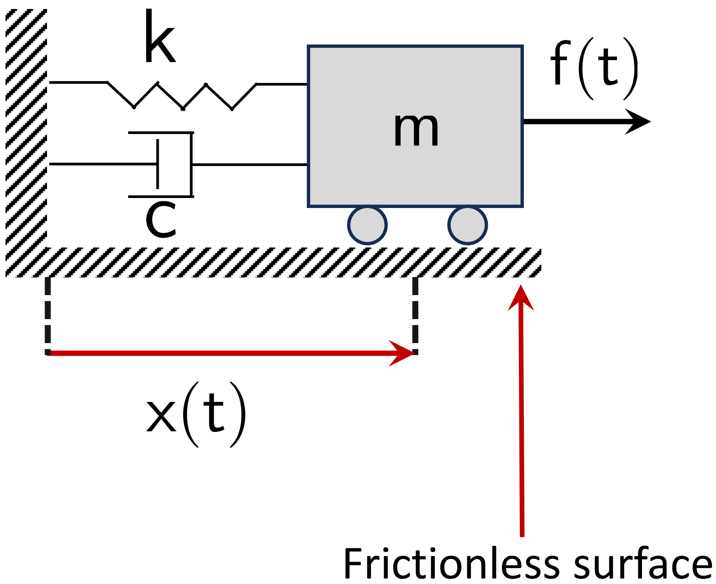
FREE VIBRATION OF A SINGLE-DEGREE-OF-FREEDOM (SDOF) MODEL

(TEACHING PRESENTATION)

FREE VIBRATION OF SDOF MODEL



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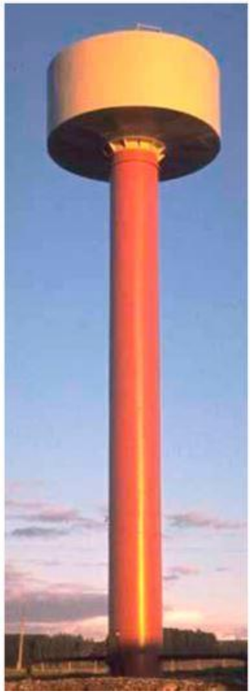

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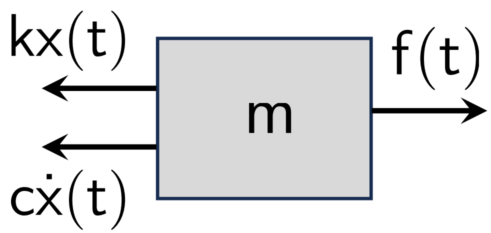

Newton's Second Law

$$\sum F_{net} = m\ddot{x}(t) \qquad (1)$$

$$f(t) - kx(t) - c\dot{x}(t) = m\ddot{x}(t) \qquad (2)$$

$$m\ddot{x}(t) + c\dot{x}(t) + k\dot{x}(t) = f(t) \qquad (3)$$

$$m\ddot{x}(t) + c\dot{x}(t) + k\dot{x}(t) = 0 \qquad (4)$$

FREE VIBRATION OF SDOF MODEL

$$m\ddot{x}(t) + c\dot{x}(t) + k\dot{x}(t) = 0 \tag{5}$$

$$\omega_n - \text{Natural Frequency} \qquad \xi - \text{Damping} \tag{6}$$

Substitute in Eq. 5

$$c = 2m\omega_n\xi \qquad k = \omega_n^2 m \tag{7}$$

$$\ddot{x}(t) + 2\omega_n\xi\dot{x}(t) + \omega_n^2 x(t) = 0 \tag{8}$$

Assumed form of solution

$$x(t) = Ae^{\lambda t} \tag{9}$$

FREE VIBRATION OF SDOF MODEL

$$\lambda^2 + 2\omega_n \xi \lambda + \omega_n^2 = 0 \tag{9}$$

$$\lambda_{1,2} = -\omega_n \xi \pm \omega_n \sqrt{1-\xi^2}\, \mathrm{i} \tag{10}$$

Two roots

$$\lambda_{1,2} = -\omega_n \xi \pm \omega_d\, \mathrm{i} \tag{11}$$

General solution

$$x(t) = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t} \tag{12}$$

$$x(t) = e^{-\omega_n \xi t}\left(A_1 e^{\omega_d \mathrm{i} t} + A_2 e^{-\omega_d \mathrm{i} t}\right) \tag{13}$$

Euler's identity

$$e^{\omega_d \mathrm{i} t} = \cos \omega_d t + \mathrm{i} \sin \omega_d t \tag{14}$$

FREE VIBRATION OF SDOF MODEL

$$x(t) = e^{-\omega_n \xi t}(a_1 \cos \omega_d t + a_2 \sin \omega_d t) \tag{15}$$

$$\text{Initial Conditions : } x(0)\ ;\ \dot{x}(0) \tag{16}$$

$$a_1 = x(0) \qquad a_2 = \frac{\dot{x}(0) + \omega_n \xi x(0)}{\omega_d} \tag{17}$$

$$\boxed{\begin{aligned} &\xi < 1 : \ \text{under damped} \\ &\xi = 1 : \ \text{critically damped} \\ &\xi > 1 : \ \text{overdamped damped} \end{aligned}}$$

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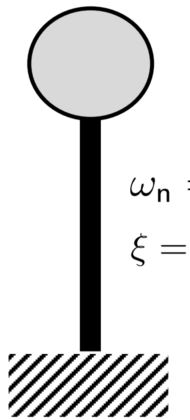
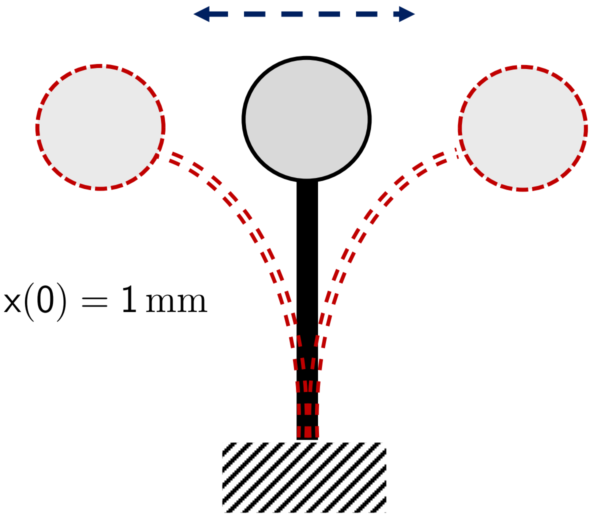
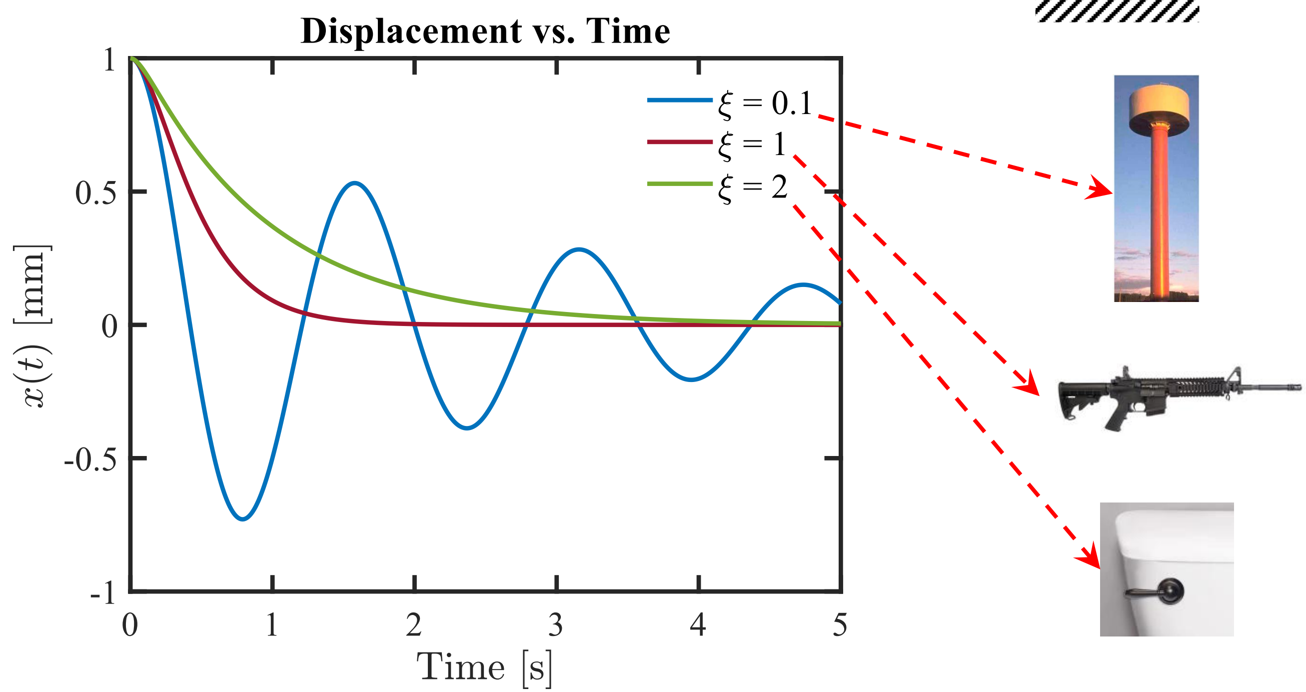


$\omega_n = 4\,\mathrm{rad/s}$

$\xi = 0.1, 1,$ and 2



$x(0) = 1\,\mathrm{mm}$



Download @ https://github.com/adarshss-github/SDOF_FREE_VIB