	$Q = (6.4 - 5.6) \times 220 = 0.8 \times 220 = 176 \text{ MeV}$	
20.	Two blocks A and B of masses $m_A = 2 \text{ kg}$ and $m_B = 4 \text{ kg}$ are kept on the table as shown in figure. The coefficient of friction between A and B is 0.2 and between B and the surface of the table is also 0.2. The maximum force F (in N) that can be	A

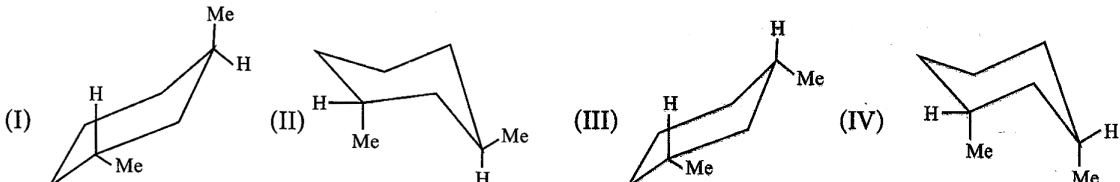
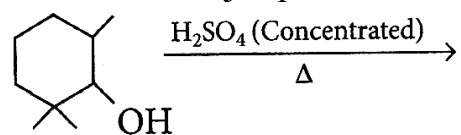
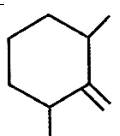
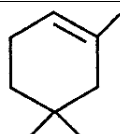
	<p>applied on B horizontally, so that the block A does not slide over the block B is (Take $g = 10 \text{ m/s}^2$)</p> 	
	24	
	20	
	25	
	30	
Sol.	$f_A = \mu m_A g = 0.2 \times 2 \times 10 = 4 \text{ N}$ $f_B = \mu(m_A + m_B)g = 0.2 \times 6 \times 10 = 12 \text{ N}$ For B to slide over table $F > f_B = 12 \text{ N}$ One B slides $a_B = \frac{F - 12}{2 + 4}$ (Assuming A & B move together) Now $f_A = m_A a_B = 2 \left[\frac{F - 12}{6} \right]$ As $(f_A)_{\max} = 4$ we have $4 = 2 \left[\frac{F - 12}{6} \right] \Rightarrow F_{\max} = 24 \text{ N}$	
21.	<p>In the given figure two wheels P & Q are connected by a belt B. The radius of P is three times that of Q. In case of same angular momentum for the two wheels about their axis of rotation, the ratio of their moment of inertia's will be</p> 	3
Sol.	$I_1 \omega_1 = I_2 \omega_2$ By constant $V_P = 3R\omega_1 = V_Q = R\omega_2$ $\Rightarrow \frac{\omega_1}{\omega_2} = \frac{1}{3} \therefore \frac{I_1}{I_2} = \frac{\omega_2}{\omega_1} = \frac{3}{1}$	
22.	<p>In a certain thermodynamic process the temperature of a gas is proportional to the square of its pressure. The workdone by one mole of the gas when the temperature increases by 80°C will be xR. $x =$</p>	40
Sol.	$T \propto P^2 \Rightarrow PV \propto P^2 \Rightarrow PV^{-1} = C$ Polytropic exponent $x = -1$ Now $W = \frac{nR}{1 - x}(T_2 - T_1) = \frac{R}{1 - (-1)} \times 80 = 40R$	
23.	In the given circuit the current through the 10 V cell is (in Amperes)	1

		
Sol.	<p>Current in 5Ω $i_1 = \frac{10}{5} = 2A$.</p> <p>Current in 10Ω $i_2 = \frac{10}{10} = 1A$</p> <p>Current through $10V$ cell is $i_1 - i_2 = 2 - 1 = 1A$</p>	
24.	<p>In the given circuit the value of current (in A) through the battery will be (the diode is ideal)</p> 	2
Sol.	<p>The equivalent</p> $i = \frac{10}{5} = 2A$	
25.	<p>A square loop of side 12 cm with its sides parallel to X and Y axes is moved with a velocity of 8 cms^{-1} in the positive x-direction in an environment containing a magnetic field in the positive z-direction. The field is neither uniform in space nor constant in time. If has a gradient of 10^{-3} Tcm^{-1} along the negative x-direction (that is it increases by 10^{-3} Tcm^{-1} as one moves in the negative x-direction), and it is decreasing in time at the rate of 10^{-3} Ts^{-1}. The magnitude of the induced current in the loop if its resistance is $4.5\text{m}\Omega$ is ----- mA.</p>	29
Sol.	$\phi = \int_0^a B(x,t) dx$ $\frac{d\phi}{dt} = a \int_0^a \frac{dB(x,t)}{dt} dx = a \int_0^a \left(\frac{\partial B(x,t)}{\partial t} + \frac{\partial B}{\partial x} \times \frac{dx}{dt} \right) dx$ $= A \left[\frac{\partial B}{\partial t} + V \frac{\partial B}{\partial x} \right] = 144 \times 10^{-4} [10^{-3} \times 8 + 10^{-3}]$ $= 1296 \times 10^{-7}$ $\therefore I = \frac{E}{R} = \frac{1296 \times 10^{-7}}{4.5 \times 10^{-3}} = 288 \times 10^{-4} = 28.8 \text{ mA}$	
26.	<p>Why is the oxidation of a primary alcohol with a mixture of sodium dichromate</p>	B

	and sulphuric acid not a good method for the preparation of corresponding aldehyde?	
	The product will be the corresponding ketone	
	Any aldehyde produced will be oxidized further	
	The product will be an alkyl sulphonate, R – SO ₃ H mixture of sodium dichromate and sulphuric acid will not oxidize a primary alcohol.	
	A mixture of sodium dichromate and sulphuric acid will not oxidize a primary alcohol	
Sol.	<p>Primary alcohols can be oxidized to either aldehydes or carboxylic acids depending on the reaction conditions.</p> <p>In the case of the formation of carboxylic acids, the alcohol is first oxidized to an aldehyde which is then oxidized further to the acid.</p> <p>Primary Alcohol $\xrightarrow{[O]}$ Aldehyde $\xrightarrow{[O]}$ Carboxylic acid</p> <p>So, Any aldehyde produced will be oxidized further during the reaction.</p>	
27.	The solubility of a sparingly soluble salt A(OH) ₂ (mol. Wt. 192.3) is 19.23 g/litre at 300 K. The pH of its saturated solution assuming 80% ionization at 300 K is :	C
	1.0970	
	12.9030	
	13.2041	
	12.0000	
Sol.	<p>$S = \frac{19.23}{192.3} \text{ mol/litre} = 0.1 \text{ M}$</p> <p>$A(OH)_2 \rightleftharpoons A^{2+} + 2OH^-$</p> <p>$\therefore [OH^-] = \frac{0.1 \times 2 \times 80}{100} = 0.16$</p> <p>$\therefore pOH = 0.7958$</p> <p>$\therefore pH = 13.2041$</p>	
28.	<p>The freezing point of a 0.1M formic acid aqueous solution is – 0.2046°C. Find equilibrium constant of the reaction –</p> <p>$HCOO^-(aq) + H_2O(l) \rightleftharpoons HCOOH(aq) + OH^-(aq)$</p> <p>Given : $K_f(H_2O) = 1.86 \text{ K-kg mole}^{-1}$</p> <p>Assume solution to be very dilute</p>	B
	$1.1 \times 10^{-3} \text{ M}$	
	$9 \times 10^{-12} \text{ M}$	
	$9 \times 10^{-13} \text{ M}$	
	$1.1 \times 10^{-11} \text{ M}$	
Sol.	<p>$HCOOH \rightleftharpoons HCOO^- + H^+$</p> <p>0.1- X X X</p> <p>$\Delta T_f = 0.2046 = 1.86 \times m$</p> <p>$m = \frac{0.2046}{1.86} = 0.1 + X = 0.11$</p> <p>$K_c = \frac{K_w}{K_a}$</p> <p>$\therefore K_a = \frac{X^2}{0.1 - X} = \frac{10^{-2}}{9}$</p> <p>$\Rightarrow K_b (HCOO^-) = 9 \times 10^{-12} \text{ M}$</p>	

29.	The compounds those would not respond to tests of both nitrogen and sulfur with sodium fusion extracts are?	A
	$\text{H}_2\text{N}-\overset{\oplus}{\text{N}}\text{H}_3\cdot\text{HSO}_4^\ominus \quad \text{H}_2\text{N}-\overset{\text{S}}{\parallel}-\text{NH}_2 \quad \text{H}_2\text{N}-\overset{\oplus}{\text{N}}\text{H}_3\cdot\text{HCO}_3^\ominus \quad \text{H}_2\text{N}-\overset{\text{O}}{\parallel}-\text{S}-\text{CH}_3$ <p style="text-align: center;">I II III IV</p>	
	I and III	
	III and IV	
	I and IV	
	II and IV	
30.	Select incorrect statement.	A
	Octahedral Co (III) complexes with strong field ligands have very high magnetic moments.	
	When $\Delta_0 < P$, then d– electron configuration is $t_{2g}^4 e_g^2$	
	Complex $[\text{Co}(\text{C}_2\text{O}_4)_3]^{3-}$ has zero magnetic moment	
	If the Δ_0 for a Co (III) complex is 18000 cm^{-1} , then Δ_t for Co (III) complex with same ligand will be 8000 cm^{-1} .	
Sol.	Octahedral Co (III) complexes with strong field ligands have zero magnetic moment.	
31.	3 moles of FeSO_4 are oxidised by a moles of $\text{K}_2\text{Cr}_2\text{O}_7$ in acidic medium whereas 3 moles of FeC_2O_4 are oxidized by b moles of $\text{K}_2\text{Cr}_2\text{O}_7$ in acidic medium, the ratio of a and b is :-	A
	$\frac{1}{3}$	
	$\frac{1}{2}$	
	$\frac{1}{4}$	
	$\frac{1}{5}$	
Sol.	<p>geq of FeSO_4 = geq of $\text{K}_2 \text{Cr}_2 \text{O}_7$</p> <p>$\therefore \text{mole} \times \text{n factor} = \text{geq}$</p> <p>$\therefore 3 \times 1 = a \times 6 \dots (i)$</p> <p>similarly geq of $\text{FeC}_2 \text{O}_4$ = geq of $\text{K}_2 \text{Cr}_2 \text{O}_7$</p> <p>$3 \times 3 = b \times 6 \dots (ii)$</p> <p>$\frac{3 \times 1}{3 \times 3} = \frac{a \times 6}{b \times 6}$</p> <p>$\left[\frac{a}{b} = \frac{1}{3} \right]$</p>	
32.	A variable, opposite external potential (E_{ext}) is applied to the cell $\text{Zn} \text{Zn}^{2+}(1 \text{ M}) \text{Cu}^{2+}(1 \text{ M}) \text{Cu}$ of potential 1.1 V. When $E_{\text{ext}} < 1.1 \text{ V}$ and $E_{\text{ext}} > 1.1 \text{ V}$, respectively electrons flow form :	B
	Anode to cathode in both cases	
	Anode to cathode and cathode to anode	
	Cathode to anode in both cases	
	Cathode to anode and anode to cathode	

Sol.	Cell is behaving as galvanic when $E_{\text{ext}} < 1.1 \text{ V}$ and a electrolytic when $E_{\text{ext}} > 1.1 \text{ V}$																					
33.	In the following statement, which combination of true (T) and false (F) options in correct? (A) Ionic mobility is highest for I^- in water as compared to other halides. (B) IF_5 is square pyramidal and IF_7 in pentagonal bipyramidal in shape (C) Reactivity order in $\text{F}_2 < \text{Cl}_2 < \text{Br}_2 < \text{I}_2$ (D) Oxidising power order is $\text{F}_2 < \text{Cl}_2 < \text{Br}_2 < \text{I}_2$	D																				
	TFTF																					
	TTFT																					
	TTTT																					
	TTFE																					
Sol.	Fluorine is most reactive amongst halides and has highest oxidising power																					
34.	An aqueous solution of a metal ion (A) on reaction with KI gives a black brownish ppt (B) and this aqueous suspension on treatment with excess KI gives orange yellowish solution. Then A is :	B																				
	Pb^{2+}																					
	Bi^{3+}																					
	Hg_2^{2+}																					
	Hg^{2+}																					
Sol.	$\text{Bi}^{3+} \xrightarrow{\text{KI}} \text{BiI}_3 \xrightarrow[\text{KI}]{\text{excess}} [\text{BiI}_4]^-$ (A) (B) Black ppt yellow brown solution																					
35.	Match List-I with List-II. <table border="1"><thead><tr><th colspan="2">List I</th><th colspan="2">List II</th></tr></thead><tbody><tr><td>A.</td><td>van't Hoff factor, i</td><td>I.</td><td>Cryoscopic constant</td></tr><tr><td>B.</td><td>k_f</td><td>II.</td><td>Isotonic solutions</td></tr><tr><td>C.</td><td>Solutions with same osmotic pressure</td><td>III.</td><td>Normal molar mass Abnormal molar mass</td></tr><tr><td>D.</td><td>Azeotropes</td><td>IV.</td><td>Solutions with same composition of vapour above it</td></tr></tbody></table> Choose the correct answer from the options given below:	List I		List II		A.	van't Hoff factor, i	I.	Cryoscopic constant	B.	k_f	II.	Isotonic solutions	C.	Solutions with same osmotic pressure	III.	Normal molar mass Abnormal molar mass	D.	Azeotropes	IV.	Solutions with same composition of vapour above it	D
List I		List II																				
A.	van't Hoff factor, i	I.	Cryoscopic constant																			
B.	k_f	II.	Isotonic solutions																			
C.	Solutions with same osmotic pressure	III.	Normal molar mass Abnormal molar mass																			
D.	Azeotropes	IV.	Solutions with same composition of vapour above it																			
	A-III, B-II, C-I, D-IV																					
	A-III, B-I, C-IV, D-II																					
	A-I, B-III, C-II, D-IV																					
	A-III, B-I, C-II, D-IV																					
Sol.	van't Hoff factor (i) = $\frac{\text{Normal molar mass}}{\text{Abnormal molar mass}}$ (A-III) Cryoscopic constant : K_f (B-I) Same osmotic pressure = Isotonic solution solution (C-II) Azeotropes – Solutions with same composition of vapours above it. (D-IV)																					
36.	Given the data at 25°C ,	D																				

	$\text{Ag} + \text{I}^- \rightarrow \text{AgI} + \text{e}^- ; E^\circ = 0.152 \text{ V}$ $\text{Ag} \rightarrow \text{Ag}^+ + \text{e}^- ; E^\circ = -0.800 \text{ V}$ <p>What is the value of $\log K_{sp}$ for AgI?</p> $\left(2.303 \frac{RT}{F} = 0.059 \text{ V}\right)$	
	- 8.12	
	+ 8.612	
	- 37.83	
	-16.13	
Sol.	$\text{AgI}_{(s)} + \text{e}^- \rightarrow \text{Ag}_{(s)} + \text{I}^- , E^\circ = -0.152 \text{ V}$ $\text{Ag}_{(s)} \rightarrow \text{Ag}^+ + \text{e}^- , E^\circ = -0.800 \text{ V}$ $\text{AgI}_{(s)} \rightarrow \text{Ag}^+ + \text{I}^- , E^\circ = -0.952 \text{ V}$ $E^\circ_{\text{cell}} = \frac{0.059}{n} \log K \Rightarrow -0.952 = \frac{0.059}{1} \log K_{sp}$ $\text{or } \log K_{sp} = -\frac{0.952}{0.059} = -16.135$	
37.	Which of the following pairs represents linkage isomers?	B
	$[\text{Cu}(\text{NH}_3)_4][\text{PtCl}_4]$ and $[\text{Pt}(\text{NH}_3)_4][\text{CuCl}_4]$	
	$[\text{Pd}(\text{PPh}_3)_2(\text{NCS})_2]$ and $[\text{Pd}(\text{PPh}_3)_2(\text{SCN})_2]$	
	$[\text{Co}(\text{NH}_3)_5(\text{NO}_3)]\text{SO}_4$ and $[\text{Co}(\text{NH}_3)_5(\text{SO}_4)]\text{NO}_3$	
	$[\text{PtCl}_2(\text{NH}_3)_4]\text{Br}_2$ and $[\text{PtBr}_2(\text{NH}_3)_4]\text{Cl}_2$	
Sol.	Linkage isomerism is exhibited by compounds containing ambidentate ligand.	
38.	Arrange in the correct order of stability (decreasing order) for the following molecules:	D
		
	(I) > (II) > (III) > (IV)	
	(IV) > (III) > (II) \approx (I)	
	(I) > (II) > \approx (III) > (IV)	
	(III) > (I) \approx (II) > (IV)	
Sol.	When two methyl groups on cyclohexane are present in axial form at 1, 3 position they exhibit 1, 3-diaxial interaction and the form becomes least stable. So, the stability order is III > I \approx II > IV	
39.	Find out the major product from the following reaction.	D
		
		
		

Sol.	<p style="text-align: center;"> $\text{1,2-dimethylcyclohexanol} \xrightarrow[\Delta]{\text{H}_2\text{SO}_4(\text{Conc.})} \text{2}^\circ \text{ carbocation} \xrightarrow{\text{1,2-methyl shift}} \text{3}^\circ \text{ carbocation} \xrightarrow{(-\text{H}^+)} \text{1,3-dimethylcyclohexene}$ </p>	
40.	<p>In the following reaction:</p> $ \begin{array}{c} \text{H}_3\text{C} \quad \text{CH}_3 \\ \diagdown \quad / \\ \text{CH} \\ \\ \text{C}_6\text{H}_5 \end{array} + \text{O}_2 \longrightarrow \begin{array}{c} \text{H}_3\text{C} \quad \text{CH}_3 \\ \diagdown \quad / \\ \text{C} - \text{O} - \text{O} - \text{H} \\ \\ \text{C}_6\text{H}_5 \end{array} \xrightarrow[\text{H}_2\text{O}]{\text{H}^+} \text{A and B} $ <p>The compounds A and B respectively are</p>	C
	, CH_3COOH	
	, CH_3COCH_3	
	, CH_3COCH_3	
	, CH_3COCH_3	
Sol.	<p style="text-align: center;"> $\begin{array}{c} \text{CH}_3 \quad \text{CH}_3 \\ \diagdown \quad / \\ \text{C} - \text{O} - \text{O} - \text{H} \\ \\ \text{C}_6\text{H}_5 \end{array} \xrightarrow[\text{H}_2\text{O}]{\text{H}^+} \text{C}_6\text{H}_5\text{OH} + \text{CH}_3 - \text{C}(=\text{O}) - \text{CH}_3$ </p>	
41.	Match List-I with List-II.	D

List-I		List-II	
(A)	$\text{C}_6\text{H}_5\text{N}_2^+\text{Cl}^- \xrightarrow{\text{Cu}_2\text{Cl}_2} \text{C}_6\text{H}_5\text{Cl} + \text{N}_2$	(i)	Wurtz reaction
(B)	$\text{C}_6\text{H}_5\text{N}_2^+\text{Cl}^- \xrightarrow{\text{Cu, HCl}} \text{C}_6\text{H}_5\text{Cl} + \text{N}_2$	(ii)	Sandmeyer reaction
(C)	$2\text{CH}_3\text{CH}_2\text{Cl} + 2\text{Na} \xrightarrow{\text{Ether}} \text{C}_2\text{H}_5 - \text{C}_2\text{H}_5 + 2\text{NaCl}$	(iii)	Fittig reaction
(D)	$2\text{C}_6\text{H}_5\text{Cl} + 2\text{Na} \xrightarrow{\text{Ether}} \text{C}_6\text{H}_5 - \text{C}_6\text{H}_5 + 2\text{NaCl}$	(iv)	Gatterman reaction

Choose the correct answer from the options given below.

(A) → (iii), (B) → (i), (C) → (iv) →, (D) → (ii)	
(A) → (iii), (B) → (iv), (C) → (i) →, (D) → (ii)	
(A) → (ii), (B) → (i), (C) → (iv) →, (D) → (iii)	
(A) → (ii), (B) → (iv), (C) → (i) →, (D) → (iii)	

42. The molar heat capacity of water at constant pressure, C_p is $75 \text{ J K}^{-1} \text{ mol}^{-1}$. When 10 kJ of heat is supplied to 1 kg water which is free to expand, the increase in temperature of water is :

A

2.4 K

4.8 K

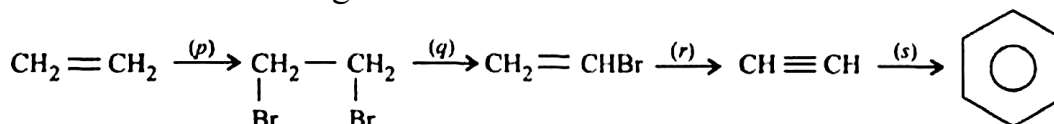
3.3 K

7.2 K

Sol. $q_m = mc \Delta T$
 $q_m = 10 \text{ kJ or } 10000 \text{ J, } m = 1000 \text{ g}$
 $c = 75 \text{ J K}^{-1} \text{ mol}^{-1}$. Number of moles
 $= \frac{1000}{18} = 55.5 \text{ mol}$
 $\Delta T = \frac{10000}{55.5 \times 75} = 2.4 \text{ K}$

43. Choose the correct reagents used in the conversion

D



p = Br₂ q = alc. KOH r = NaOH s = Al₂O₃

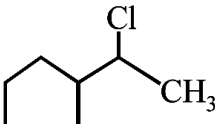
p = HBr q = alc. KOH r = CaC₂ s = KMnO₄

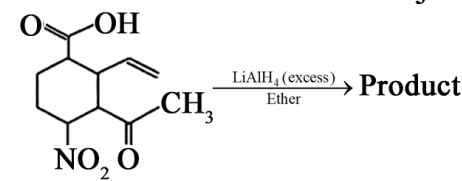
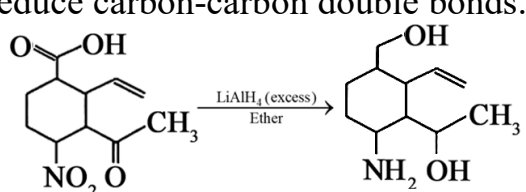
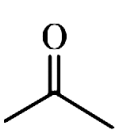
p = HBr q = alc. KOH r = NaNH₂ s = red hot iron tube

p = Br₂ q = alc. KOH r = NaNH₂ s = red hot iron tube

Sol. $\text{CH}_2=\text{CH}_2 \xrightarrow{\text{Br}_2} \underset{\text{Br}}{\text{CH}_2} - \underset{\text{Br}}{\text{CH}_2} \xrightarrow[\text{KOH}]{\text{alc.}} \text{CH}_2=\text{CHBr} \xrightarrow{\text{NaNH}_2} \text{CH}\equiv\text{CH} \xrightarrow[\text{iron tube}]{\text{red hot}} \text{Benzene ring}$

Here

	NaNH ₂ is used for second time elimination as it is stronger reagent and suitable for this step.	
44.	Intramolecular force between n-hexane and n-heptane are nearly same as between hexane and heptanes individually. When these two are mixed, which of the following is not true about the solution formed?	D
	It obeys Raoult's law, i.e., $p_A = x_A p_A^\circ$ and $p_B = x_B p_B^\circ$	
	ΔH_{mixing} is zero	
	ΔV_{mixing} is zero	
	Its forms minimum boiling azeotrope	
Sol.	n-hexane and n-heptane form an ideal solution so here azeotropic mixture is not possible.	
45.	<p>$A(g) + 2B(s) \rightleftharpoons 2C(g)$.</p> <p>Initially 2 mol A(g), 4 mole of B(s) and 1 mole of an inert gas are present in a closed container. After equilibrium has established total pressure of container becomes 9 atm.</p> <p>If A(g) is consumed 50% at equilibrium then calculate K_p for above reaction</p>	A
	9 atm	
	36/5 atm	
	12 atm	
	6 atm	
46.	<p>How many reactions giving alkyl halide as one of the final product?</p> <p>(a) $C_2H_5OH + SOCl_2 \rightarrow$</p> <p>(b) $R-I + AgF \xrightarrow{\text{Acetone}}$</p> <p>(c) $EtOH + NaI + H_2SO_4 \rightarrow$</p> <p>(d) $EtOH + NaI + H_3PO_4 \rightarrow$</p> <p>(e) $CH_3-CH=CH_2 \xrightarrow{HBr}$</p> <p>(f) $CH_3-CH_2-Cl \xrightarrow[\text{(Acetone)}]{NaI}$</p> <p>(g)  $\xrightarrow[\text{Ag}_2O]{\text{Moist}}$</p> <p>(h) $CH_3-O-C_2H_5 \xrightarrow{HI}$</p>	6
Sol.	(a), (c), (d), (e), (f), (h)	
47.	<p>One mole of a monatomic gas at pressure 2 atm, 279 K taken to final pressure 4 atm by a reversible path described by $P/V = \text{constant}$. Calculate the magnitude of $\frac{\Delta E}{w}$ of the process.</p>	3

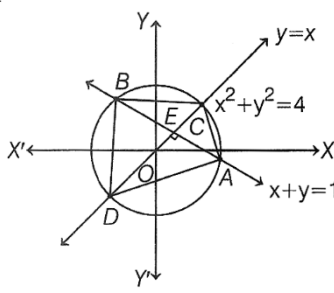
Sol.	$dE = dq + dw \text{ (F L T)}$ $P_1 = K' V_1 \text{ and } P_2 = K' V_2 \text{ Also}$ $V_1 = \frac{P_1}{K'}, V_2 = \frac{P_2}{K'}$ $T_2 = T_1 \left(\frac{P_2 V_2}{P_1 V_1} \right) = T_1 \left(\frac{P_2 P_2 / K'}{P_1 P_1 / K'} \right) = T_1 \frac{P_2^2}{P_1^2}$ $T_2 = T_1 \frac{4^2}{2^2} = 4 T_1$ $\Delta E = C_v (\Delta T) = C_v (4T_1 - T_1)$ $= 3C_v T_1$ $= 3 \frac{3}{2} R T_1 = 9 \frac{R T_1}{2}$ $w = - \int_{V_1}^{V_2} p dv = - \int_{V_1}^{V_2} K' v dv = - \left[\frac{K' v^2}{2} \right]_{V_1}^{V_2}$ $\left(K' v^2 = K' v v = P v = R T \right)$ $w = - \left[\frac{R T}{2} \right]_{T_1}^{T_2} = - \frac{R}{2} (T_2 - T_1) =$ $- \frac{R}{2} 3 T_1$ $\frac{\Delta E}{w} = \frac{9 R T_1 / 2}{-3 R T_1 / 2} = -3$	
48.	<p>The number of π bonds in the major product will be</p> <div style="text-align: center;">  </div>	1
Sol.	<p>LiAlH_4 reduces aldehydes, ketones and carboxylic acids to corresponding alcohols and nitro ($-\text{NO}_2$) group to amino ($-\text{NH}_2$) group. But LiAlH_4 cannot reduce carbon-carbon double bonds.</p> <div style="text-align: center;">  </div>	
49.	<p>Number of compounds which do not react with Fehling solution are x? (i) CH_3CHO, $\text{Ph}-\text{CHO}$, Glucose</p> <div style="text-align: center;">  </div> <p>(ii) Tollen's reagent can be used to distinguish between y number of pairs: $\text{CH}_3\text{CHOCH}_3$ and $\text{CH}_3\text{CH}(\text{OCH}_3)_2$, $\text{CH}_3\text{CCH}_2\text{OH}$ and CH_3COCH_3, CH_3CHO and HCOOH, CH_3CHO and CH_3COCH_3 [Give your answer as $x + y$]</p>	5
Sol.	<p>$x = 2; y = 3$ (i) Aromatic aldehyde and Ketones do not react with Fehling solution. (ii) CH_3CHO and HCOOH Both compounds give positive tollen's test.</p>	
50.	<p>Find out the % of oxalate ion in given sample of oxalate salt of which 0.3 g is present in 100 mL of solution required 90 mL of $\frac{N}{20}$ KMnO_4 for complete</p>	66

	oxidation. (Report your answer in two significant figures)	
Sol.	<p>The redox changes are</p> $\text{Mn}^{+7} + 5\text{e}^{-} \rightarrow \text{Mn}^{2+}$ $\left(\text{C}^{3+}\right)_2 \rightarrow 2\text{C}^{4+} + 2\text{e}^{-}$ <p>\therefore meq. of oxalate ion = meq. of KMnO_4</p> $(\text{w}/\text{E}) \times 1000 = 90 \times (1/20)$ <p>Or</p> $\left(\frac{\text{w}}{88/2}\right) \times 1000 = \frac{9}{2} \left(\because E_{\text{C}_2\text{O}_4^{2-}} = \frac{88}{2}\right)$ <p>or $\text{w}_{\text{C}_2\text{O}_4^{2-}} = 0.198\text{g}$</p> <p>$\therefore$ % of oxalate in sample</p> $= (0.198 \times 100)/0.3 = 66\%$	
51.	Let $P_n = \alpha^n + \beta^n, n \in N$. If $P_{10} = 123, P_9 = 76, P_8 = 47$ and $P_1 = 1$, then the quadratic equation having roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is	D
	$x^2 - x - 1 = 0$	
	$x^2 - x + 1 = 0$	
	$x^2 + x + 1 = 0$	
	$x^2 + x - 1 = 0$	
Sol.	<p>(d) $P_{10} = P_9 + P_8 \Rightarrow P_{10} - P_9 - P_8 = 0$</p> <p>By Newton's method, therefore, the equation is</p> $x^2 - x - 1 = 0 \text{ as } P_1 = 1$ $\alpha + \beta = 1 \text{ and } \alpha\beta = -1$ <p>\therefore Quadratic equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}$ is</p> $x^2 - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)x + \frac{1}{\alpha\beta} = 0$ $\Rightarrow x^2 + x - 1 = 0$ <p>Option (d) is correct.</p>	
52.	Number of ways of arranging 8 identical books into 4 identical shelves, where any number of shelves may remain empty is equal to	C
	12	
	16	
	15	
	18	
Sol.	<p>(c) 8 identical objects into 4 identical shelves, where any number of shelves may remain empty is given by</p> $0008, 0017, 0026, 0035, 0044 = 5$ $\begin{bmatrix} 0 & 1 & 1 & 6 \\ 0 & 1 & 2 & 5 \\ 0 & 1 & 3 & 4 \\ 0 & 2 & 2 & 4 \\ 0 & 2 & 3 & 3 \end{bmatrix} = 5$ $\begin{bmatrix} 1 & 1 & 1 & 5 \\ 1 & 1 & 2 & 4 \\ 1 & 1 & 3 & 3 \\ 1 & 2 & 2 & 3 \end{bmatrix} = 4$ $\begin{bmatrix} 2 & 2 & 2 & 2 \end{bmatrix} = 1$ <p>\therefore Total = 15</p>	

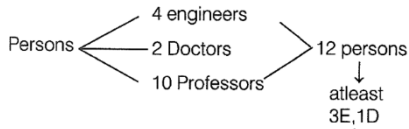
53.	Let $\alpha = \sum_{r=0}^n (4r^2 + 2r + 1)^n C_r$ and $\beta = \left(\sum_{r=0}^n \frac{{}^nC_r}{r+1} \right) + \frac{1}{n+1}$. If $140 < \frac{2\alpha}{\beta} < 281$, then the value of n is _____.	A
	5	
	10	
	20	
	25	
Sol.	$\therefore \alpha = \sum_{r=0}^n (4r^2 + 2r + 1)^n C_r$ $= \sum_{r=1}^n r^2 \frac{n}{r} {}^{n-1}C_{r-1} + 2 \sum_{r=1}^n r \frac{n}{r} {}^{n-1}C_{r-1} + \sum_{r=0}^n {}^nC_r$ $= \sum_{r=1}^n r \cdot {}^{n-1}C_{r-1} + 2n \sum_{r=1}^n {}^{n-1}C_{r-1} + \sum_{r=0}^n {}^nC_r$ $= \sum_{r=1}^n (n+1) 2^{n-2} + 2n 2^{n-1} + 2^n$ $= \sum_{r=1}^n [n(n+1) + n+1]$ $= \sum_{r=1}^n [n^2 + 2n + 1] = 2^n (n+1)^2$ <p>Now, $\beta = \sum_{r=0}^n \left(\frac{{}^nC_r}{r+1} + \frac{1}{n+1} \right)$</p> $= \sum_{r=0}^n \left(\frac{{}^{n+1}C_{r+1}}{n+1} + \frac{1}{n+1} \right) = \frac{1}{n+1} \cdot 2^{n+1}$ $= \frac{2\alpha}{\beta} = 2 \cdot \frac{2^n (n+1)^2 \times (n+1)}{2^{n+1}} = (n+1)^3$ <p>Given, that, $140 < \frac{2\alpha}{\beta} < 281$</p> $= 140 < (n+1)^3 < 281$ <p>Now, for $n = 4$; $(n+1)^3 = 5^3 = 125$ For $n = 5$; $(n+1)^3 = 6^3 = 216$ For $n = 6$; $(n+1)^3 = 7^3 = 343$ Hence, the value of $n = 5$.</p>	
54.	Let a_1, a_2, a_3, \dots be in arithmetic progression of positive terms. Let $A_k = a_1^2 - a_2^2 + a_3^2 - a_4^2 + \dots + a_{2k-1}^2 - a_{2k}^2$. If $A_3 = -153, A_5 = -435$ and $a_1^2 + a_2^2 + a_3^2 = 66$, then $a_{17} - A_7$ is equal to _____.	A
	910	
	999	
	950	
	980	

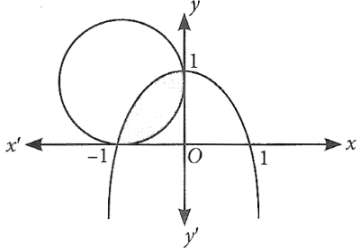
<p>Sol.</p>	<p>: a_1, a_2, a_3, \dots is an A.P.</p> <p>Let d be the common difference</p> <p>$\therefore a_2 - a_1 = a_3 - a_2 = \dots = a_{2k} - a_{2k-1} = d$</p> <p>Now,</p> $a_1^2 - a_2^2 + a_3^2 - a_4^2 + \dots + a_{2k-1}^2 - a_{2k}^2$ $= (a_1 - a_2)(a_1 + a_2) + \dots + (a_{2k-1} - a_{2k})(a_{2k-1} + a_{2k})$ $= -d[a_1 + a_2 + \dots + a_{2k-1} + a_{2k}]$ $= -d \cdot \left[\frac{2k}{2} [a_1 + a_{2k}] \right] = -dk(a_1 + a_{2k})$ <p>$\therefore A_k = -dk(a_1 + a_{2k})$</p> <p>So, $A_3 = -3d(a_1 + a_6) = -153$</p> $\Rightarrow -3d(a_1 + a_1 + 5d) = -153$ $\Rightarrow -3d(2a_1 + 5d) = -153$ $\Rightarrow 2a_1 + 5d = \frac{51}{d} \quad \dots(i)$ <p>Similarly $A_5 = -435$</p> $\Rightarrow -5d(2a_1 + 9d) = -435$ $\Rightarrow 2a_1 + 9d = \frac{87}{d} \quad \dots(ii)$ <p>Solving (i) and (ii) we get</p> $4d = \frac{36}{d} \Rightarrow d^2 = 9 \Rightarrow d = 3$ <p>[\therefore Given A.P. is a progression of positive terms]</p> $\Rightarrow a_1 = 1$ <p>Now, $a_{17} = a_1 + 16d = 1 + 48 = 49$</p> <p>and $A_7 = -3 \times 7(a_1 + a_{14})$</p> $= -21(1 + 1 + 13 \times 3) = -21(41) = -861$ <p>$\therefore a_{17} - A_7 = 49 + 861 = 910$</p>	
<p>55.</p>	<p>Let $y^2 = 12x$ be the parabola and S be its focus. Let PQ be a focal chord of the parabola such that $(SP)(SQ) = \frac{147}{4}$. Let C be the circle described taking PQ as a diameter. If the equation of a circle C is $64x^2 + 64y^2 - \alpha x - 64\sqrt{3}y = \beta$, then $\beta - \alpha$ is equal to _____.</p>	<p>B</p>
	<p>1325</p>	
	<p>1328</p>	
	<p>1340</p>	
	<p>1400</p>	

Sol.	<p>: Given, equation of parabola is</p> $y^2 = 12x$ $= S = (3, 0)$ <p>$(3t_1^2, 6t_1)$ and $Q(3t_2^2, 6t_2)$ are points on parabola</p> $t_1 t_2 = -1$ $\Rightarrow P \equiv (3t^2, 6t) \text{ and } Q \equiv \left(\frac{3}{t^2}, \frac{-6}{t}\right)$ $(SP)(SQ) = \frac{147}{4}$ $\Rightarrow (3 + 3t^2) \left(3 + \frac{3}{t^2}\right) = \frac{147}{4}$ <p>Now, $(SP) = PM$ and $(SQ) = QN$, where PM and QN are perpendicular distance from directrix)</p> $\Rightarrow \frac{(1+t^2)^2}{t^2} = \frac{49}{12} \Rightarrow 12t^4 - 25t^2 + 12 = 0$ $\Rightarrow t^2 = \frac{3}{4}, \frac{4}{3} \Rightarrow t = \pm \frac{\sqrt{3}}{2} \text{ or } t = \pm \frac{2}{\sqrt{3}}$ <p>Case I: When $t = \frac{\sqrt{3}}{2}$ or $t = -\frac{\sqrt{3}}{2}$</p> <p>Point are $P\left(\frac{9}{4}, 3\sqrt{3}\right)$ and $Q(4, -4\sqrt{3})$</p> <p>\Rightarrow equation of circle is</p> $(x-4)\left(x-\frac{9}{4}\right) + (y-3\sqrt{3})(y+4\sqrt{3}) = 0$ $\Rightarrow x^2 + y^2 - \frac{25x}{4} + \sqrt{3}y - 27 = 0$ <p>On comparing with given equation of circle</p> $64x^2 + 64y^2 - \alpha x - 64\sqrt{3}y = \beta, \text{ we get } \alpha = 400, \beta = 1728$ <p>Case II: When $t = \frac{-\sqrt{3}}{2}$ or $t = \frac{2}{\sqrt{3}}$</p> <p>Point are $P\left(\frac{9}{4}, -3\sqrt{3}\right)$ and $Q(4, 4\sqrt{3})$</p> <p>Similarly, we get $\alpha = 400, \beta = 1728$</p> $\therefore \beta - \alpha = 1728 - 400 = 1328$	
56.	<p>Let the line $x + y = 1$ meet the circle $x^2 + y^2 = 4$ at the points A and B. If the line perpendicular to AB and passing through the mid-point of the chord AB intersects the circle at C and D, then the area of the quadrilateral $ADBC$ is equal to</p>	D
	$\sqrt{14}$	
	$5\sqrt{7}$	
	$3\sqrt{7}$	
	$2\sqrt{14}$	

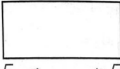
<p>Sol.</p>	<p>(d) First of all we have to write equation of line CD.</p>  <p>As $CD \perp AB$</p> $\therefore m_{CD} = \frac{-1}{m_{AB}} = \frac{-1}{-1} = 1$ <p>and CD passes through mid-point of AB.</p> <p>when we solve $x + y = 1$ with $x^2 + y^2 = 4$.</p> <p>then we get coordinats of A and B</p> <p>i.e. $A \equiv \left(\frac{(1 + \sqrt{7})}{2}, \frac{1 - \sqrt{7}}{2} \right)$ and</p> $B \equiv \left(\frac{1 - \sqrt{7}}{2}, \frac{1 + \sqrt{7}}{2} \right)$ <p>and thus, we find</p> <p>that mid-point of AB is $\left(\frac{1}{2}, \frac{1}{2} \right)$.</p> <p>So, equation of line CD will be</p> $y - \frac{1}{2} = 1 \left(x - \frac{1}{2} \right)$ $\Rightarrow y = x \quad \dots(i)$ <p>Now, solving $y = x$ with $x^2 + y^2 = 4$ to get coordinates of C and D. Then,</p> $x^2 + x^2 = 4$ $\Rightarrow x^2 = 2$ $\Rightarrow x = \pm \sqrt{2}$ $\therefore y = \pm \sqrt{2}$ <p>So, $C \equiv (\sqrt{2}, \sqrt{2})$ and $D \equiv (-\sqrt{2}, -\sqrt{2})$</p> <p>[$\because C$ lies in first quadrant and D lies in third quadrant]</p> <p>Now, area of quadrilateral $ADBC$</p> $= 2 \times (\text{area of } \triangle BCD)$ $= 2 \times \frac{1}{2} \begin{vmatrix} \sqrt{2} & \sqrt{2} & 1 \\ \frac{1 - \sqrt{7}}{2} & \frac{1 + \sqrt{7}}{2} & 1 \\ -\sqrt{2} & -\sqrt{2} & 1 \end{vmatrix}$ $= 2\sqrt{14} \text{ sq unit}$	
<p>57.</p>	<p>The set of all values of a for which $\lim_{x \rightarrow a} [x - 5] - [2x + 2] = 0$, where $[\alpha]$ denotes the greatest integer less than or equal to α is equal to</p> <p><input type="checkbox"/> $[-7.5, -6.5)$</p> <p><input type="checkbox"/> $(-7.5, -6.5)$</p> <p><input type="checkbox"/> $(-7.5, -6.5]$</p> <p><input type="checkbox"/> $[-7.5, -6.5]$</p>	<p>A</p>

Sol.	<p>(a) We have,</p> $\lim_{x \rightarrow \alpha} ([x - 5] - [2x + 2]) = 0$ $[\alpha - 5] - [2\alpha + 2] = 0$ $[\alpha - 5] = [2\alpha + 2] \dots(i)$ <p>For algebraic calculation,</p> $\alpha - 5 = 2\alpha + 2 \Rightarrow \alpha = -7$ <p>So, its value will be around -7.</p> <p>For $\alpha = -7.5$ in Eq. (i),</p> $[-7.5 - 5] = [-15 + 2]$ $\Rightarrow [-12.5] = [-13]$ $\Rightarrow -13 = -13 \Rightarrow \text{True}$ <p>For $\alpha = -6.5$ in Eq. (i),</p> $[-6.5 - 5] = [-13 + 2]$ $\Rightarrow [-11.5] = [-11]$ $\Rightarrow -12 \neq -11$ $\Rightarrow \text{Not satisfies.}$ <p>\therefore The required range will be</p> $\alpha \in [-7.5, -6.5).$	
58.	If the mean and the variance of 6, 4, a , 8, b , 12, 10, 13 are 9 and 9.25 respectively, then $a + b + ab$ is equal to	C
	106	
	100	
	103	
	105	
Sol.	<p>multiplied by c.</p> $\frac{a + b + 8 + 12 + 10 + 6 + 4 + 13}{8} = 9$ $\Rightarrow a + b = 19 \dots(i)$ $\frac{\sum x_i^2}{8} - 9^2 = 9.25$ $\Rightarrow a^2 + b^2 + 8^2 + 12^2 + 10^2 + 6^2 + 4^2 + 13^2 = (81 + 9.25) \times 8$ $\Rightarrow a^2 + b^2 = 193 \dots(ii)$ $\Rightarrow ab = \frac{(a + b)^2 - (a^2 + b^2)}{2}$ $= \frac{361 - 193}{2} = 84$ <p>[using Eqs. (i) and (ii)]</p> $\Rightarrow a + b + ab = 19 + 84 = 103$	
59.	The probability, of forming a 12 persons committee from 4 engineers, 2 doctors and 10 professors containing at least 3 engineers and at least 1 doctor, is	D
	$\frac{17}{26}$	
	$\frac{103}{182}$	
	$\frac{19}{26}$	
	$\frac{129}{182}$	

Sol.	<p>(d)</p>  <p> \therefore Number of favourable cases $3E1D8F + 3E2D7F + 4E1D7F + 4E2D6F$ $= {}^4C_3 \cdot {}^2C_1 \cdot {}^{10}C_8 + {}^4C_3 \cdot {}^2C_2 \cdot {}^{10}C_7$ $+ {}^4C_4 \cdot {}^2C_1 \cdot {}^{10}C_7 + {}^4C_4 \cdot {}^2C_2 \cdot {}^{10}C_6 = 1290$ \therefore Required probability $= \frac{1290}{{}^{16}C_{12}}$ $= \frac{1290 \times 4 \times 3 \times 2 \times 1}{16 \times 15 \times 14 \times 13} = \frac{129}{182}$ </p>	
60.	Let $S = \{x: \cos^{-1} x = \pi + \sin^{-1} x + \sin^{-1}(2x + 1)\}$. Then $\sum_{x \in S} (2x - 1)^2$ is equal to _____.	D
	20	
	10	
	25	
	5	
Sol.	<p>We have, $\cos^{-1} x = \pi + \sin^{-1} x + \sin^{-1}(2x + 1)$ $\Rightarrow 2\cos^{-1} x - \sin^{-1}(2x + 1) = \frac{3\pi}{2}$ $\Rightarrow 2\alpha - \beta = \frac{3\pi}{2}$, where $\cos^{-1} x = \alpha$, $\sin^{-1}(2x + 1) = \beta$ $\Rightarrow 2\alpha = \frac{3\pi}{2} + \beta \Rightarrow \cos 2\alpha = \sin \beta$ $\Rightarrow 2\cos^2 \alpha - 1 = \sin \beta \Rightarrow 2x^2 - 1 = 2x + 1$ $[\because x = \cos \alpha \text{ and } 2x + 1 = \sin \beta]$ $\Rightarrow x^2 - x - 1 = 0$ $\Rightarrow x = \frac{1 \pm \sqrt{5}}{2} \Rightarrow x = \frac{1 - \sqrt{5}}{2}$ $(\because x = \frac{1 + \sqrt{5}}{2} \text{ rejected as } \cos \alpha \leq 1)$ Now, $\sum_{x \in S} (2x - 1)^2 = \sum_{x \in S} (4x^2 + 1 - 4x) = 5$ </p>	
61.	Let $y = px$ be the parabola passing through points $(-1, 0)$, $(0, 1)$ and $(1, 0)$. If the area of the region $\{(x, y): (x + 1)^2 + (y - 1)^2 \leq 1, y \leq p(x)\}$ is A, then $12(\pi - 4A)$ is equal to _____.	A
	16	
	20	
	25	
	30	

Sol.	<p>There can be infinite parabola through given points. In question, it must be given that axis of parabola is parallel to y-axis.</p>  <p>Equation of parabola passing through $(-1, 0)$, $(0, 1)$ and $(1, 0)$ is $x^2 = -(y - 1)$... (i)</p> <p>\therefore Required area,</p> $A = \int_{-1}^0 \left\{ (1 - x^2) - (1 - \sqrt{1 - (x+1)^2}) \right\} dx$ $= \int_{-1}^0 \left(-x^2 + \sqrt{1 - (x+1)^2} \right) dx$ $= \left[-\frac{x^3}{3} + \frac{x+1}{2} \sqrt{1 - (x+1)^2} + \frac{1}{2} \sin^{-1} \left(\frac{x+1}{1} \right) \right]_{-1}^0$ $= \frac{1}{2} \left(\frac{\pi}{2} \right) - \frac{1}{3} = \frac{\pi}{4} - \frac{1}{3}$ <p>$\therefore 12(\pi - 4A) = 12 \left(\pi - 4 \left(\frac{\pi}{4} - \frac{1}{3} \right) \right) = 12 \left(\frac{4}{3} \right) = 16$</p>	
62.	<p>Let A and B be two square matrices of order 2. If $\det(A) = 2$, $\det(B) = 3$ and $\det((\det(5(\det A)B))A^2) = 2^a 3^b 5^c$ for some $a, b, c \in N$, then $a + b + c$ is equal to</p>	B
	10	
	12	
	13	
	14	
Sol.	<p>(b) We have, $\det(A) = 2$, $\det(B) = 3$ and $\det((\det(5(\det A)B))A^2) = 2^a 3^b 5^c$ $= \det((\det(10B))A^2)$ $= \det(100(\det B)A^2) = \det(300 \cdot A^2)$ $\Rightarrow (300)^2 \det A^2 = (300)^2 \cdot 2^2$ $\Rightarrow 2^6 \cdot 5^4 \cdot 3^2 = 2^a \cdot 3^b \cdot 5^c$ Now, $a + b + c = 12$</p>	
63.	<p>If the function $f(x) = \begin{cases} \frac{72^x - 9^x - 8^x + 1}{\sqrt{2} - \sqrt{1 + \cos x}}, & x \neq 0 \\ a \log_e 2 \log_e 3, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of a^2 is equal to</p>	A
	1152	
	1250	
	968	
	746	

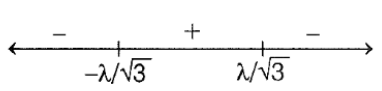
Sol.	<p>(a) Given,</p> $f(x) = \begin{cases} \frac{72^x - 9^x - 8^x + 1}{\sqrt{2} - \sqrt{1 + \cos x}}, & x \neq 0 \\ a \log_e 2 \log 3, & x = 0 \end{cases}$ <p>$\therefore f(x)$ is continuous at $x = 0$</p> $\therefore \lim_{x \rightarrow 0} f(x) = f(0)$ $\Rightarrow \lim_{x \rightarrow 0} \frac{72^x - 9^x - 8^x + 1}{\sqrt{2} - \sqrt{1 + \cos x}} = a \ln 2 \cdot \ln 3$ $\Rightarrow \lim_{x \rightarrow 0} \frac{(8^x - 1)(9^x - 1)}{\sqrt{2} - \sqrt{1 + \cos x}} \times \frac{\sqrt{2} + \sqrt{1 + \cos x}}{\sqrt{2} + \sqrt{1 + \cos x}} = a \ln 2 \cdot \ln 3$ $\Rightarrow \lim_{x \rightarrow 0} \frac{(8^x - 1)(9^x - 1)}{1 - \cos x} \times (\sqrt{2} + \sqrt{1 + \cos x}) = a \ln 2 \cdot \ln 3$ $\Rightarrow \lim_{x \rightarrow 0} \left(\frac{8^x - 1}{x} \right) \left(\frac{9^x - 1}{x} \right) \frac{x^2 (\sqrt{2} + \sqrt{1 + \cos x})}{\frac{2 \sin^2 \frac{x}{2} \times \frac{x^2}{4}}{(x/2)^2} \times \frac{x^2}{4}} = a \ln 2 \ln 3$ $\Rightarrow \ln 8 \cdot \ln 9 \cdot 2 \cdot 2\sqrt{2} = a \ln 2 \cdot \ln 3$ $\Rightarrow a = \frac{3 \ln 2 \cdot 2 \ln 3 \times 4\sqrt{2}}{\ln 2 \cdot \ln 3}$ $\Rightarrow a = 24\sqrt{2}$ $\Rightarrow a^2 = 576 \times 2 = 1152$	
64.	The integral $\int \frac{(x^8 - x^2)dx}{(x^{12} + 3x^6 + 1) \tan^{-1}\left(x^3 + \frac{1}{x^3}\right)}$ is equal to	D
	$\log_e \left(\left \tan^{-1} \left(x^3 + \frac{1}{x^3} \right) \right \right) + C$	
	$\log_e \left(\left \tan^{-1} \left(x^3 + \frac{1}{x^3} \right) \right \right)^{1/2} + C$	
	$\log_e \left(\left \tan^{-1} \left(x^3 + \frac{1}{x^3} \right) \right \right)^3 + C$	
	$\log_e \left(\left \tan^{-1} \left(x^3 + \frac{1}{x^3} \right) \right \right)^{1/3} + C$	
Sol.	<p>(d) Let</p> $I = \int \frac{(x^8 - x^2)dx}{(x^{12} + 3x^6 + 1) \tan^{-1}\left(x^3 + \frac{1}{x^3}\right)}$ <p>Put $\tan^{-1}\left(x^3 + \frac{1}{x^3}\right) = z$</p> $\Rightarrow \frac{1}{1 + \left(x^3 + \frac{1}{x^3}\right)^2} \times \left(3x^2 - \frac{3}{x^4}\right) dx = dz$ $\Rightarrow \frac{1}{1 + \left(\frac{x^6 + 1}{x^3}\right)^2} \times \left(\frac{3x^6 - 3}{x^4}\right) dx = dz$ $\Rightarrow \frac{(x^3)^2}{(x^3)^2 + (x^{12} + 1 + 2 \times 1 \times x^6)} \times \frac{3(x^6 - 1)}{x^4} dx = dz$ $\Rightarrow \frac{x^6 \times 3(x^6 - 1)}{x^4 [x^6 + x^{12} + 1 + 2x^6]} dx = dz$ $\Rightarrow \frac{(x^8 - x^2)}{(x^{12} + 3x^6 + 1)} dx = \frac{1}{3} dz$ <p>So, $I = \frac{1}{3} \int \frac{1}{z} dz = \frac{1}{3} \ln z + C$</p> $= \frac{1}{3} \log_e \left(\left \tan^{-1} \left(x^3 + \frac{1}{x^3} \right) \right \right) + C$ $\Rightarrow I = \log_e \left(\left \tan^{-1} \left(x^3 + \frac{1}{x^3} \right) \right \right)^{1/3} + C$	
65.	<p>If the points of intersection of two distinct conics $x^2 + y^2 = 4b$ and $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ lie on the curve $y^2 = 3x^2$, then $3\sqrt{3}$ times the area of the rectangle formed by the intersection points is _____.</p>	B

	440	
	432	
	448	
	460	
Sol.	<p>We have, $x^2 + y^2 = 4b$... (i)</p> <p>and $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$... (ii)</p> <p>From (i) and (ii), $x^2 = \frac{16b}{b+4}$ and $y^2 = \frac{4b^2}{b+4}$</p> <p>∴ These points lie on curve $y^2 = 3x^2$.</p> <p>So, $\frac{4b^2}{b+4} = 3 \times \frac{16b}{b+4}$</p> <p>$\Rightarrow b^2 = 12b \Rightarrow b(b-12) = 0$</p> <p>$\Rightarrow b = 12$ ($b \neq 0$)</p> <p>$(-2\sqrt{3}, 6)$ $(2\sqrt{3}, 6)$</p>  <p>$(-2\sqrt{3}, -6)$ $(2\sqrt{3}, -6)$</p> <p>So, the points are $(\pm 2\sqrt{3}, \pm 6)$</p> <p>∴ Area of rectangle = $4\sqrt{3} \times 12 = 48\sqrt{3}$ sq. units</p> <p>Thus, $3\sqrt{3}$ times of the area of rectangle</p> <p>$= 3\sqrt{3} \times 48\sqrt{3} = 432$</p>	
66.	<p>Let $y = y(x)$ be the solution of the differential equation $(1 + y^2)e^{\tan x} dx + \cos^2 x(1 + e^{2\tan x}) dy = 0, y(0) = 1$.</p> <p>Then, $y\left(\frac{\pi}{4}\right)$ is equal to</p>	D
	$\frac{2}{e}$	
	$\frac{2}{e^2}$	
	$\frac{1}{e^2}$	
	$\frac{1}{e}$	
Sol.	<p>(a) $(1 + y^2)e^{\tan x} dx + \cos^2 x(1 + e^{2\tan x}) dy = 0$</p> <p>$\Rightarrow -\frac{1}{1 + y^2} \cdot dy = \frac{e^{\tan x} \cdot \sec^2 x}{1 + (e^{\tan x})^2} \cdot dx$</p> <p>On integrating both sides,</p> <p>$\int \frac{1}{1 + y^2} dy = - \int \frac{e^{\tan x}}{1 + (e^{\tan x})^2} \sec^2 x \cdot dx$</p> <p>Let $\tan x = t \Rightarrow \sec^2 x \cdot dx = dt$</p> <p>$\Rightarrow \tan^{-1}(y) = - \int \frac{e^t}{1 + (e^t)^2} \cdot dt$</p> <p>Let $e^t = u, e^t \cdot dt = du$</p> <p>$\Rightarrow \tan^{-1}(y) = - \int \frac{1}{1 + u^2} du$</p> <p>$\Rightarrow \tan^{-1}(y) = - \tan^{-1}(u) + c$</p> <p>$\Rightarrow \tan^{-1}(y) = - \tan^{-1}(e^{\tan x}) + c$</p> <p>∵ $y(0) = 1$</p> <p>$\Rightarrow \pi/4 = - \tan^{-1}(1) + c$</p> <p>$\Rightarrow c = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$</p> <p>So, $y = \tan\left(\frac{\pi}{2} - \tan^{-1}(e^{\tan x})\right)$</p> <p>$= \cot(\tan^{-1}(e^{\tan x}))$</p> <p>At $x = \frac{\pi}{4}, y\left(\frac{\pi}{4}\right) = \cot(\tan^{-1}(e))$</p> <p>$= \cot\left(\cot^{-1}\left(\frac{1}{e}\right)\right) = \frac{1}{e}$</p>	
67.	<p>Let $\vec{a} = 9\hat{i} - 13\hat{j} + 25\hat{k}, \vec{b} = 3\hat{i} + 7\hat{j} - 13\hat{k}$ and $\vec{c} = 17\hat{i} - 2\hat{j} + \hat{k}$ be three given vectors. If \vec{r} is a vector such that $\vec{r} \times \vec{a} = (\vec{b} + \vec{c}) \times \vec{a}$ and $\vec{r} \cdot (\vec{b} - \vec{c}) = 0$, then $\frac{ 593\vec{r} + 67\vec{a} ^2}{(593)^2}$ is equal to _____.</p>	B
	600	
	569	
	550	
	580	

Sol.	<p>We have, $\vec{r} \times \vec{a} = (\vec{b} + \vec{c}) \times \vec{a}$ $\Rightarrow [\vec{r} - (\vec{b} + \vec{c})] \times \vec{a} = 0 \Rightarrow \vec{r} - (\vec{b} + \vec{c}) = \lambda \vec{a}$, for some scalar λ. $\Rightarrow \vec{r} = \lambda \vec{a} + \vec{b} + \vec{c}$ Also, $\vec{r} \cdot (\vec{b} - \vec{c}) = 0$ $\Rightarrow (\lambda \vec{a} + \vec{b} + \vec{c}) \cdot (\vec{b} - \vec{c}) = 0$ $\Rightarrow \lambda \vec{a} \cdot \vec{b} - \lambda \vec{a} \cdot \vec{c} + \vec{b} ^2 - \vec{b} \cdot \vec{c} + \vec{b} \cdot \vec{c} - \vec{c} ^2 = 0$ $\Rightarrow \lambda = \frac{ \vec{c} ^2 - \vec{b} ^2}{\vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c}} = \frac{294 - 227}{-389 - 204} = \frac{-67}{593}$ $\Rightarrow \vec{r} = \vec{b} + \vec{c} - \frac{67}{593} \vec{a}$ $\therefore \frac{ 593\vec{r} + 67\vec{a} ^2}{(593)^2} = \frac{(593(\vec{b} + \vec{c}))^2}{(593)^2}$ $= \vec{b} + \vec{c} ^2 = 20\hat{i} + 5\hat{j} - 12\hat{k} ^2 = 569$</p>	
68.	<p>Let the probability of getting head for a biased coin be $\frac{1}{4}$. It is tossed repeatedly until a head appears. Let N be the number of tosses required. If the probability that the equation $64x^2 + 5Nx + 1 = 0$ has no real root is p/q, where p and q are co-prime, then $q - p$ is equal to _____.</p>	A
	27	
	30	
	40	
	60	
Sol.	<p>Let H be the event of getting head and T be the event of getting tail. $P(H) = \frac{1}{4}$ and $P(T) = \frac{3}{4}$ Now, $64x^2 + 5Nx + 1 = 0$ $D = 25N^2 - 4(64)(1) = 25N^2 - 256$ For no real roots, $D < 0$ $\Rightarrow 25N^2 - 256 < 0$ $\Rightarrow 25N^2 < 256 \Rightarrow N^2 < \frac{256}{25} \Rightarrow N < \frac{16}{5}$ $\therefore N = 1, 2, 3$ Required probability = $H + TH + TTH$ $= \frac{1}{4} + \frac{3}{4} \times \frac{1}{4} + \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} = \frac{37}{64} = \frac{p}{q}$ [Given] $\therefore q - p = 64 - 37 = 27$</p>	
69.	<p>Let $[x]$ denote the greatest integer $\leq x$, where $x \in R$. If the domain of the real valued function $f(x) = \sqrt{\frac{ [x] - 2}{ [x] - 3}}$ is $(-\infty, a) \cup [b, c) \cup [4, \infty)$, $a < b < c$, then the value of $a + b + c$ is</p>	C
	8	
	1	
	-2	
	-3	

Sol.	<p>(c) To evaluate the domain of the function $f(x)$, $\frac{ [x] - 2}{ [x] - 3} \geq 0$</p> <p>Case I When $[x] - 2 \geq 0$ and $[x] - 3 > 0$ $\Rightarrow x \in (-\infty, -3) \cup [4, \infty)$... (i)</p> <p>Case II When $[x] - 2 \leq 0$ and $[x] - 3 < 0$ $\Rightarrow x \in [-2, -3)$... (ii)</p> <p>So, from Eqs. (i) and (ii), domain of function $f(x)$ $= (-\infty, -3) \cup [-2, 3) \cup [4, \infty)$ $\therefore (a + b + c) = -3 + (-2) + 3$ $= -2(a < b < c)$</p>	
70.	<p>If the value of $\frac{3 \cos 36^\circ + 5 \sin 18^\circ}{5 \cos 36^\circ - 3 \sin 18^\circ}$ is $\frac{a\sqrt{5}-b}{c}$, where a, b and c are natural numbers and $\gcd(a, c) = 1$, then $a + b + c$ is equal to</p> <p>40</p> <p>50</p> <p>54</p> <p>52</p>	D
Sol.	<p>(d) $\frac{3 \cos 36^\circ + 5 \sin 18^\circ}{5 \cos 36^\circ - 3 \sin 18^\circ}$</p> $= \frac{3 \left(\frac{\sqrt{5}+1}{4} \right) + 5 \left(\frac{\sqrt{5}-1}{4} \right)}{5 \left(\frac{\sqrt{5}+1}{4} \right) - 3 \left(\frac{\sqrt{5}-1}{4} \right)}$ $= \frac{3\sqrt{5} + 3 + 5\sqrt{5} - 5}{5\sqrt{5} + 5 - 3\sqrt{5} + 3} = \frac{8\sqrt{5} - 2}{2\sqrt{5} + 8}$ $= \frac{4\sqrt{5} - 1}{\sqrt{5} + 4} \times \frac{\sqrt{5} - 4}{\sqrt{5} - 4}$ $= \frac{20 - 16\sqrt{5} - \sqrt{5} + 4}{-11}$ $= \frac{24 - 17\sqrt{5}}{-11} = \frac{17\sqrt{5} - 24}{11}$ <p>Now, $\frac{17\sqrt{5} - 24}{11} = \frac{a\sqrt{5} - b}{c}$ $a = 17, b = 24$ and $c = 11$ Therefore, $a + b + c = 52$</p>	
71.	<p>Let $S = \left\{ z \in \mathbb{C} - \{i, 2i\} : \frac{z^2 + 8iz - 15}{z^2 - 3iz - 2} \in \mathbb{R} \right\}$. If $\alpha - \frac{13}{11}i \in S, \alpha \in \mathbb{R} - \{0\}$, then $242\alpha^2$ is equal to.....</p>	1680

Sol.	<p>Given that,</p> $S = \left\{ z \in \mathbb{C} - \{i, 2i\} : \frac{z^2 + 8iz - 15}{z^2 - 3iz - 2} \in \mathbb{R} \right\}$ <p>Now, $\frac{z^2 + 8iz - 15}{z^2 - 3iz - 2} \in \mathbb{R}$</p> $\Rightarrow 1 - \frac{11iz - 13}{z^2 - 3iz - 2} \in \mathbb{R}$ <p>Let, $z = \alpha - \frac{13}{11}i \in S$ [given]</p> <p>So, this implies,</p> <p>$(z^2 - 3iz - 2)$ is imaginary</p> <p>Now, put, $z = x + iy$</p> $\Rightarrow [x^2 + y^2 - 3y - 2 + (2xy - 3x)i] = 0$ <p>[If z is imaginary $\Rightarrow \text{Re } z = 0$]</p> $\Rightarrow x^2 - y^2 + 3y - 2 = 0$ $\Rightarrow x^2 = y^2 - 3y + 2 = (y - 2)(y - 1)$ <p>Since, $z = \alpha - \frac{13}{11}i$</p> <p>So, $x = \alpha, y = -\frac{13}{11}$</p> $\alpha^2 = \left(-\frac{13}{11} - 2\right) \left(\frac{-13}{11} - 1\right)$ $= \left(\frac{-35}{11}\right) \times \left(\frac{-24}{11}\right)$ $\Rightarrow \alpha^2 = \frac{24 \times 35}{121}$ $\Rightarrow 24\alpha^2 = 242 \times \frac{24 \times 35}{121}$ $= 48 \times 35 = 1680$	
72.	<p>Let $S = (-1, \infty)$ and $f: S \rightarrow \mathbb{R}$ be defined as</p> $f(x) = \int_{-1}^x (e^t - 1)^{11} (2t - 1)^5 (t - 2)^7 (t - 3)^{12} (2t - 10)^{61} dt.$ <p>Let $p = \text{Sum of squares of the value of } x$, where $f(x)$ attains local maxima on S, and $q = \text{Sum of the values of } x$, where $f(x)$ attains local minima on S. Then, the value of $p^2 + 2q$ is _____.</p>	27
Sol.	<p>Given, $f(x)$</p> $= \int_{-1}^x (e^t - 1)^{11} (2t - 1)^5 (t - 2)^7 (t - 3)^{12} (2t - 10)^{61} dt$ $f'(x) = (e^x - 1)^{11} (2x - 1)^5 (x - 2)^7 (x - 3)^{12} (2x - 10)^{61}$ <p>So, $x = 0, x = \frac{1}{2}, x = 2, x = 3, x = 5$</p> <p>$f'(x) = \begin{array}{ccccccc} + & - & + & - & - & + & \\ \leftarrow & 0 & 1/2 & 2 & 3 & 5 & \rightarrow \end{array}$</p> <p>$\therefore p = 0 + 4 = 4$</p> $q = \frac{1}{2} + 5 = \frac{11}{2}$ <p>$\therefore p^2 + 2q = 16 + 11 = 27$</p>	
73.	<p>Let $A = \{1, 2, 3, \dots, 100\}$. Let R be a relation on A defined by $(x, y) \in R$ if and only if $2x = 3y$. R_1 be a symmetric relation on A such that $R \subset R_1$ and the number of elements in R_1 is n. Then, the minimum value of n is.....</p>	66

<p>Sol.</p>	<p>(66) $A = \{1, 2, 3, \dots, 100\}$ $(x, y) \in R$ and $2x = 3y$ Then, $R = \{(3, 2), (6, 4), (9, 6), (12, 8),$ $\dots, (99, 66)\}$ Number of elements in $R = 33$ R_1 be a symmetric relation on A such that $R \subset R_1$ $\therefore R_1 = \{(3, 2), (2, 3), (6, 4), (4, 6), \dots$ $(99, 66), (66, 99)\}$ So minimum number of elements in $R_1 = 66$ So, $n = 66$</p>	
<p>74.</p>	<p>Let the set of all positive values of λ, for which the point of local minimum of the function $(1 + x(\lambda^2 - x^2))$ satisfies $\frac{x^2+x+2}{x^2+5x+6} < 0$, be (α, β). Then, $\alpha^2 + \beta^2$ is equal to.....</p>	<p>39</p>
<p>Sol.</p>	<p>(39) Given, $\frac{x^2 + x + 2}{x^2 + 5x + 6} < 0$ $\Rightarrow \frac{1}{(x+2)(x+3)} < 0$ $(\because x^2 + x + 2 > 0, \forall x \in \mathbb{R})$ $\Rightarrow x \in (-3, -2) \quad \dots(i)$ Also, $f(x) = 1 + x(\lambda^2 - x^2)$ $\Rightarrow f'(x) = (\lambda^2 - x^2) + (-2x)x$ For finding local minima, we put $f'(x) = 0$ $\lambda^2 = 3x^2 \Rightarrow x = \pm \frac{\lambda}{\sqrt{3}}$  \therefore Local minima at $x = \frac{-\lambda}{\sqrt{3}}$ So, from Eq. (i), $-3 < x < -2$ $\Rightarrow -3 < -\frac{\lambda}{\sqrt{3}} < -2$ $\Rightarrow 3\sqrt{3} > \lambda > 2\sqrt{3} \Rightarrow 2\sqrt{3} < \lambda < 3\sqrt{3}$ $\therefore \alpha = 2\sqrt{3}$ and $\beta = 3\sqrt{3}$ Hence, $\alpha^2 + \beta^2 = 12 + 27 = 39$</p>	
<p>75.</p>	<p>Let a line l pass through the origin and be perpendicular to the lines, $l_1: \vec{r} = (\hat{i} - 11\hat{j} - 7\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k}), \lambda \in \mathbb{R}$ and $l_2: \vec{r} = (-\hat{i} + \hat{k}) +$ $\mu(2\hat{i} + 2\hat{j} + \hat{k}), \mu \in \mathbb{R}$. If P is the point of intersection of l and l_1, and $Q(\alpha, \beta, \gamma)$ is the foot of perpendicular from P on l_2, then $9(\alpha + \beta + \gamma)$ is equal to _____.</p>	<p>5</p>

Sol.

Let

$$l = (0\hat{i} + 0\hat{j} + 0\hat{k}) + \gamma(a\hat{i} + b\hat{j} + c\hat{k})$$

$$= \gamma(a\hat{i} + b\hat{j} + c\hat{k})$$

Since, l is perpendicular to the lines l_1 and l_2

$$a\hat{i} + b\hat{j} + c\hat{k} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 2 & 1 \end{vmatrix}$$

$$= \hat{i}(2-6) - \hat{j}(1-6) + \hat{k}(2-4)$$

$$= -4\hat{i} + 5\hat{j} - 2\hat{k} \quad \therefore l = \gamma(-4\hat{i} + 5\hat{j} - 2\hat{k})$$

P is the point of intersection of l and l_1 .

$$-4\gamma = 1 + \lambda, 5\gamma = -11 + 2\lambda, -2\gamma = -7 + 3\lambda$$

By solving these equations, we get $\gamma = -1$.

So, $P \equiv (4, -5, 2)$

Let $Q(-1 + 2\mu, 2\mu, 1 + \mu)$

$$\text{Then, } \overrightarrow{PQ} \cdot (2\hat{i} + 2\hat{j} + \hat{k}) = 0$$

$$\Rightarrow [(2\mu - 5)\hat{i} + (2\mu + 5)\hat{j} + (\mu - 1)\hat{k}] \cdot (2\hat{i} + 2\hat{j} + \hat{k}) = 0$$

$$\Rightarrow 4\mu - 10 + 4\mu + 10 + \mu - 1 = 0$$

$$\Rightarrow 9\mu = 1 \Rightarrow \mu = \frac{1}{9}$$

$$\text{Hence, } 9(\alpha + \beta + \gamma) = 9\left(\frac{-7}{9} + \frac{2}{9} + \frac{10}{9}\right) = 5$$