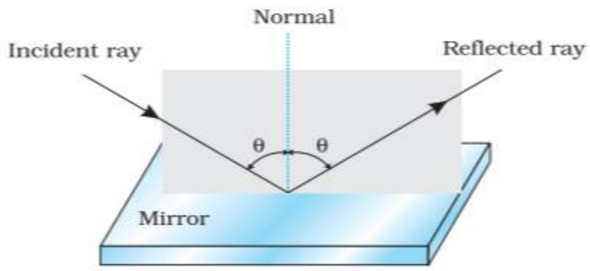
Chapter 9
Ray Optics and Optical Instruments

9.1 Introduction

Light is an electromagnetic wave. Light travels with a speed of 3 x10^8 m/s in vacuum. The speed of light in vacuum is the highest speed attainable in nature. A light wave can be considered to travel from one point to another, along a straight line joining them. The path is called a ray of light, and a bundle of such rays constitutes a beam of light.

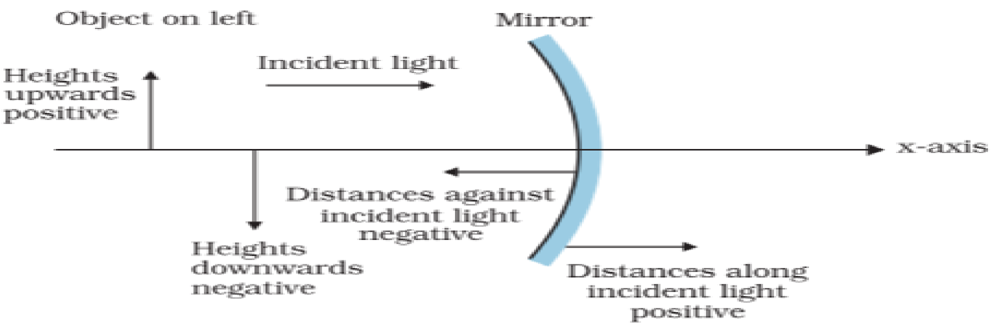
9.2 Reflection of Light by Spherical Mirrors



Laws of Reflection

1) The incident ray, reflected ray and the normal to the reflecting surface at the point of incidence lie in the same plane.
2)The angle of incidence is equal to the angle of reflection(i=r).

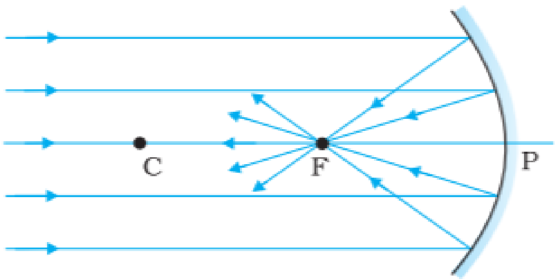
Sign Convention



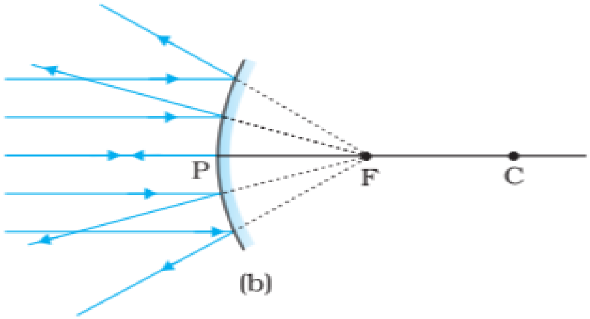
We follow the **Cartesian sign convention** to measure distances. According to this convention,

1) All distances are measured from the pole of the mirror or the optical centre of the lens.
2) The distances measured in the same direction as the incident light are taken as positive and those measured in the direction opposite to the direction of incident light are taken as negative.
3) The heights measured upwards with respect to principal axis of the mirror/ lens are taken as positive . The heights measured downwards are taken as negative.

Focal Length of Spherical Mirrors

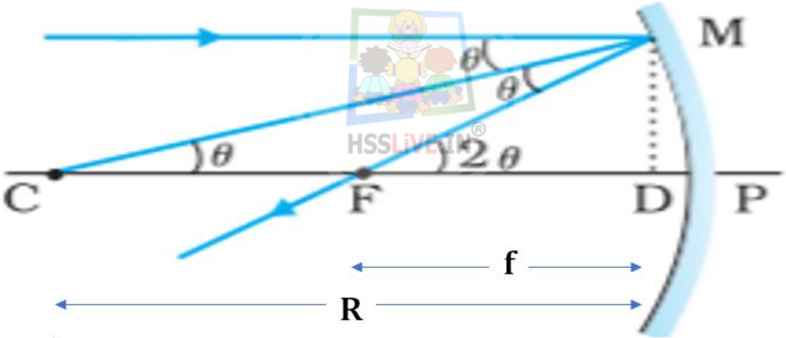


When a parallel beam of light is incident on a concave mirror, the reflected rays converge at a point F on its principal axis . The point F is called the principal focus of the concave mirror.



When a parallel beam of light is incident on a convex mirror, the reflected rays appear to diverge from a point F on its principal axis . The point F is called the principal focus of the convex mirror.

Relation between Focal Length and Radius of Curvature



Let f be the focal length and R be the radius of curvature of lens

From figure , $\tan\theta = \frac{MD}{R}$, $\theta = \frac{MD}{R}$ ------(1) (For small value of θ, $\tan\theta \approx \theta$)

$\tan 2\theta = \frac{MD}{f}$, $2\theta = \frac{MD}{f}$ -----(2) ($\tan 2\theta \approx 2\theta$)

Substituting θ from eq(1) in eq(2) $2\frac{MD}{R} = \frac{MD}{f}$

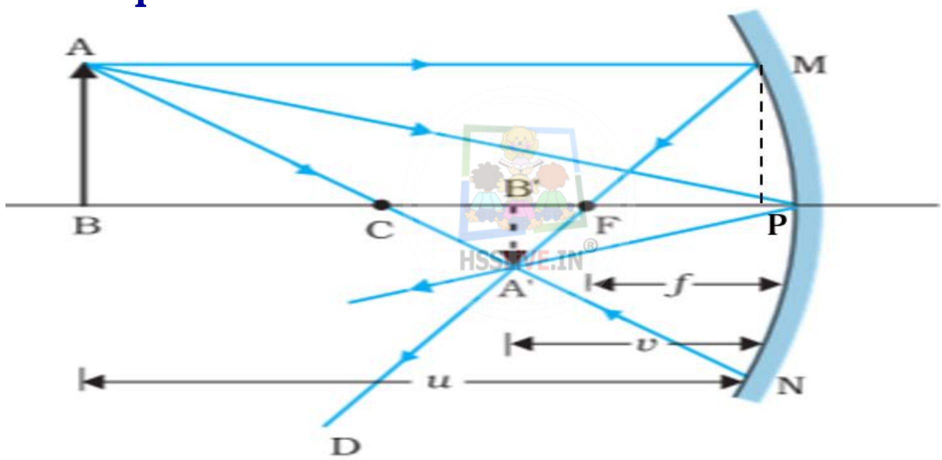
$$\frac{2}{R} = \frac{1}{f}$$

$$\mathbf{f = \frac{R}{2}}$$

Some important points to consider while image formation in spherical mirrors :-

- If rays emanating from a point actually meet at another point after reflection , that point is called the image of the first point.
- The image is real if the rays actually converge to the point.
- The image is virtual if the rays do not actually meet but appear to diverge from the point when produced backwards.
- The ray parallel to the principal axis, goes through the focus of the mirror after reflection.
- The ray passing through the centre of curvature of a concave mirror ,retraces the path.
- The ray passing through the focus of the concave mirror , after reflection ,goes parallel to the principal axis.
- The ray incident at any angle at the pole. The reflected ray follows laws of reflection.

The Mirror Equation



The two right-angled triangles A′B′F and MPF are similar

$$\frac{B'A'}{PM} = \frac{B'F}{FP}$$

$$\frac{B'A'}{BA} = \frac{B'F}{FP} \text{------------}(1) \quad (\text{since } PM = AB)$$

The right angled triangles A′B′P and ABP are also similar.

$$\frac{B'A'}{BA} = \frac{B'P}{BP} \text{------------}(2)$$

From eqns(1) and (2)

$$\frac{B'F}{FP} = \frac{B'P}{BP}$$

$B'P = v$, $BP = u$, $B'F = v-f$, $FP = f$,

$$\frac{v-f}{f} = \frac{v}{u}$$

Applying sign convention ,

$$\frac{-v- -f}{-f} = \frac{-v}{-u}$$

$$\frac{v-f}{f} = \frac{v}{u}$$

$$\frac{v}{f} - 1 = \frac{v}{u}$$

Dividing by v

$$\frac{1}{f} - \frac{1}{v} = \frac{1}{u}$$

$$\mathbf{\frac{1}{u} + \frac{1}{v} = \frac{1}{f}}$$

This relation is known as the mirror equation.

Linear Magnification (m)

Linear magnification (m) is the ratio of the height of the image (h′) to the height of the object (h).

$$\mathbf{m = \frac{h'}{h}}$$

From above figure

$$\frac{B'A'}{BA} = \frac{B'P}{BP}$$

With the sign convention,

$$\frac{-h'}{h} = \frac{-v}{-u}$$

$$\frac{h'}{h} = \frac{-v}{u}$$

$$\mathbf{m = \frac{h'}{h} = \frac{-v}{u}}$$

Example

An object is placed at (i) 10 cm, (ii) 5 cm in front of a concave mirror of radius of curvature 15 cm. Find the position, nature, and magnification of the image in each case.

Example 9.3 An object is placed at (i) 10 cm, (ii) 5 cm in front of a concave mirror of radius of curvature 15 cm. Find the position, nature, and magnification of the image in each case.

Solution

The focal length $f = -15/2$ cm $= -7.5$ cm

(i) The object distance $u = -10$ cm. Then Eq. (9.7) gives

$$\frac{1}{v} + \frac{1}{-10} = \frac{1}{-7.5}$$

or $v = \frac{10 \times 7.5}{-2.5} = -30$ cm

The image is 30 cm from the mirror on the same side as the object.

Also, magnification $m = -\frac{v}{u} = -\frac{(-30)}{(-10)} = -3$

The image is magnified, real and inverted.

(ii) The object distance $u = -5$ cm. Then from Eq. (9.7),

$$\frac{1}{v}+\frac{1}{-5}=\frac{1}{-7.5}$$

or $$v=\frac{5\times7.5}{(7.5-5)}=15 \text{ cm}$$

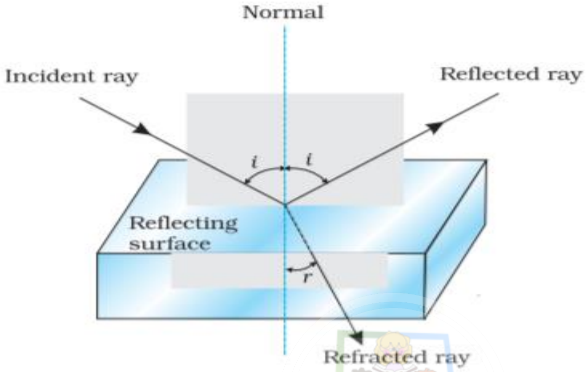
This image is formed at 15 cm behind the mirror. It is a virtual image.

Magnification $m = -\frac{v}{u} = -\frac{15}{(-5)} = 3$

The image is magnified, virtual and erect.

9.3 Refraction

The direction of propagation of an obliquely incident ray of light that enters the other medium, changes at the interface of the two media. This phenomenon is called refraction of light.



Laws of Refraction

i)The incident ray, the refracted ray and the normal to the interface at the point of incidence, all lie in the same plane.

ii)The ratio of the sine of the angle of incidence to the sine of angle of refraction is constant

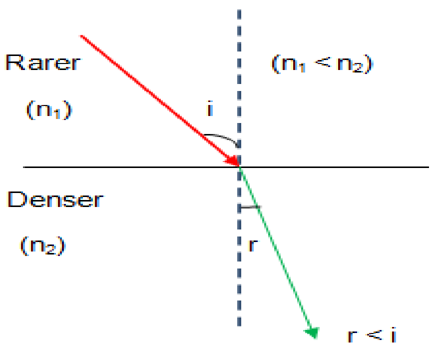
$$\frac{\sin i}{\sin r} = n_{21}$$

where n_{21} is a constant, called the refractive index of the second medium with respect to the first medium.

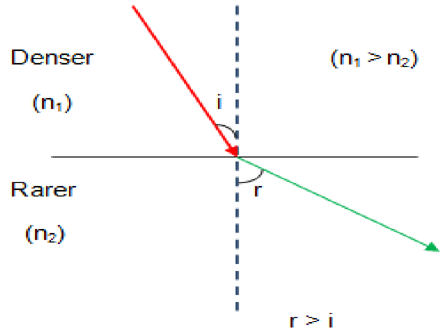
$$n_{21} = \frac{n_2}{n_1}$$

This equation is known as Snell's law of refraction.

- When a ray travels from rarer to denser medium, the refracted ray bends towards the normal.
  i.e., if $n_{21} > 1$, i.e., $n_2 > n_1$, $r < i$

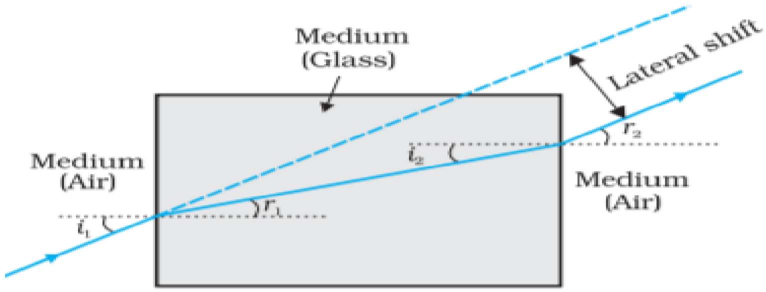


- When a ray travels from denser to rarer medium, the refracted ray bends away from the normal.
  i.e., if $n_{21} < 1$, i.e., $n_2 < n_1$, $r > i$



Some Elementary Results Based on The Laws of Refraction

(i)Lateral Shift



For a rectangular slab, refraction takes place at two interfaces (air-glass and glass-air). The emergent ray is parallel to the incident ray—there is no refraction and reflection of light, but it does suffer lateral displacement/ shift with respect to the incident ray.

(ii)Apparent depth

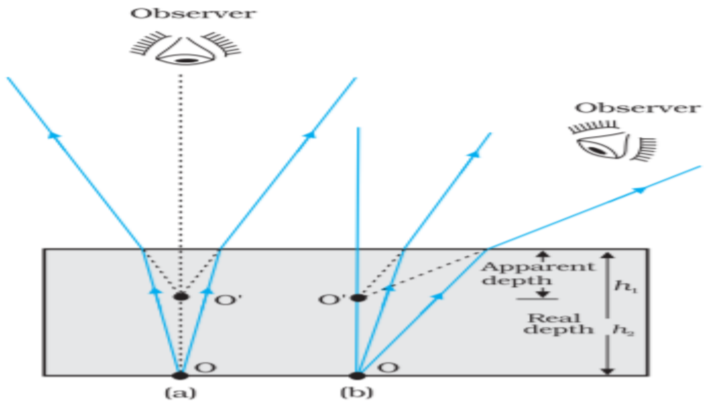


FIGURE 9.10 Apparent depth for (a) normal, and (b) oblique viewing.

The bottom of a tank filled with water appears to be raised .

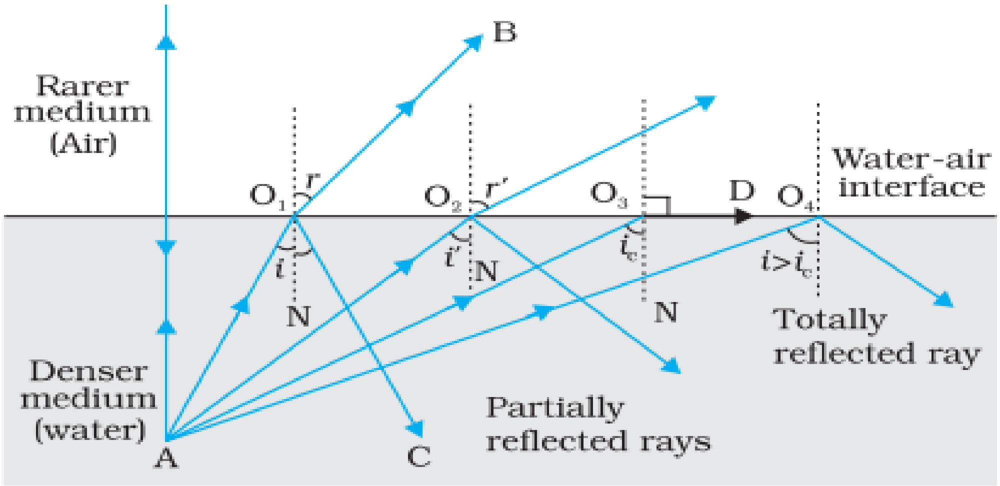
For viewing near the normal direction ,

$$\text{Apparent depth} = \frac{\textbf{real depth}}{\textbf{Refractive Index}}$$

$$h_1 = \frac{h_2}{n}$$

9.4 Total Internal Reflection

When a ray of light enters from a denser medium to a rarer medium, if the angle of incidence is greater than the critical angle (i_c) for the given pair of media, the incident ray is totally reflected. This is called total internal reflection.



Explanation:-

When a ray of light enters from a denser medium to a rarer medium, it bends away from the normal.

- The incident ray AO_1 is partially reflected (O_1C) and partially refracted(O_1B) .
- As the angle of incidence increases, the angle of refraction also increases.(for ray AO_2)
- When the angle of incidence becomes equal to the critical angle(i_c) for the given pair of media, the angle of refraction becomes 90º.(for rayAO_3)
- If the angle of incidence is increased further (ray AO_4), refraction is not possible, and the incident ray is totally reflected.

Conditions for Total Internal Reflection

- The ray of light should enter from a denser medium to a rarer medium.
- The angle of incidence should be greater than the critical angle (i_c) for the given pair of media .

Critical Angle

The angle of incidence in the denser medium, corresponding to an angle of refraction 90º, is called the critical angle (i c) for the given pair of media.

Let the second medium be air.

By Snell's law $$\frac{\sin i}{\sin r} = \frac{n_2}{n_1} = \frac{1}{n}$$

When $i = i_c$, $r = 90$

$$\frac{\sin i_c}{\sin 90} = \frac{1}{n}$$

$$\sin i_c = \frac{1}{n}$$

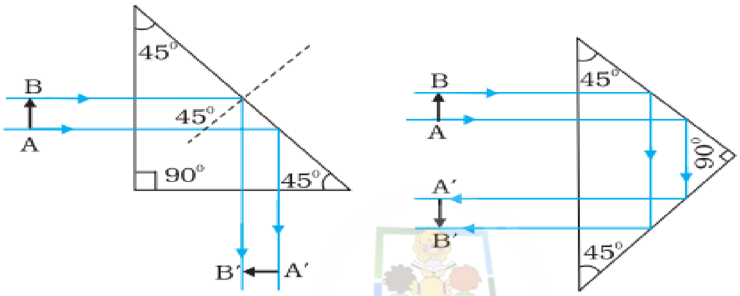
$$n = \frac{1}{\sin i_c}$$

This is the relation connecting refractive index and critical angle.

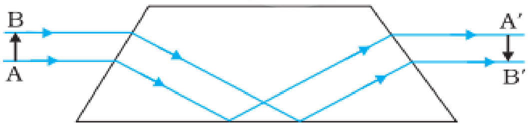
Total internal reflection in nature and its technological applications

(i)Prism:

Prisms designed to bend light by 90º or by 180º make use of total internal reflection .In these cases, the critical angle i_c for the material of the prism must be less than 45º.

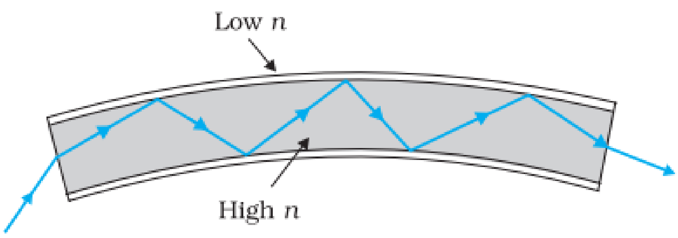


Such a prism is also used to invert images without changing their size.



(ii) Optical fibres:

Now-a-days optical fibres are extensively used for transmitting audio and video signals through long distances. Optical fibres make use of the phenomenon of total internal reflection. Optical fibres are fabricated with high quality composite glass/quartz fibres.

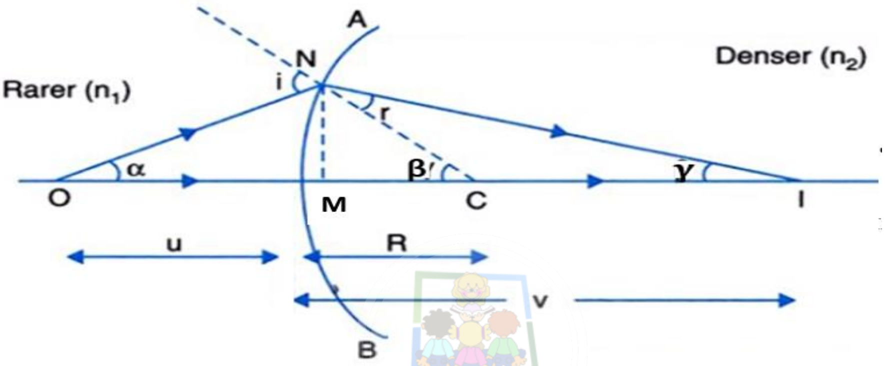


Each fibre consists of a core and cladding. The refractive index of the material of the core is higher than that of the cladding. When a signal in the form of light is directed at one end of the fibre at a suitable angle, it undergoes repeated total internal reflections along the length of the fibre and finally comes out at the other end . Since light undergoes total internal reflection at each stage, there is no appreciable loss in the intensity of the light signal.

A bundle of optical fibres can be put to several uses. Optical fibres are extensively used for transmitting and receiving electrical signals which are converted to light by suitable transducers. Obviously, optical fibres can also be used for transmission of optical signals. For example, these are used as a 'light pipe' to facilitate visual examination of internal organs like esophagus, stomach and intestines. Optical fibres are used in decorative lamps.

9.5 Refraction at Spherical Surfaces and by Lenses

Refraction at a Spherical Surface



$$\tan\alpha = \frac{MN}{OM}, \quad \alpha = \frac{MN}{OM}$$

$$\tan\beta = \frac{MN}{MC}, \quad \beta = \frac{MN}{MC}$$

$$\tan\gamma = \frac{MN}{MI}, \quad \gamma = \frac{MN}{MI}$$

From ΔNOC, $\quad i = \alpha + \beta$--------------(1)

From ΔNIC, $\quad \beta = r + \gamma$

$r = \beta - \gamma$---------------(2)

From Snell's law

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1}$$

$$n_1 \sin i = n_2 \sin r$$

For small values of i and r

$$n_1\, i = n_2 r$$

Substituting fron eqn (1) and (2)

$$n_1\,(\alpha + \beta) = n_2\,(\beta - \gamma)$$

$$n_1\left(\frac{MN}{OM} + \frac{MN}{MC}\right) = n_2\left(\frac{MN}{MC} - \frac{MN}{MI}\right)$$

$$\frac{n_1}{OM} + \frac{n_1}{MC} = \frac{n_2}{MC} - \frac{n_2}{MI}$$

$$\frac{n_1}{OM} + \frac{n_2}{MI} = \frac{n_2}{MC} - \frac{n_1}{MC}$$

$$\frac{n_1}{OM} + \frac{n_2}{MI} = \frac{n_2 - n_1}{MC}$$

Applying the Cartesian sign convention,

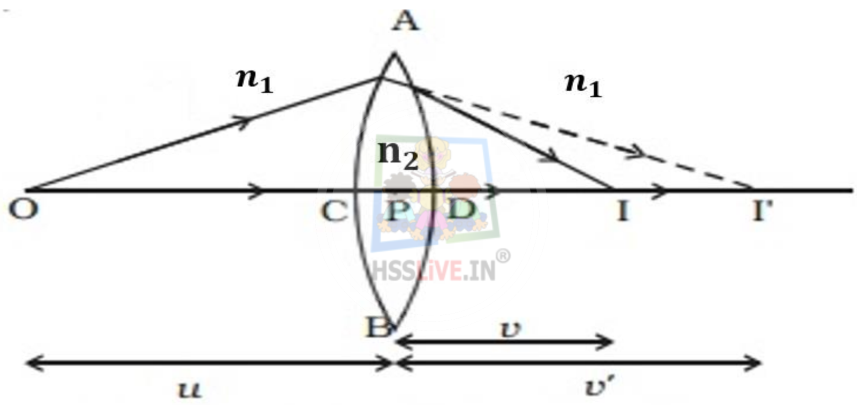
$$OM = -u,\ MI = +v,\ MC = +R$$

$$\frac{n_1}{-u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$$

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

This equation gives the relation between object and image distance in terms of refractive index of the medium and the radius of curvature of the curved spherical surface.

Refraction by a Lens -Lens Maker's Formula



For the first interface ACB of radius of curvature R_1

$$\frac{n_2}{v'} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} \text{----------(1)}$$

For the first interface ADB of radius of curvature R_2

$$\frac{n_1}{v} - \frac{n_2}{v'} = \frac{n_1 - n_2}{R_2} \text{-------------(2)}$$

Eqns (1) + (2)

$$\frac{n_2}{v'} - \frac{n_1}{u} + \frac{n_1}{v} - \frac{n_2}{v'} = \frac{n_2 - n_1}{R_1} + \frac{n_1 - n_2}{R_2}$$

$$\frac{n_1}{v} - \frac{n_1}{u} = (n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Dividing throughout by n_1

$$\frac{1}{v} - \frac{1}{u} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{v} - \frac{1}{u} = (n_{21} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \text{------------(3)}$$

When u=∞ (infinity) ,   v=f

$$\frac{1}{f} - \frac{1}{\infty} = (n_{21} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = (n_{21} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

If the lens is placed in air

$$\frac{1}{f} = (n - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \text{-----------}(4)$$

This Equation is known as the lens maker's formula.

From eq (3) and (4)

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

This equation is called thin lens formula

Image Formation by a Convex Lens

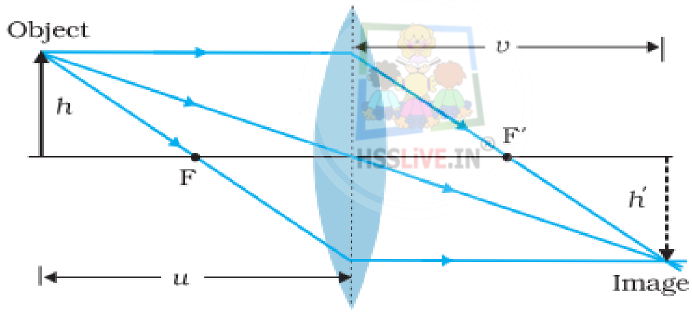
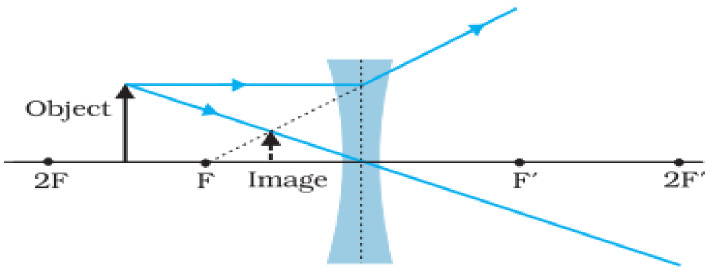


Image Formation by a Concave Lens



Magnification (m)

Magnification produced by a lens is defined as the ratio of the size of the image to that of the object.

$$m = \frac{h'}{h} = \frac{v}{u}$$

For erect (and virtual) image formed by a convex or concave lens, m is positive, while for an inverted (and real) image, m is negative.

Power of a lens

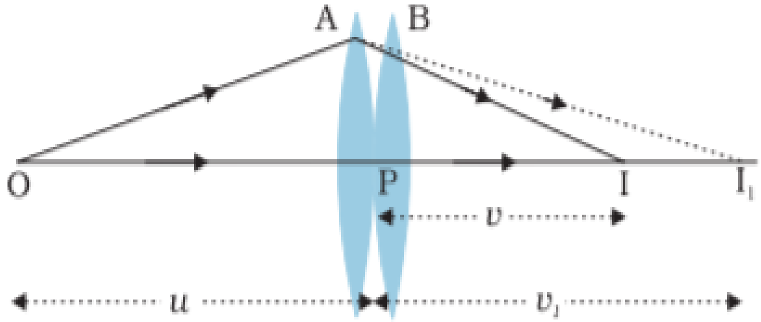
Power of a lens is the reciprocal of focal length expressed in metre

$$p = \frac{1}{f}$$

The SI unit for power of a lens is **dioptre D**. $1D = 1m^{-1}$

Power of a lens is positive for a converging lens and negative for a diverging lens.

Combination of Thin Lenses in Contact



For the image formed by the first lens A,

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1} \text{ -----------(1)}$$

For the image formed by the second lens B,

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2} \text{ -----------(2)}$$

Eqn (1) +(2)

$$\frac{1}{v_1} - \frac{1}{u} + \frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2} \text{ -----------(3)}$$

If the two lens-system is regarded as equivalent to a single lens of focal length f, we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \text{ ---------------(4)}$$

From eqn (3) and (4)

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

For a number of thin lenses in contact

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_2} + \ldots\ldots$$

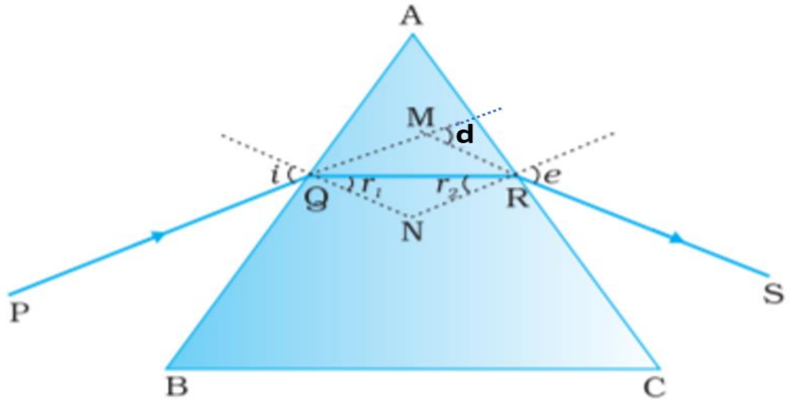
Total power,P of the combination is,

$$P = P_1 + P_2 + P_3 + \ldots\ldots$$

Total magnification, m of the combination is,

$$m = m_1\ m_2\ m_3 \ldots\ldots$$

9.6 Refraction Through a Prism



In the quadrilateral AQNR,

$$\angle A + \angle QNR = 180^\circ$$

From the triangle QNR,

$$r_1 + r_2 + \angle QNR = 180^\circ$$

Comparing these two equations, we get

$$\mathbf{r_1 + r_2 = A} \text{ ------------------(1)}$$

The total deviation δ is the sum of deviations at the two faces,

$$d = (i - r_1) + (e - r_2)$$

$$d = i + e - (r_1 + r_2)$$

$$\mathbf{d = i + e - A} \text{ ------------------(2)}$$

Thus, the angle of deviation depends on the angle of incidence

At the minimum deviation

$$d = D, \quad i = e, \quad r_1 = r_2 = r$$

From eqa (1)

$$2r = A$$

$$\mathbf{r = \frac{A}{2}} \text{ ----------------(3)}$$

From eqa (2)

$$d = 2i - A$$

$$2i = A + D$$

$$\mathbf{i = \frac{A + D}{2}} \text{ -------------(4)}$$

By Snell's law the refractive index of prism

$$n = \frac{\sin i}{\sin r}$$

$$\mathbf{n = \frac{\sin \frac{A+D}{2}}{\sin \frac{A}{2}}}$$

For a small angle prism, i.e., a thin prism, D is also very small, and we get

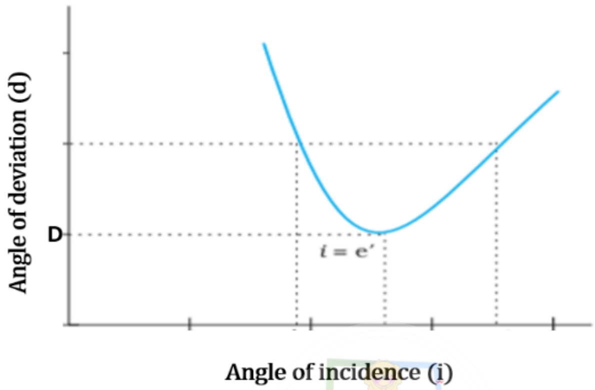
$$n = \frac{\frac{A+D}{2}}{\frac{A}{2}}$$

$$n = 1 + \frac{D}{A}$$

$$D = (n-1)A$$

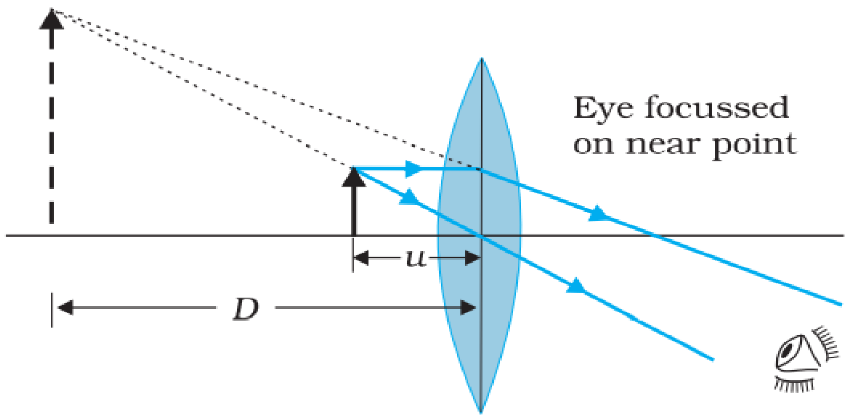
It implies that, thin prisms do not deviate light much.

Graph between the angle of deviation and angle of incidence - i-d curve



9.7 Optical Instruments

The microscope



A simple microscope or magnifier is a converging lens of small focal length.

If the object is held between the focus and optical centre of lens, an erect, magnified and virtual image is formed at near point 25 cm or more. If the object is held at focus, the image will be formed at infinity.

The linear magnification m(when image is at D)

$$m = \frac{v}{u} = v\left(\frac{1}{v} - \frac{1}{f}\right)$$

$$m = 1 - \frac{v}{f}$$

Applying sign convention , v=-D

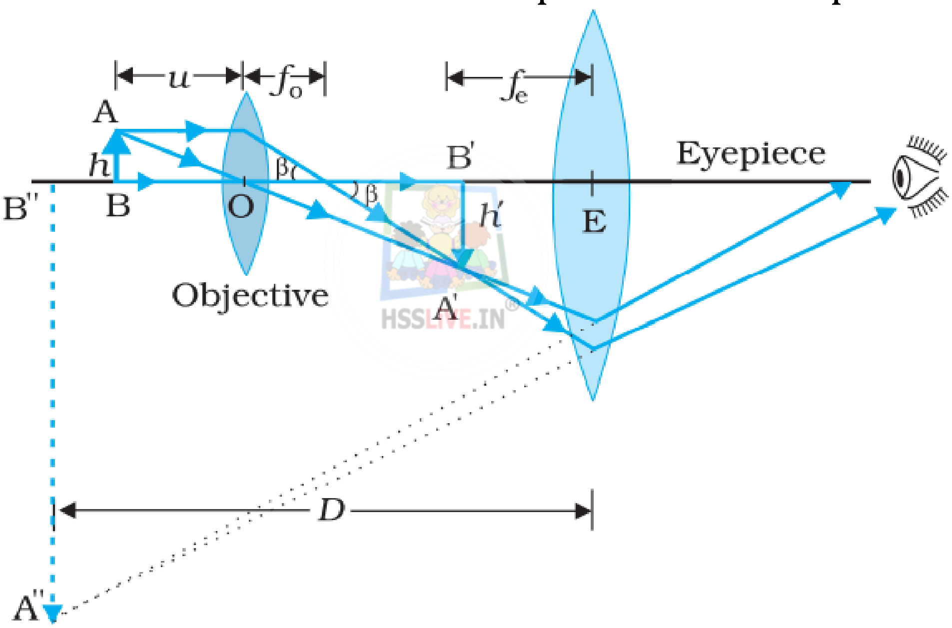
$$m = 1 + \frac{D}{f}$$

The linear magnification m(when image is at infinity)

$$m = \frac{D}{f}$$

Compound Microscope

A simple microscope has a limited maximum magnification (≤ 9). For much larger magnifications, one uses two lenses, one compounding the effect of the other. This is known as a compound microscope.



The lens near the object, called the objective. It forms a real, inverted, magnified image of the object. This serves as the object for the second lens, the eyepiece, at the focal plane (or little closer) of the eyepiece . The eyepiece functions like a simple microscope or magnifier and produces an enlarged and virtual image at infinity, or at the near point. Clearly, the final image is inverted with respect to the original object.

Magnifiction, $m = m_o \times m_e$ -----------(1)

$$m_0 = \frac{h'}{h} = \frac{L}{f_0}$$

When the final image is formed at infinity,

$$m_e = \frac{D}{f_e}$$

Substituting in eqn(1)

$$\mathbf{m = \frac{L}{f_0} \times \frac{D}{f_e}}$$

D= near point=25cm
f_0 = focal length of objective
f_e = focal length of eyepiece
L= The tube length of the compound microscope
(The distance between the second focal point of the objective and the first focal point of the eyepiece is called the tube length.)

Clearly, to achieve a large magnification of a small object , the objective and eyepiece should have small focal lengths.

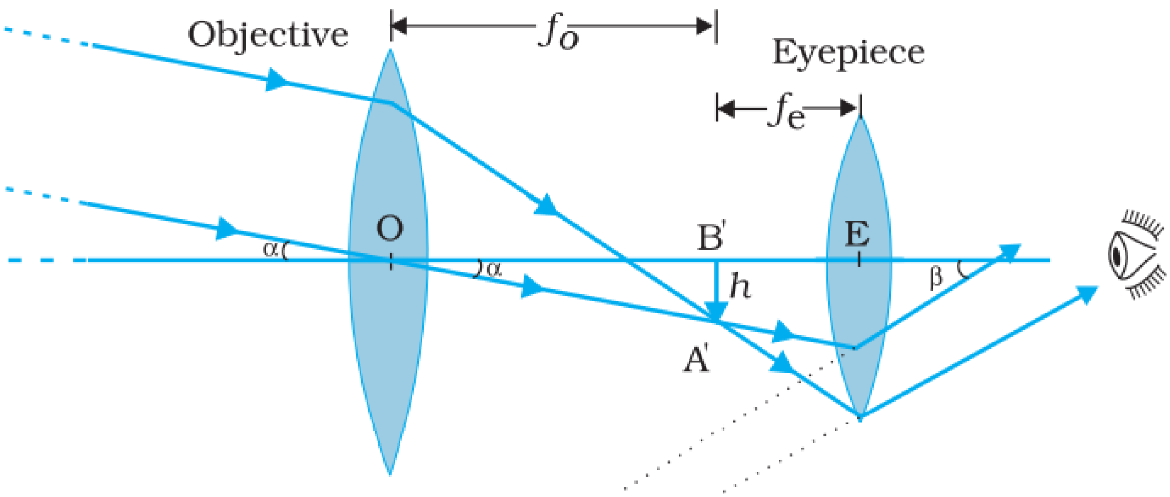
When the final image is formed at the near point,

$$m_e = 1 + \frac{D}{f_e}$$

$$\mathbf{m = \frac{L}{f_0}\left(1 + \frac{D}{f_e}\right)}$$

Telescope

Refracting Telescope



The telescope is used to provide angular magnification of distant objects . It also has an objective and an eyepiece. The objective has a large focal length and a much larger aperture than the eyepiece. Light from a distant object enters the objective and a real image is formed in the tube at its second focal point. The eyepiece magnifies this image producing a final inverted image.

The magnifying power m is the ratio of the angle β subtended at the eye by the final image to the angle α which the object subtends at the lens or the eye.

$$m \approx \frac{\beta}{\alpha} \approx \frac{h}{f_e} \cdot \frac{f_o}{h} = \frac{f_0}{f_e}$$

$$\mathbf{m = \frac{f_0}{f_e}}$$

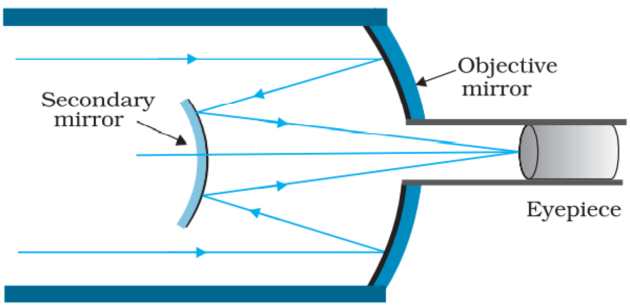
Terrestrial telescopes have, in addition, a pair of inverting lenses to make the final image erect. Refracting telescopes can be used both for terrestrial and astronomical observations.

For high resolving power, optical telescopes should have objective of large diameter. Such big lenses are very heavy and it is rather difficult and expensive to make such large sized lenses which form images that are free from any kind of chromatic aberration and distortions.So modern telescopes use a concave mirror rather than a lens for the objective.(Reflecting Telescope)

Reflecting Telescope

Telescopes with mirror objectives are called reflecting telescopes. They have several advantages.

- First, there is no chromatic aberration in a mirror.
- Second, if a parabolic reflecting surface is chosen, spherical aberration is also removed. Mechanical support is much less of a problem since a mirror weighs much less than a lens of equivalent optical quality.



A reflecting telescope is that the objective mirror focusses light inside the telescope tube. A convex secondary mirror focusses the incident light, which passes through a hole in the objective primary mirror. It has the advantages of a large focal length in a short telescope.