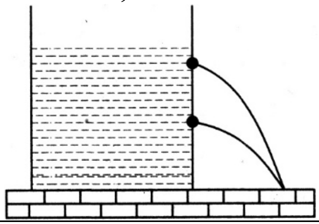
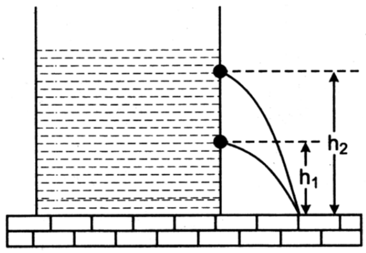
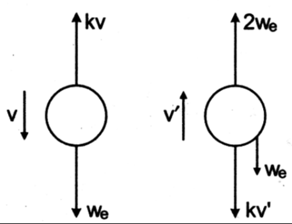
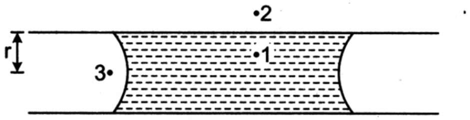
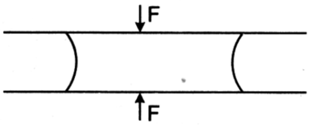
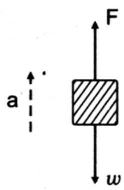
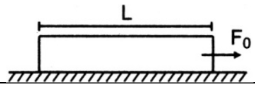
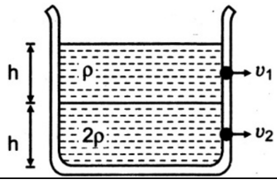
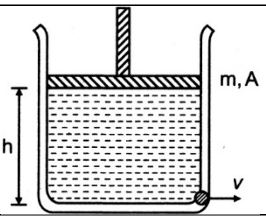
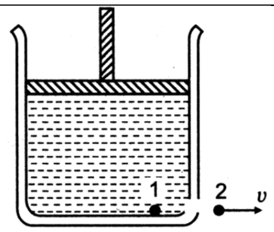
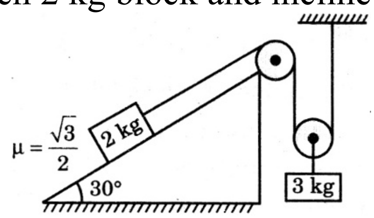
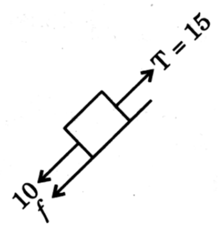
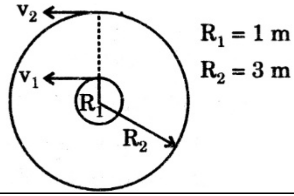
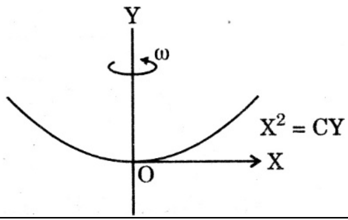
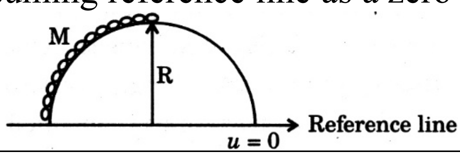
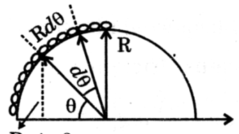
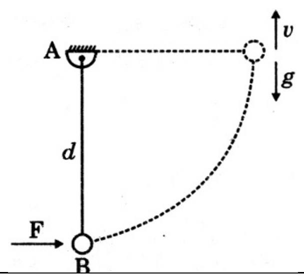
1.	In a cylindrical vessel containing liquid of density ρ, there are two holes in the side walls at heights of h_1 and h_2 respectively such that the range of efflux at the bottom of the vessel is same. The height of a hole for which the range of efflux would be maximum, will be 	**D**
	$h_2 - h_1$	
	$h_2 + h_1$	
	$\frac{h_2 - h_1}{2}$	
	$\frac{h_2 + h_1}{2}$	
Sol.	If H is the height of the liquid surface then for same range $h_2 = H - h_1$ and for maximum range $h = \frac{H}{2} = \frac{h_1 + h_2}{2}$ 	
2.	A small steel ball falls through a syrup at a constant speed of 1.0 m/s. If the steel ball is pulled upwards with a force equal to twice its effective weight, how fast will it move upward?	**A**
	1.0 m/s	
	2.0 m/s	
	0.5 m/s	
	Zero	
Sol.	w_e = effective weight $kv = w_e$ ...(1) In equilibrium $2w_e - w_e = kv'$ ...(2) From Eqs. (1) and (2) $v' = v = 1.0$ m/s 	
3.	A drop of water of mass m and density ρ is placed between two well cleaned glass plates, the distance between which is d. What is the force of attraction between the plates? (T = surface tension)	**C**
	$\frac{Tm}{2\rho d^2}$	
	$\frac{4Tm}{\rho d^2}$	

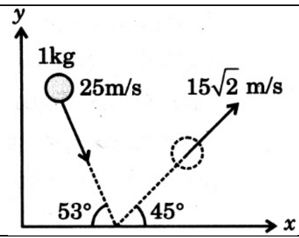
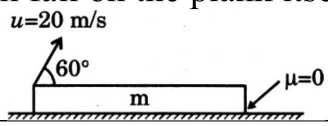
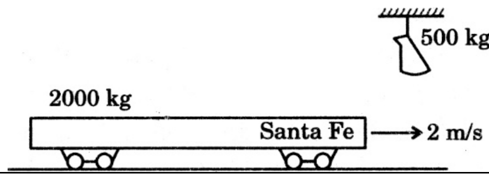
	$\frac{2Tm}{\rho d^2}$	
	$\frac{Tm}{\rho d^2}$	
Sol.	For cylindrical surface $\Delta p = \frac{T}{r}$ not $\frac{2T}{r}$  Here, $r = \frac{d}{2}$  $\therefore \quad \Delta p = \frac{2T}{d}$ $\therefore \quad p_2 - p_1 = \frac{2T}{d} = \Delta P$ $\therefore \quad F = (\Delta p)S = (\Delta p)\left(\frac{V}{d}\right) = \frac{(\Delta p)m}{\rho d}$ $\therefore \quad F = \frac{2Tm}{\rho d^2}$	
4.	A small block of wood of specific gravity 0.5 is submerged at a depth of 1.2 m in a vessel filled with water. The vessel is accelerated upwards with an acceleration $a_0 = \frac{g}{2}$. Time taken by the block to reach the surface, if it is released with zero initial velocity is (g = 10 m/s^2)	**B**
	0.6 s	
	0.4 s	
	1.2 s	
	1 s	
Sol.	Let m be the mass of block and a its acceleration in upward direction. Then $a = \frac{\text{upthrust} - \text{weight}}{m} = \frac{F - w}{m}$ $= \frac{\left(\frac{m}{0.5 \times 10^3}\right)(10^3)(g + a_0) - mg}{m}$ $= 2\left(g + \frac{g}{2}\right) - g = 2g$  Acceleration of block relative to water $a_r = a - a_0 = 2g - \frac{g}{2} = \frac{3g}{2}$ $\therefore \quad h = \frac{1}{2} a_r t^2$ or $\quad t = \sqrt{\frac{2h}{a_r}} = \sqrt{\frac{2h}{\frac{3}{2}g}}$ $= 2\sqrt{\frac{h}{3g}} = 2\sqrt{\frac{12}{3 \times 10}} = 0.4\,\text{s}$	
5.	A constant force F_0 is applied on a uniform elastic string placed over a smooth horizontal surface as shown in figure. Young's modulus of string is Y and area of cross-section is S. The strain produced in the string in the direction of force is  	**C**

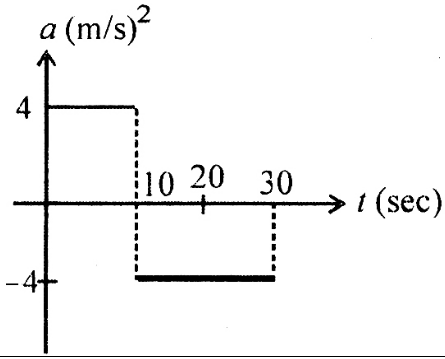
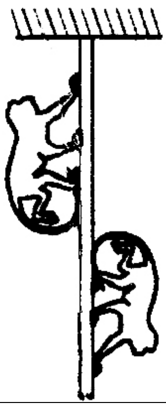
	$\frac{F_0 Y}{S}$	
	$\frac{F_0}{SY}$	
	$\frac{F_0}{2SY}$	
	$\frac{F_0 Y}{2S}$	
Sol.	Let F be the force at a distance x from the rear end. Then, $\frac{F}{x} = \frac{F_0}{L}$, $\therefore\ F = \frac{F_0 x}{L}$ $\therefore$ Increase in $dx = \frac{F_0}{LSY} x\,dx$ $\therefore$ Total increase $\Delta L = \frac{F_0}{LSY}\int_0^L x\,dx = \frac{F_0 L}{2SY}$ $\therefore$ Strain $= \frac{\Delta L}{L}$ $= \frac{F_0}{2SY}$	
6.	Equal volumes of two immiscible liquids of densities ρ and 2ρ are filled in a vessel as shown in figure. Two small holes are punched at depths h/2 and 3h/2 from the surface of lighter liquid. If v_1 and v_2 are the velocities of efflux at these two holes, then v1/v2 is 	**D**
	$\frac{1}{2\sqrt{2}}$	
	$\frac{1}{2}$	
	$\frac{1}{4}$	
	$\frac{1}{\sqrt{2}}$	
Sol.	$v_1 = \sqrt{2g\left(\frac{h}{2}\right)} = \sqrt{gh}$ ...(1) From Bernoulli's theorem $\rho gh + 2\rho g\left(\frac{h}{2}\right) = \frac{1}{2}(2\rho)v_2^2$ $\therefore\ v_2 = \sqrt{2gh}$ ...(2) $\therefore\ \frac{v_1}{v_2} = \frac{1}{\sqrt{2}}$	
7.	A cylindrical vessel contains a liquid of density ρ upto a height h. The liquid is closed by a piston of mass m and area of cross-section A. There is a small hole at the bottom of the vessel. The speed v with which the liquid comes out of the hole is	**B**

		
	$\sqrt{2gh}$	
	$\sqrt{2\left(gh+\frac{mg}{\rho A}\right)}$	
	$\sqrt{2\left(gh+\frac{mg}{A}\right)}$	
	$\sqrt{2gh+\frac{mg}{A}}$	
Sol.	Applying Bernoulli's equation at 1 and 2 : $\rho gh+\frac{mg}{A}=\frac{1}{2}\rho v^2$ or $v=\sqrt{2gh+\frac{2mg}{\rho A}}$ $=\sqrt{2\left(gh+\frac{mg}{\rho A}\right)}$ 	
8.	If the system in the adjacent figure is released from rest, then : (P) acceleration of 2 kg block = acceleration of 3 kg block = 0. (Q) frictional force between 2 kg block and incline = 5 N (R) acceleration of 2 kg block = 0.5 m/s^2, acceleration of 3 kg block = 0.25 m/s^2 (S) frictional force between 2 kg block and incline = 10 N.  Now, choose the correct option.	**A**
	(P) and (Q) are correct.	
	(Q) and (R) are correct.	
	(R) and (S) are correct.	
	(R) and (S) are correct.	
Sol.	$f=\mu_s\, mg\cos 30°=15$ N $mg\sin\theta=10$ N At rest, tension = 15 N So, system will not move.  Acceleration of 2 kg block = acceleration of 3 kg block = 0. $\Rightarrow$ 15 = 10 + f So, frictional force = 5 N.	
9.	Two particles start moving on circles of radius shown with velocity V_1 = 2 m/s, V_2 = 15 m/s. The minimum time after which the particles will again be	**D**

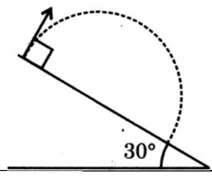
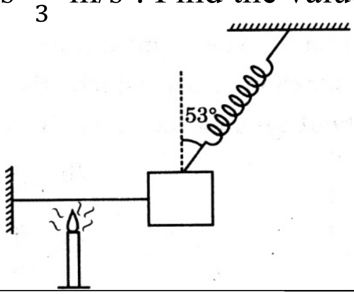
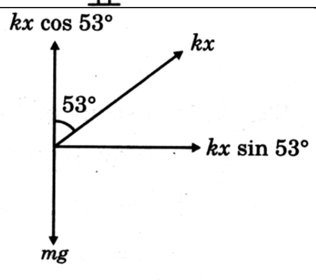
	collinear with centre is equal to: 	
	$\frac{2\pi}{3}$ sec	
	$\frac{2\pi}{5}$ sec	
	2π sec	
	$\frac{\pi}{3}$ sec	
Sol.	$\omega_1 = \frac{v_1}{R_1} = 2$ $\omega_2 = \frac{v_2}{R_2} = \frac{15}{3} = 5$ $\theta_2 = \theta_1 + \pi$ $\omega_2 t = \omega_1 t + \pi$	
10.	Vertical section of a bowl shown in figure can rotate about axis OY. The section is parabolic in shape and represented by the equationX^2= CY, where C is a positive constant. Then find angular velocity of the bowl for which a sphere of small size placed on its smooth inner surface does not slide : 	**B**
	$\omega = \sqrt{\frac{C}{2g}}$	
	$\omega = \sqrt{\frac{2g}{C}}$	
	$\omega = \frac{2g}{C}$	
	$\omega = \frac{C}{2g}$	
Ans.	$\omega = \sqrt{\frac{2g}{C}}$	
11.	A chain of mass M is kept on a hemisphere as shown. Find out potential energy of the chain assuming reference line as a zero potential energy level. 	**A**
	$Mg\frac{2R}{\pi}$	

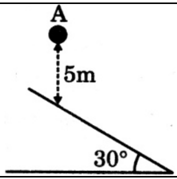
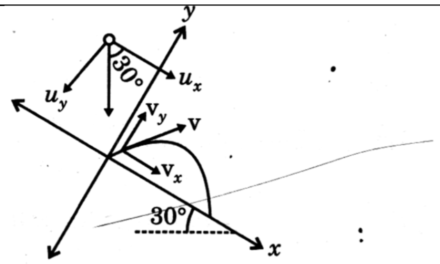
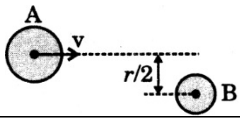
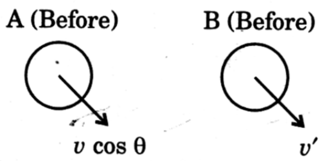
	$Mg\frac{R}{\pi}$	
	$Mg\frac{R}{2\pi}$	
	$Mg\frac{3R}{2\pi}$	
Sol.	 Consider an elemental part of chain as shown. Mass of element; $dm = \frac{M}{\frac{\pi R}{2}} Rd\theta = \frac{2M}{\pi} d\theta$ Gravitational potential energy of this element is $du = dmgh = \left(\frac{2M}{\pi} d\theta\right) g \times R\sin\theta$ $u = \int_0^u dU = \frac{2M}{\pi} Rg \int_0^{\pi/2} \sin\theta \, d\theta$ $u = \frac{2Mg}{\pi} R(-\cos\theta)\Big\|_0^{\pi/2} = Mg \times \frac{2R}{\pi}$ **It is clear from above expression that we can find the gravitational PE of extended object by placing their equivalent mass at respective centre of gravity.**	
12.	A small sphere 'B' of mass M is connected at one end of a light rigid rod whose other end is hinged So that the sphere hangs vertically. At some instant of time a strong wind begins to apply a constant horizontal force to sphere 'B'. As a result, the sphere rotates about A in a vertical plane. The speed of sphere B at the instant when the rod becomes horizontal :  	**C**
	$\sqrt{\left(\frac{F}{M} - g\right)\pi d}$	
	$\sqrt{\frac{Fd - 2Mgd}{M}}$	
	$\sqrt{\frac{2Fd - 2Mgd}{M}}$	
	$\sqrt{\frac{Fd\pi - Mgd}{M}}$	
Sol.	$W_g + W_f = \Delta KE$ $v = \sqrt{\frac{2Fd - 2Mgd}{M}}$	
13.	A ball of mass 1 kg bounces against the ground as shown in the figure. The approaching velocity is 25 m/s and the velocity after hitting the ground is $15\sqrt{2}$ m/s, the impulse exerted on the ball is :	**C**

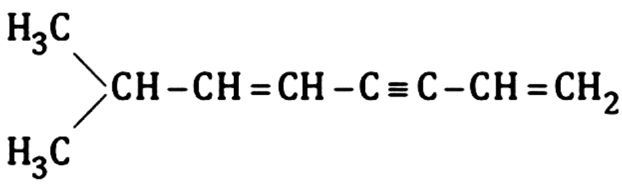
		
	5 Ns	
	$5(5 - 3\sqrt{2})$ Ns	
	35 Ns	
	None of these	
Sol.	**The momentum will be same along the ground before and after collision** **V_1 before collision perpendicular to ground** $= -25 \times \sin 53° = -20$ m/s **V_2 after collision perpendicular to ground** $= 15\sqrt{2} \times \sin 45° = 15$ m/s $\Delta P = mV_2 - mV_1$ $= 1 \times 15 - 1 \times (-20)$ $= 35$ kg m/s **Impulse exerted by ground = ΔP = 35 N-sec**	
14.	A particle of mass m is projected at an angle of 60° with a velocity of 20 m/s relative to the ground from a plank of same mass m which is placed on smooth surface. Initially plank was at rest. The minimum length of the plank for which the ball will fall on the plank itself is 	**A**
	$40\sqrt{3}$ m	
	$20\sqrt{3}$ m	
	$10\sqrt{3}$ m	
	$60\sqrt{3}$ m	
Sol.	**Applying conservation of momentum along X** $0 = m \times 20 \cos 60 + mv_p$ $\therefore \quad v_p = -10$ m/s $v_{rel} = 20$ m/s ($\rightarrow$) **Time of flight (T)** $= \dfrac{2u\sin\theta}{g} = \dfrac{2 \times 20 \times \sqrt{3}}{2 \times 10} = 2\sqrt{3}$ sec **Minimum length of plank** $= v_{rel} \times T$ $= 20 \times 2\sqrt{3} = 40\sqrt{3}$ m	
15.	A 500 kg sack of coal is dropped on a 2000 kg railroad flatcar which was initially moving at 3 m/s as shown. After the sack rests on the flatcar, the speed of the flatcar is : 	**D**
	0.6 m/s	
	1.2 m/s	

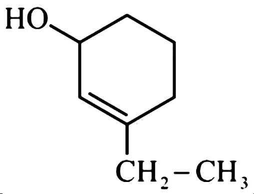
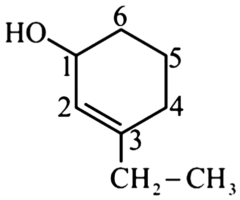
	1.8 m/s	
	2.4 m/s	
Ans.	2.4 m/s	
16.	The acceleration versus time graph for a particle moving along a straight line is shown in the figure. If the particle starts from rest at t = 0, then its velocity at t = 30 sec will be  	**C**
	20 m/sec	
	0 m/sec	
	–40 m/sec	
	40 m/sec	
Sol.	**For a–t curve, area under the graph gives change in velocity at $t = 10$ sec, $v = 40$ m/s** **For 10 – 30 sec, $\Delta v = -80$, $v_{30sec} - 40 = -80$** **Speed at 30 sec = – 40 m/s**	
17.	Two monkeys of masses 10 kg and 8 kg are moving along a vertical rope which is light and inextensible, the former climbing up with an acceleration of 2 m/s^2 while the latter coming down with a uniform velocity of 2 m/s. Find the tension (in newtons). 	**A**
	200 N	
	150 N	
	300 N	
	100 N	
Sol.	**(a)** $T = T_1 + T_2 = m_1(g + a) + m_2 g$ $= 10(10 + 2) + 8(10) = 120 + 80 = 200N$	
18.	The velocity of the bob on a pendulum of length 10 m is given by $v = v_0 \cos\frac{2\pi t}{T}$ where v_0 = 1.00 m/s and $T = 2\pi s$. The radial acceleration at $t = \frac{\pi}{4} s$ is	**C**
	1 m/s^2	
	1/10 m/s^2	

	1/20 m/s^2	
	1/30 m/s^2	
Sol.	**(c)** $v = v_0 \cos \frac{2\pi t}{T} = 1 \cos \frac{2\pi \times \frac{\pi}{4}}{2\pi} = \frac{1}{\sqrt{2}}$	
19.	A cylinder of mass m and density ρ hanging from a string is lowered into a vessel of cross-sectional area s containing a liquid of density σ(<ρ) unit it is fully immersed. The increase in pressure at the bottom of the vessel is $\frac{Nm\sigma g}{2\rho s}$. Find the value of N.	**C**
	1	
	3	
	2	
	4	
Sol.	Upthrust in cylinder $= \left(\frac{m}{\rho}\right)\sigma g$ From Newton's third law, force exerted by the cylinder on the liquid is also $\left(\frac{m}{\rho}\right)\sigma g$ ∴ Increase in pressure $= \frac{m\sigma g}{\rho s}$	
20.	A helicopter is flying horizontally at 8 m/s at an altitude 180 m when a package of emergency medical supplies is ejected horizontally backward with a speed of 12 m/s relative to the helicopter. Ignoring air resistance what is horizontal distance (in m) between the package and the helicopter when the package hits the ground?	**D**
	68 m	
	48 m	
	24 m	
	72 m	
Sol.	$V_{ph} = V_{pG} - V_{hG}$ $-12\hat{i} = \vec{V}_{pG} - 8\hat{i}$ $\vec{V}_{pG} = -12\hat{i} + 8\hat{i} = -4\hat{i}$ **Time taken by package to hit ground** $180 = \frac{1}{2} \times 10t^2$ $\Rightarrow \quad t = 6$ sec **Distance between package and helicopter** $= \lvert\vec{v}_{pG}\rvert \times t + \lvert V_{hG}\rvert t$ $= 4 \times 6 + 8 \times 6$ $= 72$ m	
21.	Two particles, one with constant velocity 50 m/s and the other with uniform acceleration 10 m/s^2, start moving simultaneously from the same place in the same direction they will be at a distance of 125 m from each other after time (in s):	**5**

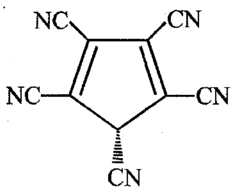
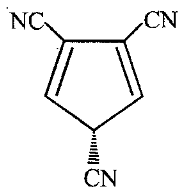
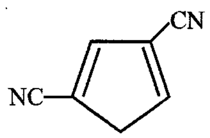
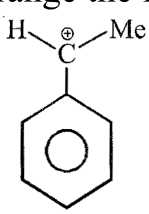
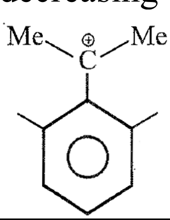
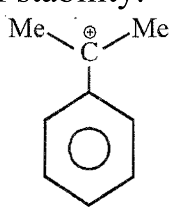
Sol.	$X_A = 50t,$ $X_B = \frac{1}{2} \times 10 \times t^2 = 5t^2$ $X_A - X_B = 50t - 5t^2 = 125,$ $5t^2 - 50t + 125 = 0$ at $t = 5$ sec $X_B - X_A = 5t^2 - 5t = 125,$ $-5t^2 + 50t + 125 = 0$ and $t = 5(\sqrt{2}+1)$ $5t^2 - 5t + 125 = 0$	
22.	A ball is projected from point A with velocity 10 m/sec perpendicular to the inclined plane as in figure. Range of the ball on the incline is $\frac{n}{3}$ m. Find the value of n.  	**40**
Sol.	$u_x = 0, u_y = u$ $a_x = g \sin 30° = \frac{g}{2}$ and $a_y = -g \cos 30° = \frac{-(g\sqrt{3})}{2}$ $0 = u_y T + \frac{1}{2} a_y T^2$ $T = \frac{4u}{\frac{g}{\sqrt{3}}}$ $\therefore$ $X = u_x T + \frac{1}{2} a_x T^2$ $X = R = \frac{1}{2} g \sin\theta \left(\frac{4u}{g\sqrt{3}}\right)^2 = \frac{40}{3}$ m	
23.	The block shown in the figure is equilibrium. The acceleration of the block just after the string burns is $\frac{ng}{3}$ m/s^2. Find the value of n.  	**4**
Sol.	Just after the string burns ; tension is n the string remains constant. $kx \cos 53° = mg$ $kx \sin 53° = ma$ $\therefore a = g \tan 53° = \frac{4g}{3}$  	
24.	A ball is released from position A and travels 5 m before striking the smooth fixed inclined plane as shown. If the coefficient of restitution in the impact is $e = \frac{1}{2}$, the time (in s) taken by the ball to strike the plane again is :	**1**

		
Sol.	 Just before first collision, Speed $u = \sqrt{2g(5)} = 10$ m/s $u_y = -10 \cos 30° = -5\sqrt{3}$ $u_x = 10 \sin 30° = 5$ m/s $v_x = u_x = 5$ m/s $v_y = -eu_y = \frac{5}{2}\sqrt{3}$ It strikes the plane again at time, $t = \frac{2v_y}{g \cos 30°} = 1$ sec	
25.	A disk A of radius r moving on perfectly smooth surface at a speed u undergoes an elastic collision with an identical stationary disk B. The velocity of thedisk B after collision if the impact parameter is $\frac{r}{2}$ as shown in the figureis $\frac{v\sqrt{n}}{4}$. Find the value of n. 	**15**
Sol.	In perfectly elastic collision velocities along common normal will interchange in case of identical objects. So, take velocity component along line of impact.  $v' = v \cos \theta$ $\cos \theta = \frac{\sqrt{15}}{9}$	
26.	Compound that is both paramagnetic and coloured is	**C**
	$K_2Cr_2O_7$	
	$(NH_4)_2[TiCl_6]$	
	$VOSO_4$	
	$K_3[Cu(CN)_4]$	

Sol.	$K_2Cr_2O_7$ contains $Cr^{+6}(3d^0)$ which is diamagnetic but coloured due to charge transfer spectra. $(NH_4)_2[TiCl_6]$ contains $Ti^{4+}(3d^0)$, which is diamagnetic and colourless. $VOSO_4$ contains $V^{4+}(3d^1)$, which is paramagnetic and coloured. $K_3[Cu(CN)_4]$ contains $Cu^+(3d^{10})$, which is diamagnetic and colourless.	
27.	Consider the methyl substituted benzoic acids. 1. PhCOOH 2. o – $CH_3C_6H_4COOH$ 3. p – $CH_3C_6H_4COOH$ 4. m – $CH_3C_6H_4COOH$ The correct sequence of acidity is	**C**
	$1 < 2 < 3 < 4$	
	$2 < 3 < 4 < 1$	
	$3 < 4 < 1 < 2$	
	$3 < 4 < 2 < 1$	
Sol.	Both p- and m-methyl groups are electron-donating and acid weakening. Hyperconjugation of $p-CH_3$ produces a little extra δ^- on the ring carbon, bearing the COO^-. This results in more effective electron-donation and acid-weakening. This makes $p-CH_3C_6H_4COOH$ less acidic than $m-CH_3C_6H_4COOH$. $o-CH_3C_6H_4COOH$ is a stronger acid than benzoic acid, due to ortho-effect. $p-CH_3C_6H_4COOH <$ $m-CH_3C_6H_4COOH <$ PhCOOH $<$ $o-CH_3C_6H_4COOH$	
28.	Consider the given statement about the molecule  **1.** Three carbon atoms are sp^3 – Hybridised **2.** Three carbon atoms are sp^2 – Hybridised **3.** Two carbon atoms are sp-hybridised of the statement	**D**
	1, 2 and 3 are correct	
	1 and 2 are correct	
	2 and 3 are correct	
	1 and 3 are correct	

<table>
<tr><td>29.</td><td>Which one of the following complexes shows optical isomerism
(en = ethylenediamine)</td><td>C</td></tr>
<tr><td></td><td>$[Co(NH_3)_4Cl_2]Cl$</td><td></td></tr>
<tr><td></td><td>$[Co(NH_3)_3Cl_3]$</td><td></td></tr>
<tr><td></td><td>Cis $[Co(en)_2Cl_2]Cl$</td><td></td></tr>
<tr><td></td><td>Trans $[Co(en)_2Cl_2]Cl$</td><td></td></tr>
<tr><td>Sol.</td><td>Complex $[Co(NH_3)_4Cl_2]Cl$ have two G.I. which are optically inactive due to presence of plane of symmetry.
Complex $[Co(NH_3)_3\ Cl_3]$ also have two optically inactive geometrical isomers due to presence of plane of symmetry.
Complex cis$[Co(en)_2Cl_2]Cl$ is optically active due to formation of non-superimposable mirror image trans$[Co(en)_2Cl_2]Cl$
Complex trans $[Co(en)_2Cl_2]Cl$ is optically inactive.</td><td></td></tr>
<tr><td>30.</td><td>IUPAC name of the following compounds is :
</td><td>C</td></tr>
<tr><td></td><td>1-Ethylcyclohex-1en-3-ol</td><td></td></tr>
<tr><td></td><td>1-Ethyl-3-hydroxy cyclohex-1-ene</td><td></td></tr>
<tr><td></td><td>3-Ethylcyclohex-2-en-1-ol</td><td></td></tr>
<tr><td></td><td>3-Ethyl-6-hydroxy cyclohexene</td><td></td></tr>
<tr><td>Sol.</td><td></td><td></td></tr>
<tr><td>31.</td><td>Match List –I with List – II.
<table>
<tr><th>List –I</th><th>List – II</th></tr>
<tr><td>A. $[Cr(NH_3)_4Cl_2]Cl$</td><td>I. Paramagnetic exhibit ionisation isomerism</td></tr>
<tr><td>B. $[Ti(H_2O)_5\ Cl](NO_3)_2$</td><td>II. Diamagnetic and exhibits cis–trans isomerism</td></tr>
<tr><td>C. $[Pt(en)(NH_3)Cl]NO_3$</td><td>III. Paramagnetic and exhibits cis– trans isomerism</td></tr>
<tr><td>D. $[CO(NH_3)_4(NO_3)_2]NO_3$</td><td>IV. Diamagnetic and exhibits ionisation isomerism</td></tr>
</table>
Choose the correct answer from the options given below:</td><td>B</td></tr>
<tr><td></td><td>A – IV, B – II, C – III, D – I</td><td></td></tr>
<tr><td></td><td>A – III, B – I, C – IV, D – II</td><td></td></tr>
<tr><td></td><td>A – II, B – I, C – III, D – IV</td><td></td></tr>
<tr><td></td><td>A – I, B – III, C – IV, D – II</td><td></td></tr>
</table>

Sol.	The correct match is A-III, B-I, C-IV, D-II. A. Cr^{3+} has $3d^3$ configuration, with 3 unpaired electrons. Hence, it shows paramagnetic behaviour. Complex of the type Ma_4b_2 shows *cis-trans* isomerism B. Ti^{3+} has $3d^1$ configuration, hence shows paramagnetic behaviour. Complex gives Cl^- and NO_3^- ions in solution hence, shows ionisation isomerism. C. Pt^{2+} has $5d^8$ configuration but ligands are strong field ligands hence, it forms square planar complex. thus, all electrons are paired and it also exhibits ionisation isomerism. D. Co^{3+} has $3d^6$ configuration. But, ligands present are strong enough to cause electron pairing, hence, it shows diamagnetic behaviour and exhibits *cis-trans* isomerism as it is Ma_4b_2 type complex.	
32.	Given below are two statements, one is labelled as Assertion(A) and the other is labelled as Reason(R). **Assertion (A)** $[Ti(H_2O)_6]^{3+}$ is coloured while $[Sc(H_2O)_6]^{3+}$ is colourless. **Reason (R)** d – d transition is not possible in $[Sc(H_2O)_6]^{3+}$. In the light of the above statements, choose the correct answer from the options given below.	**A**
	Both (A) and (R) are true and (R) is the correct explanation of (A).	
	Both (A) and (R) are true but (R) is not the correct explanation of (A).	
	(A) is true but (R) is false.	
	(A) is false but (R) is true.	
Sol.	***(a)*** Both (A) and (R) are true and (R) is the correct explanation of (A). $[Sc(H_2O)_6]^{3+}$ has no unpaired electron in its *d*-subshell and thus *d*-*d* transition is not possible. $[Ti(H_2O)_6]^{3+}$ has one unpaired electron in its *d*-subshell which gives rise to *d-d* transition.	
33.	The incorrect statement is	**A**
	The gemstone, ruby, has Cr^{3+} ions occupying the tetrahedral sites of beryl.	
	The spin-only magnetic moment of $[Ni(NH_3)_4(H_2O)_2]^{2+}$ is 2.83 B.M.	
	The colour of $[CoCl(NH_3)_5]^{2+}$ is violet as it absorbs the yellow light.	
	The spin-only magnetic moments of $[Fe(H_2O)_6]^{2+}$ and $[Cr(H_2O)_6]^{2+}$ are nearly similar.	
Sol.	The gemstone, ruby, has Cr^{3+} ions occupying the octahedral sites of aluminium oxide (Al_2O_3) while beryl is beryllium aluminium cyclosilicate ($Be_3Al_2Si_6O_{18}$) which is a variety of emerald and aquamarine.	
34.	Consider the following statements. **A.** Alcohols are weaker acids than water. **B.** Acid strength of alcohols decreases in the following. $RCH_2OH > R_2CHOH > R_3COH$ **C.** Carbon-oxygen bond length in methanol, CH_3OH is shorter than that of C – O bond length in phenol.	**A**

	O / \\ **D.** The bond angle C H in methanol is 108.9°. Choose the correct answer from the given options below.	
	A, B and D	
	A and B	
	Only C	
	B and D	
Sol.	Among the given statements A, B and D are correct while C is incorrect. The corrected form is given as : Carbon-oxygen bond length in methanol CH_3OH is longer than that of C–O bond length in phenol because of the resonance occurs in phenol.	
35.	If excess of $AgNO_3$ solution is added to 100 mL of a 0.024 M solution of dichlorobis (Ethylene diamine) cobalt (III) chloride, how many mol of AgCl be precipitated?	**C**
	0.0012	
	0.0016	
	0.0024	
	0.0048	
Sol.	$[CoCl_2(en)_2]Cl + AgNO_3 \longrightarrow AgCl\downarrow$ $100 \times 0.024 = 2.4$ $= 2.4$ mol	
36.	The decreasing order of K_a value is (I)  (II)  (III) 	**A**
	I > II > III	
	I > III > II	
	II > I > III	
	III > I > II	
37.	Arrange the following in decreasing order of stability. (I)  (II)  (III) 	**D**
	I > II > III	
	III > II > I	
	II > I > III	
	III > I > II	
38.	A solution is made by mixing one mole of volatile liquid A with 3 moles of volatile liquid B. The vapour pressure of pure A is 200 mm Hg and that of the solution is 500 mm Hg. The vapour pressure of pure B and the least volatile component of the solution, respectively, are :	**D**

	1400 mm Hg, A	
	1400 mm Hg, B	
	600 mm Hg, B	
	600 mm Hg, A	
Sol.	$P_S = P_A^o . X_A + P_B^o . X_B$ $500 = 200 \times \frac{1}{4} + P_B^o . \frac{3}{4}$ $P_B^o = 600$ mm Hg As $P_A^o < P_B^o \Rightarrow$ A is least volatile.	
39.	The correct order of basic nature on aqueous solution for the bases NH_3, $H_2N - NH_2$, $CH_3CH_2NH_2$, $(CH_3CH_2)_2NH$ and $(CH_3CH_2)_3N$ is :	**D**
	$NH_3 < H_2N - NH_2 < (CH_3CH_2)_3N < CH_3CH_2NH_2 < (CH_3CH_2)_2NH$	
	$NH_3 < H_2N - NH_2 < CH_3CH_2NH_2 < (CH_3CH_2)_2NH < (CH_3CH_2)_3N$	
	$H_2N - NH_2 < NH_3 < (CH_3CH_2)_3N < CH_3CH_2NH_2 < (CH_3CH_2)_2NH$	
	$NH_2 - NH_2 < NH_3 < CH_3CH_2NH_2 < (CH_3CH_2)_3N < (CH_3CH_2)_2NH$	
Sol.	Basic strength of amine depends on hydrogen bonding and electronic inductive effect. $NH(Et)_2 > N(Et)_3 > NH_2Et > \ddot{N}H_3 > NH_2 - NH_2$	
40.	For per gram of reactant, the maximum quantity of N_2 gas is produced in which of the following thermal decomposition reactions? • (**Given :** Atomic wt. Cr = 52 u, Ba = 137 u)	**D**
	• $2NH_4NO_3(s) \rightarrow 2N_2(g) + 4H_2O(g) + O_2(g)$	
	• $Ba(N_3)_2(s) \rightarrow Ba(s) + 3N_2(g)$	
	• $(NH_4)_2Cr_2O_7(s) \rightarrow N_2(g) + 4H_2O(g) + Cr_2O_3(s)$	
	• $2NH_3(g) \rightarrow N_2(g) + 3H_2(g)$	
Sol.	(a) $2NH_4NO_{3(s)} \longrightarrow 2N_{2(g)} + 4H_2O_{(g)} + O_{2(g)}$ Mol. mass = 80 g mol^{-1} Mol. mass = 28 g mol^{-1} 80 g of NH_4NO_3 gives 28 g of N_2 $\therefore$ 1 g of NH_4NO_3 will give $= \frac{28}{80} \times 1 = 0.35$ g (b) $Ba(N_3)_{2(g)} \longrightarrow Ba_{(s)} + 3N_{2(g)}$ Mol. mass = 221 g mol^{-1} 221 g of $Ba(N_3)_2$ gives 3×28 g of N_2 1 g of $Ba(N_3)_2$ will give $= \frac{3 \times 28}{221} \times 1 = 0.38$ g	

	(c) $(NH_4)_2Cr_2O_{7(s)} \longrightarrow N_{2(g)} + 4H_2O_{(g)} + Cr_2O_{3(s)}$ Mol. mass $= 252\ g\ mol^{-1}$ 252 g of $(NH_4)_2Cr_2O_7$ gives 28 g of N_2 1 g of $(NH_4)_2Cr_2O_7$ will give $= \frac{28}{252} \times 1 = 0.111$ g (d) $2NH_{3(g)} \longrightarrow N_{2(g)} + 3H_{2(g)}$ Mol. mass $= 17\ g\ mol^{-1}$ 2×17 g of NH_3 gives 28 g of N_2	
41.	1 g of a non-volatile non-electrolyte solute is dissolved in 100 g of two different solvents A and B whose ebullioscopy constants are in the ratio of 1 : 5. The ratio of the elevation in their boiling points, $\frac{\Delta T_b(A)}{\Delta T_b(B)}$, is	**D**
	5 : 1	
	1 : 0.2	
	10 : 1	
	1 : 5	
Sol.	**(d)** : $\Delta T_b = K_b \times m$ $\therefore \frac{\Delta T_{b(A)}}{\Delta T_{b(B)}} = \frac{K_{b(A)}}{K_{b(B)}} = \frac{1}{5}$ (as $m_A = m_B$)	
42.	A molecule M associates in a given solvent according to the equation $M \rightleftharpoons (M)_n$. For a certain concentration of M, the van't Hoff factor was found to be 0.9 and the fraction of associated molecules was 0.2. The value of n is:	**C**
	3	
	5	
	2	
	4	
Sol.	van't Hoff factor, $i = 0.9$ Fraction of association $(\alpha) = 0.2$ For association, $\alpha = \frac{i-1}{\frac{1}{n}-1}$ $0.2 = \frac{0.9-1}{\frac{1}{n}-1}$ $\frac{0.2}{n} - 0.2 = -0.1$ $\frac{0.2}{n} = 0.1 \Rightarrow n = 2$	
43.	Match List I with List II	**D**

	List I / List II table below. Choose the correct answer from the options given below :	

List I		List II	
A.	van't Hoff factor, i	I.	Cryoscopic constant
B.	k_f	II.	Isotonic solutions
C.	Solutions with same osmotic pressure	III.	$\frac{\text{Normal molar mass}}{\text{Abnormal molar mass}}$
D.	Azeotropes	IV.	Solutions with same composition of vapour above it

- Choose the correct answer from the options given below :

A-III, B-II, C-I, D-IV

A-III, B-I, C-IV, D-II

A-I, B-III, C-II, D-IV

A-III, B-I, C-II, D-IV

Sol.

$$\text{van't Hoff factor } (i) = \frac{\text{Normal molar mass}}{\text{Abnormal molar mass}}$$

(A-III)

Cryoscopic constant : K_f(B-I)

Same osmotic pressure = Isotonic solution (C-II)

Azeotropes – Solutions with same composition of vapours above it. (D – IV)

44. Consider separate solutions of 0.500 M $C_2H_5OH(aq)$, 0.100 M $Mg_3(PO_4)_2(aq)$, 0.250 M KBr(aq) and 0.125 M $Na_3PO_4(aq)$ at 25ºC. Which statement is true about these solutions, assuming all salts to be strong electrolytes? **B**

0.500 M $C_2H_5OH(aq)$ has the highest osmotic pressure.

They all have the same osmotic pressure.

0.100 M has the highest osmotic pressure.

0.125 M $Na_3PO_4(aq)$ has the highest osmotic pressure.

Sol. Applying the equation, $\pi = iCRT$

Solution	i	C	$i \times C$
$C_2H_5OH_{(aq)}$	1	0.5	0.5
$Mg_3(PO_4)_{2\,(aq)}$	5	0.1	0.5
$KBr_{(aq)}$	2	0.25	0.5
$Na_3PO_{4(aq)}$	4	0.125	0.5

The value of $i \times C$ indicates that all the solutions have same osmotic pressure.

45. Chlorine undergoes disproportionation in alkaline medium as shown below : **C**

$$aCl_2(g) + bOH^-(aq) \rightarrow c\,ClO^-(aq) + dCl^-(aq) + eH_2O(l)$$

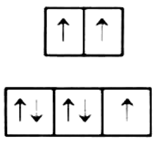
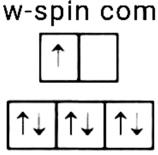
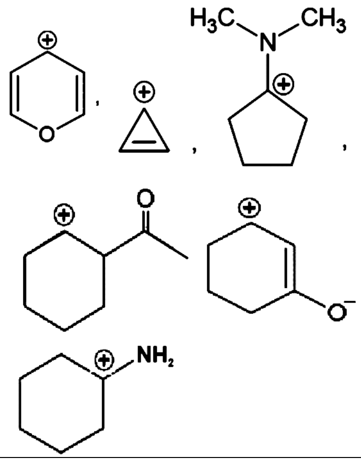
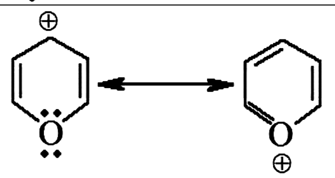
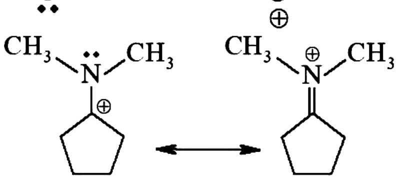
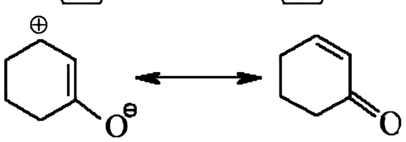
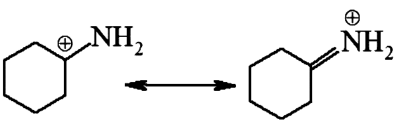
The values of a, b, c and d in a balanced redox reaction are respectively

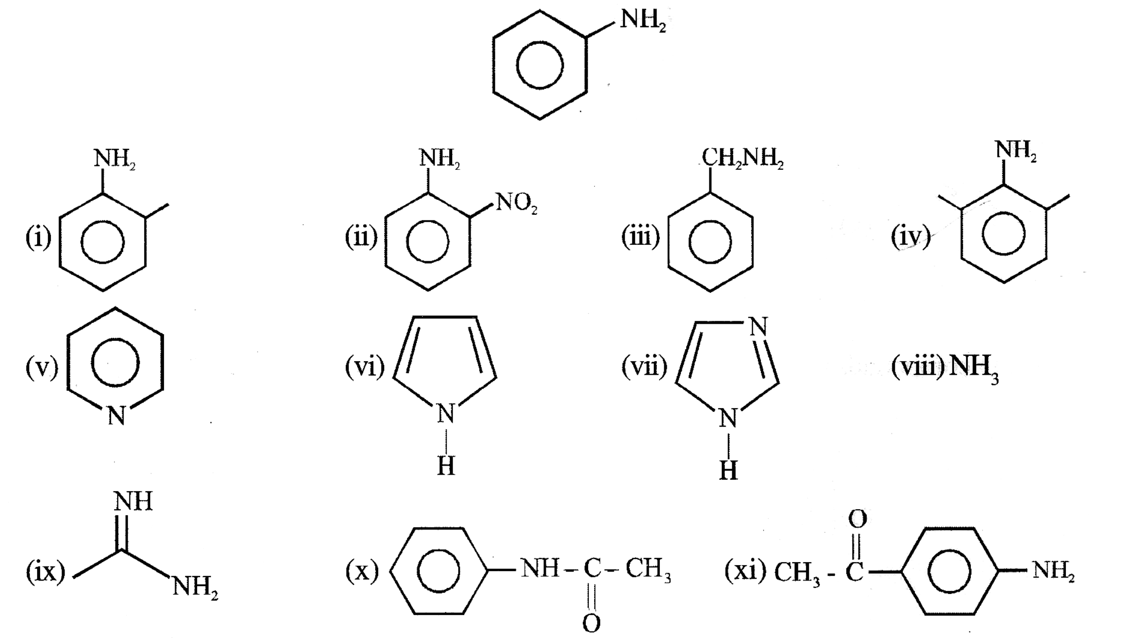
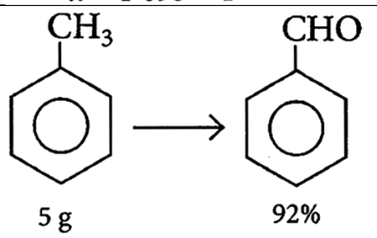
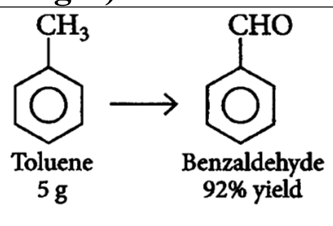
1, 2, 1 and 1

2, 4, 1 and 3

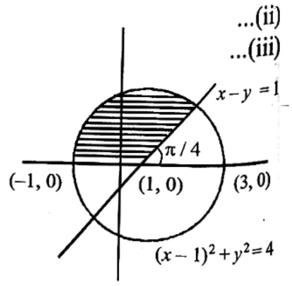
3, 4, 4 and 2

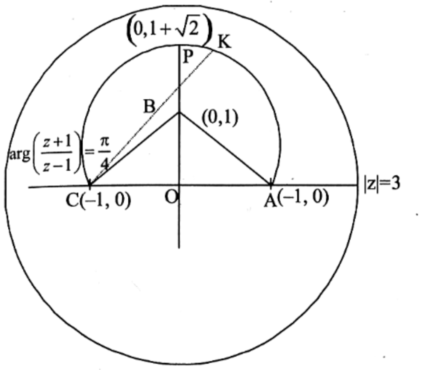
2, 2, 1 and 3

Sol.	To get the desired equation we need to add all three equations : $X \rightleftharpoons Y \quad ; \quad K_1 = 1.0$ $Y \rightleftharpoons Z \quad ; \quad K_2 = 2.0$ $Z \rightleftharpoons W \quad ; \quad K_3 = 4.0$ $X \rightleftharpoons W ; K = K_1 \times K_2 \times K_3$ $K = 1 \times 2 \times 4 = 8$	
46.	The difference in the number of unpaired electrons in Co^{2+} ion in its high-spin and low-spin octahedral complexes is …………	**2**
Sol.	$Co^{2+} \Rightarrow [Ar] 3d^7$ High-spin complex:  3 unpaired electrons Low-spin complex:  1 unpaired electron $\therefore$ Difference $= 3 - 1 = 2$	
47.	How many of the following intermediates have at least one contributing structure in which all atoms have their octet complete? 	**4**
Sol.	1.  2.  3.  4. 	
48.	Find the number of compound which are less basic than	**6**

		
49.	4 g equimolar mixture of NaOH and Na_2CO_3 contains x g of NaOH and y g Of Na_2CO_3. The value of x is ________ g. (Nearest integer)	**1**
Sol.	Given that solution is equimolar so, $\frac{x}{40} = \frac{y}{106} \Rightarrow y = \frac{106}{40}x$ Also $x + y = 4$ $x + \frac{106}{40}x = 4 \Rightarrow \frac{146}{40}x = 4 \Rightarrow x = \frac{4 \times 40}{146}$ $x = 1{\cdot}095 \approx 1$	
50.	 In the above reaction, 5 g of toluene is converted into benzaldehyde with 92% yield. The amount of benzaldehyde produced is _______ $\times 10^{-2}$ g. **(Nearest integer)**	**530**
Sol.	 Number of moles of toluene $= \frac{5}{92}$ mole 1 mol of toluene give 1 mol of benzaldehyde So, $\frac{5}{92}$ mol of toluene give $\frac{5}{92}$ mol of benzaldehyde Moles of benzaldehyde formed $= \frac{92}{100} \times \frac{5}{92} = \frac{1}{20}$ Number of moles of benzaldehyde $= \frac{1}{20}$	

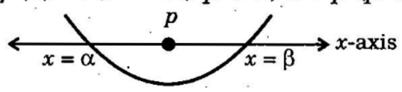
	$\frac{\text{Weight of benzaldehyde}}{\text{Molar mass}} = \frac{1}{20}$ Weight of benzaldehyde $= \frac{1}{20} \times 106$ $= 53 \times 10^{-1} = 530 \times 10^{-2}$ g The value of 'x' is 530.	
51.	Let a complex number be $w = 1 - \sqrt{3}\, i$. Let another complex number z be such that $\|zw\| = 1$ and $\arg(z) - \arg(w) = \frac{\pi}{2}$. Then the area of the triangle with vertices origin, z and w is equal to:	**A**
	$\frac{1}{2}$	
	2	
	4	
	$\frac{1}{4}$	
Sol.	(a) $\because w = 1 - \sqrt{3}i \Rightarrow \|w\| = z$ $\|wz\| = 1 \Rightarrow \|w\|\|z\| = 1 \Rightarrow \|z\| = \frac{1}{2}$ Since, $\arg(z) - \arg(w) = \frac{\pi}{2}$, then angle between z and w is $\frac{\pi}{2}$ Area of triangle $= \frac{1}{2} \cdot \|w\|\|z\| = \frac{1}{2} \cdot 2 \times \frac{1}{2} = \frac{1}{2}$	
52.	Let z satisfy $\|z\| = 1$ and $z = 1 - \bar{z}$. **Statement 1:** z is a real number. **Statement 2:** Principal argument of $z\ is \frac{\pi}{3}$	**B**
	Statement 1 is true Statement 2 is true; Statement 2 is a correct explanation for statement 1.	
	Statement 1 is false; Statement 2 is true	
	Statement 1 is true, Statement 2 is false.	
	Statement 1 is true; Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.	
Sol.	(b) Let $z = x + iy$, $\bar{z} = x - iy$ Now, $z = 1 - \bar{z}$ $\Rightarrow x + iy = 1 - (x - iy) \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$ Now, $\|z\| = 1 \Rightarrow x^2 + y^2 = 1 \Rightarrow y^2 = 1 - x^2$ $\Rightarrow y = \pm\frac{\sqrt{3}}{2}$ Now, $\tan\theta = \frac{y}{x}$ (θ is the argument) $= \frac{\sqrt{3}}{2} \div \frac{1}{2}$ (+ve since only principal argument) $= \sqrt{3} \Rightarrow \theta = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$ Hence, z is not a real number So, statement-1 is false and 2 is true.	
53.	Let α and β be the roots of $x^2 + \sqrt{3}x - 16 = 0$, and γ and δ be the roots of $x^2 + 3x - 1 = 0$. If $P_n = \alpha^n + \beta^n\ and\ Q_n = \gamma^n + \delta^n$, then $\frac{P_{25} + \sqrt{3}P_{24}}{2P_{23}} + \frac{Q_{25} - Q_{23}}{Q_{24}}$ is equal to	**C**

	3	
	4	
	5	
	7	
Sol.	**(c)** $x^2+\sqrt{3x}-16=0 \begin{smallmatrix}\alpha \\ \beta\end{smallmatrix}$; $P_n=\alpha^n+\beta^n$ $P_n+\sqrt{3}P_{n-1}-16P_{n-2}=0$ $P_{25}+\sqrt{3}P_{24}-16P_{23}=0$ $\therefore \frac{P_{25}+\sqrt{3}P_{24}}{2P_{23}}=8$ Similarly, $x^2+3x-1=0 \begin{smallmatrix}\gamma \\ \delta\end{smallmatrix}$; $Q_n=\gamma^n+\delta^n$ $\therefore \frac{Q_{25}-Q_{23}}{Q_{24}}=-3$ $\frac{P_{25}+\sqrt{3}P_{24}}{2P_{23}}+\frac{Q_{25}-Q_{23}}{Q_{24}}=8-3=5$	
54.	The area (in sq. unit s) of the region $S=\{z\in\mathbb{C}; \lvert z-1\rvert\le 2; (z+\overline{z})+i(z-\overline{z})\le 2, 1m(z)\ge 0\}\ is$	**B**
	$\frac{7\pi}{3}$	
	$\frac{3\pi}{2}$	
	$\frac{17\pi}{8}$	
	$\frac{7\pi}{4}$	
Sol.	**(b)** Put $z=x+iy$ $\lvert z-1\rvert\le 2 \Rightarrow (x-1)^2+y^2\le 4$ ...(i) $(z+\overline{z})+i(z-\overline{z})\le 2 \Rightarrow 2x+i(2iy)\le 2$ $\Rightarrow x-y\le 1$ ...(ii) Im $(z)\ge 0 \Rightarrow y\ge 0$ ...(iii) Required area = Area of semi-circle – area of sector A $=\frac{1}{2}\pi(2)^2-\frac{\frac{\pi}{4}}{2\pi}\times\pi\times 4$ $=2\pi-\frac{\pi}{2}=\frac{3\pi}{2}$ 	
55.	Let arg(z) represent the principal argument of the complex number z. The $\lvert z\rvert=3$ and arg(z – 1) – arg (z + 1) = $\frac{\pi}{4}$ intersect:	**C**
	Exactly at one point	
	Exactly at two points	
	Nowhere or no point of intersection	
	At infinitely many points	

Sol.	**(c)**  $\because \quad \arg(z-1)-\arg(z+1)=\dfrac{\pi}{4}$ $\Rightarrow \quad \angle ABO = \angle AKC = \dfrac{\pi}{4} \Rightarrow \alpha = 1 \Rightarrow B(0, 1), r=\sqrt{2}$ $\Rightarrow \quad PO = 1+\sqrt{2}$. No point of intersection.	
56.	If the cube roots of unity are $1, \omega, \omega^2$ then the roots of the equation $(x-1)^3+8=0, are$	**C**
	$-1, -1+2\omega, -1-2\omega^2$	
	$-1, -1, -1$	
	$-1, 1-2\omega, 1-2\omega^2$	
	$-1, 1+2\omega, 1+2\omega^2$	
Sol.	**(c)** $\because (x-1)^3+8=0 \Rightarrow (x-1)=(-2)\,(1)^{1/3}$ $\Rightarrow x-1=-2$ or -2ω or $-2\omega^2$ or $x=-1$ or $1-2\omega$ or $1-2\omega^2$.	
57.	The sum of the roots of the equation $x+1-2\log_2(3+2^x)+2\log_4(10-2^{-x})=0$, is	**D**
	$\log_2 14$	
	$\log_2 12$	
	$\log_2 13$	
	$\log_2 11$	
Sol.	$x+1-2\log_2(3+2^x)+2\log_4(10-2^{-x})=0$ $\therefore \quad \log_2(2^{x+1})-\log_2(3+2^x)^2+\log_2(10-2^{-x})=0$ $\Rightarrow \quad \log_2\left(\dfrac{2^{x+1}(10-2^{-x})}{(3+2^x)^2}\right)=0$ $\Rightarrow \quad \dfrac{2(10\times 2^x-1)}{(3+2^x)^2}=1$ $\Rightarrow \quad 20\times 2^x-2=9+2^{2x}+6\times 2^x$ $\therefore \quad (2^x)^2-14(2^x)+11=0$ If roots are x_1 and x_2 then, $2^{x_1}\times 2^{x_2}=11$ $\Rightarrow \quad x_1+x_2=\log_2(11)$	
58.	The product of all solutions of the equation $e^{5(\log_e x)^2+3}=x^8, x>0, is$	**C**
	e^2	
	e	
	$e^{8/5}$	
	$e^{6/5}$	

Sol.	$e^{5(\log_e x)^2+3} = x^8, x > 0$ $\Rightarrow 5(\log_e x)^2 + 3 = 8\log_e x$ $\Rightarrow 5(\log_e x)^2 - 8\log_e x + 3 = 0$ If equation has roots x_1 and x_2, then $\log_e x_1 + \log_e x_2 = \frac{8}{5}$ $\Rightarrow x_1 x_2 = e^{\frac{8}{5}}$	
59.	If α ,β, γ are the roots of the cubic $x^3 - 2x + 3 = 0$ then the value of $\frac{1}{\alpha^3+\beta^3+6} + \frac{1}{\beta^3+\gamma^3+6} + \frac{1}{\gamma^3+\alpha^3+6}$ *equals* :	**B**
	$\frac{1}{3}$	
	$\frac{-1}{3}$	
	$\frac{1}{2}$	
	$\frac{-1}{2}$	
Sol.	$x^3 - 2x + 3 = 0 \begin{matrix}\alpha\\ \beta\\ \gamma\end{matrix}$; $\alpha+\beta+\gamma = 0$; $\sum \alpha\beta = -2$ and $\alpha\beta\gamma = -3$ $\alpha^3 - 2\alpha + 3 = 0$ $\Rightarrow \quad \alpha^3 = 2\alpha - 3 \quad$ ...(i) Similarly, $\beta^3 = 2\beta - 3$ and $\quad$ ...(ii) $\gamma^3 = 2\gamma - 3 \quad$ ...(iii) $\therefore \quad \alpha^3 + \beta^3 = 2(\alpha+\beta) - 6$ $\alpha^3 + \beta^3 + 6 = 2(\alpha+\beta)$ $\therefore \quad \frac{1}{\alpha^3+\beta^3+6} = \frac{1}{2(\alpha+\beta)}$ $\therefore \sum \frac{1}{\alpha^3+\beta^3+6} = \frac{1}{2}\left[\frac{1}{\alpha+\beta+\gamma-\gamma} + \frac{1}{\beta+\gamma+\alpha-\alpha} + \frac{1}{\gamma+\alpha+\beta-\beta}\right]$ $= \frac{-1}{2}\left[\frac{1}{\gamma} + \frac{1}{\alpha} + \frac{1}{\beta}\right] \quad (\alpha+\beta+\gamma = 0)$ $= \frac{-1}{2}\left[\frac{\alpha\beta+\beta\gamma+\gamma\alpha}{\alpha\beta\gamma}\right]$ $= \frac{-1}{2}\left[\frac{-2}{-3}\right] = \frac{-1}{3}$	
60.	Given that α and β are the roots of the equation $2t^2 - 3t + 4 = 0$. The equation whose roots are $\alpha + \alpha^{-1}$ and $\beta + \beta^{-1}$ is:	**B**
	$18x^2 - 13x + 18 = 0$	
	$8x^2 - 18x + 13 = 0$	
	$18x^2 - 8x - 13 = 0$	
	$18x^2 - 8x - 13 = 0$	

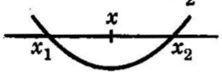
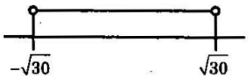
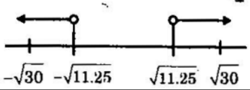
Sol	Given, $2t^2 - 3t + 4 = 0$ with roots α, β $\therefore \quad \alpha + \beta = \frac{3}{2}, \alpha\beta = 2$ Now, sum of roots, $S = \left(\alpha + \frac{1}{\alpha}\right) + \left(\beta + \frac{1}{\beta}\right)$ $= (\alpha + \beta) + \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$ $= (\alpha + \beta) + \left(\frac{\alpha + \beta}{\alpha\beta}\right) = \frac{3}{2} + \frac{3}{2} \times \frac{1}{2}$ $= \frac{3}{2} + \frac{3}{4} = \frac{9}{4}$ Also, product of roots, $P = \left(\alpha + \frac{1}{\alpha}\right)\left(\beta + \frac{1}{\beta}\right)$ $= \alpha\beta + \left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) + \frac{1}{\alpha\beta} \Rightarrow \alpha\beta + \frac{\alpha^2 + \beta^2}{\alpha\beta} + \frac{1}{\alpha\beta}$ $= \alpha\beta + \left[\frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}\right] + \frac{1}{\alpha\beta}$ $= 2 + \frac{\left(\frac{9}{4} - 4\right)}{2} + \frac{1}{2} = \frac{5}{2} + \frac{(9 - 16)}{8}$ $= \frac{5}{2} - \frac{7}{8} = \frac{13}{8}$ $\therefore$ The required quadratic equation is $x^2 - Sx + p = 0$ $\Rightarrow \quad x^2 - \frac{9}{4}x + \frac{13}{8} = 0$ $\Rightarrow \quad 8x^2 - 18x + 13 = 0$	
61.	If the roots of the quadratic equation $x^2 + px + q = 0$ are $\tan 30^o$ and $\tan 15^o$ respectively, then the value of $(2 + q - p)$ is:	**B**
	2	
	3	
	0	
	1	
Sol	$x^2 + px + q = 0$ Now, $\tan 30° + \tan 15° = -p$ and $\tan 30° \cdot \tan 15° = q$ $\therefore \quad \tan 45° = \tan(30° + 15°)$ $= \frac{\tan 30° + \tan 15°}{1 - \tan 30° \cdot \tan 15°} = \frac{-p}{1 - q} = 1$ $\Rightarrow \quad q - p = 1$ Hence, $(2 + q - p) = 3$	
62.	The range of a for which the expression $y = \frac{(x-1)(x-5)}{(x-a)}$ is capable of taking all real values for $x \in R$ (the set of all rela numbers) is;	**C**
	$(-\infty, 1)$	
	$a \in (5, \infty)$	
	$a \in (1,5)$	

	$a \in \emptyset$	
Sol.	Given, $y = \frac{(x-1)(x-5)}{(x-a)}$ $\Rightarrow\ yx - ya = x^2 - 6x + 5$ $\Rightarrow\ x^2 - (6+y)x + (5+ya) = 0$ As $x \in R$, so $D \geq 0$ $\Rightarrow\ (6+y)^2 - 4(5+ya) \geq 0$ $\Rightarrow\ y^2 + 4(3-a)y + 16 \geq 0\ \forall\ y \in R$. Now, $D \leq 0$, $16(3-a)^2 - 4 \cdot 16 \leq 0 \Rightarrow\ 1 \leq a \leq 5$ But when $a = 1$ or 5, then range of y is not R (the set of all real numbers), as numerator and denominator of expression has common factor. Hence, $1 < a < 5$	
63.	The range of $p \in R$ for which the equation $2x^2 - 2(2p+1)x + p(p+1) = 0$ have one root less than p and other root greater than p,is :	**B**
	$-1 < P < 0$	
	$P < -1$ or $P > 0$	
	$p \geq 0$	
	$P = 0$	
Sol.	We must have $f(p) < 0$, where $f(x) = 2x^2 - 2(2p+1)x + p(p+1)$  So, $f(p) < 0$ $\Rightarrow\ p^2 + p > 0$ $\Rightarrow\ p > 0$ or $p < -1$	
64.	Let λ_1 and λ_2 be two values of λ for which the expression $x^2 + (2-\lambda)x + \lambda - \frac{3}{4}$ becomes a perfect square. The value of $\lambda_1^2 + \lambda_2^2$ equals:	**C**
	8	
	25	
	50	
	100	
Sol	Given, $f(x) = x^2 + (2-\lambda)x + \left(\lambda - \frac{3}{4}\right)$ Now, for a quadratic expression to become a perfect square, its $D = 0$ $\Rightarrow\ (2-\lambda)^2 = 4\left(\lambda - \frac{3}{4}\right)$ $\Rightarrow\ 4 + \lambda^2 - 4\lambda = 4\lambda - 3$ $\Rightarrow\ \lambda^2 - 8\lambda + 7 = 0$ $\Rightarrow\ (\lambda-7)(\lambda-1) = 0$ $\therefore\ \lambda = 1, 7$ Hence, $(\lambda_1^2 + \lambda_2^2) = (1)^2 + (7)^2$ $= 1 + 49 = 50$	
65.	If $c^2 = 4d$ and the the two equations $x^2 - ax + b = 0$ and $x^2 - cx + d = 0$ have one common root, then the value of $2(b+d)$ is equal to:	**B**
	$\frac{a}{c}$	

	ac	
	2ac	
	a + c	
Sol.	With the condition, $c^2 = 4d$, the equation $x^2 - cx + d$ has equal roots. Each root $= \frac{c}{2}$ As $\frac{c}{2}$ is a root of $x^2 - ax + b = 0$ $\therefore \quad \frac{c^2}{4} - a \cdot \left(\frac{c}{2}\right) + b = 0$ $\Rightarrow \quad \frac{4d}{4} - \frac{ac}{2} + b = 0$ $\Rightarrow \quad b + d = \frac{ac}{2}$ $\Rightarrow \quad 2(b + d) = ac$	
66.	Number of terms common to the two sequences, 17, 21, 25, …. 417 and 16, 21, 26,…..466 is:	**B**
	19	
	20	
	21	
	22	
Sol.	$417 = 17 + (n-1)4$ $\Rightarrow \quad 400 = 4(n-1)$ $\Rightarrow \quad n = 101$ Similarly, $466 = 16 + (m-1)5$ $\Rightarrow \quad 450 = 5(m-1)$ $\Rightarrow \quad m = 91$ Let T_n is common to both for some n for which m is an integer. $17 + (n-1)4 = 16 + (m-1)5$ $1 + 4n - 4 = 5m - 5$ $5m = 4n + 2$ Hence, $n = 2, 7, 12, \ldots, 97 \rightarrow 20$	
67.	Let $s_1, s_2, s_3\ldots$ and $t_1, t_2, t_3\ldots\ldots$ are two arithmetic sequences such that $s_1 = t_1 \neq 0$; $s_2 = 2t_2$ and $\sum_{i=1}^{10} s_i = \sum_{i=1}^{15} t_i$. Then the value of $\frac{s_2 - s_1}{t_2 - t_1}$ *is* :	**C**
	$\frac{8}{3}$	
	$\frac{3}{2}$	
	$\frac{19}{8}$	
	2	

Sol.	Given $s_1 + s_2 + s_3 + ... + s_{10} = t_1 + t_2 + t_3 + ... + t_{15}$ Let 1st sequence is $a_1, a_1 + d_1, a_1 + 2d_1, ...$ and 2nd is $a_1, a_1 + d_2, a_1 + 2d_2, ...$ (since $s_1 = t_1$) Given $s_2 = 2t_2$ $\therefore \quad a_1 + d_1 = 2(a_1 + d_2)$ $\therefore \quad a_1 = d_1 - 2d_2$ ...(i) We have to find $\frac{s_2 - s_1}{t_2 - t_1} = \frac{d_1}{d_2} = ?$ Now $\frac{10}{2}[2a_1 + 9d_1] = \frac{15}{2}[2a_1 + 14d_2]$ This gives $a_1 = 9d_1 - 21d_2$ ...(ii) From eqs. (i) and (ii) $\frac{d_1}{d_2} = \frac{19}{8}$	
68.	Let $a_n, n \in N$ is an A.P. with common difference 'd' and all whose terms are non–zero. If n approaches infinity, then the sum $\frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \cdots + \frac{1}{a_n a_{n+1}}$ will approach:	**A**
	$\frac{1}{a_1 d}$	
	$\frac{2}{a_1 d}$	
	$\frac{1}{2a_1 d}$	
	$a_1 d$	
Sol.	$\frac{1}{d}\left[\frac{a_2 - a_1}{a_1 a_2} + \frac{a_3 - a_2}{a_2 a_3} + ... + \frac{a_{n+1} - a_n}{a_n \cdot a_{n+1}}\right]$ $= \frac{1}{d}\left[\frac{1}{a_1} - \frac{1}{a_2} + \frac{1}{a_2} - \frac{1}{a_3} + ... + \frac{1}{a_n} - \frac{1}{a_{n+1}}\right]$ $= \frac{1}{d}\left[\frac{1}{a_1} - \frac{1}{a_{n+1}}\right]$ $= \frac{1}{d}\left[\frac{a_{n+1} - a_1}{(a_1)(a_{n+1})}\right] = \frac{1}{d}\left[\frac{a_1 + nd - a}{(a_1)(a_{n+1})}\right]$ $= \frac{n}{a_1[a_1 + nd]} = \frac{1}{a_1\left[\frac{a_1}{n} + d\right]}$ $\Rightarrow$ as $n \to \infty$ then $S = \frac{1}{a_1 d}$	
69.	If $S_n = \frac{1}{1^3} + \frac{1+2}{1^3+2^3} + \cdots + \frac{1+2+3+\cdots+n}{1^3+^3+3^3+\cdots+n^3}$, n = 1,2,3,……, then S_n is not greater than:	**C**
	$\frac{1}{2}$	
	1	
	2	
	4	
Sol.	$S_n = \frac{1}{1^3} + \frac{1+2}{1^3+2^3} + \frac{1+2+3}{1^3+2^3+3^3} + ...$	

	$T_n = \dfrac{1+2+3+4+5+\ldots n}{1^3+2^3+3^3+\ldots n^3}$ $= \dfrac{\frac{n(n+1)}{2}}{\left[\frac{n(n+1)}{2}\right]^2} = \dfrac{2}{n(n+1)}$ $\Rightarrow S_n = 2\sum\left(\dfrac{1}{n} - \dfrac{1}{n+1}\right)$	
70.	For x > 0, the sum of the series $\frac{1}{1+x} - \frac{(1-x)}{(1+x)^2} + \frac{(1-x)^2}{(1+x)^3} - \cdots \infty$ is equal to:	**B**
	$\frac{1}{4}$	
	$\frac{1}{2}$	
	$\frac{3}{4}$	
	1	
Sol.	**The above series in an infinite G.P., whose first term =** $\dfrac{1}{1+x}$ **and common ratio =** $\dfrac{-(1-x)}{(1+x)}$. $\therefore\ S_\infty = \dfrac{\frac{1}{1+x}}{1+\left(\frac{1-x}{1+x}\right)} = \dfrac{1}{2}$	
71.	Let S denote sum of the series $\frac{3}{2^3} + \frac{4}{2^4.3} + \frac{5}{2^6.3} + \frac{6}{2^7.5} + \cdots \infty$ Compute the value of S^{-1}.	**2**
Sol.	Let $S = \sum_{r=1}^{\infty} \dfrac{r+2}{2^{r+1} r(r+1)} = \sum_{r=1}^{\infty} \dfrac{2(r+1)-r}{2^{r+1}\cdot r\cdot(r+1)}$ $= \sum_{r=1}^{\infty} \dfrac{1}{2^{r+1}}\left(\dfrac{2}{r} - \dfrac{1}{r+1}\right) = \sum_{r=1}^{\infty}\left(\dfrac{1}{2^r\cdot r} - \dfrac{1}{2^{r+1}(r+1)}\right)$ $= \lim_{n\to\infty}\left[\left(\dfrac{1}{2^1\cdot 1} - \dfrac{1}{2^2\cdot 2}\right) + \left(\dfrac{1}{2^2\cdot 2} - \dfrac{1}{2^3\cdot 3}\right)\right]$ $+ \left(\dfrac{1}{2^3\cdot 3} - \dfrac{1}{2^4\cdot 4}\right) + \ldots + \left(\dfrac{1}{2^n n} - \dfrac{1}{2^{n+1}\cdot(n+1)}\right)$ $= \lim_{n\to\infty}\left(\dfrac{1}{2} - \dfrac{1}{2^{n+1}9n+1}\right)$ $\therefore\ S = \dfrac{1}{2}$ Hence, $S^{-1} = 2$	
72.	Let $\alpha = 8 - 14i, A = \left\{z \in \mathbb{C}: \frac{\alpha z - \overline{\alpha z}}{z^2 - (\overline{z})^2 - 112i} = 1\right\}$ and $B = \{z \in \mathbb{C}: \lvert z + 3i\rvert = 4\}$. Then $\sum_{z\in A\cap B}(Re z - Im\ z)$ is equal to________.	**14**

Sol.	**(14)** Given that, $\alpha = 8 - 14i$ Let $z = x + iy$; $\alpha z = (8x + 14y) + i(-14x + 8y)$ $\overline{\alpha z} = (8x + 14y) - i(-14x + 8y)$ $\alpha z - \overline{\alpha z} = 2i(-14x + 8y)$ We have, $z + \bar{z} = 2x,\ z - \bar{z} = 2iy$; Set A: $\frac{2i(-14x + 8y)}{i(4xy - 112)} = 1$ $\Rightarrow\ (x - 4)(y + 7) = 0 \Rightarrow x = 4$ or $y = -7$ Set B: $x^2 + (y + 3)^2 = 16$ when $x = 4 \Rightarrow y = -3$ when $y = -7 \Rightarrow x = 0$ $\therefore\ A \cap B = \{4 - 3i, 0 - 7i\}$ So, $\sum_{z \in A \cap B} (\text{Re}\,z - \text{Im}\,z) = 4 - (-3) + (0 - (-7)) = 14$	
73.	If $5x^2 - 2kx + 1 < 0$ has exactly one integral solution then find the sum of all positive integral values of k.	**9**
Sol.	**Let** $f(x) = 5x^2 - 2kx + 1 \left\langle \begin{matrix} x_1 \\ x_2 \end{matrix} \right.$  **Now we want exactly one x for which $f(x) < 0$.** **Hence,** $1 < \lvert x_1 - x_2 \rvert < 2$ $\lvert x_1 - x_2 \rvert < 2$ $\Rightarrow\ (x_1 - x_2)^2 < 4$ $\Rightarrow\ (x_1 + x_2)^2 - 4x_1x_2 < 4$ $\Rightarrow\ \frac{4k^2}{25} - \frac{4}{5} < 4$ $\Rightarrow\ k^2 - 5 < 25$ $\Rightarrow\ k^2 < 30$ $\Rightarrow\ k \in$  **...(i)** **and** $\lvert x_1 - x_2 \rvert > 1$ $\Rightarrow\ (x_1 - x_2)^2 > 1$ $\Rightarrow\ (x_1 + x_2)^2 - 4x_1x_2 > 1$ $\Rightarrow\ \frac{4k^2}{25} - \frac{4}{5} > 1$ $\Rightarrow\ \frac{4k^2}{25} > \frac{9}{5}$ $\Rightarrow\ k^2 > \frac{45}{4}$ $\Rightarrow\ k \in$  **...(ii)**	

Eqns. (i) and (ii)

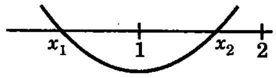
$\Rightarrow$ Positive integral values of k are 4 and 5.

Hence, sum = 9

Alternatively: $f(x) = 5x^2 - 2kx + 1$ x_1 x_2

Since k is positive.

Hence, vertex of $f(x)$ lies on positive x-axis.



Also product of root is $\frac{1}{5}$, so both roots of $f(x)$ are positive and smaller root x_1 has to be positive fraction because if $x_1 > 1$, then for $x_1x_2 = \frac{1}{5}$, x_2 has to be fraction which is not possible.

Hence, for exactly one integral solution of $f(x) < 0$,
$f(1) < 0$ and $f(2) > 0$

$$f(1) = 6 - 2k < 0$$

$$\Rightarrow \quad k > 3 \qquad \text{...(i)}$$

$$\text{and} \quad f(2) = 21 - 4k > 0$$

$$\Rightarrow \quad k < \frac{21}{4} \qquad \text{...(ii)}$$

Eqns. (i) and (ii)

$\Rightarrow$ Positive integral values of k are 4 and 5

Hence, sum = 9

74. The number of solutions of the equation $\log_{(x+1)}(2x^2+7x+5) + \log_{(2x+5)}(x+1)^2 - 4 = 0, x > 0,$ *is*______. **1**

Sol.

$\log_{(x+1)}(2x^2+7x+5) + \log_{(2x+5)}(x+1)^2 - 4 = 0$

$\therefore \quad \log_{(x+1)}(2x+5)(x+1) + 2\log_{(2x+5)}(x+1) = 4$

$\Rightarrow \quad \log_{(x+1)}(2x+5) + 1 + 2\log_{(2x+5)}(x+1) = 4$

Put $\log_{(x+1)}(2x+5) = t$

$\therefore \quad t + \frac{2}{t} = 3 \Rightarrow t^2 - 3t + 2 = 0$

$\Rightarrow \quad t = 1, 2$

$\Rightarrow \quad \log_{(x+1)}(2x+5) = 1$ or $\log_{(x+1)}(2x+5) = 2$

$\Rightarrow \quad x + 1 = 2x + 5$ or $2x + 5 = (x+1)^2$

$\Rightarrow \quad x = -4$ (rejected) or $x^2 = 4 \Rightarrow x = 2, -2$ (rejected)

So, $x = 2$

Hence, no. of solutions is 1.

75. For any $x, y \in R, xy > 0,$ then the minimum value of $\frac{2x}{y^3} + \frac{x^3y}{3} + \frac{4y^2}{9x^4}$ equals: **2**

Sol.

As $x, y \in R$ and $xy > 0$, so x and y will be of same sign.

$\therefore$ All the quantities $\frac{2x}{y^3}, \frac{x^3y}{3}, \frac{4y^2}{9x^4}$ are positive.

$\therefore$ A.M. $\geq$ G.M.

$$\Rightarrow \frac{2x}{y^3} + \frac{x^3y}{3} + \frac{4y^2}{9x^4} \geq 3\left(\left(\frac{2x}{y^3}\right)\left(\frac{x^3y}{3}\right)\left(\frac{4y^2}{9x^4}\right)\right)^{1/3}$$

$$= 3 \times \frac{2}{3} = 2$$