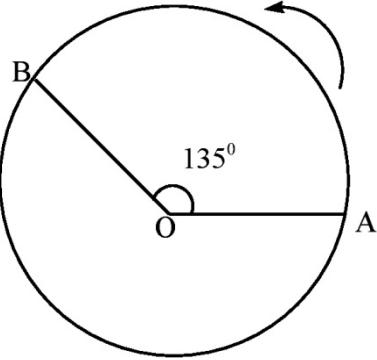
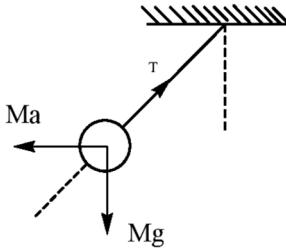
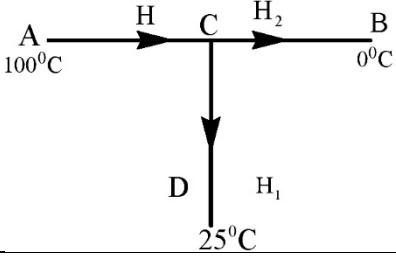
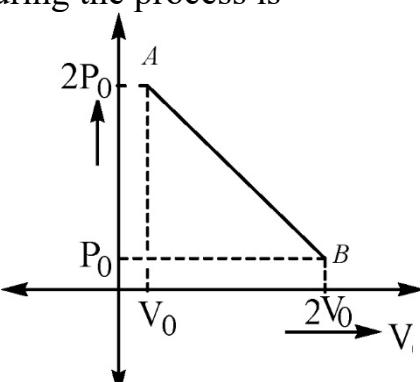


1.	<p>Match List-I with List-II</p> <p><b>List-I</b></p> <p>(Type of electromagnetic waves)</p> <p>A) Radio waves B) X-rays C) Infrared D) Ultraviolet</p> <p><b>List-II</b></p> <p>(Wavelength range &amp; production)</p> <p>I) <math>10 \text{ \AA}^\circ</math> to <math>0.01 \text{ \AA}^\circ</math>, Inner shells of electrons II) <math>10^7 \text{ \AA}^\circ</math> to <math>7000 \text{ \AA}^\circ</math>, vibrations of atoms III) <math>4000 \text{ \AA}^\circ</math> to <math>10 \text{ \AA}^\circ</math>, Inner shell electrons in atoms moving from one energy level to a lower level IV) <math>&gt;0.1 \text{ m}</math>, rapid acceleration and deceleration of Electrons in aerial</p> <p>Choose the correct answer from the option given below.</p>	C
	A-I, B-II, C-III, D-IV	
	A-II, B-III, C-IV, D-I	
	A-IV, B-I, C-II, D-III	
	A-III, B-IV, C-I, D-II	
<b>Sol.</b>	From Electromagnetic spectrum & Production, detection of the waves.	
2.	<p>Given below are two statements:</p> <p><b>Statement-I:</b> Carbon (C), Silicon (Si), Germanium (Ge) have same lattice structure. But C is semiconductor &amp; Si, Ge are insulators. <b>Statement-II:</b> Ionisation energy is least for C followed by Si, Ge.</p>	B
	Both Statements are true	
	Both Statements are false	
	Statement -I true, Statement -II false	
	Statement -I false, Statement -II true	
<b>Sol.</b>	Ionisation energy will be least for Ge, followed by Si and highest for C.	
3.	<p>A solenoid of length 0.5m has a radius of 1 cm and is made up of 500 turns. It carries a current of 10A. What is the magnitude of the magnetic field inside the solenoid :</p> <p><math>6.78 \times 10^{-3} \text{ T}</math> <math>12.56 \times 10^{-3} \text{ T}</math> <math>2.68 \times 10^{-3} \text{ T}</math> <math>6.82 \times 10^{-3} \text{ T}</math></p>	B
<b>Sol.</b>	$B = \mu_0 n i = \mu_0 \frac{N}{\ell} i = 4\pi \times 10^{-7} \left( \frac{500}{0.5} \right) 10 = 12.56 \times 10^{-3} \text{ T}$	
4.	<p>Find the torque of a force <math>7\hat{i} + 3\hat{j} - 5\hat{k}</math> about the origin. The force acts on a particle whose position vector is <math>\hat{i} - \hat{j} + \hat{k}</math>.</p> <p><math>\hat{i} + 12\hat{j} + 10\hat{k}</math> <math>2\hat{i} + 12\hat{j} + 10\hat{k}</math> <math>\hat{i} + \hat{j} + \hat{k}</math> <math>\hat{i} + \hat{j} + 10\hat{k}</math></p>	B
<b>Sol.</b>	$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 7 & 3 & -5 \end{vmatrix}$ $= \hat{i}(5-3) - \hat{j}(-5-7) + \hat{k}(3+7) = 2\hat{i} + 12\hat{j} + 10\hat{k}$	
5.	<p>The force is given in terms of time 't' and displacement 'x' by the equation <math>F = A \cos Bt + C \sin Dx</math> then the dimensions of ABCD is</p> <p><math>[M^2 LT^{-5}]</math></p>	A

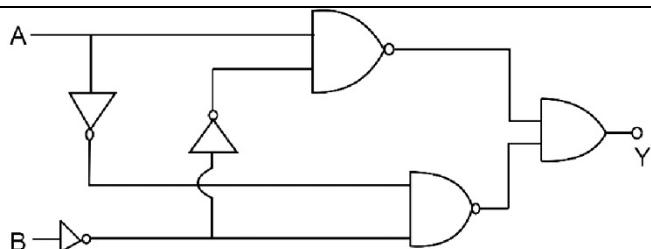
	$[MLT^{-5}]$	
	$[M^2L^2T^{-5}]$	
	$[M^2L^3T^{-5}]$	
<b>Sol.</b>	$B_t=1 \Rightarrow B=\frac{1}{T}=T^{-1}$ $D_x=1 \Rightarrow D=\frac{1}{x}=\frac{1}{L}=L^{-1}$ $F=A \Rightarrow MLT^{-2}=A$ $F=C \Rightarrow MLT^{-2}=C \quad \therefore ABCD=MLT^{-2}(MLT^{-2})T^{-1}L^{-1}=M^2LT^{-5}$	
<b>6.</b>	<p>A person moved from A to B on a circular path as shown in fig.</p>  <p>If the distance travelled by him is 90m, then the magnitude of displacement would be (Given <math>\cos 135^\circ = -0.7</math>)</p>	<b>B</b>
	42.26 m	
	70.46 m	
	19.37 m	
	40.87 m	
<b>Sol.</b>	$d=R\theta$ $90=R\left(\frac{3\pi}{4}\right) \quad R=\frac{90 \times 4}{3\pi}=\frac{120}{\pi}$ $\therefore \text{Displacement}=\sqrt{R^2+R^2-2R^2 \cos 135^\circ}$ $=\sqrt{3.4R^2}=\sqrt{3.4\left(\frac{120}{\pi}\right)^2} \approx 70.46\text{m}$	
<b>7.</b>	<p>A projectile is projected at <math>45^\circ</math> from horizontal with initial velocity 40m/s. The velocity of the projectile at <math>t=1\text{sec}</math> is from the start will be (given <math>g=10\text{m/s}^2</math>)</p>	<b>A</b>
	33.68 m/s	
	60 m/s	
	30 m/s	
	20 m/s	
<b>Sol.</b>	$V=\sqrt{V_x^2+V_y^2}=\sqrt{(ucos\theta)^2+(usin\theta-gt)^2}$ $=\sqrt{\left(40\frac{1}{\sqrt{2}}\right)^2+\left(40\frac{1}{\sqrt{2}}-10(1)\right)^2}$ $=\sqrt{(28.29)^2+(28.29-10)^2}$ $=\sqrt{800.32+334.52}=\sqrt{1134.84}=33.68\text{m/s}$	
<b>8.</b>	<p>Inside a horizontally moving box, an experimenter finds that when an object is placed on a smooth horizontal plane and is released, it moves with an acceleration of <math>10\text{m/s}^2</math>. In this box if 1kg body is suspended with a light string, find the tension in the string in equilibrium position with respect to experimenter (<math>g=10\text{m/s}^2</math>)</p>	<b>B</b>
	10 N	

	10 $\sqrt{2}$ N	
	10 $\sqrt{3}$ N	
	100 N	
<b>Sol.</b>	$T = \sqrt{(mg)^2 + (ma)^2} = \sqrt{(10)^2 + (10)^2} = 10\sqrt{2} \text{ N}$ 	
<b>9.</b>	The time taken by an object to slide down $60^\circ$ rough inclined plane is $n$ times as it takes to slide down a perfectly smooth $60^\circ$ inclined plane. The coefficient of kinetic friction b/w the object and the inclined plane is	<b>B</b>
	$\sqrt{\frac{1}{1-n^2}}$	
	$\sqrt{3} \left(1 - \frac{1}{n^2}\right)$	
	$\frac{1}{\sqrt{3}(1-n^2)}$	
	$3 \left(\frac{n^2-1}{n}\right)$	
<b>Sol.</b>	$\mu = \tan\theta \left(1 - \frac{1}{n^2}\right)$ $= \tan 60^\circ \left(1 - \frac{1}{n^2}\right) = \sqrt{3} \left(1 - \frac{1}{n^2}\right)$	
<b>10.</b>	If momentum of a body increased by 30%. Then its K.E is increased by	<b>A</b>
	69%	
	44%	
	32%	
	60%	
<b>Sol.</b>	$P' = P + \frac{30}{100} P = \frac{130}{100} P = 1.3P$ % increase in K.E $= \frac{K' - K}{K} \times 100$ $= \frac{P'^2 - P^2}{P^2} \times 100 = \left((1.3)^2 - 1^2\right) \times 100 = 69\%$	
<b>11.</b>	A rod of length 'L' has non - uniform linear mass density given by $\rho(x) = a + b \left(\frac{x}{L}\right)$ when $a, b$ are constants and $0 \leq x \leq L$ the value of 'x' for the centre of mass of the rod is at	<b>D</b>
	$\frac{L(2a+b)}{3(3a+2b)}$	
	$3L \frac{(2a+b)}{(3a+2b)}$	

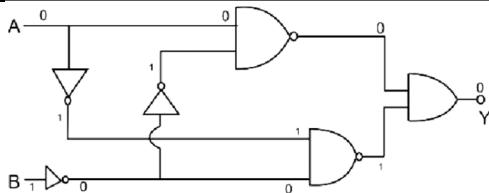
	$2L \frac{(2a+b)}{(3a+2b^2)}$	
	$\frac{L}{3} \left( \frac{3a+2b}{2a+b} \right)$	
<b>Sol.</b>	<p>Position of com is at</p> $X_{cm} = \frac{\int_0^L x dm}{\int_0^L dm} = \frac{\int_0^L x \rho(x) dx}{\int_0^L \rho(x) dx}$ $= \frac{\int_0^L x \left( a+b \frac{X}{L} \right) dx}{\int_0^L \left( a+b \frac{X}{L} \right) dx} = \frac{\int_0^L \left( ax + \frac{bx^2}{L} \right) dx}{\int_0^L \left( a + \frac{bx}{L} \right) dx} = \frac{\left( a \frac{L^2}{2} + \frac{b}{L} \cdot \frac{L^3}{3} \right)}{aL + \frac{b}{L} \cdot \frac{L^2}{2}}$ $= \frac{\frac{aL^2}{2} + \frac{bL^2}{3}}{aL + \frac{bL}{2}} = \frac{aL + \frac{bL}{3}}{a + \frac{b}{2}} = \frac{2 \left( \frac{3aL + 2bL}{6} \right)}{(2a+b)}$	
<b>12.</b>	Consider a Uniform wire of mass 'M' and length 'L'. It is bent in to a circle. It's moment of Inertia about a line perpendicular to the plane of the wire passing through the centre is	<b>D</b>
	$\frac{ML^2}{\pi^2}$	
	$\frac{ML^2}{3\pi^2}$	
	$\frac{ML^2}{2\pi^2}$	
	$\frac{ML^2}{4\pi^2}$	
<b>Sol.</b>	$I = MR^2$ $L = 2\pi R$ $= M \left( \frac{L}{2\pi} \right)^2$ $R = \frac{L}{2\pi}$ $= \frac{ML^2}{4\pi^2}$	
<b>13.</b>	The angular momentum of planet of mass M moving around the sun in an elliptical orbit is $\vec{L}$ . Then torque acting on the planet is	<b>B</b>
	1 N-m	
	0	
	5 N-m	
	10 N-m	
<b>Sol.</b>	Planet's follow law of conservation of angular momentum.	
<b>14.</b>	A man grows in to giant such that his linear dimension increases by a factor of 7. Assuming that his density remains same, the stress in the leg will change by a factor of	<b>A</b>
	7	
	9	
	8	
	6	
<b>Sol.</b>	Stream $\sigma = \frac{F}{A} = \frac{Mg}{A}$ $\sigma \propto \frac{M}{A} \propto \frac{Vol}{A}$ $\sigma \propto l$ $\sigma_2 = 7\sigma_1$	
<b>15.</b>	A square loop of side 10cm and resistance 0.5Ω is placed vertically in the east-west plane. A uniform magnetic field of 1T is setup across the plane in the north-east direction. The magnetic field is decreased to zero in 0.7sec at a	<b>C</b>

	steady rate. The magnitude of current during this time-interval is 0.6 mA 1 mA $2.02 \times 10^{-2}$ A $3.02 \times 10^{-2}$ A	
<b>Sol.</b>	$I = \frac{e}{R} = \frac{\frac{d\phi}{dt}}{R} = \frac{BA\cos\theta}{tR} = \frac{1(10^{-2})}{0.7(0.5)} \frac{1}{\sqrt{2}} = 2.02 \times 10^{-2} A$	
<b>16.</b>	A rod CD of thermal resistance $5\text{K/w}$ is joined at the middle of an identical rod AB as shown in fig. The ends A,B D are maintained at $100^\circ\text{C}$ , $0^\circ\text{C}$ , $25^\circ\text{C}$ respectively. Find the heat current in CD	<b>A</b>
		
	4 W 5 W 6W 7W	
<b>Sol.</b>	$R_{AC} = R_{CB} = \frac{R_{CD}}{2} = \frac{5}{2} = 2.5 \text{ k/w}$ $H_1 = \frac{T_C - T_D}{R_{CD}} = \frac{T_C - 25}{5}$ $H_2 = \frac{T_C - T_B}{R_{CB}} = \frac{T_C - 0}{2.5} \quad H = \frac{T_A - T_C}{R_{AC}} = \frac{100 - T_C}{2.5} \quad H = H_1 + H_2$ $\frac{100 - T_C}{2.5} = \frac{T_C - 25}{5} + \frac{T_C}{2.5} \quad T_C = 45^\circ\text{C}$ $H_1 = \frac{45 - 25}{5} = \frac{20}{5} = 4 \text{ W}$	
<b>17.</b>	n-moles of an ideal gas undergoes a process A $\rightarrow$ B as shown. The maximum temperature during the process is	<b>C</b>
		
	$\frac{8P_0V_0}{3nR}$	
	$\frac{16P_0V_0}{3nR}$	
	$\frac{9P_0V_0}{4nR}$	

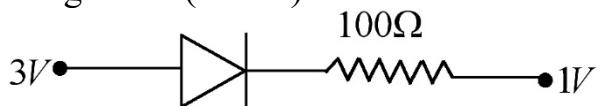
	$\frac{25 P_0 V_0}{4 nR}$	
<b>Sol.</b>	<p>Equation of line is <math>P = \frac{-P_0}{V_0}V + 3P_0</math></p> $PV_0 = P_0 V = 3P_0 V_0 \quad \frac{nRT}{V} + P_0 V = 3P_0 V_0$ $nRT V_0 + P_0 V^2 = 3P_0 V_0 V$ <p>For temp max <math>\frac{dT}{dV} = 0 \quad \therefore V = \frac{3V_0}{2} \quad \therefore T_{\max} = \frac{9P_0 V_0}{4nR}</math></p>	
<b>18.</b>	<p>A particle of mass <math>10^{-3}</math> kg and charge <math>1.0 \text{ C}</math> initially at rest. At <math>t = 0</math>, the particle comes under the influence of an electric field <math>E(t) = E_0 \sin \omega t</math> where <math>E_0 = 1 \text{ N/C}</math> and <math>\omega = 10^3 \text{ SI units}</math>. The maximum speed attained by the particle is</p> <p>2 m/s</p> <p>3 m/s</p> <p>4 m/s</p> <p>1 m/s</p>	<b>A</b>
<b>Sol.</b>	$F = Eq \quad ma = Eq \quad m \frac{dv}{dt} = E_0 q \sin \omega t$ $dv = \frac{E_0 q}{m} \sin \omega t dt \quad V_{\max} = \frac{E_0 q}{m} \left( \frac{-\cos \omega t}{\omega} \right)_0^{\pi/\omega} = 2 \text{ m/s}$	
<b>19.</b>	<p>The resistance <math>R = \frac{V}{I}</math> where <math>V = (50 \pm 2) \text{ V}</math> and <math>I = (20 \pm 0.2) \text{ A}</math>. The percentage error in <math>R</math> is <math>x\%</math>. The value of <math>x</math> to the nearest integer is</p> <p>4</p> <p>5</p> <p>3</p> <p>1</p>	<b>B</b>
<b>Sol.</b>	$R = \frac{V}{I} \quad \frac{\Delta R}{R} \times 100 = \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100$ $= \frac{2}{50} \times 100 + \frac{0.2}{20} \times 100 = 5\%$	
<b>20.</b>	<p>Unpolarized light of intensity <math>40 \text{ W m}^{-2}</math> passes through three polarizers such that the transmission axis of the last polarizer is crossed with first. If the angle between the axes of second and third polarizers is <math>45^\circ</math>. Then the intensity of the emerging light is <math>\text{W m}^{-2}</math></p> <p>3</p> <p>4</p> <p>5</p> <p>6</p>	<b>C</b>
<b>Sol.</b>	$I = \frac{I_0}{8} \sin^2 2\theta$ Here $I_0 = 40 \text{ W m}^{-2} \quad \therefore I = \frac{40}{8} \times 1 = 5 \text{ W m}^{-2} \quad \theta = 45^\circ$	
<b>21.</b>	<p>In the logic Circuit Shown in fig . If input A and B are 0 &amp; 1 respectively, the out put at y would be 'X' the value of X is</p>	<b>1</b>



**Sol.**



- 22.** Assuming that the junction diode is ideal, the current in the arrangement shown in the circuit diagram is (in mA) 20



**Sol.**

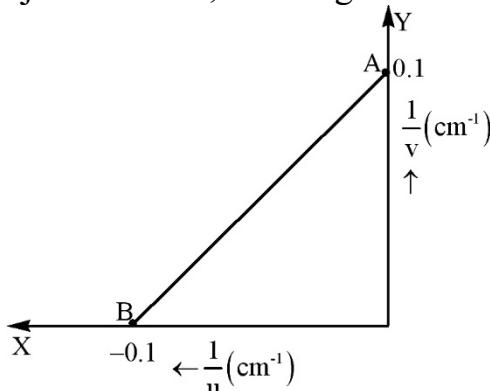
$$I = \frac{\Delta V}{R} = \frac{3-1}{100} = \frac{2}{100} = 20mA$$

- 23.** A uniform string of length 10 m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is (in sec) : (take  $g = 10 \text{ ms}^{-2}$ ) 2

**Sol.**

$$t = 2\sqrt{\frac{\ell}{g}} = 2\sqrt{\frac{10}{10}} = 2 \text{ sec}$$

- 24.** The graph between  $\frac{1}{u}$  and  $\frac{1}{v}$  for a thin convex lens in order to determine its focal length is plotted as shown in fig. The refractive index of lens is 1.55 and its both surface has same radius of curvatures R. Then the value of R will be \_\_\_\_\_ cm (When u= object distance, v= Image distance) 11



**Sol.**

$$\text{For point B; } \frac{1}{u} = -0.1, \frac{1}{v} = 0$$

$$\therefore u = -10 \text{ cm}$$

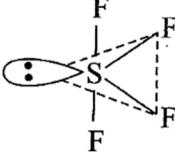
$$v = \infty$$

$$\therefore f = 10 \text{ cm}$$

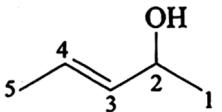
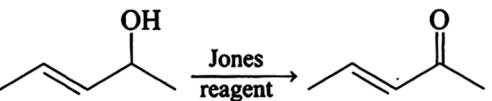
$$\frac{1}{10} = (1.55-1) \left( \frac{2}{R} \right)$$

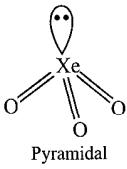
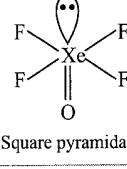
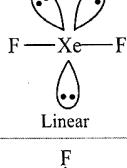
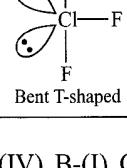
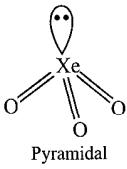
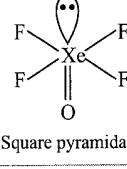
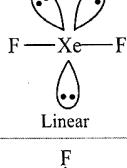
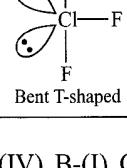
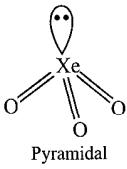
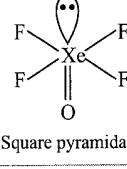
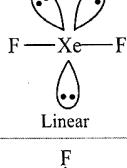
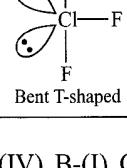
$$R = 11 \text{ cm}$$

- 25.** An Electric charge  $2 \times 10^{-8} \text{ C}$  is placed at the point (1,2,4). At the point (4,2,0) the electric potential is (in Volt) 36

<b>Sol.</b>	$\vec{r}_0 = \hat{i} + 2\hat{j} + 4\hat{k}$ $\vec{r} = 4\hat{i} + 2\hat{j} \quad \vec{r} - \vec{r}_0 = 3\hat{i} - 4\hat{k}$ $V = \frac{1}{4\pi\epsilon_0} \frac{q}{ \vec{r} - \vec{r}_0 }$ $V = \frac{9 \times 10^9 (2) 10^{-8}}{5} = 36 \text{ V}$	
<b>26.</b>	Equivalent conductivity of $\text{BaCl}_2$ , $\text{H}_2\text{SO}_4$ and $\text{HCl}$ are $y_1$ , $y_2$ and $y_3 \text{ Scm}^{-1} \text{ eq}^{-1}$ , respectively at infinite dilution. If conductivity of saturated $\text{BaSO}_4$ solution is $y \text{ S cm}^{-1}$ , then $K_{\text{sp}}$ of $\text{BaSO}_4$ will be	<b>D</b>
	$\frac{y^2}{(y_1 + y_2 - 2y_3)^2}$	
	$\frac{2.5 y^{-2}}{(y_1 + y_2 - 2y_3)^2}$	
	$\frac{500}{(y_1 + y_2 - 2y_3)}$	
	$\frac{2.5 \times 10^5 y^2}{(y_1 + y_2 - 2y_3)^2}$	
<b>Sol.</b>	$\Lambda_m^\infty (\text{BaSO}_4) = 2\Lambda_{\text{eq}}^\infty (\text{BaSO}_4)$ $\Lambda_{\text{eq}}^\infty (\text{BaSO}_4) = \Lambda_{\text{eq}}^\infty (\text{Ba}^{2+}) + \Lambda_{\text{eq}}^\infty (\text{SO}_4^{2-})$ $= \Lambda_{\text{eq}}^\infty (\text{BaCl}_2) + \Lambda_{\text{eq}}^\infty (\text{H}_2\text{SO}_4) - 2\Lambda_{\text{eq}}^\infty (\text{HCl})$ $\Lambda_{\text{eq}}^\infty (\text{BaSO}_4) = y_1 + y_2 - 2y_3$ $\therefore \Lambda_m^\infty = 2(y_1 + y_2 - 2y_3)$ <p>For sparingly soluble salt, <math>\Lambda_m^\infty = \frac{\kappa}{M} \times 1000</math></p> <p>or <math>M = \frac{y}{2(y_1 + y_2 - 2y_3)} \times 1000 = \frac{500y}{y_1 + y_2 - 2y_3}</math></p> $K_{\text{sp}} = M^2 = \frac{2.5 \times 10^5 y^2}{(y_1 + y_2 - 2y_3)^2}$	
<b>27.</b>	The shape of $\text{SF}_4$ is	<b>C</b>
	T-shaped	
	linear	
	see-saw	
	octahedral	
<b>Sol.</b>	<p>The hybridisation of iodine in <math>\text{SF}_4</math> is <math>sp^3d</math> and its structure is</p>  <p>The shape of <math>\text{SF}_4</math> is <i>see-saw</i>.</p>	
<b>28.</b>	Order of paramagnetism. <ul style="list-style-type: none"> <li>A. <math>[\text{Fe}(\text{CN})_6]^{3-} &gt; [\text{Co}(\text{NH}_3)_6]^{3+}</math></li> <li>B. <math>[\text{Fe}(\text{H}_2\text{O})_6]^{2+} &gt; [\text{Zn}(\text{H}_2\text{O})_6]^{2+}</math></li> <li>C. <math>\text{K}_3[\text{Cu}(\text{CN})_4] &gt; \text{K}_3[\text{Cr}(\text{oxalate})_3]</math></li> <li>D. <math>[\text{Fe}(\text{CN})_6]^{3-} &gt; [\text{Cr}(\text{H}_2\text{O})_6]^{3+}</math></li> <li>E. <math>[\text{K}_3[\text{Cu}(\text{CN})_4]] &gt; [\text{CuF}_6]^{3-}</math></li> </ul>	<b>A</b>

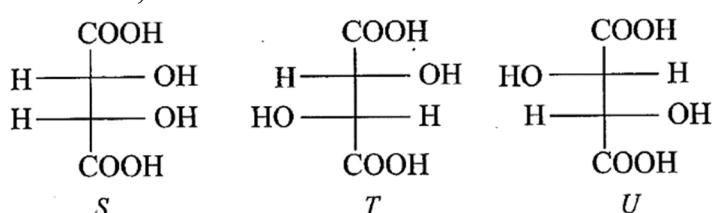
	Choose the correct answer from the options given below.																
	A and B																
	D and E																
	B, A and C																
	B, C and D																
<b>Sol.</b>	<p>Paramagnetism is due to the presence of unpaired electrons of metal.</p> <p>A. <math>[\text{Fe}(\text{CN})_6]^{3-}</math>, <math>[\text{Co}(\text{NH}_3)_6]^{3+}</math>  <math>\text{Fe}^{3+} = 3d^5</math>   <math>\text{Co}^{3+} = 3d^6</math></p> <p>B. <math>[\text{Fe}(\text{H}_2\text{O})_6]^{2+}</math>, <math>[\text{Zn}(\text{H}_2\text{O})_6]^{2+}</math>  <math>\text{Fe}^{2+} = 3d^6</math>   <math>\text{Zn}^{2+} = 3d^{10}</math></p> <p>C. <math>\text{K}_3[\text{Cu}(\text{CN})_4]</math>, <math>\text{K}_3[\text{Cr}(\text{oxalate})_3]</math>  <math>\text{Cu}^+ = 3d^{10}</math>   <math>\text{Cr}^{3+} = 3d^3</math></p> <p>D. <math>[\text{Fe}(\text{CN})_6]^{3-}</math>, <math>[\text{Cr}(\text{H}_2\text{O})_6]^{3+}</math>  <math>\text{Fe}^{3+} = 3d^5</math>   <math>\text{Cr}^{3+} = 3d^3</math></p> <p>E. <math>\text{K}_3[\text{Cu}(\text{CN})_4]</math>, <math>[\text{CuF}_6]^{3-}</math>  <math>\text{Cu}^+ = 3d^{10}</math>   <math>\text{Cu}^{3+} = 3d^8</math></p> <p>Hence, the correct option is (a).</p>																
<b>29.</b>	In which of the following transformations, the bond order has increased and the magnetic behaviour has changed?	<b>B</b>															
	$\text{C}_2 \rightarrow \text{C}_2^+$																
	$\text{NO} \rightarrow \text{NO}^+$																
	$\text{O}_2 \rightarrow \text{O}_2^+$																
	$\text{N}_2 \rightarrow \text{N}_2^+$																
<b>Sol.</b>	<table border="1"> <thead> <tr> <th><b>Process</b></th> <th><b>Change in magnetic nature</b></th> <th><b>Bond order change</b></th> </tr> </thead> <tbody> <tr> <td><math>\text{C}_2 \rightarrow \text{C}_2^+</math></td> <td>Diamagnetic-Paramagnetic</td> <td>2-1.5</td> </tr> <tr> <td><math>\text{O}_2 \rightarrow \text{O}_2^+</math></td> <td>Paramagnetic-Paramagnetic</td> <td>2-2.5</td> </tr> <tr> <td><math>\text{NO} \rightarrow \text{NO}^+</math></td> <td>Paramagnetic-Diamagnetic</td> <td>2.5-3</td> </tr> <tr> <td><math>\text{N}_2 \rightarrow \text{N}_2^+</math></td> <td>Diamagnetic-Paramagnetic</td> <td>3-2.5</td> </tr> </tbody> </table> <p>Hence, in <math>\text{NO} \rightarrow \text{NO}^+</math> the bond order increased and the magnetic behaviour also changed.</p>	<b>Process</b>	<b>Change in magnetic nature</b>	<b>Bond order change</b>	$\text{C}_2 \rightarrow \text{C}_2^+$	Diamagnetic-Paramagnetic	2-1.5	$\text{O}_2 \rightarrow \text{O}_2^+$	Paramagnetic-Paramagnetic	2-2.5	$\text{NO} \rightarrow \text{NO}^+$	Paramagnetic-Diamagnetic	2.5-3	$\text{N}_2 \rightarrow \text{N}_2^+$	Diamagnetic-Paramagnetic	3-2.5	
<b>Process</b>	<b>Change in magnetic nature</b>	<b>Bond order change</b>															
$\text{C}_2 \rightarrow \text{C}_2^+$	Diamagnetic-Paramagnetic	2-1.5															
$\text{O}_2 \rightarrow \text{O}_2^+$	Paramagnetic-Paramagnetic	2-2.5															
$\text{NO} \rightarrow \text{NO}^+$	Paramagnetic-Diamagnetic	2.5-3															
$\text{N}_2 \rightarrow \text{N}_2^+$	Diamagnetic-Paramagnetic	3-2.5															
<b>30.</b>	Match List – I with List – II	<b>C</b>															
	<b>List-I</b>	<b>List-II</b>															
	A. Nessler's reagent	I. Halogen specifically															
	B. Cu(II) oxide	II. $\text{Ni}^{2+}$															
	C. Silver nitrate	III. $\text{NH}_3$ or $\text{NH}_4^+$															
	D. Dimethyl glyoxime	IV. Carbon															
	Choose the most appropriate answer from the options given below.																
	A – II, B – IV, C – III, D – I																
	A – I, B – III, C – IV, D – II																

	A – III, B – IV, C – I, D – II	
	A – III, B – II, C – IV, D – I	
<b>Sol.</b>	<p>The correct order is A-III, B-IV, C-I, D-II.</p> <ul style="list-style-type: none"> <li>• Nessler's reagent is used to detect <math>\text{NH}_3</math> or <math>\text{NH}_4^+</math>.</li> <li>• Cu(II) oxide is used to detect carbon and hydrogen.</li> <li>• Silver nitrate is used to detect specifically halogens.</li> <li>• Dimethyl glyoxime is used to detect nickel ion (<math>\text{Ni}^{2+}</math>).</li> </ul>	
<b>31.</b>	<p>An amino acid contain a carboxyl group and amino group whose <math>\text{pK}_a</math> values are 3.30 and 9.0, respectively. The value of pH at this isoelectric point is.....and it is a.....amino acid.</p> <p>Choose the <b>correct</b> option that fill in the given blanks.</p>	<b>A</b>
	6.15, neutral	
	2.34, acidic	
	9.60, basic	
	6.67, basic	
<b>Sol.</b>	<p>Amino acid has two functional groups carboxylic (COOH) and amino (<math>\text{NH}_2</math>). It means that, it has an acidic and a basic group. The acidic group loses hydrogen and basic group gains <math>\text{H}^+</math>. The nature of pH at which there is no net charge on the amino acid is known as isoelectric point.</p> <p>Isoelectric point may be calculated as</p> $\text{Isoelectric point} = \frac{\text{p}K_{a_1} + \text{p}K_{a_2}}{2} = \frac{9.00 + 3.30}{2} = 6.15$ <p>∴ This is a neutral amino acid. The range of pH of neutral amino acid is 5.5 to 6.3.</p>	
<b>32.</b>	What will be the product when an optically active compound having IUPAC name pent-3-ene-2-ol is treated with Jones reagent?	<b>B</b>
	Pentanone	
	Pent-3-ene-2-one	
	Pentan-2-ol	
	Propanoic acid	
<b>Sol.</b>	<p>Structure of pent-3-ene-2-ol is</p>  <p>Jones reagent is <math>\text{CrO}_3</math> dissolved in aqueous sulphuric acid. On treatment with allylic alcohol it produces allylic ketone. Jones reagent does not react with double bond, it only oxidises the alcoholic group.</p>  <p><b>Study Tactics</b>  This problem includes conceptual mixing of nomenclature' and 'oxidation by Jones reagent. This problem can be solved by using following steps. Write the structural formula of compound.  Now look at the effective site where the reagent will attack then</p>	

	complete the reaction.											
33.	<p>Match List-I with List-II</p> <table> <thead> <tr> <th style="text-align: center;">List-II</th> <th style="text-align: center;">List-II</th> </tr> </thead> <tbody> <tr> <td>A. <math>\text{XeO}_3</math></td> <td>I. Square pyramidal</td> </tr> <tr> <td>B. <math>\text{XeOF}_4</math></td> <td>II. Bent T-shaped</td> </tr> <tr> <td>C. <math>\text{XeF}_2</math></td> <td>III. Linear</td> </tr> <tr> <td>D. <math>\text{ClF}_3</math></td> <td>IV. Pyramidal</td> </tr> </tbody> </table> <p>Choose the most appropriate answer from the codes given below:</p> <p>A-(I), B-(III), C-(II), D-(IV)</p> <p>A-(III), B-(II), C-(IV), D-(I)</p> <p>A-(IV), B-(I), C-(III), D-(II)</p> <p>A-(I), B-(IV), C-(II), D-(III)</p>	List-II	List-II	A. $\text{XeO}_3$	I. Square pyramidal	B. $\text{XeOF}_4$	II. Bent T-shaped	C. $\text{XeF}_2$	III. Linear	D. $\text{ClF}_3$	IV. Pyramidal	C
List-II	List-II											
A. $\text{XeO}_3$	I. Square pyramidal											
B. $\text{XeOF}_4$	II. Bent T-shaped											
C. $\text{XeF}_2$	III. Linear											
D. $\text{ClF}_3$	IV. Pyramidal											
<b>Sol.</b>	<p><b>(c) Shape of molecule and VSEPR theory</b> According to this theory, the exact shape of any molecule is determined on the basis of repulsion between lone pairs, and bond pairs. The shape having lesser value of energy is the acquired shape of compound/ion. The repulsion between lone pair and bond pair are as follows</p> $bp - bp < bp - lp < lp - lp$ <p>where, <math>bp</math> = bond pair  <math>lp</math> = lone pair</p> <p>According to this theory, exact shape of molecule can be determined from the hybridisation as follows,</p> $\text{XeO}_3 \longrightarrow \frac{8+0}{2} = 4, \text{sp}^3; \text{XeOF}_4 \longrightarrow \frac{8+4}{2} = 6, \text{sp}^3d$ $\text{XeF}_2 \longrightarrow \frac{8+2}{2} = 5, \text{sp}^3d; \text{ClF}_3 \longrightarrow \frac{7+3}{2} = 5, \text{sp}^3d$ <p style="text-align: right;">2</p> <table style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="text-align: center; border-bottom: 1px solid black;">Molecule</th> <th style="text-align: center; border-bottom: 1px solid black;">Shape</th> </tr> </thead> <tbody> <tr> <td style="text-align: center;">A. <math>\text{XeO}_3</math></td> <td style="text-align: center;">I. </td> </tr> <tr> <td style="text-align: center;">B. <math>\text{XeOF}_4</math></td> <td style="text-align: center;">II. </td> </tr> <tr> <td style="text-align: center;">C. <math>\text{XeF}_2</math></td> <td style="text-align: center;">III. </td> </tr> <tr> <td style="text-align: center;">D. <math>\text{ClF}_3</math></td> <td style="text-align: center;">IV. </td> </tr> </tbody> </table> <p>Hence, the correct match is A-(IV), B-(I), C-(III), D-(II).</p> <p><b>Study Tactics</b></p> <p>Calculate value of <math>H = \frac{V+M-C+A}{2}</math></p> <p>where, <math>V</math> = valence electrons, <math>M</math> = Mono- valent atoms, <math>C</math> = (+) charges, <math>A</math> = (-) charges and then determine the state of hybridisation.</p> <p>Now draw the structure using concept of bent rule. Then, interpret their shape.</p>	Molecule	Shape	A. $\text{XeO}_3$	I. 	B. $\text{XeOF}_4$	II. 	C. $\text{XeF}_2$	III. 	D. $\text{ClF}_3$	IV. 	
Molecule	Shape											
A. $\text{XeO}_3$	I. 											
B. $\text{XeOF}_4$	II. 											
C. $\text{XeF}_2$	III. 											
D. $\text{ClF}_3$	IV. 											

34. P and Q are isomers of a dicarboxylic acid  $C_4H_4O_4$ . Both decolourise  $Br_2/H_2O$ . On heating, P forms a cyclic anhydride. **B**

Upon treatment with dilute alkaline  $KMnO_4$ , P as well as Q could produce one or more than one form S, T and U.



Compounds formed from P and Q are respectively

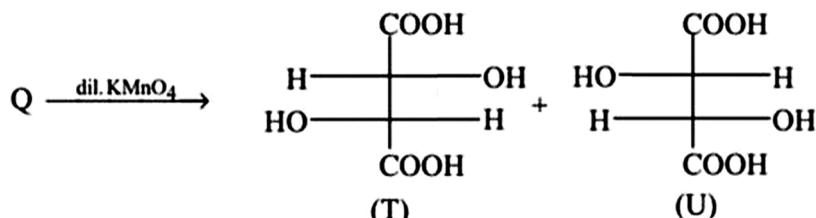
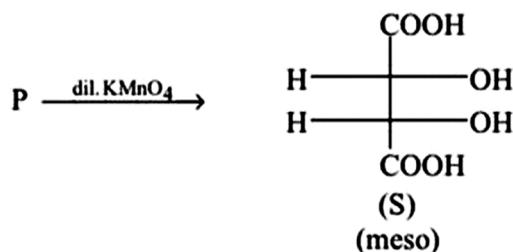
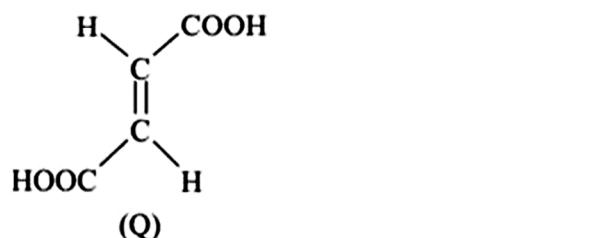
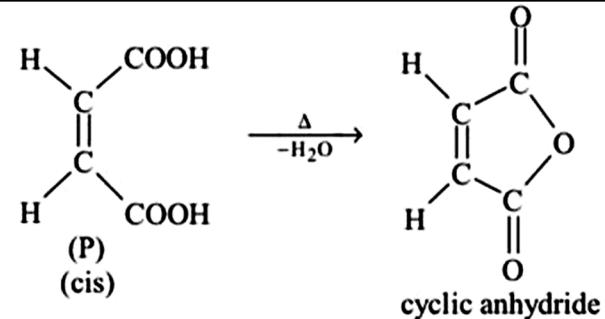
Optically active S and optically active pair (T, U).

Optically inactive S and optically active pair (T, U).

Optically active pair (T, U) and optically active S.

Optically inactive pair (T, U) and optically inactive S.

**Sol.**



35. Which of the following is the correct order of stability of conformational isomers of 2-amino ethan-1-ol? **B**

Gauche > eclipsed > anti

Gauche > anti > eclipsed

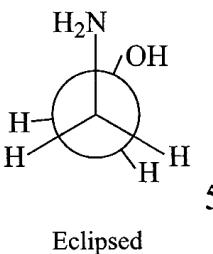
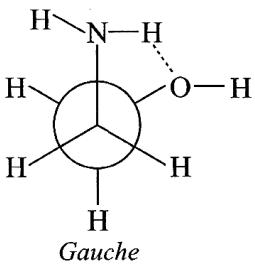
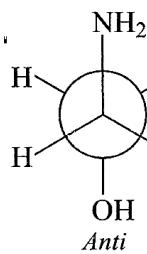
Eclipsed > Gauche > anti

Anti > eclipsed > Gauche

**Sol.**

**Hydrogen bonding** The presence of hydrogen bonding stabilises the structure.

The conformations of 2-amino-ethan-1-ol are



5

Out of above, gauche form have more stability due to hydrogen bonding. So, the correct order is gauche > anti > eclipsed.

### Study Tactics

This problem includes conceptual mixing of conformational structure of molecule and H-bonding.

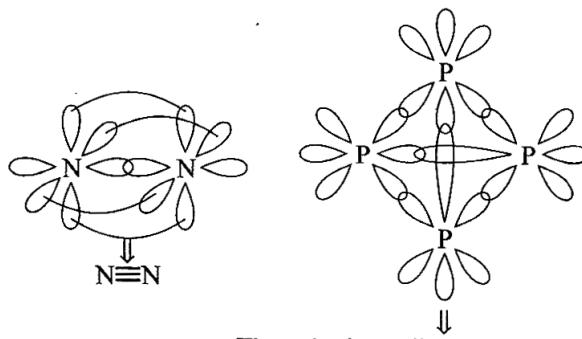
Students are advised to draw the conformational structure of organic molecule first followed by analysing the effect of H-bonding on molecule.

36.	<p>Given below are two statements.</p> <p><b>Statement I :</b> The graph of molar conductivity for KI increases steeply with dilution.</p> <p><b>Statement II :</b> The graph of molar conductivity for carbonic acid increases slowly with dilution.</p> <p>In the light of the above statements, choose the correct answer from the options given below.</p>	B
	Both Statement I and Statement II are correct.	
	Both Statement I and Statement II are incorrect.	
	Statement I is correct and Statement II is incorrect.	
	Statement I is incorrect and Statement II is correct.	

Sol.	<p>Both Statement I and Statement II are incorrect.</p> <p>For KI, molar conductivity increases slowly with dilution. For carbonic acid, molar conductivity increases steeply with dilution.</p>	
------	--	--

37.	<p>Nitrogen exist as N<sub>2</sub> but phosphorus exist as P<sub>4</sub> this is due to</p> <p>Triple bond exist between phosphorus atom</p> <p>p<math>\pi</math> – p<math>\pi</math> bond is weak in P<sub>4</sub></p> <p>p<math>\pi</math> – p<math>\pi</math> bond is weak in N<sub>2</sub></p> <p>Multiple bonds form easily</p>	B
-----	--	---

Sol.	<p>Existence of nitrogen as N<sub>2</sub> is due to strong p<math>\pi</math> – p<math>\pi</math> bonding between smaller sized p-orbitals of N.</p> <p>Existence of phosphorus as P<sub>4</sub> is due to existence of weak p<math>\pi</math> – p<math>\pi</math> bonding due to large size of p-orbital of phosphorous atom.</p>	
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Thus, it shows discrete unit of  $P_4$ , (due to large size of  $p$ -orbital or phosphorus)

38. Match List-I and List-II.

**List-I**

- A. Alcoholic potassium hydroxide
- B.  $Pd/BaSO_4$
- C. BHC (Benzene hexachloride)
- D.  $LiAlH_4$

**List-II**

- I. Strong reducing agent
- II. Obtained by addition reaction
- III. Used for  $\beta$ -elimination reaction
- IV. Lindlar's Catalyst

Choose the correct answer from the options given below.

A-III, B-IV, C-II, D-I

A-III, B-I, C-IV, D-II

A-II, B-III, C-I, D-IV

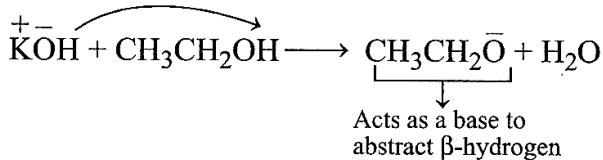
A-II, B-IV, C-III, D-I

A

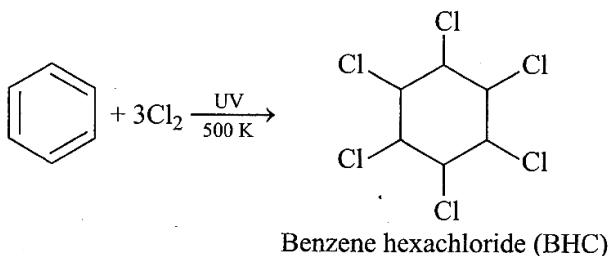
**Sol.** The correct match is

$A \rightarrow (III)$ ,  $B \rightarrow (IV)$ ,  $C \rightarrow (II)$ ,  $D \rightarrow (I)$ .

- Alcoholic KOH is used for  $\beta$ -elimination reaction.



- $Pd/BaSO_4$  known as Lindlar's catalyst.
- Benzene hexachloride (BHC) is obtained by addition reaction.



- $LiAlH_4$  is a strong reducing agent.

39. Which carbonyl compound gives tertiary alcohol with Grignard reagent followed by acidic hydrolysis?

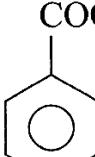
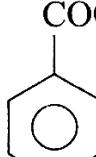
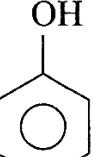
C

Formaldehyde

Acetaldehyde

Acetone

Methyl formate

Sol.	<p>The reaction between Grignard reagent and ketone produces tertiary alcohol as a product. The reaction is carried out in a single step and involves the formation of a six-membered transition state, which is formed when the Grignard reagent attacks the carbonyl centre of the ketone.</p>	
40.	<p>When aqueous NaOH is added to an aqueous solution of chromium (III) ion, a green blue precipitate is formed first which re-dissolves to give a green solution. The green colour of the solution is due to the formation of</p> <p><math>[\text{Cr}(\text{H}_2\text{O})_6]^{3+}</math></p> <p><math>[\text{Cr}(\text{OH})_4]^-</math></p> <p><math>\text{CrO}_4^{2-}</math></p> <p><math>[\text{Cr}(\text{OH})_3(\text{H}_2\text{O})_3]</math></p>	B
Sol.	<p>When aqueous solution of NaOH is added to aqueous solution of Cr (III) ion it produces <math>[\text{Cr}(\text{H}_2\text{O})_3(\text{OH})_3]</math>.</p> $[\text{Cr}(\text{H}_2\text{O})_6]^{3+} + \text{NaOH} \longrightarrow [\text{Cr}(\text{OH})_3(\text{H}_2\text{O})_3] + \text{H}_2\text{O}$ <p style="text-align: center;">Green blue ppt.</p> <p>Which on further redissolves in <i>aq.</i> NaOH (in excess) to produce dark green solution due to formation of</p> <p><math>[\text{Cr}(\text{OH})_4]^-</math></p> $[\text{Cr}(\text{OH})_3(\text{H}_2\text{O})_3] + \text{OH}^- \xrightarrow{\text{Excess}} [\text{Cr}(\text{OH})_4]^-$ <p style="text-align: center;">Dark green solution</p>	
41.	<p>Which of the following is the correct order acidity of the given Compounds?</p> <p>COOH      COOH      OH</p> <p>   I         </p> <p>   II         </p> <p>   III         </p>	C
	III > II > I	
	II > I > III	
	I > II > III	
	III > I > II	
Sol.	<p>The decreasing order of acidic strength of given compound is I &gt; II &gt; III.</p> <p>The benzoate ion is more stabilised because the negative charge is on the more electronegative oxygen atom, whereas in phenoxide ion, it is on the less electronegative carbon atoms. This causes benzoic acid to be a stronger acid than phenol.</p> <p>The alkyl group attached to para position to carboxylic group in aromatic carboxylic acid reduces the acidity.</p> <p>Similarly, electron releasing group, i.e. <math>-\text{CH}_3</math> present in the para position to <math>-\text{OH}</math> group decreases its acidity.</p> <p><b>Study Tactics</b></p> <p>The more stable is the conjugate base the stronger is the acid. Also, ERG decreases the acidity while EWG increases the acidity.</p>	
42.	<p>There are some statements regarding the periodic classification of elements.</p> <p><b>A.</b> Non-metallic elements are less in number than metallic elements.</p> <p><b>B.</b> The first ionization enthalpies of elements generally increases with an increase in atomic number as we go along a period.</p>	A

	<p><b>C.</b> For transition elements, 3d-orbitals are filled with electrons after 3p-orbitals and before 4s-orbitals.</p> <p>Choose the correct statement from the options given below:</p>									
	A and B									
	A and C									
	B and C									
	A, B and C									
<b>Sol.</b>	<p>In case of transition elements (or any other elements), the order of filling of electrons in various orbital is <math>3p &gt; 4s &gt; 3d</math>.</p> <p>So, according to Aufbau principle, 3d-orbital is filled when 4s-orbital gets completely filled.</p> <p>Hence, among the given statements, only (C) is incorrect while (A) and (B) are correct.</p>									
<b>43.</b>	<p>The formula of A and B for the following reaction sequence are</p> <p style="text-align: center;">     Fructose <math>\xrightarrow{\substack{\text{HCN} \\ \text{H}_3\text{O}^+}} A</math>  <math>\xrightarrow{\substack{\text{(i) NaBH}_4 \\ \text{(ii) HI/P}}}</math> B   </p>	<b>A</b>								
	$A = \text{C}_7\text{H}_{14}\text{O}_8, B = \text{C}_6\text{H}_{14}$									
	$A = \text{C}_7\text{H}_{13}\text{O}_7, B = \text{C}_7\text{H}_{14}\text{O}$									
	$A = \text{C}_7\text{H}_{12}\text{O}_8, B = \text{C}_6\text{H}_{14}$									
	$A = \text{C}_7\text{H}_{14}\text{O}_8, B = \text{C}_6\text{H}_{14}\text{O}_6$									
<b>Sol.</b>	<p>The reaction is represented as</p> <p style="text-align: center;"> <math>\begin{array}{c} \text{CH}_2\text{OH} \\   \\ \text{C}=\text{O} \\   \\ (\text{CHOH})_3 \\   \\ \text{CH}_2\text{OH} \end{array} \xrightarrow{\text{HCN}} \begin{array}{c} \text{CH}_2\text{OH} \\   \\ \text{HO}—\text{C}—\text{CN} \\   \\ (\text{CHOH})_3 \\   \\ \text{CH}_2\text{OH} \end{array} \xrightarrow{\text{H}_3\text{O}^+} \begin{array}{c} \text{CH}_2\text{OH} \\   \\ \text{HO}—\text{C}—\text{COOH} \\   \\ (\text{CHOH})_3 \\   \\ \text{CH}_2\text{OH} \end{array}</math>  <math>\downarrow \begin{array}{l} \text{(i) NaBH}_4 \\ \text{(ii) HI/P} \end{array}</math>  <math>n\text{-hexane (B)}</math>  <math>\text{C}_6\text{H}_{14}</math> </p> <p style="text-align: right;"><math>(A) \text{C}_7\text{H}_{14}\text{O}_8</math></p>									
<b>44.</b>	$\text{CH}_3\text{CH}=\text{CH}_2 \xrightarrow{\text{A}} \text{CH}_3\text{CH}_2\text{CH}_2\text{OH}$ . <p>The reagent A is</p>	<b>C</b>								
	$\text{H}_2\text{O} / \text{H}_2\text{SO}_4$									
	$\text{Hg(OAc)}_2 / \text{H}_2\text{O}$ followed by $\text{NaBH}_4$									
	$\text{B}_2\text{H}_6$ followed by $\text{H}_2\text{O}_2$									
	$\text{CH}_3\text{CO}_2\text{H}/\text{H}_2\text{SO}_4$									
<b>Sol.</b>	$\text{CH}_3\text{—CH}=\text{CH}_2 + \text{B}_2\text{H}_6 \xrightarrow{\text{H}_2\text{O}_2}$ <p style="text-align: center;">Propene</p> $\text{CH}_3\text{—CH}_2\text{—CH}_2\text{OH}$ <p style="text-align: center;">Propan-1-ol</p> <p>When propene reacts with diborane followed by <math>\text{H}_2\text{O}_2</math>, it gives primary alcohols which is in accordance with anti-Markownikoff's rule.</p>									
<b>45.</b>	<p>Match List – I with List – II.</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="text-align: center;">List – I</th> <th style="text-align: center;">List – II</th> </tr> </thead> <tbody> <tr> <td style="text-align: center;">A. <math>\text{BF}_4^-</math></td> <td style="text-align: center;">I. Square planar</td> </tr> <tr> <td style="text-align: center;">B. <math>\text{BrF}_4^-</math></td> <td style="text-align: center;">II. Bent shape</td> </tr> <tr> <td style="text-align: center;">C. <math>\text{SF}_4</math></td> <td style="text-align: center;">III. Tetrahedral</td> </tr> </tbody> </table>	List – I	List – II	A. $\text{BF}_4^-$	I. Square planar	B. $\text{BrF}_4^-$	II. Bent shape	C. $\text{SF}_4$	III. Tetrahedral	<b>C</b>
List – I	List – II									
A. $\text{BF}_4^-$	I. Square planar									
B. $\text{BrF}_4^-$	II. Bent shape									
C. $\text{SF}_4$	III. Tetrahedral									

D.  $\text{BrF}_2^+$ 

IV. See-saw

Choose the correct answer from the options given below.

A – II, B – III, C – I, D – IV

A – III, B – IV, C – IV, D – II

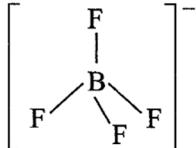
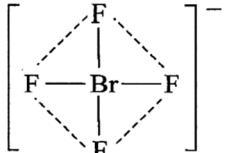
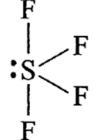
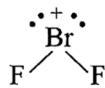
A – III, B – I, C – IV, D – II

A – IV, B – III, C – II, D – I

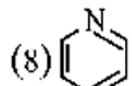
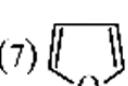
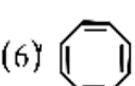
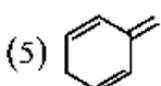
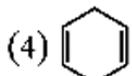
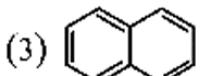
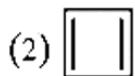
**Sol.**

The correct match is A-III, B-I, C-IV, D-II.

The molecules with their shapes are given below.

(A)  $\text{BF}_4^-$  – Tetrahedral(B)  $\text{BrF}_4^-$  – Square planar(C)  $\text{SF}_4$  – See-saw(D)  $\text{BrF}_2^+$  – Bent-shape**46.**

Number of aromatic compounds in the following :

**3****Sol.**

The requirements for a compound to be aromatic are:

The compound must be cyclic

The molecule is planar. This means that the ring cannot contain a neutral  $\text{sp}^3$  carbon.The compound must follow Huckel's Rule (the ring has to contain  $4n + 2$  p-orbital electrons).

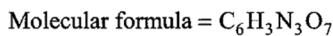
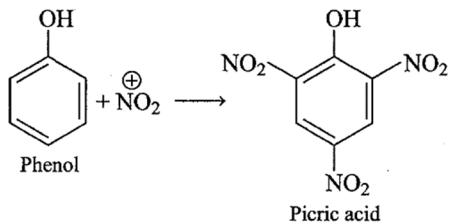
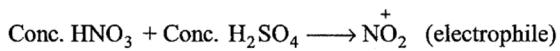
Among the given compounds naphthalene, furan and pyridine are aromatic compounds.

**47.**Among the following reagents, alkaline  $\text{KMnO}_4$ ,  $\text{H}_2\text{O}_2$ ,  $\text{Cl}_2$ ,  $\text{O}_3$ ,  $\text{Na}_2\text{S}_2\text{O}_3$  and  $\text{HNO}_3$ , the total number of reagents that can oxidise aqueous iodide to iodine is .....**4**

Sol.	<p><math>\text{H}_2\text{O}_2</math>, <math>\text{Cl}_2</math>, <math>\text{O}_3</math> and <math>\text{HNO}_3</math> oxidise aqueous iodide to iodine whereas alkaline <math>\text{KMnO}_4</math> oxidise aqueous iodide to <math>\text{IO}_3^-</math>.</p> <p>However, <math>\text{Na}_2\text{S}_2\text{O}_3</math>, being a strong reducing agent, on reaction with <math>\text{I}_2</math> produces <math>\text{I}^-</math>.</p> $\text{Na}_2\text{S}_2\text{O}_3 + \text{I}_2 \longrightarrow 2\text{I}^- + \text{Na}_2\text{S}_4\text{O}_6$ <p>Thus, no reaction takes place between <math>\text{Na}_2\text{S}_2\text{O}_3</math> and <math>\text{I}^-</math>.</p>	
48.	<p>The energy of an electron is given by <math>E = -2.178 \times 10^{-18} \text{ J} \left( \frac{z^2}{n^2} \right)</math> Wavelength of light required to excite an electron in a hydrogen atom from level <math>n = 1</math> to <math>n = 2</math> will be ..... <math>\times 10^{-7} \text{ m}</math>. [Nearest integer]</p> <p>[Given; <math>h = 6.62 \times 10^{-34} \text{ Js}</math> and <math>c = 3 \times 10^8 \text{ ms}^{-1}</math>]</p>	1
Sol.	<p>The energy difference between two levels will be</p> $E_2 - E_1 = \frac{-2.178 \times 10^{-18}}{4} + 2.178 \times 10^{-18}$ $= \frac{3}{4} \times 2.178 \times 10^{-18} \text{ J}$ <p>Now, the wavelength of light required to excite an electron,</p> $\lambda = \frac{hc}{E_2 - E_1} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{\frac{3}{4} \times 2.178 \times 10^{-18}}$ $= 1.22 \times 10^{-7} \text{ m}$ $\approx 1 \times 10^{-7} \text{ m}$	
49.	One atom of an element, $y$ = weighs $6.64 \times 10^{-23} \text{ g}$ . Then, the number of moles of atom in 20 kg is ..... .	500
Sol.	<p>Atomic weight of element,  <math>y = 6.64 \times 10^{-23} \times N_A \approx 40</math></p> <p>Number of moles of <math>y = \frac{20 \times 1000}{40} = 500</math></p>	
50.	Phenol on reaction with a mixture of conc. $\text{HNO}_3$ and conc. $\text{H}_2\text{SO}_4$ produces a compound. The degree of unsaturation of product obtained will be:	7

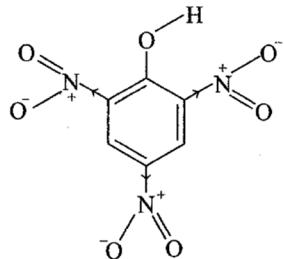
**Sol.**

Phenol undergoes electrophilic substitution reaction on reaction with a mixture of conc.  $\text{HNO}_3$  and conc.  $\text{H}_2\text{SO}_4$  (known as nitrating mixture). This reaction produces nitro compound and on successive nitration it produces trinitrophenol.



$$\begin{aligned}\text{Degree of unsaturation} &= (C + 1) - \frac{H}{2} - \frac{X}{2} + \frac{N}{2} \\ &= (6 + 1) - \frac{3}{2} + \frac{3}{2} = 7 - 0 = 7\end{aligned}$$

**Nature of compound** The compound is acidic in nature due to presence of three strong electron withdrawing groups ( $\text{NO}_2$ ).

**51.**

Let  $A = \left\{ \theta \in [0, 2\pi] : 1 + 10\text{Re} \left( \frac{2\cos\theta + i\sin\theta}{\cos\theta - 3i\sin\theta} \right) = 0 \right\}$ . Then,  $\sum_{\theta \in A} \theta^2$  is equal to

**D**

$$\frac{27}{4}\pi^2$$

$$8\pi^2$$

$$6\pi^2$$

$$\frac{21}{4}\pi^2$$

<b>Sol.</b>	<p>(d) Given,</p> $1 + 10 \operatorname{Re} \left( \frac{2\cos \theta + i\sin \theta}{\cos \theta - 3i\sin \theta} \right) = 0$ $\Rightarrow \operatorname{Re} \left( \frac{2\cos \theta + i\sin \theta}{\cos \theta - 3i\sin \theta} \right) = -\frac{1}{10}$ $\Rightarrow \operatorname{Re} \left[ \frac{(2\cos \theta + i\sin \theta)(\cos \theta + 3i\sin \theta)}{\cos^2 \theta + 9\sin^2 \theta} \right] = -\frac{1}{10}$ $\Rightarrow \frac{2\cos^2 \theta - 3\sin^2 \theta}{\cos^2 \theta + 9\sin^2 \theta} = -\frac{1}{10}$ $\Rightarrow 20\cos^2 \theta - 30\sin^2 \theta = -\cos^2 \theta - 9\sin^2 \theta$ $\Rightarrow 21\cos^2 \theta = 21\sin^2 \theta$ $\Rightarrow \sin^2 \theta = 1 - \sin^2 \theta$ $\Rightarrow \sin^2 \theta = \sin^2 \frac{\pi}{4}$ $\Rightarrow \theta = n\pi \pm \frac{\pi}{4}$ <p>So, possible values of <math>\theta</math> are</p> $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ $\sum_{\theta \in A} \theta^2 = \frac{\pi^2}{16} (1^2 + 3^2 + 7^2 + 5^2)$ $= \frac{84\pi^2}{16} = \frac{21\pi^2}{4}$	
<b>52.</b>	<p>Let <math>p</math> and <math>q</math> be real numbers such that <math>p \neq 0, p^3 \neq q</math> and <math>p^3 \neq -q</math>. If <math>\alpha</math> and <math>\beta</math> are non-zero complex numbers satisfying <math>\alpha + \beta = -p</math> and <math>\alpha^3 + \beta^3 = q</math>, then a quadratic equation having <math>\frac{\alpha}{\beta}</math> and <math>\frac{\beta}{\alpha}</math> as its roots is</p>	<b>B</b>
	$(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$	
	$(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$	
	$(p^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$	
	$(p^3 - q)x^2 - (5p^3 + 2q)x + (p^3 - q) = 0$	
<b>Sol.</b>	<p>(b) Sum of roots = <math>\frac{\alpha^2 + \beta^2}{\alpha\beta}</math>  and product = 1  Given, <math>\alpha + \beta = -p</math> and <math>\alpha^3 + \beta^3 = q</math>  <math>\Rightarrow (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2) = q</math>  <math>\therefore \alpha^2 + \beta^2 - \alpha\beta = \frac{-q}{p}</math> ... (i)  and <math>(\alpha + \beta)^2 = p^2</math>  <math>\Rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = p^2</math> ... (ii)  From Eqs. (i) and (ii), we get  <math>\alpha^2 + \beta^2 = \frac{p^3 - 2q}{3p}</math> and <math>\alpha\beta = \frac{p^3 + q}{3p}</math>  <math>\therefore</math> Required equation is,  <math display="block">x^2 - \frac{(p^3 - 2q)x}{(p^3 + q)} + 1 = 0</math>  <math display="block">\Rightarrow (p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0</math> </p>	
<b>53.</b>	A five digits number divisible by 3 is to be formed using the numbers 0, 1, 2,	<b>A</b>

	3, 4 and 5, without repetition. The total number of ways this can be done, is 216 240 600 3125	
<b>Sol.</b>	<p>(a) Since, a five-digit number is formed by using the digits 0,1,2,3,4 and 5 divisible by 3 i.e. only possible when sum of the digits is multiple of three.</p> <p><b>Case I</b> Using digits 0, 1, 2, 4, 5 number of ways = <math>4 \times 4 \times 3 \times 2 \times 1 = 96</math></p> <p><b>Case II</b> Using digits 1, 2, 3, 4, 5 number of ways <math>= 5 \times 4 \times 3 \times 2 \times 1 = 120</math></p> <p>Total numbers formed <math>= 120 + 96 = 216</math></p>	
<b>54.</b>	If $\sum_{r=1}^n T_r = \frac{(2n-1)(2n+1)(2n+3)(2n+5)}{64}$ , then $\lim_{n \rightarrow \infty} \sum_{r=1}^n \left(\frac{1}{T_r}\right)$ is equal to	<b>D</b>
	0	
	1/3	
	1	
	2/3	
<b>Sol.</b>	<p>(d)</p> $\sum_{r=1}^n T_r = \frac{(2n-1)(2n+1)(2n+3)(2n+5)}{64}$ $T_n = S_n - S_{n-1} = \frac{(2n-1)(2n+1)}{64}$ $- \frac{(2n-3)(2n-1)(2n+1)(2n+3)}{64}$ $= \frac{(2n-1)(2n+1)(2n+3)}{64}$ $= \frac{(2n-1)(2n+1)(2n+3)}{8}$ <p><math>\therefore</math></p> $\sum_{r=1}^n \frac{1}{T_r} = 8 \sum_{r=1}^n \frac{1}{(2r-1)(2r+1)(2r+3)}$ $= \frac{8}{4} \left( \sum_{r=1}^n \frac{1}{(2r-1)(2r+1)} - \frac{1}{(2r+1)(2r+3)} \right)$ $= 2 \left[ \left( \frac{1}{1 \cdot 3} - \frac{1}{3 \cdot 5} \right) + \left( \frac{1}{3 \cdot 5} - \frac{1}{5 \cdot 7} \right) \dots \right.$ $\left. \left( \frac{1}{(2n-1)(2n+1)} - \frac{1}{(2n+1)(2n+3)} \right) \right]$ $= 2 \left( \frac{1}{3} - \frac{1}{(2n+1)(2n+3)} \right)$ <p>At <math>n \rightarrow \infty</math> required value = <math>2 \times \frac{1}{3} = \frac{2}{3}</math></p>	
<b>55.</b>	The point (4, 1) undergoes the following three transformations successively I. Reflection about the line $y = x$ . II. Transformation through a distance 2 units along the positive direction of X-axis. III. Rotation through an angle $\pi/4$ about the origin in the counter clockwise direction.	<b>C</b>

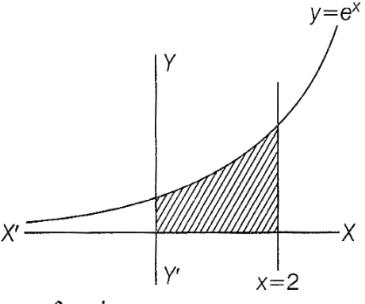
	Then, the final position of the point is given by the coordinates $\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ $(-\sqrt{2}, 7\sqrt{2})$ $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ $(\sqrt{2}, 7\sqrt{2})$	
<b>Sol.</b>	<p>(c) Let <math>B, C, D</math> be the position of the point <math>A(4, 1)</math> after the three operations I, II and III, respectively. Then, <math>B</math> is <math>(1, 4)</math>, <math>C(1+2, 4)</math> i.e. <math>(3, 4)</math>. The point <math>D</math> is obtained from <math>C</math> by rotating the coordinate axes through an angle <math>\pi/4</math> in anti-clockwise direction.</p> <p>Therefore, the coordinates of <math>D</math> are given by</p> $X = 3\cos\frac{\pi}{4} - 4\sin\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$ <p>and <math>Y = 3\sin\frac{\pi}{4} + 4\cos\frac{\pi}{4} = \frac{7}{\sqrt{2}}</math></p> <p><math>\therefore</math> Coordinates of <math>D</math> are <math>\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)</math></p>	
<b>56.</b>	The $\Delta PQR$ is inscribed in the circle $x^2 + y^2 = 25$ . If $Q$ and $R$ have coordinates $(3, 4)$ and $(-4, 3)$ respectively, then $\angle QPR$ is equal to	<b>C</b>
	$\pi/2$	
	$\pi/3$	
	$\pi/4$	
	$\pi/6$	
<b>Sol.</b>	<p>(c) Let <math>O</math> be the point at centre and <math>P</math> be the point at circumference. Therefore, angle <math>QOR</math> is double the angle <math>QPR</math>. So, be is sufficient to find the angle <math>QOR</math>.</p> <p>Now, slope of <math>OQ</math>, <math>m_1 = 4/3</math>,  slope of <math>OR</math>, <math>m_2 = -3/4</math>  Here, <math>m_1 m_2 = -1</math>  Therefore, <math>\angle QOR = \pi/2</math>  which implies that <math>\angle QPR = \pi/4</math></p>	
<b>57.</b>	Let $f: R \rightarrow R$ be such that $f(1) = 3$ and $f'(1) = 6$ . Then, $\lim_{x \rightarrow 0} \left[ \frac{f(1+x)}{f(1)} \right]^{1/x}$ equals	<b>C</b>
	1	
	$e^{1/2}$	
	$e^2$	

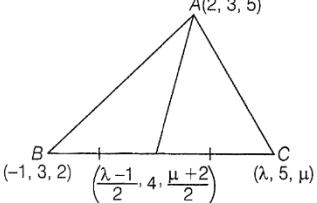
	$e^3$	
<b>Sol.</b>	<p>(c) Let <math>y = \left[ \frac{f(1+x)}{f(1)} \right]^{1/x}</math></p> $\Rightarrow \log y = \frac{1}{x} [\log f(1+x) - \log f(1)]$ $\Rightarrow \lim_{x \rightarrow 0} \log y = \lim_{x \rightarrow 0} \left[ \frac{1}{f(1+x)} \cdot f'(1+x) \right]$ <p style="text-align: center;">[using L' Hospital's rule]</p> $= \frac{f'(1)}{f(1)} = \frac{6}{3}$ $\Rightarrow \log \left( \lim_{x \rightarrow 0} y \right) = 2$ $\Rightarrow \lim_{x \rightarrow 0} y = e^2$	
<b>58.</b>	Let $S$ be the set of all the words that can be formed by arranging all the letters of the words GARDEN. From the set $S$ , one word is selected at random. The probability that the selected word will not have vowels in alphabetical order is.	<b>B</b>
	1/3	
	1/2	
	1/4	
	2/3	
<b>Sol.</b>	<p>(b) There are 2 vowels <math>A</math> and <math>E</math> only  So, probability the selected word will not have vowels in alphabetical order</p> $1 - \frac{\binom{6!}{2!}}{6!} = \frac{1}{2}$	
<b>59.</b>	Let $E = \{1, 2, 3, 4\}$ and $F = \{1, 2\}$ . Then, the number of onto functions from $E$ to $F$ is	<b>A</b>
	14	
	16	
	12	
	8	
<b>Sol.</b>	<p>(a) The number of onto functions from <math>E = \{1, 2, 3, 4\}</math> to <math>F = \{1, 2\}</math>  = Total number of functions which map <math>E</math> to <math>F</math> – Number of functions for which map <math>f(x) = 1</math> and <math>f(x) = 2</math> for all <math>x \in E</math> = <math>2^4 - 2 = 14</math></p>	
<b>60.</b>	If $a = \sin^{-1}(\sin(5))$ and $b = \cos^{-1}(\cos(5))$ , then $a^2 + b^2$ is equal to	<b>D</b>
	$4\pi^2 - 25$	
	$4\pi^2 - 20\pi + 50$	
	25	
	$8\pi^2 - 40\pi + 50$	
<b>Sol.</b>	<p>(d) We have, <math>a = \sin^{-1}(\sin 5) = 5 - 2\pi</math>  and <math>b = \cos^{-1}(\cos 5) = 2\pi - 5</math>  So, <math>a^2 + b^2 = (5 - 2\pi)^2 + (2\pi - 5)^2</math>  = <math>50 + 8\pi^2 - 40\pi</math></p>	

61.	<p>Let <math>A = [a_{ij}]</math> be a <math>3 \times 3</math> matrix such that <math>A \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}</math>, <math>A \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}</math> and <math>A \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}</math>, then <math>a_{23}</math> equals</p>	B
	2	
	-1	
	1	
	0	
Sol.	<p>b) Let <math>A = \begin{bmatrix} a_{11} &amp; a_{12} &amp; a_{13} \\ a_{21} &amp; a_{22} &amp; a_{23} \\ a_{31} &amp; a_{32} &amp; a_{33} \end{bmatrix}</math></p> $\because A \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $\Rightarrow a_{12} = 0, a_{22} = 0, a_{32} = 1$ <p>and <math>A \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}</math></p> $4a_{11} + a_{12} + 3a_{13} = 0$ $= 4a_{11} + 3a_{13} = 0 \quad \dots(i)$ $4a_{21} + a_{22} + 3a_{23} = 1$ $= 4a_{21} + 3a_{23} = 1 \quad \dots(ii)$ $4a_{31} + a_{32} + 3a_{33} = 0$ $= 4a_{31} + 3a_{33} = 0 \quad \dots(iii)$ <p>and <math>A \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}</math></p> $2a_{11} + a_{12} + 2a_{13} = 1$ $= 2a_{11} + 2a_{13} = 1 \quad \dots(iv)$ $2a_{21} + a_{22} + 2a_{23} = 0$ $= a_{21} + a_{23} = 0 \quad \dots(v)$ $2a_{31} + a_{32} + 2a_{33} = 0$ $\Rightarrow a_{31} + a_{33} = 0 \quad \dots(vi)$ <p>Solving Eqs. (ii) and (v), we get  <math>-a_{23} = 1 \Rightarrow a_{23} = -1</math></p>	
62.	<p>Let <math>f: [-1, 21] \rightarrow R</math> be given by <math>f(x) = 2x^2 + x + [x^2] - [x]</math>, where <math>[t]</math> denotes the greatest integer less than or equal to <math>t</math>. The number of points, where <math>f</math> is not continuous, is</p>	C
	3	
	6	
	4	
	5	

<b>Sol.</b>	<p>(c) <math>f(x) = 2x^2 + x + [x^2] - [x]</math>  <math>f : [-1, 2] \rightarrow \mathbb{R}</math>  <math>\because [x]</math> is discontinuous at integral point  <math>\therefore -1 \leq x \leq 2, 0 \leq x^2 \leq 4</math>  <math>\therefore</math> Point where <math>f(x)</math> can be discontinuous are  <math>\Rightarrow x = 1, 0, 1, \sqrt{2}, \sqrt{3}, 2</math></p> <p>At end points we have to check  At <math>x = -1</math>  <math>\therefore f(-1) = 3</math>  <math>f(-1^+) = 2 - 1 + 0 - (-1) = 2</math>  <math>\therefore</math> Discontinuous at <math>x = -1</math></p> <p>At <math>x = 0, f(0) = 0</math>  <math>f(0^-) = 0 + 0 + 0 - (-1) = 1</math>  <math>\therefore</math> Discontinuous at <math>x = 0</math></p> <p>At <math>x = 2</math>  <math>f(2) = 8 + 2 + 4 - 2 = 12</math>  <math>f(2^-) = 8 + 2 + 3 - 1 = 12</math>  <math>\therefore</math> Continuous at <math>x = 2</math></p> <p>At <math>x = 1</math>  <math>f(1) = 3 - 1 + 1 = 3</math>  <math>f(1^+) = 2 + 1 - 1 + 1 = 3</math>  <math>f(1^-) = 2 + 1 - 0 - 0 = 3</math>  <math>\therefore</math> Continuous at <math>x = 1</math></p> <p>Similarly, <math>f(x)</math> will be discontinuous at <math>x = \sqrt{2}</math> and <math>\sqrt{3}</math> also.</p> <p>Number of points of discontinuity = 4</p> <p>● <b>Concept Enhancer</b>  Polynomial function is continuous everywhere and Greatest Integer Function is discontinuous at integer.</p>	
<b>63.</b>	<p>If <math>f(x) = x^3 + bx^2 + cx + d</math> and <math>0 &lt; b^2 &lt; c</math>, then in <math>(-\infty, \infty)</math></p> <p><math>f(x)</math> is strictly increasing function</p> <p><math>f(x)</math> has a local maxima</p> <p><math>f(x)</math> is strictly decreasing function</p> <p><math>f(x)</math> is bounded</p>	<b>A</b>
<b>Sol.</b>	<p>(a) Given, <math>f(x) = x^3 + bx^2 + cx + d</math>  <math>\Rightarrow f'(x) = 3x^2 + 2bx + c</math></p> <p>As we know that, if <math>ax^2 + bx + c &gt; 0</math>,  <math>\forall x</math>, then <math>a &gt; 0</math> and <math>D &lt; 0</math>.</p> <p>Now, <math>D = 4b^2 - 12c = 4(b^2 - c) - 8c</math>  [where, <math>b^2 - c &lt; 0</math> and <math>c &gt; 0</math>]</p> <p><math>\Rightarrow D &lt; 0</math></p> <p><math>\Rightarrow f'(x) = 3x^2 + 2bx + c &gt; 0</math>,  <math>\forall x \in (-\infty, \infty)</math> [as <math>D &lt; 0</math> and <math>a &gt; 0</math>]</p> <p>Hence, <math>f(x)</math> is strictly increasing function.</p>	
<b>64.</b>	<p>Let <math>\int \frac{2 - \tan x}{3 + \tan x} dx = \frac{1}{2}(\alpha x + \log_e  \beta \sin x + \gamma \cos x ) + C</math>, where <math>C</math> is the constant of integration. Then, <math>\alpha + \frac{\gamma}{\beta}</math> is equal to</p>	<b>D</b>
		1

	3	
	7	
	4	
<b>Sol.</b>	$(d) \int \frac{2 - \tan x}{3 + \tan x} dx = \int \frac{2 \cos x - \sin x}{3 \cos x + \sin x} dx$ $2 \cos x - \sin x = A(3 \cos x + \sin x)$ $+ B(\cos x - 3 \sin x)$ <p>On comparing coefficient of <math>\sin x</math> and <math>\cos x</math> from both sides, we get</p> $3A + B = 2 \quad \dots(i)$ $A - 3B = -1 \quad \dots(ii)$ <p>On solving Eqs. (i) and (ii), we get</p> $A = \frac{1}{2}, B = \frac{1}{2}$ <p>So, <math display="block">\int \frac{2 \cos x - \sin x}{3 \cos x + \sin x} dx</math></p> $= \frac{x}{2} + \frac{1}{2} \ln  3 \cos x + \sin x  + C$ $= \frac{1}{2} (x + \ln  3 \cos x + \sin x ) + C$ $= \frac{1}{2} (\alpha x + \ln  \beta \sin x + \gamma \cos x ) + C$ $\alpha = 1, \beta = 1, \gamma = 3$ $\therefore \alpha + \frac{\gamma}{\beta} = 1 + \frac{3}{1} = 4$	
65.	<p>Let <math>f: R \rightarrow R</math> be a twice differentiable function such that <math>f(x + y) = f(x)f(y)</math>, <math>\forall x, y \in R</math>. If <math>f'(0) = 4a</math> and <math>f</math> satisfies <math>f''(x) - 3af'(x) - f(x) = 0</math>, <math>a &gt; 0</math>, then the area of the region <math>R = \{(x, y)   0 \leq y \leq f(ax), 0 \leq x \leq 2\}</math> is</p> <p><b>A</b></p>	
	$e^2 - 1$	
	$e^4 + 1$	
	$e^4 - 1$	
	$e^2 + 1$	

<b>Sol.</b>	$  \begin{aligned}  f(x+y) &= f(x) \cdot f(y), \forall x, y \in R \\  f(x) &= k^x \\  f'(x) &= k^x \ln k \\  \Rightarrow f'(0) &= \ln k = 4a \Rightarrow k = e^{4a} \\  f(x) &= (e^{4a})^x = e^{4ax} \\  f''(x) &= 4a e^{4ax} \\  f'''(x) &= 16a^2 e^{4ax} \\  \text{Now, } f'''(x) - 3af'(x) - f(x) &= 0 \\  16a^2 e^{4ax} - 12a^2 e^{4ax} - e^{4ax} &= 0 \\  (4a^2 - 1)e^{4ax} &= 0 \Rightarrow 4a^2 = 1 \\  a &= 1/2 \quad [\because a > 0] \\  \therefore y &= e^{2x} \Rightarrow f(ax) = e^x  \end{aligned}  $  <p>Area of region</p> $  \begin{aligned}  R &= \{(x, y); 0 \leq y \leq f(ax), 0 \leq x \leq 2\} \\  A &= \int_0^2 e^x dx = [e^x]_0^2 = e^2 - 1  \end{aligned}  $	
<b>66.</b>	<p>If the points with position vectors <math>\alpha\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + 13\hat{\mathbf{k}}</math>, <math>6\hat{\mathbf{i}} + 11\hat{\mathbf{j}} + 11\hat{\mathbf{k}}</math>, <math>\frac{9}{2}\hat{\mathbf{i}} + \beta\hat{\mathbf{j}} - 8\hat{\mathbf{k}}</math> are collinear, then <math>(19\alpha - 6\beta)^2</math> is equal to</p> <p>36</p> <p>25</p> <p>49</p> <p>16</p>	<b>A</b>
<b>Sol.</b>	<p>(a) Let <math>\mathbf{a} = \alpha\hat{\mathbf{i}} + 10\hat{\mathbf{j}} + 13\hat{\mathbf{k}}</math></p> $  \begin{aligned}  \mathbf{b} &= 6\hat{\mathbf{i}} + 11\hat{\mathbf{j}} + 11\hat{\mathbf{k}} \\  \mathbf{c} &= \frac{9}{2}\hat{\mathbf{i}} + \beta\hat{\mathbf{j}} - 8\hat{\mathbf{k}}  \end{aligned}  $ <p>If <math>P(\mathbf{a})</math>, <math>Q(\mathbf{b})</math>, <math>R(\mathbf{c})</math> are collinear, then</p> $  \mathbf{PQ} \parallel \mathbf{QR} \parallel \mathbf{PR}  $ <p>Now, <math>\mathbf{PQ} = (6 - \alpha)\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}</math></p> $  \mathbf{QR} = -\frac{3}{2}\hat{\mathbf{i}} + (\beta - 11)\hat{\mathbf{j}} - 19\hat{\mathbf{k}}  $ $  \mathbf{PQ} \parallel \mathbf{QR} \Rightarrow \frac{6 - \alpha}{-\frac{3}{2}} = \frac{1}{\beta - 11} = \frac{2}{19}  $ $  \Rightarrow 19(6 - \alpha) = 117 \Rightarrow 6\beta = 123  $ $  \therefore (19\alpha - 6\beta)^2 = (117 - 123)^2 = 36  $	
<b>67.</b>	<p><math>ABC</math> is triangle in a plane with vertices <math>A(2, 3, 5)</math>, <math>B(-1, 3, 2)</math> and <math>C(\lambda, 5, \mu)</math>. If the median through <math>A</math> is equally inclined to the coordinate axes, then the value of <math>(\lambda^3 + \mu^3 + 5)</math> is</p>	<b>B</b>

	1130	
	1348	
	1077	
	676	
<b>Sol.</b>	<p>(b) DR's of <math>AD</math> are <math>\frac{\lambda-1}{2} - 2, 4 - 3, \frac{\mu+2}{2} - 5</math>  i.e. <math>\frac{\lambda-5}{2}, 1, \frac{\mu-8}{2}</math></p> <p><math>\therefore</math> This median is making equal angles with coordinate axes. Therefore,</p>  $\frac{\lambda-5}{2} = 1 = \frac{\mu-8}{2}$ $\Rightarrow \lambda = 7 \text{ and } \mu = 10$ $\therefore \lambda^3 + \mu^3 + 5 = 1348$	
<b>68.</b>	<p><math>A</math> and <math>B</math> alternately throw a pair of dice. <math>A</math> wins if he throws a sum of 5 before <math>B</math> throws a sum of 8 and <math>B</math> wins if he throws a sum of 8 before <math>A</math> throws a sum of 5. The probability that <math>A</math> wins if <math>A</math> makes the first throw, is</p>	<b>C</b>
	$\frac{8}{17}$	
	$\frac{8}{19}$	
	$\frac{9}{19}$	
	$\frac{9}{17}$	
<b>Sol.</b>	<p>(c) For sum = 5, <math>\{(1, 4), (2, 3), (3, 2), (4, 1)\}</math></p> $\therefore P(A) = \frac{4}{36} = \frac{1}{9}$ <p>For sum = 8</p> $\Rightarrow \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$ $\therefore P(B) = 5/36$ <p>Now,</p> $P(\bar{A}) = 1 - P(A) = 1 - \frac{4}{36} = \frac{32}{36}$ $\text{and } P(\bar{B}) = 1 - P(B) = 1 - \frac{5}{36} = \frac{31}{36}$ $P(A \text{ win } S) = P(A) + P(\bar{A}) \cdot P(\bar{B}) \cdot P(A)$ $+ \frac{(P(\bar{A}) \cdot P(\bar{B}))^2}{P(\bar{A}) P(\bar{B}) P(\bar{A}) P(\bar{B})} \cdot P(A) \dots$ <p>takes the form sum of infinite GP</p> $S_\infty = \frac{a}{1-r}$ $= \frac{P(A)}{1 - P(\bar{A}) P(\bar{B})}$ $= \frac{\frac{4}{36}}{1 - \frac{32}{36} \times \frac{31}{36}} = \frac{9}{19}$	
<b>69.</b>	<p>An organization awarded 48 medals in event <math>A</math>, 25 in event '<math>B</math>' and 18 in event '<math>C</math>'. If these medals went to total 60 men and only five men got medals in all the three events, then how many received medals in exactly two of three events?</p>	<b>C</b>
	10	
	9	
	21	

	15	
<b>Sol.</b>	<p>(c) We have, <math>n(A) = 48</math>,  <math>n(B) = 25, n(C) = 18</math>  <math>n(A \cup B \cup C) = 60</math>  and <math>n(A \cap B \cap C) = 5</math></p> <p><math>e = 5</math>,  <math>a + c + e + d = 48 \Rightarrow a + c + d = 43 \dots (i)</math>  <math>b + c + e + f = 25</math>  <math>\Rightarrow b + c + f = 20 \dots (ii)</math>  <math>d + e + f + g = 18</math>  <math>\Rightarrow d + f + g = 13 \dots (iii)</math>  <math>a + b + c + d + e + f + g = 60</math>  <math>\Rightarrow a + b + c + d + f + g = 55 \dots (iv)</math>  Adding Eqs. (i), (ii) and (iii),  <math>a + b + 2c + 2d + 2f + g = 76 \dots (v)</math>  Subtract Eq. (iv) from Eq. (v), we get  <math>c + d + f = 21</math>  <math>\therefore</math> Number of men who received exactly 2 medals <math>= c + d + f = 21</math></p>	
<b>70.</b>	The value of $36(4 \cos^2 9^\circ - 1)(4 \cos^2 27^\circ - 1)(4 \cos^2 81^\circ - 1)(4 \cos^2 243^\circ - 1)$ is	<b>B</b>
	27	
	36	
	18	
	54	
<b>Sol.</b>	<p>(b) <math>4 \cos^2 \theta - 1 = 4 - 4 \sin^2 \theta - 1</math>  <math>= 3 - 4 \sin^2 \theta</math>  <math>= \frac{3 \sin \theta - 4 \sin^3 \theta}{\sin \theta} = \frac{\sin 3\theta}{\sin \theta}</math></p> <p><math>\therefore</math> Given expression,</p> $36 \cdot \frac{\sin(3 \times 9^\circ)}{\sin 9^\circ} \times \frac{\sin(3 \times 27^\circ)}{\sin 27^\circ} \times \frac{\sin(3 \times 81^\circ)}{\sin 81^\circ} \times \frac{\sin(3 \times 243^\circ)}{\sin 243^\circ}$ $= 36 \cdot \frac{\sin 729^\circ}{\sin 9^\circ} = 36 \times \frac{\sin(2 \times 360^\circ + 9^\circ)}{\sin 9^\circ}$ $= 36 \times \frac{\sin 9^\circ}{\sin 9^\circ} = 36$	
<b>71.</b>	If $\sum_{r=0}^{10} \left( \frac{10^{r+1} - 1}{10^r} \right) \cdot {}^{11}C_{r+1} = \frac{\alpha^{11} - 11^{11}}{10^{10}}$ , then $\alpha$ is equal to	<b>20</b>
<b>Sol.</b>	<p>(b) <math>\sum_{r=0}^{10} \frac{10^{r+1} - 1}{10^r} \cdot {}^{11}C_{r+1}</math>  <math>= \sum_{r=0}^{10} 10 \cdot {}^{11}C_{r+1} - \sum_{r=0}^{10} \left( \frac{1}{10} \right)^r \cdot {}^{11}C_{r+1}</math>  <math>= 10 \cdot (2^{11} - 1) - 10 \left[ \left( 1 + \frac{1}{10} \right)^{11} - 1 \right]</math>  <math>= 10 \cdot 2^{11} - \frac{11^{11}}{10^{10}} = \frac{20^{11} - 11^{11}}{10^{10}}</math>  <math>\therefore \alpha = 20</math></p>	

72.	Consider the data on $x$ taking the values $0, 2, 4, 8, \dots, 2^n$ with frequencies ${}^nC_0, {}^nC_1, {}^nC_2, \dots, {}^nC_n$ respectively. If the mean of this data is $\frac{728}{2^n}$ , then $n$ is equal to .....	6
Sol.	<p>(6) Mean = <math>\frac{\sum x_i f_i}{\sum f_i} = \frac{0 \cdot {}^nC_0 + 2 \cdot {}^nC_1 + 2^2 \cdot {}^nC_2 + \dots + 2^n \cdot {}^nC_n}{{}^nC_0 + {}^nC_1 + \dots + {}^nC_n}</math></p> <p>To find sum of numerator consider</p> $(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_n x^n \quad \dots(1)$ <p>Put <math>x=2 \Rightarrow 3^n - 1 = 2 \cdot {}^nC_1 + 2^2 \cdot {}^nC_2 + \dots + 2^n \cdot {}^nC_n</math></p> <p>To find sum of denominator, put <math>x=1</math> in</p> <p>Eq. (i), we get</p> $2^n = {}^nC_0 + {}^nC_1 + \dots + {}^nC_n$ $\therefore \frac{3^n - 1}{2^n} = \frac{728}{2^n} \Rightarrow 3^n = 729$ $\Rightarrow n = 6$	
73.	The number of singular matrices of order 2, whose elements are from the set $\{2, 3, 6, 9\}$ , is .....	36
Sol.	<p>(36) Let <math>A = \begin{pmatrix} a &amp; b \\ c &amp; d \end{pmatrix}</math></p> <p>for <math>A</math> to be singular matrix</p> $ad = bc$ <p><b>Case I</b> Exactly 1 number in used</p> $\Rightarrow {}^4C_1 \text{ ways}$ <p><b>Case II</b> Exactly 2 numbers are used</p> $\Rightarrow {}^4C_2 \text{ ways}$ <p><b>Case III</b> Exactly 3 numbers are used</p> $\Rightarrow \text{none will be singular}$ <p><b>Case IV</b> Exactly 4 number are used</p> $\Rightarrow ab = cd \Rightarrow 2 \times 9 = 3 \times 6$ $\Rightarrow {}^4C_1 \times 2! = 8 \text{ matrices}$ <p><math>\therefore</math> Total ways = <math>4 + 6 \times 4 + 8 = 36</math> matrices.</p>	
74.	Let $f: R \rightarrow R$ be a thrice differentiable odd function satisfying $f'(x) \geq 0, f''(x) = f(x), f(0) = 0, f'(0) = 3$ . Then, $9f(\ln 3)$ is equal to.....	36
Sol.	<p>(36) <math>f'(x) \geq 0</math></p> $f''(x) = f(x)$ <p>Second order differential equation</p> $f(x) = Ae^x + Be^{-x}$ $f(0) = 0 \Rightarrow A = -B$ $f(x) = A(e^x - e^{-x})$ $\Rightarrow f'(x) = A(e^x + e^{-x})$ $f'(0) = 3 = A(e^0 + e^{-0}) \Rightarrow A = \frac{3}{2}$ $f(x) = \frac{3}{2}(e^x - e^{-x})$ $\Rightarrow f(\ln 3) = \frac{3}{2}(e^{\ln 3} - e^{-\ln 3})$ $\Rightarrow 9f(\ln 3) = \frac{27}{2} \left( 3 - \frac{1}{3} \right) = 36$	

75.	Let $A = \{1, 2, 3, \dots, 7\}$ and let $P(A)$ denote the power set of $A$ . If the number of functions $f: A \rightarrow P(A)$ such that $a \in f(a), \forall a \in A$ is $m^n \cdot m$ and $n \in N$ and $m$ is least, then $m + n$ is equal to.....	51
Sol.	<p>(51) Given, <math>A = \{1, 2, 3, \dots, 7\}</math>      Then, <math>n(P(A)) = 2^7 = 128</math>  <math>f: A \rightarrow P(A)</math>      So, number of function  <math>= 128 \times 128 \times \dots \times 128</math>  <math>= 128^7 = (2^7)^7 = 2^{49}</math>  <math>\Rightarrow m^n = 2^{49}</math>  <math>\therefore m = 2</math> and <math>n = 49</math>  <math>\therefore m + n = 51</math></p>	