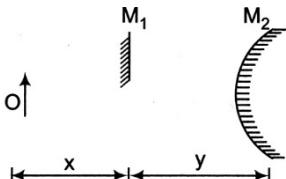


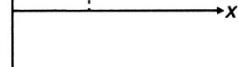
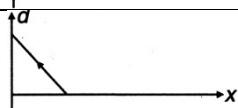
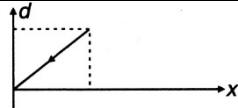
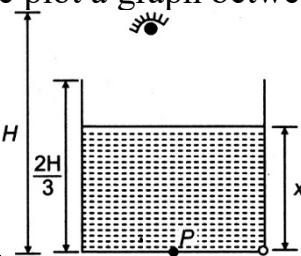
1.	Two satellites of same mass are orbiting round the earth at the heights of R and $4R$ above the earth's surface; R being the radius of the earth. The kinetic energies are in the ratio	D
	4 : 1	
	3 : 2	
	4 : 3	
	5 : 2	
Sol.	$m_1 = m_2 = m$ $n_1 = R; n_2 = 4R$ $KE = \frac{GMm}{2(R+h)}$ $KE_1 = \frac{GMm}{2(R+R)} = \frac{GMm}{4R}$ $KE_2 = \frac{GMm}{2(R+4R)} = \frac{GMm}{10R}$ $\therefore KE_1 : KE_2 = \frac{GMm}{4R} : \frac{GMm}{10R} = 5 : 2$	
2.	The ground state and first excited state energies of hydrogen atom are -13.6 eV and -3.4 eV respectively. If potential energy in ground state is taken to be zero. Then	A
	Total energy in the first excited state would be 23.8 eV.	
	Total energy in the first excited state is -3.4 eV.	
	Total energy in the first excited state is 20.4 eV.	
	None of these	
Sol.	Under normal conditions total energy, potential energy and kinetic energy in ground state and first excited state are -13.6 eV, -27.2 eV, 13.6 eV, -3.4 eV, -6.8 eV and 3.4 eV respectively. If potential energy in ground state is taken to be zero, then kinetic energy will remain unchanged but potential and total energies are increased by 27.2 eV. Therefore, the new values are 13.6 eV, 0 , 13.6 eV, 23.8 eV, 20.4 eV and 3.4 eV respectively.	
3.	An object O is placed in front of a small plane mirror M_1 and a large convex mirror M_2 of focal length f . The distance between O and M_1 is x and the distance between M_1 and M_2 is y . The images of O formed by M_1 and M_2 coincide. The magnitude of f is	B
	$x - y$	
	$\frac{x^2 - y^2}{2y}$	
	$\frac{x^2 + y^2}{2y}$	
	$\frac{x^2 + y^2}{x - y}$	
Sol.	$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$	



$$\frac{1}{f} = \frac{1}{x-y} - \frac{1}{x+y}$$

$$f = \frac{x^2 - y^2}{2y}$$

4. Liquid is filled in a vessel of height $\frac{2H}{3}$. At the bottom of the vessel there is a spot P and a hole from which liquid is coming out. Let d be the distance of image of P from an eye at height H from bottom at an instant when level of liquid in the vessel is x. If we plot a graph between d and x it will be like



Sol.
$$d = (H - x) + \frac{x}{\mu} = H - x \left(1 - \frac{1}{\mu}\right) = H - \frac{x}{4}$$

d - x graph is a straight line with positive intercept and negative slope.

5. A ray of light is incident on a parallel slab of thickness t and refractive index n. If the angle of incidence θ is small, then the lateral displacement in the incident and emergent ray will be

$$\frac{t\theta(n-1)}{n}$$

$$\frac{t\theta}{n}$$

$$\frac{t\theta n}{n-1}$$

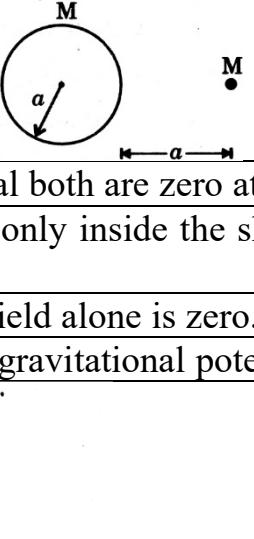
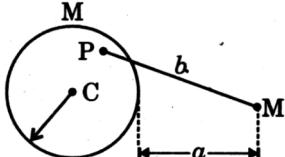
$$\frac{t\theta(n+1)}{n}$$

C

A

Sol.	<p>Lateral shift is given by</p> $d = \left[1 - \frac{\cos \theta}{\sqrt{n^2 - \sin^2 \theta}} \right] t \sin \theta$ <p>If angle of incidence θ is small, then we can write</p> $\cos \theta \approx 1, t \sin \theta \approx t \theta$ <p>and $\sqrt{n^2 - \sin^2 \theta} \approx n$</p> $\therefore d = \frac{t \theta (n-1)}{n}$	
6.	<p>In the figure, a convex mirror of radius of curvature 20 cm is shown. An object O is placed in front of this mirror. Its ray diagram is shown. How many mistakes are there in the ray diagram (AB is its principal axis)</p>	B
3		
2		
1		
0		
Sol.	<p>Ray diagram of the ray incident at B is only correct.</p>	
7.	<p>The ratio between total acceleration of the electron in singly ionized helium atom and hydrogen atom (both in ground state) is</p>	B
1		
8		
4		
16		
Sol.	$a = \frac{v^2}{r}$ $\therefore a \propto \frac{(z)^2}{(1/z)} \quad (\text{for } n=1)$ <p>or $a \propto z^3$</p> $\therefore \frac{a_1}{a_2} = \left(\frac{2}{1} \right)^3 = 8$	
8.	<p>Let K_1 be the maximum kinetic energy of photoelectrons emitted by light of wavelength λ_1 and K_2 corresponding to wavelength λ_2. If $\lambda_1 = 2\lambda_2$ then</p>	C
2 $K_1 = K_2$		
K ₁ = 2 K ₂		
K ₁ < $\frac{K_2}{2}$		
K ₁ > 2 K ₂		
Sol.	$K_1 = \frac{hc}{\lambda_1} - W \quad \dots (1)$ <p>and $K_2 = \frac{hc}{\lambda_2} - W \quad \dots (2)$</p>	
	<p>Substituting $\lambda_1 = 2\lambda_2$ in Eq. (1), we get</p>	
	$K_1 = \frac{hc}{2\lambda_2} - W$ $= \frac{1}{2} \left(\frac{hc}{\lambda_2} \right) - W = \frac{1}{2} (K_2 + W) - W$ $\therefore K_1 = \frac{K_2}{2} - \frac{W}{2} \quad \text{or} \quad K_1 < \frac{K_2}{2}$	

9.	Angular momentum (L) and radius (r) of a hydrogen atom are related as Lr = constant Lr ² = constant Lr ⁴ = constant None of these	D
Sol.	$r = \frac{\epsilon_0 n^2 h^2}{e^2 \pi m}$ $= \frac{\epsilon_0 (2\pi L)^2}{e^2 \pi m} \quad \left(L = n \frac{h}{2\pi} \text{ or } nh = 2\pi L \right)$ $\therefore Lr^{-\frac{1}{2}} = \text{constant}$	
10.	In a hypothetical system, a particle of mass m and charge $-3q$ is moving around a very heavy particle having charge q . Assuming Bohr's model to be true to this system, the orbital velocity of mass m when it is nearest to heavy particle is	A
	$\frac{3q^2}{2\epsilon_0 h}$	
	$\frac{3q^2}{4\epsilon_0 h}$	
	$\frac{3q}{2\epsilon_0 h}$	
	$\frac{3q}{4\epsilon_0 h}$	
Sol.	$\frac{1}{4\pi\epsilon_0} \frac{(3q)(q)}{r^2} = \frac{mv^2}{r} \quad \dots(1)$ $mvr = \frac{h}{2\pi} \quad \dots(2)$ <p>Solving these two equations we get,</p> $v = \frac{3q^2}{2\epsilon_0 h}$	
11.	A ray of light moving along the vector $(\hat{i} - 2\hat{j})$ undergoes refraction at an interference of two media which is x-z plane μ for $y > 0$ is 2 while $Y < 0$ it is $\frac{\sqrt{5}}{2}$. The unit vector along which the refracted ray moves is	D
	$\left(\frac{-3\hat{i} - 5\hat{j}}{\sqrt{34}} \right)$	
	$\left(\frac{-4\hat{i} - 3\hat{j}}{5} \right)$	
	$\left(\frac{-3\hat{i} - 4\hat{j}}{5} \right)$	
	$\left(\frac{4\hat{i} - 3\hat{j}}{5} \right)$	
Sol.	$\frac{\sin i}{\sin r} = \frac{\sqrt{5}}{2\sqrt{2}}$ $\left[\sin r = \frac{4}{5} \right]$	
12.	The width of depletion region in a p-n junction diode is 500 nm and an	A

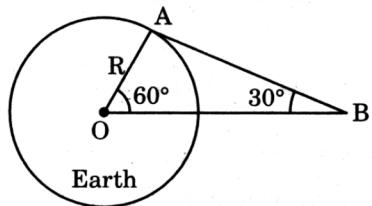
	<p>intense electric field of $6 \times 10^5 \text{ Vm}^{-1}$ is also found to exist. The height of potential barrier is</p> <p>0.30 V</p> <p>0.40 V</p> <p>3 V</p> <p>4 V</p>	
Sol.	$V = Ed = (6 \times 10^5) (500 \times 10^{-9}) = 0.30 \text{ V}$	
13.	<p>A particle of mass M is at a distance 'a' from surface of a thin spherical shell of uniform equal mass and having radius a.</p> 	D
	<p>Gravitational field and potential both are zero at centre of the shell.</p> <p>Gravitational field is zero not only inside the shell but at a point outside the shell also.</p> <p>Inside the shell, gravitational field alone is zero.</p> <p>Neither gravitational field nor gravitational potential is zero inside the shell.</p>	
Sol.	 <p>At centre $V_c = -\frac{GM}{a} - \frac{GM}{2a}; E_c = \frac{GM}{(2a)^2}$</p> <p>At any point P inside $V_p = -\frac{GM}{a} - \frac{GM}{b}$</p> <p>$E_p = \frac{GM}{b^2}$ {only due to outside mass M}</p>	
14.	<p>A satellite is launched in the equatorial plane in such a way that it can transmit signals upto 60° latitude on the earth. Then the angular velocity of the satellite is:</p> <p>$\sqrt{\frac{GM}{8R^3}}$</p> <p>$\sqrt{\frac{GM}{2R^3}}$</p> <p>$\sqrt{\frac{GM}{4R^3}}$</p> <p>$\sqrt{\frac{3\sqrt{3}GM}{8R^3}}$</p>	A

Sol.

In ΔAOB ,

$$\cos 60^\circ = \frac{R}{OB} \Rightarrow OB = 2R$$

Here gravitational force will provide the required centripetal force.



$$\text{Hence } \frac{GMm}{(OB)^2} = m(OB) \omega^2$$

$$\Rightarrow \omega = \sqrt{\frac{GM}{(OB)^3}} = \sqrt{\frac{GM}{(2R)^3}} = \sqrt{\frac{GM}{8R^3}}$$

15.

Three identical stars, each of mass M , form an equilateral triangle (stars are positioned at the corners) that rotates around the centre of the triangle. The system is isolated and edge length of the triangle is L . The amount of work done, that is required to dismantle the system, is:

$$\frac{3GM^2}{L}$$

$$\frac{3GM^2}{2L}$$

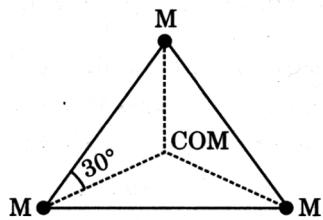
$$\frac{3GM^2}{4L}$$

$$\frac{GM^2}{2L}$$

Sol.

Stars move around COM. Distance of each star

$$\text{from COM is } \frac{2}{3} \times L \cos 30^\circ = \frac{L}{\sqrt{3}}$$



Force on each star M due to the other two

$$\Rightarrow 2 \frac{GMM}{L^2} \cos 30^\circ = \sqrt{3} \frac{GM^2}{L^2}$$

This acts as centripetal force.

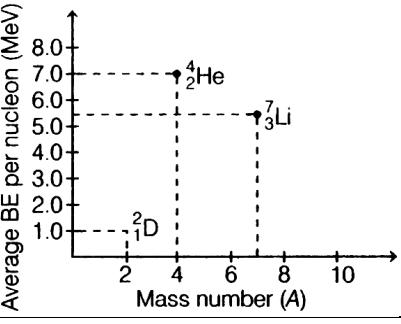
$$\text{Therefore, } \sqrt{3} \frac{GM^2}{L^2} = \frac{mv^2}{\left(\frac{L}{\sqrt{3}}\right)}$$

$$\Rightarrow V = \sqrt{\frac{GM}{L}}$$

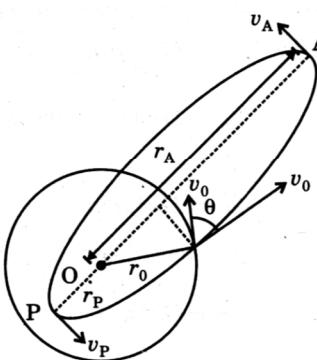
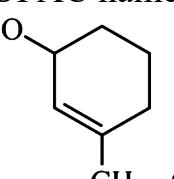
To dismantle the system, energy equal to total energy of the system must be provided.

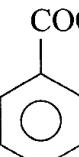
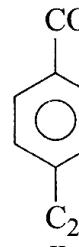
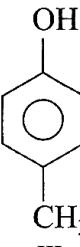
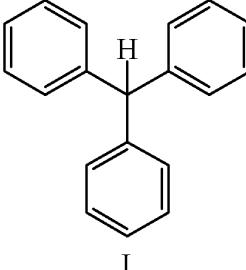
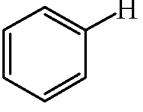
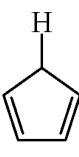
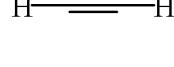
$$\text{i.e., } 3 \times \frac{1}{2} MV^2 + \left(3 \times \frac{GM^2}{L}\right) = \frac{3}{2} \frac{GM^2}{L}$$

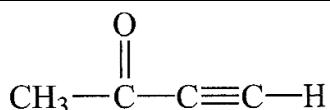
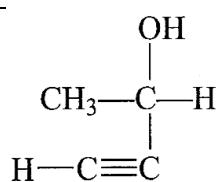
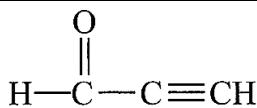
B

16.	<p>The positions of 2_1D, 4_2He, 7_3Li are shown on the binding energy curve as shown in figure. The energy released in the fusion reaction.</p> $^1_1D + ^7_3Li \longrightarrow 2 ^4_2He + ^1_0n$ <p>will be closest to</p> 	B
	20 MeV	
	16 MeV	
	8 MeV	
	4 MeV	
Sol.	$\text{Energy released} = (\text{BE})_{\text{products}} - (\text{BE})_{\text{reactants}}$ $= 2(7 \times 4) - (2 \times 1 + 7 \times 5.5) = 15.5 \text{ MeV}$	
17.	<p>If the binding energy per nucleon in 7_3Li and 4_2He nuclei are 5.60 MeV and 7.06 MeV, respectively, then in the reaction $p + ^7_3Li \longrightarrow 2 ^4_2He$ energy of proton must be</p>	C
	39.2 MeV	
	28.24 MeV	
	17.28 MeV	
	1.46 MeV	
Sol.	$\text{Energy of proton} = (\text{BE})_{\text{He}} - (\text{BE})_{\text{Li}}$ $= 2 \times 4 \times 7.06 - 7 \times 5.60$ $= 17.28 \text{ MeV}$	
18.	<p>An element A decays into an element C by a two step process. $A \rightarrow B + He_2^4$ and $B \rightarrow C + 2e_{-1}^0$. Then,</p>	A
	A and C are isotopes	
	A and C are isobars	
	B and C are isotopes	
	A and B are isobars	
Sol.	<p>The first process is α-decay. So, A loses two protons. The second process is β^--decay. So, C gains two protons. Hence, atomic number Z is same for A and C and thus they are isotopes.</p>	
19.	<p>Pure Si at 500 K has equal number of electron (n_e) and hole (n_h) concentrations of $1.5 \times 10^{16} \text{ m}^{-3}$. Doping by indium increases n_h to $4.5 \times 10^{22} \text{ m}^{-3}$. The doped semiconductor is of</p>	B
	n-type having electron concentration $n_e = 2.5 \times 10^{23} \text{ m}^{-3}$	
	p-type having electron concentration $n_e = 5 \times 10^9 \text{ m}^{-3}$	
	n-type with electron concentration $n_e = 5 \times 10^{22} \text{ m}^{-3}$	
	p-type with electron concentration $n_e = 2.5 \times 10^{10} \text{ m}^{-3}$	
Sol.	$n_i^2 = n_e n_h \Rightarrow (1.5 \times 10^{16})^2 = n_e (4.5 \times 10^{22})$ $\Rightarrow n_e = 0.5 \times 10^{10} = 5 \times 10^9$ $n_h = 4.5 \times 10^{22}$ $n_h \gg n_e, \text{ Semiconductor is p-type and } n_e = 5 \times 10^9 \text{ m}^{-3}$	

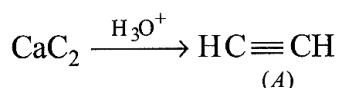
20.	From the circuit shown below, the maximum and minimum values of Zener diode current are	C
	6 mA, 5 mA	
	14 mA, 5 mA	
	9 mA, 1 mA	
	3 mA, 2 mA	
Sol.	$I_{\max} = \frac{120 - 50}{5} - \frac{50}{10} = 9 \text{ mA}$ $I_{\min} = \frac{80 - 50}{5} - \frac{50}{10} = 1 \text{ mA}$	
21.	The shortest wavelength of the Brackett series of a hydrogen like atom (atomic number = Z) is the same as the shortest wavelength of the Balmer series of hydrogen atom. The value of Z is	2
Sol.	<p>Shortest wavelength of Brackett series corresponds to the transition of electron between $n_1 = 4$ and $n_2 = \infty$ and the shortest wavelength of Balmer series corresponds to the transition of electron between $n_1 = 2$ and $n_2 = \infty$. So,</p> $(Z^2) \left(\frac{136}{16} \right) = \left(\frac{136}{4} \right)$ $\therefore Z^2 = 4 \quad \text{or} \quad Z = 2$	
22.	<p>A light ray is incident on a plane mirror M. The mirror is rotated in direction as shown in figure by an arrow at frequency $(9/\pi)$ revolution/sec. The light reflected by the mirror is received on the wall W at a distance of 10 m from axis of rotation. Speed of the spot on wall when angle of incident = 37° is $n \times 100 \text{ m/s}$. Find the value of n</p>	10
Sol.	<p>We know that the reflected ray rotates by twice the angle of rotation of the mirror. Since the angular frequency is directly proportional to angle of rotation so if the plane mirror rotates by an angle θ or with angular velocity ω then the reflected ray rotates by an angle 2θ or with angular velocity, $\omega' = 2\omega$</p> $\omega' = 36 \text{ rad sec}^{-1}$ <p>Since, $\omega' = \left \frac{d\theta}{dt} \right$</p> <p>So from trigonometric ratio we can see from diagram, $h = 10 \cot \theta$</p>	

	$\left \frac{dh}{dt} \right = \left -10 \operatorname{cosec}^2 37 \frac{d(\theta)}{dt} \right $ $\left \frac{dh}{dt} \right = \left \frac{10}{0.36} \times 36 \right $ $\therefore \left \frac{dh}{dt} \right = 1000 \text{ msec}^{-1}$	
23.	When a centimeter thick surface is illuminated with light of wavelength λ , the stopping potential is V . When the same surface is illuminated by light of wavelength 2λ , the stopping potential is $V/3$. The threshold wavelength for the surface is $n\lambda$. Find the value of n .	4
Sol.	$\left[\frac{hc}{\lambda} = \omega_f + ev \right]$ $\left[\frac{hc}{2\lambda} = \omega_f + \frac{eV}{3} \right]$ $\omega_f = \frac{hc}{4\lambda}, \lambda_0 = 4\lambda$	
24.	A small satellite of mass 'm' is revolving around earth in a circular orbit of radius r_0 with speed v_0 . At certain point of its orbit, the direction of motion of satellite is suddenly changed by angle $\theta = \cos^{-1}\left(\frac{3}{5}\right)$ by turning its velocity vector in the same plane of motion, such that speed remains constant. The satellite, consequently goes to elliptical orbit around earth. The ratio of speed at perigee to speed at apogee is :	9
Sol.	<p>Using conservation of angular momentum about O.</p> $mV_P r_P = mV_A r_A = mV_0 r_0 \cos \theta$ $V_A r_A = V_P r_P = \frac{3v_0 r_0}{5}$ <p>Using conservation of energy</p> $\frac{1}{2} mV_A^2 + \frac{-GMm}{r_A} = \frac{-GMm}{r_0} + \frac{1}{2} mV_0^2$ $\Rightarrow \frac{9V_0^2 r_0^2}{50r_A^2} - \frac{V_0^2 r_0}{r_A} + \frac{V_0^2}{2} = 0 \quad \left[\text{Let } \frac{r_0}{r_A} = x \right]$ $\Rightarrow 9x^2 - 50x + 25 = 0$ $\Rightarrow x = 5 \text{ or } \left(\frac{5}{9}\right)$ $\Rightarrow r_A = \frac{9r_0}{5}; r_P = \frac{r_0}{5}$ $\Rightarrow \frac{V_P}{V_A} = \frac{r_A}{r_P} = 9$ 	
25.	In an α - decay the kinetic energy of α - particle is 48 MeV and Q-value of the reaction is 50 MeV. The mass number of the mother nucleus is $24n$. Find the value of n . (Assume that daughter nucleus is in ground state)	4
Sol.	$K_\alpha = \left(\frac{A-4}{A} \right) Q$ $\Rightarrow 48 = \left(\frac{A-4}{A} \right) 50 \Rightarrow A = 100$	
26.	IUPAC name of the following compounds is : 	C

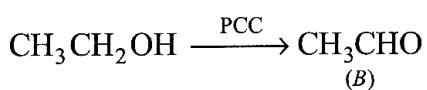
	1-Ethylcyclohex-1-en-3-ol 1-Ethyl-3-hydroxy cyclohex-1-ene 3-Ethylcyclohex-2-en-1-ol 3-Ethyl-6-hydroxy cyclohexene	
Sol.		
27.	<p>Which of the following is the correct order acidity of the given Compounds?</p> <p>    </p> <p> I II III </p>	C
	III > II > I	
	II > I > III	
	I > II > III	
	III > I > II	
Sol.	<p>The decreasing order of acidic strength of given compound is I > II > III.</p> <p>The benzoate ion is more stabilised because the negative charge is on the more electronegative oxygen atom, whereas in phenoxide ion, it is on the less electronegative carbon atoms. This causes benzoic acid to be a stronger acid than phenol.</p> <p>The alkyl group attached to <i>para</i> position to carboxylic group in aromatic carboxylic acid reduces the acidity. Similarly, electron releasing group, i.e. $-\text{CH}_3$ present in the <i>para</i> position to $-\text{OH}$ group decreases its acidity.</p> <p>Study Tactics The more stable is the conjugate base the stronger is the acid. Also, ERG decreases the acidity while EWG increases the acidity.</p>	
28.	<p>With respect to the compounds I-V, choose the incorrect statement(s).</p> <p>      </p> <p> I II III IV V </p>	D
	The acidity of compound I is due to delocalization in the conjugate base.	
	The conjugate base of compound IV is aromatic.	
	Compound II becomes more acidic, when it has a $-\text{NO}_2$ substituent.	
	The acidity of compounds follows the order I > IV > V > II > III.	
29.	<p>What will be the product formed by reaction between A and B in presence of NaOEt? A is prepared by hydrolysis of CaC_2. B is prepared by reaction of $\text{CH}_3\text{CH}_2\text{OH}$ with PCC?</p> <p>$\text{CH}_3\text{CH}_2\text{COOCH}_3$</p>	C



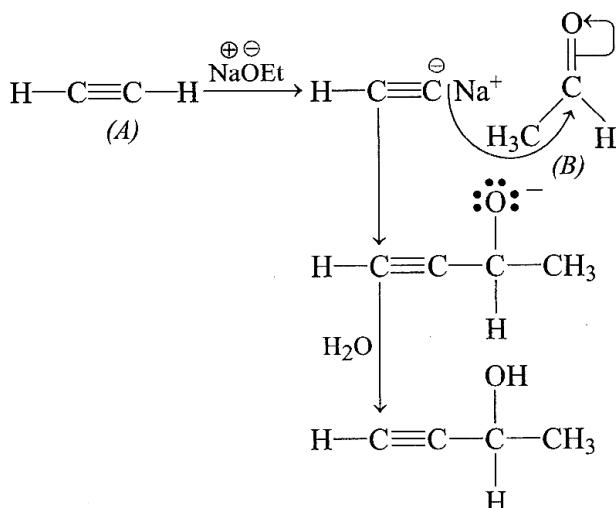
Sol. **Preparation of alkyne** When CaC_2 is hydrolysed, it produces acetylene gas A .



Preparation of aldehyde When alcohol is oxidised by PCC, it produces aldehyde (B).



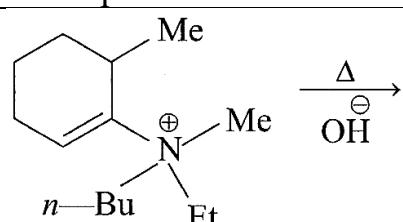
Reaction between *A* and *B* in presence of base NaOEt



Study Tactics

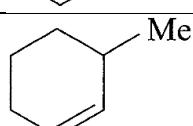
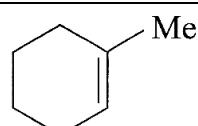
This problem is based on conceptual mixing of preparation of alkyne and reaction of alkyne with carbonyl compound. Complete the reaction using information supplied in the question then undergo sequential oxidation and nucleophilic addition reaction to get the final product.

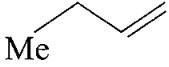
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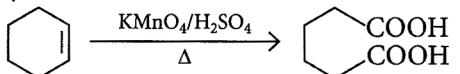
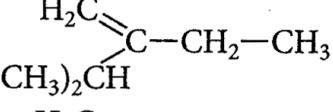
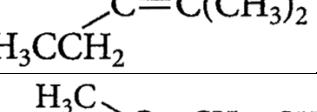
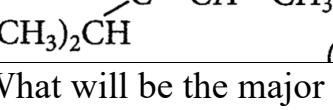
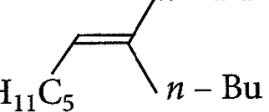
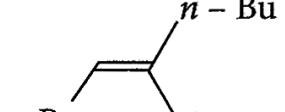
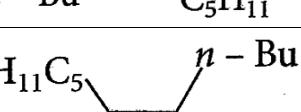
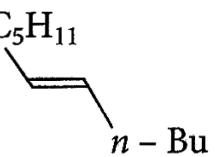
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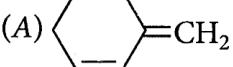
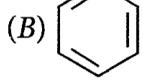
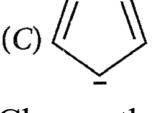
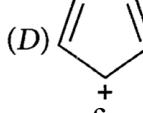
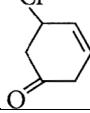
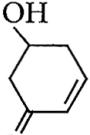
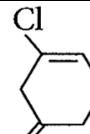
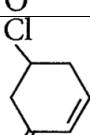
The alkene formed as a major product in the above elimination reaction is



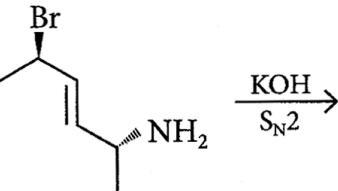
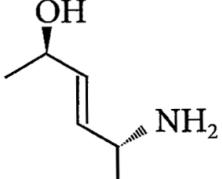
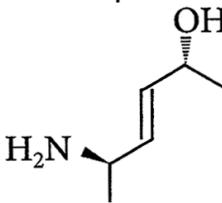
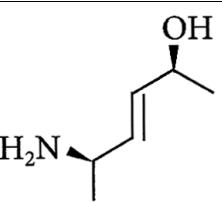
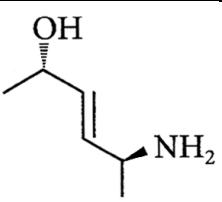
		
	$\text{CH}_2 = \text{CH}_2$	
Sol.	Hofmann's rule When theoretically more than one type of alkenes are possible, the alkene containing least alkylated double bond is formed.	
31.	Identify the (A) and (B) in the following reaction.	B
	$\text{Ph}-\underset{\substack{ \\ \text{CH}_3}}{\text{CH}}-\text{CH}=\text{CH}-\text{CH}_3 \xrightarrow[\Delta]{\text{NBS}} (\text{A}) \xrightarrow{\text{Alc. KOH}} (\text{B})$	
	$\text{A} = \text{Ph}-\text{C}(\text{Br})(\text{CH}_3)\text{CH}=\text{CH}-\text{CH}_3,$ $\text{B} = \text{Ph}-\text{C}(\text{CH}_3)(\text{Br})\text{CH}=\text{CH}-\text{CH}_3$	
	$\text{A} = \text{Ph}-\text{C}(\text{Br})(\text{CH}_3)-\text{CH}=\text{CH}-\text{CH}_3,$ $\text{B} = \text{Ph}-\underset{\substack{ \\ \text{CH}_2}}{\text{C}}-\text{CH}=\text{CH}-\text{CH}_3$	
	$\text{A} = \text{Ph}-\text{C}(\text{Br})(\text{CH}_3)\text{CH}-\text{CH}_2-\text{CH}_3,$ $\text{B} = \text{Ph}-\text{C}(\text{CH}_3)(\text{Br})\text{CH}=\text{CH}-\text{CH}_3$	
	$\text{A} = \text{Ph}-\text{C}(\text{Br})(\text{CH}_3)\text{CH}-\text{CH}_2-\text{CH}_3,$ $\text{B} = \text{Ph}-\text{C}(\text{OH})(\text{CH}_3)\text{CH}=\text{CH}-\text{CH}_3$	
Sol.	$\text{Ph}-\underset{\substack{ \\ \text{CH}_3}}{\text{CH}}-\text{CH}=\text{CH}-\text{CH}_3 \xrightarrow[\Delta]{\text{NBS}} \text{Ph}-\underset{\substack{ \\ \text{Br}}}{\text{C}}-\text{CH}=\text{CH}-\text{CH}_3$ $(\text{A}) \xrightarrow{\text{Alc. KOH}} \text{Ph}-\underset{\substack{ \\ \text{CH}_2}}{\text{C}}-\text{CH}=\text{CH}-\text{CH}_3$ (B)	
	<p>In this reaction, compound (A) is optically active and compound (B) shows geometrical isomerism.</p> <p>However, both (A) and (B) are stereoisomers.</p>	

32.	<p>A is :</p> <p>Sol.</p>	A
	<p>None of these</p>	
33.	<p>The major organic compound formed by the reaction of 1,1,1-trichloroethane with silver powder is</p> <p>2-butene</p> <p>Acetylene</p> <p>Ethane</p> <p>2-butyne</p>	D
	<p>Sol.</p>	
34.	 <p>For the above chemical reactions, identify the correct statement from the following.</p> <p>Compound 'A' is diol and compound 'B' is dicarboxylic acid</p> <p>Both compound 'A' and compound 'B' are dicarboxylic acids.</p> <p>Compound 'A' is dicarboxylic acid and compound 'B' is diol.</p> <p>Both compound 'A' and compound 'B' are diols.</p>	C

Sol.	<p>Hot KMnO_4 in presence of H_2SO_4 acts as strong oxidising agent. It reacts with alkenes to give dicarboxylic acid.</p> <p></p>	
35.	<p>The major product [r] in the following sequence of reactions is C</p> <p>$\text{HC}\equiv\text{CH} \xrightarrow[\text{(ii) H}_3\text{C}]{\text{(i) LiNH}_2/\text{ether}} [\text{P}] \xrightarrow[\text{(ii) NaBH}_4]{\text{(i) HgSO}_4/\text{H}_2\text{SO}_4} [\text{Q}]$</p> <p>$\xrightarrow[\Delta]{\text{Conc. H}_2\text{SO}_4} [\text{R}]$</p>	
	<p></p>	
	<p></p>	
	<p></p>	
	<p></p>	
36.	<p>What will be the major product of following sequence of reactions? C</p> <p>$n - \text{Bu} - \equiv \xrightarrow[\text{(ii) Lindlar cat, H}_2]{\text{(i) } n - \text{BuLi, } n - \text{C}_5\text{H}_11\text{Cl}} \dots$</p>	
	<p></p>	
	<p></p>	
	<p></p>	
	<p></p>	
Sol.	<p>$n - \text{Bu} - \equiv \xrightarrow{\text{H}} n - \text{Bu} - \equiv \text{Li}$</p> <p>$n - \text{Bu} - \equiv \text{Li} \xrightarrow{n - \text{C}_5\text{H}_11\text{Cl}} n - \text{Bu} - \equiv - \text{C}_5\text{H}_11$</p> <p>$n - \text{Bu} - \equiv - \text{C}_5\text{H}_11 \xleftarrow[\text{catalyst, H}_2]{\text{Lindlar's}} \text{C}_5\text{H}_11 - \text{CH}_2 - \text{CH} - \text{CH}_2 - \text{CH} - \text{CH}_2 - \text{C}_5\text{H}_11$</p> <p>Hence, correct option is (c).</p>	
37.	<p>The treatment of CH_3MgX with $\text{CH}_3\text{C} \equiv \text{C} - \text{H}$ produces A</p> <p>CH_4</p> <p>$\text{CH}_3 - \text{CH} = \text{CH}_2$</p> <p>$\text{CH}_3\text{C} \equiv \text{C} - \text{CH}_3$</p>	

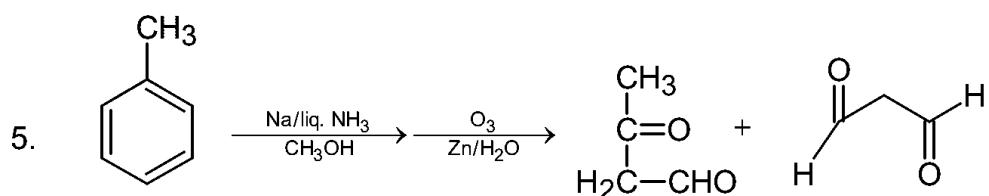
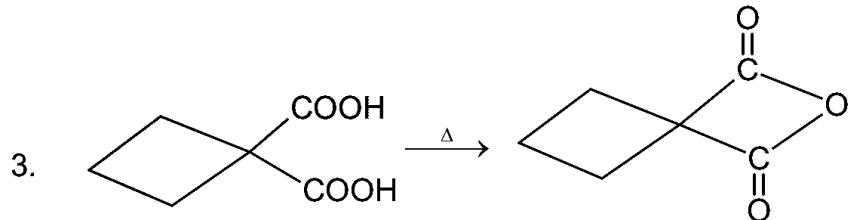
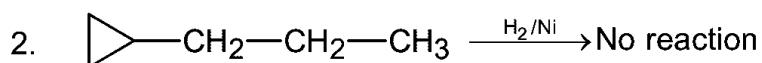
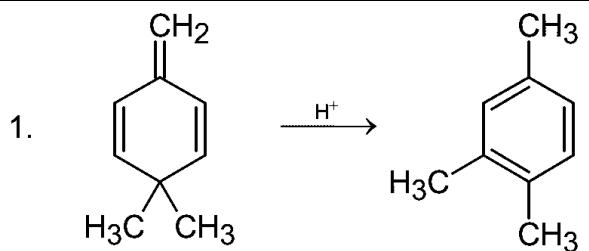
	$ \begin{array}{c} \text{H} \quad \text{H} \\ \quad \\ \text{CH}_3 - \text{C} = \text{C} - \text{CH}_3 \end{array} $	
Sol.	<p>Grinard reagent reacts with compounds having active or acidic hydrogen atom to give alkane.</p> $\text{CH}_3\text{MgX} + \text{CH}_3\text{C}\equiv\text{C} - \text{H} \rightarrow \text{CH}_4 + \text{CH}_3\text{C}\equiv\text{C}\text{MgX}$	
38.	<p>Among the following, the aromatic compounds are</p> <p>(A)  (B) </p> <p>(C)  (D) </p> <p>Choose the correct answer from the following options.</p> <p>(B) and (C) only</p> <p>(A) And (B) only</p> <p>(A), (B) and (C) only</p> <p>(B), (C) and (D) only</p>	A
Sol.	<p>A compound should be aromatic if it has planarity, complete delocalization of π-electrons in ring and follows Hückel's rule $(4n + 2)\pi$ electrons.</p> <p> $6\pi e^-s$ Aromatic ;  $6\pi e^-s$ Aromatic</p> <p> $4\pi e^-s$ Antiaromatic</p> <p> Not complete delocalization (Non-aromatic)</p>	
39.	<p>The major product of the following reaction is.</p> <p> $\xrightarrow{\text{(i) HBr}}$ $\xrightarrow{\text{(ii) alc. KOH}}$</p>	A
		
		
		
		

Sol.		
40.	<p>The major product of the following reactions is</p>	D
Sol.		
41.	<p>Which among the following halide/s will not show S_N1 reaction?</p> <p>(A) $H_2C = CH - CH_2Cl$ (B) $CH_3 - CH = CH - Cl$ (C) (D) </p> <p>Choose the most appropriate answer from the options given below:</p> <p>(A) and (B) only (B) only (A), (B) and (D) only (B) and (C) only</p>	B

Sol.	<p>Carbocation intermediates are formed during S_N1 mechanism. (A) forms stable allyl carbocation $CH_2=CH-\overset{+}{CH}_2$, (B) forms highly unstable vinyl carbocation $CH_3-CH=\overset{+}{CH}$, (C) forms benzyl carbocation $\overset{+}{CH}_2$ which is stable due to resonance, and</p> <p style="text-align: center;"></p> <p>(D) forms secondary carbocation $CH_3-\overset{+}{CH}-CH_3$.</p> <p>Hence, (B) will not show S_N1 reaction.</p>	
42.	<p>Given below are two statements :</p> <p>Statement I: S_N2 reactions are 'stereospecific, indicating that they result in the formation of only one stereo-isomer as the product.</p> <p>Statement II: S_N1 reactions generally result in formation of product as racemic mixtures. In the light of the above statements, choose the correct answer from the options given below.</p>	D
	Statement I is false but Statement II is true	
	Statement I is true but Statement II is false.	
	Both Statement I and Statement II are false.	
	Both Statement I and Statement II are true	
Sol.	Inversion of configuration occurs in S_N2 reaction and S_N1 reaction proceeds via carbocation intermediate, it produces racemic mixture. Hence, both statements are true.	
43.	<p>The major product of the following reaction is</p> <p style="text-align: center;"></p>	B
	<p style="text-align: center;"></p>	
	<p style="text-align: center;"></p>	
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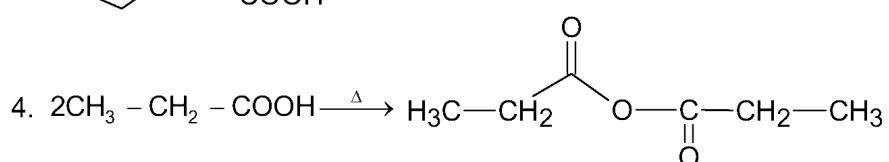
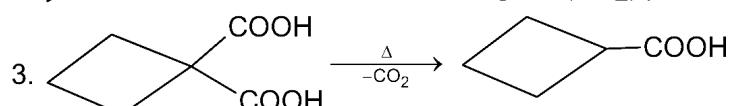
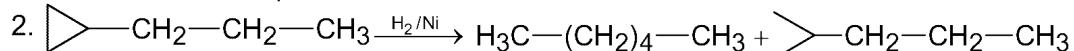
Sol.		
44.	<p>The number of optical isomers in following compound is _____.</p>	C
8		
16		
32		
28		
Sol.	<p>The total number of chiral centres in the given molecule is 5.</p> <p>Hence, $n = 5$</p> <p>Number of optical isomer = $2^n = 2^5 = 32$</p>	
45.	<p>The major product of the following reaction is P.</p>	A
$\text{CH}_3\text{C}\equiv\text{C}-\text{CH}_3 \xrightarrow[\substack{\text{(ii) dil. KMnO}_4 \\ 273 \text{ K}}]{\substack{\text{(i) Na/liq. NH}_3}} (P)$		
Number of oxygen atom(s) present in product 'P' is _____ (Nearest integer)		
2		
3		
4		
6		
Sol.	$\text{CH}_3-\text{C}\equiv\text{C}-\text{CH}_3 \xrightarrow[\substack{\text{(ii) dil. KMnO}_4 \\ 273 \text{ K}}]{\substack{\text{(i) Na/liq. NH}_3}} \begin{array}{c} \text{H}_3\text{C} \quad \text{H} \\ \diagdown \quad \diagup \\ \text{C}=\text{C} \\ \diagup \quad \diagdown \\ \text{H} \quad \text{CH}_3 \\ \text{trans-alkene} \end{array}$ $\text{H}_3\text{C}-\text{HC}(\text{OH})-\text{CH}(\text{OH})-\text{CH}_3 \leftarrow$ <p>Diol (P)</p>	
INTEGER TYPE	Number of moles of AgCl formed in the following reaction is _____.	2

	$\text{Reaction: } \text{Structure} + \text{AgNO}_3 \rightarrow (A) + X \text{AgCl} \downarrow$	
Sol.	<p>There are only two replaceable Cl^- ions which form white ppt. of AgCl with AgNO_3.</p>	
47.	<p>In how many reactions the 1st reaction is faster than 2nd reaction?</p> <p>(P) $\xleftarrow[(1)]{\text{CF}_3\text{CH}_2\text{O}^\ominus} \boxed{\text{CH}_3 - \text{CH}_2 - \text{Br}} \xrightarrow[(2)]{\text{CH}_3\text{CH}_2\text{O}^\ominus}$</p> <p>(Q) (Alkene) $\xleftarrow[(1)]{(\text{CH}_3)_3\text{CBr}/\Delta} \boxed{\text{C}_2\text{H}_5\text{O}^\ominus} \xrightarrow[(2)]{(\text{CH}_3)_2\text{CHCH}_2\text{Br}/\Delta} \text{(Alkene)}$</p> <p>(R) $\xleftarrow[(1)]{} \text{Br} \quad \boxed{\text{aqueous acetone}} \quad \xrightarrow[(2)]{}$</p> <p>(S) $\xleftarrow[(1)]{\text{HI}} \quad \boxed{\text{OH}} \quad \xrightarrow[(2)]{\text{Lucas reagent}}$</p>	3
Sol.	<p>(P) $\xleftarrow[(1)]{\text{CF}_3\text{CH}_2\text{O}^\ominus} \boxed{\text{CH}_3 - \text{CH}_2 - \text{Br}} \xrightarrow[(2)]{\text{CH}_3\text{CH}_2\text{O}^\ominus}$</p> <p>In reaction 2nd Nucleophile is stronger.</p> <p>(Q) (Alkene) $\xleftarrow[(1)]{(\text{CH}_3)_3\text{CBr}/\Delta} \boxed{\text{C}_2\text{H}_5\text{O}^\ominus} \xrightarrow[(2)]{(\text{CH}_3)_2\text{CHCH}_2\text{Br}/\Delta} \text{(Alkene)}$</p> <p>In reaction 1st 3° alkyl halide $1 > 2$</p> <p>(R) In reaction 1st stable carbocation is formed $(\text{Cyclopropyl methyl C}^+)$ $1 > 2$</p> <p>(S) In reaction 1st reagent in HI. HI is more ionised, gives sufficient concentration of H^+ ion and I^- is a good Nucleophile also. > 2</p>	
48.	<p>The sum of reaction numbers for which the major product is/are correctly mentioned is:</p>	6



Sol.

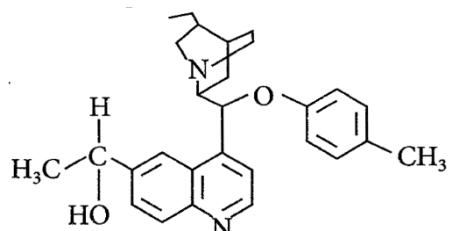
1 and 5 have correct products.



49.

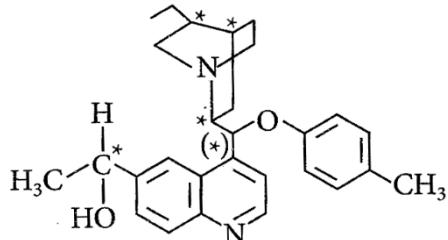
The number of chiral carbons present in the molecule given below is ____.

5



Sol.

Only 2-butanol is chiral.



50.

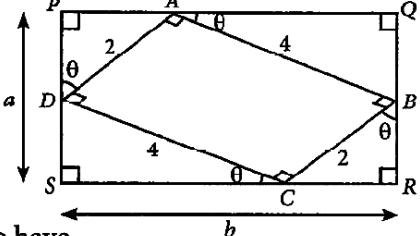
In the presence of sunlight, benzene reacts with Cl_2 to give product, X. The number of hydrogen in X is ____.

6

Sol.	$ \begin{array}{c} \text{C}_6\text{H}_6 \\ \text{Benzene} \end{array} + 3\text{Cl}_2 \xrightarrow{h\nu} \begin{array}{c} \text{Cl} \\ \\ \text{Cl}-\text{C}-\text{C}-\text{C}-\text{C}-\text{Cl} \\ \quad \quad \\ \text{Cl} \quad \text{Cl} \quad \text{Cl} \\ (\text{B.H.C.}) \\ (\text{X}) \end{array} $ <p>Product 'X' contains 6 hydrogen atom.</p>	
51.	<p>In bromination of propyne, with bromine ; 1, 1, 2, 2-tetrabromopropane is obtained in 27% yield. The amount of 1, 1, 2, 2-tetrabromopropane obtained from 1 g of bromine in this reaction is _____ 10^{-1} g. (Nearest integer) (Molar Mass: Bromine = 80 g/mol)</p>	3
Sol.	$ \begin{array}{c} \text{CH}_3-\text{C}\equiv\text{CH} \\ 40 \text{ g} \end{array} + 2\text{Br}_2 \longrightarrow \begin{array}{c} \text{Br} \\ \\ \text{CH}_3-\text{C}-\text{CH}-\text{Br} \\ \quad \\ \text{Br} \quad \text{Br} \\ 360 \text{ g} \end{array} $ <p>320 g bromine (with 100% yield) gives = 360 g 1, 1, 2, 2 tetrabromopropane</p> <p>1 g bromine will gives $\frac{360}{320} = 1.125$ g product</p> <p>If yield is only 27% then 1, 1, 2, 2-tetrabromopropane formed is $= 1.125 \times \frac{27}{100} = 0.304$ g or 3×10^{-1} g</p>	
51.	<p>If $f(x) = \{x^2\} - (\{x\})^2$, where $\{x\}$ denotes the fractional part of x then</p> <p>$f(x)$ is continuous at $x = -2$ but not at $x = 2$</p> <p>$f(x)$ is continuous at $x = 2$ but not $x = -2$</p> <p>$f(x)$ continuous at $x = 2$ and at $x = -2$</p> <p>$f(x)$ discontinuous at $x = -2$ and at $x = 2$</p>	B
Sol.	$f(2) = 0$ $f(2^+) = \{4^+\} - \{2^+\}^2 = 0 - 0 = 0$ $f(2^-) = \{4^-\} - \{2^-\}^2 = 1 - 1 = 0$ <p>Hence, $f(x)$ is continuous at $x=2$</p> $f(-2) = 0$ $f(-2^+) = \{4^-\} - \{-2^+\}^2 = 1 - 0 = 0 \neq f(-2^-)$ <p>Hence, $f(x)$ is discontinuous at $x=-2$</p>	
52.	<p>The function $f(x) = \frac{\log(1+ax) - \log(1-bx)}{x}$ is not defined at $x = 0$, the value of which should be assigned to f at $x = 0$, so that it is continuous at $x = 0$, is</p> <p>a - b</p> <p>a + b</p> <p>$\log a + \log b$</p> <p>$\log a - \log b$</p>	B

Sol.	<p>Given $f(x) = \frac{\log(1+ax) - \log(1-bx)}{x}$</p> <p>Now, At $x = 0$, it is in $\frac{0}{0}$ form.</p> <p>\therefore By applying L' Hopital Rule.</p> <p>We get: $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{\frac{1}{1+ax} \cdot a - \frac{1}{1-bx} \cdot (-b)}{1} = a+b.$</p>	
53.	$f(x) = \begin{cases} 2 - x^2 + 5x + 6 & x \neq -2 \\ a^2 + 1 & x = -2 \end{cases}$ <p>Then the range of a so that $f(x)$ has a maximum at $x = -2$</p>	A
	$ a \geq 1$	
	$ a < 1$	
	$a > 1$	
	$a < 1$	
Sol.	$a^2 + 1 \geq 2 \Rightarrow a \geq 1$	
54.	<p>Let function $f : R \rightarrow R$ be defined by $f(x) = 2x + \sin x$ for $x \in R$, then f is</p>	A
	<p>One –one and onto</p>	
	<p>One – one but not onto</p>	
	<p>Onto but not one – one</p>	
	<p>Neither one – one nor onto</p>	
Sol.	<p>Given that $f(x) = 2x + \sin x$, $x \in R$</p> <p>$\Rightarrow f'(x) = 2 + \cos x$</p> <p>But $-1 \leq \cos x \leq 1$</p> <p>$\Rightarrow 1 \leq 2 + \cos x \leq 3 \Rightarrow 1 \leq 2 + \cos x \leq 3$</p> <p>$\therefore f'(x) > 0$, $\forall x \in R$</p> <p>$\Rightarrow f(x)$ is strictly increasing and hence one-one</p> <p>Also as $x \rightarrow \infty$, $f(x) \rightarrow \infty$ and $x \rightarrow -\infty$, $f(x) \rightarrow -\infty$</p> <p>\therefore Range of $f(x) = R$ = domain of $f(x) \Rightarrow f(x)$ is onto.</p> <p>Thus, $f(x)$ is one-one and onto.</p>	
55.	<p>Let $f: [0, 2] \rightarrow R$ be a twice differentiable function such that $f''(x) > 0$, for all $x \in (0, 2)$. If $\phi(x) = f(x) + f(2-x)$, then ϕ is:</p>	C
	<p>Increasing on $(0, 1)$ and decreasing on $(1, 2)$</p>	
	<p>Decreasing on $(0, 2)$</p>	
	<p>Decreasing on $(0, 1)$ and increasing on $(1, 2)$</p>	
	<p>Increasing on $(0, 2)$</p>	

Sol.	$\phi(x) = f(x) + f(2-x) \Rightarrow \phi'(x) = f'(x) - f'(2-x)$ Given that $f''(x) > 0, \forall x \in (0, 2)$ $\therefore f'(x)$ is increasing function on $(0, 2)$ for increasing $f'(x) > 0$ $f'(x) - f'(2-x) > 0$ $\Rightarrow f'(x) > f'(2-x) \quad (\because f'(x) \text{ is increasing})$ $\Rightarrow x > 2-x \quad \Rightarrow x > 1 \quad \Rightarrow x \in (1, 2)$ For decreasing $\phi(x) < 0 \Rightarrow f'(x) < f'(2-x) \Rightarrow x < 2-x \Rightarrow x < 1 \Rightarrow x \in (0, 1)$	
56.	$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan^3 x - \tan x}{\cos(x + \frac{\pi}{4})} =$	D
	4	
	2	
	-2	
	-4	
Sol.	$\alpha = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan^3 x - \tan x}{\cos(x + \pi/4)} = \frac{0}{0}$ form Use L' HOSPITAL $= \lim_{x \rightarrow \pi/4} \frac{3\tan^2 x \sec^2 x - \sec^2 x}{-\sin(x + \pi/4)} = -4$	
57.	Let the function $f(x) = \begin{cases} \frac{\log_e(1+5x) - \log_e(1+\alpha x)}{x}, & \text{if } x \neq 0 \\ 10, & \text{if } x = 0 \end{cases}$ Be continuous at $x = 0$. Then α is equal to	D
	10	
	-10	
	5	
	-5	
Sol.	Use L hospital	
58.	For the function $f(x) = (\cos x) - x + 1, x \in R$, between the following two statements (S1) $f(x) = 0$ for only one value of x in $[0, \pi]$. (S2) $f(x)$ is decreasing in $[0, \frac{\pi}{2}]$ and increasing in $[\frac{\pi}{2}, \pi]$.	C
	Only (S2) is correct	
	Both (S1) and (S2) are incorrect	
	Only (S1) is correct	
	Both (S1) and (S2) are correct.	
Sol.	We have, $f(x) = \cos x - x + 1, x \in R$ $\Rightarrow f'(x) = -\sin x - 1 < 0 \quad \forall x \in R$ $\Rightarrow f(x)$ is decreasing $\forall x \in R$ For $f(x) = 0$ Now, $f(0) = 2$ $f(\pi) = -\pi$ $f(0) f(\pi) < 0$ So, f is strictly decreasing in $[0, \pi]$.	

	So, $f(x) = 0$ has one solution in $[0, \pi]$. (S1) is correct but (S2) is incorrect.	
59.	Let a rectangle ABCD of sides 2 and 4 be inscribed in another rectangle PQRS such that the vertices of the rectangle ABCD lie on the sides of the rectangle PQRS. Let a and b be the sides of the rectangle PQRS when its area is maximum. Then $(a + b)^2$ is equal to	D
	80	
	60	
	64	
	72	
Sol.	<p>In $\triangle CDS$, we have</p> $\cos \theta = \frac{CS}{CD}$ $\Rightarrow \cos \theta = \frac{CS}{4}$ $\Rightarrow CS = 4 \cos \theta$  <p>Similarly, in $\triangle ABCR$, we have</p> $\sin \theta = \frac{CR}{2}$ $\Rightarrow CR = 2 \sin \theta$ $\therefore SR = CS + CR = 4 \cos \theta + 2 \sin \theta$ <p>Similarly, $PS = 4 \sin \theta + 2 \cos \theta$</p> $\therefore \text{Area of } PQRS = PS \times SR$ $= 16 \cos \theta \sin \theta + 8 \cos^2 \theta + 8 \sin^2 \theta + 4 \sin \theta \cos \theta$ $= 8(\cos^2 \theta + \sin^2 \theta) + 10(2 \sin \theta \cos \theta)$ $= 8 + 10 \sin 2\theta$ <p>Area is maximum, when $\sin 2\theta = 1 \Rightarrow \theta = 45^\circ$</p> $\therefore \text{Side } a = 4 \sin \theta + 2 \cos \theta \quad [\because \theta = 45^\circ]$ $= \frac{4}{\sqrt{2}} + \frac{2}{\sqrt{2}} = \frac{6}{\sqrt{2}} = 3\sqrt{2}$ <p>and $b = 4 \cos \theta + 2 \sin \theta = 3\sqrt{2}$</p> <p>Now, $(a + b)^2 = (6\sqrt{2})^2 = 72$</p>	
60.	Let $f: R \rightarrow R$ be a thrice differentiable function such that $f(0) = 0, f(1) = 1, f(2) = -1, f(3) = 2$ and $f(4) = -2$. Then, the minimum number of zeros of $(3f'f'' + ff''')(x)$ is	C
	3	
	2	
	5	
	None of these	
Sol.	<p>$\because f: R \rightarrow R$ and $f(0) = 0, f(1) = 1, f(2) = -1, f(3) = 2$ and $f(4) = -2$, then</p> <p>$\therefore f(x)$ has atleast 4 real roots.</p> <p>Then $f'(x)$ has atleast 3 real roots and $f''(x)$ has atleast 2 real roots.</p> $\frac{d}{dx}(f^3 \cdot f'') = 3f^2 \cdot f' \cdot f'' + f^3 \cdot f''' = f^2(3f' \cdot f'' + f \cdot f''')$ <p>Hence, $f^3 f''$ has atleast 6 roots</p> <p>Then its differentiation has atleast 5 distinct roots.</p>	
61.	If $f(x) = \begin{cases} x^3 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$, then	D

	$f''(0) = 0$	
	$f''(0) = 1$	
	$f''\left(\frac{2}{\pi}\right) = \frac{12-\pi^2}{2\pi}$	
	$f''\left(\frac{2}{\pi}\right) = \frac{24-\pi^2}{2\pi}$	
Sol.	$f(x) = \begin{cases} x^3 \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ <p>It is a differentiable function.</p> $\begin{aligned} \therefore f'(x) &= 3x^2 \sin \frac{1}{x} + x^3 \cos \frac{1}{x} \left(\frac{-1}{x^2} \right) \\ &= 3x^2 \sin \frac{1}{x} - x \cos \frac{1}{x} \\ f''(x) &= 6x \sin \frac{1}{x} - 3 \cos \frac{1}{x} - \cos \frac{1}{x} - \frac{\sin \frac{1}{x}}{x} \\ f''\left(\frac{2}{\pi}\right) &= \frac{6 \times 2}{\pi} - \frac{\pi}{2} = \frac{24-\pi^2}{2\pi} \end{aligned}$	
62.	<p>If $a = \lim_{x \rightarrow 0} \frac{\sqrt{1+\sqrt{1+x^4}} - \sqrt{2}}{x^4}$ and $b = \lim_{x \rightarrow 0} \frac{\sin^2 x}{\sqrt{2} - \sqrt{1+\cos x}}$, then the value of ab^3 is :</p> <p>C</p>	
	25	
	36	
	32	
	30	
Sol.	$a = \lim_{x \rightarrow 0} \frac{\sqrt{1+\sqrt{1+x^4}} - \sqrt{2}}{x^4}$ <p>Rationalising numerator, we get</p> $\begin{aligned} a &= \lim_{x \rightarrow 0} \frac{1 + \sqrt{1+x^4} - 2}{x^4 (\sqrt{1+\sqrt{1+x^4}} + \sqrt{2})} \\ &= \lim_{x \rightarrow 0} \frac{\sqrt{1+x^4} - 1}{x^4 (\sqrt{1+\sqrt{1+x^4}} + \sqrt{2})} \end{aligned}$ <p>Again rationalising numerator, we get</p> $= \lim_{x \rightarrow 0} \frac{1}{(\sqrt{1+x^4} + 1)(\sqrt{1+\sqrt{1+x^4}} + \sqrt{2})} = \frac{1}{2 \times 2\sqrt{2}} = \frac{1}{4\sqrt{2}}$ <p>Now, $b = \lim_{x \rightarrow 0} \frac{\sin^2 x}{\sqrt{2} - \sqrt{1+\cos x}}$</p> $\begin{aligned} &= \lim_{x \rightarrow 0} \frac{(1-\cos^2 x)(\sqrt{2} + \sqrt{1+\cos x})}{[2 - (1+\cos x)]} \\ &\quad \text{[By rationalising denominator]} \\ &= \lim_{x \rightarrow 0} \frac{(1+\cos x)(1-\cos x)(\sqrt{2} + \sqrt{1+\cos x})}{1-\cos x} \\ &= \lim_{x \rightarrow 0} (1+\cos x)(\sqrt{2} + \sqrt{1+\cos x}) = 4\sqrt{2} \\ \therefore ab^3 &= \frac{1}{4\sqrt{2}} \times (4\sqrt{2})^3 = 32 \end{aligned}$	
63.	<p>Let for a differentiable function $f: (0, \infty) \rightarrow R$, $f(x) - f(y) \geq \log_e \left(\frac{x}{y}\right) +$</p> <p>A</p>	

	$x - y, \forall x, y \in (0, \infty)$. Then $\sum_{n=1}^{20} f' \left(\frac{1}{n^2} \right)$ is equal to	
	2890	
	2590	
	2980	
	2590	
Sol.	$f(x) - f(y) \geq \log_e \left(\frac{x}{y} \right) + x - y \quad \forall (x, y) \in (0, \infty) \quad \dots \text{(i)}$ $\text{Now, } f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \log_e \frac{\left(\frac{x+h}{x} \right) + h}{h}$ $= \lim_{h \rightarrow 0} \left[\frac{\log_e \left(1 + \frac{h}{x} \right)}{h} + 1 \right] = \frac{1}{x} + 1 \quad \dots \text{(ii)}$ $\text{Now, } \sum_{n=1}^{20} f' \left(\frac{1}{n^2} \right) = \sum_{n=1}^{20} \frac{1}{\frac{1}{n^2}} + 1 \quad [\text{Using (i)}]$ $= \sum_{n=1}^{20} (n^2 + 1) = \left[\frac{n(n+1)(2n+1)}{6} + n \right]_{n=20}$ $= \frac{20 \times 21 \times 41}{6} + 20 = 2870 + 20 = 2890$	
64.	If the function $f(x) = \left(\frac{1}{x} \right)^{2x}$; $x > 0$ attains the maximum value at $x = \frac{1}{e}$ then	C
	$(2e)^\pi > \pi^{(2e)}$	
	$e^{2\pi} < (2\pi)^e$	
	$e^\pi > \pi^e$	
	$e^\pi < \pi^e$	
Sol.	Not available	
65.	The function $f(x) = \frac{x^2 + 2x - 15}{x^2 - 4x + 9}$, $x \in R$ is	B
	Both one-one and onto	
	Neither one-one nor onto	
	Onto but not one-one	
	One-one but not onto	
Sol.	$\text{We have, } f(x) = \frac{x^2 + 2x - 15}{x^2 - 4x + 9}$ $f(x) = \frac{(x+5)(x-3)}{x^2 - 4x + 9}$ $f(x) \text{ has two roots as } x = -5, 3$ $\text{So } f(-5) = 0 \text{ and } f(3) = 0$ $\text{So } f \text{ can not be one-one.}$ $\text{Now consider, } x^2 - 4x + 9$ $D = 16 - 36 = -20 < 0$ $\text{Now, Let } y = \frac{x^2 + 2x - 15}{x^2 - 4x + 9}$ $\Rightarrow yx^2 - 4xy + 9y = x^2 + 2x - 15$ $\Rightarrow x^2(y-1) - 2x(2y+1) + (9y+15) = 0$ $D = 4(2y+1)^2 - 4(y-1)(9y+15) \geq 0 \quad [\because x \in R]$ $\Rightarrow 4(4y^2 + 1 + 4y) - (36y^2 - 36y + 60y - 60) \geq 0$ $\Rightarrow 16y^2 + 4 + 16y - 36y^2 - 24y + 60 \geq 0$ $\Rightarrow -20y^2 - 8y + 64 \geq 0$ $\Rightarrow 5y^2 + 2y - 16 \leq 0$ $\Rightarrow (5y-8)(y+2) \leq 0$ $\Rightarrow y \in \left[-2, \frac{8}{5} \right] \text{ which is range of function } f$ $\text{So, } f \text{ is not onto if } f: R \rightarrow R.$ $\text{Here in given question co-domain is not defined.}$	
66.	Let M and m respectively be the maximum and minimum values of the function $f(x) = \tan^{-1}(\sin x + \cos x)$ in $\left[0, \frac{\pi}{2}\right]$ then the value of M + m	C

	$-Tan^{-1}\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$	
	$\pi + Tan^{-1}(3 + 2\sqrt{2})$	
	$\pi - Tan^{-1}(3 + 2\sqrt{2})$	
	$Tan^{-1}\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$	
Sol.	$f(x) = Tan^{-1}(\sin x + \cos x) \text{, where } x \in \left[0, \frac{\pi}{2}\right]$ <p>w.k.t. $1 \leq \sin x + \cos x \leq 2$ $\Rightarrow Tan^{-1}(1) \leq f(x) \leq Tan^{-1}\sqrt{2}$</p> $\therefore m = Tan^{-1}(1) \quad M = Tan^{-1}(\sqrt{2})$ $M + m = Tan^{-1}\sqrt{2} + Tan^{-1}(1)$ $= \pi + Tan^{-1}\left(\frac{\sqrt{2}+1}{1-\sqrt{2}}\right) \quad (\because xy > 1) \quad = \pi + Tan^{-1}\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right) \quad = \pi - Tan^{-1}(3 + 2\sqrt{2})$	
67.	$Lt_{x \rightarrow 2} \left[\left(\frac{x^3 - 4x}{x^3 - 8} \right)^{-1} \left(\frac{x + \sqrt{2x}}{x - 2} - \frac{\sqrt{2}}{\sqrt{x} - \sqrt{2}} \right)^{-1} \right]$ is equal to	A
	1/2	
	2	
	1	
	2/3	
Sol.	$Lt_{x \rightarrow 2} \left[\frac{(x-2)(x^2 + 2x + 4)}{x(x-2)(x+2)} - \left(\frac{\sqrt{x}}{\sqrt{x} - \sqrt{2}} - \frac{\sqrt{2}}{\sqrt{x} - \sqrt{2}} \right)^{-1} \right]$ $Lt_{x \rightarrow 2} \left(\frac{x^2 + 2x + 4}{x(x+2)} - 1 \right) = \frac{12}{8} - 1 = \frac{1}{2}$	
68.	<p>If $f(x) = \begin{cases} 1 - 4x^2 , & 0 \leq x < 1 \\ [x^2 - 2x], & x \leq x < 2 \end{cases}$, where $[.]$ denotes the greatest integer function, then $f(x)$ is</p> <p>Differentiable for all x</p> <p>Non-differentiable at $x = 1$</p> <p>Continuous at $x = 1$</p> <p>Discontinuous at $x = \frac{1}{2}$</p>	B
Sol.	$f(x) = \begin{cases} 1 - 4x^2 & 0 \leq x \leq \frac{1}{2} \\ 4x^2 - 1 & \frac{1}{2} \leq x \leq 1 \\ -1 & 1 \leq x \leq 2 \end{cases}$ $f\left(\frac{1}{2}^-\right) = f\left(\frac{1}{2}^+\right) = f\left(\frac{1}{2}\right)$ <p>& $f(1) \neq f(1^-) \Rightarrow f$ is not continuous at $x = 1$</p> <p>$\therefore f$ is non differentiable at $x = 1$</p>	
69.	If $lt_{x \rightarrow 0} \frac{ae^x - b \cos x + ce^{-x}}{x \sin x} = 2$, $a, b, c \in R$ then $(a + b - c)^2 =$	D
	1	

	2	
	3	
	4	
Sol.	$\text{Lt}_{x \rightarrow 0} \left(\frac{a \left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \right) - b \left(1 - \frac{x^2}{2!} + \frac{x}{4} \dots \right) + c \left(1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots \right)}{x^2 \cdot \frac{\sin x}{x}} \right) = 2$ $a - b + c = 0 \rightarrow (1)$ $a - c = 0 \Rightarrow a = c \Rightarrow (2)$ $\frac{a}{2} + \frac{b}{2} + \frac{c}{2} = 2 \Rightarrow a + b + c = 4 \rightarrow (3) \quad \therefore a = 1, b = 2, c = 1$	
70.	If $\sin^{-1} \left(\frac{3}{x} \right) + \sin^{-1} \left(\frac{4}{x} \right) = \frac{\pi}{2}$, then x equals	B
	-5	
	5	
	7	
	12	
Sol.	<p>Given, $\sin^{-1} \left(\frac{3}{x} \right) + \sin^{-1} \left(\frac{4}{x} \right) = \frac{\pi}{2}$... (i)</p> <p>Also, $\sin^{-1} \left(\frac{3}{x} \right) + \cos^{-1} \left(\frac{3}{x} \right) = \frac{\pi}{2}$... (ii)</p> <p>From (i) and (ii),</p> $\sin^{-1} \left(\frac{4}{x} \right) = \cos^{-1} \left(\frac{3}{x} \right)$ $\sin^{-1} \left(\frac{4}{x} \right) = \sin^{-1} \sqrt{1 - \left(\frac{3}{x} \right)^2}$ $\Rightarrow \frac{4}{x} = \sqrt{1 - \left(\frac{3}{x} \right)^2}$ $\Rightarrow \frac{4^2}{x^2} + \frac{3^2}{x^2} = 1$ $\Rightarrow x = \pm 5$ $\Rightarrow x = 5 \text{ is a solution}$	
71.	If $\text{Lt}_{x \rightarrow 0} \frac{\cos 4x + a \cos 2x + b}{x^4}$ is finite then this limit value is	8
Sol.	$\text{Lt}_{x \rightarrow 0} \frac{\left(1 - \frac{(4x)^2}{2!} \dots \right) + a \left(1 - \frac{(2x)^2}{2!} \dots \right) + b}{x^4}$ $1 + a + b = 0, -8 - 2a = 0 \Rightarrow a = -4, b = 3 \quad L = \frac{32}{3} + \frac{2}{3}a = 8$	
72.	If $x = \cot^{-1} \left(\frac{31}{25} \right) + \cot^{-1} \left(\frac{91}{25} \right) + \cot^{-1} \left(\frac{201}{25} \right) + \cot^{-1} \left(\frac{361}{25} \right) + \dots$ then $15 \cot x$ is equal to	3

Sol.	$\cot^{-1}\left(\frac{25r^2 - 15r + 21}{25}\right) = \tan^{-1}\frac{1}{r^2 - \frac{3}{5}r + \frac{21}{25}}$ $= \tan^{-1}\left(r + \frac{1}{5}\right) - \tan^{-1}\left(r - \frac{4}{5}\right)$ $S_n = \tan^{-1}\left(\frac{25m}{5m + 26}\right) = \tan^{-1}5$	
73.	Let $f: R \rightarrow R$ and $f(x) = \min(x + 2 , e^x)$, then the number of points where function $f(x)$ is non-differentiable, is(are)	4
Sol.	Not available	
74.	Let $f(x) = x^2 - 4x - 3, x > 2$ and $g(x)$ be the inverse of $f(x)$. Then the value of $600g^1(2)$ (where $g^1(x)$ represents first order derivative)	100
Sol.	$g(x) = f^{-1}(x)$ $f(g(x)) = x$ $f^1(g(x))g^1(x) = 1$ $g^1(x) = \frac{1}{f^1(g(x))}$ $g^1(2) = \frac{1}{f^1(g(2))} = \frac{1}{f^1(5)}$ $f(5) = 2$ $5 = f^{-1}(2) = g(2)$ $f^1(x) = 2x - 4$ $f^1(5) = 6$	
75.	Let $A = \{(x, y) : 2x + 3y = 23, x, y \in N\}$ and $B = \{x : (x, y) \in A\}$. Then the number of one-one functions from A to B is equal to	24
Sol.	<p>We have, $A = \{(x, y) : 2x + 3y = 23, x, y \in N\}$</p> <p>$B = \{x : (x, y) \in A\}$</p> <p>$\therefore A = \{(1, 7), (4, 5), (7, 3), (10, 1)\}$</p> <p>and $B = \{1, 4, 7, 10\}$</p> <p>Total number of one-one functions from A to B = $4! = 24$</p>	