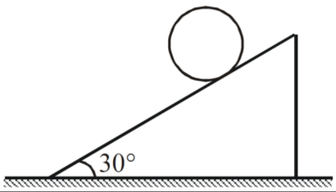
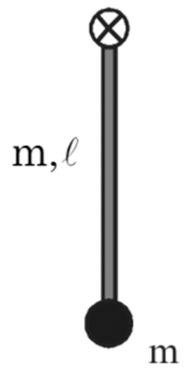
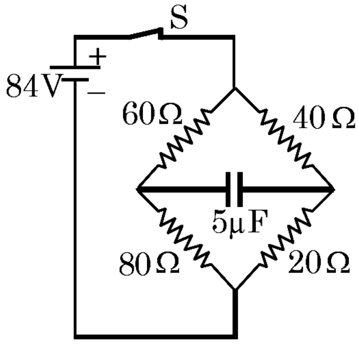
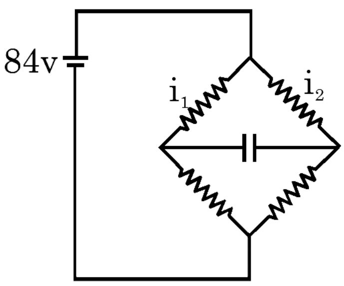
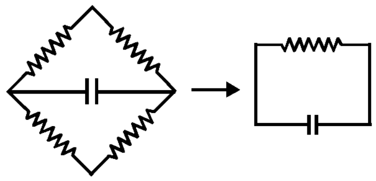
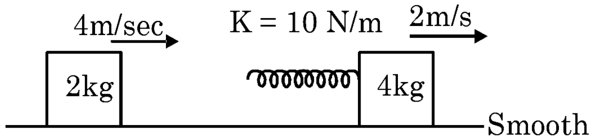
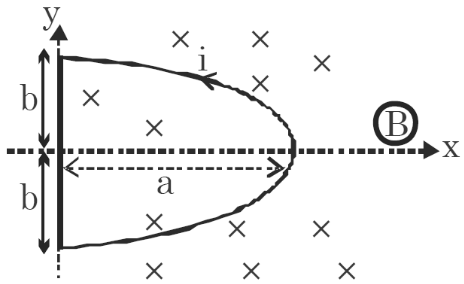
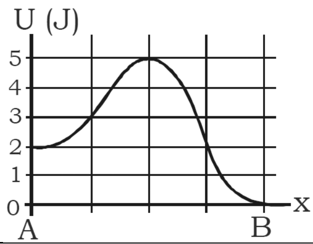
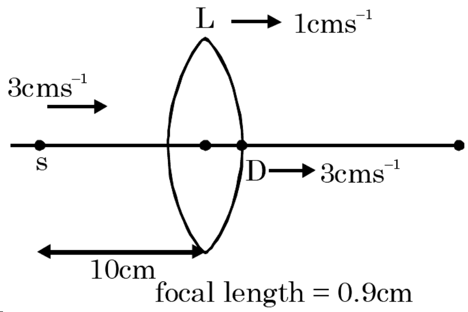
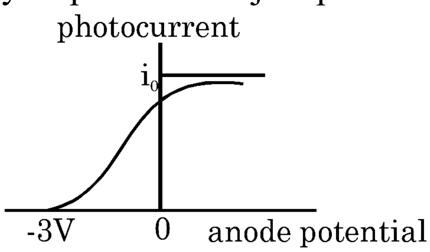
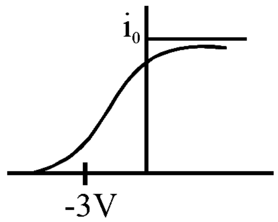
1.	The electric field inside a sphere having charge density related to the distance from the centre as $\rho = \alpha\ r$ (α is a constant) is :	**B**
	$\frac{\alpha r^3}{4\epsilon_0}$	
	$\frac{\alpha r^2}{4\epsilon_0}$	
	$\frac{\alpha r^2}{3\epsilon_0}$	
	None of these	
Sol.	From Gauss's Law $\oint \vec{E} \cdot \overrightarrow{dA} = \frac{q_{in}}{\epsilon_0}$ $E \cdot 4\pi r^2 = \frac{\int_0^r (4\pi r^2 \cdot dr)(\alpha r)}{\epsilon_0}$ $E \cdot 4\pi r^2 = \frac{4\pi \cdot \alpha \cdot r^4}{4\ \epsilon_0}$ $E = \left(\frac{\alpha r^2}{4\ \epsilon_0}\right)$	
2.	A uniform disc of mass 2kg and radius 50cm is released from rest on a smooth incline plane as shown. If the length of incline equals 160 cm, the time taken by the disc to arrive at bottom equals 	**B**
	1 s	
	0.8 s	
	0.6 s	
	0.5 s	

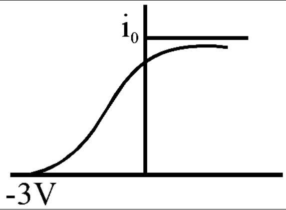
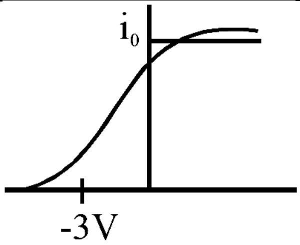
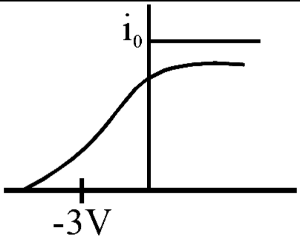
Sol.	Since plane is smooth hence motion is only translational $a = g\sin\theta = 10 \times \frac{1}{2} = 5\ m/s^2$ $s = ut + \frac{1}{2}at^2$ $1.6 = 0 + \frac{1}{2} \times 5 \times t^2$ $t^2 = \sqrt{\frac{3.2}{5}} = \sqrt{0.64} = 0.8$ sec.	
3.	Which of the following transitions of He^+ ion will give rise to spectral line which has same wavelength as some spectral line in hydrogen atom?	**A**
	n = 4 to n = 2	
	n = 6 to n = 5	
	n = 6 to n = 3	
	None of these	
Sol.	If $n_2 \to n_1$ in H(Z = 1) gives λ then $zn_2 \to zn_1$ give same λ H-like ion for He^+ ion (z = 2)	
4.	A particle is attached to the lower end of a uniform rod which is hinged at its other end as shown in the figure. Another identical particle moving horizontally, collides inelastically and sticks to it. The minimum speed of moving particle so that the rod with particles performs circular motion in a vertical plane will be : [length of the rod is ℓ, consider masses of both particles and rod to be same]  	**C**
	$\sqrt{10g\ell}$	
	$\sqrt{20g\ell}$	
	$\sqrt{\frac{70}{3}g\ell}$	
	$\sqrt{\frac{175}{3}g\ell}$	

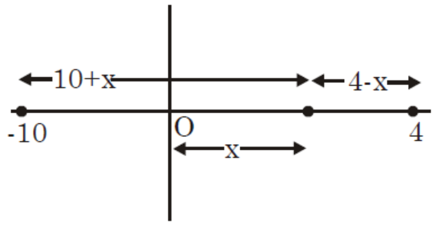
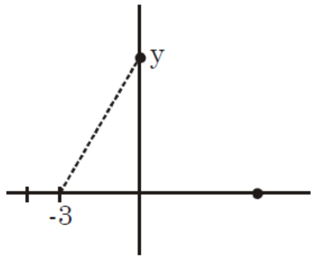
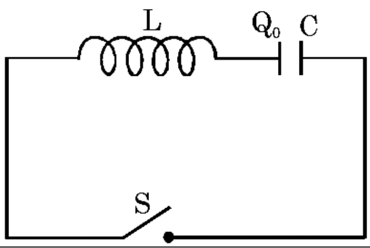
Sol.	From conservation of Angular momentum about hinged point $\mathrm{mv}\ell = \left(\frac{\mathrm{m}\ell^2}{3} + 2\mathrm{m}\cdot\ell^2\right)\omega$ $\omega = \left(\frac{3}{7}\frac{\mathrm{v}}{\ell}\right)$ Now from conservation of Energy. $\frac{1}{2}\times\frac{7}{3}\mathrm{m}\ell^2\times\omega^2 = \mathrm{mgl} + 2\mathrm{mg}\cdot 2\ell$ $\mathrm{v} = \sqrt{\frac{70}{3}\mathrm{g}\ell}$	
5.	The circuit shown in the figure is in steady state for a long time. The connection to battery is suddenly broken (switch S is opened up). What is the charge (in μC) on the capacitor after 0.001 sec.? 	**C**
	100	
	100 e^{-2}	
	100 e^{-4}	
	0	
Sol.	$i_1 = \frac{84}{140} = 0.6\mathrm{A}$ $i_2 = \frac{84}{60} = 1.4\mathrm{A}$ 	

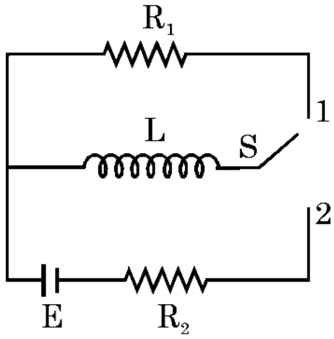
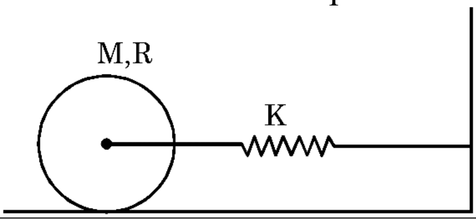
	p.d across capacitor = 20 v $Q = CV = 5 \times 20 = 100\ \mu C$ After S is open $Q = Q \cdot e^{-t/RC}$ $= 100\ e^{-4}$ 	
6.	Figure shows two blocks A and B having mass 2 kg and 4 kg moving with a speed4 m/sec and 2 m/sec respectively. The maximum compression in the spring and final velocity of 2kg block are respectively. 	**C**
	$\sqrt{\frac{24}{45}}$m, 3 m/s	
	$\sqrt{\frac{24}{25}}$m, 3 m/s	
	$\sqrt{\frac{24}{45}}$m, 4/3 m/s	
	$\sqrt{\frac{24}{25}}$m, 8/3 m/s	
Sol.	$\frac{1}{2}\mu\, v_r^2 = \frac{1}{2}kx^2$ $x = V_{rer}{}^{2}\sqrt{\frac{\mu}{k}}$ $= 2 \times \sqrt{\frac{8/6}{10}} = \sqrt{\frac{8}{15}}$ From conservation of momentum $2 \times 4 + 4 \times 2 = 2v_1 + 4v_2$ $8 = v_1 + 2v_2$ ....(1) From conservation of energy $\frac{1}{2} \times 2 \times 4^2 + \frac{1}{2} \times 4 \times 2^2 = \frac{1}{2} \times 2 \times v_1^2 + \frac{1}{2} \times 4 \times v_2^2$ ...(ii) On solving $v_1 = \frac{4}{3}$ m/s, $v_2 = \frac{10}{3}$ m/s	
7.	In the figure, there is a conducting wire having current i and which has a	**D**

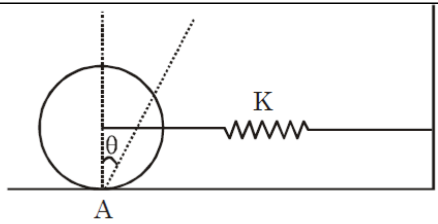
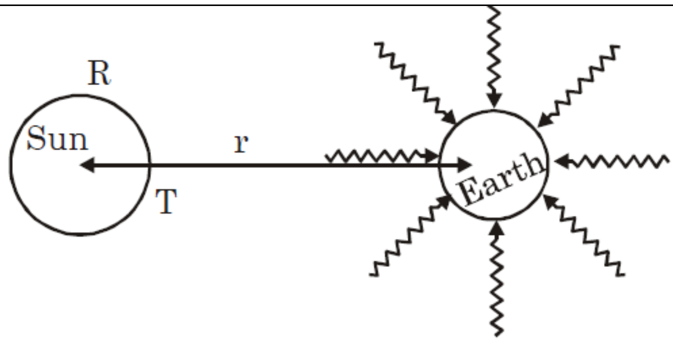
	shape of closed half ellipse $\left[\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\right]$ is kept in a uniform magnetic field B as shown. The magnitude of magnetic dipole moment of loop and torque acting on it are- 	
	π abi, 0	
	π abi, π abi B	
	$\frac{\pi\, abi}{2}, \frac{\pi\, abi B}{2}$	
	$\frac{\pi\, abi}{2}$, 0	
Sol.	Area of ellipse = πab Magnetic dipole moment = $\frac{\pi abi}{2}$ Torque $\vec{\tau} = \vec{M} \times \vec{B}$ $\tau = MB\sin\theta$ $\because \quad \vec{M} \,\|\, \vec{B} \qquad \boxed{\tau = 0}$	
8.	What minimum speed does a 100 g particle need at point B to reach point A? The graph shows potential energy versus position 	**C**
	$\sqrt{40}$ m/s	
	$\sqrt{60}$ m/s	
	10 m/s	
	5 m/s	

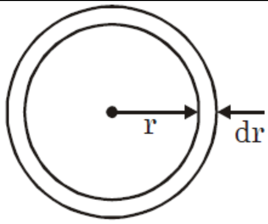
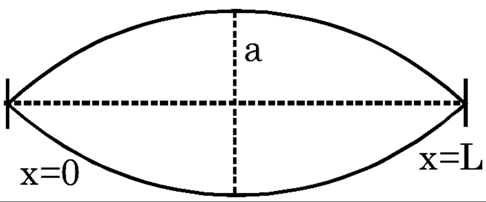
Sol.	To reach at point A particle must cross the peak point. Loss in KE = gain in PE $\frac{1}{2}\times\frac{100}{1000}\times v^2=(5-0)$ $v^2 = 100$ $v = 10$ m/s	
9.	The figure shows the initial position of a point source of light s, a detector D and lens L. Now at t = 0, all three starts moving towards right with different velocity as shown in figure. The times at which detector receives the maximum light, is(Assume that the detector is initially just touching the lens). 	**B**
	5 s and 10 s	
	0.5 s and 4.5 s	
	2 s and 3 s	
	Only 10 s	
Sol.	$\frac{1}{0.9}=+\frac{1}{(10-2t)}+\frac{1}{2t}$ t = 0.5 sec and t = 4.5 sec.	
10.	Figure shows the anode potential vs photo-current graph when photons of 6 eVare incident on cathode in a photo electric experiment set-up. In the same experimental set-up, if photons of 8 eV and same intensity are incident on cathode, then anode-potential vs photo-current graph will be (assume 100%efficiency of photons to eject photo-electrons in both cases) 	**D**
		

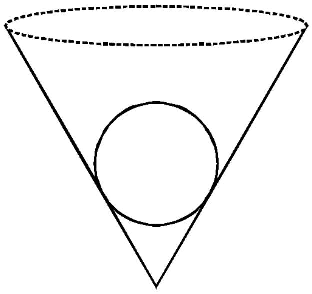
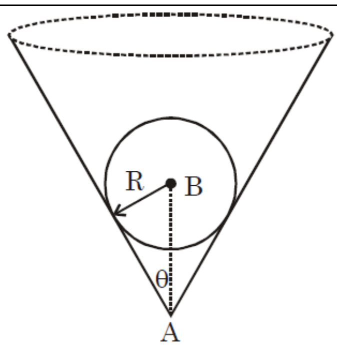
		
		
		
Sol.	Stopping potential increases and number of photons decreases. $I = \frac{hc\,n_p}{\lambda t}$	
11.	A sound source is located somewhere along the x-axis. Experiment shows that the same wavefront simultaneously reaches listeners at x = –10m and x = +4.0 m. A third listener is positioned along the positive y-axis. What is y-coordinate (in m) if the same wavefront reaches at him at the same instant it does the first two listeners ?	**A**
	$\sqrt{40}$	
	3	
	7	
	$\sqrt{30}$	

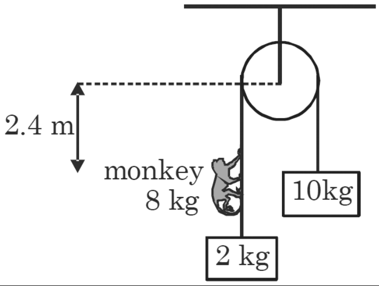
Sol.	  $t = \frac{10+x}{C} = \frac{4-x}{C}$ $\therefore\ -6 = 2x$ $t = \frac{10-3}{C} \quad \therefore\ x = -3$ $\therefore\ t = \frac{7}{C}$ $\therefore\ 7 = \sqrt{3^2 + y^2}$ $\therefore\ y = \sqrt{40}$	
12.	At $t < 0$, the capacitor is charged and the switch is opened. At $t = 0$ the switch is closed. The shortest time T at which the charge on the capacitor will be zero is given by :  	**C**
	$\pi\sqrt{LC}$	
	$\frac{3}{2}\pi\sqrt{LC}$	
	$\frac{\pi}{2}\sqrt{LC}$	
	$2\pi\sqrt{LC}$	
Sol.	The time period of LC oscillations, $T = 2\pi\sqrt{LC}$ The time at which charge on the capacitor will be zero is $\frac{T}{4}$. So $t = \frac{\pi}{2}\sqrt{LC}$	
13.	In the circuit shown swith S is connected to position 2 for a long time and then joined to position 1. The total heat produced in resistance R_1 is:	**A**

		
	$\frac{LE^2}{2R_2^{\ 2}}$	
	$\frac{LE^2}{2R_1^{\ 2}}$	
	$\frac{LE^2}{2R_1R_2}$	
	$\frac{LE^2(R_1+R_2)^2}{2R_1^{\ 2}R_2^{\ 2}}$	
Sol.	$\boxed{I=\frac{E}{R_2}}$ $U_{Inductor}$ = Heat loss in $R_1=\frac{1}{2}L\left(\frac{E}{R_2}\right)^2$	
14.	A solid cylinder attached to horizontal massless spring can roll without slipping along horizontal surface. Find time period of oscillation. 	**D**
	$2\pi\sqrt{\frac{M}{2k}}$	
	$\pi\sqrt{\frac{3M}{2k}}$	
	$\pi\sqrt{\frac{2M}{3k}}$	
	$2\pi\sqrt{\frac{3M}{2k}}$	

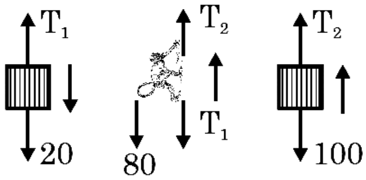
Sol.	 $\tau_A = I\alpha$ $K x R = \frac{3}{2} MR^2 \alpha$ $KR\theta R = \frac{3}{2} MR^2 \alpha$ $\alpha = \frac{2K}{3M}$ $\therefore \boxed{T = 2\pi\sqrt{\frac{3M}{2K}}}$	
15.	Assuming the sun to be a spherical body($e = 1$) of radius R at a temperature of T K, evaluate the total radiant power, incident on Earth having radius r_0, at a distance r from the Sun, where r_0 is the radius of the earth and σ is Stefan's constant.	**A**
	$\frac{\pi r_0^2 R^2 \sigma T^4}{r^2}$	
	$\frac{\pi_0^2 R^2 \sigma T^4}{4\pi r^2}$	
	$\frac{R^2 \sigma T^4}{r^2}$	
	$\frac{4\pi r_0^2 R^2 \sigma T^4}{r^2}$	
Sol.	 $\frac{e A_{sun} \sigma T^4}{4\pi r^2} \times A_{proj\,Earth}$ $\frac{4\pi R^2}{4\pi r^2} \sigma T^4 \pi r_0^2$ $\pi \frac{R^2}{r^2} r_0^2 \sigma T^4$	
16.	A plastic circular disc of radius 10 cm is placed on a thin oil film of thickness 2 mm, spread over a flat horizontal surface. The torque (N-m) required to spin the disc about its central vertical axis with a constant angular velocity 8 rad/sec is (coefficient of viscosity of oil is 1.0 kg/m-s)	**B**

	(take $\pi = 3.14$)	
	0.12	
	0.63	
	1.26	
	0.31	
Sol.	 $\tau = \int F \cdot r = \int_0^R \eta (2\pi r\, dr)\left(\frac{rw}{t}\right) \cdot r = \frac{2\pi\eta W R^4}{4t} = \frac{2 \times 3.14 \times 1 \times 8 \times 10^{-4}}{4 \times 2 \times 10^{-3}}$ $= 0.625\,N - m \simeq 0.63\,N - m$	
17.	A railway track (made of iron) is laid in winter when the average temperature is 18°C. The track consists of sections of 12.0 m placed on after the other. How much gap (in cm) should be left between two such sections so that there is no compression during summer when the maximum temperature goes to48°C ? Coefficient of linear expansion of iron = 11×10^{-6}/°C.	**A**
	0.4	
	0.8	
	0.2	
	0.1	
Sol.	$\Delta L = L_0\, \alpha\, \Delta T$ $= 12 \times 11 \times 10^{-6} \times 30$ $= 3960 \times 10^{-6}$ m	
18.	A string of mass per unit length m is clamped at both ends such that one end of the string is at x = 0 and the other end at x = L. When string vibrates in fundamental mode, amplitude of the midpoint of string is a and tension in string is F. Find the total oscillation energy (in J) stored in the string. (Use $L = 1m$, $F = 10$ N, $a = \frac{1}{\pi}m$) 	**C**
	1.5	
	3.5	
	2.5	
	5.5	
Sol.	$\frac{\pi^2 a^2 F}{4L} = \pi^2 \times \frac{1}{\pi^2} \frac{10}{4 \times 1} = 2.5J$	
19.	A monochromatic light of wavelength 6500Åis used in YDSE. Now both slits are covered by two thin slab of refractive index 3.2 and2.7 respectively. By doing so central bright fringe shifts to original 7^{th} bright fringe. If both slabs have same thickness then thickness of slab is (in μm).	**B**
	3.1	

	9.1	
	4.5	
	7.3	
Sol.	$7\times\frac{D\lambda}{d}=\frac{(\mu_1-\mu_2)t\cdot D}{d}$ $t = 9.1\ \mu m$	
20.	A hollow sphere (mass m, radius R) is put inside a hollow cone (mass m, radius 3R and semi-angle 30°) as shown in the figure. The whole arrangement is rigidly fixed to the ground at the apex of the cone. Both the sphere and the cone are made of the same material ($\alpha = 10^{-4}/°C$). Considering that major changes in dimensions could happen only due to heating effects, find the increase in potential energy (in Joule) of the system if it's temperature is increased by 100°C.  $m = 0.5kg$ $R=\frac{\sqrt{3}-1}{2}m$	**B**
	0.05	
	0.10	
	0.15	
	0.20	
Sol.	 $\theta = 30°$ $\therefore\ AB=\frac{R}{\sin\theta}=2R$ Also, height of cone $H = 3R\cot\theta$ $\therefore\ \Delta U = mg\Delta(2R)+mg\Delta\left(\frac{2H}{3}\right)$ But $\Delta R = R\alpha\Delta T$ and $\Delta H = H\alpha\Delta T$ Putting values, $\Delta U = +0.1\ J$	
21.	Two blocks of mass 10 kg and 2 kg respectively are connected by an ideal string passing over a fixed smooth pulley as shown in figure. A monkey of 8kg started climbing the string with constant acceleration $2ms^{-2}$ with respect to string at t = 0. Initially the monkey is 2.4 m from the pulley. Find the time taken by the monkey to reach the pulley.	**2**



Sol. Let the acceleration of string = a



For 2 kg $20 - T_1 = 2a$ ....(i)

For monkey $T_2 - 80 - T_1 = 8(2 - a)$ .....(ii)

For 10 kg $T_2 - 100 = 10a$ ....(iii)

$\therefore$ $a = 0.8\ m/s^2$

Acceleration of monkey w.r. to ground

$= 2 - 0.8 = 1.2\ m/s^2$

$$s = ut + \frac{1}{2}at^2$$

$$2.4 = 0 + \frac{1}{2} \times 1.2 \times t^2$$

t = 2 sec.

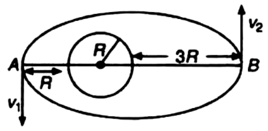
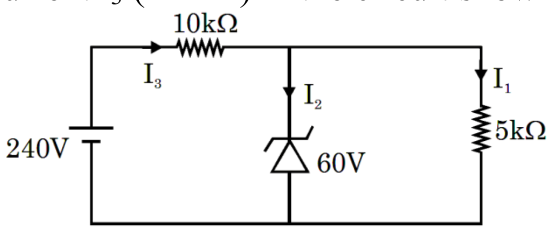
22. A parallel beam of monochromatic light is incident on a narrow rectangular slit of width 1 mm. When the diffraction pattern is seen on a screen placed at a distance of2 m, the width of principal maxima is found to be 2.5 mm. The wavelength of light is (in Å) **6250**

Sol. Here the width of principal maxima is 2.5 mm

Then its half width is $= \frac{\beta}{2}$

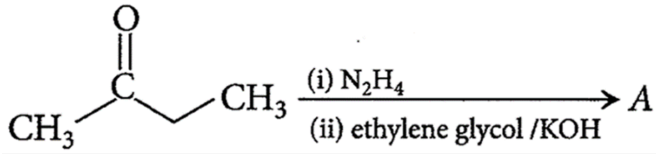
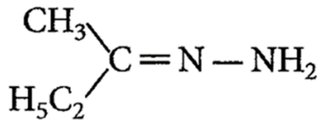
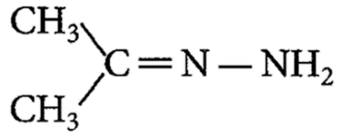
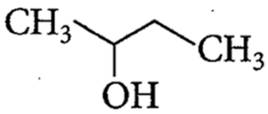
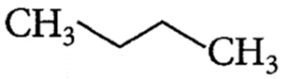
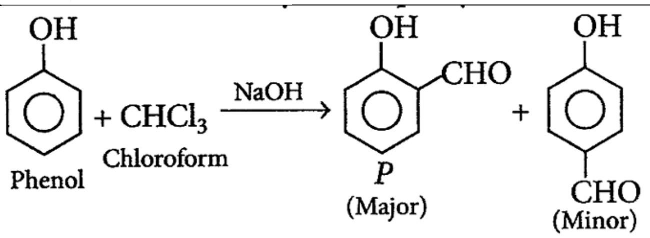
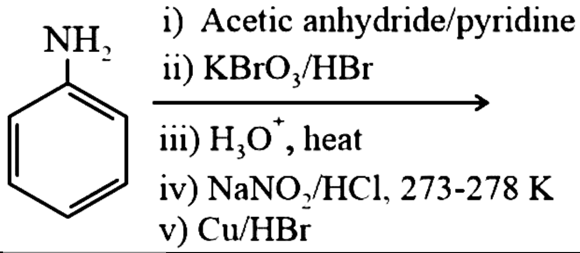
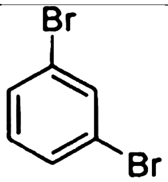
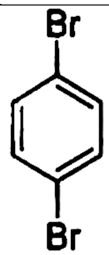
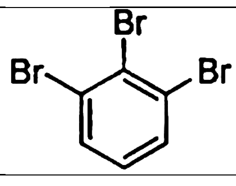
$$= \frac{2.5}{2} = 1.25 \times 10^{-3} m$$

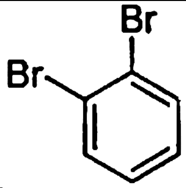
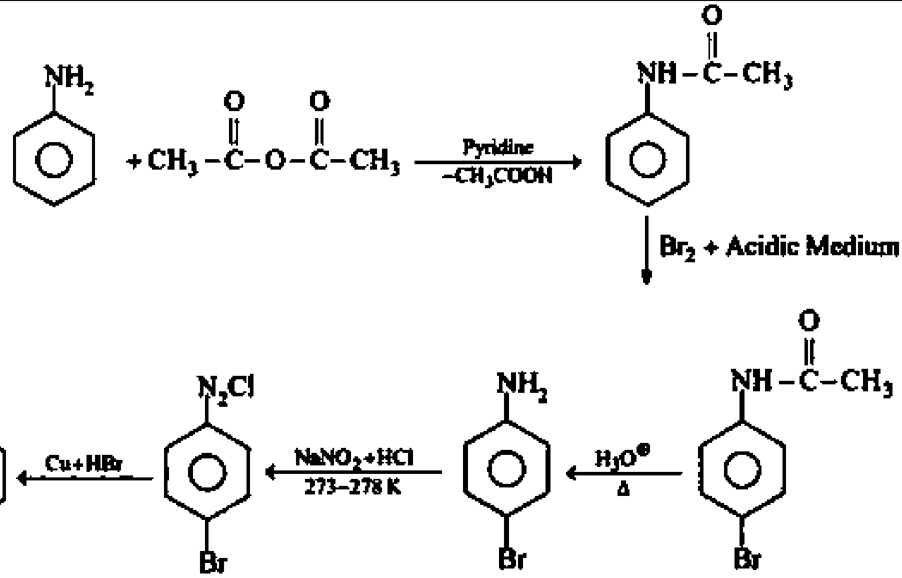
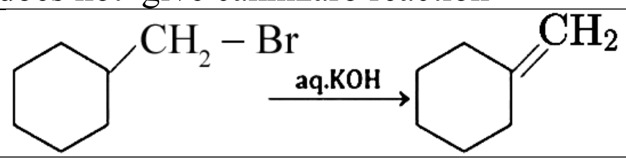
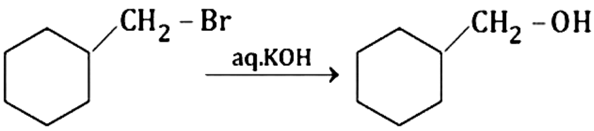
Diffraction angle $\theta = \frac{\left(\frac{\beta}{2}\right)}{D} = \frac{1.25 \times 10^{-3}}{2}$ ....(i)

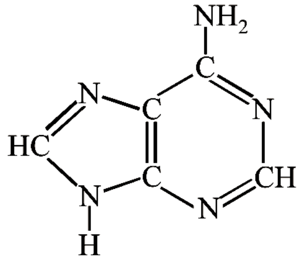
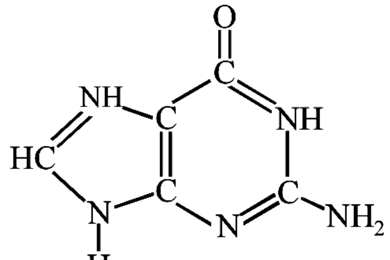
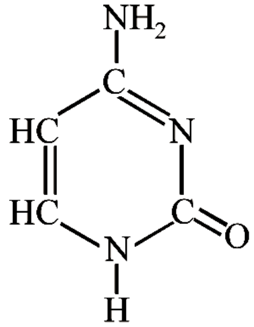
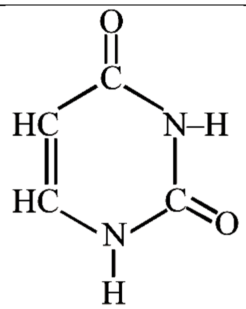
	$\therefore\ a\theta = \lambda \qquad \therefore\ \theta = \dfrac{\lambda}{a} \qquad ....(ii)$ From (i) and (ii) $\lambda = \theta a$ $= \dfrac{1.25\times10^{-3}}{2}\times10^{-3}$ $= 6.25 \times 10^{-7}$ m $\therefore\ \lambda = 6250\,\text{Å}$	
23.	The minimum and maximum distance of a satellite from the centre of the earth are 2Rand 4R, respectively, where R is the radius of earth and M is the mass of earth. The radius of curvature at the point of maximum distance is $\dfrac{xR}{3}$, then find the value of x.	**8**
Sol.	 (a) Applying conservation of angular momentum $mv_1(2R) = mv_2(4R)$ $v_1 = 2v_2 \quad ...(i)$ From conservation of energy $\frac{1}{2}mv_1^2 - \frac{GMm}{2R} = \frac{1}{2}mv_2^2 - \frac{GMm}{4R} \quad ...(ii)$ Solving Eqs. (i) and (ii), we get $v_2 = \sqrt{\frac{GM}{6R}}, \quad v_1 = \sqrt{\frac{2GM}{3R}}$ (b) If r is tl point A $\frac{mv_1^2}{r} = \frac{GM}{(2R)}$ $r = \frac{4v_1^2R^2}{GM} = \frac{8R}{3}$	
24.	In Young's double slit experiment, the wavelength of red light is 7800 Å and that of blue light is 5200 Å. The minimum value of n for which nth bright band due to red light coincides with (n + 1) the bright band due to blue light, is :	**2**
Sol.	n (7800) = (n + 1) 5200	
25.	Write the value of current i_3 (in mA) in the circuit shown in the figure. 	**18**

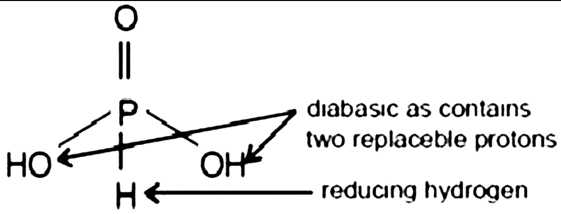
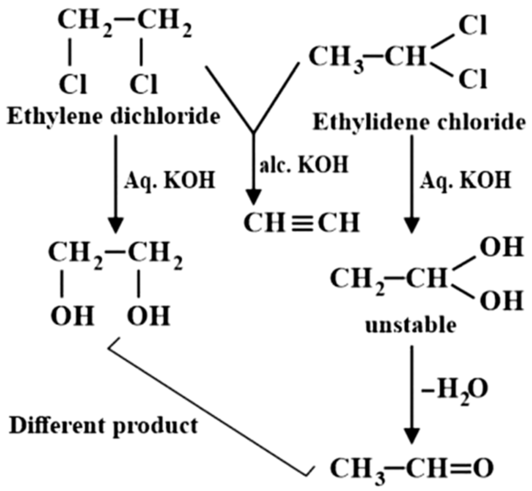
Sol.	$5 \times 10^3 I_1 = 60$ $\Rightarrow I_1 = \frac{60}{5\times10^3} = 12mA$ Potential difference across $10\ k\Omega = 240 - 60 = 180\ V$ So, $I_3 \times 10 \times 10^3 = 180$ $\Rightarrow I_3 = 18\ mA$ $I_2 = I_3 - I_1 = 6\ mA$	
26.	Which product will be obtained by Grignard reaction, when Formaldehyde reacts and Ethyl magnesium iodide?	**B**
	2-Propanol	
	1-Propanol	
	Ethanol	
	2-Methyl, 2-Propanol	
Sol.	$HCHO + C_2H_5MgI \rightarrow C_2H_5-\underset{\displaystyle H}{\overset{\displaystyle OH}{\underset{\vert}{\overset{\vert}{C}}}}-H$ (1 - propanol) $C_2H_5 - CH_2 - OH$ is formed in the reaction	
27.	Pick out **incorrect** statement	**A**
	In a electrolysis experiment, α-amino acids migrate at the isoelectric point towards electrodes	
	p-aminobenzenesulphonic acid as a dipolar ion; while p-aminobenzoic acid does not	
	Sulphanilic acid is soluble in base, but not in acid	
	$H_3\overset{+}{N}CH_2COOH$ is more acidic than RCH_2COOH	
Sol.	A. The pH at which Anion = Cation is called isoelectric point. At isoelectric point, α -amino acids do not migrate when electric field is applied. B. $-SO_3H$ is strongly acidic and donates H^+ to weakly basic arylamino group. ArCOOH is not acidic enough to transfer H^+ to the arylamino group. C. In $p-H_3\overset{+}{N}C_6H_4SO_3, H_3\overset{+}{N}$ is acid enough to transfer H^+ to bases to give the soluble anion, $p-H_2N-C_6H_4SO_3$. $-SO_3$ is too feebly basic and cannot accept H^+ from acids. D. α-$H_3\overset{+}{N}$ group increases acidity, because of its electron-withdrawing inductive effect.	
28.	Four successive members of the first row transition elements are listed below with atomic numbers. Which one of them is expected to have to	**D**

	highest $E^o_{M^{3+}/M^{2+}}$, value?	
	Fe(Z = 26)	
	Mn (Z = 25)	
	Cr (Z = 24)	
	Co(Z = 27)	
Sol.	$E^o_{Cr^{3+}/Cr^{2+}} = -0.41V; E^o_{Mn^{3+}/Mn^{2+}} = +1.57V;$ $E^o_{Fe^{3+}/Fe^{2+}} = +0.77V;$ $E^o_{Co^{3+}/Co^{2+}} = +1.97V$ Standard reduction potential (SRP) value normally increases from left to right in the period of d-block elements. Some SRP values are exceptionally higher due to stability of the product ion. For e.g., $E^o_{Mn^{3+}/Mn^{2+}} = +1.57V; E^o_{Co^{3+}/Co^{2+}} = +1.97V$	
29.	$(X) + K_2CO_3 + Air \xrightarrow{heat} (Y)$ $(Y) + Cl_2 \rightarrow (Z)$ Pink Which of the following is correct?	**C**
	X = Black, MnO_2, Y = Blue, K_2CrO_4, Z = MnO_4	
	X = Green, Cr_2O_3, Y = Yellow, K_2CrO_4, Z = $K_2Cr_2O_7$	
	X = Black, MnO_2, Y = Green, K_2MnO_4, Z = $KMnO_4$	
	X = Black, Bi_2O_3, Y = Colourless, $KBiO_2$, Z = $KBiO_3$	
Sol.	$\underset{(X)}{2MnO_2} + 2K_2CO_3 + \underset{(air)}{O_2} \xrightarrow{\Delta} \underset{\substack{green\\(Y)}}{2K_2MnO_4} + 2CO_2\,(g)$ $\underset{(Y)}{2K_2MnO_4} + Cl_2 \longrightarrow \underset{(Z)\ Pink}{2KMnO_4} + 2KCl$	
30.	The enthalpy changes for the following processes are listed below: $Cl_2(g) = 2Cl(g)$, 242.3 kJ mol^{-1} $I_2(g) = 2I(g)$, 151.0 kJ mol^{-1} $ICl(g) = I(g) + Cl(g)$, 211.3 kJ mol^{-1} $I_2(s) = I_2(g)$, 62.76 kJ mol^{-1} Given that the standard states for iodine and chlorine are $I_2(s)$ and $Cl_2(g)$, the standard enthalpy of formation for ICl(g) is	**C**
	– 14.6 kJ mol^{-1}	
	– 16.8 kJ mol^{-1}	
	+ 16.8 kJ mol^{-1}	
	+ 244.8 kJ mol^{-1}	
Sol.	$\frac{1}{2}I_{2\,(s)} + \frac{1}{2}Cl_{2\,(g)} \rightarrow ICl_{\,(g)}$ $\Delta H_{ICl(g)} = \left[\frac{1}{2}\Delta H_{I_2(s)\rightarrow I_2(g)} + \frac{1}{2}\Delta H_{I-I} + \frac{1}{2}\Delta H_{Cl-Cl}\right] - [\Delta H_{I-Cl}]$ $= \left[\frac{1}{2}\times 62.76 + \frac{1}{2}\times 151.0 + \frac{1}{2}\times 242.3\right] - [211.3]$ $= [31.38 + 75.5 + 121.15] - 211.3 = 228.03 - 211.3$ $= 16.73$ kJ/mol	

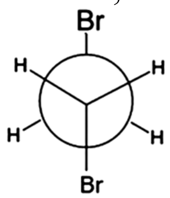
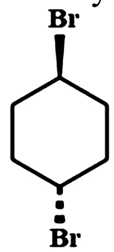
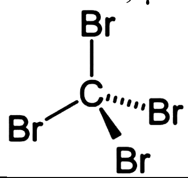
31.	Identify A in the following reaction.  	**D**
		
		
		
		
Sol.	 Molecular formula of major product, $P = C_7H_6O_2$ Molar mass of product, $P = (7 \times 12 + 6 \times 1 + 16 \times 2)$ $= 122$ g/mol $\therefore$ The mass percentage of carbon in P $= \frac{84}{122} \times 100 = 68.85\% \approx 69\%$	
32.	The product (s) of the following reaction sequence is (are)  	**B**
		
		
		

		
Sol.		
33.	Among the following which statement is incorrect?	**D**
	$CH_3-\overset{\overset{\large O}{\Vert}}{C}-OH \xrightarrow{X_2/OH^-}$ Will not respond to haloform test	
	$CH_2=CH-CH=O \xrightarrow{\overset{\oplus}{O}H/\Delta}$ gives cannizaro reaction	
	$Cl-\underset{\underset{\large Cl}{\vert}}{\overset{\overset{\large Cl}{\vert}}{C}}-CH=O \xrightarrow{\overset{\oplus}{O}H/\Delta}$ does not give cannizaro reaction	
	 (Major)	
Sol.	(i) is correct (ii) $CH_2=CH-CH=O \xrightarrow{OH^-}$ give cannizzaro (iii) $CCl_3-CHO \xrightarrow{H^-\Delta} CHCl_3 + HCOO^-Na^+$ (iv) 	
34.	Which of the nitrogen containing heterocyclic compound (BASE) is not present in DNA :	**D**

	 Adenine	
	 Guanine	
	 Cytosine	
	 Uracil	
35.	10 mL of 1mM surfactant solution forms a monolayer covering 0.24 cm^2 on a polar substrate. If the polar head is approximated as cube, what is its edge length	**A**
	2.0 pm	
	2.0 nm	
	1.0 pm	
	0.1 nm	
Sol.	Millimoles $= 10 \times 10^{-3} = 10^{-2}$ Moles $= 10^{-5}$ No. of molecules $= 6 \times 10^{23} \times 10^{-5} = 6 \times 10^{18}$ Surface area ocupied by one molecule $= \frac{0.24}{6\times10^{18}} = 0.04 \times 10^{-18} cm^2$ $4 \times 10^{-20} = x^2 \left[\because x = \text{edge length}\right]$ $x = 2 \times 10^{-10}$ cm = 2 pm	
36.	For H_3PO_3 and H_3PO_4 correct choice is	**A**
	H_3PO_3 is dibasic and reducing agent	

	H_3PO_3 is diabasic and non-reducing agent	
	H_3PO_4 is tribasic and reducing agent	
	H_3PO_4 is dibasic and non-reducing agent.	
Sol.	 As oxidation state of P is + 3 can be decreased to – 3 and increased to + 5 so it can act both as a reducing agent as well as an oxidising agent.	
37.	Which of the following statements is **incorrect** for ethylene dichloride and ethylidene chloride?	**C**
	These are structural isomers	
	Both of these yields same product on reaction with alcoholic KOH solution	
	Both of these yields same product on treatment with aqueous KOH solution	
	Both of these yields same product on reduction	
Sol.		
38.	A compound (X) of molecular formula C_3H_6O forms bisulphate complex, gives iodoform test but does not reduce Tollens reagent. (X) on reaction with CH_3MgBr/H_3O^+ gives a compound (Y) that **cannot**	**C**
	Give red colour with cerium Ammonium Nitrate (CAN)	
	Give white turbidity immediately with Lucas reagent	
	Give iodoform test	
	Be dehydrated to alkene on reaction with heated Cu	
Sol.	In given conditions X (C_3H_6O) should be a methyl ketone	
39.	Calcium lactate is a salt of weak acid i.e., lactic acid having general formula $Ca(LaC)_2$. Aqueous solution of salt has 0.3 M concentration. pOH of solution is 5.60 if 90% of the salt is dissociated then what will be the value of pK_a?	**A**
	2.8 – log (0.54)	
	2.8 + log (0.54	
	2.8 + log (0.27)	
	None of these	

Sol.	$Ca(LaC)_2 \rightarrow Ca^{2+} + 2LaC^-$ Concentration of salt $= \frac{0.6}{2} = 0.3$ M Concentration of (LaC^-) after dissociation $= 2 \times 0.3 \times 0.9 = 0.54$ M pH after salt hydrolysis may be calculated as, $pH = \frac{1}{2}[(pK_w + pK_a + \log(LaC^-)]$ $14 - 5.6 = \frac{1}{2}[14 + pK_a + \log 0.54]$ $pK_a = 2.8 - \log 0.54$	
40.	A white solid on reaction with dil. H_2SO_4 gives a gas (B) and a compound (C). Compound (B) gives lime water test and compound (C) gives Brick red colour in flame test. Identify A.	**B**
	$CaSO_4$	
	$CaCO_3$	
	$BaCl_2$	
	$SrCrO_4$	
41.	Which of the following statement is incorrect for ligand-metal complex	**B**
	Larger the donor atom of the ligand the more stable is the metal-ligand bond	
	Highly charged ligand forms strong bond	
	Larger the permanent dipole moment of ligand, the more stable is the bond	
	Greater the ionization potential of central metal, the stronger is the bond	
Sol.	The charge does not decide the formation of bond but availability of lone pair decide the formation of Co-ordinate bond.	
42.	**Assertion :** Azeotropic mixture are formed only by non-ideal solution and they may have boiling points either greater than both the components or less than both the components. **Reason :** The composition of the vapour phase is same as that of the liquid phase of an azeotropic mixture.	**B**
	If both assertion and reason are true and the reason is correct explanation of the assertion	
	If both assertion and reason are true and the reason is not the correct explanation of the assertion	
	It assertion is true but reason is false.	
	It assertion is false but reason is true.	
Sol.	Non-ideal solutions with positive deviation i.e., having more vapour pressure than expected. Boil at lower temperature while those with negative deviation boil at higher temperature than those of the compounds.	
43.	Which of the following molecules have zero dipole moment? **(I)** Gauche conformation of 1, 2-dibromoethane **(II)** Anti conformation of 1, 2-dibromoethane. **(III)** Trans-1, 4-dibromocyclohexane **(IV)** cis-1, 4-dibromocylohexane **(V)** Tetrabromomethane **(VI)** 1, 1-dibromocyclohexane	**B**

	I and II	
	II, III and V	
	II and V	
	I, IV and VI	
Sol.	(II) Anti conformation of 1, 2-dibromoethane, $\mu = 0$  (III) Trans-1, 4-dibromocyclohexane, $\mu = 0$  (v) tetrabromoethane, $\mu = 0$ 	
44.	Arrange the following molecules from most to least polar. CH_4 (**I**) CF_2Cl_2 (**II**) CF_2H_2 (**III**) CCl_4 (**IV**) CH_2Cl_2 (**V**)	**A**
	III > V > II > I = IV	
	II > IV > III > IV > I	
	III > II > V > IV > I	
	V > III > II > IV = I	
Sol.	(i) In CH_4 and CCl_4 being tetrahedral (Symmetrical molecules), bond dipoles cancel one other resulting in zero dipole moment. (ii) As electro negativity difference between two bonded atoms increases, the bond polarity increases resulting into the increase in dipole moment.	
45.	In latent heat of fusion of ice is 80 cals per g at 0°C, Calculate molal depression constant for water.	**C**
	18.63	
	186.3	
	1.863	
	0.1863	
Sol.	$K_f = \frac{RT_f^2}{1000 \times l_V}$ Here $R = 2$ cals , $T_f = 0 + 273 = 273,\ l_f = 80$ cals $K_f = \frac{2 \times 273 \times 273}{1000 \times 80} = 1.863$	
46.	Among the following, how many reagents 'A' are used to give black precipitate of metal sulphide ? $AgNO_3$, $BaCl_2$, $Pb(OAC)_2$, $HgCl_2$, $CaCl_2$, $CuSO_4$, $SbCl_3$.	**4**
Sol.	[$AgNO_3$, $Pb(OAc)_2$, $HgCl_2$, $CuSO_4$]	
47.	The weight of glucose dissolved in 100 gm of water produce same lowering	**6**

	of vapour pressure as 1 gm urea dissolved in 50 gm of water at same temperature is	
Sol.	since same lowering in vapour pressure. $X_{\text{solute of urea}} = X_{\text{solute of glucose}}$ $\frac{\frac{1}{60}}{\frac{1}{60}+\frac{50}{18}} = \frac{\frac{w}{180}}{\frac{w}{180}+\frac{100}{18}} \Rightarrow \frac{\frac{1}{60}}{\frac{50}{18}} = \frac{\frac{w}{180}}{\frac{100}{18}}$ (For dilute solution) $\Rightarrow \quad w = 6$ of glucose	
48.	One mole of a monatomic gas at pressure 2 atm, 279 K taken to final pressure 4 atm by a reversible path described by P/V = constant. Calculate the magnitude of $\frac{\Delta E}{w}$ of the process.	**3**
Sol.	$dE = dq + dw$ (F L T) $P_1 = K'V_1$ and $P_2 = K'V_2$ Also $V_1 = \frac{P_1}{K'}, V_2 = \frac{P_2}{K'}$ $T_2 = T_1\left(\frac{P_2V_2}{P_1V_1}\right) = T_1\left(\frac{P_2P_2/K'}{P_1P_1/K'}\right) = T_1\frac{P_2^2}{P_1^2}$ $T_2 = T_1\frac{4^2}{2^2} = 4T_1$ $\Delta E = C_v(\Delta T) = C_v(4T_1 - T_1)$ $= 3C_vT_1$ $= 3\frac{3}{2}RT_1 = 9\frac{RT_1}{2}$ $w = -\int_{V_1}^{V_2} pdv = -\int_{V_1}^{V_2} K'vdv = -\left\|\frac{K'v^2}{2}\right\|_{V_1}^{V_2}$ $\left(K'v^2 = K'vv = Pv = RT\right)$ $w = -\left\|\frac{RT}{2}\right\|_{T_1}^{T_2} = -\frac{R}{2}(T_2 - T_1) =$ $-\frac{R}{2}3T_1$ $\frac{\Delta E}{w} = \frac{9RT_1/2}{-3RT_1/2} = -3$	
49.	The number of essential amino acids among the following ……………. Histidine, Glycine, valine, Alanine, Asoartic acid, Lysine, Methionine.	**4**
Sol.	Histidine, Valine, Lysine, Methionine.	
50.	A given sample of milk turns sour at room temperature (27°C) in 16 hours. In a refrigerator at 7°C, milk can be stored three times as long before it sours. How long (in hours) it takes milk to sour at 57°C? [Given : log 3 = 0.5, $10^{(7/11)}$ = 100.64 = 4]	**4**

Sol.	$\ell n\frac{K_2}{K_1}=\ell n\frac{t_1}{t_2}=\frac{E_a}{R}\left(\frac{1}{T_1}-\frac{1}{T_2}\right)$ $\ell n\frac{1}{3}=\frac{E_a}{R}\left(\frac{1}{300}-\frac{1}{280}\right)$ and $\ell n\frac{16}{t}=\frac{E_a}{R}\left(\frac{1}{300}-\frac{1}{330}\right)$ t = 4 hrs.	
51.	If the set $\left\{Re\left(\frac{z-\bar{z}+z\bar{z}}{2-3z+5\bar{z}}\right): z \epsilon C, Re(z)=3\right\}$ is equal to the interval $(\alpha,\beta]$, then $24(\beta-\alpha)$ is equal to	**B**
	36	
	30	
	27	
	42	

Sol.	(b) Let $w=\left(\frac{z-\bar{z}+z\bar{z}}{2-3z+5\bar{z}}\right), z\in C$ where $z=x+iy$ Since, $\text{Re}(z)=3 \Rightarrow z=3+iy$ $\Rightarrow \bar{z}=3-iy$ $w=\left(\frac{z-\bar{z}+z\bar{z}}{2-3z+5\bar{z}}\right)$ $=\left(\frac{(3+iy)-(3-iy)+(3+iy)(3-iy)}{2-3(3+iy)+5(3-iy)}\right)$ $=\left(\frac{2iy+9+y^2}{2-9-3iy+15-5iy}\right)$ $=\left(\frac{(9+y^2)+i(2y)}{8-8iy}\right)$ $=\frac{[(9+y^2)+i(2y)](1+iy)}{8(1-iy)(1+iy)}$ $=\frac{(9+y^2)+i(2y)+iy(9+y^2)-2y^2}{8(1+y^2)}$ $\therefore w=\frac{(9+y^2-2y^2)+i(2y+9y+y^3)}{8(1+y^2)}$ $\text{Re}(w)=\frac{9-y^2}{8(1+y^2)}$ $=\frac{1}{8}\left[\frac{10-(1+y^2)}{1+y^2}\right]=\frac{1}{8}\left[\frac{10}{1+y^2}-1\right]$ Since, $1+y^2\in[1,\infty)$ $\Rightarrow \frac{1}{1+y^2}\in(0,1] \Rightarrow \frac{10}{1+y^2}\in(0,10]$ $\Rightarrow \frac{10}{1+y^2}-1\in(-1,9]$ $\Rightarrow \frac{1}{8}\left(\frac{10}{1+y^2}-1\right)\in\left(-\frac{1}{8},\frac{9}{8}\right]$ $\because \frac{1}{8}\left(\frac{10}{1-y^2}-1\right)\in(\alpha,\beta]$ [given] $\therefore \alpha=-\frac{1}{8}, \beta=\frac{9}{8}$ Now, $24(\beta-\alpha)=24\left(\frac{9}{8}+\frac{1}{8}\right)=24\left(\frac{10}{8}\right)$ $=30$	
52.	A candidate is required to answer 6 out of 10 questions, which are divided into two groups, each containing 5 questions. He is not permitted to attempt more than 4 questions from either group. The number of different ways in which the candidate can choose 6 questions is	**C**
	50	
	150	
	200	
	250	

Sol.	The number of ways the candidate can choose questions under the given conditions is enumerated below.	

Group 1	Group 2	Number of ways
4	2	$({}^5C_4)({}^5C_2) = 50$
3	3	$({}^5C_3)({}^5C_3) = 100$
2	4	$({}^5C_2)({}^5C_4) = 50$
	Total number of ways	200

53.	Let the binomial coefficient of three consecutive terms T_r, T_{r+1} & T_{r+2} in the binomial expansion of $(a+b)^{12}$ be in a G.P and let p be the number of all possible values of r. Let q be the sum of all rational terms in the binomial expansion of $\left(\sqrt[4]{3}+\sqrt[3]{4}\right)^{12}$. Then $p+q$ is equal to______.	**D**
	299	
	287	
	295	
	283	
Sol.	$\boxed{p=0}$ $\left(3^{\frac{1}{4}}+4^{\frac{1}{3}}\right)^{12}$ ${}^{12}C_r\, 3^{\frac{12-r}{4}} \cdot 4^{\frac{r}{3}}$ $r = 0, 3, 6, 9, 12$ $q = {}^{12}C_0\, 3^3 + {}^{12}C_{12}\, 4^4$ 283 $27+256$ $T_r = {}^{12}C_{r-1}\, a\, b$ $T_{r+1} = {}^{12}C_r\, a\, b$ $T_{r+2} = {}^{12}C_{r+1}\, a\, b$ $\frac{{}^{12}C_r}{{}^{12}C_{r-1}} = \frac{{}^{12}C_{r+1}}{{}^{12}C_r}$ $\frac{13-r}{r} = \frac{12-r}{r+1}$ $(13-r)(r+1) = (12-r)\cdot r$ $13\cancel{r} - \cancel{r^2} - \cancel{r} + 13 = \cancel{12r} - \cancel{r^2}$ No value of r exist $(13=0)$	
54.	Let $a_1 = b_1 = 1$ and $a_n = a_{n-1} + (n-1), b_n = b_{n-1} + a_{n-1}, \forall\, n \geq 2$. If $S = \sum_{n=1}^{10} \frac{b_n}{2^n}$ and $T = \sum_{n=1}^{8} \frac{n}{2^{n-1}}$, then $2^7(2S-T)$ is equal to______.	**A**
	461	
	450	
	660	
	750	

Sol.

Given, $S=\sum_{n=1}^{10}\frac{b_n}{2^n}$

$=\frac{b_1}{2}+\frac{b_2}{2^2}+.......+\frac{b_9}{2^9}+\frac{b_{10}}{2^{10}}$

$\Rightarrow \quad \frac{S}{2}=\frac{b_1}{2^2}+\frac{b_2}{2^3}+.......+\frac{b_9}{2^{10}}+\frac{b_{10}}{2^{11}}$

Subtracting, we get

$\frac{S}{2}=\frac{b_1}{2}+\left(\frac{a_1}{2^2}+\frac{a_2}{2^3}+......+\frac{a_9}{2^{10}}\right)-\frac{b_{10}}{2^{11}}$

$\Rightarrow \quad S=b_1-\frac{b_{10}}{2^{10}}+\left(\frac{a_1}{2}+\frac{a_2}{2^2}+.....+\frac{a_9}{2^9}\right)$

$\Rightarrow \quad \frac{S}{2}=\frac{b_1}{2}-\frac{b_{10}}{2^{11}}+\left(\frac{a_1}{2^2}+\frac{a_2}{2^3}+.....+\frac{a_9}{2^{10}}\right)$

Subtracting, we get

$\frac{S}{2}=\frac{b_1}{2}-\frac{b_{10}}{2^{11}}+\left(\frac{a_1}{2}-\frac{a_9}{2^{10}}\right)$

$+\left(\frac{1}{2^2}+\frac{2}{2^3}+......+\frac{8}{2^9}\right)$

$\Rightarrow \quad \frac{S}{2}=\frac{a_1+b_1}{2}-\frac{(b_{10}+2a_9)}{2^{11}}+\frac{T}{4}$

$\Rightarrow \quad 2S=2(a_1+b_1)-\frac{b_{10}+2a_9}{2^9}+T$

$\Rightarrow \quad 2^7(2S-T)=2^8(a_1+b_1)-\frac{(b_{10}+2a_9)}{4}$

Given $a_n-a_{n-1}=n-1$

$\therefore \quad a_2-a_1=1$

$a_3-a_2=2$

$a_9-a_8=8$

$a_9-a_1=1+2+.......8=36$

$\Rightarrow \quad a_9=37$

Also, $b_n-b_{n-1}=a_{n-1}$

$\therefore \quad b_{10}-b_1=a_1+a_2+......+a_9$

$=1+2+4+7+11+16+22+29+37$

$\Rightarrow \quad b_{10}=130$

$\therefore \quad 2^7(2S-T)=2^8(1+1)-\frac{(130+2\times 37)}{4}$

$=461$

55. Let a conic C pass through the point $(4,-2)$ and $P(x,y), x\geq 3$, be any point on C. Let the slope of the line touching the conic C only at a single point P be half the slope of the line joining the points P and $(3,-5)$. If the focal distance of the point
(7, 1) on C is d, then $12d$ is equals_______. **D**

	80
	85
	90
	75

Sol.

Slope of C at $P=\frac{1}{2}\left(\frac{y+5}{x-3}\right)$

$\Rightarrow \quad \frac{dy}{dx}=\frac{1}{2}\left(\frac{y+5}{x-3}\right)$

$\Rightarrow \quad \frac{2dy}{y+5}=\frac{1}{x-3}dx$

On integrating, we get

$2\ln(y+5)=\ln(x-3)+C$

Since, C passes through $(4,-2)$

$\Rightarrow \quad 2\ln 3=C$

$\Rightarrow \quad 2\ln(y+5)=\ln(x-3)+2\ln 3$

$\Rightarrow \quad 2\ln\left(\frac{y+5}{3}\right)=\ln(x-3)$

$\Rightarrow \quad \left(\frac{y+5}{3}\right)^2=x-3$

$\Rightarrow \quad (y+5)^2=9(x-3)$, which represent a parabola so, $4a=9$

$\Rightarrow \quad a=\frac{9}{4}$

Focus $=\left(3+\frac{9}{4},-5\right)=\left(\frac{21}{4},-5\right)$

$\therefore \quad d=\sqrt{\left(\frac{21}{4}-7\right)^2+(-5-1)^2}$

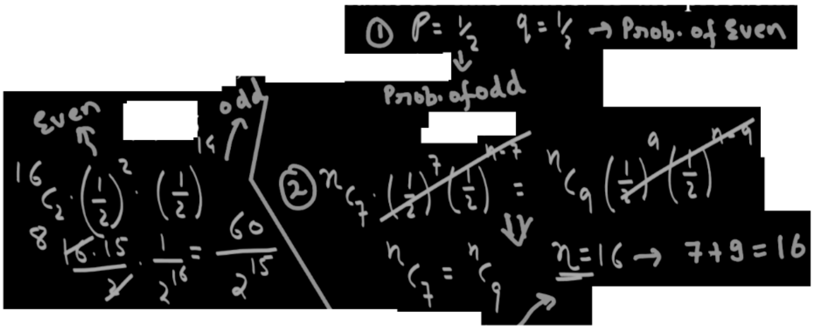
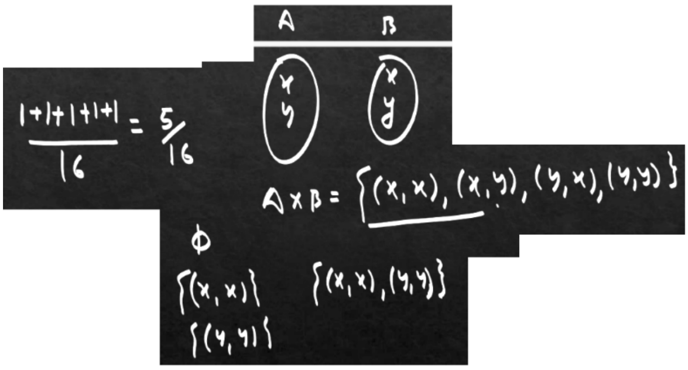
$=\sqrt{\left(-\frac{7}{4}\right)^2+(-6)^2}=\frac{25}{4}$

$\Rightarrow \quad 12d=\frac{25}{4}\times 12=75$

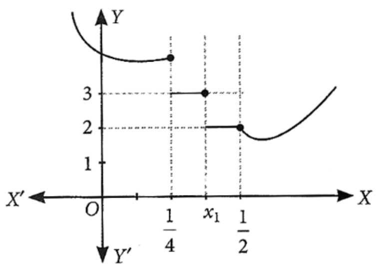
56. If $\lim_{n\to\infty}\left(\sqrt{n^2-n-1}+n\alpha+\beta\right)=0$, then $8(\alpha+\beta)$ is equal to **C**

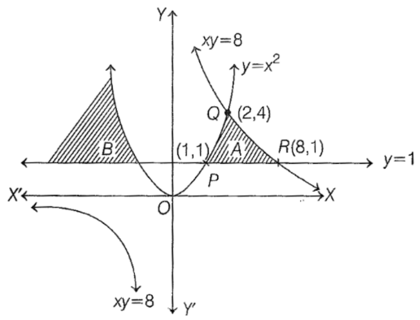
	4

	–8	
	–4	
	8	
Sol.	(c) Using binomial $\lim\limits_{n \to \infty} n\left(1-\frac{n+1}{n^2}\right)^{\frac{1}{2}} + \alpha n + \beta = 0$ $\lim\limits_{n \to \infty} n\left\{1-\frac{1}{2}\left(\frac{n+1}{n^2}\right) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(\frac{n+1}{n^2}\right)^2 + \ldots\right\} + \alpha n + \beta = 0$ $\lim\limits_{n \to \infty}\left(n - \frac{1}{2} + \frac{1}{n} + \ldots + n\alpha + \beta\right) = 0$ $\lim\limits_{n \to \infty}\left[(\alpha+1)n + \left(\beta - \frac{1}{2}\right) + \frac{1}{n} + \ldots\right]$ $\alpha = -1, \beta = \frac{1}{2} \Rightarrow 8(\alpha+\beta) = -4$	
57.	Given, $a < -1$ and $a < -1$ with ABCD and *r*. Let the slope of the line AB equals $2r$. Point BD lies on the line ABD such that the slope of CBD equals c_1 where C. If the area of the c_1 can be expressed as $\frac{2\sqrt{5}r}{5}$, then the largest possible value of $\frac{2\sqrt{10}r}{5}$ is	**D**
	$\frac{\sqrt{85}r}{5}$	
	$\frac{4r}{5}$	
	$\angle ABC_1 = \theta$	
	cos θ	
Sol.	Let the coordinates of C be $(1, c)$ Here, $m_2 = \frac{c-y}{1-x} \Rightarrow m_2 = \frac{c-m_1x}{1-x}$ (Let $y = mx$) $\Rightarrow m_2 - m_2x = c - m_1x \quad \Rightarrow c = (m_1 - m_2)x + m_2$ ....(i) Now, area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ x & m_1x & 1 \\ 1 & c & 1 \end{vmatrix} = \frac{1}{2}(cx - m_1x)$ $= \frac{1}{2}\left[\left((m_1-m_2)x + m_2\right)x - m_1x\right] = \frac{1}{2}\left[(m_1-m_2)x^2 + m_2x - m_1x\right]$ $= \frac{1}{2}(m_1-m_2)(x-x^2) \qquad \left[\because x > x^2 \text{ in } (0,1)\right]$ $\therefore f(x) = \frac{1}{2}(x-x^2) \qquad \Rightarrow \max f(x) = \frac{1}{8}$ when $x = \frac{1}{2}$	
58.	Let a die be rolled *n* times. Let the probability of getting odd numbers seven times be equal to the probability of getting odd numbers nine times. If the probability of getting even numbers twice is $\frac{k}{2^{15}}$, then *k* is equal to	**A**
	60	
	15	
	90	
	30	

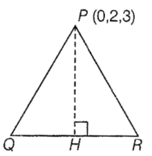
Sol.		
59.	The probability that a relation R from $\{x, y\}$ to $\{x, y\}$ is both symmetric and transitive, is equal to:	**A**
	$\frac{5}{16}$	
	$\frac{9}{16}$	
	$\frac{11}{16}$	
	$\frac{13}{16}$	
Sol.		
60.	Let $A = \begin{bmatrix} 1 & \frac{1}{51} \\ 0 & 1 \end{bmatrix}$. If $B = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} A \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$, then the sum of all the elements of the matrix $\sum_{n=1}^{50} B^n$ is equal to	**A**
	100	
	75	
	125	
	50	

Sol.	*(a)* Given, $A=\begin{bmatrix}1 & \frac{1}{51}\\ 0 & 1\end{bmatrix}$ $\Rightarrow A^2=\begin{bmatrix}1 & \frac{1}{51}\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & \frac{1}{51}\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1 & \frac{2}{51}\\ 0 & 1\end{bmatrix}$ and so on $\Rightarrow A^n=\begin{bmatrix}1 & \frac{n}{51}\\ 0 & 1\end{bmatrix}$ ...(i) $B=\begin{bmatrix}1 & 2\\ -1 & -1\end{bmatrix}A\begin{bmatrix}-1 & -2\\ 1 & 1\end{bmatrix}$ $B^2=\begin{bmatrix}1 & 2\\ -1 & -1\end{bmatrix}A\begin{bmatrix}-1 & -2\\ 1 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\ -1 & -1\end{bmatrix}A\begin{bmatrix}-1 & -2\\ 1 & 1\end{bmatrix}$ $=\begin{bmatrix}1 & 2\\ -1 & -1\end{bmatrix}A\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}A\begin{bmatrix}-1 & -2\\ 1 & 1\end{bmatrix}$ $=\begin{bmatrix}1 & 2\\ -1 & -1\end{bmatrix}A^2\begin{bmatrix}-1 & -2\\ 1 & 1\end{bmatrix}$ and so on $\Rightarrow B^n=\begin{bmatrix}1 & 2\\ -1 & -1\end{bmatrix}A^n\begin{bmatrix}-1 & -2\\ 1 & 1\end{bmatrix}$ $\therefore \sum_{n=1}^{50}B^n=\begin{bmatrix}1 & 2\\ -1 & -1\end{bmatrix}(A+A^2+\ldots+A^{50})\begin{bmatrix}-1 & -2\\ 1 & 1\end{bmatrix}$ $=\begin{bmatrix}1 & 2\\ -1 & -1\end{bmatrix}\left(\begin{bmatrix}1 & 1/51\\ 0 & 1\end{bmatrix}+\begin{bmatrix}1 & 2/51\\ 0 & 1\end{bmatrix}+\ldots+\begin{bmatrix}1 & 50/51\\ 0 & 1\end{bmatrix}\right)\begin{bmatrix}-1 & -2\\ 1 & 1\end{bmatrix}$ $=\begin{bmatrix}1 & 2\\ -1 & -1\end{bmatrix}\begin{bmatrix}50 & \frac{1+2+\ldots+50}{51}\\ 0 & 50\end{bmatrix}\begin{bmatrix}-1 & -2\\ 1 & 1\end{bmatrix}$ $=\begin{bmatrix}1 & 2\\ -1 & -1\end{bmatrix}\begin{bmatrix}50 & 25\\ 0 & 50\end{bmatrix}\begin{bmatrix}-1 & -2\\ 1 & 1\end{bmatrix}$ $=\begin{bmatrix}75 & 25\\ -25 & 25\end{bmatrix}$; $\therefore$ Sum of elements $=100$	
61.	Number of values of θ lying in $[0, 100\pi]$ for which the system of equations, $(sin\ 3\theta)x - y + z = 0; (cos\ 2\theta)x + 4y + 3z = 0; 2x + 7y + 7z = 0$ has non-trivial solution is_______.	**A**
	201	
	205	
	210	
	220	

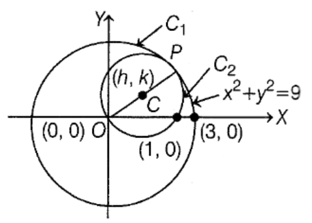
Sol.	The system has a non-trivial solution if $\Delta = \begin{vmatrix} \sin 3\theta & -1 & 1 \\ \cos 2\theta & 4 & 3 \\ 2 & 7 & 7 \end{vmatrix} = 0$ $\Rightarrow$ $(28-21)\sin 3\theta - (-7-7)\cos 2\theta + 2(-3-4) = 0$ $\Rightarrow$ $\sin 3\theta + 2\cos 2\theta - 2 = 0$ $\Rightarrow$ $3\sin\theta - 4\sin^3\theta + 2 - 4\sin^2\theta - 2 = 0$ $\Rightarrow$ $4\sin^3\theta + 4\sin^2\theta - 3\sin\theta = 0$ $\Rightarrow$ $\sin\theta(4\sin^2\theta + 4\sin\theta - 3) = 0$ $\Rightarrow$ $\sin\theta(2\sin\theta + 3)(2\sin\theta - 1) = 0$ So, $\sin\theta = 0; 1/2$, since $\sin\theta \neq -3/2$ Now, $\sin\theta = 0$ is for $0, \pi, 2\pi, \ldots, 100\pi$; total 101 values $\sin\theta = 1/2$ for $\theta = \pi/3$ and $2\pi/3$. So, there will be 100 values lying in $[0, 100\pi]$ Therefore, total number of values of θ is 201.	
62.	Let $f(x) = \begin{cases} \lvert 4x^2 - 8x + 5 \rvert, & if\ 8x^2 - 6x + 1 \geq 0 \\ [4x^2 - 8x + 5], & if\ 8x^2 - 6x + 1 < 0 \end{cases}$, where $[\alpha]$ denotes the greatest integer less than or equal to α. Then the number of points in R where f is not differentiable is ______.	**B**
	6	
	3	
	9	
	12	
Sol.	From the question, we have $f(x) = \begin{cases} \lvert 4x^2 - 8x + 5 \rvert, \\ x \in \left(-\infty, \frac{1}{4}\right) \cup \left(\frac{1}{2}, \infty\right) \\ [4x^2 - 8x + 5], x \in \left(\frac{1}{4}, \frac{1}{2}\right) \end{cases}$ $f(x) = \begin{cases} 4x^2 - 8x + 5, & x \in \left(-\infty, \frac{1}{4}\right) \cup \left(\frac{1}{2}, \infty\right) \\ 3, & x \in \left(\frac{1}{4}, x_1\right) \\ 2, & x \in \left(x_1, \frac{1}{2}\right) \end{cases}$  Total number of non differentiable points = 3	
63.	Let $x = -1$ and $x = 2$ be the critical points of the function $f(x) = x^3 + ax^2 + b\log_e\lvert x\rvert + 1, x \neq 0$. Let $m\ and\ M$ respectively be the absolute minimum and the absolute maximum values of f in the interval $\left[-2, -\frac{1}{2}\right]$. Then $\lvert M + m \rvert$ is equal to $(take \log_e 2 = 0.7)$:	**C**
	19.8	
	22.1	
	21.1	
	20.9	

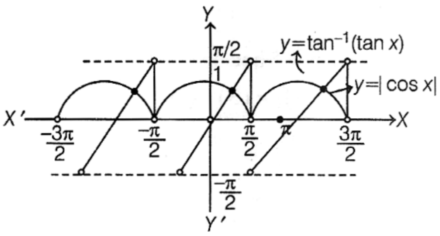
Sol.	We have, $f(x) = x^3 + ax^2 + b\log_e\|x\| + 1, x \neq 0$ $f'(x) = 3x^2 + 2ax + \frac{b}{x}$ $f'(-1) = 3 - 2a - b = 0$ $f'(2) = 12 + 4a + \frac{b}{2} = 0 \Rightarrow a = \frac{-9}{2}, b = 12$ $\therefore \quad f(x) = x^3 - \frac{9}{2}x^2 + 12\log_e\|x\| + 1$ $f(-1) = -1 - \frac{9}{2} + 1 = -\frac{9}{2}$ $M = -4.5$ Min. value at $x = -2$ $f(-2) = -8 - 18 + 12\log_e 2 + 1$ $m = -25 + 12(0.7) = -16.6$ $\therefore \quad \|M + m\| = 21.1$	
64.	The area of the region $\{(x, y): xy \leq 8, 1 \leq y \leq x^2\}$ is	**C**
	$8\log_e 2 - \frac{14}{3}$	
	$8\log_e 2 - \frac{7}{3}$	
	$16\log_e 2 - \frac{14}{3}$	
	$16\log_e 2 - 6$	
Sol.	*(c)* The given region $\{(x, y): xy \leq 8, 1 \leq y \leq x^2\}$. From the figure, region A and B satisfy the given region but only A is bounded region, so area of bounded region  $A = \int_1^2 (x^2 - 1)\,dx + \int_2^8 \left(\frac{8}{x} - 1\right) dx$ [$\because$ Points $P(1, 1), Q(2, 4)$ and $R(8, 1)$ $= \left[\frac{x^3}{3} - x\right]_1^2 + [8\log\|x\| - x]_2^8$ $= \left(\frac{8}{3} - 2 - \frac{1}{3} + 1\right) + 8\log 8 - 8 - 8\log 2 + 2$ $= -\frac{14}{3} + 16\log 2 = 16\log 2 - \frac{14}{3}$ sq units	
65.	Let $y = y(x)$ be the solution of the differential equation $sec\ x\ dy + [2(1 - x)tan\ x + x(2 - x)]dx = 0$ such that $y(0) = 2$. Then , $y(2)$ is equal to	**A**
	2	
	$2\{1 - \sin(2)\}$	
	1	
	$2\{\sin(2) + 1\}$	

Sol.	(a) We have, $\sec x\, dy + \{2(1-x)\tan x + x(2-x)\}\, dx = 0$ $\Rightarrow \frac{dy}{dx} = -[2(1-x)\sin x + x(2-x)\cos x]$ On variable separable and integration, $\int dy = -\int (2(1-x)\sin x + x(2-x)\cos x)\, dx$ $y = 2\int (x-1)\sin x\, dx + \int (x-2)x\cos x\, dx$ $= 2[-(x-1)\cos x + \int \cos x\, dx] + \int (x-2)x\cos x\, dx$ $= 2[\sin x - (x-1)\cos x] + (x-2)x\sin x - \int (2x-2)\sin x\, dx$ $= 2(\sin x - (x-1)\cos x) + (x-2)x\sin x - 2[-(x-1)\cos x + \sin x] + C$ $y = (x-2)x\sin x + C$ Given, $y(0) = 2 \Rightarrow 2 = c$ $\therefore \quad y(x) = (x-2)x\sin x + 2$ So, $\quad y(2) = 2$	
66.	Let a unit vector $\hat{u} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ make angles $\frac{\pi}{2}, \frac{\pi}{3}\ and\ \frac{2\pi}{3}$ with the vectors $\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{k}, \frac{1}{\sqrt{2}}\hat{\jmath} + \frac{1}{\sqrt{2}}\hat{k}$ and $\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}$ respectively. If $\vec{v} = \frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath} + \frac{1}{\sqrt{2}}\hat{k}$, then $\|\hat{u} - \vec{v}\|^2$ is equal to	**D**
	9	
	$\frac{11}{2}$	
	7	
	$\frac{5}{2}$	
Sol.	Let $\vec{a} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k},\ \vec{b} = \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$ and $\vec{c} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$ Now, $\hat{u}\cdot\vec{a} = \|\hat{u}\|\cdot\|\vec{a}\|\cos\theta$, $\Rightarrow \quad \frac{x}{\sqrt{2}} + \frac{z}{\sqrt{2}} = 1\cdot 1\cdot\cos\frac{\pi}{2}$ $\Rightarrow \quad x + z = 0 \quad$ ...(i) Again $\hat{u}\cdot\vec{b} = \frac{y}{\sqrt{2}} + \frac{z}{\sqrt{2}} = 1\cdot 1\cdot\cos\frac{\pi}{3}$ $\Rightarrow \quad y + z = \frac{1}{\sqrt{2}} \quad$ ...(ii) and $\hat{u}\cdot\vec{c} = \frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = 1\cdot 1\cdot\cos\frac{2\pi}{3}$ $\Rightarrow \quad x + y = \frac{-1}{\sqrt{2}} \quad$ ...(iii) Solving (i), (ii) and (iii), we get $x = \frac{-1}{\sqrt{2}},\ y = 0,\ z = \frac{1}{\sqrt{2}}\ \therefore\ \hat{u} = \frac{-1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}$ So, $\left\|\hat{u} - \vec{v}\right\|^2 = \left\|-\sqrt{2}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}\right\|^2 = 2 + \frac{1}{2} = \frac{5}{2}$	
67.	Let $a\ and\ b$ be the vectors of the same magnitude such that $\frac{\|a+b\| + \|a-b\|}{\|a+b\| - \|a-b\|} = \sqrt{2} + 1$.	**A**

	Then, $\frac{\|a+b\|^2}{\|a\|^2}$ is	
	$2+\sqrt{2}$	
	$2+4\sqrt{2}$	
	$4+2\sqrt{2}$	
	$1+\sqrt{2}$	
Sol.	(a) $\frac{\|\mathbf{a}+\mathbf{b}\|+\|\mathbf{a}-\mathbf{b}\|}{\|\mathbf{a}+\mathbf{b}\|-\|\mathbf{a}-\mathbf{b}\|}=\frac{\sqrt{2}}{1}$ Using componendo and dividendo, $\frac{2\|\mathbf{a}+\mathbf{b}\|}{2\|\mathbf{a}-\mathbf{b}\|}=\frac{\sqrt{2}+1}{\sqrt{2}-1}$ $\Rightarrow \frac{\|\mathbf{a}+\mathbf{b}\|^2}{\|\mathbf{a}-\mathbf{b}\|^2}=(1+\sqrt{2})^2=3+2\sqrt{2}$ $\frac{a^2+b^2+2ab\cos\theta}{a^2+b^2-2ab\cos\theta}=3+2\sqrt{2}$ $\frac{a^2+a^2+2aa\cos\theta}{a^2+a^2-2aa\cos\theta}=3+2\sqrt{2}$ $[\because \|\mathbf{a}\|=\|\mathbf{b}\|]$ $\frac{2\|\mathbf{a}\|^2(1+\cos\theta)}{2\|\mathbf{a}\|^2(1-\cos\theta)}=3+2\sqrt{2}$ $(1+\cos\theta)=(3+2\sqrt{2})(1-\cos\theta)$ $\Rightarrow \cos\theta=\frac{2+2\sqrt{2}}{4+2\sqrt{2}}$ $\cos\theta=\frac{1+\sqrt{2}}{2+\sqrt{2}}=\frac{\sqrt{2}}{2}$ $\Rightarrow \cos\theta=\frac{1}{\sqrt{2}}$ Thus, $\frac{\|\mathbf{a}+\mathbf{b}\|^2}{\|\mathbf{a}\|^2}=\frac{a^2+a^2+2aa\cos\theta}{a^2}$ $=\frac{2a^2(1+\cos\theta)}{a^2}=2+\sqrt{2}$	
68.	Let the vertices Q and R of the ΔPQR lie one the line $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$, $QR=5$ and the coordinates of the point P be $(0,2,3)$. If the area of the ΔPQR is $\frac{m}{n}$, then	**B**
	$5m-2\sqrt{21}n=0$	
	$2m-5\sqrt{21}n=0$	
	$5m-21\sqrt{2}n=0$	
	$m-5\sqrt{21}n=0$	
Sol.	(b) $H:(5\lambda-3, 2\lambda+1, 3\lambda-4)$ $<$ Direction ratios of PH $>$ $<5\lambda-3, 2\lambda-1, 3\lambda-7>$  $\mathbf{PH}\cdot\mathbf{QR}=0$ $\Rightarrow 5(5\lambda-3)+2(2\lambda-1)+3(3\lambda-7)=0$ $\Rightarrow \lambda=1$ $\therefore H\equiv(2,3,-1)$ $PH=\sqrt{4+1+16}=\sqrt{21}$ Area $=\frac{1}{2}(PH)(QR)$ $=\frac{1}{2}\sqrt{21}\times 5$ $=\frac{5}{2}\sqrt{21}=\frac{m}{n}$ $\Rightarrow 2m-5\sqrt{21}n=0$	
69.	Let $g(x)=3f\left(\frac{x}{3}\right)+f(3-x)\forall\ x\ \epsilon\ (0,3)$ and $f''(x)>0\ \forall\ x\ \epsilon(0,3)$, then	**C**

	$g(x)$ decreases in interval $(0, a)$, then a is-	
	$\frac{7}{4}$	
	$\frac{2}{3}$	
	$\frac{9}{4}$	
	$\frac{7}{4}$	
Sol.	$g'(x) = \cancel{3}f'(\frac{x}{3})\cdot\frac{1}{\cancel{3}} + f'(3-x)(-1)$ $= f'(\frac{x}{3}) - f'(3-x) < 0$ $f'(x) \uparrow$ $\frac{x}{3} < 3-x$ $x < 9-3x$ $4x < 9$ $x < \frac{9}{4}$ $(0, \frac{9}{4})$	
70.	Let a be the sum of all coefficients in the expansion of $(1-2x+2x^2)^{2023}(3-4x^2+2x^3)^{2024}$ and $b = \lim_{x\to 0}\left(\frac{\int_0^x \frac{\log(1+t)}{t^{2024}+1}dt}{x^2}\right)$. If the equations $cx^2+dx+e=0$ and $2bx^2+ax+4=0$ have a common root, where $c, d, e \in R$, then $d : c : e$ equals	**C**
	2: 1: 4	
	4: 1: 4	
	1: 1: 4	
	1: 2: 4	
Sol.	$: a = 1,\ b = \lim_{x\to 0}\left(\frac{\int_0^x \frac{\log(1+t)}{t^{2024}+1}}{x^2}\right)$ Using L-Hopital's rule $b = \lim_{x\to 0}\frac{\log(1+x)}{2x(x^{2024}+1)} = \frac{1}{2}.$ Then, $x^2+x+4=0$ (non-real) ...(i) and $cx^2+dx+e=0$ ...(ii) Both equation (i) and (ii) have a common root So, $\frac{c}{1} = \frac{d}{1} = \frac{e}{4}$ So, $d : c : e$ is equal to 1 : 1 : 4	
71.	Let m and n be the numbers of real roots of the quadratic equations $x^2 - 12x + [x] + 31 = 0$ and $x^2 - 5\lvert x+2\rvert - 4 = 0$ respectively, where $[x]$ denotes the greatest integer $\leq x$. Then, $m^2 + mn + n^2$ is equal to ………..	**9**

Sol.	(9) $x^2-12x+[x]+31=0$ $\Rightarrow x^2-11x+31+[x]-x=0$ $\Rightarrow x^2-11x+31=[x]+x$ $0\le x^2-11x+31<1$ $x^2-11x+30<0$ and $x^2-11x+31\ge 0$ $\Rightarrow x\in R \Rightarrow 5<x<6 \Rightarrow [x]=5$ $\therefore$The given equation becomes $x^2-12x+5+31=0$ $\Rightarrow x^2-12x+36=0 \Rightarrow x=6$ Which is not possible $[\because 5<x<6]$ $\therefore$There is no value of x i.e. $m=0$ Now, $x^2-5\|x+2\|-4=0$ Case-I Case -II $x\ge -2$ $x<-2$ $x^2-5x-14=0$ $x^2+5x+6=0$ $\Rightarrow x=7,-2$ and $x=-3,-2$ $\therefore x=7,-2,-3 \Rightarrow n=3$ $\therefore m^2+mn+n^2=0+0+3^2=9$	
72.	Let the centre of a circle, passing through the points (0, 0), (1, 0) and touching the circle $x^2+y^2=9$, be (h,k). Then, for all possible values of the coordinates of the centre (h,k), $4(h^2+k^2)$ is equal to……...	**9**
Sol.	(9) Let $C_1: x^2+y^2=9$ and $C_2:(x-h)^2+(y-k)^2=h^2+k^2$ $\Rightarrow x^2+y^2-2hx-2ky=0$ ...(i)  $\because$ Eq. (i) passes through (1, 0), then $1+0-2h=0$ $\Rightarrow h=\frac{1}{2}$ $\because OC=\frac{OP}{2}$ $\sqrt{\left(\frac{1}{2}\right)^2+k^2}=\frac{3}{2}$ $\Rightarrow \frac{1}{4}+k^2=\frac{9}{4}$ $\Rightarrow k^2=2 \Rightarrow k=\pm\sqrt{2}$ $\therefore$Possible values of (h,k) are $\left(\frac{1}{2},\sqrt{2}\right)$, $\left(\frac{1}{2},-\sqrt{2}\right)$ Thus, $4(h^2+k^2)=4\left(\frac{1}{4}+2\right)=4\left(\frac{9}{4}\right)$ $=9$	
73.	Let $\tan^{-1}(x)\epsilon\left(-\frac{\pi}{2},\frac{\pi}{2}\right), for\ x\ \epsilon\ R$. Then, the number of real solutions of the equation $\sqrt{1+\cos(2x)}=\sqrt{2}\tan^{-1}(tan\ x)$ in the set	**3**

	$\left(\frac{-3\pi}{2}, \frac{-\pi}{2}\right) \cup \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ is equal to	
Sol.	**(3)** Given $\sqrt{1+\cos 2x}$ $=\sqrt{2}\tan^{-1}(\tan x)$ $\Rightarrow \quad \sqrt{2\cos^2 x} = \sqrt{2}\tan^{-1}(\tan x)$ $\Rightarrow \quad \lvert\cos x\rvert = \tan^{-1}(\tan x) \quad \text{...(i)}$  Now, our job is to find total number of real solution(s) of Eq. (i). For this graphical approach is much more convinent So, we draw the graph of $y = \lvert\cos x\rvert$ and $y = \tan^{-1}(\tan x)$ in the given domain and hence we count total number of points of intersection. Hence, total number of real solutions = 3	
74.	If $\int \frac{1}{\sqrt[5]{(x-1)^4(x+3)^6}}\,dx = A\left(\frac{\alpha x - 1}{\beta x + 3}\right)^B + C$, where C is the constant of integration, then the value of $\alpha + \beta + 20AB$ is ………..	**7**
Sol.	**(7)** Let $I = \int \frac{1}{(x-1)^{4/5}(x+3)^{6/5}}\,dx$ $= \int \frac{1}{\left(\frac{x-1}{x+3}\right)^{4/5} \cdot (x+3)^2}\,dx$ Let $\frac{x-1}{x+3} = t \Rightarrow \frac{4}{(x+3)^2}\,dx = dt$ $\Rightarrow \quad I = \frac{1}{4}\int \frac{1}{t^{4/5}}\,dt$ $= \frac{1}{4}\left[\frac{t^{1/5}}{1/5}\right] + C = \frac{5}{4}t^{1/5} + C$ $\Rightarrow \quad I = \frac{5}{4}\left(\frac{x-1}{x+3}\right)^{1/5} + C$ Comparing with $A\left(\frac{\alpha x - 1}{\beta x + 3}\right)^B + C$, we get $A = \frac{5}{4}, \alpha = \beta = 1, B = \frac{1}{5}$ Then, $\alpha + \beta + 20AB$ $= 2 + 20 \times \frac{5}{4} \times \frac{1}{5} = 7$	
75.	25% of the population are smokers. A smoker has 27 times more chances to develop lung cancer than a non-smoker. A person is diagnosed with lung cancer and the probability that this person is a smoker is $\frac{k}{10}$. Then, the value of k is …….	**9**

Sol.	(9) Probability of a person being a smoker = 25% $P(S)=\frac{25}{100} \Rightarrow P(S)=\frac{1}{4}$ So, probability of a person being non- smoker $=P(N)=1-\frac{1}{4}=\frac{3}{4}$ And probability of a man to be a smoker to get a lung cancer $=P\left(\frac{L}{S}\right)$ $\because$ Probability of a non-smoker to get a lung cancer $=P\left(\frac{L}{N}\right)$ and $P\left(\frac{L}{S}\right)=27P\left(\frac{L}{N}\right)$ and $P\left(\frac{S}{L}\right)=\frac{k}{10}$ $P\left(\frac{S}{L}\right)=\frac{P(S)\cdot P\left(\frac{L}{S}\right)}{P(L)}=\frac{k}{10}$ where, $P(L)=P(N)\cdot P\left(\frac{L}{N}\right)+P(S)\cdot P\left(\frac{L}{S}\right)$ $=\frac{3}{4}P\left(\frac{L}{N}\right)+\frac{1}{4}\left\{27P\left(\frac{L}{N}\right)\right\}$ So, $P(L)=\frac{30}{4}P\left(\frac{L}{N}\right)$ So, $P\left(\frac{S}{L}\right)=\frac{\frac{1}{4}\left\{27P\left(\frac{L}{N}\right)\right\}}{\frac{30}{4}P\left(\frac{L}{N}\right)}=\frac{k}{10}$ $\Rightarrow \quad P\left(\frac{S}{L}\right)=\frac{9}{10}=\frac{k}{10} \Rightarrow k=9$	