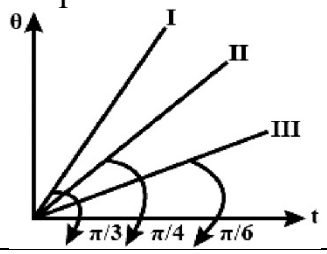
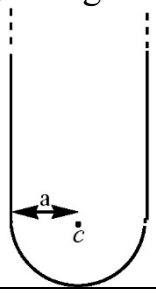
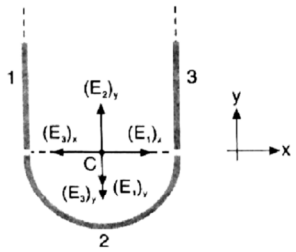
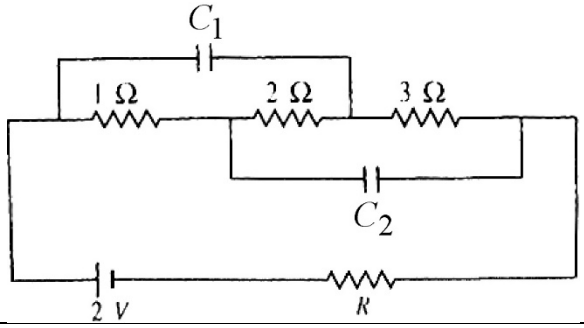
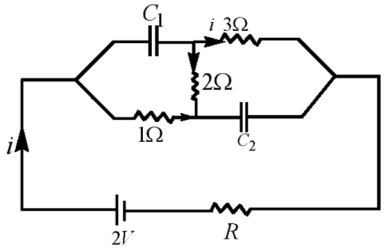
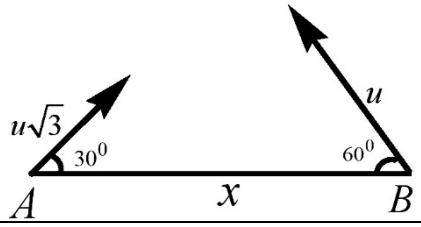
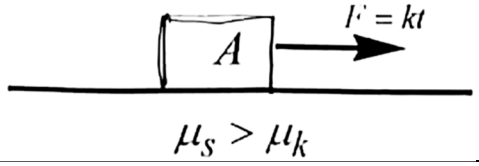
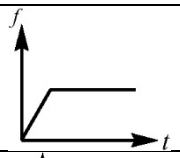
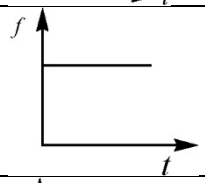
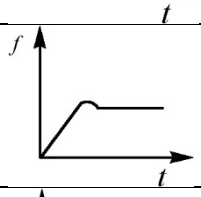
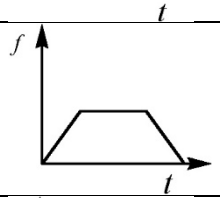
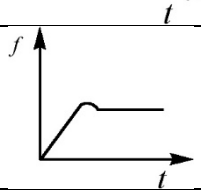
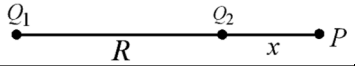
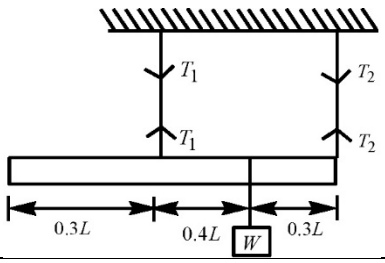
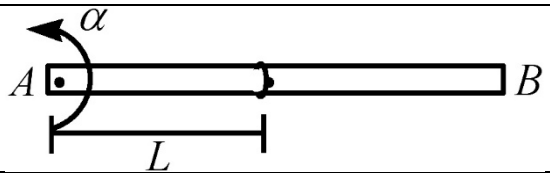
1.	A microscope is focused on an object at the bottom of a bucket. if liquid with refractive index $\frac{5}{3}$is poured inside the bucket, then microscope have to be raised by 30cm to focus the object again. The height of the liquid in the bucket is:	**A**
	75 cm	
	50 cm	
	18 cm	
	12 cm	
Sol.	As, apparent shift $= t\left(1-\frac{1}{\mu}\right)$ $\Rightarrow 30 = t\left(1-\frac{3}{5}\right) \Rightarrow 30 = \frac{2t}{5} \Rightarrow t = 75\,cm$	
2.	Three bodies A, B and C of masses $\frac{m}{\sqrt{3}}$, m and $\sqrt{3}m$respectively are supplied heat at a constant rate. The change in temperature θ versus time t graph for A, B and C are shown by I, II and III respectively. If their specific heat capacities are S_A, S_B and S_C respectively, then which of the following relations is correct? (Initial temperature of the body is 0º C)  	**C**
	$S_A > S_B > S_C$	
	$S_B = S_C < S_A$	
	$S_A = S_B = S_C$	
	$S_B = S_C > S_A$	
Sol.	$\frac{Q}{t} = \frac{ms\Delta t}{t}; \frac{Q}{t} = slope, \frac{\Delta t}{t} = const$ $slope \propto ms$	
3.	The work function of a metal is 12.4eV. The wavelength of photon that can initiate photo electric effect is	**B**
	1200 Å	
	990 Å	
	1330 Å	
	1440 Å	
Sol.	$\lambda = \frac{12400}{E_{in}eV}$	
4.	A double convex lens has power P and same radii of curvature R of both the surfaces. The radius of curvature of a surface of a plano-convex lens made of the same material with power 1.5P is:	**D**
	2R	
	$\frac{R}{2}$	

	$\frac{3R}{2}$	
	$\frac{R}{3}$	
Sol.	Using lens maker's formula $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$	
5.	If M is mass of water that rises in the capillary tube of radius r, the mass of water which will rise in same metal glass tube but of radius 2r is	**A**
	2M	
	M	
	$\frac{M}{2}$	
	4M	
Sol.	$2\pi RT\cos\theta=\mu g\Rightarrow R\times m$	
6.	In the given arrangement find the electric field at C in the figure. Here the U–shaped infinite wire uniformly charged with linear charge density λ. 	**D**
	$\frac{\lambda}{2\pi\varepsilon_0 a}(\hat{\imath})$	
	$\frac{\lambda}{2\pi\varepsilon_0 a}(-\hat{\imath})$	
	$\frac{\lambda}{4\pi\varepsilon_0 a}(\hat{\jmath})$	
	0	
Sol.	Here we shall divide the arrangement into three parts as shown and then apply the principle of superposition. $(E_1)_x=\frac{\lambda}{4\pi\varepsilon_0 a}(along+x\ axis)$  Similarly $(E_3)_x=\frac{\lambda}{4\pi\varepsilon_0 a}$ (along $-x$ axis) And $(E_2)_x=0$ $\quad(E_2)_y=\frac{\lambda}{2\pi\varepsilon_0 a}$ (along $+y$ axis) So, $\vec{E}_C=\vec{E}_1+\vec{E}_2+\vec{E}_3\quad\Rightarrow\quad\vec{E}_C=\vec{0}$	

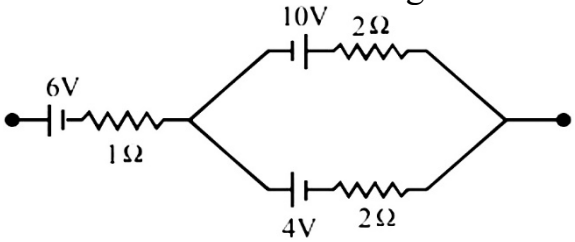
7.	In the shown network charges in capacitors are same, then $\frac{C_1}{C_2}$ is 	**A**
	$\frac{5}{3}$	
	1	
	$\frac{1}{3}$	
	$\frac{1}{5}$	
Sol.	$C_1V_1 = C_2V_2 \therefore \quad \frac{C_1}{C_2} = \frac{V_2}{V_1} \quad = \frac{5i}{3i} = \frac{5}{3}$ 	
8.	Which of the following has the dimensions of pressure?	**D**
	$\frac{M}{L^2T^2}$	
	$\frac{M}{LT}$	
	$\frac{ML}{T^2}$	
	$\frac{M}{LT^2}$	
Sol.	[Pressure] $= \left[\frac{Force}{Area}\right] = ML^{-1}T^{-2}$	
9.	A force $\vec{F} = 3\hat{\imath} + c\hat{\jmath} + 2\hat{k}$acting on a particle causes a displacement $\vec{d} = -4\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$. If the work done is 6J then the value of 'c' is:	**C**
	12	
	0	
	6	
	4	
Sol.	$\omega = \overline{F}.\overline{d} \qquad -12 + 2c + 6 = 6 \qquad 2c = 12$ $c = 6$	
10.	Two particles A and B are separated by a horizontal distance x . They are projected at the same instant towards each other with speeds $u\sqrt{3}$and u at angle of projections 30º and 60º respectively as shown in the figure. The	**B**

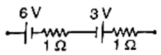
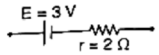
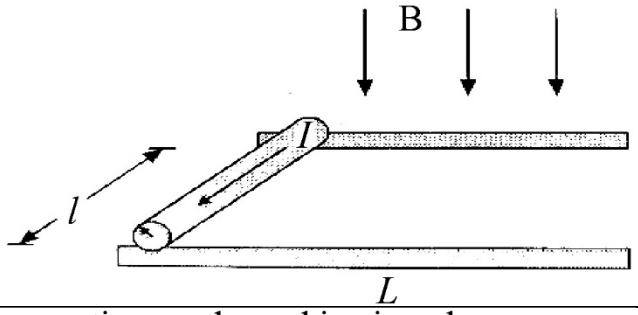
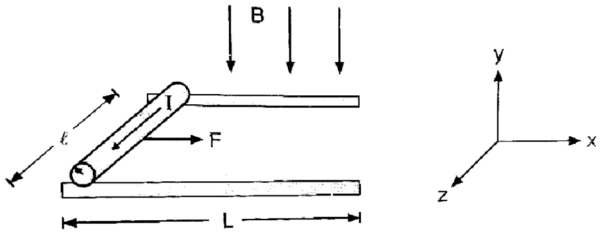
	time after which the horizontal distance between them becomes zero is:  	
	$\frac{x}{u}$	
	$\frac{x}{2u}$	
	$\frac{2x}{u}$	
	$\frac{4x}{u}$	
Sol.	Their velocity of approach is $v = u\sqrt{3}\cos 30^0 + u\cos 60^0 . \qquad = 2u .$ $\therefore \qquad t = \frac{x}{v} = \frac{x}{2u}$	
11.	A force F = kt is applied to a block A as shown in figure. Where t is time in second. The force is applied at t = 0, when the system was at rest. Which of the following graphs correctly gives the frictional force on block A as a function of time? (k is a positive constant)  	**C**
	 	
	 	
	 	
	 	
Ans.	 	
12.	The charge Q_1 = 18 μC and Q_2 = –2 μC are separated by a distance R, and Q_1 is on the left of Q_2. The distance of the point where the net electric field is zero is	**D**

	Between Q_1 and Q_2	
	Left of Q_1 at R/2	
	Right Q_2 at R	
	Right Q_2 at R/2	
Sol.	$\frac{kQ_2}{x^2}=\frac{kQ_1}{(x+R)^2}$ or $x=\frac{R}{2}$ 	
13.	A stretched wire of some length under a tension is vibrating with its fundamental frequency. Its length is decreased by 45% and tension is increased by 21%. Now fundamental frequency	**B**
	increases by 50%	
	increases by 100%	
	decreases by 50%	
	decreases by 25%	
Sol.	$n_1=\frac{1}{2l_1}\sqrt{\frac{T_1}{m}}, n_2=\frac{1}{2l_2}\sqrt{\frac{T_2}{m}} \quad \therefore \quad \frac{n_2}{n_1}=\frac{l_1}{l_2}\sqrt{\frac{T_2}{T_1}}$ Let $l_1=100l, l_2=55l$ $T_1=100T, T_2=121T$ $\therefore \quad \frac{n_2}{n_1}=\frac{100l}{55l}\sqrt{\frac{121T}{100T}}=\frac{100}{55}\times\frac{11}{10}=2 \quad \Rightarrow \quad n_2=2n_1$	
14.	In figure, the bar is uniform and weighing 500 N. How large must W be if T_1 and T_2 are to be equal? 	**D**
	500 N	
	300 N	
	750 N	
	1500 N	
Sol.	Taking torque about the attachment point for W, we get $-T_1(0.4L)+T_2(0.3L)+500(0.2L)=0$ $T=1000N$, where $T_1=T_2=T$ $\Sigma F_y=0 \Rightarrow 2T-W-500=0$ $\Rightarrow \quad W=1500\ N$	
15.	A long horizontal rod has a bead which can slide along its length and initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with constant angular acceleration α. If the coefficient of friction between the rod and the bead is μ, and gravity is neglected, then the time after which the bead starts slipping is	**D**

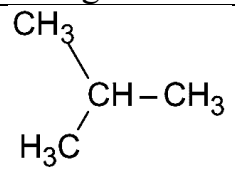
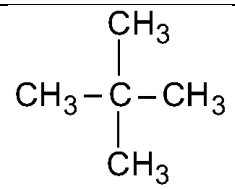
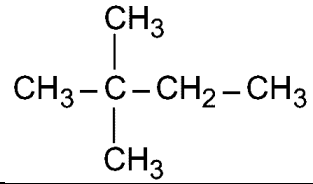
		
	$\frac{\sqrt{\mu\alpha}}{1}$	
	$\frac{\mu}{\sqrt{\alpha}}$	
	$\frac{1}{\sqrt{\mu\alpha}}$	
	$\sqrt{\frac{\mu}{\alpha}}$	
Sol.		
16.	A particle executes SHM of amplitude A. the distance from the mean position when it's kinetic energy becomes equal to its potential energy is:	**A**
	$\frac{1}{\sqrt{2}}A$	
	$2A$	
	$\sqrt{2A}$	
	$\frac{1}{2}A$	
Sol.	Let the distance from the mean position is X. Given KE= PE So, $\frac{1}{2}M\omega^2\left(A^2-x^2\right)=\frac{1}{2}M\omega^2x^2$ $A^2-x^2=x^2\Rightarrow A^2=2\times2 \quad \therefore \quad x=\pm\frac{A}{\sqrt{2}}$	
17.	**Statement I:** Energy is released when heavy nuclei undergo fission or light nuclei undergo fusion **Statement II:** For heavy nuclei, binding energy per nucleon increases with increasing Z while for light nuclei it decreases with increasing Z.	**D**
	Statement – I is false, Statement – II is true	
	Statement – I is true, Statement – II is true; Statement – II is the correct explanation of Statement – I	
	Statement – I is true, Statement – II is true; Statement – II is not the correct explanation of Statement – I	
	Statement – I is true, statement – II is false	
Sol.	We know that energy is released when heavy nuclei undergo fission or light nuclei undergo fusion. Therefore statement (I) is correct. The second statement is false because for heavy nuclei the binding energy per nucleon decreases with increasing Z and for light nuclei, B. E/nucleon increases with increasing Z.	
18.	Match List I with List II:	**A**

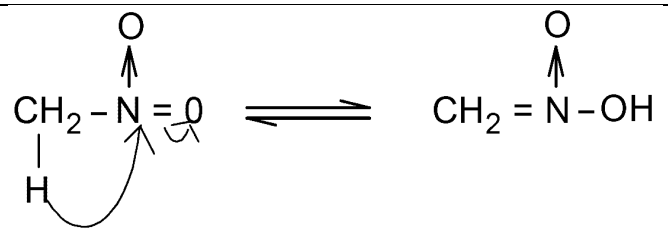
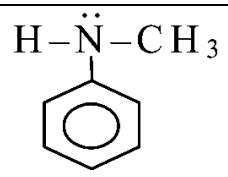
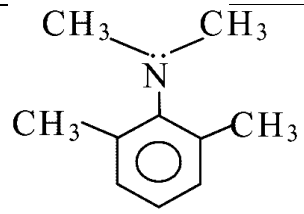
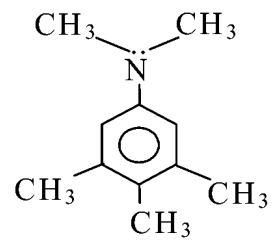
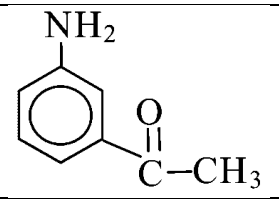
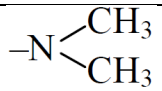
List I		**List II**	
A	Intrinsic semiconductor	(i)	Fermi-level near the valence band
B	n – type semiconductor	(ii)	Fermi-level in the middle of valence and conduction band

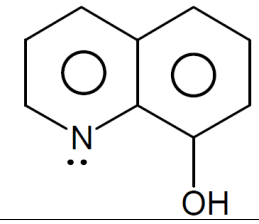
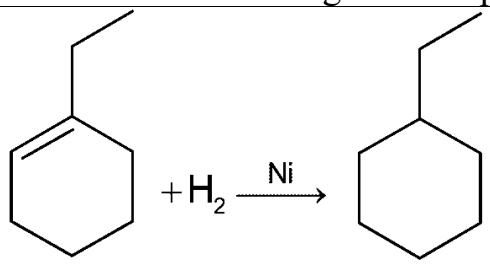
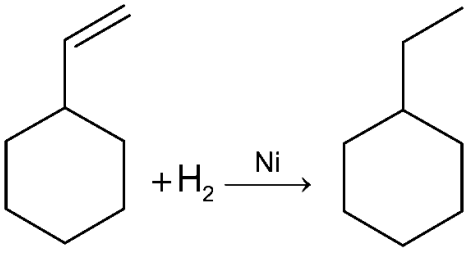
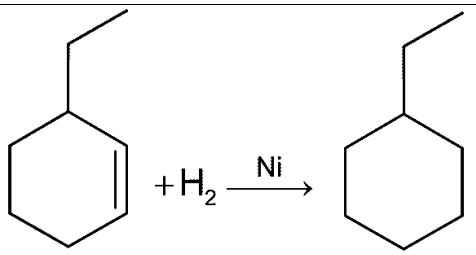
	C	p –type semiconductor	(iii)	Fermi-level near the conduction band	
	D	Metals	(iv)	Fermi-level inside the conduction band	
	Choose the correct answer from the options given below:				
	A-ii, B-iii, C-i, D-iv				
	A-i, B-ii, C-iii, D-iv				
	A-iii, B-i, C-ii, D-iv				
	A-ii, B-i, C-iii, D-iv				
Sol.	A. In instrinsic semiconductor, Fermi level is between the two bands. B. In n –type semiconductor, Fermi level is close to conduction band. C. In p –type semiconductor, fermi level is near to valence band. D. In metal, Fermi level inside the conduction band.				
19.	When current in a coil changes from 5A to 2A in 0.1s, average voltage of 50 V is produced. The self-inductance (in H) of the coil is:				**A**
	1.67				
	1.87				
	1.97				
	1.57				
Sol.	Induced emf, $e = \frac{Ldi}{dt}$ $50 = L\left(\frac{5-2}{0.1\text{sec}}\right) \Rightarrow L = \frac{50\times 0.1}{3} = \frac{5}{3} = 1.67\,H$				
20.	If the terminal speed of a sphere of gold (density = 19.5 kg/m^3) is 0.2 m/s in a viscous liquid (density = 1.5 kg/m^3), find the terminal speed of the sphere of silver (density = 10.5 kg/m^3) of the same size in the same liquid				**C**
	0.4 m/s				
	0.133 m/s				
	0.1 m/s				
	0.2 m/s				
Sol.	Terminal velocity, $v_T = \frac{2r^2(\rho-\sigma)g}{9\eta}$ $\therefore \frac{v_{T_2}}{0.2}\frac{(10.5-1.5)}{(19.5-1.5)} \Rightarrow v_{T_2} = 0.2\times\frac{9}{18} = 0.1 m/s$				
21.	Find the magnitude of emf in volts of a single battery which is equivalent to a combination of three batteries as shown in figure.  				**3**
Sol.	The given combination consists of two batteries in parallel and resultant of these two in series with the third one. For parallel combination we can apply,				

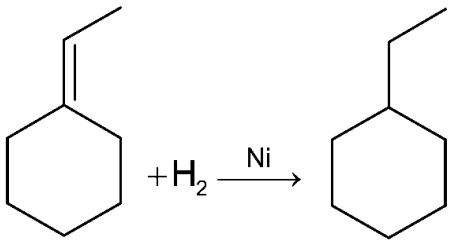
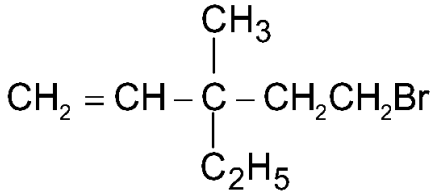
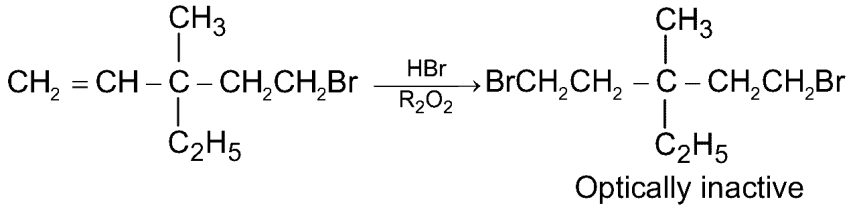
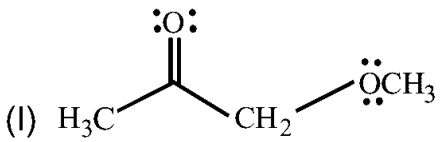
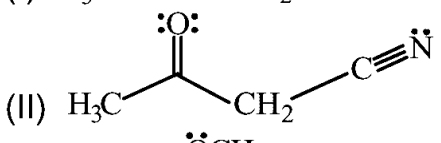
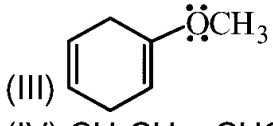
	$E_{eq}=\dfrac{\frac{E_1}{r_1}-\frac{E_2}{r_2}}{\frac{1}{r_1}+\frac{1}{r_2}}=\dfrac{\frac{10}{2}-\frac{4}{2}}{\frac{1}{2}+\frac{1}{2}}=3V$  Further, $\dfrac{1}{r_{eq}}=\dfrac{1}{r_1}+\dfrac{1}{r_2}=\dfrac{1}{2}+\dfrac{1}{2}=1$  $\Rightarrow \quad r_{eq}=1\Omega$ Now this resistance is in series with the third resistance so the equivalent emf of these two is $(6-3)V=3V$ and the internal resistance will be $(1+1)=2\Omega$.	
22.	A rod of mass m and radius R rests on two parallel rails that are a distance l apart and have a length L. The rod carries a current I (in the direction shown) and rolls along the rails without slipping. A uniform magnetic field B is directed perpendicular to the rod and the rails. If it starts from rest, the speed of the rod as it leaves the rails $V=\sqrt{\frac{xBIlL}{3m}}$. Then the value of x is ----------. 	**4**
Sol.	The magnetic force acting on the rod is given by $\vec{F}=I\left(\vec{l}\times\vec{B}\right)=Il\left(\hat{k}\right)\times B\left(-\hat{j}\right)=\left(BIl\right)\hat{i}$  From Work Energy Theorem we have $\left(K_{trans}+K_{rot}\right)_{initial}+\Delta E=\left(K_{trans}+K_{rot}\right)_{final}$ $\Rightarrow \quad 0+0+Fs\cos\theta=\frac{1}{2}m\upsilon^2+\frac{1}{2}I\omega^2$, where $I=\frac{1}{2}mR^2$ $\Rightarrow \quad \left(BIl\right)L\cos 0^0=\frac{1}{2}m\upsilon^2+\frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{\upsilon}{R}\right)^2$ $\Rightarrow \quad BIlL=\frac{3}{4}m\upsilon^2 \quad \Rightarrow \quad \upsilon=\sqrt{\frac{4BIlL}{3m}}$	
23.	A body rotating with an angular speed of 600 rpm is uniformly accelerated to 1800 rpm in 10 sec. The number of rotations made in the process is_____.	**200**

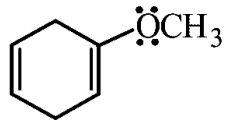
Sol.	Final angular velocity $\omega_i = 2\pi f = \frac{2\pi \times 1800}{60} = 60\pi rad/s$ Initial angular velocity $\omega_f = \frac{2\pi \times 600}{60} = 20\pi\ rad/s$ Using $\omega_f = \omega_i + \alpha t$ $\quad 60\pi = 20\pi + \alpha(10)$ Angular acceleration, $\alpha = 4\pi rad/s^2$ We have, $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ $\theta = 20\pi(10) + \frac{1}{2}(4\pi)(10)^2 = 400\pi$ radian. Number of rotations $\quad n = \frac{\theta}{2\pi} = \frac{400\pi}{2\pi} = 200$	
24.	A body of mass 1 kg begins to move under the action of a time dependence force $\vec{F} = (t\hat{\imath} + 3t^2\hat{\jmath})$ N. Where $\hat{\imath}$ and $\hat{\jmath}$ are then unit vectors along x and y axis. The power developed by above force, at the time t = 2s . Will be_____W	**100**
Sol.	$\vec{F} = t\hat{i} + 3t^2\hat{j}$ $\quad \frac{m d\vec{v}}{dt} = t\hat{i} + 3t^2\hat{j}$ $m = 1\ kg, \int_0^v dv = \int_0^t t dt\hat{i} + \int_0^t 3t^2 dt\hat{j}$ $\vec{v} = \frac{t^2}{2}\hat{i} + t^3\hat{j}$ $\quad$ Power $= \vec{F}.\vec{v} = \frac{t^3}{2} + 3t^5$ At $t = 2$, power $= \frac{8}{2} + 3 \times 32 = 100$	
25.	The radius in kilometer to which the present radius of earth (R = 6400 km) to be compressed so that the escape velocity is increased 10 times is ____.	**64**
Sol.	$v_e = \sqrt{\frac{2GM}{R}}$ $v' = 10 v_e$ $\therefore R' = \frac{R}{100} = \frac{6400}{100} = 64\,km$	
26.	Molar heat capacity of an ideal gas at constant volume is given by $C_v = 2 \times 10^{-2}$ J (in Joule). If 3.5 mole of this ideal gas are heated at constant volume from 300 K to 400 K the change in internal energy will be :	**A**
	7	
	4	
	8	
	6	

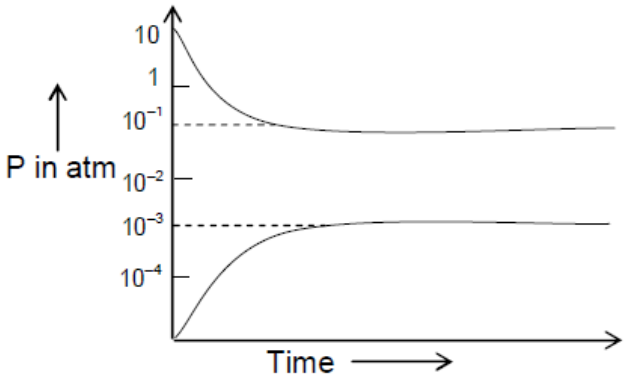
Sol.	Using $\Delta U = nC_V \times \Delta T$ $\Delta T = 100$ $n = 3.5$ $C_V = 2 \times 10^{-2}$ $\Delta U = 7$	
27.	The compound X – CH_2COOH will be the strongest acid if X is:	**B**
	CH_3	
	NO_2	
	F	
	OH	
Sol.	NO_2 is the strongest electron withdrawing group among the given groups or atoms.	
28.	In the detection of nitrogen, blue/green colour is due to formation of Prussian blue. It is:	**A**
	$Fe_4^{III}[Fe^{II}(CN)_6]$	
	$NaFe^{II}[Fe^{III}(CN)_6]$	
	$Na_4[Fe(CN)_6]$	
	$Na_3[Fe(CN)_6]$	
29.	Atomic radii of Ti, Zr and Hf vary :	**D**
	Ti > Zr > Hf	
	Ti < Zr < Hf	
	Ti < Hf < Zr	
	Ti < Zr = Hf	
Sol.	This is due to lanthanoid contraction on 6th period.	
30.	Which of the following hydrocarbon can form one monochloro product with Cl_2/light?	**B**
		
		
		
	$CH_3CH_2CH_3$	
Sol.	It is most symmetrical.	
31.	Dehydration of alcohols produces :	**B**
	Alkanes	
	Alkenes	
	Alkynes	
	Aldehydes	
Sol.	$RCH_2CH_2OH \xrightarrow[\Delta]{\text{Conc. } H_2SO_4} R-CH=CH_2 + H_2O$	

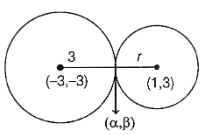
32.	The shape of $[Co(NH_3)_6]^{3+}$ is:	**C**
	Tetrahedral	
	Trigonal bipyramidal	
	Octahedral	
	Square planar	
Sol.	Octahedral complex with C.N number 6.	
33.	$AgNO_3$ solution will not given white precipitate with :	**A**
	$[Co(NH_3)_3Cl_3]$	
	$[Co(NH_3)_4Cl_2]Cl$	
	$[Co(NH_3)_5Cl]Cl_2$	
	$[Co(NH_3)_6]Cl_3$	
Sol.	No free Cl^- in the ionisation sphere.	
34.	Which of the following compound exhibits tautomerism?	**D**
	Chloroethane	
	Ethanol	
	Ethoxyethane	
	Nitroethane	
Sol.		
35.	Which of the following is most basic?	**B**
		
		
		
		
Sol.	Steric hinderence of two – CH_3 groups makes  group out of plane of benzene ring. So, lone pair on nitrogen atom is not involved in resonance, therefore, this amine is most basic.	
36.	In which combinations, all the ligands are ambient in nature?	**A**

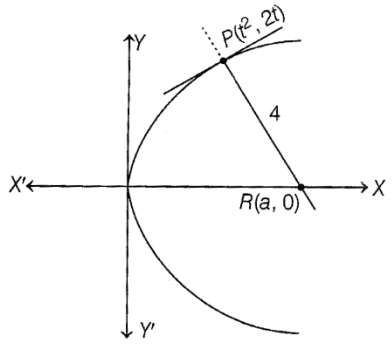
	$N\bar{O}_2, \bar{S}CN$ and $\bar{C}N$	
	EDTA, $\bar{S}CN$	
	H_2O, 	
	All of these	
Sol.	Having two possible donating site but donate from one only.	
37.	$CH_3CH=CH-\underset{\displaystyle CH_3}{\underset{\vert}{CH}}-CH=CHCH_3$ How many stereoisomer(s) is/are possible for the above compound?	**C**
	2	
	3	
	4	
	6	
Sol.	Due to double bond, the geometrical isomers are: $CH_3CH=CH-\underset{\displaystyle CH_3}{\underset{\vert}{CH}}-CH=CH-CH_3$ Cis Cis Trans Trans Cis Trans } Identical Trans Cis } When the double bonds are Cis and trans, the molecule becomes optically active. $\therefore$ For trans Cis(two optical isomers are possible) $\therefore$ Total no. of stereoisomers are: Geometrical (2) + Optical (2) = 4	
38.	Which of the following reaction produces maximum heat?	**B**
		
		
		

		
Sol.	It is least substituted alkene. Hence it is unstable.	
39.	Which of the following substance can decrease the ionization of CH_3COOH in water? $CH_3COOH(I) \rightleftharpoons CH_3COO^-(aq) + H^+(aq)$	**B**
	NaOH	
	NH_4Cl	
	K_2CO_3	
	NaCl	
Sol.	$NH_4Cl \longrightarrow NH_4^+ + Cl^-$ $NH_4^+ + H_2O \longrightarrow NH_4OH + H^+$ H^+ shift the ionization reaction toward backward direction due to common ion effect.	
40.	 Which of the following substance converts the above optically active compound to optically inactive compound?	**C**
	HBr	
	Br_2/CCl_4	
	HBr/R_2O_2	
	Br_2/H_2O	
Sol.	 Optically inactive	
41.	The molecule/molecules that has/have delocalised lone pair(s) of electrons is/are  (I)  (II)  (III) (IV) $CH_3CH = CHCH_2CH_3$	**D**
	I, II and III	
	I, II and IV	

	I and III	
	Only III	
Sol.	In  the lone pair of oxygen get delocalised on the π bond located on the next carbon.	
42.	$Ca \xrightarrow{Air} (P+Q) \xrightarrow{H_2O} R + S\uparrow$ $(P+Q) \xrightarrow{CO_2} \underset{\text{White ppt.}}{T}$ $S\uparrow + CO_2 \longrightarrow U + H_2O$ Which of the following is 'U'?	**B**
	CH_3COONH_4	
	NH_2CONH_2	
	NH_2COONH_4	
	NH_2COOH	
Sol.	P = CaO, Q = Ca_3N_2, R = $Ca(OH)_2$, S = NH_3, T = $CaCO_3$, U = NH_2CONH_2	
43.	Which of the following orbital has the maximum number of radial nodes?	**A**
	4s	
	4p	
	4d	
	4f	
Sol.	No. of radial nodes = (n – l – 1) For 4s orbital, it is 3 For 4p orbital, it is 2 For 4d orbital, it is 1 For 4f orbital, it is 0	
44.	In which case bond length of C – O bond is maximum	**B**
	Carboxylate ion	
	Phenoxide ion	
	p-nitro phenoxide ion	
	2, 4 di-nitro phenoxide ion	
Sol.	More double bond character, smaller bond length	
45.	Which of the following overlap forms the strongest sigma bond?	**B**
	2s – 2s	
	$2p_x - 2p_x$	
	$3p_y - 3p_y$	
	$2p_x - 3p_x$	
Sol.	Small size atomic orbitals undergo better overlap than larger atomic orbitals.	
46.	k for a zero order reaction is 2×10^{-2} $mol^{-1}s^{-1}$. If the concentration of the reactant after 25 s is 0.5 M, the initial concentration must have been	**1**
Sol.	For a zero order reaction, $C_0 - C_t = Kt$ $\therefore C_0 = C_t + Kt = 0.5 + 2 \times 10^{-2} \times 25 = 1.0$ M	
47.	56.7 g anhydrous oxalic acid ($H_2C_2O_4$) completely decolourises 200 mL of acidified $KMnO_4$ solution. If the molarity of the $KMnO_4$ solution is	**106**

	expressed as $\left(\frac{20+x}{100}\right)$ M. What is the value of x?	
Sol.	M_{eq} of $KMnO_4$ = M_{eq} of $H_2C_2O_4$ $(V) \times (M)(n) = \frac{W}{E} \times 1000$ Or, $(200)(M)(5) = \frac{56.7}{90/2} \times 1000$ $\therefore$ M = 1.26 $\therefore \frac{20+x}{100} = 1.26 \Rightarrow x = 106$	
48.	 The plot of partial pressure of the product and reactant is given above. What is the value of ΔG^o in L atm unit at temperature 'T'? [Assume 2.303 RT = 4 L atm mol^{-1}]	**8**
Sol.	$K_P = \frac{10^{-3}}{10^{-1}} = 10^{-2}$ ΔG^o = -2.303 RTlogK_P = -2.303 RTlog10^{-2} = (-4)(2 log10) = 8	
49.	AB_2 is 10% dissociated in water to A^{2+} and B^-. The boiling point of a 10.0 molal aqueous solution of AB_2 is _____ºC. (Round off to the nearest integer). [Given: Molal elevation constant of water K_b = 0.5 K kg mol^{-1} boiling point of pure water = 100ºC]	**106**
Sol.	$AB_2 \longrightarrow A^{2+} + 2B^-$ t = 0 a 0 0 t = t a - aα aα 2 aα nT = a - aα + aα + 2 aα = a(1 + 2aα) So, i = 1 + 2α Now ΔT_b = i × m × K_b ΔT_b =(1 + 2α) × m × K_b α = 0.1, m = 10, K_b = 0.5 ΔT_b = 1.2 × 10 × 0.5 = 6 So boiling point = 106	
50.	A certain orbital as n = 4 and m_ℓ = – 3. The number of radial nodes in this orbital is _____(Round off to the nearest integer)	**0**
Sol.	n = 4 and m_ℓ = – 3 Hence, ℓ value must be 3. Now number of radial nodes = n – ℓ – 1 = 4 – 3 – 1 = 0	

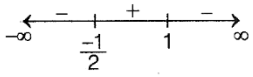
51.	Let the range of the function f(x) = 6 + 16 cosx. $\cos\left(\frac{\pi}{3}-x\right).\cos\left(\frac{\pi}{3}+x\right)$.sin 3x. cos 6x, $x \in R$ *be* $[\alpha, \beta]$. Then, the distance of the point (α, β) from the line 3x + 4y + 12 = 0 is	**D**
	9	
	10	
	8	
	11	
Sol.	*(d)* $f(x)=6+16\cos x\cos\left(\frac{\pi}{3}-x\right)\cos\left(\frac{\pi}{3}+x\right)\times\sin 3x\cos 6x$ $\because \cos A\cos(60°-A)\cos(60°+A)$ $=\frac{1}{4}\cos 3A$ $\therefore f(x)=6+16\left(\frac{1}{4}\cos 3x\right)\sin 3x\cos 6x$ $=6+4\cos 3x\sin 3x\cos 6x$ $=6+2\sin 6x\cos 6x$ $=6+\sin 12x$ $\therefore$Range of $f(x)=[6-1,\ 6+1]$ $=[5, 7]$ $\therefore(\alpha,\beta)$ is (5, 7). $\therefore$Distance between (5, 7) and line $3x+4y+12=0$ is $=\left\lvert\frac{3(5)+4(7)+12}{\sqrt{3^2+4^2}}\right\rvert$ $=\frac{15+28+12}{5}=11$	
52.	Let C_1 be the circle in the third quadrant of radius 3, that touches both coordinate axes. Let C_2 be the circle with centre (1, 3) that touches C_1 externally at the point (α, β). If $(\beta-\alpha)^2=\frac{m}{n}.\gcd(m,n)=1, then\ m+n$ is equal to	**C**
	9	
	13	
	22	
	31	
Sol.	*(c)*  $\sqrt{16+36}=r+3$ $r=\sqrt{52}-3$ $\Rightarrow \frac{3-3r}{r+3}=\alpha$ $\frac{9-3r}{3+r}=\beta$ $\Rightarrow (\beta-\alpha)^2=\frac{m}{n}$ [given] $\Rightarrow \frac{(9-3r-3+3r)^2}{(r+3)^2}=\frac{m}{n}$ $\Rightarrow \frac{36}{52}=\frac{m}{n}$ $\Rightarrow \frac{m}{n}=\frac{9}{13}$ $\Rightarrow m+n=22$	
53.	Let the shortest distance from (a, 0) a > 0, to the parabola $y^2 = 4x$ be 4. Then, the equation of the circle passing through the point (a, 0) and the focus of the parabola and having its centre on the axis of the parabola is	**C**

	$x^2 + y^2 - 8x + 7 = 0$	
	$x^2 + y^2 - 10x + 9 = 0$	
	$x^2 + y^2 - 6x + 5 = 0$	
	$x^2 + y^2 - 4x + 3 = 0$	
Sol.	*(c)* Parabola $y^2 = 4x$ Normal at $P(t)$ to the parabola $y + tx = 2t + t^3$ $\because$ It passes through $R(a, 0)$.  $\Rightarrow \quad at = 2t + t^3$ $\Rightarrow \quad a = 2 + t^2 \Rightarrow a - t^2 = 2$ $\because \quad PR = 4$ $PR^2 = 16$ $\Rightarrow \quad (a - t^2)^2 + (2t)^2 = 16$ $\Rightarrow \quad (2)^2 + 4t^2 = 16 \Rightarrow t^2 = 3$ $\therefore \quad t = \pm\sqrt{3}$ $a = 2 + 3 = 5 \Rightarrow R(5, 0)$ $\because$ Focus of parabola = (1, 0) $\because$ (1, 0) and (5, 0) are end points of diameter. $\therefore$ Equation of circle $(x - 1)(x - 5) + (y - 0)(y - 0) = 0$ $x^2 + y^2 - 6x + 5 = 0$	
54.	Let the product of the focal distances of the point $\left(\sqrt{3}, \frac{1}{2}\right)$ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, (a > b)$ $be \frac{7}{4}$. Then, the absolute difference of the eccentricities of two such ellipse is	**A**
	$\frac{3-2\sqrt{2}}{2\sqrt{3}}$	
	$\frac{1-2\sqrt{2}}{\sqrt{3}}$	
	$\frac{1-\sqrt{3}}{\sqrt{2}}$	
	$\frac{3-2\sqrt{2}}{3\sqrt{2}}$	

Sol.	(a) As, we know that focal distance of ellipse $= a \pm ex$ $\therefore$ Its product $= (a+ex)(a-ex)$ $= a^2 - e^2x^2$ $= a^2 - e^2(\sqrt{3})^2$ $= a^2 - 3e^2 = \frac{7}{4}$ [given] $\Rightarrow 4a^2 = 7 + 12e^2$ ...(i) Also, $(\sqrt{3}, 1/2)$ lies on ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ So, $\frac{3}{a^2} + \frac{1}{4b^2} = 1$ $\Rightarrow \frac{3}{a^2} + \frac{1}{4a^2(1-e^2)} = 1$ $[\because b^2 = a^2(1-e^2)]$ $\Rightarrow 3(1-e^2) + 1 = 4a^2(1-e^2)$ $\Rightarrow 3 - 12e^2 = (7+12e^2)(1-e^2)$ (using Eq. (i)) $\Rightarrow 3 - 12e^2 = 7 - 7e^2 + 12e^2 - 12e^4$ $\Rightarrow 12e^4 - 17e^2 + 6 = 0$ $\Rightarrow e^2 = \frac{17 \pm \sqrt{289-288}}{24} = \frac{17 \pm 1}{24}$ $\Rightarrow e^2 = \frac{3}{4}, \frac{2}{3}$ $\Rightarrow e = \frac{\sqrt{3}}{2}$ and $\sqrt{2/3}$ $[\because 0 < e < 1$ always for an ellipse$]$ Absolute difference $= \left\| \frac{\sqrt{3}}{2} - \sqrt{2/3} \right\|$ $= \left\| \sqrt{2/3} - \frac{\sqrt{3}}{2} \right\| = \frac{3-2\sqrt{2}}{2\sqrt{3}}$	
55.	$\lim_{n\to\infty} \left\{ \left(2^{\frac{1}{2}} - 2^{\frac{1}{3}}\right)\left(2^{\frac{1}{2}} - 2^{\frac{1}{5}}\right) \ldots \left(2^{\frac{1}{2}} - 2^{\frac{1}{2n-1}}\right) \right\}$ is equal to	**C**
	$1/\sqrt{2}$	
	$\sqrt{2}$	
	0	
	1	
Sol.	Let $L = \lim_{n\to\infty} \{(2^{\frac{1}{2}} - 2^{\frac{1}{3}})(2^{\frac{1}{2}} - 2^{\frac{1}{5}}) \ldots (2^{\frac{1}{2}} - 2^{\frac{1}{2n-1}})\}$ Here, $(2^{\frac{1}{2}} - 2^{\frac{1}{3}})^n \le (2^{\frac{1}{2}} - 2^{\frac{1}{3}})(2^{\frac{1}{2}} - 2^{\frac{1}{5}}) \ldots (2^{\frac{1}{2}} - 2^{\frac{1}{2n-1}}) \le (2^{\frac{1}{2}} - 2^{\frac{1}{2n-1}})^n$ $\Rightarrow \lim_{n\to\infty} (2^{\frac{1}{2}} - 2^{\frac{1}{3}})^n \le L \le \lim_{n\to\infty} (2^{\frac{1}{2}} - 2^{\frac{1}{2n-1}})^n$ Since, $\lim_{n\to\infty} (2^{\frac{1}{2}} - 2^{\frac{1}{3}})^n = 0$ and $\lim_{n\to\infty} (2^{\frac{1}{2}} - 2^{\frac{1}{2n-1}})^n = 0$ $\therefore \quad L = 0$	
56.	Let $f: (-\infty, \infty) - \{0\} \to R$ be a differentiable function such that $f'(1) = \lim_{a\to\infty} a^2 f\left(\frac{1}{a}\right)$. *Then,* $\lim_{a\to\infty} \frac{a(a+1)}{2} \tan^{-1}\left(\frac{1}{a}\right) + a^2 - 2\log_e a$ is equal to	**A**

	$\frac{5}{2}+\frac{\pi}{8}$	
	$\frac{3}{2}+\frac{\pi}{4}$	
	$\frac{3}{4}+\frac{\pi}{8}$	
	$\frac{3}{8}+\frac{\pi}{4}$	
Sol.	*(a)* Given, $f:(-\infty,\infty)-\{0\}\to R$ $$f'(1)=\lim_{a\to\infty} a^2 f\left(\frac{1}{a}\right)$$ Now, we have to find $$\lim_{a\to\infty}\frac{a(a+1)}{2}\tan^{-1}\left(\frac{1}{a}\right)+a^2-2\ln(a)$$ $$=\lim_{a\to\infty} a^2\left(\frac{\left(1+\frac{1}{a}\right)}{2}\tan^{-1}\left(\frac{1}{a}\right)+1+\frac{2}{a^2}\ln\frac{1}{a}\right)$$ $$\Rightarrow f(x)=\frac{1}{x^2}(1+x)\tan^{-1}x+1+2x^{-}\ln(x)$$ $$f'(x)=\frac{1}{2}\left(\frac{1+x}{1+x^2}+\tan^{-1}x\right)+4x\ln x$$ $$f'(1)=\frac{1}{2}\left(1+\frac{\pi}{4}\right)+2=\frac{5}{2}+\frac{\pi}{8}$$	
57.	The domain of the function $f(x)=\sin^{-1}[2x^2-3]+\log_2\left(\log_{\frac{1}{2}}(x^2-5x+5)\right)$ where, [t] is the greatest integer function, is	**C**
	$\left(-\sqrt{\frac{5}{2}},\frac{5-\sqrt{5}}{2}\right)$	
	$\left(\frac{5-\sqrt{5}}{2},\frac{5+\sqrt{5}}{2}\right)$	
	$\left(1,\frac{5-\sqrt{5}}{2}\right)$	
	$\left[1,\frac{5+\sqrt{5}}{2}\right)$	

Sol.	(c) $f(x)=\sin^{-1}[2x^2-3]$ $+\log_2\left[\log_{\frac{1}{2}}(x^2-5x+5)\right]$ Take angle of $\sin^{-1}$ as T_1, which lies between -1 and 1. $T_1: -1\le[2x^2-3]\le 1$ $\Rightarrow -1\le(2x^2-3)\le 2 \Rightarrow 2\le 2x^2<5$ $\Rightarrow 1\le x^2<\frac{5}{2} \Rightarrow T_1: x\in\left(-\frac{5}{2},-1\right]\cup\left[1,\frac{5}{2}\right)$ Similarly, $T_2: x^2-5x+5>0$ $\Rightarrow\left(x-\left(\frac{5-\sqrt{5}}{2}\right)\right)\left(x-\left(\frac{5+\sqrt{5}}{2}\right)\right)>0$ $T_3: \log_{\frac{1}{2}}(x^2-5x+5)>0$ $\Rightarrow \quad x^2-5x-5<1 \Rightarrow x^2-5x+4<0$ $\Rightarrow \quad T_3: x\in(1,4)$ Now, take intersection of T_1, T_2 and T_3. $T_1\cap T_2\cap T_3=\left(1,\frac{5-\sqrt{5}}{2}\right)$	
58.	Let $A=\begin{bmatrix}2 & -2\\ 1 & -1\end{bmatrix}$ *and* $B=\begin{bmatrix}-1 & 2\\ -1 & 2\end{bmatrix}$. Then, the number of elements in the set $\{(n,m): n,m\in\{1,2,\ldots\ldots,10\}$and $nA^n+mB^m=I\}$ *is* ……	**B**
	5	
	1	
	3	
	2	
Sol.	(1) Given matrices A and B with 2×2 order. It can be observed that $A^2=A$ and $B^2=B$ and it is true for every m and n. Also, $A+B=I$ Therefore, equation $nA^n+nB^m=I$ becomes $nA+mB=I$, which gives $m=n=1$ Only one element possible.	
59.	Let $f: R\to R$ be a continuous function satisfying f(0) = 1 and f(2x) – f(x) = x for all $x\in R$. If $\lim_{n\to\infty}\left\{f(x)-f\left(\frac{x}{2^n}\right)\right\}=G(x), then\ \sum_{r=1}^{10}G(r^2)$ is equal to	**D**
	540	
	420	
	215	
	385	

Sol.	(d) $\because f(2x)-f(x)=x$ Put $x\to\frac{x}{2}$ $f(x)-f\left(\frac{x}{2}\right)=\frac{x}{2}$ $x\to\frac{x}{2}$ $f\left(\frac{x}{2}\right)-f\left(\frac{x}{4}\right)=\frac{x}{2^2}$ $\vdots$ $f\left(\frac{x}{2^{n-1}}\right)-f\left(\frac{x}{2^n}\right)=\frac{x}{2^n}$ $f(x)-f\left(\frac{x}{2^n}\right)=\frac{x}{2}\frac{\left(1-\left(\frac{1}{2}\right)^h\right)}{1-\frac{1}{2}}$ $f(x)-f\left(\frac{x}{2^n}\right)=x\left(1-\left(\frac{1}{2}\right)^n\right)$ $\because \lim_{n\to\infty} f(x)-f\left(\frac{x}{2^n}\right)$ $\Rightarrow \lim_{n\to 0} x\left(1-\left(\frac{1}{2}\right)^n\right)=x$ $\therefore G(x)=x$ $\therefore \sum_{r=1}^{10} G(r^2)=\sum_{r=1}^{10} r^2=\frac{10}{6}(11)$ $=385$	
60.	The function $f(x)=xe^{x(1-x)}, x\in R$, is	**A**
	Increasing in $\left(-\frac{1}{2},1\right)$	
	Decreasing in $\left(\frac{1}{2},2\right)$	
	Increasing in $\left(-1,-\frac{1}{2}\right)$	
	Decreasing in $\left(-\frac{1}{2},\frac{1}{2}\right)$	
Sol.	(a) Given $f(x)=xe^{x(1-x)}$ $f'(x)=-e^{x(1-x)}(2x+1)(x-1)=0$ Now, $f'(x)=0$, so that $\Rightarrow x=1,-\frac{1}{2}$  $\therefore f(x)$ is increasing in $\left(-\frac{1}{2},1\right)$.	
61.	The integral $\int_{\pi/4}^{\pi/2}(2cosec\ x)^{17}dx$ *is equal to*	**A**
	$\int_0^{\log(1+\sqrt{2})} 2(e^u+e^{-u})^{16}du$	
	$\int_0^{\log(1+\sqrt{2})} (e^u+e^{-u})^{17}du$	
	$\int_0^{\log(1+\sqrt{2})} (e^u-e^{-u})^{17}du$	
	$\int_0^{\log(1+\sqrt{2})} 2(e^u-e^{-u})^{16}du$	

Sol.

(a) Given, $I = \int_{\pi/4}^{\pi/2} (2\,\text{cosec}\,x)^{17}\,dx$

$$= \int_{\pi/4}^{\pi/2} \frac{2^{17}(\text{cosec}\,x)^{16}\,\text{cosec}\,x\,(\text{cosec}\,x + \cot x)}{(\text{cosec}\,x + \cot x)}\,dx$$

Let $\text{cosec}\,x + \cot x = t$

$\Rightarrow (-\text{cosec}\,x \cdot \cot x - \text{cosec}^2 x)\,dx = dt$

and $\text{cosec}\,x - \cot x = 1/t$

$\Rightarrow 2\,\text{cosec}\,x = t + \frac{1}{t}$

$\text{cosec}x = \frac{1}{2}\left(t + \frac{1}{t}\right)$

When $x = \frac{\pi}{2}$, $t = 1$

and when $x = \frac{\pi}{4}$, $t = \sqrt{2} + 1$

$\therefore\ I = -\int_{\sqrt{2}+1}^{1} 2^{17} \left(\frac{t + \frac{1}{t}}{2}\right)^{16} \frac{dt}{t}$

Let $t = e^u \Rightarrow dt = e^u du.$

When $t = 1, e^u = 1 \Rightarrow u = 0$

and when $t = \sqrt{2} + 1, e^u = \sqrt{2} + 1$

$\Rightarrow u = \ln(\sqrt{2} + 1)$

$\Rightarrow I = -\int_{\ln(\sqrt{2}+1)}^{0} 2(e^u + e^{-u})^{16} \frac{e^u du}{e^u}$

$= 2\int_0^{\ln(\sqrt{2}+1)} (e^u + e^{-u})^{16}\,du$

62.	$96 \cos\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{4\pi}{33}\cos\frac{8\pi}{33}\cos\frac{16\pi}{33}$ is equal to	**B**
	4	
	3	
	2	
	1	

Sol.

(b) Let $A = 96\cos\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{4\pi}{33}\cos\frac{8\pi}{33}\cos\frac{16\pi}{33}$

$\Rightarrow 2A = 96 \times 2\left(\cos\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{4\pi}{33}\cos\frac{8\pi}{33}\cos\frac{16\pi}{33}\right)$

$\Rightarrow 2A \times \sin\frac{\pi}{33} = 96 \times \left(2\sin\frac{\pi}{33}\cos\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{4\pi}{33}\cdot\cos\frac{8\pi}{33}\cos\frac{16\pi}{33}\right)$

$\Rightarrow 2A \times \sin\frac{\pi}{33} = 6 \times \sin\frac{32\pi}{33}$

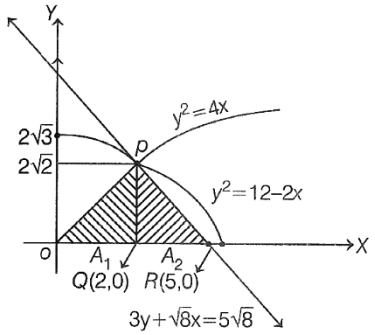
$= 6 \times \sin\frac{\pi}{33} \Rightarrow 2A = 6$

$\Rightarrow A = 3$

63.	If z_1 and z_2 are two non–zero complex numbers such that $\|z_1 + z_2\| = \|z_1\| + \|z_2\|$, then $\arg(z_1) - \arg(z_2)$ is equal to	**C**
	$-\pi$	
	$-\frac{\pi}{2}$	
	0	
	$\frac{\pi}{2}$	
Sol.	(c) Given, $\|z_1 + z_2\| = \|z_1\| + \|z_2\|$ On squaring both sides, we get $\|z_1\|^2 + \|z_2\|^2 + 2\|z_1\|\|z_2\|$ $\cos(\arg z_1 - \arg z_2)$ $= \|z_1\|^2 + \|z_2\|^2 + 2\|z_1\|\|z_2\|$ $\Rightarrow 2\|z_1\|\|z_2\|\cos(\arg z_1 - \arg z_2)$ $= 2\|z_1\|\|z_2\|$ $\Rightarrow \cos(\arg z_1 - \arg z_2) = 1$ $\Rightarrow \arg(z_1) - \arg(z_2) = 0$	
64.	Let R be a relation from the set {1, 2, 3,…..,60} to itself such that $R = \{(a, b): b = pq, where\ p, q \geq 3\ are\ prime\ numbers\}$. Then, the number of elements in R is	**B**
	600	
	660	
	540	
	720	
Sol.	(b) Now, number of possible values of $a = 60$, for $b = pq$, If $p = 3$, $q = 3, 5, 7, 11, 13, 17, 19$, then total cases = 7 If $p = 5$, $q = 5, 7, 11$, then total If $p = 7$, $q = 7$, then total cases Total cases $= 60 \times (7 + 3 + 1) = 660$	
65.	Let α, β, γ and δ be the coefficients of $x^7, x^5, x^3 and\ x$ respectively in the expansion of $\left(x + \sqrt{x^3 - 1}\right)^5 + \left(x - \sqrt{x^3 - 1}\right)^5, x > 1$. If u and v satisfy the equations $\alpha u + \beta v = 18, \gamma u + \delta v = 20$, then u + v equals	**D**
	4	
	8	
	3	
	5	

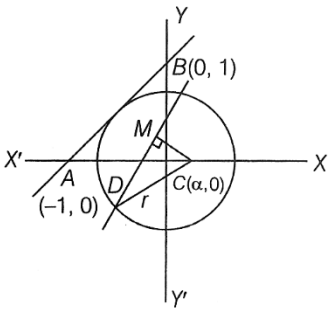
Sol.	*(d)* $\because (a+b)^n + (a-b)^n$ $= 2$ (sum of all odd terms) $\therefore (x+\sqrt{x^3-1})^5 + (x-\sqrt{x^3-1})^5$ $= 2[{}^5C_0x^5 + {}^5C_2x^3(\sqrt{x^3-1})^2$ $+ {}^5C_4x^1(\sqrt{x^3-1})^4]$ $= 2[x^5 + 10x^3(x^3-1)$ $+ 5x(x^6 - 2x^3 + 1)]$ $\alpha =$ Coefficient of $x^7 = 2\times 5 = 10$ $\beta =$ Coefficient of $x^5 = 2\times 1 = 2$ $\gamma =$ Coefficient of $x^3 = 2(-10) = -20$ $\delta =$ Coefficient of $x = 2\times 5 = 10$ $\because \quad \alpha u + \beta v = 18 \Rightarrow 10u + 2v = 18$ $\Rightarrow \quad 5u + v = 9 \quad \ldots(i)$ and $\quad \gamma u + \delta v = 20$ $\Rightarrow \quad -20u + 10v = 20$ $\Rightarrow \quad -2u + v = 2 \quad \ldots(ii)$ By solving Eqs. (i) and (ii) we get $7u = 7 \Rightarrow u = 1$ and $\quad v = 2 + 2 = 4$ $\therefore \quad u + v = 5$	
66.	Let the sum of an infinite GP, whose first term is a and the common ratio is r, be 5. Let the sum of its first five terms be 98/25. Then, the sum of the first 21 term of an AP, whose first term is 10ar, n^{th} term is a^n and the common difference is $10ar^2$, is equal to	**A**
	$21a_{11}$	
	$22a_{11}$	
	$15a_{16}$	
	$14a_{16}$	
Sol.	*(a)* Given, sum of infinite GP is 5. $S_5 = \frac{a(-r^5+1)}{1-r} = \frac{98}{25}$ (Given) ...(i) $S_\infty = \frac{a}{1-r} = 5$ ...(ii) Now, on dividing Eq. (i) by Eq. (ii), we get $1 - r^5 = \frac{98}{125}, r^5 = \frac{27}{125}$ $S_{21} = \frac{21}{2}[20ar + 20\cdot 10ar^2]$ $= 21[10ar + 100ar^2] = 21a_{11}$	
67.	The value(s) of $\int_0^1 \frac{x^4(1-x)^4}{1+x^2}\, dx$ *is (are)*	**A**
	$\frac{22}{7} - \pi$	
	$\frac{2}{105}$	
	0	
	$\frac{71}{15} - \frac{3\pi}{2}$	

Sol.	(a) Let $I = \int_0^1 \frac{x^4(1-x)^4}{1+x^2}dx$ $= \int_0^1 \frac{(x^4-1)(1-x)^4 + (1-x)^4}{(1+x^2)}dx$ $= \int_0^1 (x^2-1)(1-x)^4\,dx + \int_0^1 \frac{(1+x^2-2x)^2}{(1+x^2)}dx$ $= \int_0^1 \{(x^2-1)(1-x)^4 + (1+x^2) - 4x + \frac{4x^2}{(1+x^2)}\}dx$ $= \int_0^1 ((x^2-1)(1-x)^4 + (1+x^2) - 4x + 4 - \frac{4}{1+x^2})dx$ $= \int_0^1 (x^6 - 4x^5 + 5x^4 - 4x^2 + 4 - \frac{4}{1+x^2})dx$ $= \left[\frac{x^7}{7} - \frac{4x^6}{6} + \frac{5x^5}{5} - \frac{4x^3}{3} + 4x - 4\tan^{-1}x\right]_0^1$ $= \frac{1}{7} - \frac{4}{6} + \frac{5}{5} - \frac{4}{3} + 4 - 4\left(\frac{\pi}{4} - 0\right) = \frac{22}{7} - \pi$	
68.	Let $S = \{(x,y) \in R \times R: x \geq 0, y \geq 0, y^2 \leq 4x, y^2 \leq 12 - 2x \text{ and } 3y + \sqrt{8}\,x \leq 5\sqrt{8}\}$. If the area of the region S is α is equal to	**B**
	17/2	
	17/3	
	17/4	
	17/5	

Sol.	*(b)* Point of intersection of the curves $y^2 = 4x$ and $y^2 = 12 - 2x$ is $(2, \sqrt{8})$ i.e., $(2, 2\sqrt{2})$ which also satisfies the equation of straight line $3y + \sqrt{8}x = 5\sqrt{8}$.  Now, required area $(S) = A_1 + A_2$ $\Rightarrow \quad \alpha\sqrt{2} = \int_0^2 2\sqrt{x}\, dx + \frac{1}{2} \times 3 \times 2\sqrt{2}$ $\left(\because \text{ Area of } \Delta PQR = \frac{1}{2} \times QR \times PQ \right.$ $\left. = \frac{1}{2} \times (5-2) \times 2\sqrt{2}\right)$ $= 2 \cdot \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^2 + 3\sqrt{2}$ $\Rightarrow \alpha\sqrt{2} = \frac{17\sqrt{2}}{3}$ $\Rightarrow \quad \alpha = \frac{17}{3}$	
69.	Suppose the solution of the differential equation $\frac{dy}{dx} = \frac{(2+\alpha)x - \beta y + 2}{\beta x - 2\alpha y - (\beta y - 4\alpha)}$ represents a circle passing through origin. Then, the radius of this circle is	**B**
	2	
	$\sqrt{17}/2$	
	1/2	
	$\sqrt{17}$	

Sol.	(b) $\frac{dy}{dx}=\frac{(2+\alpha)x-\beta y+2}{\beta x-2\alpha y-(\beta y-4\alpha)}$ $\Rightarrow (\beta x-2\alpha y-(\beta y-4\alpha))\,dy$ $=[(2+\alpha)x-\beta y+2]\,dx$ $\Rightarrow \beta x\,dy-2\alpha y\,dy-[\beta y-4\alpha]\,dy$ $=(2+\alpha)\,x\,dy-\beta y\,dx+2\,dx$ $\Rightarrow \beta[x\,dy+y\,dx]-2\alpha y\,dy-[\beta y-4\alpha]\,dy$ $=[2+\alpha]\,xdx+2dx$ $\Rightarrow \beta[d(xy)]-2\alpha y\,dy-[\beta y-4\alpha]\,dy$ $=[2+\alpha]\,xdx+2\,dx$ On integrating, $\beta xy-\alpha y^2-[\beta y-4\alpha]y$ $=[2+\alpha]\frac{x^2}{2}+2x+C$ This passes through (0, 0), $\therefore \quad C=0$ $\beta xy-\alpha y^2-[\beta y-4\alpha]y$ $=[2+\alpha]\frac{x^2}{2}+2x$ $\Rightarrow \frac{[2+\alpha]x^2}{2}+\alpha y^2+2x+[\beta y-4\alpha]y$ $-\beta xy=0$ Put $\beta=0$ for equation of circle and $\frac{2+\alpha}{2}=\alpha$ $\Rightarrow \alpha=2$ and $\beta=0$ $\Rightarrow 2x^2+2y^2+2x-8y=0$ $\Rightarrow x^2+y^2+x-4y=0$ $\Rightarrow \left(x+\frac{1}{2}\right)^2+(y-2)^2=\frac{1}{4}+4$ $\Rightarrow \left(x+\frac{1}{2}\right)^2+(y-2)^2=\frac{17}{4}$ $\Rightarrow \left(x+\frac{1}{2}\right)^2+(y-2)^2=\left(\frac{\sqrt{17}}{2}\right)^2$ Hence, radius of circle is $\frac{\sqrt{17}}{2}$.	
70.	$f(x)=4\log_e(x-1)-2x^2+4x+5, x>1$, which one of the following is not correct?	**C**
	f is increasing in (1, 2) and decreasing in (2, ∞)	
	f(x) = –1 has exactly two solutions	
	f'(e) – f''(2) < 0	
	f (x) = 0 has a root in the interval (e, e + 1)	

Sol.	*(c)* Given function is $f(x)=4\log_e(x-1)-2x^2+4x+5, x>1$ On differentiating w.r.t. 'x', we get $f'(x)=\frac{4}{x-1}-4(x-1)$ ...(i) Check by option, Now, take $1<x<2 \Rightarrow f'(x)>0$ Also, take $x>2$ $\Rightarrow \quad f'(x)<0$ So, option (a) is correct. Take $f(x)=-1$ We have, $\log_e(x-1)^2=(x-3)(x+1)$ Therefore, it has two solutions. Take $f(e)>0, f(e+1)<0$ $f(e)\cdot f(e+1)<0$ So, option (d) is correct. Now, $f''(x)=\frac{-4}{(x-1)^2}-4x$ $\Rightarrow \quad f''(2)=-8$ ...(ii) Now, put $x=e$ in Eqs. (i) and (ii), we get $f'(e)-f''(2)=\frac{4}{e-1}-4(e-1)+8>0$ Therefore, option (c) is incorrect.	
71.	Let the circle C touches the line x – y + 1 = 0 have the centre on the positive X–axis and cut off a chord of length $\frac{4}{\sqrt{13}}$ along the line –3x + 2y = 1. Let H be the hyperbola $\frac{x^2}{\alpha^2}-\frac{y^2}{\beta^2}=1$, whose one of the foci is the centre of C and the length of the transverse axis is the diameter of C. Then, $2\alpha^2 + 3\beta^2$ is equal to……	**19**

Sol.	*(19)* Given that $x - y + 1 = 0$ …(i) $\text{CM} = \left\lvert\frac{-3\alpha - 1}{\sqrt{13}}\right\rvert$  $r = \left\lvert\frac{\alpha + 1}{\sqrt{2}}\right\rvert$ $\Rightarrow \quad (\alpha + 1)^2 = 2r^2$ …(i) Now, in ΔCMD $CD^2 = DM^2 + CM^2$ $r^2 = \frac{(3\alpha + 1)^2}{13} + \frac{4}{13}$ $(3\alpha + 1)^2 + 4 = 13r^2$ …(ii) From Eqs. (i) and (ii), we get $(3\alpha + 1)^2 + 4 = 13\frac{(\alpha + 1)^2}{2}$ $\Rightarrow 18\alpha^2 + 12\alpha + 10 = 13\alpha^2 + 26\alpha + 13$ $\Rightarrow \; 5\alpha^2 - 14\alpha - 3 = 0$ $\Rightarrow 5\alpha^2 - 15\alpha + \alpha - 3 = 0$ $\alpha = \frac{-1}{5}, 3 \quad [\because \alpha > 0 \Rightarrow \alpha = 3]$ From Eq. (i), we get $r = 2\sqrt{2}$ Now, $\alpha e = 3, 2\alpha = 4\sqrt{2}$ $\Rightarrow \quad \alpha = 2\sqrt{2}$ $\alpha^2 e^2 = 9$ $\Rightarrow \; \alpha^2\left(1 + \frac{\beta^2}{\alpha^2}\right) = 9$ $\Rightarrow \alpha^2 + \beta^2 = 9 \Rightarrow \; \beta^2 = 1$ $\therefore 2\alpha^2 + 3\beta^2 = 2(8) + 3 = 19$	
72.	Let {x} denote the fractional part of x and $f(x) = \frac{\cos^{-1}(1-\{x\}^2)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3}, x \neq 0$. If L and R respectively denotes the left hand limit and the right hand limit of f(x) at x = 0, then $\frac{32}{\pi^2}(L^2 + R^2)$ is equal to…….	**18**

Sol.

(18) Finding right hand limit

$$\lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} f(h)$$

$$= \lim_{h\to 0} \frac{\cos^{-1}(1-h^2)\sin^{-1}(1-h)}{h(1-h^2)}$$

$$= \lim_{h\to 0} \frac{\cos^{-1}(1-h^2)}{h}\left(\frac{\sin^{-1}1}{1}\right)$$

Let $\cos^{-1}(1-h^2)=\theta$

$$\Rightarrow \quad \cos\theta = 1-h^2 = \frac{\pi}{2}\lim_{\theta\to 0}\frac{\theta}{\sqrt{1-\cos\theta}}$$

$$= \frac{\pi}{2}\lim_{\theta\to 0}\frac{1}{\sqrt{\frac{1-\cos\theta}{\theta^2}}} = \frac{\pi}{2}\frac{1}{\sqrt{\frac{1}{2}}}$$

$$\therefore \quad R = \frac{\pi}{\sqrt{2}}$$

$$\therefore \quad L = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(-h)$$

$$= \lim_{h\to 0} \frac{\cos^{-1}(1-\{-h\}^2)\sin^{-1}(1-\{-h\})}{\{-h\}-\{-h\}^3}$$

$$= \lim_{h\to 0} \frac{\cos^{-1}(1-(-h+1)^2)}{\sin^{-1}(1-(-h+1))}\frac{}{(-h+1)-(-h+1)^3}$$

$$= \lim_{h\to 0} \frac{\cos^{-1}(-h^2+2h)\sin^{-1}h}{(1-h)(1-(1-h)^2)}$$

$$= \lim_{h\to 0}\left(\frac{\pi}{2}\right)\frac{\sin^{-1}h}{(1-(1-h)^2)}$$

$$= \frac{\pi}{2}\lim_{h\to 0}\left(\frac{\sin^{-1}h}{-h^2+2h}\right)$$

$$= \frac{\pi}{2}\lim_{h\to 0}\left(\frac{\sin^{-1}h}{h}\right)\left(\frac{1}{-h+2}\right)$$

$$\therefore \quad L = \pi/4$$

$$\Rightarrow \frac{32}{\pi^2}(L^2+R^2) = \frac{32}{\pi^2}\left(\frac{\pi^2}{2}+\frac{\pi^2}{16}\right)$$

$$= 16+2 = 18$$

73.	Let $P = \begin{bmatrix} -30 & 20 & 56 \\ 90 & 140 & 112 \\ 120 & 60 & 14 \end{bmatrix}$ *and* $A = \begin{bmatrix} 2 & 7 & \omega^2 \\ -1 & \omega^2+1 & 1 \\ 0 & -\omega & -\omega^2+1 \end{bmatrix}$ where, $\omega = \frac{-1+i\sqrt{3}}{3}$ *and* I_3 be the identity matrix of order 3. If the determinant of the matrix $(P^{-1}AP - I_3)^2$ *is* $\alpha\omega^2$, then the value α is equal to…….	**36**

Sol.

(36) $P^{-1}AP - I_3 = P^{-I}AP - P^{-1}P$

$$= P^{-1}(A - I_3)P$$

$$\Rightarrow |P^{-1}AP - I_3| = |P^{-1}||A - I_3||P|$$

$$= |A - I_3|$$

$$\left[\because |P^{-1}| = \frac{1}{P}\right]$$

$$\because \quad |A - I_3| = \begin{vmatrix} 1 & 7 & \omega^2 \\ -1 & \omega^2 & 1 \\ 0 & -\omega & -\omega \end{vmatrix} = -6\omega$$

$$\Rightarrow |P^{-1}AP - I_3|^2 = (-6\omega)^2$$

$$= 36\omega^2$$

$$\therefore \quad \alpha = 36$$

74.	If α satisfies the equation $x^2+x+1=0$ and $(1+\alpha)^7 = A + B\alpha + C\alpha^2$,	**5**

	$A, B, C \geq 0$, then 5(3A – 2B – C) is equal to……	
Sol.	*(5)* Since, α satisfies the equation $x^2 + x + 1 = 0$. So, $\alpha = \omega, \omega^2$ [which are roots of $x^2 + x + 1 = 0$] Let $\alpha = \omega$. Then, $(1+\alpha)^7 = -\omega^{14}$ $= -\omega^2 = 1 + \omega$ $= A + B\alpha + C\alpha^2 = A + B\omega + C\omega^2$ So, $A = 1, B = 1$ and $C = 0$ $\therefore 5(3A - 2B - C) = 5(3 - 2 - 0) = 5$	
75.	The number of points, where the curve $y = x^5 - 20x^3 + 50x + 2$ crosses the X-axis, is……..	**5**
Sol.	*(5)* Given equation of curve $y = x^5 - 20x^3 + 50x + 2$ $\Rightarrow \frac{dy}{dx} = 5x^4 - 60x^2 + 50$ On putting $dy/dx = 0$ $\Rightarrow 5(x^4 - 12x^2 + 10) = 0$ $\Rightarrow x^2 = \frac{12 \pm \sqrt{144 - 40}}{2} = 6 \pm \sqrt{26}$ $\Rightarrow x^2 = 6 - \sqrt{26}, 6 + \sqrt{26}$ $\Rightarrow x^2 = 6 - 5.10, 6 + 5.10$ $\Rightarrow x^2 = 0.9, 11.1$ $\Rightarrow x = \pm\sqrt{0.9}, \pm\sqrt{11.1}$ $\Rightarrow x = -0.95, 0.95, -3.33, 3.33$ Now, $y(0) = 2(+\text{ve})$ $\Rightarrow y(1) = +\text{ve}$ $y(2) = -\text{ve} \Rightarrow y(3.3) = -\text{ve}$ $y(-1) = -\text{ve} \Rightarrow y(-2) = +\text{ve}$ $y(-3.3) = -\text{ve}$ So, the given curve cuts the X-axis, 5 times.	