Chapter 1
Electric Charges and Fields

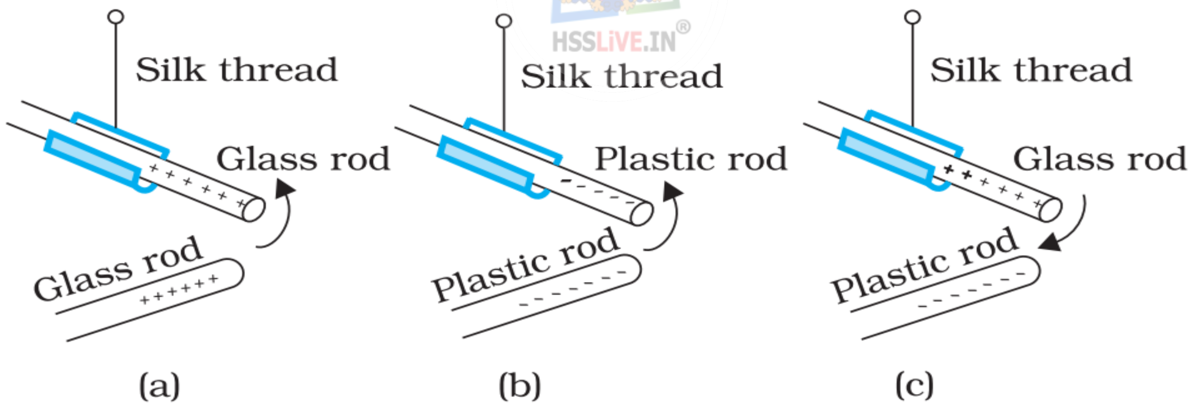
1.1 Introduction

A common example of electric discharge is the lightning that we see in the sky during thunderstorms. We also experience a sensation of an electric shock either while opening the door of a car or holding the iron bar of a bus after sliding from our seat. The reason for these experiences is discharge of electric charges through our body, which were accumulated due to rubbing of insulating surfaces. This is due to generation of static electricity. Static means anything that does not move or change with time.

Electrostatics deals with the study of forces, fields and potentials arising from static charges.

1.2 Electric charge

Historically the credit of discovery of the fact that amber rubbed with wool or silk cloth attracts light objects goes to Thales of Miletus, Greece, around 600 BC. The name electricity is coined from the Greek word **elektron meaning amber.**



If Two glass rods rubbed with wool or silk cloth are brought close to each other, they repel each other [Fig. 1.1(a)]. Similarly, two plastic rods rubbed with cat's fur repelled each other [Fig. 1.1(b)] . On the other hand, the plastic rod attracts the glass rod [Fig. 1.1(c)] .

There are two kinds of electrification and we find that

(i) like charges repel and

(ii) unlike charges attract each other.

The property which differentiates the two kinds of charges is called the polarity of charge.

The charges were named as positive and negative by the American scientist Benjamin Franklin.

On rubbing electrons are transferred from one body to the other. The body, which loses electrons, will become positively charged and which gains electrons becomes negatively charged.

- When a glass rod is rubbed with silk, glass rod becomes positively charged and silk negative.
- When a plastic rod is rubbed with fur, plastic rod becomes negatively charged and fur positive.

1.3 Conductors and Insulators

Conductors

Conductors are those substances which allow passage of electricity through them. Eg. Metals, human and animal bodies and earth are conductors.

- They have electric charges (electrons) that are comparatively free to move inside the material.
- When some charge is transferred to a conductor, it readily gets distributed over the entire surface of the conductor.
- Metals cannot be charged by friction,because the charges transferred to the metal leak through our body to the ground as both are conductors of electricity.

Insulators

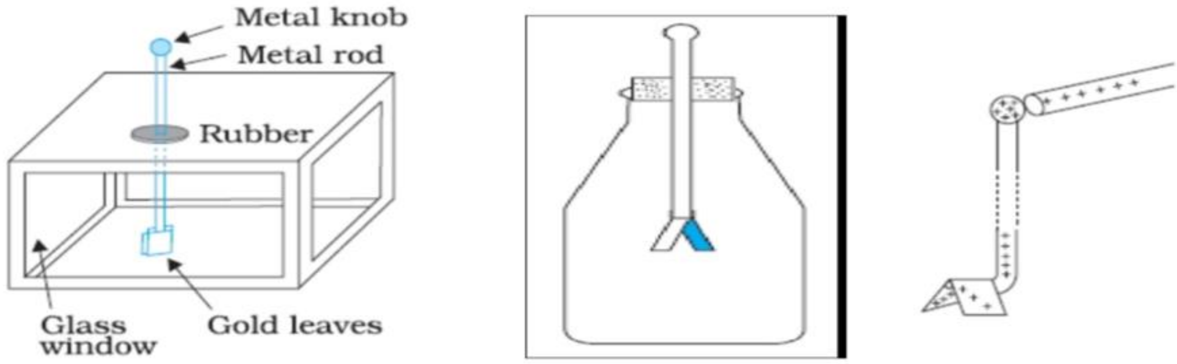
The substances which offer high resistance to the passage of electricity through them are called Insulators . Eg. glass, porcelain, plastic, nylon, wood

- If some charge is put on an insulator, it stays at the same place. So insulators gets electrified on combing dry hair or on rubbing.

Gold Leaf Electroscope

A simple apparatus to detect charge on a body is called a gold-leaf electroscope.

Apparatus It consists of a vertical metal rod placed in a box. Two thin gold leaves are attached to its bottom end as shown in figure.



Working

When a charged object touches the metal knob at the top of the rod, charge flows on to the leaves and they diverge. The degree of divergence is an indicator of the amount of charge.

1.4 Basic properties of electric charges

1. Additivity of charge: The total charge on a surface is the algebraic sum of individual charges present on that surface.

If a system contains n charges q_1, q_2, q_3....................., q_n then the total charge of the system is,

$$q = q_1 + q_2 + q_3 + \ldots\ldots\ldots + q_n$$

2. Charge is conserved: It means that total charge of an isolated system remains constant. It s not possible to create or destroy net charge carried by an isolated system although the charge carrying particles may be created or destroyed in a process.

3. Quantization of charge : According to quantisation of electric charge, charge of a body is an integral multiple of a basic charge, which is the electronic charge.

Charge on a body, **$q = \pm ne$;** **where, $n = 1,2,3$.........**

e is the electronic charge. $e = 1.602 \times 10^{-19}$ C

Example 1

How many electronic charges form 1 C of charge?

$$q = \pm ne,$$

$$n = \frac{q}{e}$$

$$n = \frac{1}{1.602 \times 10^{-19}} = 6.25 \times 10^{18}$$

Example 2

A comb drawn through person's hair causes 10^{22} electrons to leave the 2person's hair and stick to the comb. Calculate the charge carried by the comb.

$$q = ne,$$
$$q = 10^{22} \times 1.602 \times 10^{-19} C$$
$$= -1.602 \times 10^{3} C$$

As the comb gains electrons it gets negatively charged.

1.5 Coulomb's Law

The force of attraction or repulsion between two stationery electric charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

If charges are placed in free space, $\mathbf{F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}}$

If charges are placed in a medium, $\mathbf{F = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{q_1 q_2}{r^2}}$

- Where ε_0 - permittivity of free space, $\varepsilon_0 = 8.854 \times 10^{-12} C^2 N^{-1} m^{-2}$

  ε_r - relative permittivity.

  $\varepsilon = \varepsilon_0 \varepsilon_r$ $\quad$ $\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$

  ε - Permittivity of the medium.

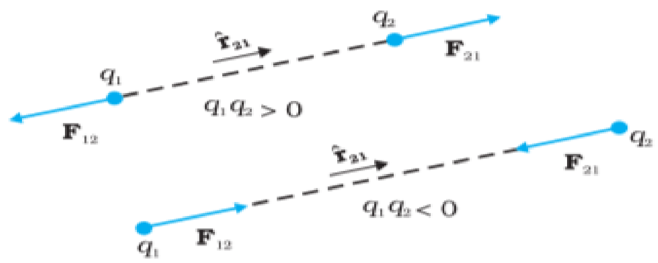
Thus $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^{9} C^{-2} N^{1} m^{2}$

Definition of coulomb

$$\mathbf{F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}}$$

- When $q_1 = q_2 = 1$ C, r = 1 m , $F = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^{9}$ N
- 1 C is the charge that when placed at a distance of 1 m from another charge of the same magnitude in vacuum experiences an electrical force of repulsion of magnitude 9×10^{9} N.

Coulomb's Law in vector form

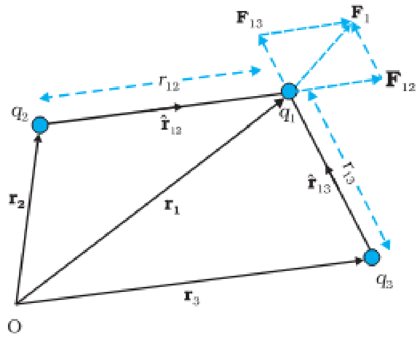


$$\mathbf{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}^2} \hat{\mathbf{r}}_{12}$$

$$\mathbf{F}_{21} = \frac{1}{4\pi\varepsilon_o} \frac{q_1 q_2}{r_{21}^2} \hat{\mathbf{r}}_{21}$$

Thus $F_{12} = -F_{21}$, Coulomb's law agrees with Newton's third law.

1.6 Forces between Multiple charges



consider a system of three charges q_1, q_2 and q_3 , as shown in figure. The force on one charge q_1 , due to two other charges q_2 , q_3 is obtained by performing a vector addition of the forces due to each one of these charges.

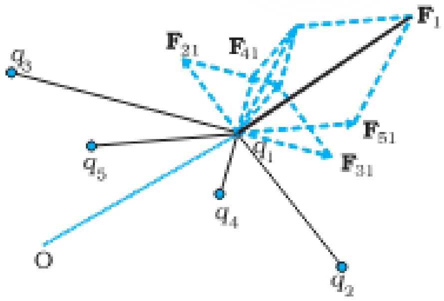
$$\mathbf{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12}$$

$$\mathbf{F}_{13} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}^2}\hat{\mathbf{r}}_{13}$$

$$\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}^2}\hat{\mathbf{r}}_{13}$$

Super position principle

Force on a charge due to a number of charges is the vector sum of forces due to individual charges.



For a system of n charges,

$$\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13} + \ldots + \mathbf{F}_{1n} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12} + \frac{q_1 q_3}{r_{13}^2}\hat{\mathbf{r}}_{13} + \ldots + \frac{q_1 q_n}{r_{1n}^2}\hat{\mathbf{r}}_{1n}\right]$$

1.7 Electric Field

Electric field is the region around a charge where its effect can be felt.

Intensity of electric field at a point is the force per unit charge.

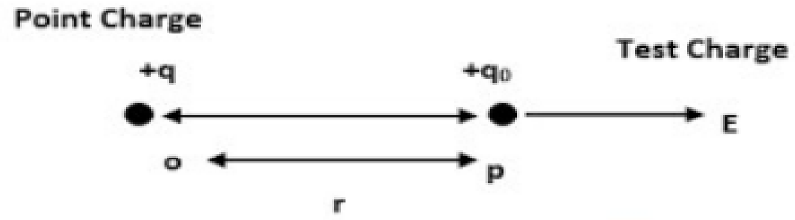
$$\mathbf{E} = \frac{\mathbf{F}}{\mathbf{q}}$$

F= qE

Unit of electric field is N/C or V/m.

It is a vector quantity.

Electric field due to a point charge



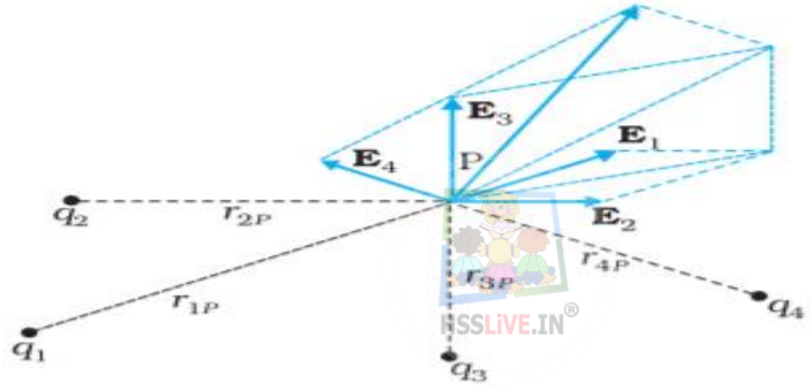
By Coulomb's law, $$F = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}$$

$$E = \frac{F}{q_0}$$

$$\mathbf{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}}$$

Electric field due to a system of charges

Electric field at a point due to a system of charges is the vector sum of the electric fields at the point due to individual charges.



$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots\ldots\ldots\ldots + \vec{E}_n$$

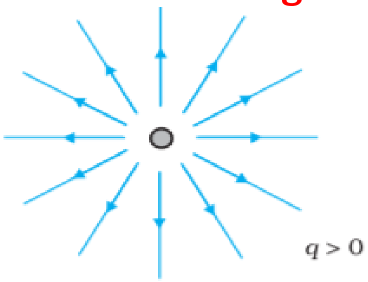
$$= \frac{1}{4\Pi\epsilon_0}\frac{q_1}{r_{1p}^2}\hat{r}_{1p} + \frac{1}{4\Pi\epsilon_0}\frac{q_2}{r_{2p}^2}\hat{r}_{2p} + \ldots\ldots + \frac{1}{4\Pi\epsilon_0}\frac{q_n}{r_{np}^2}\hat{r}_{np}$$

1.8 Electric Field Lines

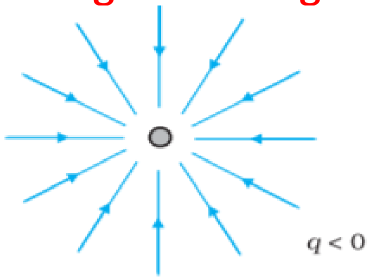
An electric field line is a curve drawn in such a way that the tangent to it at each point is in the direction of the net field at that point.

- Electric Field lines start from positive charge and end at negative charge.
- Electric field lines of a positive charge are radially outwards and that of a negative charge is radially inwards
- Electric field lines do not form closed loops.
- In a charge free region field lines are continuous.
- Two field lines never intersect.(Two directions for electric field is not possible at a point)
- Field lines are parallel ,equidistant and in same direction in uniform electric field.

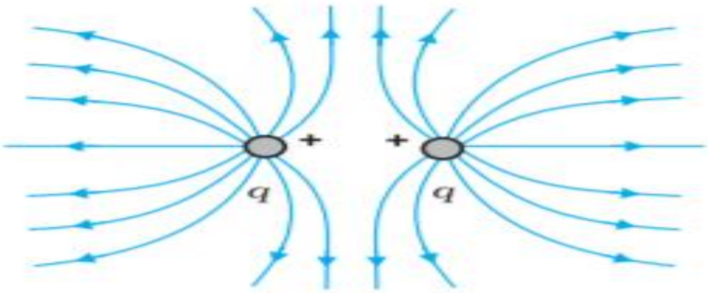
Positive Charge



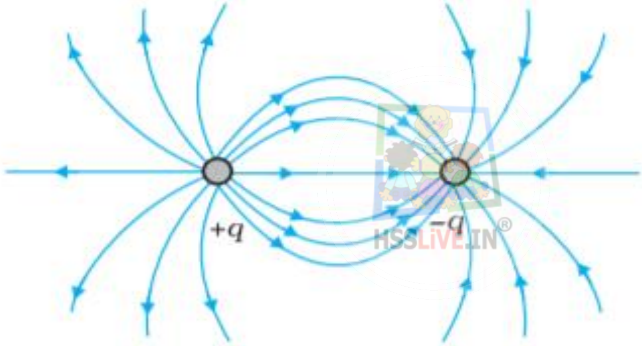
Negative Charge



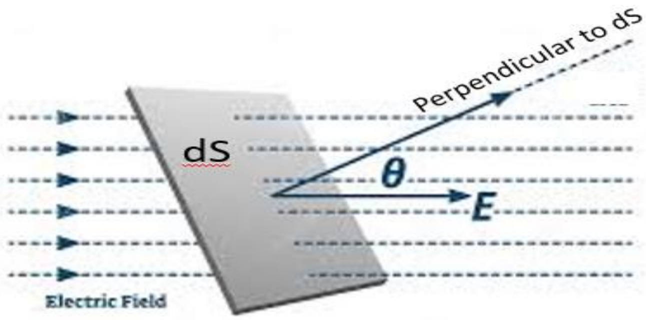
Two positive Charges



Dipole - Positive and Negative charge



1.9 Electric Flux



The electric flux associated with a surface is the number of electric field lines passing normal through a surface.

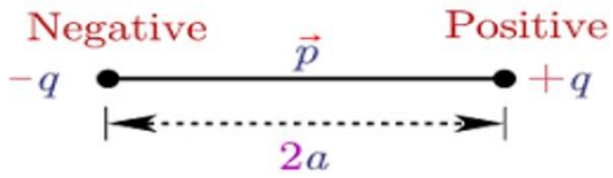
$$\phi = \int \mathbf{E} \cdot \mathbf{dS}$$

$$\phi = \int E\, dS \cos\theta$$ (θ is the angle between E and normal to dS)

- Unit – Nm^2 / C
- It is a scalar quantity

1.10 Electric Dipole

An electri dipole is a pair of equal and opposite charges separated by a distance



The total charge of the system is +q + -q =0

Electric Dipole moment ($\vec{p}$)

Electric Dipole moment(p) is the product of magnitude of one of the charges and the distance between charges.

$$\boxed{p = q \times 2\,a}$$

q- magnitude of charge

2a- distance between the charges or dipole length

Unit of dipole moment is Cm

Dipole moment is a vector quantity.

Dipole moment is directed from the negative charge to the positive charge along the dipole axis.

Physical significance of electric dipole

Non Polar molecules

In non polar molecules the centres of positive charges and negative charges lie at the same place. Dipole moment is zero for a non polar molecule in the absence of an external field.

They develop a dipole moment when an electric field is applied.

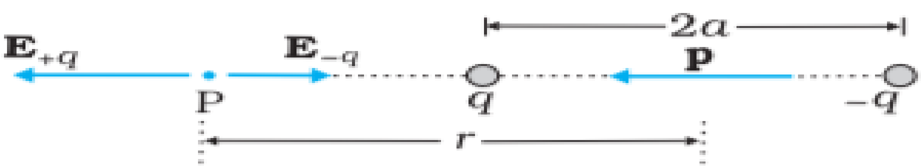
- Eg: CO_2 , CH_4

Polar molecules

The molecules in which the centres of negative charges and of positive charges do not coincide. . Therefore they have a permanent electric dipole moment, even in the absence of an electric field.

- Eg: water (H_2O)

Electric Field due to a Dipole along the Axial Line



The electric field at P due to +q

$$E_{+q} = \frac{1}{4\pi\varepsilon_0} \frac{q}{(r-a)^2} \quad \text{(in the direction of dipole moment } \vec{p}\text{)}$$

The electric field at Pdue to -q

$$E_{-q} = \frac{1}{4\pi\varepsilon_0} \frac{q}{(r+a)^2} \quad \text{(opposite to the direction of dipole moment } \vec{p}\text{)}$$

Total field, $E = E_{+q} - E_{-q}$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{(r-a)^2} - \frac{1}{4\pi\varepsilon_0} \frac{q}{(r+a)^2}$$

Thus the total electric field at P is

$$E = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]$$

Simplifying

$$E = \frac{q}{4\pi\varepsilon_0}\left[\frac{4ar}{(r^2-a^2)^2}\right]$$

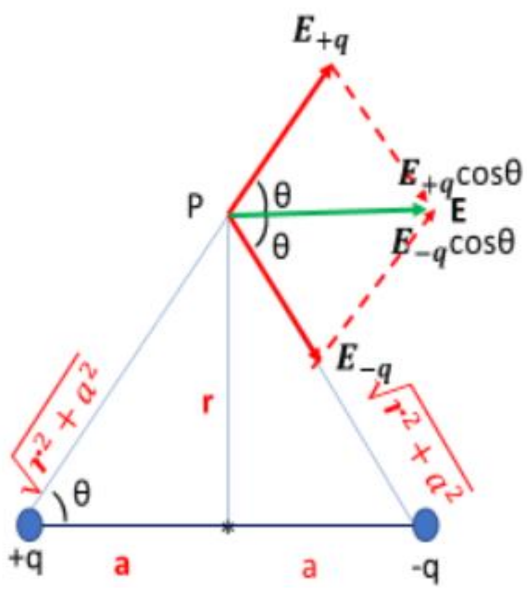
For r≫ a ,we get

$$E = \frac{1}{4\pi\varepsilon_0}\left[\frac{4qa}{r^3}\right]$$

2qa=$\vec{p}$ (dipole moment)

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{2\vec{p}}{r^3}\right]$$

Electric Field due to a Dipole along the Equatorial Line



The magnitude of electric field at P due to +q

$$E_{+q} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2 + a^2} \quad \text{----------------(1)}$$

The magnitude of electric field at P due to -q

$$E_{-q} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2 + a^2} \quad \text{------------------(2)}$$

The vertical componennts cancel each other and horizontal components add up

Total electric field at P,

$$E = E_{+q}cos\theta + E_{-q}cos\theta$$

But , $E_{+q} = E_{-q}$

$$E = 2E_{+q}cos\theta \quad \text{--------------(3)}$$

$$Cos\theta = \frac{a}{\sqrt{r^2+a^2}} = \frac{a}{(r^2+a^2)^{1/2}} \quad \text{---------------(4)}$$

Substituting eq(1) and (4) in eq(3)

$$E = 2\,x \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2 + a^2}\; x \frac{a}{(r^2+a^2)^{1/2}}$$

p=2qa (dipole moment)

$$E = \frac{1}{4\pi\varepsilon_0}\left[\frac{p}{(r^2+a^2)^{3/2}}\right]$$

For r≫ a ,we get

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{\vec{p}}{r^3}\right]$$

Relation connecting Axial field and Equatorial field of a Dipole

Axial field, $\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{2\vec{p}}{r^3}\right]$

Equatorial field , $\vec{E} = \frac{1}{4\pi\varepsilon_0}\left[\frac{\vec{p}}{r^3}\right]$

Axial field = 2 x Equatorial field

1.11 Dipole in a Uniform External field

In a uniform electric field there will be a net torque on the dipole, but the net force will be zero. Due to the torque ,the dipole rotates. There will be no translatory motion as the net force is zero.

Torque on a Dipole in a Uniform External field

Torque, τ = one of the forces x perpendicular distance between them.

τ = qE x 2a sinθ

τ =pE sinθ

$$\vec{\tau} = \vec{P} \times \vec{E}$$

• When p and E are in the same direction or opposite direction(θ=0 or180)

$$\tau = pE \sin 0 = 0$$

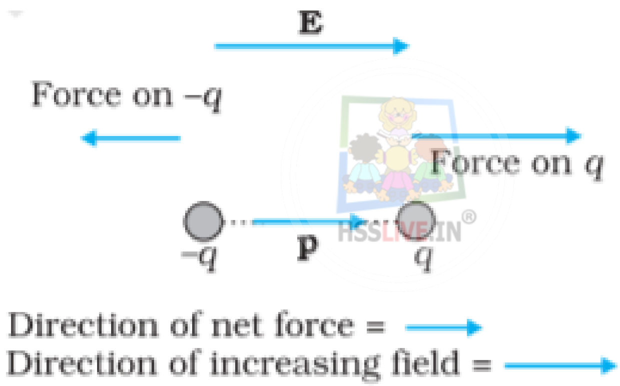
• Torque is maximum , when p and E are perpendicular. (θ=90)

$$\tau = pE \sin 90 = pE$$

Dipole in a non uniform electric field

In a non uniform electric field the dipole experiences a net force as well as a net torque in general .

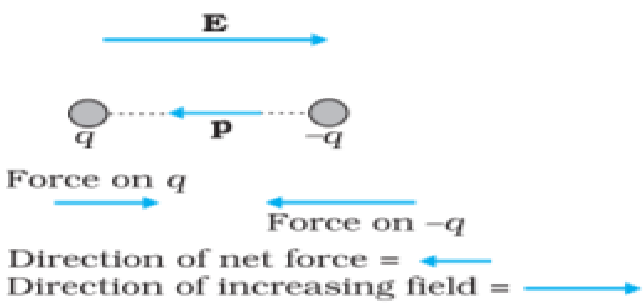
Case 1 -when p is parallel to E



when p is parallel to E, the dipole has a net force in the direction of increasing field.

But the net torque will be zero τ =pE sin0 =0

Case 2-When p is antiparallel to E.

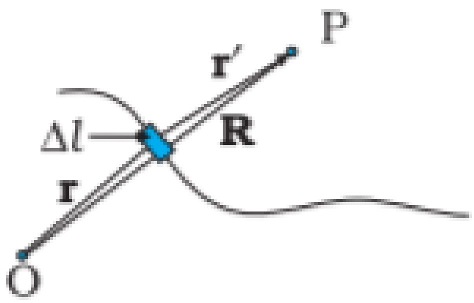


When p is antiparallel to E, the net force on the dipole is in the direction of decreasing field.

But the net torque will be zero, τ =pE sin 180 =0

1.12 Continuous Charge Distribution

Linear charge density



The linear charge density λ of a wire is defined as

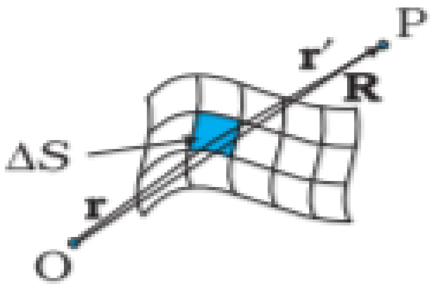
$$\lambda = \frac{\Delta q}{\Delta l}$$

$$\boldsymbol{\lambda = \frac{q}{l}}$$

The unit of λ is C/m

Line charge $\mathbf{q = \lambda l}$

Surface charge density



The surface charge density σ of a area element is defined as

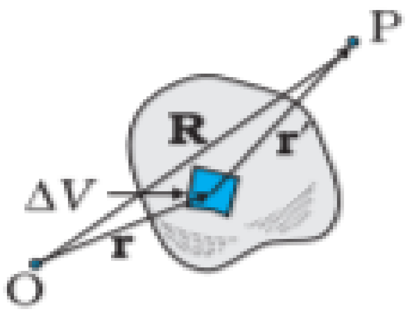
$$\sigma = \frac{\Delta q}{\Delta S}$$

$$\boldsymbol{\sigma = \frac{q}{S}}$$

The units for σ is C/m^2

Surface charge, $\mathbf{q = \sigma S}$

Volume charge density



The volume charge density ρ of a volume element is defined as

$$\rho = \frac{\Delta q}{\Delta V}$$

$$\boldsymbol{\rho = \frac{q}{V}}$$

The units for ρ is C/m^3

Volume charge , $\mathbf{q = \rho V}$

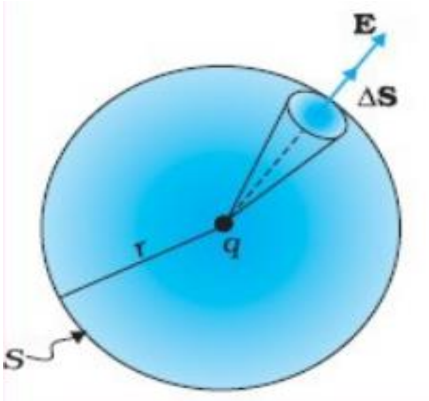
1.13 Gauss's Law

Gauss's law states that the total electric flux through a closed surface is equal to $\frac{1}{\varepsilon_0}$ times the total charge enclosed by the surface.

$$\boldsymbol{\phi = \oint E \cdot dS = \frac{q}{\varepsilon_0}}$$

The surface over which we calculate the flux is called Gaussian surface.

Proof



Consider a sphere of radius r enclosing a point charge q. the electric flux through the surface dS

$$\phi = \int E \cdot dS$$

$$\phi = \int E\, dS \cos 0 = \int E\, dS = E\int dS$$

$$\phi = ES$$

$$\phi = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \text{ x } 4\pi r^2$$

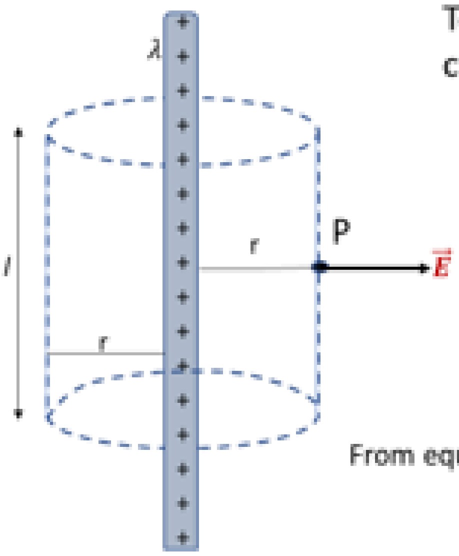
$$\boldsymbol{\phi = \frac{q}{\varepsilon_0}}$$

Features of Gauss's Law

- Gauss's law is true for any surface irrespective of the size and shape.
- The charge includes the sum of all charges enclosed by the surface.
- The surface that we choose for the application of Gauss's law is called the Gaussian Surface.
- Gauss's law is applicable to both symmetric and asymmetric system, but it will be much easier if the system has some symmetry.
- Gauss's law is based on inverse square dependence on distance contained in the Coulomb's law.

1. 14 Applications of Gauss's Law

1) Electric field due to an infinitely long straight uniformly charged wire



To find the electric field at point P at distance r consider cylindrical Gaussian surface of radius r

$$\phi = E\,S$$

$$\phi = E \times 2\pi r l \longrightarrow (1)$$

By Gauss's law $\phi = \frac{q}{\varepsilon_0}$

$\lambda = \frac{q}{l}$

$q = \lambda l$

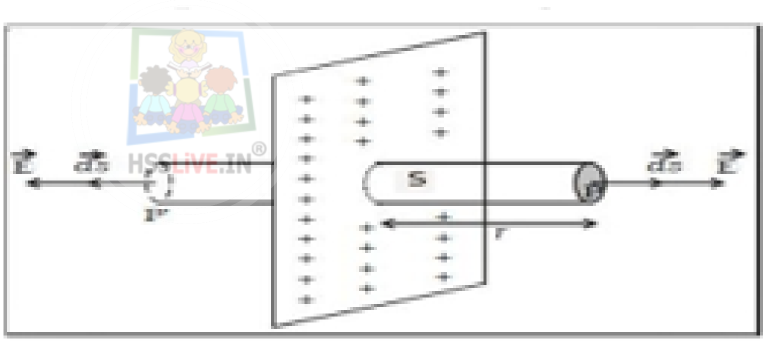
$$\phi = \frac{\lambda l}{\varepsilon_0} \longrightarrow (2)$$

From equations (1) and (2) $E \times 2\pi r l = \frac{\lambda l}{\varepsilon_0}$

$$\boxed{E = \frac{\lambda}{2\pi r \varepsilon_0}}$$

$E \alpha \frac{1}{r}$

2. Electric field due to a uniformly charged infinite planesheet



The field lines crosses only the 2 end faces of the Gaussian surface. So the flux through the Gaussian surface

$$\phi = 2\,E\,S \text{ ------------------ } (1)$$

By Gauss law

$$\phi = \frac{q}{\varepsilon_0} \qquad (\sigma = q/S)$$

$$\phi = \frac{\sigma S}{\varepsilon_0} \text{ ---------------------- } (2)$$

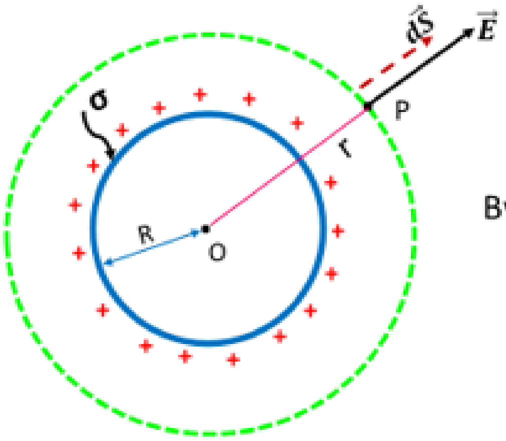
From equations (1) and (2)

$$2\,E\,S = \frac{\sigma S}{\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

Electric field is independent of distance r

3) Electric field due to a uniformly charged thin spherical shell



a) Field outside the shell

$$\phi = ES$$

$$\phi = E \times 4\pi r^2 \longrightarrow (1)$$

By Gauss's law $\phi = \frac{q}{\varepsilon_0}$

$$\sigma = \frac{q}{A}$$
$$q = \sigma A$$

$$\phi = \frac{\sigma A}{\varepsilon_0}$$

$$\phi = \frac{\sigma \times 4\pi R^2}{\varepsilon_0} \longrightarrow (2)$$

From equations (1) and (2) $E \times 4\pi r^2 = \frac{\sigma \times 4\pi R^2}{\varepsilon_0}$

$$\boxed{E = \frac{\sigma R^2}{\varepsilon_0 r^2}}$$

$$E \propto \frac{1}{r^2}$$

b) field on the surface of the shell

On the surface of shell $r = R$

$$E = \frac{\sigma R^2}{\varepsilon_0 r^2}$$

$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

E is maximum at the surface of the shell

c) field inside the shell

$$\phi = ES \longrightarrow (1)$$

Inside the shell q=0

By Gauss's law $\phi = \frac{q}{\varepsilon_0} = 0 \longrightarrow (2)$

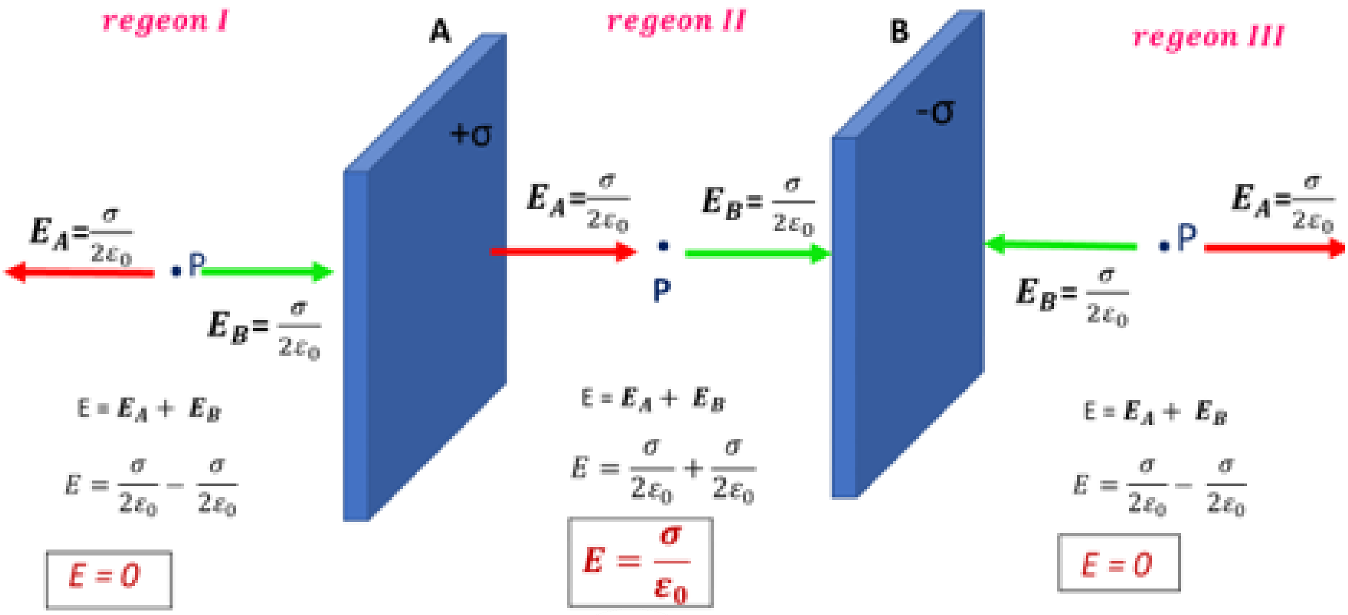
From equations (1) and (2)

$$ES = 0 \qquad (S \neq 0)$$

$$\boxed{E = 0}$$

Example

Find the electric field due two plane sheets of charge in regions I ,II and III



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Chapter 2
Electrostatic Potential and Capacitance

2.1 Introduction
Electrostatic Potential Energy at a point

Electric potential energy at a point P in an electric field is defined as the work done by the external force in bringing the charge q from infinity to that point.

$$U_{P\infty} = U_P - U_\infty = U_P - 0 = U_P$$

$$\mathbf{U_P = -\int_\infty^P F \cdot dr}$$

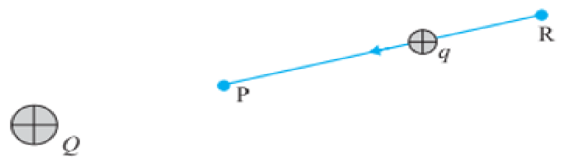
- The work done by an electrostatic field in moving a charge from one point to another depends only on the initial and the final points and is independent of the path taken to go from one point to the other.

Electrostatic potential energy difference between two points

Electric potential energy difference between two points in an electric field , can be defined as the work done by an external force in moving (without accelerating) charge q from one point to another in the electric field .

$$\Delta U = U_P - U_R = -\int_R^P F \cdot dr$$

Derivation:-



$$W_{RP} = \int_R^P F_{ext} \cdot dr$$

External force, F_{ext} is equal and opposite to repulsive electric force F.

$$F_{ext} = -F$$

$$W_{RP} = -\int_R^P F \cdot dr$$

This work done is stored as electrostatic potential energy.

So the potential energy difference between points R and P,

$$\Delta U = \boldsymbol{U_P - U_R = -\int_R^P F \cdot dr}$$

2.2 Electrostatic Potential

Electrostatic Potential at a point P

Electrostatic Potential at a point P in an electric field is the work done by an external force in bringing a unit positive charge from infinity to that point.

$$\mathbf{V_P = -\int_{\infty}^{P} E \,.\, dr}$$

Electrostatic Potential difference between two points

Electrostatic Potential difference between two points in an electric field is the work done by an external force in bringing a unit positive charge from one point to other in that field.

$$V_P - V_R = \frac{W_{RP}}{q}$$

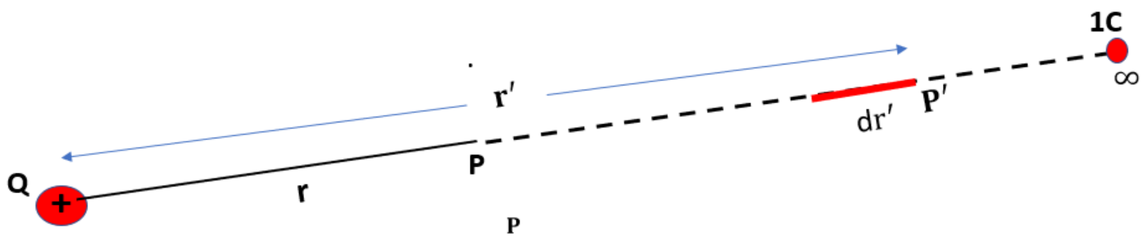
$$V_P - V_R = \frac{-\int_R^P F\,.dr}{q}$$

$$F = qE$$

$$V_P - V_R = \frac{-\int_R^P qE\,.dr}{q}$$

$$\mathbf{V_P - V_R = -\int_R^P E \,.\, dr}$$

2.3 Potential due to a Point Charge



$$V = -\int_{\infty}^{P} \mathbf{E} \cdot \mathbf{dr'}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r'^2}$$

$$V = -\int_{\infty}^{r} \frac{1}{4\,\pi\varepsilon_0}\frac{Q}{r'^2}\,dr'$$

$$V = -\frac{Q}{4\pi\varepsilon_0} \times \left[\frac{-1}{r'}\right]_{\infty}^{r}$$

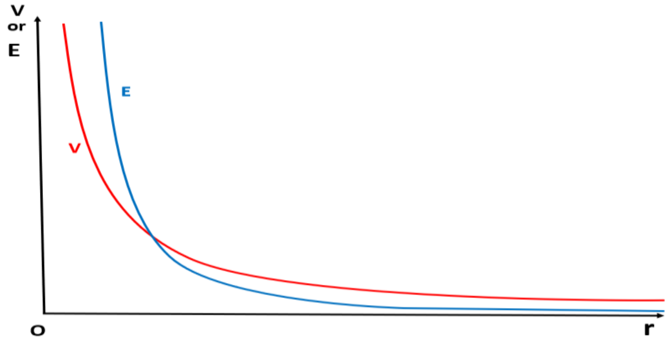
$$V = -\frac{Q}{4\pi\varepsilon_0}\left[\frac{-1}{r} - \frac{-1}{\infty}\right]$$

$$V = -\frac{Q}{4\pi\varepsilon_0}\left[\frac{-1}{r} - 0\right]$$

$$\mathbf{V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}}$$

$$V \propto \frac{1}{r}$$

Variation of potential V with r and Electric field with r for a point charge Q



Variation of potential V with r

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

$$V \propto \frac{1}{r}$$

Variation of Electric field E with r

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

$$E \propto \frac{1}{r^2}$$

Example

(a) Calculate the potential at a point P due to a charge of 4×10^{-7}C,located 9 cm away

(b) Hence obtain the work done in bringing a charge of 2×10^{-9} C from infinity to the point P. Does the answer depend on the path along which the charge is brought?

(a) $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$

$= 9 \times 10^9 \text{ x } \frac{4\ x\ 10^{-7}}{0.09}$

$\mathbf{V = 4\ x\ 10^4 V}$

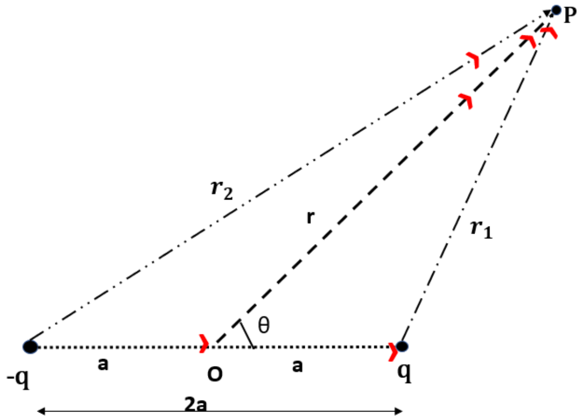
(b) W= qV

$= 2\ x\ 10^{-9}\ \text{ x } 4\ x\ 10^4$

$\mathbf{W = 8\ x\ 10^{-5}\ J}$

No, work done will be path independent. Any arbitrary infinitesimal path can be resolved into two perpendicular displacements: One along r and another perpendicular to r. The work done corresponding to the later will be zero.

2.4 Potential due to an Electric Dipole



$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_1}$$

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{-q}{r_2}$$

$$V = V_1 + V_2$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{-q}{r_2}$$

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\frac{1}{r_1} - \frac{1}{r_2} = \frac{2a\cos\theta}{r^2}$$

$$V = \frac{q}{4\pi\varepsilon_0}\frac{2a\cos\theta}{r^2}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

Potential along the axial line

$$\theta = 0$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos 0}{r^2}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$$

Potential along the equatorial line

$$\theta = 90$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos 90}{r^2}$$

$$V = 0$$

$$\vec{r} = \vec{r_1} + \vec{a}$$

$$\vec{r_1} = \vec{r} - \vec{a}$$

$$\vec{r_1} = \vec{r} + (-\vec{a})$$

$$\vec{R} = \vec{A} + \vec{B}$$

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$R^2 = A^2 + B^2 + 2AB\cos\theta$$

$$r_1^2 = r^2 + a^2 - 2ar\cos\theta$$

Since a‹‹ r, a can be neglected

$$r_1^2 = r^2 - 2ar\cos\theta$$

$$r_1^2 = r^2\left(1 - \frac{2a\cos\theta}{r}\right)$$

$$r_1 = r\left(1 - \frac{2a\cos\theta}{r}\right)^{\frac{1}{2}}$$

$$\frac{1}{r_1} = \frac{1}{r}\left(1 - \frac{2a\cos\theta}{r}\right)^{\frac{-1}{2}}$$

$$\frac{1}{r_1} = \frac{1}{r}\left(1 + \frac{a\cos\theta}{r}\right)$$

$$\vec{r_2} = \vec{r} + (\vec{a})$$

$$\vec{R} = \vec{A} + \vec{B}$$

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$R^2 = A^2 + B^2 + 2AB\cos\theta$$

$$r_2^2 = r^2 + a^2 + 2ar\cos\theta$$

Since a‹‹ r, a can be neglected

$$r_2^2 = r^2 + 2ar\cos\theta$$

$$r_2^2 = r^2\left(1 + \frac{2a\cos\theta}{r}\right)$$

$$r_2 = r\left(1 + \frac{2a\cos\theta}{r}\right)^{\frac{1}{2}}$$

$$\frac{1}{r_2} = \frac{1}{r}\left(1 + \frac{2a\cos\theta}{r}\right)^{\frac{-1}{2}}$$

$$\frac{1}{r_2} = \frac{1}{r}\left(1 - \frac{a\cos\theta}{r}\right)$$

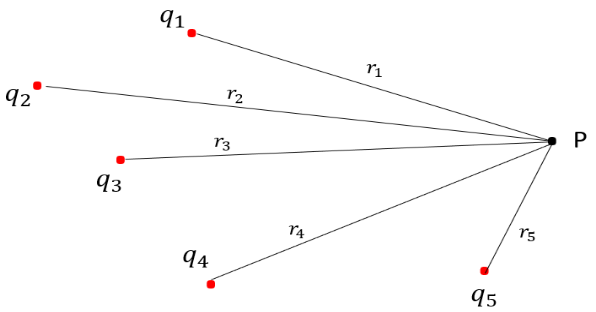
$$\frac{1}{r_1} - \frac{1}{r_2} = \frac{1}{r}\left(1 + \frac{a\cos\theta}{r}\right) - \frac{1}{r}\left(1 - \frac{a\cos\theta}{r}\right)$$

$$\frac{1}{r_1} - \frac{1}{r_2} = \frac{1}{r} + \frac{a\cos\theta}{r^2} - \frac{1}{r} + \frac{a\cos\theta}{r^2}$$

$$\frac{1}{r_1} - \frac{1}{r_2} = \frac{2a\cos\theta}{r^2}$$

2.5 Potential due to a System of Charges

By the superposition principle, the potential at a point due to a system of charges is the algebraic sum of the potentials due to the individual charges.

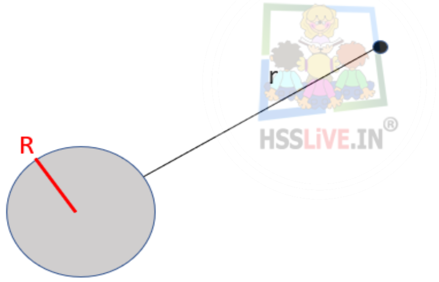


$$V = V_1 + V_2 + \cdots \quad + V_n$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2} + \ldots\ldots\ldots\ldots + \frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_n}$$

$$\mathbf{V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \ldots\ldots\ldots\ldots\ldots + \frac{q_n}{r_n}\right)}$$

Potential due to a uniformly charged spherical shell



a)The potential at a distance r, from the shell ,where r≥ R

(R-radius of sphere)

For a uniformly charged spherical shell, the electric field outside the shell is as if the entire charge is concentrated at the centre

$$\mathbf{V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}} \quad (r \geq R)$$

b) Inside the shell

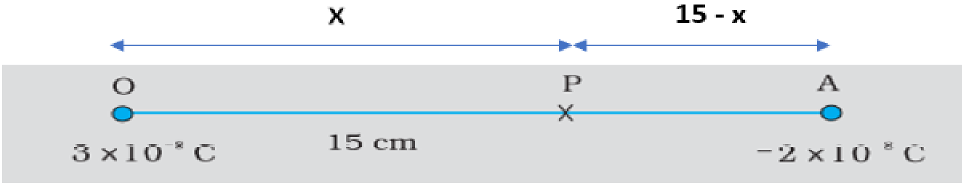
Inside the shell the electric field is zero. This implies that the potential is constant inside the shell ,which is equal to the value of potential at the surface

$$\mathbf{V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}}$$

Example

Two charges $3 \ x \ 10^{-8}$C and $-2 \ x \ 10^{-8}$ C are located 15 cm apart. At what point on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

Let P lies between O and A at a distance x from O,



Potential at P due to charge $3\,x\,10^{-8}$C

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{3\,x\,10^{-8}}{x}$$

Potential at P due to charge $-2\,x\,10^{-8}$C

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{-2\,x\,10^{-8}}{15-x}$$

Total potential at P , $\mathrm{V}=V_1 + V_2=0$

$$\frac{1}{4\pi\varepsilon_0}\frac{3\,x\,10^{-8}}{x} - \frac{1}{4\pi\varepsilon_0}\frac{2\,x\,10^{-8}}{15-x}=0$$

$$\frac{1}{4\pi\varepsilon_0}\left[\frac{3\,x\,10^{-8}}{x} - \frac{2\,x\,10^{-8}}{15-x}\right]=0$$

$$\left[\frac{3}{x} - \frac{2}{15-x}\right]=0$$

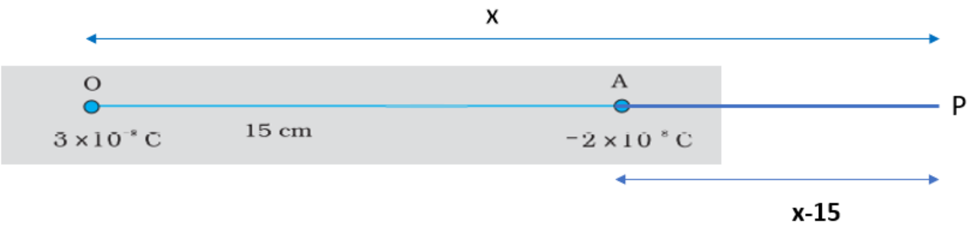
$$\frac{3}{x} = \frac{2}{15-x}$$

$$45\text{-}3x=2x$$

$$45=5x$$

$$x=9\text{cm}$$

If P lies on the extended line OA,



$$\frac{1}{4\pi\varepsilon_0}\frac{3\,x\,10^{-8}}{x} - \frac{1}{4\pi\varepsilon_0}\frac{2\,x\,10^{-8}}{x-15}=0$$

$$\frac{3}{x} = \frac{2}{x-15}$$

$$3x\text{-}45=2x$$

$$x=45\text{cm}$$

Thus, electric potential is zero at 9 cm and 45 cm away from the positive charge on the side of the negative charge.

2.6 Equipotential Surfaces

An equipotential surface is a surface with a constant value of potential at all points on the surface.

- As there is no potential difference between any two points on an equipotential surface, no work is required to move a test charge on the surface.
- For any charge configuration, equipotential surface through a point is normal to the electric field at that point

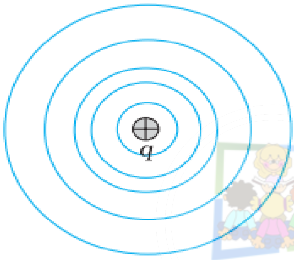
Equipotential surfaces for a single point charge

For a single charge q, the potential is

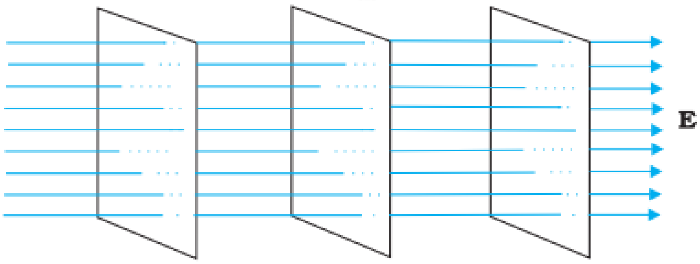
$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

V is a constant if r is constant .

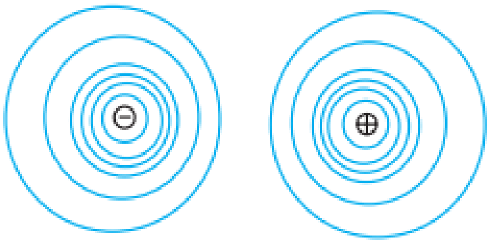
Thus, equipotential surfaces of a single point charge are concentric spherical surfaces centred at the charge.



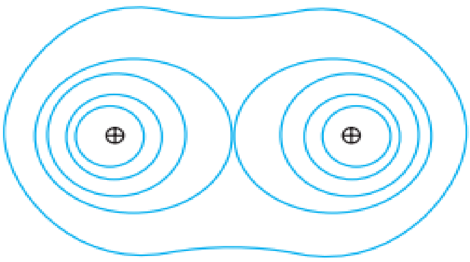
Equipotential surfaces for a uniform electric field.



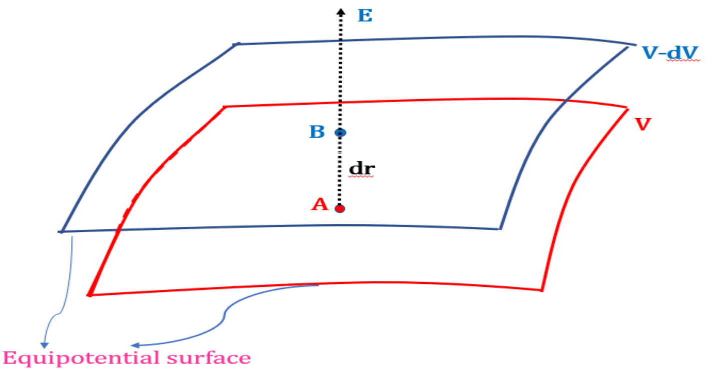
Equipotential surfaces for a dipole



Equipotential surfaces for two identical positive charges.



Relation between electric field and potential



The work done to move a unit positive charge from B to A is

$$dW = F.\,dr = F\,dr \cos 180 = -F\,dr$$

(But $F = qE$
$q = 1$
$F = E$)

$$dW = -E\,dr$$

This work equals the potential difference,

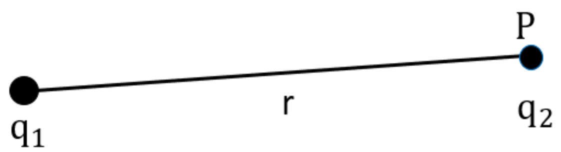
$$V_A - V_B = V - (V - dV) = dV$$

$$-E\,dr = dV$$

$$\boldsymbol{E = \frac{-dV}{dr}}$$

2.7 Potntial energy of a system of Charges

a) For a system of Two charges



The potential due to the charge q_1 at point P

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}$$

The work done in bringing charge q_2 from infinity to the point P is

$$W = qV$$

$$W = q_2 V_1$$

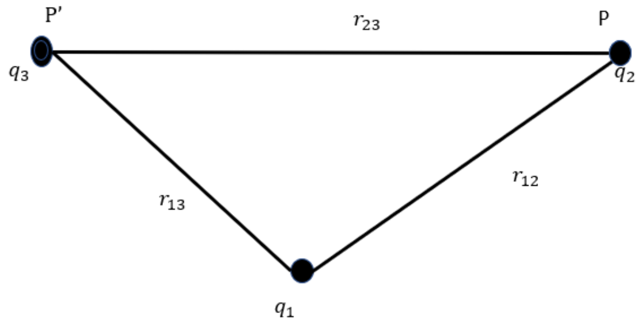
$$W = q_2 \text{ x } \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}$$

$$W = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

This work gets stored in the form of potential energy of the system. Thus, the potential energy

$$\mathbf{U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}}$$

b)For a system of three charges



The potential due to the charge q_1 at point P

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{12}}$$

The work done to bring the charge q_2 from infinity to P

$$W_1 = q_2\, V_1$$

$$W_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

The total potential at P' due to the charges q_1 and q_2

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{23}}$$

$$V_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right)$$

The work done to bring the charge q_3 from infinity to P'

$$W_2 = q_3 V_2$$

$$W_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_3}{r_{13}} + \frac{q_2\, q_3}{r_{23}}\right)$$

The toal workdone in assembling the charges

$$W = W_1 + W_2$$

$$W = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

This work is equal to the potential energy of the system of three charges q_1 , q_2 and q_3

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

2.8 Potential Energy in an External Field

1)Potential energy of a single charge

The potential energy of a point charge in an external field is the workdone in bringing the charge from infinity to the point.

The external electric field E and corresponding external potential V vary from point to point. The potential at infinity is zero.

Thus the work done in bringing the charge q from infinity to the point P in the external field is qV.

This work is stored as potential energy of charge q.

Potential energy of q at r in an external field =q $V_{(r)}$

Electron volt

The energy gained by an electron, when it is accelerated by a potential difference of 1 volt is called electron volt (I eV)

I eV=1.602 x 10^{-19} coulomb x 1volt

I eV = 1.602 x 10^{-19} J

2)Potential energy of a system of two charges in an external field

Consider a system of two charges q_1 and q_2 located at r_1 and r_2.

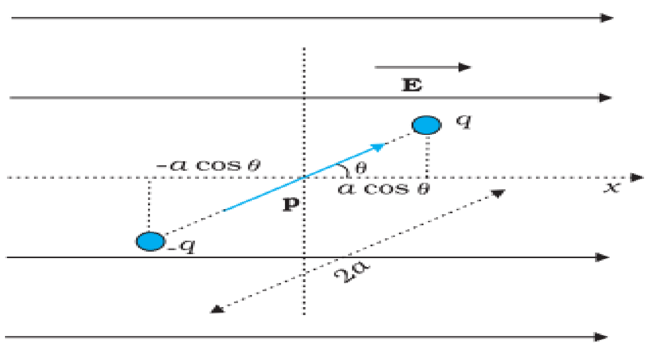
The work done in bringing the charge q_1 from infinity to r_1 =q_1 V(r_1)

For bringing the charge q_2 to r_2 work has to be done against the external field and also against the field to q_1.

The work done in bringing the charge q_2 to r_2 =q_2 V(r_2) + $\frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$

The potential energy of the system = q_1 V(r_1) + q_2 V(r_2) + $\frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$

2.8 Potential energy of a dipole in an external field



Torque acting on the dipole

$$\vec{\tau} = \vec{p} \times \vec{E}$$
$$\vec{\tau} = pE\sin\theta$$

The workdone by the external torque

$$dW = \tau_{ext}\, d\theta$$
$$dW = pE\sin\theta\, d\theta$$
$$W = \int_{\theta_0}^{\theta_1} pE\sin\theta\, d\theta$$
$$W = pE[-\cos\theta]_{\theta_0}^{\theta_1}$$
$$W = pE(-\cos\theta_1 - -\cos\theta_0)$$

$$W = pE(\cos\theta_0 - \cos\theta_1)$$

This work is stored as potential energy of the system

$$\mathbf{U = pE(\cos\theta_0 - \cos\theta_1)}$$

If we take $\theta_0 = \frac{\pi}{2}$,

$$U = -pE\cos\theta$$

$$\mathbf{U = -\vec{p} \cdot \vec{E}}$$

2.9 Electrostatics of conductors

1.Inside a conductor, electrostatic field is zero

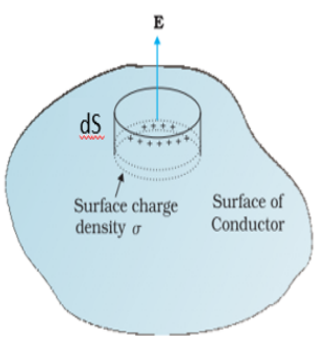
A conductor has free electrons. In the static situation, the free charges have so distributed themselves that the electric field is zero everywhere inside.

2. At the surface of a charged conductor, electrostatic field must be normal to the surface at every point.

3. The interior of a conductor can have no excess charge in the static situation.

4. Electrostatic potential is constant throughout the volume of the conductor and has the same value (as inside) on its surface.

5.Electric field at the surface of a charged conductor $E = \frac{\sigma}{\varepsilon_0}$



Consider a pill box shaped Gaussian surface to find the electric field at the surface of a charged conductor.

The total flux through the pill box comes only from the outside (circular) cross-section of the pill box

By Gauss's law,

$$E\, dS = \frac{q}{\varepsilon_0}$$
$$EdS = \frac{\sigma\, dS}{\varepsilon_0}$$
$$E = \frac{\sigma}{\varepsilon_0}$$

6.Electrostatic shielding

The electric field inside a cavity of any conductor is zero. This is known as electrostatic shielding. All charges reside only on the outer surface of a conductor with cavity.

The effect can be made use of in protecting sensitive instruments from outside electrical influence.

Why it is safer to be inside a car during lightning?

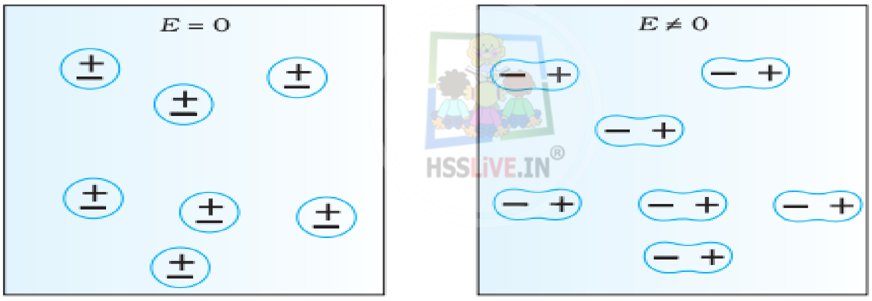
Due to Electrostatic shielding, electricfield E=0 inside the car.
So it is safer to sit inside a car than standing outside during lightening.

2.10 Dielectrics and polarisation

Dielectrics

Dielectrics are non-conducting substances. In contrast to conductors, the Dielectric substances may be made of polar or non polar molecules.

Non polar molecules

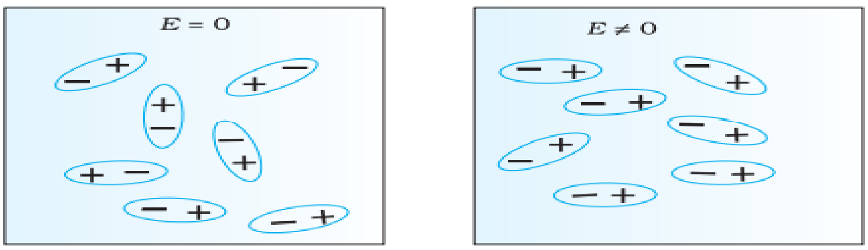


In a non-polar molecule, the centres of positive and negative charges coincide. The molecule then has no permanent (or intrinsic) dipole moment.

Eg: oxygen (O_2) , hydrogen (H_2)

In an external electric field, the positive and negative charges of a nonpolar molecule are displaced in opposite directions. The non-polar molecule thus develops an induced dipole moment. The dielectric is said to be polarised by the external field

Polar molecules



In polar molecules, the centres of positive and negative charges are separated (even when there is no external field). Such molecules have a permanent dipole moment.

Eg: HCl , H_2O

In the absence of any external field, the different permanent dipoles are oriented randomly ; so the total dipole moment is zero. When an external field is applied, the individual dipole moments tend to align with the field. A dielectric with polar molecules also develops a net dipole moment in an external field.

Polarisation(P)

The dipole moment per unit volume is called polarisation .
For linear isotropic dielectrics,

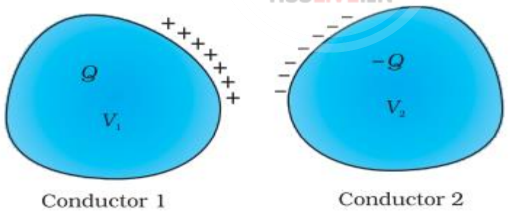
$$\mathbf{P} = \chi_e \mathbf{E}$$

where χ_e is the electric **susceptibility** of the dielectric medium.

2.11 Capacitors and Capacitance

Capacitor

A capacitor is a system of two conductors separated by an insulator. Capacitor is a charge storing device.



Capacitance

The potential difference, V between the two conductors is proportional to the charge , Q.

$$Q \propto V$$
$$Q = C\,V$$
$$\mathbf{C} = \frac{\mathbf{Q}}{\mathbf{V}}$$

The constant C is called the capacitance of the capacitor.

C is independent of Q or V.

The capacitance C depends only on the geometrical configuration (shape, size, separation) of the system of two conductors .

SI unit of capacitance is farad.

$$1 \text{ farad} = 1 \text{ coulomb volt}^{-1}$$

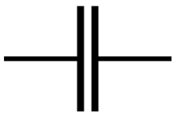
$$1 \text{ F} = 1 \text{ C V}^{-1}$$

Other units are,

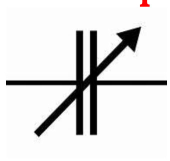
$1\ \mu\text{F} = 10^{-6}$ F , 1 nF $= 10^{-9}$ F , 1 pF $= 10^{-12}$ F, etc.

Symbol of capacitor

Fixed capacitance



Variable capacitance

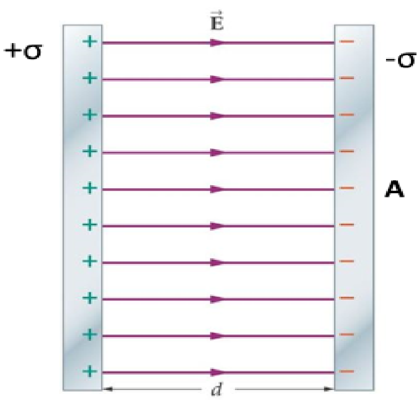


- $C = \frac{Q}{V}$. For large C, V is small for a given Q. This means a capacitor with large capacitance can hold large amount of charge Q at a relatively small V
- High potential difference implies strong electric field around the conductors. A strong electric field can ionise the surrounding air and accelerate the charges so produced to the oppositely charged plates, thereby neutralising the charge on the capacitor plates, at least partly.
- The maximum electric field that a dielectric medium can withstand without break-down (of its insulating property) is called its dielectric strength; for air it is about $3 \times 10^6\ Vm^{-1}$

2.12 The parallel plate capacitor

A parallel plate capacitor consists of two large plane parallel conducting plates separated by a small distance.

Capacitance of a parallel plate capacitor



Capacitance, $C = \frac{Q}{V}$

$Q = \sigma A$

$V = Ed$

$E = \frac{\sigma}{\varepsilon_0}$

$V = \frac{\sigma d}{\varepsilon_o}$

$C = \frac{\sigma A}{\frac{\sigma d}{\varepsilon_0}}$

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$

Capacitance can be increased,

- By increasing the area of the plates.
- By decreasing the distance between the plates.
- By introducing a dielectric medium between the plates.

2.13 Effect of dielectric on capacitance

The capacitance of a parallel plate capacitor when the medium between the plates is air,

$$C_{air} = \frac{\varepsilon_0 A}{d}$$

When dielectric medium of dielectric constant K is placed between the plates, the capacitance ,

$$C_{med} = \frac{K\varepsilon_0 A}{d}$$

The capacitance increases K times, where K is the dielectric constant.

$$\mathbf{C_{med} = K\, C_{air}}$$

Definition of dielectric constant in terms of capacitance

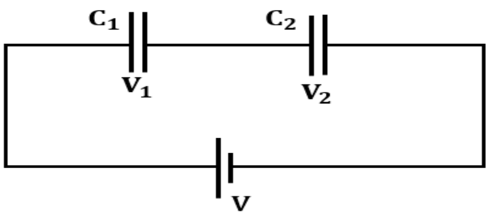
$$\frac{C_{med}}{C_{air}} = \frac{\frac{K\varepsilon_0 A}{d}}{\frac{\varepsilon_0 A}{d}} = K$$

$$\mathbf{K = \frac{C_{med}}{C_{air}}}$$

The dielectric constant of a substance is the factor by which the capacitance increases from its vacuum value, when a dielectric is inserted between the plates.

2.14 Combination of Capacitors

Capacitors In Series



In series combination the charge Q is same and potential drop is different in each capacitor. The total potential drop V across the combination is

$$V = V_1 + V_2$$

$$\mathbf{V = \frac{Q}{C_1} + \frac{Q}{C_2}} \text{ ---------(1)}$$

If the two capacitors are replaced by a single capacitor of capacitance C with the same charge Q and potential difference V.

$$V = \frac{Q}{C} \text{ ------------ (2)}$$

Equating eq (1) & $\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2}$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

- For n capacitors in series, $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \text{..........} + \frac{1}{C_n}$
- If all the capacitors have the same value, $C_1 = C_2 = \text{.........} = C_n = C$

$$\frac{1}{C'} = \frac{n}{C} \quad , \quad C' = \frac{C}{n}$$

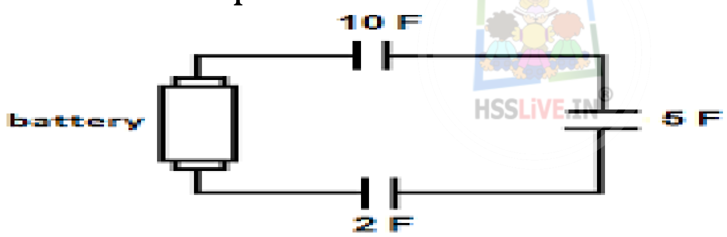
eg:- If C=1μF n=10

$$C' = \frac{C}{n} = \frac{1\mu F}{10} = 0.1\mu F$$

The effective capacitance decreases when capacitors are connected in series. In series combination the effective capacitance will be smaller than the smallest among individual capacitors.

Example

Find the effective capacitance of the combination.

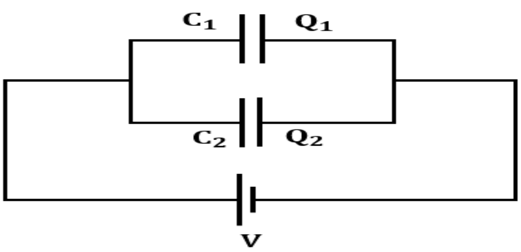


$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C} = \frac{1}{2F} + \frac{1}{5F} + \frac{1}{10F} = \frac{8}{10F}$$

$$C = \frac{10F}{8} = 1.25F$$

Capacitors In Parallel



In parallel connection, the same potential drop across both the capacitors, but the charges are different.

$$Q = Q_1 + Q_2$$

$$Q = C_1V + C_2V \text{ ------------ (1)}$$

If the two capacitors are replaced by a single capacitor of capacitance C with the same charge Q and potential difference V.

$$\mathbf{Q=CV} \text{ ------------------ } \mathbf{(2)}$$

From eq(1) & (2) $CV = C_1V + C_2V$

$$C = C_1 + C_2$$

- For n capacitors in parallel , $\mathbf{C = C_1 + C_2 + C_3 + \ldots\ldots + C_n}$
- If all the capacitors have the same value, $C_1 = C_2 = C_3 = \ldots\ldots = C_n = C$

$$\mathbf{C' = nC}$$

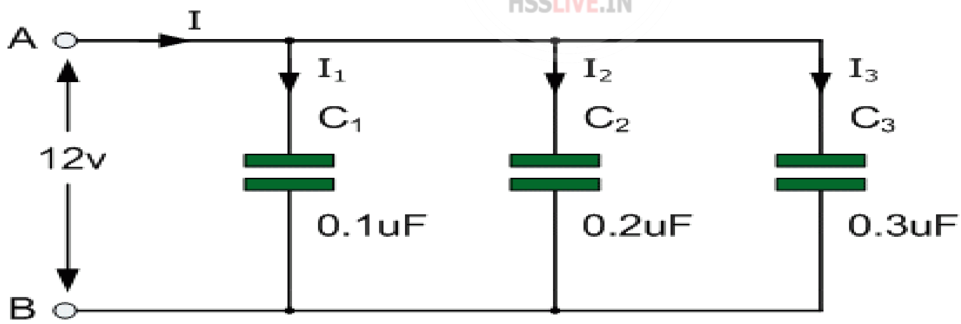
Eg: If C=1μF n=10

$C' = nC = 10 \text{ x } 1\mu F = 10\mu F$

The effective capacitance increases when capacitors are connected in parallel. In parallel combination the effective capacitance will be greater than the greatest among individual capacitors.

Example

Find the effective capacitance of the combination.

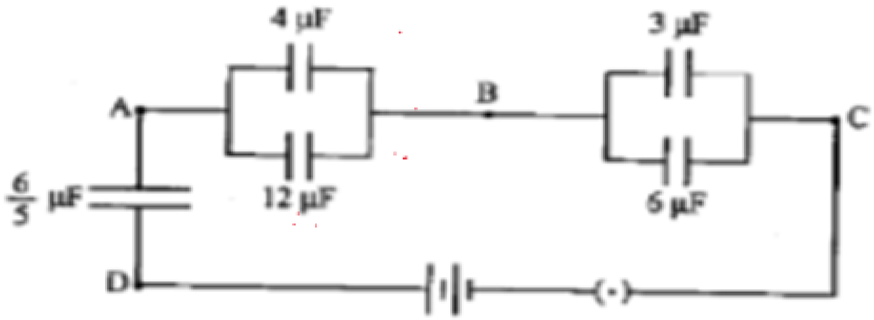


$C = C_1 + C_2 + C_3$

$C = 0.1\ \mu F + 0.2\ \mu F + 0.3\ \mu F$

$C = 0.6\ \mu F$

1)Find the equivalent capacitance of capacitors given in the network



4µF and 12µF are connected in paraliel.

$C' = C_1 + C_2$

$C' = 4\mu F + 12\mu F$

$C' = 16\mu F$

3µF and 6µF are connected in paraliel.

$C'' = C_1 + C_2$

$C'' = 3\mu F + 6\mu F$

$C'' = 9\mu F$

6/5 µF , 16µF and 9µF are connected in series

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

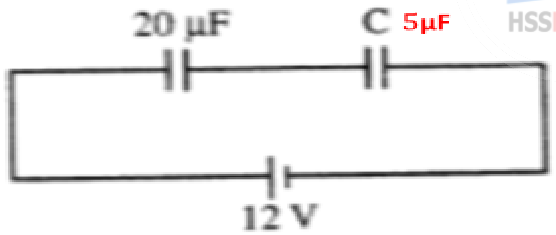
$$\frac{1}{C} = \frac{1}{6/5\ \mu F} + \frac{1}{16\mu F} + \frac{1}{9\mu F}$$

$$\frac{1}{C} = \frac{5}{6\ \mu F} + \frac{1}{16\mu F} + \frac{1}{9\mu F} = 1.0069$$

$$C = \frac{1}{1.0069}$$

$C = 0.99\mu F$

2)Two capacitors are connected as shown in figure .The equivalent capacitance of the combination is 4µF .



(a) Calculate the value of C

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{4\mu F} = \frac{1}{20\mu F} + \frac{1}{C}$$

$$\frac{1}{C} = \frac{1}{4\mu F} - \frac{1}{20\mu F}$$

$$\frac{1}{C} = \frac{4}{20\mu F}$$

C = 5µF

(b)Calculate the charge on each capacitor.

$$C' = \frac{Q}{V}$$

$Q = C'V$

$Q = 4\mu F \times 12$

Q= 48 µ C

(c) What will be the potential drop across each capacitor?

The potential drop across 20µF

$$V_1 = \frac{Q}{C_1}$$

$$V_1 = \frac{48\mu C}{20\mu\ F}$$

$V_1 = 2.4$ V

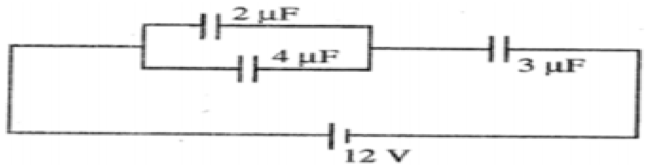
The potential drop across 5µF

$$V_2 = \frac{Q}{C_2}$$

$$V_2 = \frac{48\mu C}{5\mu\ F}$$

$V_2 = 9.6$ V

3)Three capacitors are connected to a 12V battery as shown in figure



a)What is the effective capacitance of the combination?

2μF and 4μF are connected in paralle.

$C' = C_1 + C_2$

C' = 2μF + 4μF

C' = 6μF

6μF and 3μF are connected in series

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \qquad C = \frac{C_1 C_2}{C_1 + C_2} = \frac{6\mu \times 3\mu}{6\mu + 3\mu}$$

$$C = \frac{18\mu \times \mu}{9\mu}$$

C = 2μF

b)What is the potential difference across the 2μF capacitor?

$$V = \frac{Q}{C}$$

Q= CV

Q = 2μ x12

Q= 24 μ C

$$V = \frac{Q}{C}$$

$$V = \frac{24\mu}{6\mu}$$

V = 4 V

4)What is area of plates of a 0.1μF parallel plate air capacitor, given that the separation between the plates is 0.1mm

$$C = \frac{\varepsilon_0 A}{d}$$

$$A = \frac{Cd}{\varepsilon_0} = \frac{0.1 \times 10^{-6} \times 0.1 \times 10^{-3}}{8.85 \times 10^{-12}}$$

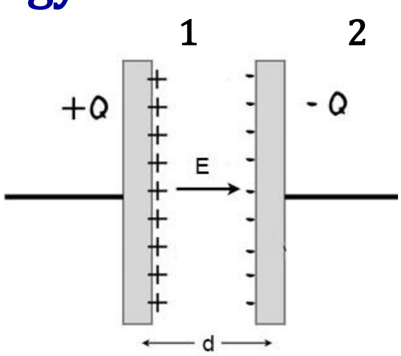
$$= 10.47 \times 10^3 \ m^2$$

5)A parallel plate capacitor with air between plates has a capacitance of 8μF . What will be the capacitance if distance between the plates is reduced by half and the space between is filled with a medium of dielectric constant 5.

$$C = \frac{\varepsilon_0 A}{d} = 8\mu F$$

$$C' = \frac{K\varepsilon_0 A}{d/2} = 2K\frac{\varepsilon_0 A}{d} = 2 \times 5 \times 8\mu F = 80\mu F$$

2.15 Energy Stored in a Capacitor



Work done to move a charge dq from conductor 2 to conductor 1

$$dW = \text{Potential} \times \text{Charge}$$
$$dW = V\,dq$$
$$dW = \frac{q}{C}\,dq$$

The total work done to attain a charge Q on conductor 1, is

$$W = \int_0^Q dW = \int_0^Q \frac{q}{C} \times dq$$

$$W = \frac{Q^2}{2C}$$

The work stored as **potential energy in the electric field** between the plates.

$$\textbf{Energy} \quad U = \frac{Q^2}{2C}$$

Energy stored in a capacitor can also be expressed as

$$\boxed{U = \frac{Q^2}{2C}}$$

$$C = \frac{Q}{V}$$

$$Q = CV$$

$$U = \frac{C^2V^2}{2\,C}$$

$$\boxed{U = \frac{1}{2}CV^2}$$

$$U = \frac{Q^2}{2C}$$

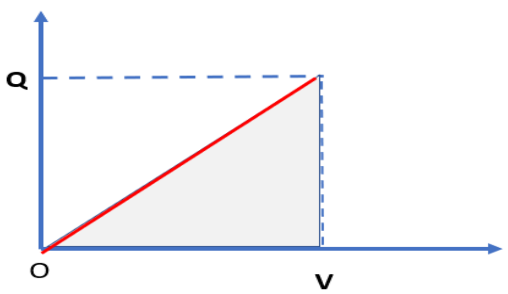
$$C = \frac{Q}{V}$$

$$U = \frac{Q^2}{2\,\frac{Q}{V}}$$

$$\boxed{U = \frac{1}{2}QV}$$

Energy Stored in a Capacitor
Graphical method

$$Q \propto V$$



Area under the graph $= \frac{1}{2}QV =$ Energy

Area under Q-V graph gives energy stored in a capacitor

Energy Density of a capacitor

Energy density is the energy density is the energy stored per unit volume

$$U = \frac{1}{2} CV^2$$

$$C = \frac{\varepsilon_0 A}{d} \text{ ----------(1)}$$

$$V = E\, d$$

$$E = \frac{\sigma}{\varepsilon_0}$$

$$V = \frac{\sigma}{\varepsilon_0}\, d \text{ ----------(2)}$$

$$U = \frac{1}{2}\ \frac{\varepsilon_0 A}{d}\ \ \frac{\sigma^2\, d^2}{\varepsilon_0^2}$$

$$U = \frac{1}{2} \varepsilon_0\ \frac{\sigma^2}{\varepsilon_0^2} Ad$$

$$U = \frac{1}{2}\ \varepsilon_0 E^2\, Ad$$

$$\text{Energy density} = \frac{\text{Energy stored}}{\text{volume}}$$

$$u = \frac{U}{Ad}$$

$$u = \frac{\frac{1}{2} \varepsilon_0\, E^2\, Ad}{Ad}$$

$$u = \frac{1}{2} \varepsilon_0 E^2$$

Chapter 3
Current Electricity

3.1 Introduction

Charges in motion constitute an electric current. Such currents occur naturally in many situations. Lightning is one such phenomenon in which charges flow from the clouds to the earth through the atmosphere. The flow of charges in lightning is not steady, but in our everyday life we see many devices where charges flow in a steady manner. A torch and a cell-driven clock are examples of such devices.

3.2 Electric Current

When current steady ,

The rate of flow of charge through any cross-section of a conductor is called electric current flowing through it.

$$I = \frac{q}{t}$$

$$\text{Unit of electric current} = \frac{\text{coulomb}}{\text{second}} = \text{C/s} = \text{ampere (A)}$$

When current is not steady,

The current at time t across the cross-section of the conductor is defined as the ratio of ΔQ to Δt in the limit of Δt tending to zero,

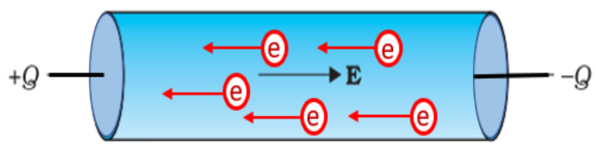
$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

3.3 Electric Currents in Conductors

When no electric field is present:-

The electrons will be moving due to thermal motion . During motion electrons collide with the fixed ions. The direction of its velocity after the collision is completely random. The average velocity of electrons will be zero. So, there will be no net electric current.

When an electric field is present:-



The electrons will be accelerated due to this field towards +Q. They will thus move to neutralise the charges and constitute an electric current.

Hence there will be a current for a very short while and no current thereafter.

To maintain a steady electric field in the body of the conductor we use cells or batteries.

3.4 Ohm's Law

A basic law regarding flow of currents was discovered by G.S. Ohm in 1828.

At constant temperature ,the current flowing through a conductor is directly proportional to the potential difference between the ends of the conductor.

$$V \propto I$$

$$V = RI$$

$$\mathbf{R = \frac{V}{I}}$$

The constant of proportionality R is called the resistance of the conductor

The SI units of resistance is ohm and is denoted by the symbol Ω.

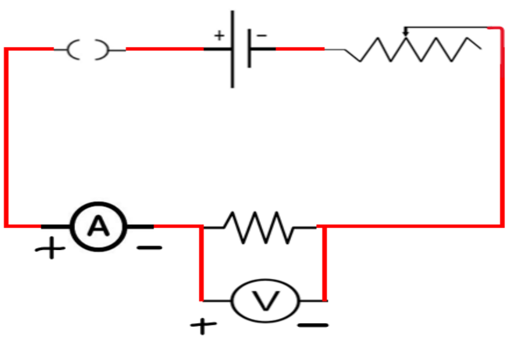
Conductance

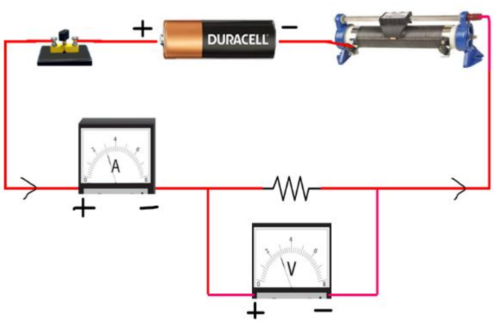
The reciprocal of resistance is called Conductance.

$$\mathbf{C = \frac{I}{V}}$$

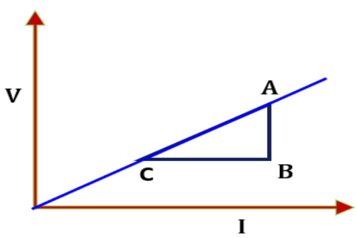
Unit of conductance is ohm^{-1} (Ω^{-1} or mho) or =siemens

Ohm's Law : Experimental verification





Voltage –Current Graph (V-I Graph)

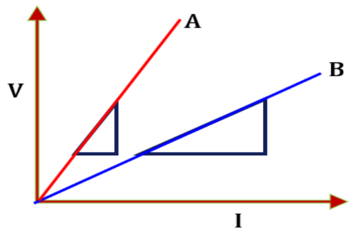


$$\text{Slope} = \frac{AB}{BC}$$

$$\mathbf{Slope = \frac{V}{I} = R}$$

Slope of V-I graph gives Resistance.
Its reciprocal gives conductance.

Which material has more resistance?



Slope of V-I graph gives Resistance. Slope of A is greater than that of B.
So material A has more resistance than B.

Factors on which the Resistance of a Conductor Depends:-

1)The material of the conductor

2)The dimensions of the conductor

a)Length of the conductor

The resistance of a conductor is directly proportional to length l of the conductor.

$$\mathbf{R \propto \mathit{l}}$$

b) The area of cross section of the conductor

The resistance of a conductor is inversely proportional to the cross-sectional area, A.

$$\mathbf{R \propto \frac{1}{A}}$$

Resistivity of a Conductor

The resistance of a conductor is directly proportional to length l of the conductor and inversely proportional to the cross-sectional area, A.

$$R \propto \frac{l}{A}$$

$$\mathbf{R = \frac{\rho \mathit{l}}{A}}$$

where the constant of proportionality ρ is called resistivity.

Resistivity depends on the material of the conductor but not on its dimensions.

$$\boldsymbol{\rho = \frac{RA}{\mathit{l}}}$$

$$\text{Unit of resistivity} = \frac{\Omega m^2}{m} = \Omega m$$

Ohm's Law in Vector Form

$V = IR$

$R = \frac{\rho l}{A}$

$V = \frac{I\rho l}{A}$

$\mathbf{V = j\rho \mathit{l}}$

Current density

Current per unit area (taken normal to the current), is called current density and is denoted by j.

Current density $j = \frac{I}{A}$

Unit of current density $= A/ m^2$

Current density is a vector quantity.

If E is the magnitude of uniform electric field in the conductor whose length is l, then the potential difference V across its ends , $V = E\,l$.

$E\,l = j\rho l$

$\vec{E} = \vec{j}\rho$

This is the vector form of Ohm's law. Here electric field and current density are vector quantities

$\vec{j} = \frac{\vec{E}}{\rho}$

$\vec{j} = \sigma\vec{E}$

This is another equation for Ohm's law in vector form.

Conductivity

Conductivity is the reciprocal of resistivity

$\sigma = \frac{1}{\rho}$

Unit of conductivity is $\Omega^{-1}\, m^{-1}$

3.5 Drift of Electrons and the Origin of Resistivity

Drift Velocity

The average velocity attained by electrons in a conductor due to an electric field is called Drift velocity.

The force acting on the electron due to the electric field,

$$F = qE = -eE$$

The acceleration of the electron,

$$a = \frac{F}{m} = \frac{-eE}{m}$$

If t is the time between two successive collissions, $v = at$

Then the velocity gained by an electron, $v = \frac{-eEt}{m}$

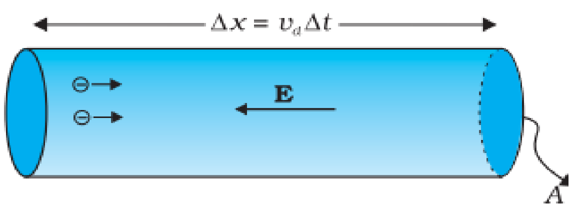
Drift velocity , $v_d = -\frac{eE}{m}(t)_{average}$

$$\mathbf{v_d = -\frac{eE}{m}\tau}$$

Relaxation Time

The average time interval between two successive collisions is called relaxation time(τ) $\quad \mathbf{v_d \propto \tau}$

Relation connecting Drift Velocity and Current



Distance Travelled by an electron in time $\Delta t = v_d \Delta t$

Volume of conductor $= A\ v_d\ \Delta t$

Let n be the number of electrons per unit volume of conductor

The number of electrons in the conductor$= n\ A\ v_d\ \Delta t$

Total charge of electrons in the conductor , $q= n\ e\ A\ v_d\ \Delta t$

$$\text{Current } I = \frac{q}{\Delta t}$$

$$I = \frac{n\ e\ A\ v_d\ \Delta t}{\Delta t}$$

$$\mathbf{I= n\ e\ A\ v_d}$$

Current density

$$I= n\ e\ A\ v_d$$

$$v_d = \frac{eE}{m}\tau$$

$$I= n\ e\ A\ \frac{eE}{m}\tau$$

$$I= \frac{ne^2A\tau E}{m}$$

$$j = \frac{I}{A} \qquad \mathbf{j = \frac{ne^2\tau E}{m}}$$

Conductivity

Comparing with, $j = \sigma E$

$$\boldsymbol{\sigma = \frac{ne^2\tau}{m}}$$

Mobility

Conductivity arises from mobile charge carriers.

In metals, these mobile charge carriers are electrons.

In an ionised gas, they are electrons and positive charged ions.

In an electrolyte, these can be both positive and negative ions.

Mobility μ defined as the magnitude of the drift velocity per unit electric field.

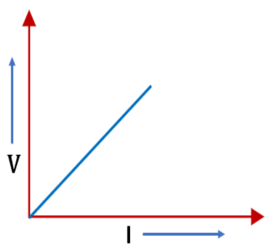
$$\boldsymbol{\mu} = \frac{\mathbf{v_d}}{\mathbf{E}} = \frac{\frac{eE}{m}\tau}{E}$$

$$\boldsymbol{\mu = \frac{e}{m}\tau}$$

3.6 Limitations of Ohm's Law

Ohmic Conductors

Conductors which obey Ohm's law are called Ohmic conductors.The Voltage – Current graph of such conductors will be linear .



Eg:- metals ,Nichrome

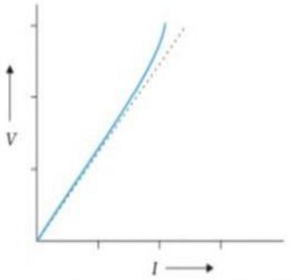
Non - Ohmic Conductors

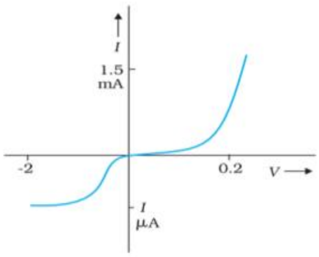
The materials and devices used in electric circuits which do not obey Ohm's law are called Non – Ohmic conductors. So V-I grapis not linear.

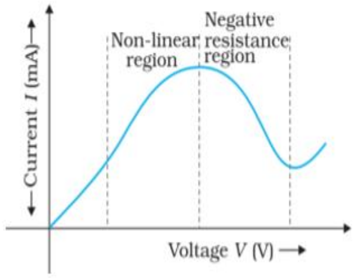
Eg:- Semi conductors, Diodes , Transistors.

The deviations broadly are one or more of the following types:

a) The value of V stops to be proportional to I.
b) The value of current changes when we reverse the direction of V.
c) The relation between V and I is not unique, i.e., there is more than one value of V for the same current I.







3.7 Resistivity of Various Materials

The materials are classified as conductors, semiconductors and insulators depending on their resistivities, in an increasing order of their values.

- Metals have low resistivities in the range of 10^{-8} Ωm to 10^{-6} Ωm.
- Insulators like ceramic, rubber and plastics having resistivities 10^{18} times greater than metals or more.
- In between the two are the semiconductors

3.8 Temperature Dependence Of Resistivity

The resistivity of a material is found to be dependent on the temperature. The resistivity of a metallic conductor is approximately given by,

$$\boldsymbol{\rho_T = \rho_0[1 + \alpha(T - T_0)]}$$

Where ρ_0 is the resistivity at a reference temperature T_0.
ρ_T is the resistivity at a temperature T
α is called the temperature co-efficient of resistivity

$$\frac{\rho_T}{\rho_0} = [1 + \alpha(T - T_0)]$$

$$\frac{\rho_T}{\rho_0} - 1 = \alpha(T - T_0)$$

$$\frac{\rho_T - \rho_0}{\rho_0} = \alpha(T - T_0)$$

$$\alpha = \frac{\rho_T - \rho_0}{\rho_0 (T - T_0)}$$

The dimension of α is $[\text{Temperature}]^{-1}$ and unit is K^{-1}.

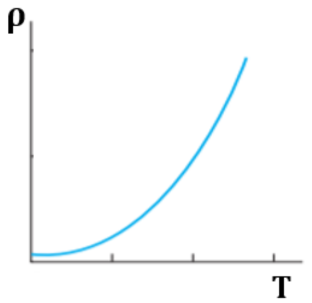
When temp increases, if the resistivity increases, then α is positive
When temp increases, if the resistivity decreases, then α is negative

For metals

For metals, α is positive ie, when temp increases, the resistivity also increases.

$$\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau} \qquad \rho \propto \frac{1}{\tau}$$

When temperature increases, the collisions of free electrons increases, relaxation time decreases and hence the resistivity increases.

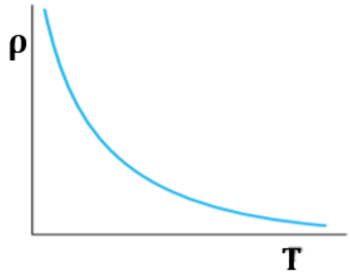


For insulators and semiconductors,

For insulators and semiconductors , α is negative ie., when temp increases, the resistivity decreases.

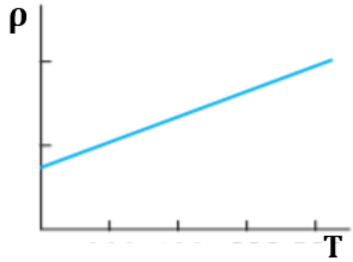
$$\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau} \qquad \rho \propto \frac{1}{n}$$

When temp increases, the number n of free electrons per unit volume increases , and hence the resistivity decreases.



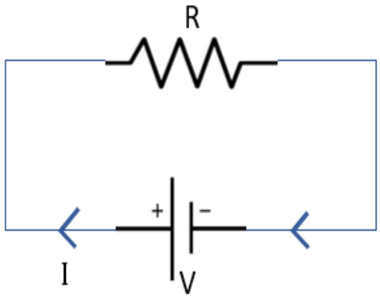
For Nichrome ,Manganin and constantan

Nichrome (which is an alloy of nickel, iron and chromium), Manganin and constantan exhibit a very weak dependence of resistivity with temperature. These materials are thus widely used in wire bound standard resistors since their resistance values would change very little with temperatures.



3.9 Electrical Energy, Power

Electrical Energy



Work done by the cell on a charge q under a pd of V volt

$$W = q V$$

$$q = It$$

$$W = VI\,t$$

This is work is same as the electricl energy supplied by the cell to the charges

$$E = VI\,t$$

The electrons move with increased KE and make collisions with atoms.
The energy gained by the charges is shared with the atoms.
The atoms vibrate more vigorously, i.e., the conductor heats up.

The amount of energy dissipated as heat in the conductor during the time interval t is

$$E = VI\,t$$

Using Ohm's law V = IR,

$$E = VI\,t$$
$$E = IR\,x\,It$$
$$E = I^2Rt$$

$$I = \frac{V}{R}$$
$$E = VI\,t = V\,x\frac{V}{R}t$$
$$E = \frac{V^2t}{R}$$

Power

Power is the energy dissipated per unit time

$$P = \frac{E}{t}$$

Unit of power is watt(W)

$$E = VI\,t \qquad E = \frac{V^2t}{R} \qquad E = I^2Rt$$

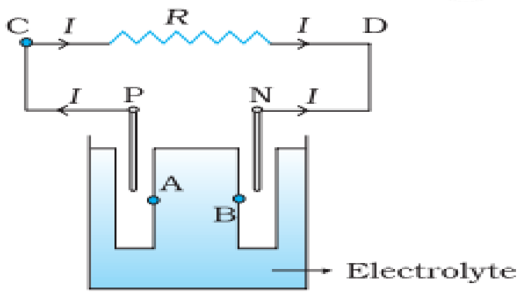
$$P = VIt/t \qquad P = \frac{V^2t}{R}/t \qquad P = \frac{I^2Rt}{t}$$

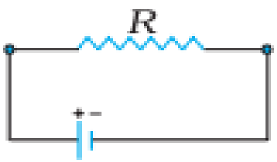
$$P = VI \qquad P = \frac{V^2}{R} \qquad P = I^2R$$

The power loss or "ohmic loss" in a conductor of resistance R carrying a current I is given by these equations. It is this power which heats up, the coil of an electric bulb to incandescence, radiating out heat and light.The external source ,that is the cell supplies this power.The chemical energy of the cell supplies this power for as long as it can.

3.10 Cells, Emf, Internal Resistance

Cell





Symbol

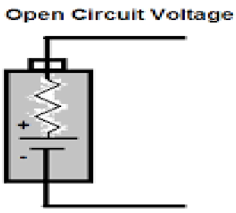
A simple device which maintain a steady current in an electric circuit is the electrolytic cell.

Basically a cell has two electrodes, called the positive (P) and the negative(N) .They are immersed in an electrolytic solution. The electrodes exchange charges with the electrolyte.

Internal resistance of a cell (r)

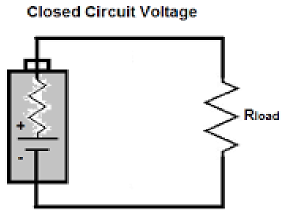
Resistance offered by the electrolytes to the flow of current through it is called internal resistance of the cell

E.M.F -Electro Motive Force (ε)



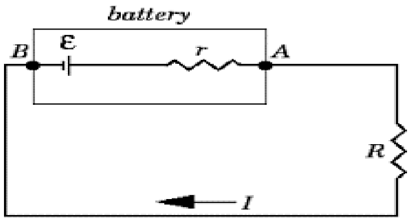
The emf ε is the potential difference between the positive and negative electrodes of a cell in an open circuit, i.e., when no current is flowing through the cell.
Note that ε is, actually, a potential difference and not a force.

Voltage (V)



The Voltage (V) is the potential difference between the positive and negative electrodes of a cell in a closed circuit, i.e., when current is flowing through the cell.

Relation connecting emf and Voltage



$$\text{Current I} = \frac{\text{emf}}{\text{Total Resistance}}$$

$$\mathbf{I = \frac{\varepsilon}{R+r}}$$

$$\varepsilon = I\,(R + r)$$

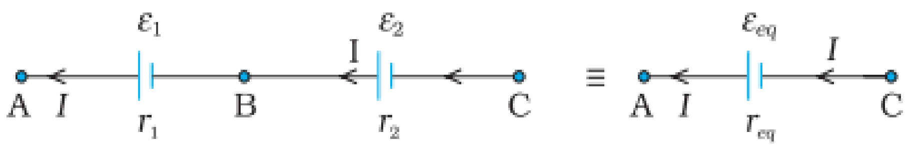
$$\varepsilon = IR + Ir$$

$$\varepsilon = V + Ir$$

$$\mathbf{V = \varepsilon - Ir}$$

3.11 Cells in Series and Parallel

Cells in Series



$$V_{AC} = \varepsilon_{eq} - I\,r_{eq}$$

$$\boldsymbol{\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2}$$

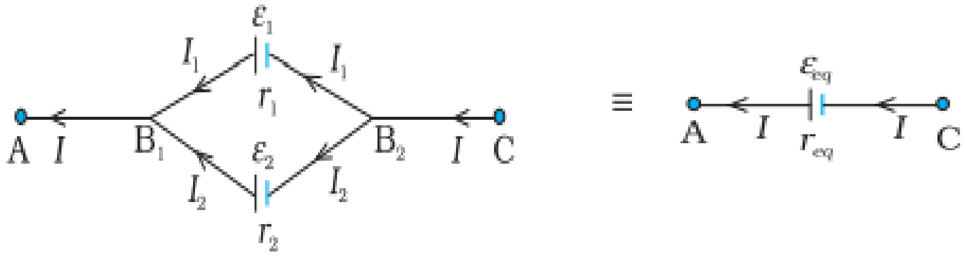
$$\mathbf{r_{eq} = r_1 + r_2}$$

For n cells in series,

$$\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2 + \ldots\ldots\ldots + \varepsilon_n$$

$$r_{eq} = r_1 + r_2 + \ldots\ldots\ldots\ldots + r_n$$

Cells in parallel



$$V_{AC} = \varepsilon_{eq} - I\, r_{eq}$$

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}$$

$$\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}$$

For n cells in parallel

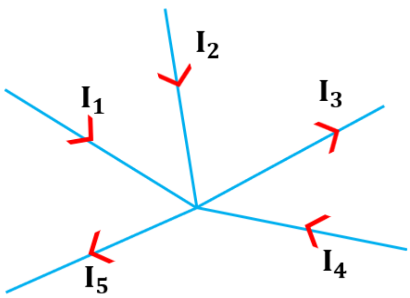
$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \ldots\ldots\ldots\ldots + \frac{1}{r_n}$$

$$\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} + \ldots\ldots\ldots\ldots + \frac{\varepsilon_n}{r_n}$$

3.12 Kirchhoff's Rules

(a)Kirchhoff's First Rule - Junction Rule:

At any junction, the sum of the currents entering the junction is equal to the sum of currents leaving the junction . $\sum I = 0$.

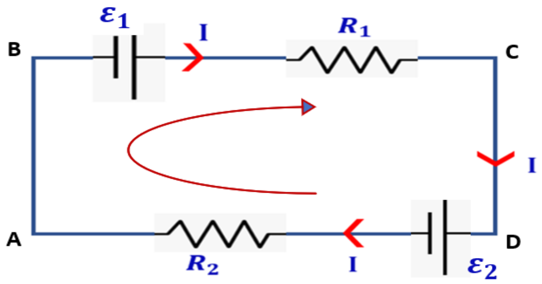


$$I_1 + I_2 + I_4 = I_3 + I_5$$
$$I_1 + I_2 - I_3 + I_4 - I_5 = 0$$
$$\sum I = 0$$

Kirchhoff's junction rule is in accordance with law of conservation of charge.

(b)Kirchhoff's Second Rule –Loop Rule

The algebraic sum of changes in potential around any closed loop is zero. $\sum \Delta V = 0$



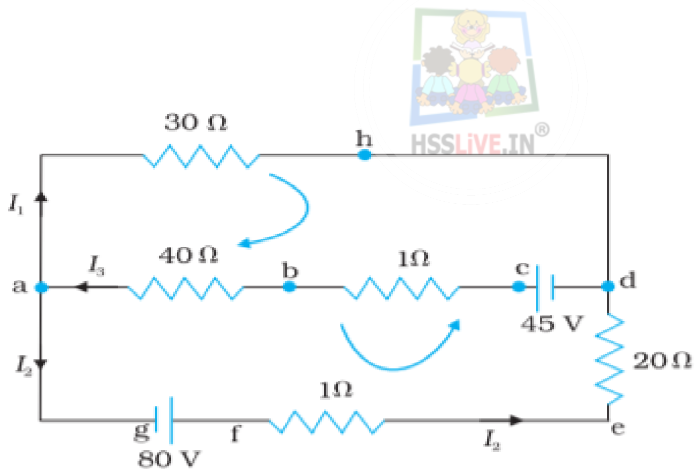
For Loop ABCDA

$$\varepsilon_1 - IR_1 - \varepsilon_2 - IR_2 = 0$$

For Cell
If Path from –ve to +ve terminal, $\Delta V = +\varepsilon$
If Path from +ve to -ve terminal $\Delta V = -\varepsilon$
For resistor
If path is in same direction of current $\Delta V = -IR$
If path is in opposite direction of current $\Delta V = +IR$

Kirchhoff's Loop rule is in accordance with Law of conservation of energy.

Example



Applying Junction rule at junction 'a'

$$I_3 = I_1 + I_2$$

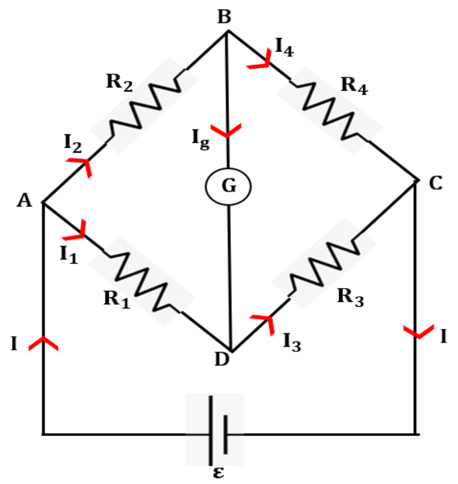
Applying Loop rule for the loops 'ahdcba' ,

$$-30\ I_1 + 45 - 1\ I_3 - 40\ I_3 = 0$$

Applying Loop rule for the loop 'ahdefga',

$$-30\ I_1 + 20\ I_2 + 1\ I_2 - 80 = 0.$$

3.13 Wheatstone Bridge



For a balanced Wheatstone's bridge , the resistors are such that the current through th galvanometer $I_g = 0$.

Apply Kirchhoff's junction rule to junctions B

$$\mathbf{I_2 = I_4 \text{ -------------(1)}}$$

Apply Kirchhoff's junction rule to junctions D

$$\mathbf{I_1 = I_3 \text{ -------------(2)}}$$

Apply Kirchhoff's loop rule to closed loop ABDA

$$\mathbf{I_1 R_1 = I_2 R_2 \text{ ----------------(3)}}$$

Apply Kirchhoff's loop rule to closed loop CBDC

$$\mathbf{I_3 R_3 = I_4 R_4 \text{ ---------------(4)}}$$

$$\frac{eq\ (3)}{eq\ (4)} \text{ -----} \frac{I_1 R_1}{I_3 R_3} = \frac{I_2 R_2}{I_4 R_4}$$

$$\frac{R_1}{R_3} = \frac{R_2}{R_4}$$

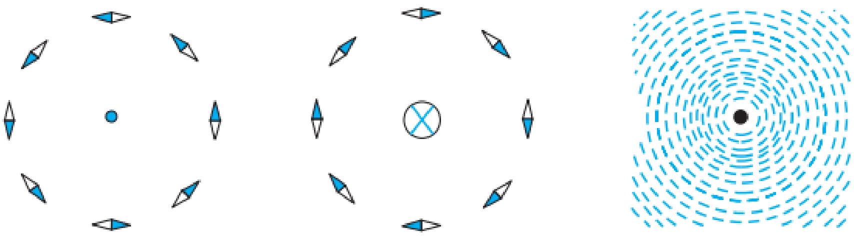
$$\mathbf{\frac{R_2}{R_1} = \frac{R_4}{R_3}}$$

This is the balance condition for the galvanometer to give zero or null deflection.

Chapter 4
Moving Charges and Magnetism

4.1 Introduction

Christian Oersted discovered that moving charges or currents produce a magnetic field in the surrounding space. The direction of the magnetic field depends on the direction of current.



The magnetic field due to a straight long current-carrying wire. The wire is perpendicular to the plane of the paper. A ring of compass needles surrounds the wire. The orientation of the needles is shown when
(a) the current emerges out of the plane of the paper,
(b) the current moves into the plane of the paper.
(c) The arrangement of iron filings around the wire.

*The darkened ends of the needle represent north poles.

*A current or a field (electric or magnetic) emerging out of the plane of the paper is depicted by a dot (.)

*A current or a field going into the plane of the paper is depicted by a cross ($\otimes$).

4.2 Magnetic Force

Sources and fields

A static charge q is the source of electric field(E) .
Moving charges or currents produces a magnetic field (B), in addition to electric field(B).

- Magnetic field is a vector field.
- It obeys the principle of superposition: the magnetic field of several sources is the vector addition of magnetic field of each individual source.

Lorentz Force

The total force acting on a charge q moving with a velocity v in presence of both the electric field E and the magnetic field B is called Lorentz force.

$$F = F_{electric} + F_{magnetic}$$

$$\vec{\mathbf{F}} = \mathbf{q}\vec{\mathbf{E}} + \mathbf{q}(\vec{\mathbf{v}} \times \vec{\mathbf{B}})$$

$$\vec{\mathbf{F}} = \mathbf{q}\left[\,\vec{\mathbf{E}} + (\vec{\mathbf{v}} \times \vec{\mathbf{B}})\right]$$

Electric Lorentz force

$$\vec{\mathbf{F}} = \mathbf{q}\vec{\mathbf{E}}$$

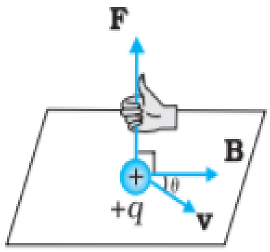
Magnetic Lorentz force

$$q(\vec{v} \times \vec{B})$$

$\vec{F} = qvB\sin\theta$ where θ is the angle between v and B

(i) Magnetic Lorentz force depends on q, v and B (charge of the particle, the velocity and the magnetic field). Force on a negative charge is opposite to that on a positive charge.

(ii) The magnetic force $\vec{F} = qvB\sin\theta$
If velocity and magnetic field are parallel ($\theta = 0$) or anti-parallel($\theta = 180$), $F = 0$.



(iii) The direction of magnetic force is perpendicular to both the velocity and the magnetic field.
Its direction is given by the screw rule or right hand rule.

(iii) The magnetic force is zero if charge is not moving (v= 0).
Only a moving charge feels the magnetic force.

Unit of B

$$F = qvB\sin\theta$$

$$B = \frac{F}{qv}$$

$$\text{Unit of B} = \frac{\text{newton second}}{\text{coulomb metre}} = \text{tesla (T)}$$

tesla is a large unit. A smaller unit (non-SI) called gauss is also often used.

$$1 \text{ gauss} = 10^{-4} \text{ tesla}$$

$$\mathbf{1\ G = 10^{-4}\ T}$$

The earth's magnetic field is about 3.6×10^{-5} T

Magnetic force on a current-carrying conductor

Consider a rod of a uniform cross-sectional area A and length l .
The total number of mobile charge carriers in it is nA l
Let e be the charge on each charge carrier .
Then q=neA l
Let each mobile carrier has an average drift velocity v_d.

$$\vec{F} = q\,(\vec{v} \times \vec{B})$$

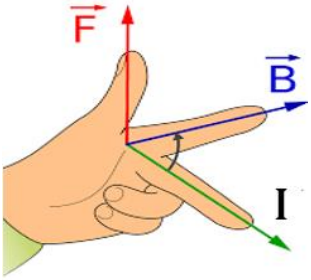
$$\vec{F} = neAl\,(\vec{v_d} \times \vec{B})$$

$$\vec{F} = (ne\,A\,v_d)\,\vec{l} \times \vec{B} \qquad (neAv_d = I)$$

$$\vec{F} = I\,(\vec{l} \times \vec{B})$$

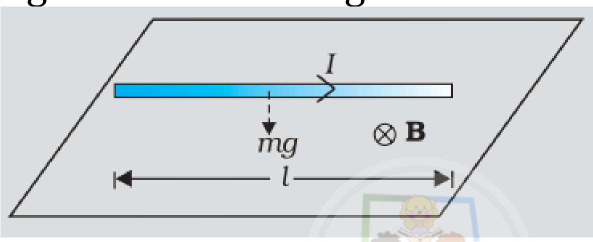
Fleming's left hand rule

Stretch the fore finger , middle finger and thumb of left hand in three mutually perpendicular directions, such that fore finger in the direction of magnetic fileld, the middle finger in the direction of current ,then the thumb gives the direction of force.



Example

1 A straight wire of mass 200 g and length 1.5 m carries a current of 2 A. It is suspended in mid-air by a uniform horizontal magnetic field B . What is the magnitude of the magnetic field?



There is an upward force F, of magnitude I l B,. For mid-air suspension, this must be balanced by the force due to gravity:

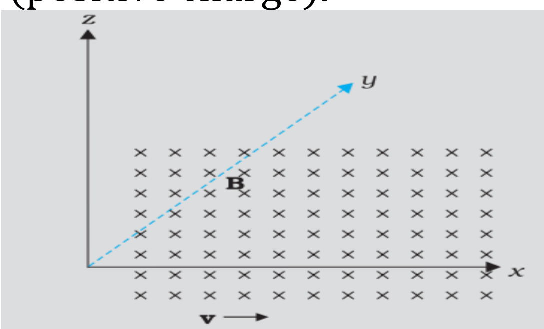
$$m\,g = I\,lB$$

$$B = \frac{mg}{Il} = \frac{2 \times 9.8}{2 \times 1.5} = 0.65\ \text{T}$$

Example

The magnetic field is parallel to the positive y-axis and the charged particle is moving along the positive x-axis (which way would the Lorentz force be for (a) an electron (negative charge),

(b) a proton (positive charge).



The velocity v of particle is along the x-axis, while B, the magnetic field is along the y-axis, so $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ is along the z-axis (screw rule or right-hand thumb rule).

(a) for electron it will be along –z axis.

(b) for a positive charge (proton) the force is along +z axis.

4.3 Motion of a charged particle in a Magnetic field

Case 1 - When $\theta = 0^0$ or $\theta = 180^0$

i.e. the charge is moving in the same direction or opposite direction of magnetic field (parallel or antiparallel)

$$F = qvB \sin 0 = 0$$
$$F = qvB \sin 180 = 0$$

Thus there is no magnetic force on the charge and **the charge moves undeflected.**

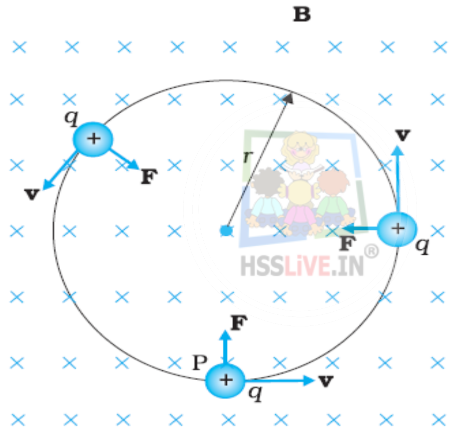
Case 2 - When $\theta = 90^0$

i.e. the charged particle entering perpendicular to a magnetic field.

$$F = qvB \sin 90$$
$$F = qvB$$

The perpendicular force, F=q v B, acts as a centripetal force and produces a circular motion perpendicular to the magnetic field.

The particle will describe a circle if v and B are perpendicular to each other



$$\frac{mv^2}{r} = qvB$$
$$v = \frac{qBr}{m}$$

angular frequency, $\omega = \frac{v}{r} = \frac{qB}{m}$

$$\text{Period } T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{qB}{m}}$$

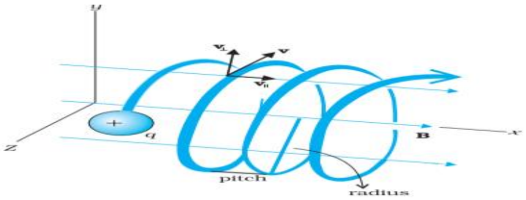
$$T = \frac{2\pi m}{qB}$$

$$\text{Frequency } \nu = \frac{1}{T}$$
$$\nu = \frac{qB}{2\pi m}$$

Case 3- When θ between 0^0 and 90^0

i.e. when the charged particle moves at an arbitrary angle θ with the field direction, **it undergoes helical path.**

Here velocity has one component along B, and the other perpendicular to B. The motion in the direction of field is unaffected by magnetic field, as the magnetic force is zero. The motion in a plane perpendicular to B is circular , thereby producing a helical motion.

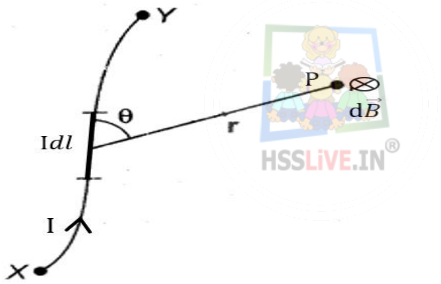


The distance moved along the magnetic field in one rotation is called pitch p.

$$p = v_{parallel} \text{ x } T$$

$$\mathbf{P = \frac{qB}{2\pi m} v_{parallel}}$$

4.4 Magnetic Field due to a Current Element – Biot-Savart Law



The magnetic field due to a small element of a current carrying conductor is directly proportional to the current (I) ,the length of the element dl , sine of the angle between dl and r and inversely proportional to the square of the distance r.

$$\mathbf{dB = \frac{\mu_0}{4\pi}\frac{I\,dl\,\sin\theta}{r^2}}$$

μ_0 = permeability of free space

$\mu_0 = 4\pi \text{ x } 10^{-7}\ \text{Tm/A}$

$\frac{\mu_0}{4\pi} = 10^{-7}\ \text{Tm/A}$

In vector form Biot – Savart law can be written as

$$\mathbf{d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\bar{l}\times\bar{r}}{r^3}}$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Idl\,\sin\theta}{r^2}$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\bar{l}\times\hat{r}}{r^2}$$

$$\hat{r} = \frac{\bar{r}}{r}$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\bar{l}\times\bar{r}}{r^3}$$

Comparison between Coulomb's law and Biot -Savart's law

(i)Coulombs law $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$. Biot-Savart law $dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$

(ii) Both are long range. The principle of superposition applies to both fields.

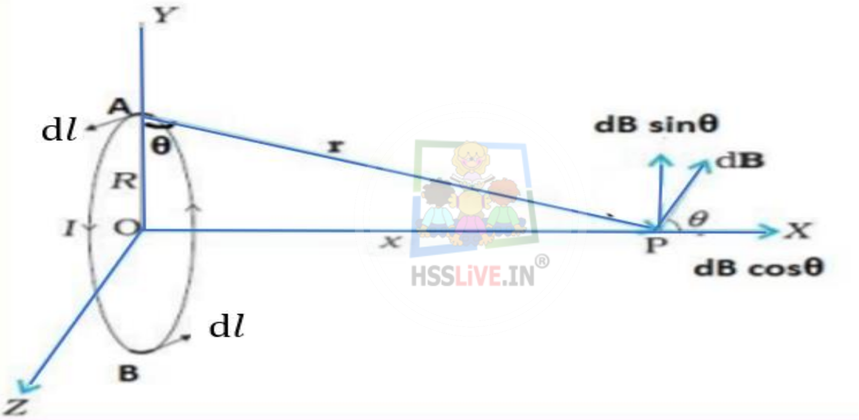
(iii) The electrostatic field is produced by a scalar source, i.e., the electric charge. The magnetic field is produced by a vector source i.e., current element Idl.

(iv) The electrostatic field is along the displacement vector joining the source and the field point. The magnetic field is perpendicular to the plane containing the displacement vector r and the current element Idl.

(v)There is an angle dependence in the Biot-Savart law which is not present in the electrostatic case.

4.5 Applications of Biot-Savart law

Magnetic Field on the Axis of a Circular Current Loop



$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin 90}{r^2}$$

$$dB = \frac{\mu_0}{4\pi}\frac{Idl}{r^2}$$

$$r^2 = x^2 + R^2$$

$$dB = \frac{\mu_0}{4\pi}\frac{Idl}{x^2+R^2} \text{ ----------------(1)}$$

Total field $B = \int dB\cos\theta$ ----------------(2)

$$\cos\theta = \frac{R}{r} = \frac{R}{(x^2+R^2)^{1/2}}$$

Substituting for dB and $\cos\theta$

$$B = \int \frac{\mu_0}{4\pi}\frac{Idl}{x^2+R^2}\frac{R}{(x^2+R^2)^{1/2}} \text{ ----------(3)}$$

$$B = \frac{\mu_0}{4\pi}\frac{IR}{(x^2+R^2)^{3/2}}\int dl$$

$$B = \frac{\mu_0}{4\pi}\frac{IR}{(x^2+R^2)^{3/2}} \times 2\pi R$$

$$B = \frac{\mu_0 IR^2}{2(x^2+R^2)^{3/2}}$$

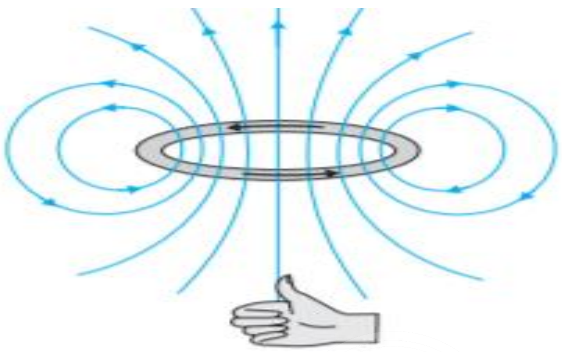
Magnetic field at the centre of the loop

At the centre x=0

$$B = \frac{\mu_0 I R^2}{2R^3}$$

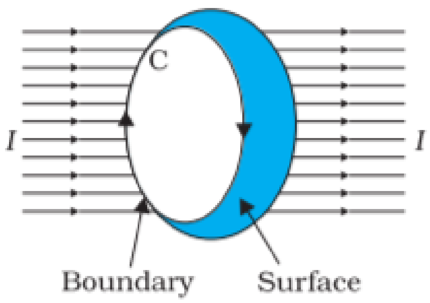
$$\mathbf{B = \frac{\mu_0 I}{2R}}$$

The direction of the magnetic field is given by **right-hand thumb rule .** Curl the palm of your right hand around the circular wire with the fingers pointing in the direction of the current. The right-hand thumb gives the direction of the magnetic field.



The upper side of the loop(current is anticlockwise) may be thought of as the north pole and the lower side(current is clockwise) as the south pole of a magnet.

4.6 Ampere's Circuital Law

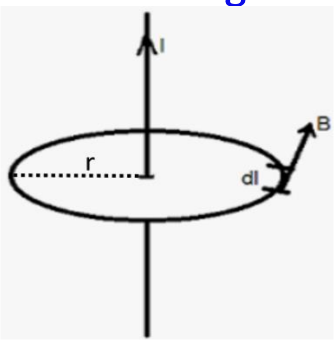


The line integral of magnetic field over a closed loop is equal to μ_0 times the total current passing through the surface.
The closed loop is called Amperian Loop.

$$\oint \vec{\mathbf{B}}.\vec{dl} = \mu_0 \mathbf{I}$$

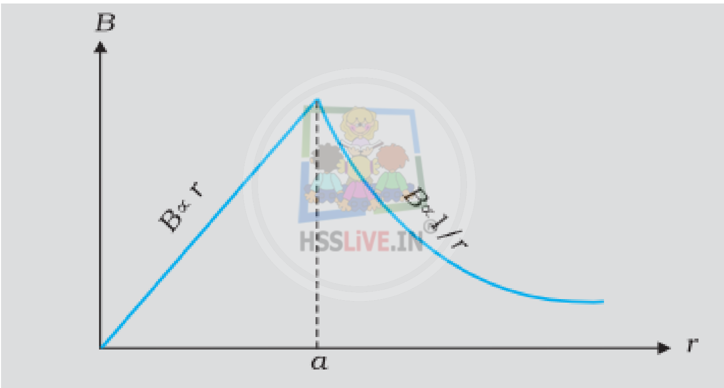
Applications of Ampere's Circuital Law

1.Magnetic field due to a straight infinite current-carrying wire



By Ampere's Circuital Law

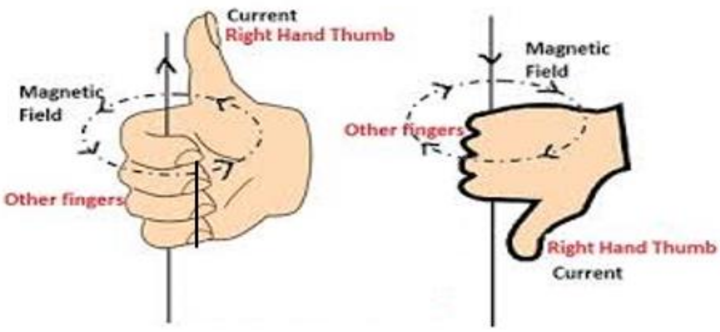
$$\oint \vec{B}.d\vec{l} = \mu_0 I$$
$$\oint Bdl \cos 0 = \mu_0 I$$
$$B\oint dl = \mu_0 I$$
$$B \text{ x } 2\pi r = \mu_0 I$$
$$\mathbf{B = \frac{\mu_0 I}{2\pi r}}$$



A plot of the magnitude of B with distance r from the centre of the wire having radius a

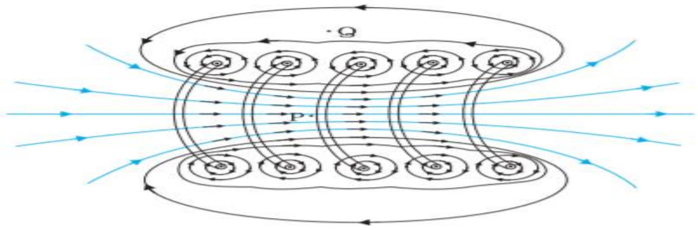
Right-hand rule

There exists a simple rule to determine the direction of the magnetic field due to a long wire ,called the right-hand rule. Grasp the wire in your right hand with your extended thumb pointing in the direction of the current. Your fingers will curl around in the direction of the magnetic field.

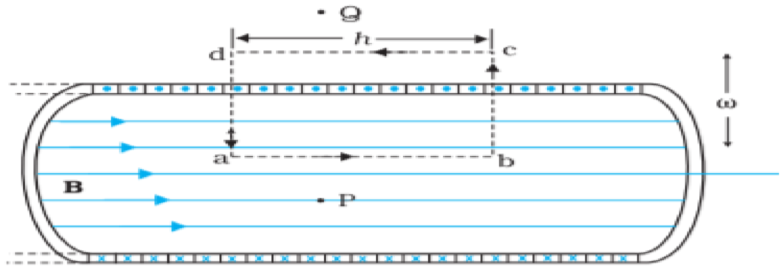


4.7 Solenoid

A solenoid consists of a long wire wound in the form of a helix where the neighbouring turns are closely spaced. The field between two neighbouring turns vanishes and the field at the interior mid-point P is uniform. The field outside the solenoid approaches zero.



2.Mangnetic field due to a Solenoid



$$\oint_{abcd}\vec{B}.d\vec{l} = \oint_{ab}\vec{B}.d\vec{l} + \oint_{bc}\vec{B}.d\vec{l} + \oint_{cd}\vec{B}.d\vec{l} + \oint_{da}\vec{B}.d\vec{l} \text{ ------------- } \mathbf{(1)}$$

$$\oint_{ab}\vec{B}.d\vec{l} = \oint_{ab} \mathrm{B\, d}l \cos 0 = \oint_{ab} \mathrm{B\, d}l = \mathrm{B}\oint_{ab} \mathrm{d}l = \mathrm{B}l$$

$$\oint_{bc}\vec{B}.d\vec{l} = \oint_{ab} \mathrm{B\, d}l \cos 90 = 0$$

$$\oint_{cd}\vec{B}.d\vec{l} = 0 \qquad \text{(since B = 0 outside)}$$

$$\oint_{da}\vec{B}.d\vec{l} = \oint_{ab} \mathrm{B\, d}l \cos 90 = 0$$

Substituting in eqn (1)

$$\oint_{abcd}\vec{B}.d\vec{l} = \mathrm{B}l + 0 + 0 + 0$$

$$\oint_{abcd}\vec{B}.d\vec{l} = \mathrm{B}l \text{ -------------(2)}$$

By Ampere's Circuital Law for N turns of solenoid

$$\oint \mathrm{B}.d\vec{l} = \mu_0 \mathrm{NI} \text{ ---------------(3)}$$

From eqns (2) and (3)

$$\mathrm{B}l = \mu_0\mathrm{NI}$$

$$\mathrm{B} = \frac{\mu_0 \mathrm{NI}}{l}$$

$$\mathbf{B = \mu_0 nI} \qquad\qquad \textbf{where } \mathbf{n} = \frac{\mathbf{N}}{l}$$

N=number of turns of solenoid

l= length of solenoid

n=number of turns per unit length of solenoid

Example

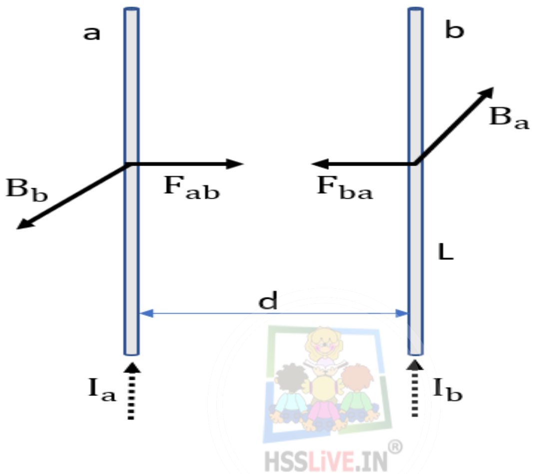
A solenoid of length 0.5 m has a radius of 1 cm and is made up of 500 turns. It carries a current of 5 A. What is the magnitude of the magnetic field inside the solenoid?

The number of turns per unit length , $n=\frac{N}{l}$

$$=\frac{500}{0.5}=1000$$

$$B = \mu_0 nI$$
$$= 4\pi \times 10^{-7} x1000x5$$
$$=6.28 \times 10^{-3}\ T$$

4.8 Force between Two Parallel Current Carrying Condutors



Two long parallel conductors a and b separated by a distance d and carrying (parallel) currents I_a and I_b, respectively.

Magnetic field produced by conductor a along the conductor 'b'

$$B_a = \frac{\mu_0 I_a}{2\pi d}$$

Force acting on conductor b due to this field B_a,

$$\boxed{\vec{F} = I(\vec{l} \times \vec{B})}$$

$$F_{ba} = I_b\, LB_a$$
$$F_{ba} = I_b\, L\frac{\mu_0 I_a}{2\pi d}$$

$$F_{ba} = \frac{\mu_0 I_a I_b\, L}{2\pi d}$$

The force F_{ba} per unit length,

$$\mathbf{f_{ba} = \frac{\mu_0 I_a I_b}{2\pi d}}$$

Similarly the force on 'a' due to 'b'

$$F_{ab} = -F_{ba}$$

- Biot-Savart law and the Lorentz force yield results in accordance with Newton's third Law.
- Parallel currents attract, and antiparallel currents repel.

Definition of ampere

$$f_{ba}=\frac{\mu_0 I_a I_b}{2\pi d}$$

If $I_a = I_b = 1A$

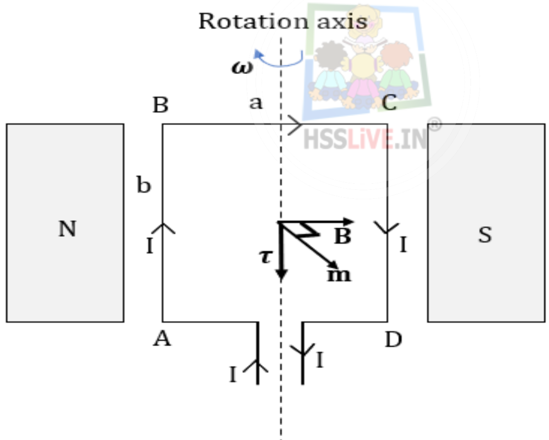
and , $d=1m$

$$f_{ba}=\frac{\mu_0}{2\pi}=\frac{4\pi \times 10^{-7}}{2\pi}=2 \times 10^{-7} N/m$$

The ampere is that current which, when flaws through two very long, straight, parallel conductors placed one metre apart in vacuum, would produce a force equal to2 x 10^{-7}N/m on each other.

4.9 Torque on Current Loop, Magnetic Dipole

Torque on a rectangular current loop in a uniform magnetic field



A rectangular loop carrying a steady current I is placed in a uniform magnetic field B,which is applied in the plane of the loop.

Force on AD and BC is zero

Force on BC , F=IaBsin0=0
Force on AD , F=IaBsin 180=0

Force on AB =Force on CD = IbB sin 90=IbB

Forces on AB and CD are equal and oppsite. So the coil does not experience a net force, but it experiences a torque.

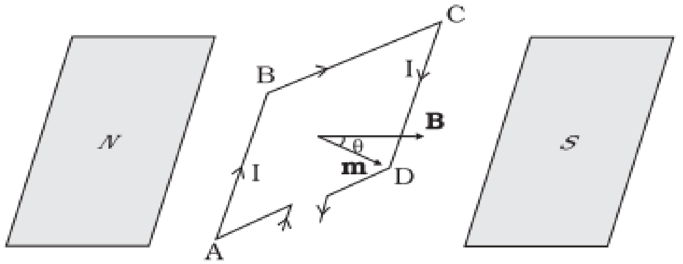
Torque , τ =Force x perpendicular distance

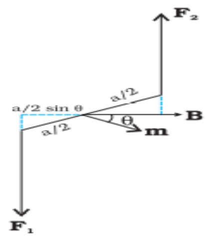
$\tau = IbB \times a = IabB$

$\tau = IAB$

where A = ab is the area of the rectangle.

When the plane of the loop, makes an angle with the magnetic field. We take the angle between the field and the normal to the coil to be angle θ.





$$\tau = IbB\ x\ a\sin\theta$$
$$\tau = IAB\sin\theta$$

For N turns of the coil

$$\tau = NIAB\sin\theta$$

We define the magnetic moment of the current loop as, **m = I A**

For N turns, **m=NIA**

Unit of magnetic moment is Am^2 and dimensions are AL^2

$$\vec{\tau} = mB\sin\theta$$
$$\vec{\tau} = \vec{m} \times \vec{B}$$

Circular current loop as a magnetic dipole

Magnetic field on the axis of circular loop

$$B = \frac{\mu_0 IR^2}{2(x^2+R^2)^{3/2}}$$

For x >> R , $$B = \frac{\mu_0 IR^2}{2x^3}$$

$$A = \pi R^2$$

$$B = \frac{\mu_0 IA}{2\pi x^3}$$

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{2\mathbf{m}}{x^3}$$

Comparing with electric field along the axial line of a an electric dipole

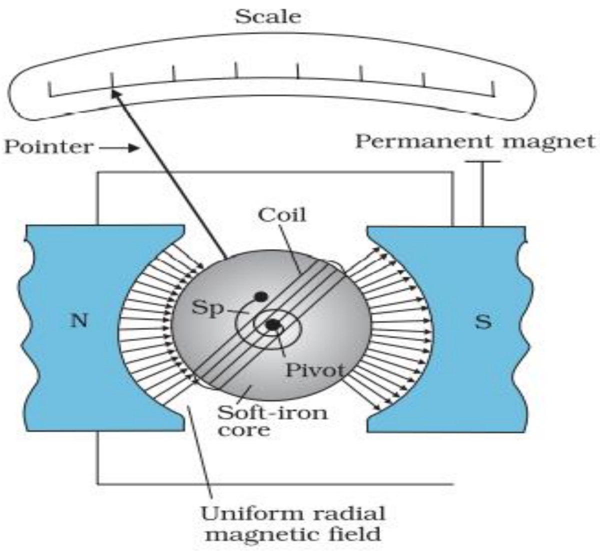
$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \frac{2\mathbf{p}}{x^3}$$

$$\mu_0 \rightarrow \frac{1}{\varepsilon_0}$$

m→ **p** (electrostatic dipole moment)

B → **E** (electrostatic field)

4.10 The Moving Coil Galvanometer



The moving coil galvanometer(MCG) consists of a coil, with many turns, free to rotate about a fixed axis , in a uniform radial magnetic field. There is a cylindrical soft iron core which not only makes the field radial but also increases the strength of the magnetic field.

When a current flows through the coil, a torque acts on it.

$$\tau = NI\,AB \text{---------------(1)}$$

The magnetic torque NIAB tends to rotate the coil. A spring Sp provides a counter torque.

$$\tau = k\phi \text{-----------------(2)}$$

where k is the torsional constant of the spring; i.e. the restoring torque per unit twist.

ϕ is the deflection is indicated on the scale by a pointer attached to the spring.

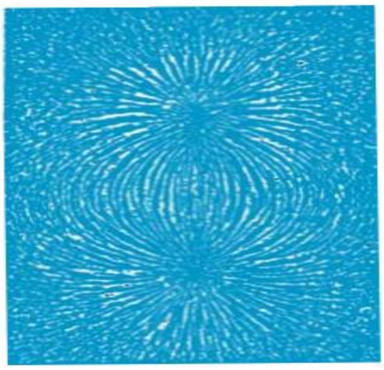
In equilibrium,

$$k\phi = NI\,AB \text{-------------(3)}$$

$$\boldsymbol{\phi = \left(\frac{N\,AB}{k}\right) I}$$

The quantity in brackets is a constant for a given galvanometer.

$$\boldsymbol{\phi \propto I}$$



Thus the deflection produced in the coil is directly proportional to the current through the coil.

Current Sensitivity of the Galvanometer

Current sensitivity of the galvanometer is defined as the deflection per unit current.

$$\frac{\phi}{I} = \left(\frac{N\,AB}{k}\right)$$

A convenient way for the manufacturer to increase the sensitivity is to increase the number of turns N.

Voltage sensitivity of the galvanometer

Voltage sensitivity of the galvanometer is defined as the deflection per unit voltage.

$$\frac{\phi}{V} = \left(\frac{N\,AB}{k}\right)\frac{I}{V} = \left(\frac{N\,AB}{k}\right)\frac{1}{R}$$

$$\frac{\phi}{V} = \left(\frac{N\,AB}{k}\right)\frac{1}{R}$$

Increasing the current sensitivity may not necessarily increase the voltage sensitivity.

If N → 2N, i.e., we double the number of turns, then current sensitivity,

$$\frac{\phi}{I} = \left(\frac{2N\,AB}{k}\right) \rightarrow 2\frac{\phi}{I}$$

Thus, the current sensitivity doubles.

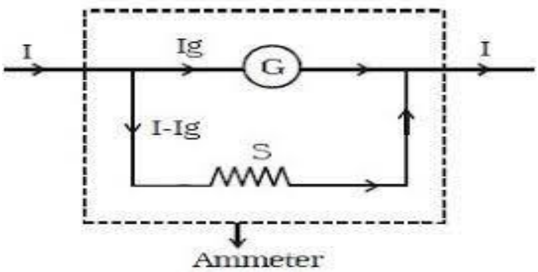
If N → 2N, then R → 2R then the voltage sensitivity,

$$\frac{\phi}{V} = \left(\frac{2N\,AB}{k}\right)\frac{1}{2R} = \left(\frac{N\,AB}{k}\right)\frac{1}{R} = \frac{\phi}{V}$$

Thus, the voltage sensitivity remains unchanged..

Conversion of Galvanometer to Ammeter

To convert a Galvanometer to an Ammeter a small resistance , called shunt resistance S ,is connected in parallel with the galvanometer coil.

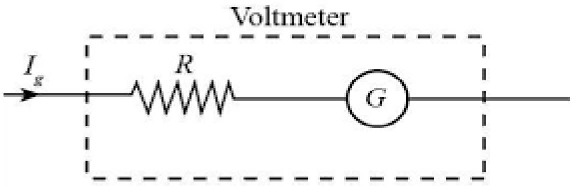


$$I_g G = (I - I_g)S$$

$$S = \frac{I_g G}{I - I_g}$$

Conversion of Galvanometer to Voltmeter

To convert a Galvanometer to a volteter a high resistance , R is connected in series with the galvanometer coil.



$$V = I_g(R + G)$$

$$R + G = \frac{V}{I_g}$$

$$\mathbf{R = \frac{V}{I_g} - G}$$

Example

A galvanometer with coil resistance 12Ω shows full scale deflection for a current of 2.5mA. How will you convert it into an ammeter of range 0 – 7.5 A?

$$S = \frac{I_g G}{I - I_g}$$

$$S = \frac{2.5 \times 10^{-3} \times 12}{7.5 - 2.5 \times 10^{-3}} = \frac{2.5 \times 10^{-3} \times 12}{7.5 - 0.0025} = 4 \times 10^{-3}\ \Omega$$

A resistance of 4×10^{-3} Ω is to be connected in parallel to the galvanometer coil to convert it into an ammeter.

Example

A galvanometer with coil resistance 12Ω shows full scale deflection for a current of 3mA. How will you convert it into a voltmeter of range 0 – 18V?

$$R = \frac{V}{I_g} - G$$

$$R = \frac{18}{3 \times 10^{-3}} - 12 = 6 \times 10^{3} - 12 = 6000 - 12 = 5988\ \Omega$$

A resistance of 5988 Ω is to be connected in seriesl to the galvanometer coil to convert it into a voltmeter.

Chapter 5
Magnetism and Matter

5.1 Introdution

The word magnet is derived from the name of an island in Greece called **magnesia** where magnetic ore deposits were found, as early as 600 BC.

Some of the commonly known ideas regarding magnetism are:

- The earth behaves as a magnet with the magnetic field pointing approximately from the geographic south to the north.
- When a bar magnet is freely suspended, it points in the north-south direction. The tip which points to the geographic north is called the north pole and the tip which points to the geographic south is called the south pole of the magnet.
- **Similar poles repel and opposite poles attract.**
- We cannot isolate the north, or south pole of a magnet. If a bar magnet is broken into two halves, we get two similar bar magnets with somewhat weaker properties. Unlike electric charges, isolated magnetic north and south poles known **as magnetic monopoles do not exist.**
- It is possible to make magnets out of iron and its alloys

5.2 The Bar Magnet

The magnet has two poles similar to the positive and negative charge of an electric dipole -one pole is designated the North pole and the other, the South pole. When suspended freely, these poles point approximately towards the geographic north and south poles, respectively.

The arrangement of iron filings surrounding a bar magnet. The pattern mimics magnetic field lines. The pattern suggests that the bar magnet is a magnetic dipole. A similar pattern of iron filings is observed around a current carrying solenoid.

The Magnetic Field Lines

- The magnetic field lines of a magnet (or a solenoid) form continuous closed loops.

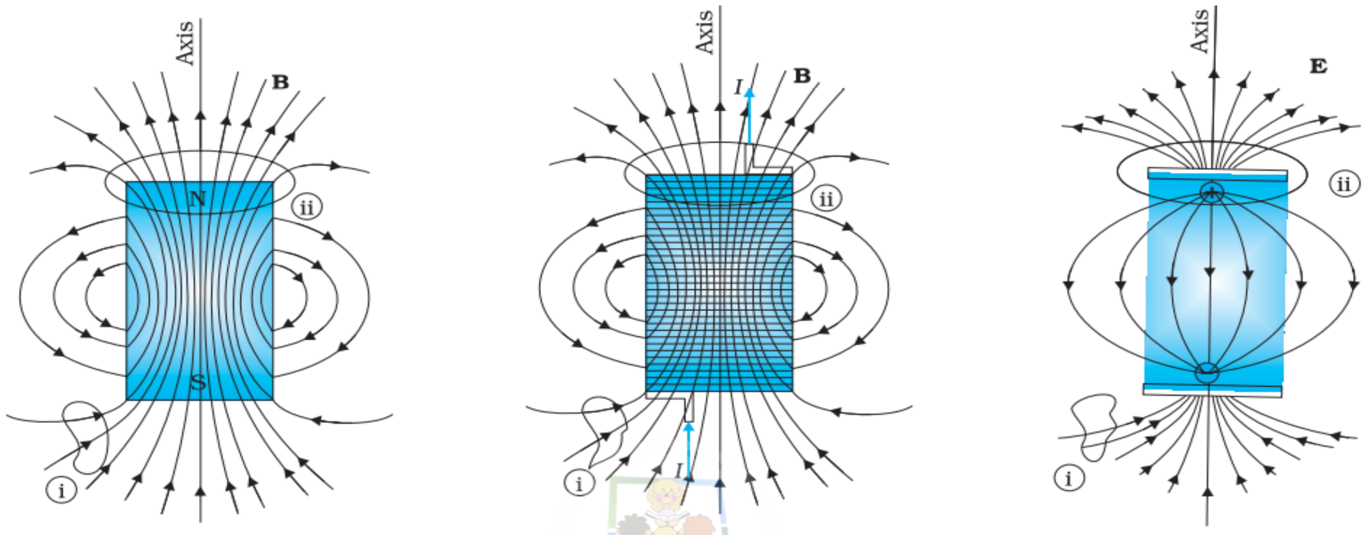
  (This is unlike the electric dipole where these field lines begin from a positive charge and end on the negative charge or escape to infinity.)
- The tangent to the field line at a given point represents the direction of the net magnetic field B at that point.

- The larger the number of field lines crossing per unit area, the stronger is the magnitude of the magnetic field B.
- The magnetic field lines do not intersect.

(If they intersect, there would be more than one direction for magnetic field at the point of intersection, which is not possible)

The Magnetic field lines of

(a) a bar magnet, (b) a current-carrying finite solenoid and (c) electric dipole



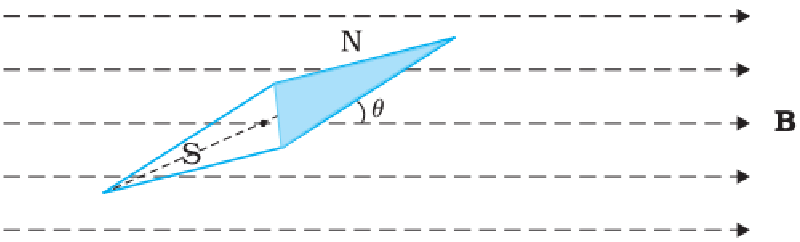
(a) (b) (c)

Note:-The magnetic field lines can not be called as magnetic lines of force. Unlike electrostatics (**F =qE**) the field lines in magnetism do not indicate the direction of the force on a moving charge(**F=q(vxB)**)

Bar magnet as an equivalent solenoid

The resemblance of magnetic field lines for a bar magnet and a solenoid suggest that a bar magnet may be thought of as a large number of circulating currents in analogy with a solenoid.

The dipole in a uniform magnetic field



When a small compass needle of magnetic moment m and moment of inertia I is allowed it to oscillate in a magnetic field B, it executes simple harmonic motion.

Magnetic potential energy

$$\mathbf{U_m} = \int \boldsymbol{\tau}\, \mathbf{d\theta}$$
$$\mathbf{U_m} = \int \mathrm{mBsin\theta d}\boldsymbol{\theta} = -\mathrm{mBcos\theta}$$
$$\mathbf{U_m} = -\mathrm{m.B}$$

Example

(a) What happens if a bar magnet is cut into two pieces:
(i) transverse to its length, (ii) along its length?
In either case, one gets two magnets, each with a north and south pole.

(b) A magnetised needle in a uniform magnetic field experiences a torque but no net force. An iron nail near a bar magnet, however, experiences a force of attraction in addition to a torque. Why?
No force if the field is uniform. The iron nail experiences a non uniform field due to the bar magnet. There is induced magnetic moment in the nail, therefore, it experiences both force and torque. The net force is attractive because the induced south pole (say) in the nail is closer to the north pole of magnet than induced north pole.

(c) Must every magnetic configuration have a north pole and a south pole? What about the field due to a toroid
Not necessarily. True only if the source of the field has a net non zero magnetic moment. This is not so for a toroid or even for a straight infinite conductor.

(d) Two identical looking iron bars A and B are given, one of which is definitely known to be magnetised. (We do not know which one.) How would one ascertain whether or not both are magnetised? If only one is magnetised, how does one ascertain which one? [Use nothing else but the bars A and B.]
Try to bring different ends of the bars closer. A repulsive force in some situation establishes that both are magnetised. If it is always attractive, then one of them is not magnetised.
In a bar magnet the intensity of the magnetic field is the strongest at the two ends (poles) and weakest at the central region. This fact may be used to determine whether A or B is the magnet. In this case, to see which one of the two bars is a magnet, pick up one, (say, A) and lower one of its ends; first on one of the ends of the other (say, B), and then on

the middle of B. If you notice that in the middle of B, A experiences no force, then B is magnetised. If you do not notice any change from the end to the middle of B, then A is magnetised.

The electrostatic analog

$\vec{E} \rightarrow \vec{B}, \quad \vec{p} \rightarrow \vec{m}, \quad \frac{1}{\varepsilon_0} \rightarrow \mu_0 \,, \quad \frac{1}{4\pi\varepsilon_0} \rightarrow \frac{\mu_0}{4\pi}$

The magnetic field along the axial line of a bar magnet,

$$\text{Axial fileld}, \mathbf{B_A} = \frac{\mu_0}{4\pi}\frac{2\vec{m}}{r^3}$$

The magnetic field along the equatorial l line of a bar magnet,

$$\text{Equatorial field}, \mathbf{B_E} = \frac{\mu_0}{4\pi}\frac{\vec{m}}{r^3}$$

The Dipole Analogy

	Electrostatics	**Magnetism**
	$\frac{1}{\varepsilon_0}$	μ_0
Dipole moment	$\vec{p}$	$\vec{m}$
Axial Field for a short dipole	$\frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3}$	$\frac{\mu_0}{4\pi}\frac{2\vec{m}}{r^3}$
Equatorial Field for a short dipole	$\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{r^3}$	$\frac{\mu_0}{4\pi}\frac{\vec{m}}{r^3}$
Torque in an external field	$\vec{\tau} = \vec{p} \times \vec{E}$	$\vec{\tau} = \vec{m} \times \vec{B}$
Energy in an external field	$\vec{U} = -\vec{p}.\vec{E}$	$\vec{U} = -\vec{m}.\vec{B}$

Example

What is the magnitude of the equatorial and axial fields due to a bar magnet of length 5 cm at a distance of 50 cm from its mid-point? The magnetic moment of the bar magnet is 0.40 A m^2 ,

$$B_E = \frac{\mu_0}{4\pi}\frac{m}{r^3} = \frac{10^{-7} x\, 0.40}{(0.5)^3} = 3.2\ x10^{-7}\ T$$

$$B_A = \frac{\mu_0}{4\pi}\frac{2m}{r^3} = 2x\ 3.2\ x10^{-7} = 6.4\ x10^{-7}\ T$$

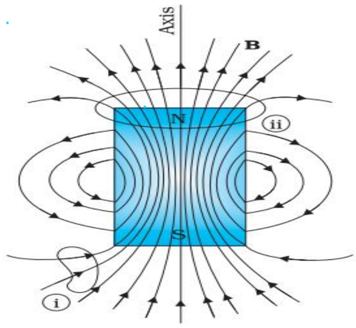
5.3 Magnetism and Gauss's Law

Gauss's law for magnetism states that the net magnetic flux through any closed surface is zero.

$$\boldsymbol{\phi} = \oint \vec{\mathbf{B}} \cdot \mathbf{d\vec{s}} = \mathbf{0}$$

The difference between the Gauss's law of magnetism and that for electrostatics is due to the fact that **isolated magnetic poles (also called monopoles) do not to exist.**

There are no sources or sinks of B; the simplest magnetic element is a dipole or a current loop.



(a)

For the Gaussian surfaces represented by i or ii , the number of magnetic field lines leaving the surface is balanced by the number of lines entering it. The net magnetic flux is zero for both the surfaces.

Example

(a) Magnetic field lines show the direction (at every point) along which a small magnetised needle aligns (at the point). Do the magnetic field lines also represent the lines of force on a moving charged particle at every point?

No. The magnetic force is always normal to B (remember magnetic force = qv × B). It is misleading to call magnetic field lines as lines of force.

(b) Magnetic field lines can be entirely confined within the core of a toroid, but not within a straight solenoid. Why?

If field lines were entirely confined between two ends of a straight solenoid, the flux through the cross-section at each end would be non-zero. But the flux of field B through any closed surface must always be zero. For a toroid, this difficulty is absent because it has no 'ends'.

(c) If magnetic monopoles existed, how would the Gauss's law of magnetism be modified?

By Gauss's law for magnetism

$$\boldsymbol{\phi = \oint \vec{B} \cdot d\vec{s} = 0}$$

If monopoles existed, the right hand side would be equal to the monopole (magnetic charge) q_m enclosed by S.

$$\boldsymbol{\phi = \oint \vec{B} \cdot d\vec{s} = \mu_0 q_m}$$

{Analogous to Gauass law in electrostatics , $\phi = \oint \vec{E} \cdot d\vec{s} = \frac{q}{\varepsilon_0}$}

(d) Does a bar magnet exert a torque on itself due to its own field? Does one element of a current-carrying wire exert a force on another element of the same wire?

No. There is no force or torque on an element due to the field produced by that element itself. But there is a force (or torque) on an element of the same wire. (For the special case of a straight wire, this force is zero.)

(e) Magnetic field arises due to charges in motion. Can a system have magnetic moments even though its net charge is zero

Yes. The average of the charge in the system may be zero. Yet, the mean of the magnetic moments due to various current loops may not be zero. We will come across such examples in connection with paramagnetic material where atoms have net dipole moment through their net charge is zero

5.4 Magnetisation and Magnetic Intensity

Magnetisation(M)

The net magnetic dipole moment developed per unit volume of a material is called Magnetisation(M).

$$\mathbf{M} = \frac{\mathbf{m_{net}}}{\mathbf{V}}$$

Magnetisation is a vector quantity, its unit is Am^{-1}, dimensions AL^{-1}

Consider a long solenoid of n turns per unit length and carrying a current I. If the interior of the solenoid is filled with a material with non-zero magnetisation, the total field inside the solenoid will be

$$B = B_0 + B_m \text{---------------(1)}$$

Here B_0 is the field due to the current in the solenoid and B_m is the field contributed by the material core which is proportional to the magnetisation M of the material.

$$B_0 = \mu_0 nI$$
$$B_m = \mu_0 M$$

$$B = \mu_0 nI + \mu_0 M$$
$$B = \mu_0 H + \mu_0 M$$
$$B = \mu_0 (H + M) \quad \text{------------(2)}$$

Here M is called magnetisation and H is called magnetic intensity

The total magnetic field inside the sample has two parts: one, due to external factors such as the current in the solenoid. This is represented by H. The other is due to the specific nature of the magnetic material, namely M.

Magnetic intensity(H)

The magnetic intensity can be defined as

$$\mathbf{H = \frac{B}{\mu_0} - M}$$

$$B = \mu_0(H + M)$$
$$\frac{B}{\mu_0} = H + M$$
$$H = \frac{B}{\mu_0} - M$$

H has the same unit and dimensions as M .Its unit is Am^{-1},dimensions AL^{-1}

Magnetic Susceptibility(χ)

The magnetisation can be influenced by external factors(H which is equal to nI). This influence is mathematically expressed as

$$M = \chi H$$
$$\boldsymbol{\chi = \frac{M}{H}}$$

where χ is a dimensionless quantity called as magnetic susceptibility. It is a mesure of how a magnetic material responds to an external field.

- χ is large and positive for ferromagnetic materials.
- χ is small and positive for paramagnetic materials.
- χ is small and negative for diamagnetic materials. For diamagnetic materials M and H are opposite in direction.

Relation connecting Susceptibility and permeability

$$B = \mu_0(H + M)$$
$$B = \mu_0(H + \chi H)$$
$$B = \mu_0(1 + \chi)H) \text{ --------------(1)}$$
$$B = \mu_0\mu_r H \quad \text{-----------------(2)}$$
$$\mathbf{B = \mu H}$$

From (1) and (2)

$$\boldsymbol{\mu_r = 1 + \chi}$$
$$\boldsymbol{\chi = \mu_r - 1}$$

$\mu_r$is a dimensionless quantity called the relative magnetic permeability of the substance.

The magnetic permeability of the substance is μ can be written as

$$\mu = \mu_0\mu_r = \mu_0(\mathbf{1} + \boldsymbol{\chi})$$

Magnetic permeability

$$B = \mu H$$
$$\boldsymbol{\mu = \frac{B}{H}}$$

Example

A solenoid has a core of a material with relative permeability 400. The windings of the solenoid are insulated from the core and carry a current of 2A.If the number of turns is 1000 per metre,calculate (a) H,(b) M,(c) B

a) $H = nI = 1000 \times 2 = 2000$ A/m

b) $M = \chi H = (\mu_r - 1)H$

$= (400-1)2000 = 399 \times 2000$

$= 7.98 \times 10^5 \cong 8 \times 10^5$ A/m

c) $B = \mu_0 \mu_r H = 4\pi \times 10^{-7} \times 400 \times 2000$

$= 100.48 \times 10^{-2} T = 1T$

5.5 Magnetic Properties of Materials

In terms of the susceptibility χ , a material is diamagnetic if χ is negative, para- if χ is positive and small, and ferro- if χ is large and positive.

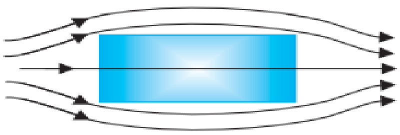
Diamagnetic	Paramagnetic	Ferromagnetic
$-1 \le \chi < 0$	$0 < \chi < \varepsilon$	$\chi >> 1$
$0 \le \mu_r < 1$	$1 < \mu_r < 1+\varepsilon$	$\mu_r >> 1$
$\mu < \mu_0$	$\mu > \mu_0$	$\mu >> \mu_0$

Diamagnetism

- Diamagnetic substances are those which get weakly magnetised opposite to the direction of external magnetic field.
- Diamagnetic substances move from stronger to the weaker part of the external magnetic field, i.e.,a magnet would repel a diamagnetic substance.
- Susceptibility **χ** is small and negative for diamagnetic materials.

$$\chi < 0$$

- Relative permeability , μ_r is positive and less than one for diamagnetic materials. $\mu_r < 1$
- When a diamagnetic material is placed in an external magnetic field, the field lines are repelled or expelled and the field inside the material is reduced.



- The resultant magnetic moment of an individual atom of a diamagnetic substance is zero.

- When a magnetic field is applied, the diamagnetic substance develops a net magnetic moment opposite to the direction of applied field and hence repulsion.
- Some diamagnetic materials are bismuth, copper, diamond, gold, lead, mercury,silver, silicon, nitrogen (at STP), water and sodium chloride.
- Super coductors exhibits perfect diamagnetism. Here the field lines are completely expelled. $\chi = -1$ and $\mu_r = 0$.

Super conductors

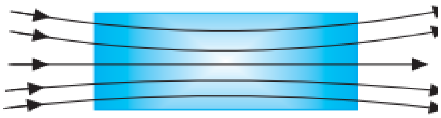
These are metals, cooled to very low temperatures which exhibits both perfect conductivity and perfect diamagnetism. Here the field lines are completely expelled. $\chi = -1$ and $\mu_r = 0$. The phenomenon of perfect diamagnetism in superconductors is called the Meissner effect.

Paramagnetism

- Paramagnetic substances are those which get weakly magnetised in the direction of external magnetic field.
- Paramagnetic substances move from a region of weak magnetic field to strong magnetic field, i.e., they get weakly attracted to a magnet.
- Susceptibility $\boldsymbol{\chi}$ is small and positive for paramagnetic materials.

$$\chi > 0$$

- Relative permeability is positive and greater than one for diamagnetic materials. $\mu_r > 1$
- When a paramagnetic material placed in an external field , the field lines gets concentrated inside the material, and the field inside is enhanced.



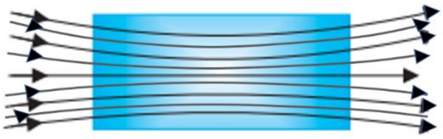
- The individual atoms of a paramagnetic material possess a permanent magnetic dipole moment of their own.
- When a magnetic field is applied, the individual atomic dipole moments align in the same direction and a net magnetic moment in the direction of applied field and hence attraction.
- Some paramagnetic materials are aluminium, sodium, calcium, chromium, lithium , magnesium, oxygen (at STP) ,copper chloride, platinum, tungsten, niobium.

Ferromagnetism

- Ferromagnetic substances are those which gets strongly magnetised when placed in an external magnetic field.
- Ferromagnetic substances have strong tendency to move from a region of weak magnetic field to strong magnetic field, i.e., they get strongly attracted to a magnet.
- Susceptibility **χ** is large and positive for ferromagnetic materials.

$$\chi >> 1$$

- Relative permeability is greater than one and large. $\mu_r >> 1$
- When a ferromagnetic material placed in an external field , the field lines gets highly concentrated inside the material, and the field inside is enhanced.



- The individual atoms (or ions or molecules) in a ferromagnetic material possess a dipole moment as in a paramagnetic material.
- When a magnetic field is applied, the individual atomic dipole moments align in the same direction and a net magnetic moment in the direction of applied field and hence attraction.
- The ferromagnetic property depends on temperature.At high enough temperature, a ferromagnet becomes a paramagnet.
- Some ferromagnetic materials are iron, cobalt, nickel, gadolinium,Fe_2O_3.

Hard ferromagnets and Soft ferromagnets

The ferromagnetic materials in which the magnetisation persists, ,even when the external field is removed are called **hard magnetic materials or hard ferromagnets**. Such materials are used to make permanent magnets.

Eg: Alnico(an alloy of iron, aluminium, nickel, cobalt & copper),lodestone

The ferromagnetic materials in which the magnetisation disappears on the removal of the external field are called **soft ferromagnetic materials.**

Eg: Soft iron .

Chapter 6
Electromagnetic Induction

6.1 Introduction

In the early decades of the nineteenth century, experiments on electric current by Oersted, Ampere and a few others established the fact that electricity and magnetism are inter-related. They found that moving electric charges produce magnetic fields.

Is the converse effect possible?
The experiments of Michael Faraday in England and Joseph Henry in USA, demonstrated that electric currents were induced in closed coils when subjected to changing magnetic fields. The pioneering experiments of Faraday and Henry have led directly to the development of modern day generators and transformers.

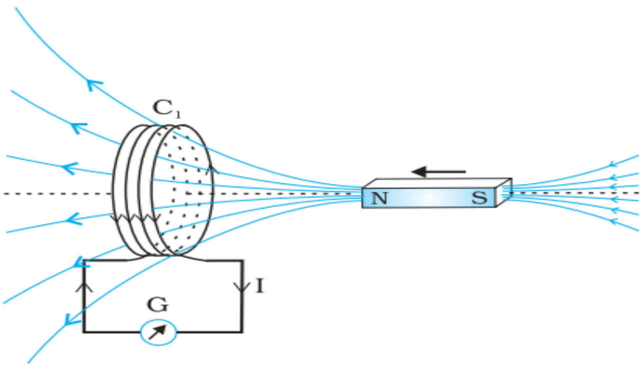
Electromagnetic Induction

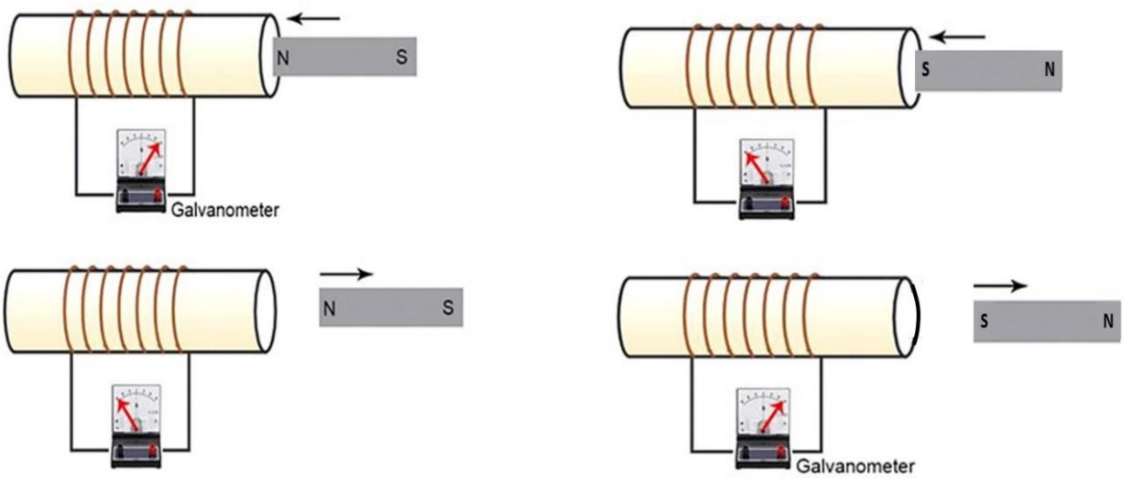
The phenomenon in which electric current is generated by varying magnetic fields is appropriately called electromagnetic induction.

6.2 The Experiments of Faraday and Henry

Experiment 1

A coil C_1 is connected to a galvanometer G.





Observations

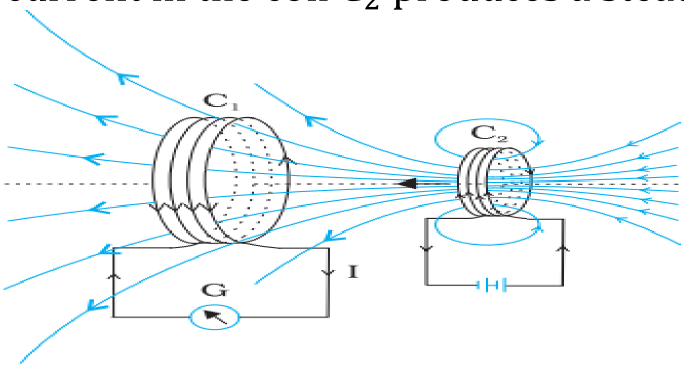
- When the North-pole of a bar magnet is pushed towards the coil, the pointer in the galvanometer deflects, indicating the presence of electric current in the coil.
- The deflection lasts as long as the bar magnet is in motion.
- The galvanometer does not show any deflection when the magnet is held stationary.
- When the magnet is pulled away from the coil, the galvanometer shows deflection in the opposite direction, which indicates reversal of the current's direction.
- Moreover, when the South-pole of the bar magnet is moved towards or away from the coil, the deflections in the galvanometer are opposite to that observed with the North-pole.
- Further, the deflection (and hence current) is found to be larger when the magnet is pushed towards or pulled away from the coil faster.
- When the bar magnet is held fixed and the coil C_1 is moved towards or away from the magnet, the same effects are observed.

Conclusion

It shows that it is the relative motion between the magnet and the coil that is responsible for generation (induction) of electric current in the coil.

Experiment 2

The bar magnet is replaced by a second coil C_2 connected to a battery. The steady current in the coil C_2 produces a steady magnetic field.



Observations

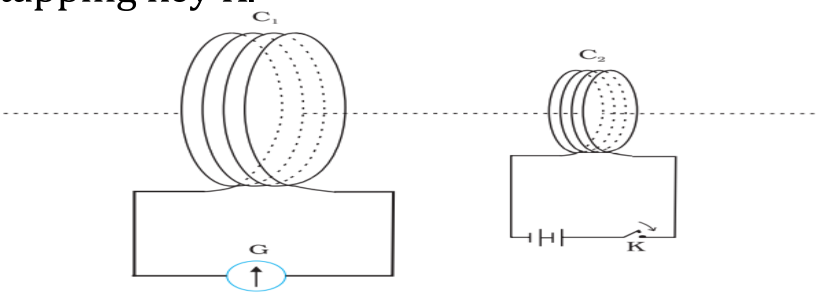
- As coil C_2 is moved towards the coil C_1, the galvanometer shows a deflection. This indicates that electric current is induced in coil C_2.
- When C_2 is moved away, the galvanometer shows a deflection in the opposite direction.
- The deflection lasts as long as coil C_2 is in motion.

- When the coil C_2 is held fixed and C_1 is moved, the same effects are observed.

Again, it is the relative motion between the coils that induces the electric current.

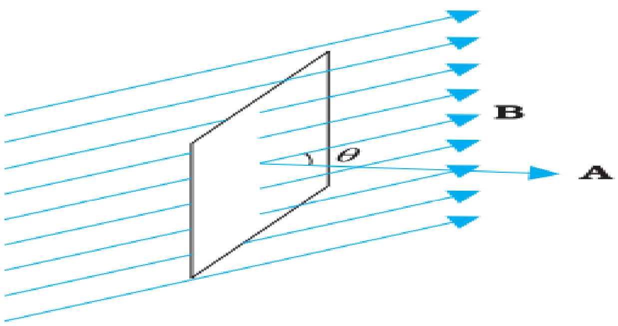
Experiment 3

Two coils C_1 and C_2 are held stationary. Coil C_1 is connected to galvanometer G while the second coil C_2 is connected to a battery through a tapping key K.



- The galvanometer shows a momentary deflection when the tapping key K is pressed. The pointer in the galvanometer returns to zero immediately.
- If the key is held pressed continuously, there is no deflection in the galvanometer.
- When the key is released, a momentory deflection is observed again, but in the opposite direction.
- It is also observed that the deflection increases dramatically when an iron rod is inserted into the coils along their axis.

6.3 Magnetic Flux



Magnetic flux through a plane of area A placed in a uniform magnetic field B can be written as

$$\phi_B = \mathbf{B} \cdot \mathbf{A} = BA\cos\theta$$

where θ is angle between B and A.

The SI unit of magnetic flux is weber(Wb) or tesla meter squared($T\ m^2$). Magnetic flux is a scalar quantity.

The flux can be varied by changing any one or more of the terms B, A and θ .

6.4 Faraday's law of electromagnetic induction

The magnitude of the induced emf in a circuit is equal to the time rate of change of magnetic flux through the circuit.

$$\varepsilon = -\frac{d\phi_B}{dt}$$

The negative sign indicates the direction of ε and hence the direction of current in a closed loop.

In the case of a closely wound coil of N turns, the total induced emf ,

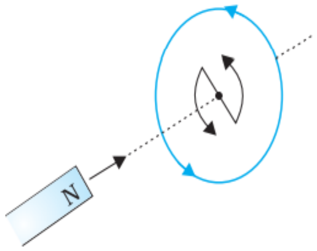
$$\varepsilon = -N\frac{d\phi_B}{dt}$$

The induced emf can be increased by increasing the number of turns N of a closed coil.

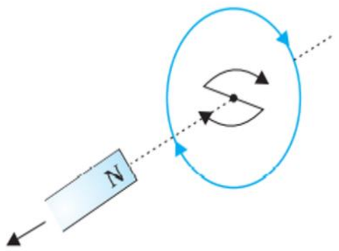
Lenz's Law

German physicist Heinrich Friedrich Lenz deduced a rule, known as Lenz's law which gives the polarity of the induced emf .

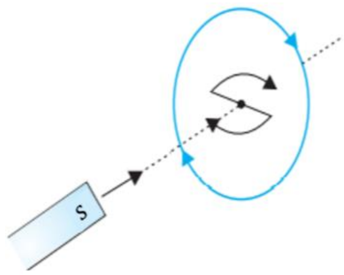
The statement of the law is: **The polarity of induced emf is such that it tends to produce a current which opposes the change in magnetic flux that produced it.**



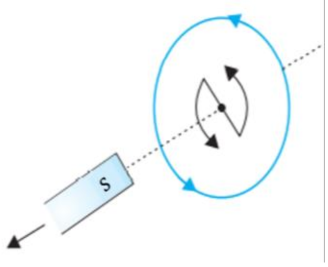
When North-pole of a bar magnet is moved towards the coil, a current is induced in the coil in such a direction that it opposes the increase in flux. That means the face of coil (towards magnet) should have North-polarity .So current in that face will be anti clockwise (counter- clockwise) .



When North-pole of a bar magnet is moved away from the coil, a current is induced in the coil in such a direction that it opposes the decrease in flux. That means the face of coil (towards magnet) should have South-polarity.So current in that face will be clockwise.



When South-pole of a bar magnet is moved towards the coil, the face of coil (towards magnet) should have South-polarity. So current in that face will be clockwise.



When South-pole of a bar magnet is moved away from the coil, the face of coil (towards magnet) should have North-polarity .So current in that face will be anti clockwise (counter- clockwise) .

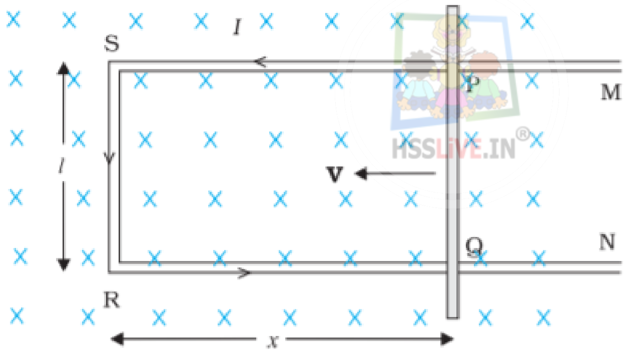
6.5 Lenz's Law and Conservation of Energy

If the induced current was in the same direction changing magnetic flux, the front face of coil gets South polarity ,when the north pole of bar magnet is pushed into the coil .The bar magnet will then be attracted towards the coil at a increasing acceleration and kinetic energy will continuously increase without expending any energy. This violates the law of conservation of energy and hence can not happen.

The current induced in the coil is opposite to the direction of changing magnetic flux. Then the bar magnet experiences a repulsive force due to the induced current. Therefore, a person has to do work in moving the magnet. This energy(work) is dissipated by Joule heating produced by the induced current.Thus Lenz's law is in accordance with law of conservation of energy.

6.6 Motional Electromotive Force

When a conducting rod is moved through a constant magnetic field, an emf is developed between the ends of the rod. This emf is known as Motional Emf.



Consider a straight conductor moving in a uniform and time independent magnetic field.The magnetic flux Φ enclosed by the loop PQRS,

$$\phi = Blx$$

Since x is changing with time, the rate of change of flux Φ will induce an emf given by:

$$\varepsilon = -\frac{d\phi_B}{dt} = \frac{d}{dt}(Blx)$$

$$\varepsilon = -Bl\frac{dx}{dt}$$

$$v = \frac{-dx}{dt} \text{ is the speed of the conductor}$$

$$\boldsymbol{\varepsilon = Blv}$$

The induced emf Blv is called motional emf.

6.7 Inductance

An electric current can be induced in a coil by flux change produced by the same coil or a flux change produced by a neighbouring coil .These phenomenon are respectively called self induction and mutual induction. In both the cases, the flux through a coil is proportional to the current.

$$\phi \propto I$$

$$\phi = L\,I$$

The constant of proportionality, in this relation, is called inductance. Inductance is a scalar quantity. It has the dimensions of $[M\,L^2\,T^{-2}\,A^{-2}]$.
The SI unit of inductance is henry and is denoted by H

Self-Induction

The phenomenon of production of induced emf in an isolated coil by varying current through the same coil is called self-induction.
The flux linked with the coil is proportional to the current through the coil.

$$\phi \propto I$$

$$\phi = L\,I \text{ -------------(1)}$$

where constant of proportionality L is called self-inductance of the coil. It is also called the coefficient of self-induction of the coil.
When the current is varied, the flux linked with the coil also changes and an emf is induced in the coil.

For N turns, $\varepsilon = -\frac{d\phi}{dt}$

$$\varepsilon = -\frac{dLI}{dt}$$

$$\varepsilon = -L\frac{dI}{dt} \qquad \text{-----------(2)}$$

Thus, the self-induced emf always opposes any change (increase or decrease) of current in the coil.

Self-Inductance of a Long Solenoid

Consider a solenoid of cross sectional area A and length l, having n turns per unit length.
The total flux linked with N turns of the solenoid ,

$$\phi = NB\,A$$

$$B = \mu_0\, n\, I$$
$$N = nl$$

$$\phi = nl\,(\mu_0\, n\, I\,)\,A$$

$$\phi = \mu_0\, n^2\, AlI \text{ ------------(1)}$$

But, $\phi = LI$ -----------------(2)

From eq (1) and (2)

$$LI = \mu_0 n^2 AlI$$

$$\mathbf{L = \mu_0 n^2 Al} \quad \text{-----------(3)}$$

If we fill the inside of the solenoid with a material of relative permeability μ_r (for example soft iron, which has a high value of relative permiability), then,

$$\mathbf{L = \mu_r \mu_0 n^2 Al} \text{-----------(4)}$$

The self-inductance depends on geometry of coil and on the permeability of the medium.

Back emf

The self-induced emf is also called the back emf as it opposes any change in the current in a circuit. **Physically, the self-inductance plays the role of inertia. Self inductance is the electromagnetic analogue of mass in mechanics.** So, work needs to be done against the back emf (ε) in establishing the current. This work done is stored as magnetic potential energy.

Energy stored in an inductor

W=qV

W=It|ε|

$dW = |\varepsilon| I\, dt$, $\frac{dW}{dt} = |\varepsilon| I$

$$\frac{dW}{dt} = |\varepsilon| I$$

$$\text{But, } |\varepsilon| = L\frac{dI}{dt}$$

$$\frac{dW}{dt} = LI\frac{dI}{dt}$$

$$W = \int dW = \int_0^I LI\frac{dI}{dt}$$

$$\mathbf{W = \frac{1}{2}LI^2}$$

This expression is analogous to $\frac{1}{2}mv^2$**,** kinetic energy of a particle of mass m, and shows that **L is analogus to m** (i.e., L is electrical inertia and opposes growth and decay of current in the circuit).

Mutual induction

The phenomenon of production of induced emf in a coil by varying the current through a neighbouring coil is called mutual-induction.

The flux linked with the coil is proportional to the current through the neighbouring coil.

$$\phi \propto I$$

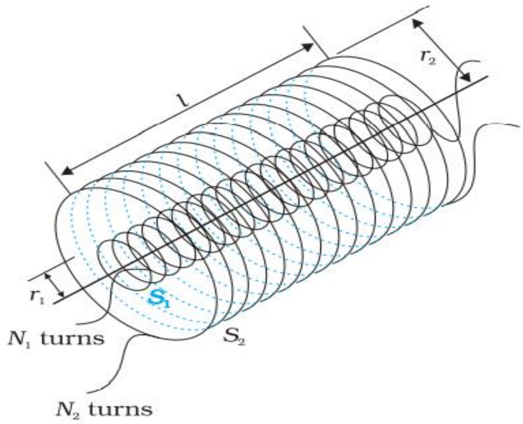
$$\mathbf{\phi = MI}$$

where constant of proportionality M is called **mutual-inductance** of the coil. It is also called **the coefficient of mutual-induction** of the coil.

When the current in the nighbouring coil is varied, the flux linked with the first coil changes and an emf is induced in the coil.

$$\boldsymbol{\varepsilon = -M\frac{dI}{dt}}$$

Mutual inductance of two co-axial solenoids



Two long co-axial solenoids of same length l.
Inner solenoid S_1 of radius r_1 and the number of turns per unit length n_1.
Outer solenoid S_2 of radius r_2 and the number of turns per unit length n_2.

The current I_2 in S_2 sets up a magnetic flux in S_1.

$$\phi_1 = N_1 B_2 A_1$$

$$B_2 = \mu_0 n_2 I_2$$
$$N_1 = n_1 l$$

$$\phi_1 = (n_1 l)(\mu_0 n_2 I_2) A_1$$

$$\phi_1 = \mu_0 n_1 n_2 A_1 \; l I_2 \quad \text{-------------(1)}$$

But, $\phi_1 = M_{12} I_2$ -------------(2)

From eq(1) and (2)

$$M_{12} I_2 = \mu_0 n_1 n_2 A_1 \; l I_2$$
$$\boldsymbol{M_{12} = \mu_0 n_1 n_2 A_1 \; l}$$

Similarly we get

$$\boldsymbol{M_{21} = \mu_0 n_1 n_2 A_1 \; l} \qquad \text{where } A_1 = \pi r_1^2$$

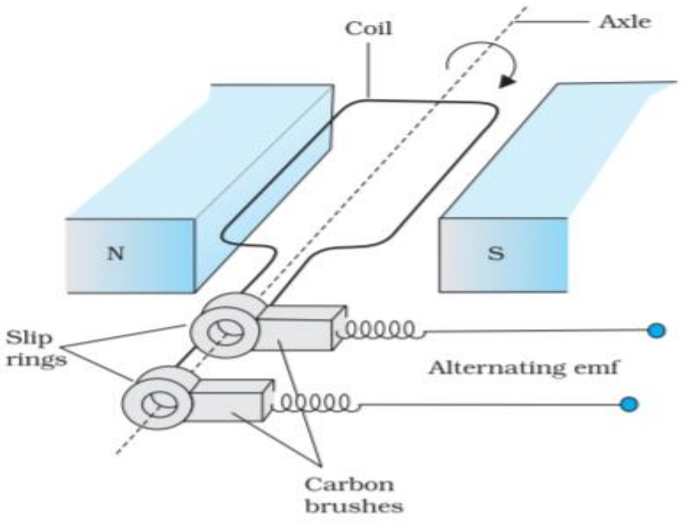
(The flux due to the current I_1 in S_1 can be assumed to be confined solely inside S_1 since the solenoids are very long. So we can take A= πr_1^2 itself)

$$\boldsymbol{M_{12} = M_{21} = M}$$

If a medium of relative permeability μ_r is introduced inside the solenoid

$$\boldsymbol{M = \mu_r \mu_0 n_1 n_2 \; A_1 l}$$

66.8 AC Generator



An ac generator converts mechanical energy into electrical energy. It consists of a coil which is mechanically rotated in the uniform magnetic field by some external means. The rotation of the coil causes the magnetic flux through it to change, so an emf is induced in the coil.

The magnetic flux at any time t is

$$\phi = BA\cos\theta = BA\cos\omega t$$

From Faraday's law, the induced emf for the rotating coil of N turns is

$$\varepsilon = -N\frac{d\phi}{dt}$$

$$\varepsilon = -N\frac{d}{dt}BA\cos\omega t$$

$$\varepsilon = -NBA\frac{d}{dt}\cos\omega t$$

$$\varepsilon = NBA\omega\sin\omega t$$

$$\boldsymbol{\varepsilon = \varepsilon_0 \sin\omega t}$$

where ε_0 =NBAω is the maximum value of the emf.

$\omega = 2\pi\nu$, ν=frequency of revolution of the generator's coil

The direction of the current changes periodically and therefore the current is called alternating current (ac).

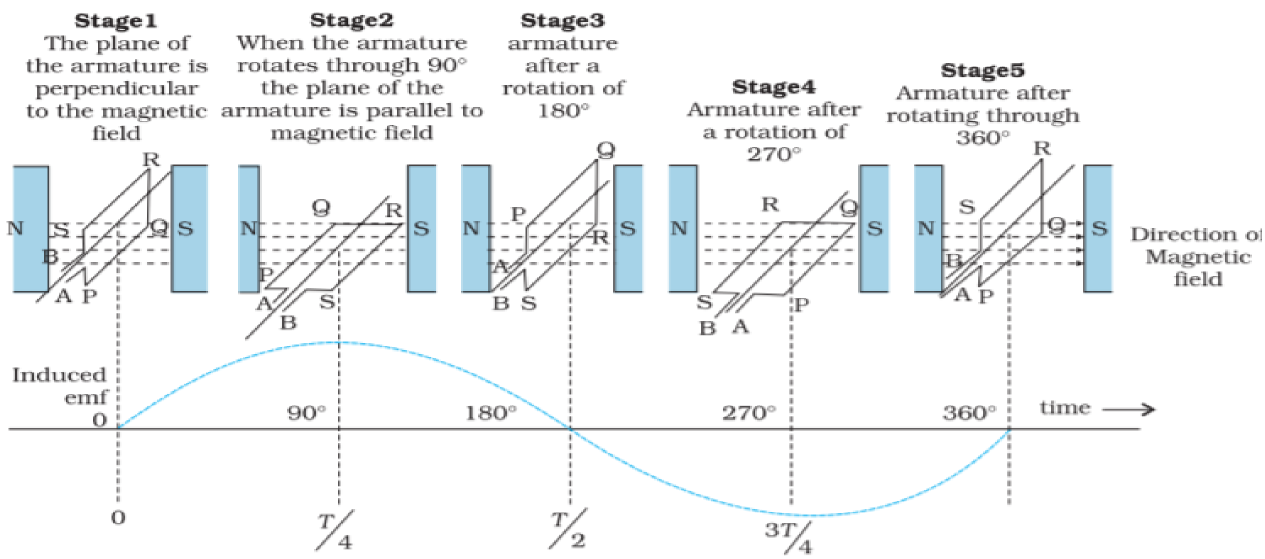
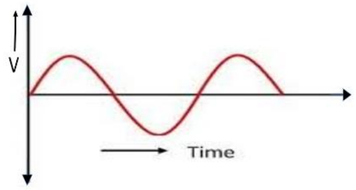


FIGURE 6.17 An alternating emf is generated by a loop of wire rotating in a magnetic field.

Chapter 7
Alternating Current

7.1 Introduction

The electric mains supply in our homes and offices is a voltage that varies like a sine function with time. Such a voltage is called alternating voltage (ac voltage) and the current driven by it in a circuit is called the alternating current (ac current)

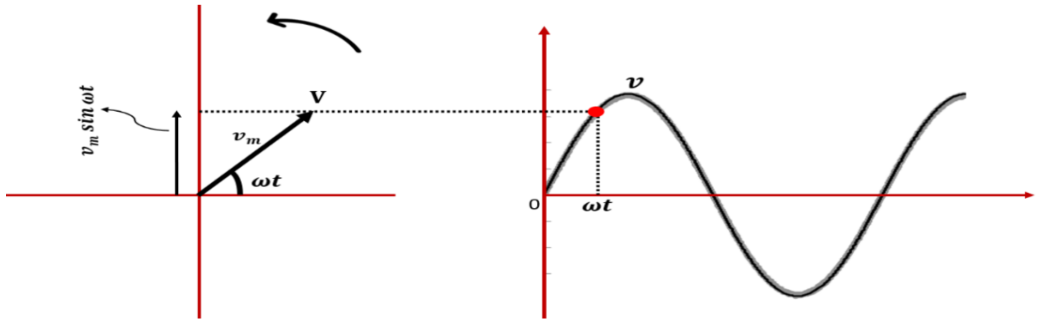


Today, most of the electrical devices we use require ac voltage. This is mainly because most of the electrical energy sold by power companies is transmitted and distributed as alternating current. The main reason for preferring use of ac voltage over dc voltage is that ac voltages can be easily and efficiently converted from one voltage to the other by means of transformers. Further, electrical energy can also be transmitted economically over long distances.

7.2 Representation of ac current and voltage by rotating vectors — phasors

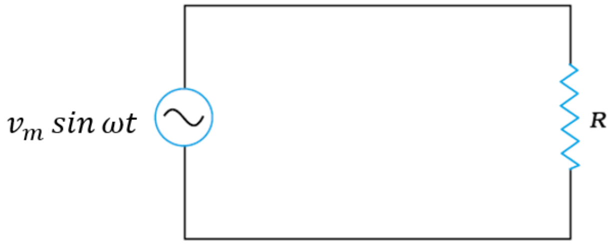
In order to show phase relationship between voltage and current in an AC circuit, we use the notion of phasors.

A phasor is a vector which rotates about the origin in anticlockwise direction with angular speed ω.



- The length of each phasor represents the amplitude or peak value of the voltage or current.
- The projection of each phasor on the vertical axis gives the instantaneous value of the quantity that the phasor represents.
- The rotation angle of each phasor is equal to the phase of alternating quantity at that instant t.
- The angle between two phasors will give you the phase difference between the corresponding quantities

7.3 AC Voltage Applied to a Resistor



Apply Kirchhoff's Loop rule ,$\Sigma\varepsilon(t) = 0$

$$v_m \sin \omega t - i R = 0$$

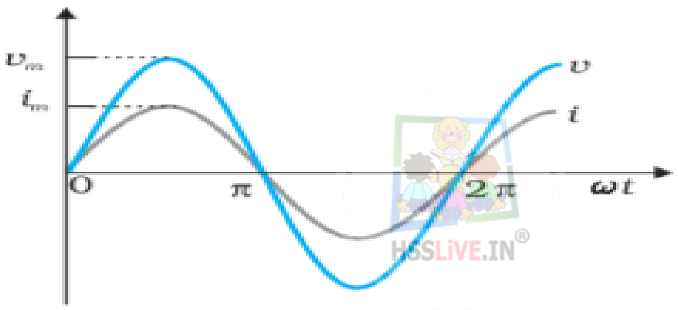
$$v_m \sin \omega t = iR$$

$$i = \frac{v_m}{R} \sin \omega t$$

$$\mathbf{i = i_m \sin \omega t} \qquad \text{where } \mathbf{i_m = \frac{v_m}{R}}$$

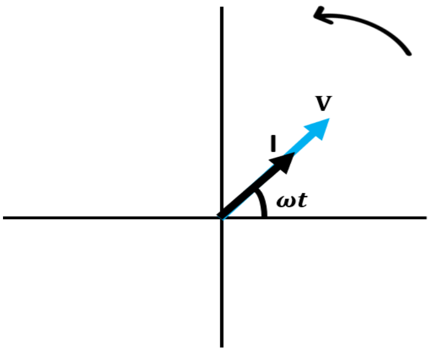
i_m is called amplitude of current

Graph of voltage and current across a pure resistor versus ωt



In a pure resistor, the voltage and current are in phase. The minima, zero and maxima occur at the same.

Phasor diagram for the circuit



Power Dissipated in the Resistor

The ac current varies sinusoidally and has corresponding positive and negative values during each cycle. Thus, the sum of the instantaneous current values over one complete cycle is zero, and the average current is zero.The fact that the average current is zero, however, does not mean that the average power consumed is zero and that there is no dissipation of electrical energy.

The instantaneous power dissipated in the resistor i

$$p = vi$$
$$p = v_m \sin \omega t \, i_m \sin \omega t$$
$$p = v_m i_m \sin^2 \omega t$$

Avearage power consumed over one complete cycle

$$\bar{p} = \langle v_m i_m \sin^2 \omega t \rangle$$
$$\bar{p} = v_m i_m \langle \sin^2 \omega t \rangle$$

$$\langle sn^2 \omega t \rangle = \frac{1}{2}$$

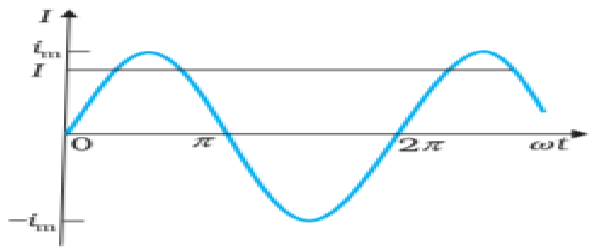
$$\bar{p} = \frac{1}{2} v_m i_m$$
$$P = \left(\frac{v_m}{\sqrt{2}}\right)\left(\frac{i_m}{\sqrt{2}}\right)$$
$$\mathbf{P = VI}$$

Where I or I_{rms} is called rms current and V or V_{rms} is called rms voltage.

The rms current (Root Mean Square Current) or Effective Current

To express **AC power** $\bar{p} = \frac{1}{2} v_m i_m$ in the same form as dc power $P = VI$, a special value of current is defined and used. It is called, root mean square (rms) or effective current and is denoted by $\mathbf{I_{rms}}$ or I.

$$I = \sqrt{\langle i^2 \rangle}$$
$$I = \sqrt{\langle (i_m \sin \omega t)^2 \rangle}$$
$$I = i_m \sqrt{\langle \sin^2 \omega t \rangle}$$
$$I = i_m \sqrt{\frac{1}{2}}$$
$$\mathbf{I = \frac{i_m}{\sqrt{2}} = 0.707\, i_m}$$



The rms current is the equivalent dc current that would produce the same average power loss as the alternating current.

Similarly ,rms voltage or Effective voltage

$$\mathbf{V = V_{rms} = \frac{v_m}{\sqrt{2}}}$$

$$\mathbf{V = \frac{v_m}{\sqrt{2}} = 0.707\, v_m}$$

Why a shock from 220V ac is more fatal than that from 220Vdc?

The household line voltage of 220 V is an rms value.

$$V = 220V$$

Its peak voltage $v_m = \sqrt{2}\,V$

$$= 1.414 \times 220\ V$$

$$= 311\ V$$

At some instant peak value of ac may reach upto 311V .So a shock from 220V ac is more fatal than that from 220Vdc.

Example

A light bulb is rated at 100W for a 220 V supply. Find
(a) the resistance of the bulb
(b) the peak voltage of the source
(c) the rms current through the bulb.

(a) We are given P = 100 W and V = 220 V. The resistance of the bulb is

$$R = \frac{V^2}{P} = \frac{(220\ \text{V})^2}{100\ \text{W}} = 484\Omega$$

(b) The peak voltage of the source is

$$v_m = \sqrt{2}V = 311\ \text{V}$$

(c) Since, $P = I\ V$

$$I = \frac{P}{V} = \frac{100\ \text{W}}{220\ \text{V}} = 0.450\text{A}$$

7.4 AC Voltage Applied to an Inductor

Apply Kirchhoff's Loop rule ,$\Sigma\varepsilon(t) = 0$

$$v_m \sin \omega t - L\frac{di}{dt} = 0$$

$$v_m \sin \omega t = L\frac{di}{dt}$$

$$\frac{di}{dt} = \frac{v_m \sin \omega t}{L}$$

$$di = \frac{v_m}{L} \sin \omega t\, dt$$

$$i = \frac{v_m}{L} \int \sin \omega t\, dt$$

$$i = \frac{v_m}{L} \times \frac{-\cos \omega t}{\omega}$$

$$i = -\frac{v_m}{\omega L}\cos \omega t$$

$$i = -i_m \cos \omega t$$

$\cos\theta = \sin(90-\theta)$
$-\cos\theta = -\sin(90-\theta)$
$-\sin\theta = \sin(-\theta)$
$-\cos\theta = \sin(-90+\theta)$
$-\cos\theta = \sin(\theta-90)$
$-\cos\omega t = \sin\left(\omega t - \frac{\pi}{2}\right)$

$$\mathbf{i = i_m \sin\left(\omega t - \frac{\pi}{2}\right)} \quad \text{where} \quad \mathbf{i_m = \frac{v_m}{\omega L}}$$

In a pure inductor, the current lags the voltage by π/2 or one-quarter (1/4) cycle.

Inductive Reactance (X_L)

The current amplitude, $i_m = \frac{v_m}{\omega L}$

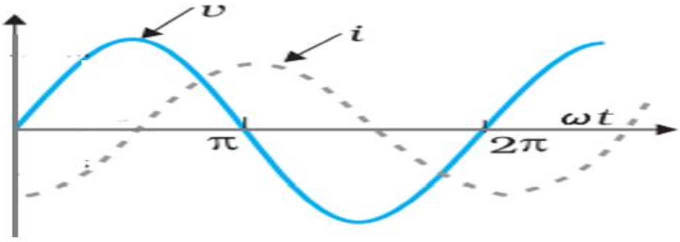
$$i_m = \frac{v_m}{X_L}$$

The quantity ω L is analogous to the resistance and is called inductive reactance, denoted by X_L

$$\mathbf{X_L = \omega L = 2\pi f L}$$

- The dimension of inductive reactance is the same as that of resistance and its SI unit is ohm (Ω).
- The inductive reactance limits the current in a purely inductive circuit in the same way as the resistance limits the current in a purely resistive circuit.
- The inductive reactance is directly proportional to the inductance and to the frequency of the current.
- **For DC , f=0 and so X_L =0 i.e., an inductor offers an easy path to DC.**
- **The value of X_L increases as frequency is increased, hence offers a resistive path to AC.**

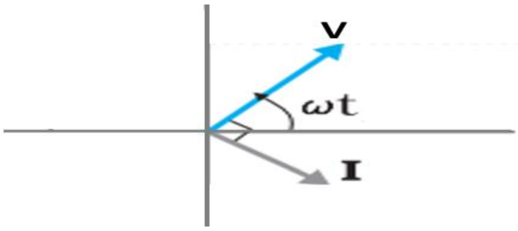
Graph of v and i versus ωt



$$\mathbf{v = v_m \sin \omega t}$$

$$\mathbf{i = i_m \sin\left(\omega t - \frac{\pi}{2}\right)}$$

Phasor diagram



The current lags the voltage by $\pi/2$.

Power Dissipated in the Inductor

Instantaneous power $p = iv$

$$p = -i_m \cos\omega t \times v_m \sin\omega t$$
$$p = -i_m v_m \cos\omega t \sin\omega t$$
$$p = -\frac{i_m v_m}{2} 2\cos\omega t \sin\omega t$$
$$p = -\frac{i_m v_m}{2} \sin(2\omega t)$$

The average power over a complete cycle

$$\bar{p} = P = \left\langle -\frac{i_m v_m}{2} \sin(2\omega t) \right\rangle$$
$$P = -\frac{i_m v_m}{2} \langle \sin(2\omega t) \rangle$$

$$\langle \sin(2\omega t) \rangle = 0$$

$$\mathbf{P = 0}$$

The average power supplied to an inductor over one complete cycle is zero.

Example

A pure inductor of 25.0 mH is connected to a source of 220 V. Find the inductive reactance and rms current in the circuit if the frequency of the source is 50 Hz.

Inductive reactance, $X_L = \omega L = 2\pi f L$

$$= 2\text{x } 3.14\text{x}50\text{x}25\text{x}10^{-3}$$
$$= 7.85\Omega$$

The rms current in the circuit is, $I = \frac{V}{X_L}$

$$I = \frac{220}{7.85} = 28\text{A}$$

7.5 AC Voltage Applied to a Capacitor

Applying Kirchhoff's Loop rule $\Sigma\varepsilon(t) = 0$

$$v_m \sin \omega t - \frac{q}{C} = 0$$

$$v_m \sin \omega t = \frac{q}{C}$$

$$q = C\, v_m \sin \omega t$$

$$i = \frac{d}{dt}(C\, v_m \sin \omega t)$$

$$i = C\, v_m \frac{d}{dt}(\sin \omega t)$$

$$i = C\, v_m\, \omega \cos \omega t$$

$$i = \omega C\, v_m \cos \omega t$$

$$i = i_m \cos \omega t$$

$\cos\theta = \sin(90+\theta)$
$\cos\theta = \sin(\theta + 90)$
$\cos \omega t = \sin\left(\omega t + \frac{\pi}{2}\right)$

$$\mathbf{i = i_m \sin\left(\omega t + \frac{\pi}{2}\right)}$$

where $i_m = \omega C\, v_m$

or $i_m = \dfrac{v_m}{\left(\frac{1}{\omega C}\right)}$

In a purely capacitive circuit, the current leads the voltage by π/2 or one-quarter (1/4) cycle.

Capacitive Reactance

Current amplitude, $i_m = \dfrac{v_m}{\left(\frac{1}{\omega C}\right)} = \dfrac{v_m}{X_C}$

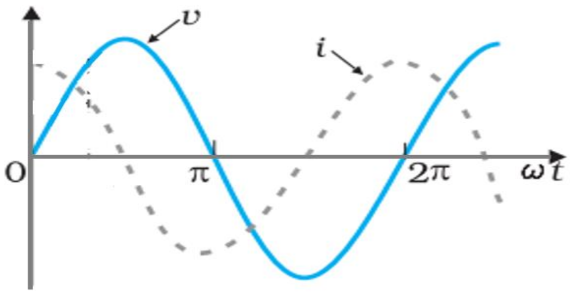
The quantity $\left(\frac{1}{\omega C}\right)$ is analogous to the resistance and is called capacitive reactance, denoted by X_C

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

- The dimension of capacitive reactance is the same as that of resistance and its SI unit is ohm (Ω).
- The capacitive reactance limits the amplitude of the current in a purely capacitive circuit in the same way as the resistance limits the current in a purely resistive circuit.
- Capacitive reactance is inversely proportional to the frequency and the capacitance.

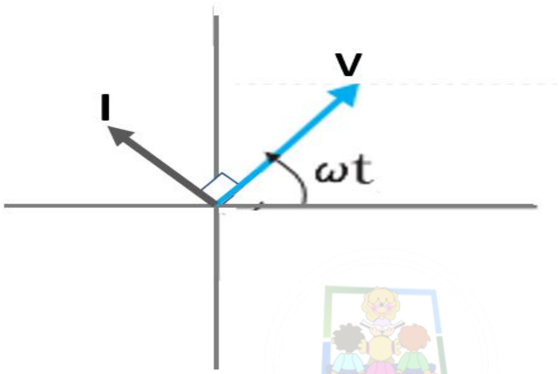
- For DC , f=0 and hence X_C = infinite i.e., the capacitor blocks DC.
- For AC, as the frequency increases, X_C decreases and hence capacitor allows AC to flow through it.

Graph of v and i versus ωt



$$v = v_m \sin \omega t$$
$$i = i_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

Phasor diagram



Power Dissipated in the Capacitor

$$P = iv$$
$$p = i_m \cos \omega t \ x \ v_m \sin \omega t$$
$$p = \frac{i_m v_m}{2} \sin(2\omega t)$$

The average power over a complete cycle

$$\bar{p} = P = \left\langle \frac{i_m v_m}{2} \sin(2\omega t) \right\rangle$$
$$P = \frac{i_m v_m}{2} \langle \sin(2\omega t) \rangle$$

$$\langle \sin(2\omega t) \rangle = 0$$

$$P = 0$$

The average power supplied to a capacitor over one complete cycle is zero.

Example

A 15.0 μF capacitor is connected to a 220 V, 50 Hz source. Find the capacitive reactance and the current (rms and peak) in the circuit. If the frequency is doubled, what happens to the capacitive reactance and the current?

The capacitive reactance $X_C = \frac{1}{\omega C}$

$$= \frac{1}{2\pi f C}$$

$$= \frac{1}{2x\ 3.14x50x15x10^{-6}} = 212\Omega$$

The rms current is , $I = \frac{V}{X_C}$

$$I = \frac{220}{212} = 1.04A$$

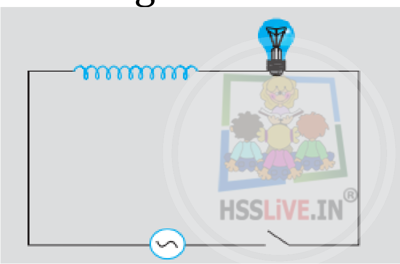
The peak current is $i_m = \sqrt{2}\ I$

$$= 1.414\ x1.04\ = 1.47A$$

If the frequency is doubled, the capacitive reactance is halved , and consequently, the current is doubled.

Example

A light bulb and an open coil inductor are connected to an ac source through a key as shown in Figure.



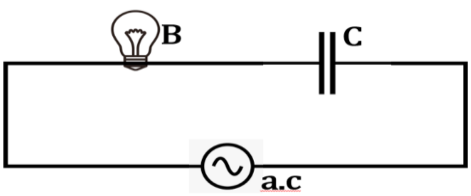
The switch is closed and after sometime, an iron rod is inserted into the interior of the inductor. The glow of the light bulb
(a)increases; (b) decreases; (c) is unchanged, as the iron rod is inserted.
Give your answer with reasons.

Solution:

As the iron rod is inserted, the magnetic field inside the coil magnetizes the iron increasing the magnetic field inside it. Hence, the inductance of the coil increases. Consequently, the inductive reactance of the coil increases. As a result, a larger fraction of the applied ac voltage appears across the inductor, leaving less voltage across the bulb. Therefore, the glow of the light bulb decreases.

Example

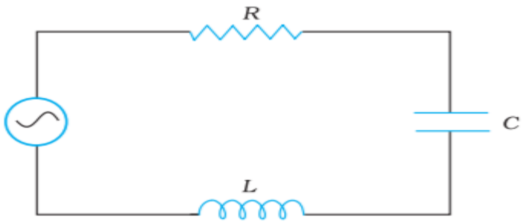
An electric bulb B and a parallel plate capacitor C are connected in series as shown in figure.



The bulb glows with some brightness. How will the glow of the bulb affected on introducing a dielectric slab between the plates of the capacitor? Give reason in support of your answer

When a dielectric slabis introduced between the plates the capacitance increases. Then capacitive reactance decreases. As a result, a smaller fraction of the applied ac voltage appears across the capacitor, leaving large voltage across the bulb. Therefore, the glow of the light bulb increases.

7.6 AC Voltage Applied to a Series LCR Circuit



Applying Kirchhoff's Loop rule $\Sigma\varepsilon(t) = 0$

$$v_m \sin \omega t - iR - L\frac{di}{dt} - \frac{q}{C} = 0$$

$$\mathbf{v_m \sin \omega t = iR + L\frac{di}{dt} + \frac{q}{C}}$$

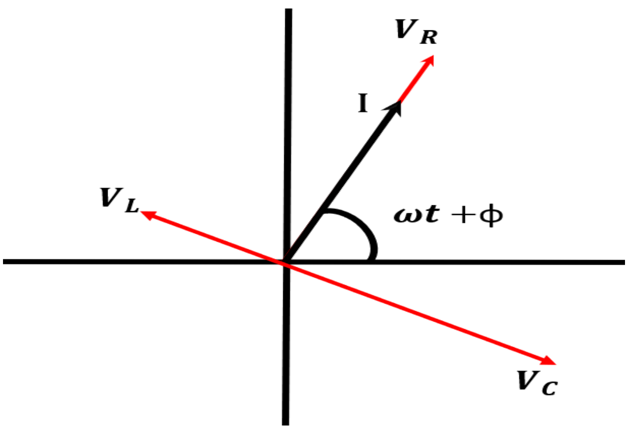
Phasor-diagram solution

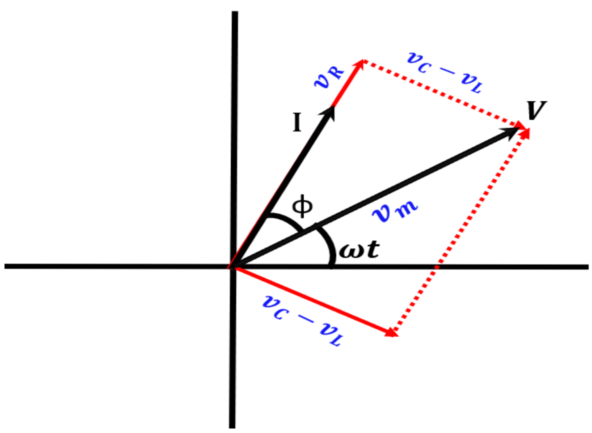
Since L,C and R are in series the ac current i in each element is the same.

Let the current be $\mathbf{i = i_m \sin(\omega t + \phi)}$

Further, let V_R , V_L , V_C , and V represent the voltage phasors across the resistor, inductor , capacitor and the source, respectively.

For resistor , V_R and I are in phase.
For inductor , V_L leads I by $\pi/2$.
For capacitor , V_C lags I by $\pi/2$.





To find the value of i_m

$$v_m^2 = v_R^{\ 2} + (v_C - v_L)^2$$
$$v_m^2 = (i_m R)^2 + (i_m X_C - i_m X_L)^2$$
$$v_m^2 = i_m^2 \ [(R)^2 + (X_C - X_L)^2]$$
$$i_m^2 = \frac{v_m^2}{(R)^2+(X_C-X_L)^2}$$
$$i_m = \frac{v_m}{\sqrt{(R)^2+(X_C-X_L)^2}}$$
$$i_m = \frac{v_m}{Z}$$

The quantity $\sqrt{(R)^2 + (X_C - X_L)^2}$ is analogous to resistance and is called impedance Z in an ac circuit.

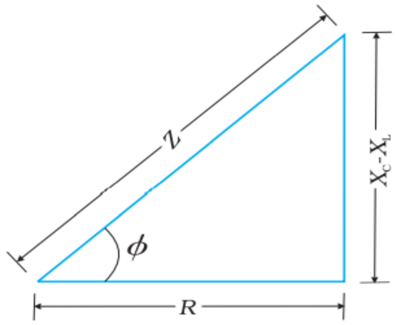
Impedance, $Z = \sqrt{(R)^2 + (X_C - X_L)^2}$

SI unit of Z is Ohm

The phase difference ϕ between voltage and current is ,

$$\tan\phi = \frac{v_{Cm}-v_{Lm}}{v_{Rm}}$$
$$\tan\phi = \frac{i_m X_C - i_m X_L}{i_m R}$$
$$\tan\phi = \frac{X_C - X_L}{R}$$
$$\phi = \tan^{-1}\frac{X_C-X_L}{R}$$

Impedance diagram



The phase difference ϕ can be obtained using impedance diagram.

$$\tan\phi = \frac{X_C - X_L}{R}$$
$$\phi = \tan^{-1}\frac{X_C-X_L}{R}$$

Example

A resistor of 200 Ω and a capacitor of 15.0 μF are connected in series to a 220 50 Hz ac source.

(a) Calculate the current in the circuit

(b)Calculate the voltage (rms) across the resistor and the capacitor. Is the algebraic sum of these voltages more than the source voltage? If yes, resolve the paradox.

a) R = 200Ω , C =15.0 μF =15x 10^{-6} F , V = 220 V, f =50Hz

$$Z = \sqrt{R^2 + {X_C}^2}$$

$$Z = \sqrt{R^2 + \left(\frac{1}{2\pi fC}\right)^2}$$

$$Z = \sqrt{200^2 + \left(\frac{1}{2x3.14x50x15x\,10^{-6}}\right)^2}$$

$$Z = \sqrt{200^2 + 212.3^2}$$

$$\mathbf{Z = 291.5\Omega}$$

The current in the circuit is

$$I = \frac{V}{Z}$$

$$\mathbf{I = \frac{220}{291.5} = 0.755A}$$

(b) The current is the same throughout the circuit.

$$V_R = IR = 0.755\ Ax200\ \Omega = 151V$$

$$V_C = IX_C = 0.755\ A\ x\ 212.3\ \Omega = 160.3V$$

Algebraic sum of V_R and V_C =151V+160.3V=311.3V

This is more than source voltage and is not possible.

There is a phase difference of 90^0 betweenV_R and V_C . Therefore, the total of these voltages must be obtained using the Pythagorean theorem.

$$\mathbf{V = \sqrt{{V_R}^2 + {V_C}^2} = \sqrt{151^2 + 160.3^2} = 220V}$$

Resonance

A system oscillating with its natural frequency is driven by an energy source at a frequency that is near the natural frequency, the amplitude of oscillation is found to be large. This phenomenon is called resonance.

A familiar example of this phenomenon is a child on a swing. If the child pulls on the rope at regular intervals and the frequency of the pulls is almost the same as the frequency of swinging, the amplitude of the swinging will be large.

Condition for resonance in an LCR circuit

For an LCR circuit the current amplitude is given by

$$i_m = \frac{v_m}{Z} = \frac{v_m}{\sqrt{(R)^2+(X_C-X_L)^2}}$$

For resonance to happen impedance should be minimum and current maximum. So the condition for resonance is,

$$X_C = X_L$$

Impedance at resonance

$$Z = \sqrt{(R)^2 + (X_C - X_L)^2}$$

$$Z = \sqrt{(R)^2 + 0^2}$$

$$Z = R$$

Impedance is minimum at resonance.

Current Amplitude at Resonance

$$i_m = \frac{v_m}{Z}$$

$$i_m^{max} = \frac{v_m}{R}$$

Current amplitude is maximum at resonance.

Resonant Frequency

The condition for resonance ,$X_C = X_L$

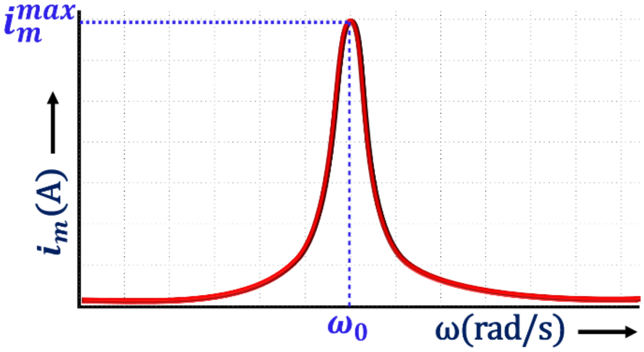
$$\frac{1}{\omega_0 C} = \omega_0 L$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

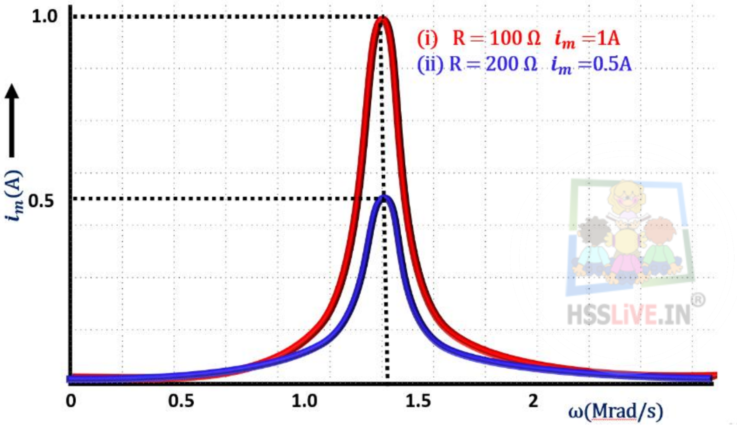
ω_0 is called Resonant frequency

Variation of i_m with ω



Example

Figure shows the variation of i m with ω in a RLC series circuit with L = 1.00 mH, C = 1.00 nF for two values of R:
(i) R = 100 Ω and (ii) R = 200 Ω. For the source applied vm = 100 V.



For R = 100 Ω

$$i_m = \frac{v_m}{R} = \frac{100}{100} = 1\text{A}$$

For R = 200 Ω

$$i_m = \frac{v_m}{R} = \frac{100}{200} = 0.5\text{A}$$

Tuning of a radio or TV

Resonant circuits have a variety of applications, for example, in the tuning mechanism of a radio or a TV set. The antenna of a radio accepts signals of dfferent frequencies from many broadcasting stations . But to hear one particular radio station, we tune the radio. In tuning, we vary the capacitance of a capacitor in the tuning circuit such that the resonant frequency of the circuit becomes nearly equal to the frequency of the radio signal received. When this happens, the amplitude of the current with the frequency of the signal of the particular radio station in the circuit is maximum.

Resonance phenomenon is exhibited by a circuit only if both L and C are present .Only then do the voltages across L and C cancel each other.

<u>We cannot have resonance in RL and RC circuit.</u>

7.7 Power In AC Circuit: The Power Factor

$$p = v\,i$$

$$p = v_m \sin\omega t \; i_m \sin(\omega t + \phi)$$

$$P = \frac{v_m i_m}{2}\langle\cos\phi - \cos(2\omega t + \phi)\rangle$$

$$P = \frac{v_m i_m}{2}\cos\phi$$

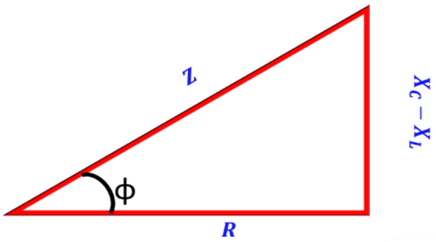
$$P = \frac{v_m}{\sqrt{2}}\,\frac{i_m}{\sqrt{2}}\cos\phi$$

$$P = V\;I\cos\phi$$

So, the average power dissipated depends not only on the voltage and current but also on the cosine of the phase angle ϕ between them.

The quantity $\cos\phi$ is called the **power factor.**

Power factor can be obtained from impedance diagram.



$$\cos\phi = \frac{R}{Z}$$

Case (i) Resistive circuit:

$$\phi = 0,$$

$$P = V\,I\cos 0 = VI$$

There is maximum power dissipation.

Case (ii) Purely inductive or capacitive circuit:

$$\phi = \pi/2$$

$$P = V\,I\cos\pi/2 = 0$$

No power is dissipated even though a current is flowing in the circuit.

This current is sometimes referred to as **wattless current.**

Case (iii) LCR series circuit:

$$\phi = \tan^{-1}\frac{X_C - X_L}{R}$$

$$P = V\;I\cos\phi$$

So, ϕ may be non-zero and power may dissipate in a RL or RC or RCL circuit. Even in such cases, power is dissipated only in the resistor.

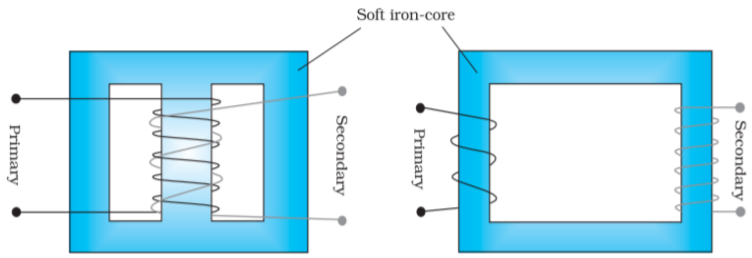
Case (iv) Power dissipated at resonance in LCR circuit:

At resonance $\mathbf{X_C} - \mathbf{X_L} = 0$, and $\phi = 0$.

$$P = V\;I\cos 0 = PV$$

That is, maximum power is dissipated in a circuit (through R) at resonance.

7.8 Transformer



A transformer consists of two sets of coils, insulated from each other. They are wound on a soft-iron core, One of the coils called the primary coil has $N_P$turns. The other coil is called the secondary coil; it has N_S turns. Often the primary coil is the input coil and the secondary coil is the output coil of the transformer.

Transformer works on the Principle of Mutual Induction

When an alternating voltage is applied to the primary, the resulting current produces an alternating magnetic flux which links the secondary and induces an emf in it.

The emf induced in the primary

$$\varepsilon_P = -N_P\frac{d\phi}{dt}$$

If the primary coil has negligible resistance $\varepsilon_P = V_P$(input voltage)

$$V_P = -N_P\frac{d\phi}{dt} \text{-------------}(1)$$

The emf induced in thesecondary

$$\varepsilon_s = -N_s\frac{d\phi}{dt}$$

If the secondary coil has negligible resistance $\varepsilon_s = V_s$(output voltage)

$$V_s = -N_s\frac{d\phi}{dt}\text{-----------------}(2)$$

$$\frac{\mathbf{eq\ (1)}}{\mathbf{eq\ (2)}} \text{ - - - } \qquad \frac{\mathbf{V_s}}{\mathbf{V_P}} = \frac{\mathbf{N_s}}{\mathbf{N_P}} \text{--------------------- } \mathbf{(3)}$$

If the transformer is 100% efficient
power input =power output

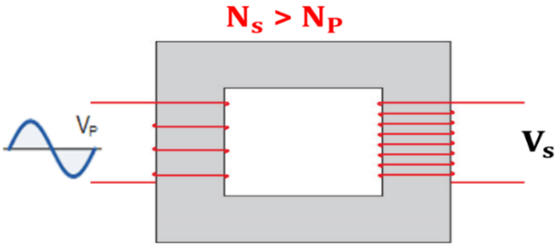
$$I_P V_P = I_s V_s$$

$$\frac{\mathbf{I_P}}{\mathbf{I_s}} = \frac{\mathbf{V_s}}{\mathbf{V_P}} \text{--------------}\mathbf{(4)}$$

Combining equations (3)*and* (4)

$$\frac{\mathbf{I_P}}{\mathbf{I_s}} = \frac{\mathbf{V_s}}{\mathbf{V_P}} = \frac{\mathbf{N_s}}{\mathbf{N_P}}$$

Step-up Transformer



For a step up transformer the number of turns in the secondary will be greater than that in the primary($N_s > N_P$)

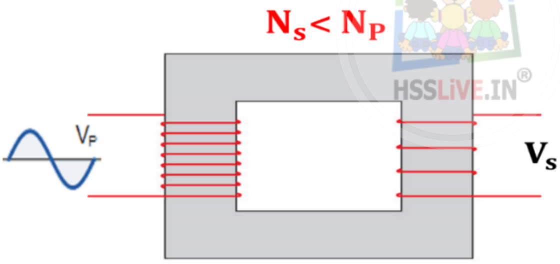
$$V_s = \left(\frac{N_s}{N_P}\right) V_p \qquad I_s = \left(\frac{N_P}{N_s}\right) V_p$$

$$\left(\frac{N_s}{N_P}\right) > 1 \qquad \left(\frac{N_P}{N_s}\right) < 1$$

$$V_s > V_p \qquad I_s < I_p$$

Thus for a step up transformer secondary voltage will be greater than primary voltage,but the secondary current will be less than primary current.

Step-down Transformer



For a step down transformer the number of turns in the secondary will be less than that in the primary($N_s < N_P$)

$$V_s = \left(\frac{N_s}{N_P}\right) V_p \qquad I_s = \left(\frac{N_P}{N_s}\right) V_p$$

$$\left(\frac{N_s}{N_P}\right) < 1 \qquad \left(\frac{N_P}{N_s}\right) > 1$$

$$V_s < V_p \qquad I_s > I_p$$

Thus for a step up transformer secondary voltage will be less than primary voltage, but the secondary current will be greater than primary current.

Energy Losses in a Transformer

(i)Flux Leakage:

There is always some flux leakage; that is, not all of the flux due to primary passes through the secondary due to poor design of the core or the air gaps in the core. It can be reduced by winding the primary and secondary coils one over the other.

(ii)Resistance of the windings :

The wire used for the windings has some resistance and so, energy is lost due to heat produced in the wire(I^2R). In high current, low voltage windings, these are minimised by using thick wire.

(iii)Eddy currents loss:

The alternating magnetic flux induces eddy currents in the iron core and causes heating. The effect is reduced by having a laminated core.

(iv)Hysteresis loss:

The magnetisation of the core is repeatedly reversed by the alternating magnetic field. This produces hysteresis and energy is lost as heat. This can be minimised by using a magnetic material which has a low hysteresis loss(e.g- soft iron core)

Chapter 8
Electromagnetic Waves

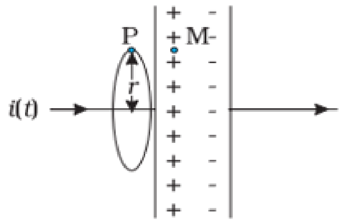
8.1 Introduction

An electrical current produces a magnetic field around it. Further, a magnetic field changing with time gives rise to an electric field. Is the converse also true? Does an electric field changing with time give rise to a magnetic field?

According to James Clerk Maxwell , time-varying electric field generates magnetic field. Maxwell formulated a set of equations involving electric and magnetic fields, known as Maxwell's equations. Maxwell's equations predicted the existence of electromagnetic waves, which are (coupled) timevarying electric and magnetic fields that propagate in space. Hertz, in 1885, experimentally demonstrated the existence of electromagnetic waves. Its technological use by Marconi and others led in due course to the revolution in communication that we are witnessing today.

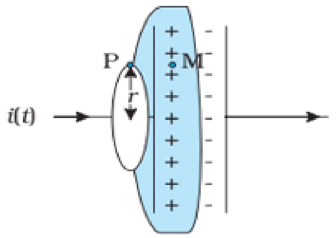
Inconsistency in the Ampere's circuital law and Displacement current

While applying the Ampere's circuital law to find magnetic field at a point outside a capacitor connected to a time-varying current, Maxwell noticed an inconsistency in the Ampere's circuital law.



To find the magnetic field at a point such as P, in a region outside the parallel plate capacitor, consider a plane circular loop of radius r. A current i(t) passes through this surface.

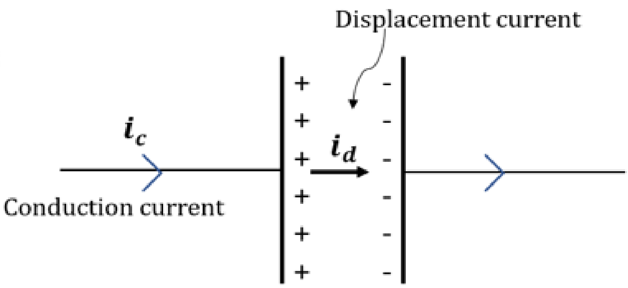
$$\oint B \cdot dl = \mu_0 i(t)$$



To find the magneticfield at the same point P, now consider a pot-shaped surface passing through the interior between the capacitor plates with the same rim as in first case. No current passes through this surface.

$$\oint B \cdot dl = 0 \qquad \text{(This is a contradiction)}$$

To remove this to inconsistency, Maxwell suggested the existence of an additional current , called, the displacement current .



8.2 Displacement Current

The current due to changing electric field or electric flux is called called displacement current or Maxwell's displacement current.

Electric flux $\phi_E = \frac{q}{\varepsilon_0}$

$$\frac{d\phi_E}{dt} = \frac{1}{\varepsilon_0}\frac{dq}{dt}$$

$$\frac{d\phi_E}{dt} = \frac{1}{\varepsilon_0} i_d$$

Displacement current $\mathbf{i_d = \varepsilon_0 \frac{d\phi_E}{dt}}$

Ampere-Maxwell law

According to Maxwell the source of a magnetic field is not just the conduction electric current due to flowing charges, but also the time rate of change of electric field.

The total current i is the sum of the conduction current (i_c) and displacement current (i_d)

$$\mathbf{i = i_c + i_d}$$

$$i = i_c + \varepsilon_0 \frac{d\phi_E}{dt}$$

Ampere's theorem become

$$\oint B \cdot dl = \mu_0 (i_C + i_d)$$

$$\oint B \cdot dl = \mu_0 \left(i_c + \varepsilon_0 \frac{d\phi_E}{dt}\right)$$

$$\boldsymbol{\oint B \cdot dl = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}}$$

This is known as Ampere-Maxwell law.

MAXWELL'S EQUATIONS

1. $\oint \mathbf{E} \cdot d\mathbf{A} = Q/\varepsilon_0$ (Gauss's Law for electricity)
2. $\oint \mathbf{B} \cdot d\mathbf{A} = 0$ (Gauss's Law for magnetism)
3. $\oint \mathbf{E} \cdot d\mathbf{l} = \frac{-d\Phi_B}{dt}$ (Faraday's Law)
4. $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$ (Ampere – Maxwell Law)

8.3 Electromagnetic waves

Sources of Electromagnetic Waves

- A stationary charge produces only electrostatic fields.
- Charges in uniform motion (steady currents) can produce magnetic fields that, do not vary with time.
- An oscillating charge(accelerating charge) produces an oscillating electric field in space, which produces an oscillating magnetic field, which in turn, is a source of oscillating electric field, and so on. The oscillating electric and magnetic fields thus regenerate each other, as the electro magnetic wave propagates through the space.
  Thus an oscillating charge(accelerating charge) is the source of electromagnetic waves.

An electric charge oscillating harmonically with frequency v, produces electromagnetic waves of the same frequency v.

- The experimental demonstration of electromagnetic wave in the radio wave region was done by **Hertz** in1887.
- Seven years after Hertz, **Jagdish Chandra Bose**, succeeded in producing and observing electromagnetic waves of much shorter wavelength (25 mm to 5 mm).
- At around the same time, **Guglielmo Marconi** succeeded in transmitting electromagnetic waves over distances of many kilometres. Marconi's experiment marks the beginning of the field of communication using electromagnetic waves.

Nature of Electromagnetic Waves

1) In an e.m waves are transverse waves in which the electric and magnetic fields are perpendicular to each other, and also to the direction of propagation.

2) The speed of e.m.wave in vacuum is,

$$\mathbf{c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}}$$

3)The speed of of electromagnetic waves in a material medium is
The speed of of electromagnetic waves in a material medium is

$$\mathbf{v = \frac{1}{\sqrt{\mu\varepsilon}} \quad or \quad v = \frac{1}{\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}} \quad or \quad v = \frac{c}{\sqrt{\mu_r\varepsilon_r}}}$$

4) The electric and the magnetic fields in an electromagnetic wave are related as

$$\mathbf{\frac{E_0}{B_0} = c}$$

5) No material medium is required for the propagation of e.m.wave.

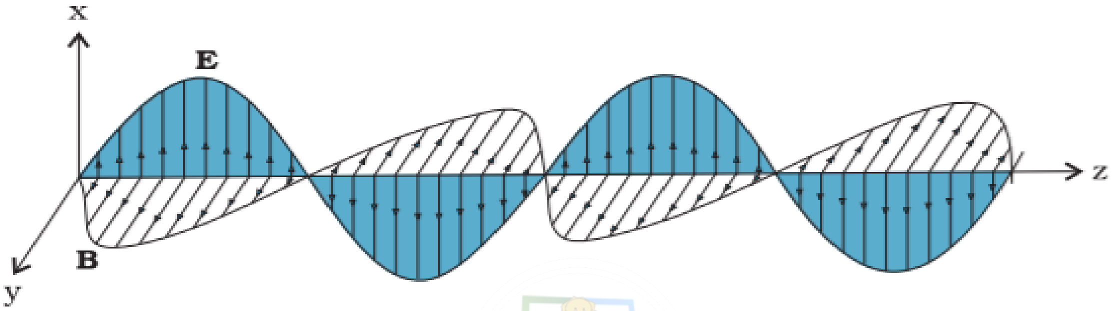
6) Electromagnetic waves carry energy as they travel through space and this energy is shared equally by the electric and magnetic fields.

7)Electromagnetic waves transport momentum as well. When these waves strike a surface, total momentum delivered to this surface is,

$$\mathbf{p = \frac{U}{c}}$$, where U is the energy

Expression for electric field and magnetic field

Consider an electromagnetic wave propagating along the z direction. Let the electric field E_x is along the x-axis and the magnetic field B_y is along the y-axis. Then



$$\mathbf{E_x = E_0 \ \sin(k\,z - \omega t)}$$
$$\mathbf{B_y = B_0 \ \sin(k\,z - \omega t)}$$

Here $\mathbf{k = \frac{2\pi}{\lambda}}$

k is the propagation constant

$$\omega = 2\pi\nu$$

ω is the angular frequency

$$\frac{\omega}{k} = \frac{2\pi\nu}{\frac{2\pi}{\lambda}} = \nu\lambda = c$$

Speed, $\mathbf{c = \frac{\omega}{k}}$

Example

A plane electromagnetic wave of frequency 25 MHz travels in free space along the x-direction. At a particular point in space and time, E = 6.3ĵ V/m. What is B at this point?

$$\frac{\mathbf{E_0}}{\mathbf{B_0}} = \mathbf{c}$$

$$\mathbf{B_0} = \frac{\mathbf{E_0}}{c} = \frac{6.3}{3 \times 10^8} = 2.1 \times 10^{-8}\ \mathrm{T}$$

E is along y-direction and the wave propagates along x-axis. Therefore, B should be in a direction perpendicular to both x- and y-axes. **i.e., B is along z-axis.**

Example

The magnetic field in a plane electromagnetic wave is given by

$B_y = 2 \times 10^{-7} \sin(0.5\times10^3 x + 1.5\times10^{11}\ t)$ T.

a) What is the wavelength and frequency of the wave?

b) Write an expression for the electric field.

(a) $$B_y = 2 \times 10^{-7} \sin(0.5\times10^3 x + 1.5\times10^{11}\ t)$$

Comparing with general expression for magnetic field of an em wave travelling in x direction,

$$B_y = B_0 \sin(kx - \omega t)$$

$$k = 0.5\times10^3$$

$$k = \frac{2\pi}{\lambda} = 0.5\times10^3$$

$$\lambda = \frac{2\pi}{0.5\times10^3}$$

$$= 12.56 \times 10^{-3}\ \mathrm{m}$$

$$\omega = 1.5\times10^{11}$$

$$\omega = 2\pi v = 1.5\times10^{11}$$

$$v = \frac{1.5\times10^{11}}{2\pi}$$

$$= 0.24\ \mathrm{x}10^{11}\ \mathrm{Hz}$$

b) B is along y-direction and the wave propagates along x-axis. Therefore, E should be in a direction perpendicular to both x- and y-axes. **i.e., E is along z-axis.**

So expression for electric field is ,

$$E_z = E_0 \sin(k\,x - \omega t)$$

$$\frac{E_0}{B_0} = c$$

$$E_0 = B_0\ x\ c$$

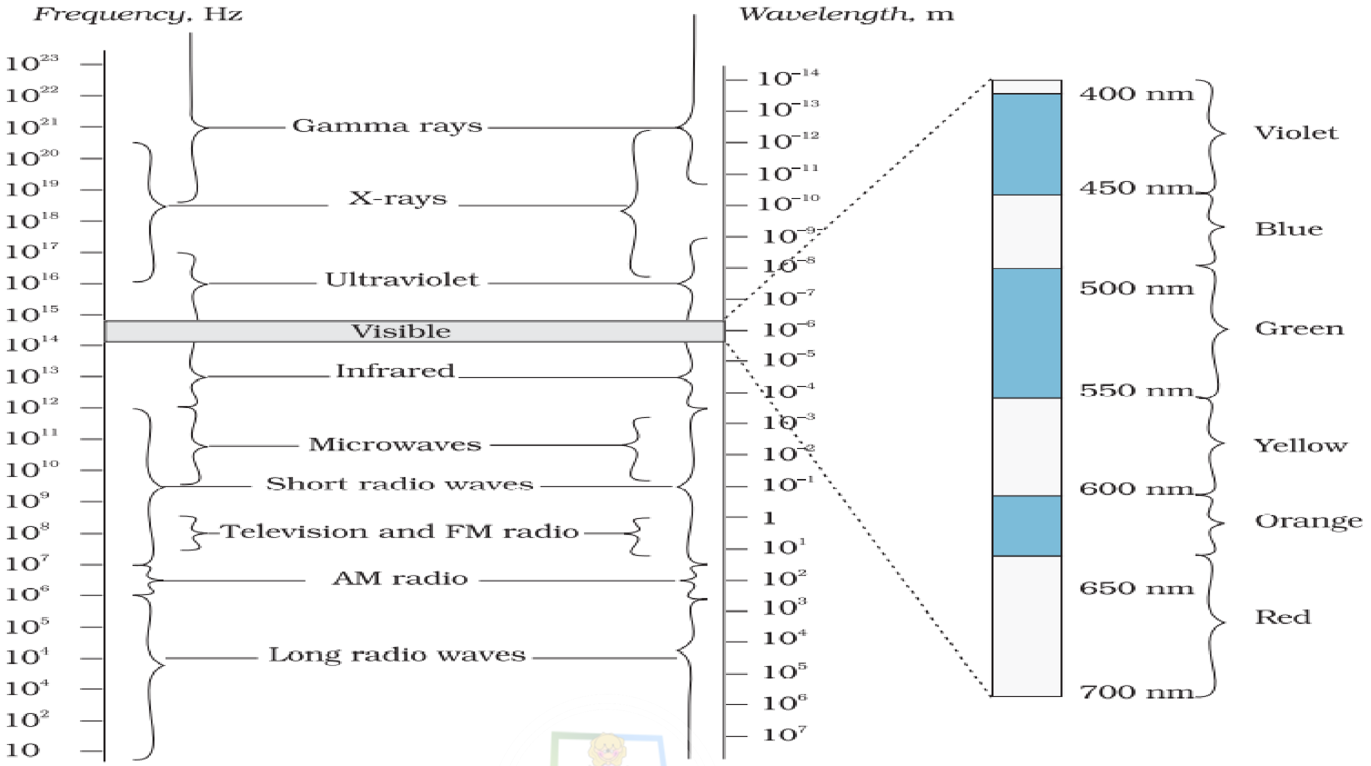
$$= 2 \times 10^{-7}\ \mathrm{x}\ 3 \times 10^8$$

$$= 60\ \mathrm{V/m}$$

$$\mathbf{E_z = 60 \sin(0.5\times10^3 x + 1.5\times10^{11}\ t)\ V/m}$$

8.4 Electromagnetic Spectrum

The classification of em waves according to frequency is the electromagnetic spectrum. There is no sharp division between one kind of wave and the next.



Radio waves

- Radio waves are produced by the accelerated motion of charges in conducting wires.
- Frequency range from 500 kHz to about 1000 MHz.
- (i)They are used in radio and television communication systems.
  (ii)Cellular phones use radio waves.

> The AM (amplitude modulated) band 530 kHz - 1710 kHz.
> Short wave bands -frequencies upto 54 MHz .
> TV waves - 54 MHz - 890 MHz.
> The FM (frequency modulated) radio band - 88 MHz - 108 MHz.
> Cellular phones - ultrahigh frequency (UHF) band.

Microwaves

- Microwaves (short-wavelength radio waves), are produced by special vacuum tubes called, klystrons, magnetrons and Gunn diodes.
- Frequencies in the gigahertz (GHz) range,
- (i)Used for radar systems used in aircraft navigation .
  (ii)Used in speed guns used to time fast balls, tennisserves, and automobiles.
  (iii) Microwaves are used in microwave ovens , for cooking.

How is food cooked in microwave ovens?

In microwave ovens, the frequency of the microwaves is selected to match the resonant frequency of water molecules so that energy from the waves is transferred efficiently to the kinetic energy of the molecules. This raises the temperature of any food containing water.

Infrared waves

- Infrared waves are produced by hot bodies and molecules.
- (i) Infrared lamps are used in physical therapy.
  (ii) Infrared radiation plays an important role in maintaining the earth's warmth or average temperature through the greenhouse effect.
  (iii)Infrared detectors are used in Earth satellites, both for military purposes and to observe growth of crops.
  (iv)LEDs emit infrared waves, which are used in the remote switches of TV sets, video recorders and hi-fi systems.

Why IR waves are called heat waves?

Infrared waves are sometimes referred to as heat waves. This is because water molecules present in most materials readily absorb infrared waves
(CO_2, NH_3, also absorb infrared waves). After absorption, their thermal motion increases, that is, they heat up and heat their surroundings.

Greenhouse Effect

Incoming visible light is absorbed by the earth's surface and reradiated as infrared (longer wavelength) radiations. This radiation is trapped by greenhouse gases such as carbon dioxide and water vapour. This trapped Infrared radiation maintains the earth's warmth.

Visible rays

- Electrons in atoms emit The eye light when they move from Photocells one energy level to a Photographic film lower energy level'
- Frequency range of 4×10^{14} Hz to 7×10^{14} Hz
  Wavelength range of about 700 – 400 nm.

Our eyes are sensitive to this range of wavelengths. Different animals are sensitive to different range of wavelengths. For example, snakes can detect infrared waves, and the 'visible' range of many insects extends well into the utraviolet.

Ultraviolet rays

- Ultraviolet (UV) radiation is produced by special lamps and very hot bodies. The sun is an important source of ultraviolet light.
- Wavelength range of (400 nm) to (0.6 nm).
- (i)UV radiations are used in LASIK
  (Laser assisted in situ keratomileusis) eye surgery.
  (ii) UV lamps are used to kill germs in water purifiers.

Why is depletion of ozone layer , a matter of international concern?

Most of the UV rays from sun is absorbed in the ozone layer in the atmosphere at an altitude of about 40 – 50 km. UV light in large quantities has harmful effects on humans. Exposure to UV radiation induces the production of more melanin, causing tanning of the skin. Ozone layer in the atmosphere plays a protective role, and hence its depletion by chlorofluoro- carbons (CFCs) gas (such as freon) is a matter of international concern.

UV radiation is absorbed by ordinary glass. Hence, one cannot get tanned or sunburn through glass windows. Welders wear special glass goggles or face masks with glass windows to protect their eyes from large amount of UV produced by welding arcs.

X-rays

- One common way to generate X-rays is to bombard a metal target by high energy electrons.
- Wavelengths from about (10 nm) to (10^{-4} nm).
- X-rays are used as a diagnostic tool in medicine and as a treatment for certain forms of cancer.

As X-rays damage or destroy living tissues and organisms, care must be taken to avoid unnecessary or over exposure.

Gamma rays

- This high frequency radiation is produced in nuclear reactions and also emitted by radioactive nuclei.
- Gamma rays are the highest frequency range of the electromagnetic spectrum and have wavelengths of from about 10^{-10}m to 10^{-14}m.
- They are used in medicine to destroy cancer cells.

TABLE 8.1 DIFFERENT TYPES OF ELECTROMAGNETIC WAVES

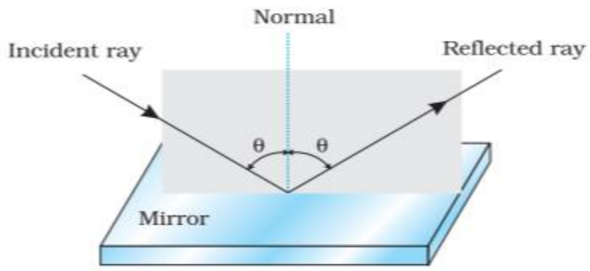
Type	Wavelength range	Production	Detection
Radio	> 0.1 m	Rapid acceleration and decelerations of electrons in aerials	Receiver's aerials
Microwave	0.1m to 1 mm	Klystron valve or magnetron valve	Point contact diodes
Infra-red	1mm to 700 nm	Vibration of atoms and molecules	Thermopiles Bolometer, Infrared photographic film
Light	700 nm to 400 nm	Electrons in atoms emit light when they move from one energy level to a lower energy level	The eye Photocells Photographic film
Ultraviolet	400 nm to 1nm	Inner shell electrons in atoms moving from one energy level to a lower level	Photocells Photographic film
X-rays	1nm to 10^{-3} nm	X-ray tubes or inner shell electrons	Photographic film Geiger tubes Ionisation chamber
Gamma rays	$<10^{-3}$ nm	Radioactive decay of the nucleus	-do-

Chapter 9
Ray Optics and Optical Instruments

9.1 Introduction

Light is an electromagnetic wave. Light travels with a speed of 3 x10^8 m/s in vacuum. The speed of light in vacuum is the highest speed attainable in nature. A light wave can be considered to travel from one point to another, along a straight line joining them. The path is called a ray of light, and a bundle of such rays constitutes a beam of light.

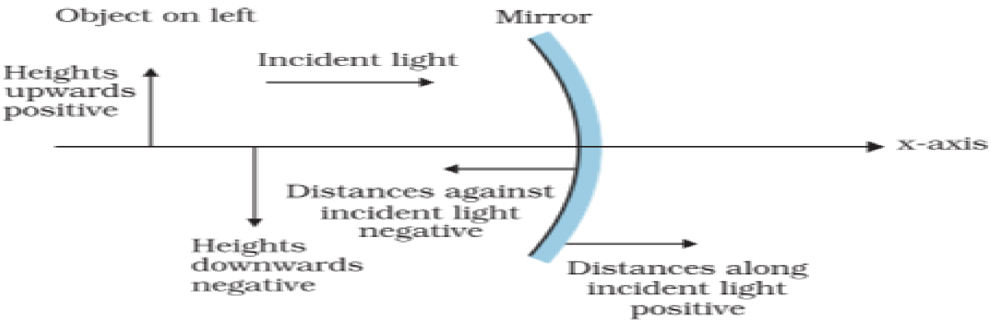
9.2 Reflection of Light by Spherical Mirrors



Laws of Reflection

1) The incident ray, reflected ray and the normal to the reflecting surface at the point of incidence lie in the same plane.
2)The angle of incidence is equal to the angle of reflection(i=r).

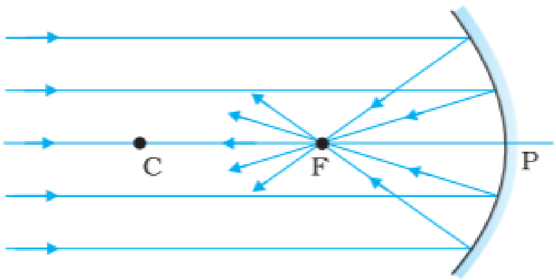
Sign Convention



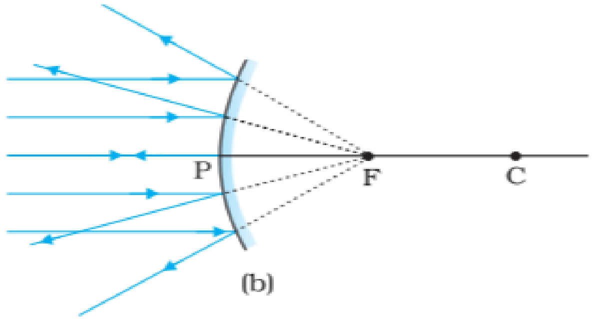
We follow the **Cartesian sign convention** to measure distances. According to this convention,

1) All distances are measured from the pole of the mirror or the optical centre of the lens.
2) The distances measured in the same direction as the incident light are taken as positive and those measured in the direction opposite to the direction of incident light are taken as negative.
3) The heights measured upwards with respect to principal axis of the mirror/ lens are taken as positive . The heights measured downwards are taken as negative.

Focal Length of Spherical Mirrors



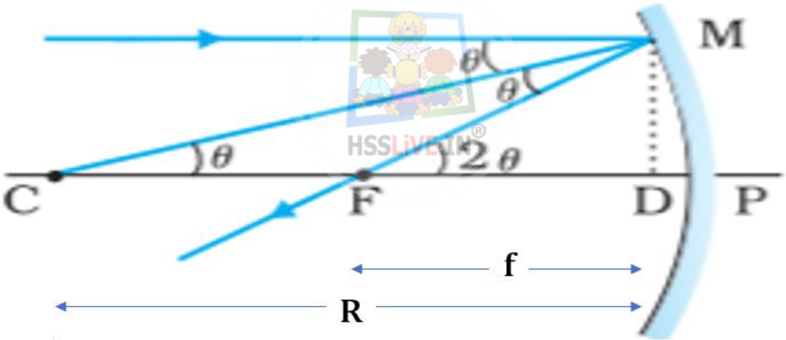
When a parallel beam of light is incident on a concave mirror, the reflected rays converge at a point F on its principal axis . The point F is called the principal focus of the concave mirror.



(b)

When a parallel beam of light is incident on a convex mirror, the reflected rays appear to diverge from a point F on its principal axis . The point F is called the principal focus of the convex mirror.

Relation between Focal Length and Radius of Curvature



Let f be the focal length and R be the radius of curvature of lens

From figure , $\tan\theta = \frac{MD}{R}$, $\theta = \frac{MD}{R}$ ------(1) (For small value of θ, $\tan\theta \approx \theta$)

$\tan 2\theta = \frac{MD}{f}$, $2\theta = \frac{MD}{f}$ -----(2) $(\tan 2\theta \approx 2\theta)$

Substituting θ from eq(1) in eq(2) $2\frac{MD}{R} = \frac{MD}{f}$

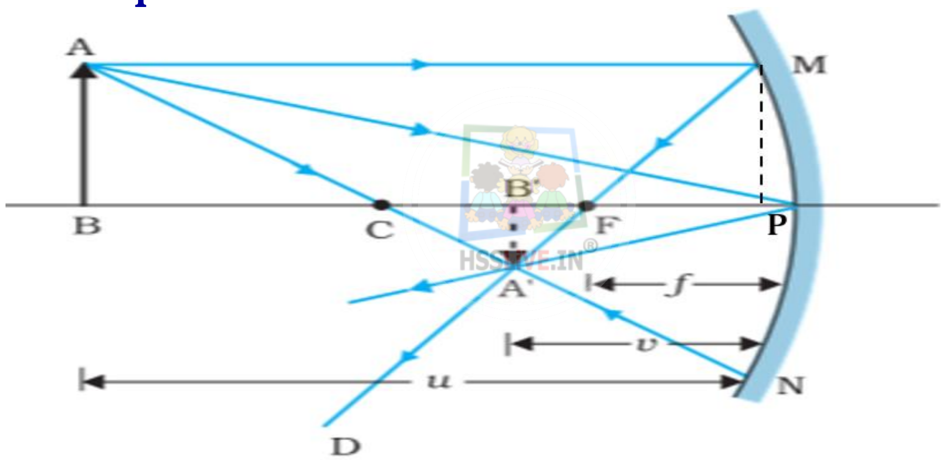
$$\frac{2}{R} = \frac{1}{f}$$

$$\mathbf{f = \frac{R}{2}}$$

Some important points to consider while image formation in spherical mirrors :-

- If rays emanating from a point actually meet at another point after reflection , that point is called the image of the first point.
- The image is real if the rays actually converge to the point.
- The image is virtual if the rays do not actually meet but appear to diverge from the point when produced backwards.
- The ray parallel to the principal axis, goes through the focus of the mirror after reflection.
- The ray passing through the centre of curvature of a concave mirror ,retraces the path.
- The ray passing through the focus of the concave mirror , after reflection ,goes parallel to the principal axis.
- The ray incident at any angle at the pole. The reflected ray follows laws of reflection.

The Mirror Equation



The two right-angled triangles A′B′F and MPF are similar

$$\frac{B'A'}{PM} = \frac{B'F}{FP}$$

$$\frac{B'A'}{BA} = \frac{B'F}{FP} \text{ ------------(1)} \quad \text{(since PM = AB)}$$

The right angled triangles A′B′P and ABP are also similar.

$$\frac{B'A'}{BA} = \frac{B'P}{BP} \text{ ------------(2)}$$

From eqns(1) and (2)

$$\frac{B'F}{FP} = \frac{B'P}{BP}$$

$B'P = v$, $BP = u$, $B'F = v\text{-}f$, $FP = f$,

$$\frac{v-f}{f} = \frac{v}{u}$$

Applying sign convention ,

$$\frac{-v- -f}{-f} = \frac{-v}{-u}$$

$$\frac{v-f}{f} = \frac{v}{u}$$

$$\frac{v}{f} - 1 = \frac{v}{u}$$

Dividing by v

$$\frac{1}{f} - \frac{1}{v} = \frac{1}{u}$$

$$\mathbf{\frac{1}{u} + \frac{1}{v} = \frac{1}{f}}$$

This relation is known as the mirror equation.

Linear Magnification (m)

Linear magnification (m) is the ratio of the height of the image (h′) to the height of the object (h).

$$\mathbf{m = \frac{h'}{h}}$$

From above figure

$$\frac{B'A'}{BA} = \frac{B'P}{BP}$$

With the sign convention,

$$\frac{-h'}{h} = \frac{-v}{-u}$$

$$\frac{h'}{h} = \frac{-v}{u}$$

$$\mathbf{m = \frac{h'}{h} = \frac{-v}{u}}$$

Example

An object is placed at (i) 10 cm, (ii) 5 cm in front of a concave mirror of radius of curvature 15 cm. Find the position, nature, and magnification of the image in each case.

Example 9.3 An object is placed at (i) 10 cm, (ii) 5 cm in front of a concave mirror of radius of curvature 15 cm. Find the position, nature, and magnification of the image in each case.

Solution

The focal length $f = -15/2$ cm $= -7.5$ cm

(i) The object distance $u = -10$ cm. Then Eq. (9.7) gives

$$\frac{1}{v} + \frac{1}{-10} = \frac{1}{-7.5}$$

or $v = \frac{10 \times 7.5}{-2.5} = -30$ cm

The image is 30 cm from the mirror on the same side as the object.

Also, magnification $m = -\frac{v}{u} = -\frac{(-30)}{(-10)} = -3$

The image is magnified, real and inverted.

(ii) The object distance $u = -5$ cm. Then from Eq. (9.7),

$$\frac{1}{v}+\frac{1}{-5}=\frac{1}{-7.5}$$

or $$v=\frac{5\times7.5}{(7.5-5)}=15 \text{ cm}$$

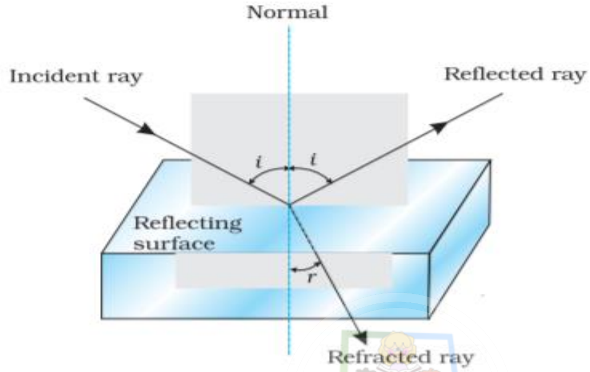
This image is formed at 15 cm behind the mirror. It is a virtual image.

Magnification $m = -\frac{v}{u}=-\frac{15}{(-5)}=3$

The image is magnified, virtual and erect.

9.3 Refraction

The direction of propagation of an obliquely incident ray of light that enters the other medium, changes at the interface of the two media. This phenomenon is called refraction of light.



Laws of Refraction

i)The incident ray, the refracted ray and the normal to the interface at the point of incidence, all lie in the same plane.

ii)The ratio of the sine of the angle of incidence to the sine of angle of refraction is constant

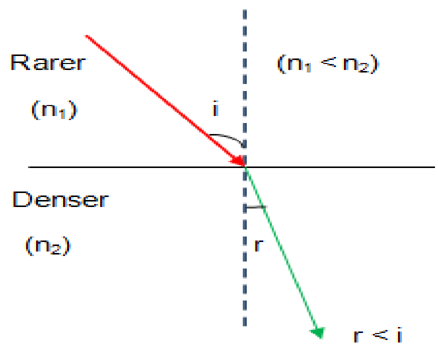
$$\frac{\sin i}{\sin r} = n_{21}$$

where n_{21} is a constant, called the refractive index of the second medium with respect to the first medium.

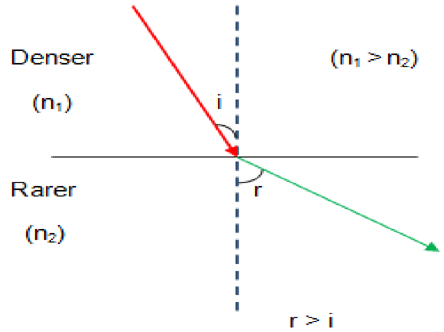
$$n_{21} = \frac{n_2}{n_1}$$

This equation is known as Snell's law of refraction.

- When a ray travels from rarer to denser medium, the refracted ray bends towards the normal.
  i.e., if $n_{21} > 1$, i.e., $n_2 > n_1$, $r < i$

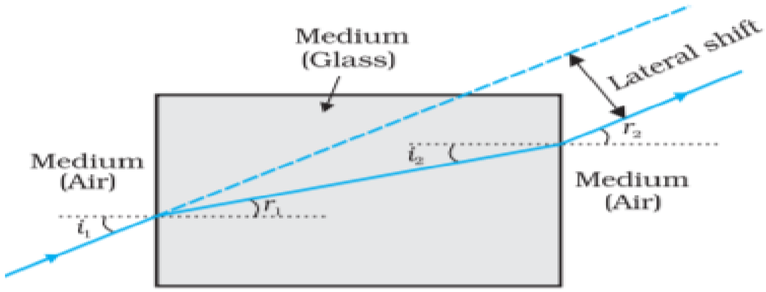


- When a ray travels from denser to rarer medium, the refracted ray bends away from the normal.
  i.e., if $n_{21} < 1$, i.e., $n_2 < n_1$, $r > i$



Some Elementary Results Based on The Laws of Refraction

(i)Lateral Shift



For a rectangular slab, refraction takes place at two interfaces (air-glass and glass-air). The emergent ray is parallel to the incident ray—there is no refraction and reflection of light, but it does suffer lateral displacement/ shift with respect to the incident ray.

(ii)Apparent depth

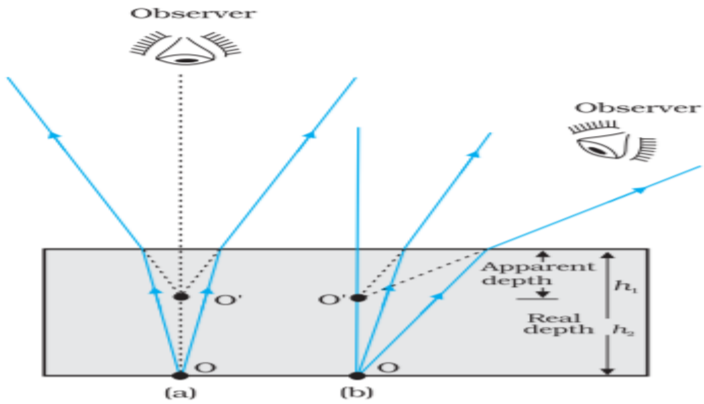


FIGURE 9.10 Apparent depth for (a) normal, and (b) oblique viewing.

The bottom of a tank filled with water appears to be raised .

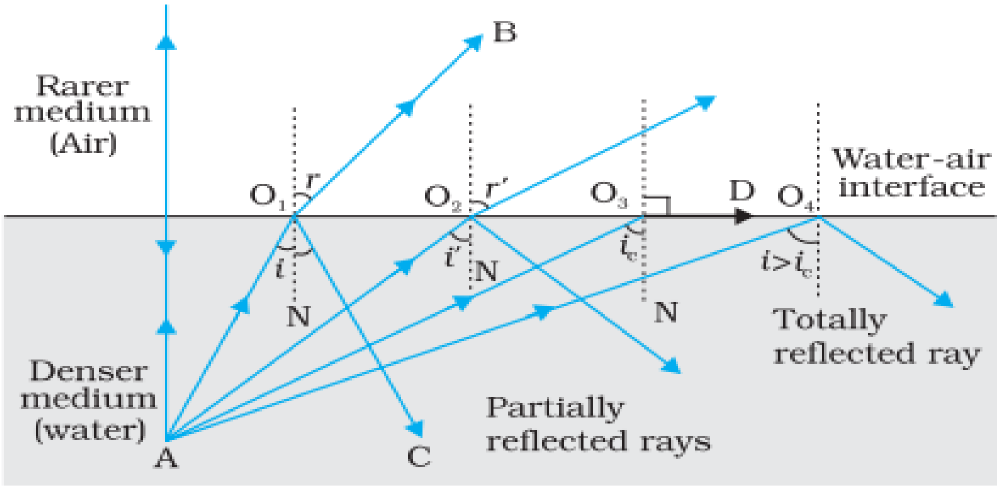
For viewing near the normal direction ,

$$\text{Apparent depth} = \frac{\textbf{real depth}}{\textbf{Refractive Index}}$$

$$h_1 = \frac{h_2}{n}$$

9.4 Total Internal Reflection

When a ray of light enters from a denser medium to a rarer medium, if the angle of incidence is greater than the critical angle (i_c) for the given pair of media, the incident ray is totally reflected. This is called total internal reflection.



Explanation:-

When a ray of light enters from a denser medium to a rarer medium, it bends away from the normal.

- The incident ray AO_1 is partially reflected (O_1C) and partially refracted(O_1B) .
- As the angle of incidence increases, the angle of refraction also increases.(for ray AO_2)
- When the angle of incidence becomes equal to the critical angle(i_c) for the given pair of media, the angle of refraction becomes 90º.(for rayAO_3)
- If the angle of incidence is increased further (ray AO_4), refraction is not possible, and the incident ray is totally reflected.

Conditions for Total Internal Reflection

- The ray of light should enter from a denser medium to a rarer medium.
- The angle of incidence should be greater than the critical angle (i_c) for the given pair of media .

Critical Angle

The angle of incidence in the denser medium, corresponding to an angle of refraction 90º, is called the critical angle (i c) for the given pair of media.

Let the second medium be air.

By Snell's law $\frac{\sin i}{\sin r} = \frac{n_2}{n_1} = \frac{1}{n}$

When $i = i_c$, $r = 90$

$$\frac{\sin i_c}{\sin 90} = \frac{1}{n}$$

$$\sin i_c = \frac{1}{n}$$

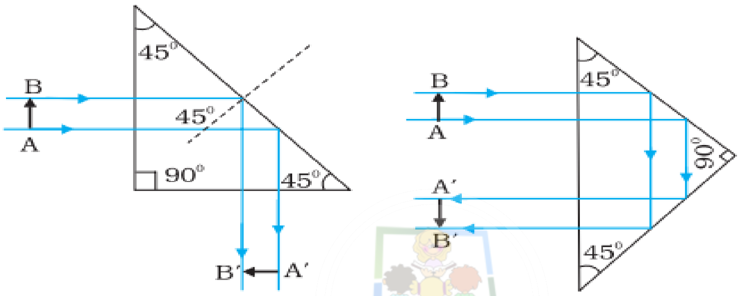
$$\mathbf{n = \frac{1}{\sin i_c}}$$

This is the relation connecting refractive index and critical angle.

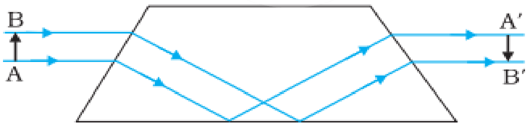
Total internal reflection in nature and its technological applications

(i)Prism:

Prisms designed to bend light by 90º or by 180º make use of total internal reflection .In these cases, the critical angle i_c for the material of the prism must be less than 45º.

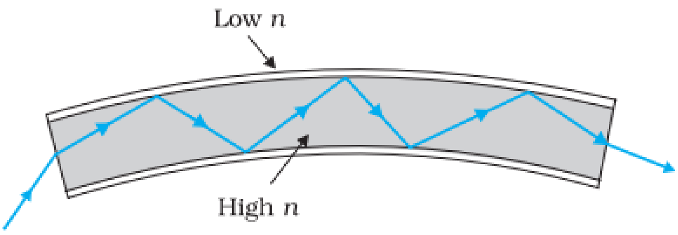


Such a prism is also used to invert images without changing their size.



(ii) Optical fibres:

Now-a-days optical fibres are extensively used for transmitting audio and video signals through long distances. Optical fibres make use of the phenomenon of total internal reflection. Optical fibres are fabricated with high quality composite glass/quartz fibres.

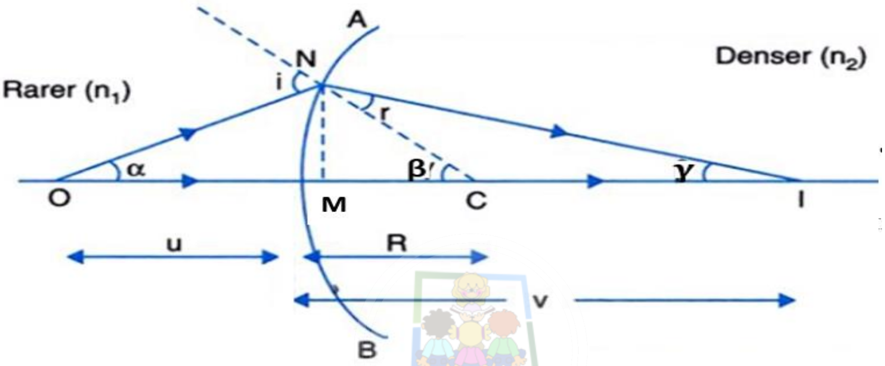


Each fibre consists of a core and cladding. The refractive index of the material of the core is higher than that of the cladding. When a signal in the form of light is directed at one end of the fibre at a suitable angle, it undergoes repeated total internal reflections along the length of the fibre and finally comes out at the other end . Since light undergoes total internal reflection at each stage, there is no appreciable loss in the intensity of the light signal.

A bundle of optical fibres can be put to several uses. Optical fibres are extensively used for transmitting and receiving electrical signals which are converted to light by suitable transducers. Obviously, optical fibres can also be used for transmission of optical signals. For example, these are used as a 'light pipe' to facilitate visual examination of internal organs like esophagus, stomach and intestines. Optical fibres are used in decorative lamps.

9.5 Refraction at Spherical Surfaces and by Lenses

Refraction at a Spherical Surface



$$\tan\alpha = \frac{MN}{OM}\ ,\qquad \alpha = \frac{MN}{OM}$$

$$\tan\beta = \frac{MN}{MC}\ ,\qquad \beta = \frac{MN}{MC}$$

$$\tan\gamma = \frac{MN}{MI}\ ,\qquad \gamma = \frac{MN}{MI}$$

From ΔNOC, $\quad i = \alpha + \beta$--------------(1)

From ΔNIC, $\quad \beta = r + \gamma$

$r = \beta - \gamma$---------------(2)

From Snell's law

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1}$$

$$n_1 \sin i = n_2 \sin r$$

For small values of i and r

$$n_1\, i = n_2 r$$

Substituting fron eqn (1) and (2)

$$n_1\,(\alpha + \beta) = n_2\,(\beta - \gamma)$$

$$n_1\left(\frac{MN}{OM} + \frac{MN}{MC}\right) = n_2\left(\frac{MN}{MC} - \frac{MN}{MI}\right)$$

$$\frac{n_1}{OM} + \frac{n_1}{MC} = \frac{n_2}{MC} - \frac{n_2}{MI}$$

$$\frac{n_1}{OM} + \frac{n_2}{MI} = \frac{n_2}{MC} - \frac{n_1}{MC}$$

$$\frac{n_1}{OM} + \frac{n_2}{MI} = \frac{n_2 - n_1}{MC}$$

Applying the Cartesian sign convention,

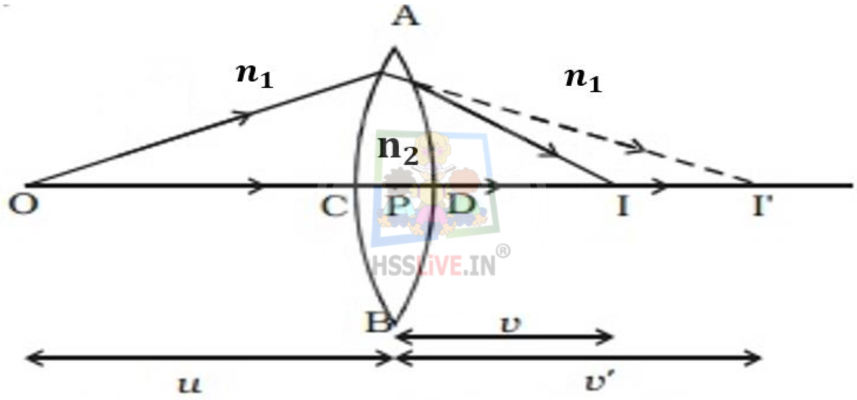
$$OM = -u, \ MI = +v, \ MC = +R$$

$$\frac{n_1}{-u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$$

$$\mathbf{\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}}$$

This equation gives the relation between object and image distance in terms of refractive index of the medium and the radius of curvature of the curved spherical surface.

Refraction by a Lens -Lens Maker's Formula



For the first interface ACB of radius of curvature R_1

$$\frac{n_2}{v'} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} \text{ ----------(1)}$$

For the first interface ADB of radius of curvature R_2

$$\frac{n_1}{v} - \frac{n_2}{v'} = \frac{n_1 - n_2}{R_2} \text{ -------------(2)}$$

Eqns (1) + (2)

$$\frac{n_2}{v'} - \frac{n_1}{u} + \frac{n_1}{v} - \frac{n_2}{v'} = \frac{n_2 - n_1}{R_1} + \frac{n_1 - n_2}{R_2}$$

$$\frac{n_1}{v} - \frac{n_1}{u} = (n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

Dividing throughout by n_1

$$\frac{1}{v} - \frac{1}{u} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{v} - \frac{1}{u} = (n_{21} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2} \right) \text{------------(3)}$$

When u=∞ (infinity) , v=f

$$\frac{1}{f} - \frac{1}{\infty} = (n_{21} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = (n_{21} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

If the lens is placed in air

$$\frac{1}{f} = (n - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \text{----------}(4)$$

This Equation is known as the lens maker's formula.

From eq (3) and (4)

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

This equation is called thin lens formula

Image Formation by a Convex Lens

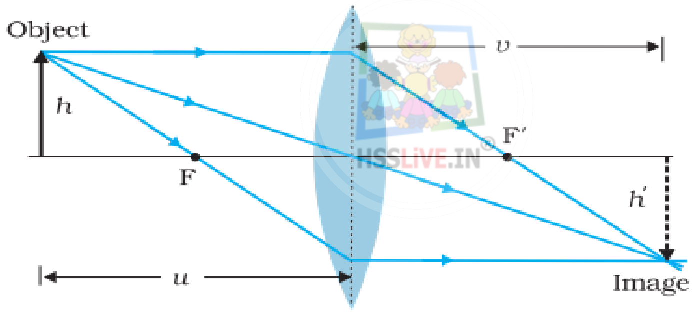
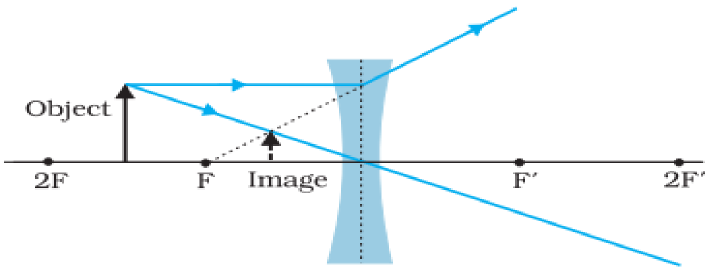


Image Formation by a Concave Lens



Magnification (m)

Magnification produced by a lens is defined as the ratio of the size of the image to that of the object.

$$m = \frac{h'}{h} = \frac{v}{u}$$

For erect (and virtual) image formed by a convex or concave lens, m is positive, while for an inverted (and real) image, m is negative.

Power of a lens

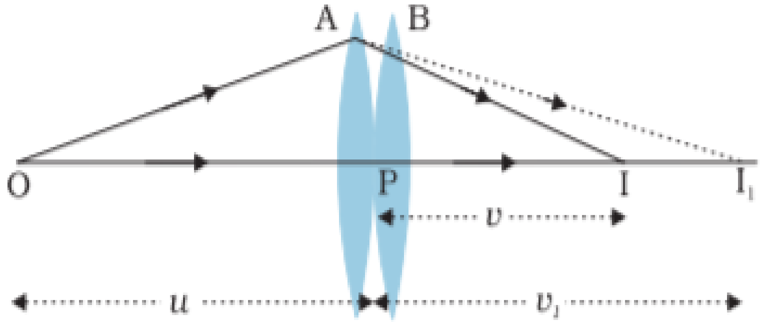
Power of a lens is the reciprocal of focal length expressed in metre

$$\mathbf{p = \frac{1}{f}}$$

The SI unit for power of a lens is **dioptre D**. $1D = 1m^{-1}$

Power of a lens is positive for a converging lens and negative for a diverging lens.

Combination of Thin Lenses in Contact



For the image formed by the first lens A,

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1} \text{ -----------(1)}$$

For the image formed by the second lens B,

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2} \text{ -----------(2)}$$

Eqn (1) +(2)

$$\frac{1}{v_1} - \frac{1}{u} + \frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2} \text{ -----------(3)}$$

If the two lens-system is regarded as equivalent to a single lens of focal length f, we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \text{ ---------------(4)}$$

From eqn (3) and (4)

$$\mathbf{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}}$$

For a number of thin lenses in contact

$$\mathbf{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_2} + \ldots\ldots}$$

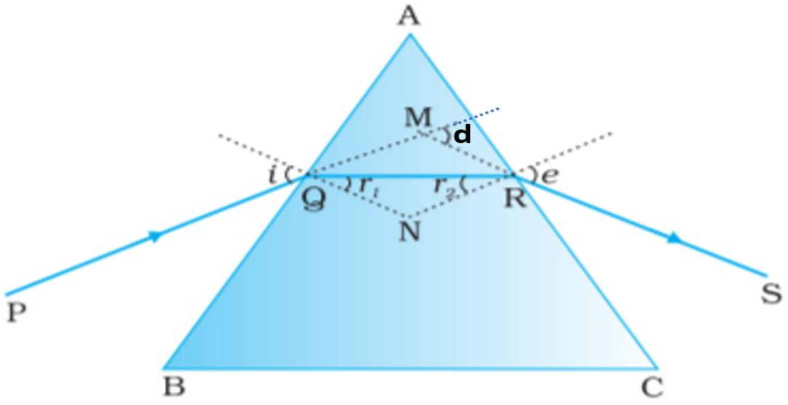
Total power,P of the combination is,

$$\mathbf{P = P_1 + P_2 + P_3 + \ldots\ldots\ldots}$$

Total magnification, m of the combination is,

$$\mathbf{m = m_1\ m_2\ m_3 \ldots\ldots\ldots\ldots}$$

9.6 Refraction Through a Prism



In the quadrilateral AQNR,

$$\angle A + \angle QNR = 180^\circ$$

From the triangle QNR,

$$r_1 + r_2 + \angle QNR = 180^\circ$$

Comparing these two equations, we get

$$\mathbf{r_1 + r_2 = A} \text{ ------------------(1)}$$

The total deviation δ is the sum of deviations at the two faces,

$$d = (i - r_1) + (e - r_2)$$

$$d = i + e - (r_1 + r_2)$$

$$\mathbf{d = i + e - A} \text{ ------------------(2)}$$

Thus, the angle of deviation depends on the angle of incidence

At the minimum deviation

$$d = D, \quad i = e, \quad r_1 = r_2 = r$$

From eqa (1)

$$2r = A$$

$$\mathbf{r = \frac{A}{2}} \text{ -----------------(3)}$$

From eqa (2)

$$d = 2i - A$$

$$2i = A + D$$

$$\mathbf{i = \frac{A + D}{2}} \text{ -------------(4)}$$

By Snell's law the refractive index of prism

$$n = \frac{\sin i}{\sin r}$$

$$\mathbf{n = \frac{\sin \frac{A + D}{2}}{\sin \frac{A}{2}}}$$

For a small angle prism, i.e., a thin prism, D is also very small, and we get

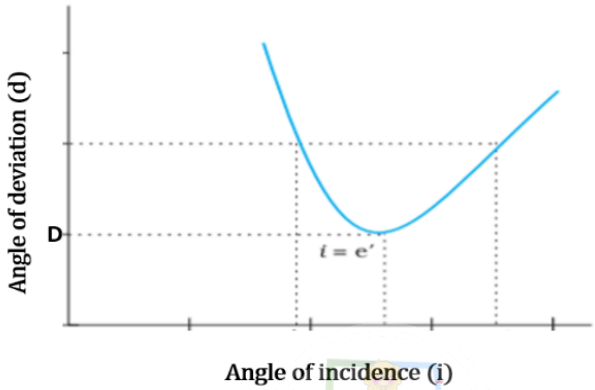
$$n = \frac{\frac{A+D}{2}}{\frac{A}{2}}$$

$$n = 1 + \frac{D}{A}$$

$$D = (n-1)A$$

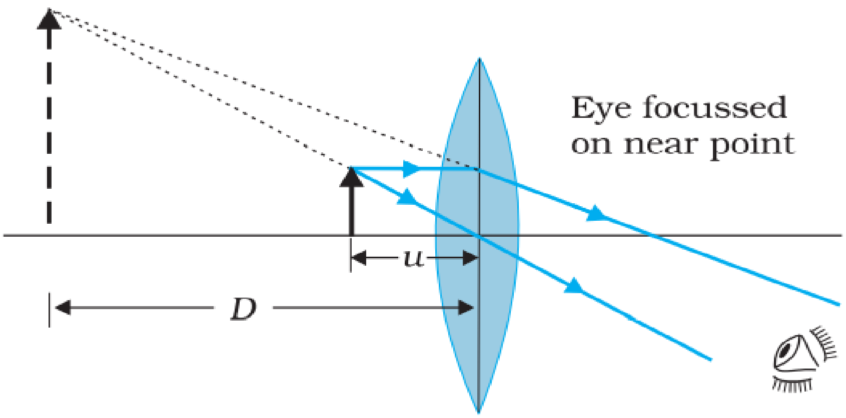
It implies that, thin prisms do not deviate light much.

Graph between the angle of deviation and angle of incidence - i-d curve



9.7 Optical Instruments

The microscope



A simple microscope or magnifier is a converging lens of small focal length.

If the object is held between the focus and optical centre of lens, an erect, magnified and virtual image is formed at near point 25 cm or more. If the object is held at focus, the image will be formed at infinity.

The linear magnification m(when image is at D)

$$m = \frac{v}{u} = v\left(\frac{1}{v} - \frac{1}{f}\right)$$

$$m = 1 - \frac{v}{f}$$

Applying sign convention , v=-D

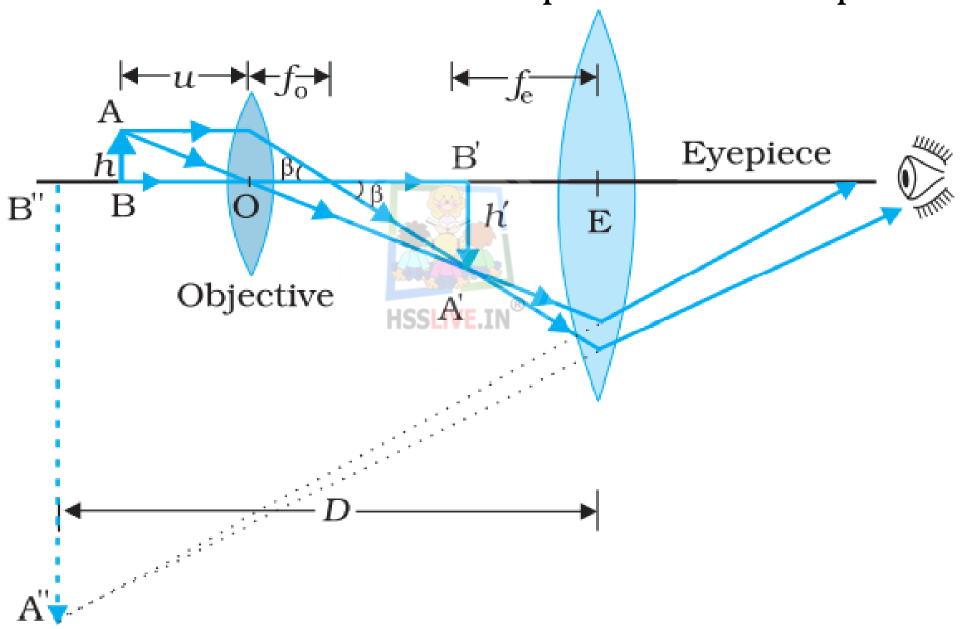
$$\mathbf{m = 1 + \frac{D}{f}}$$

The linear magnification m(when image is at infinity)

$$\mathbf{m = \frac{D}{f}}$$

Compound Microscope

A simple microscope has a limited maximum magnification (≤ 9). For much larger magnifications, one uses two lenses, one compounding the effect of the other. This is known as a compound microscope.



The lens near the object, called the objective. It forms a real, inverted, magnified image of the object. This serves as the object for the second lens, the eyepiece, at the focal plane (or little closer) of the eyepiece . The eyepiece functions like a simple microscope or magnifier and produces an enlarged and virtual image at infinity, or at the near point. Clearly, the final image is inverted with respect to the original object.

Magnifiction, $m = m_o \times m_e$ -----------(1)

$$m_0 = \frac{h'}{h} = \frac{L}{f_0}$$

When the final image is formed at infinity,

$$m_e = \frac{D}{f_e}$$

Substituting in eqn(1)

$$\mathbf{m = \frac{L}{f_0} \times \frac{D}{f_e}}$$

D= near point=25cm
f_0 = focal length of objective
f_e = focal length of eyepiece
L= The tube length of the compound microscope
(The distance between the second focal point of the objective and the first focal point of the eyepiece is called the tube length.)

Clearly, to achieve a large magnification of a small object , the objective and eyepiece should have small focal lengths.

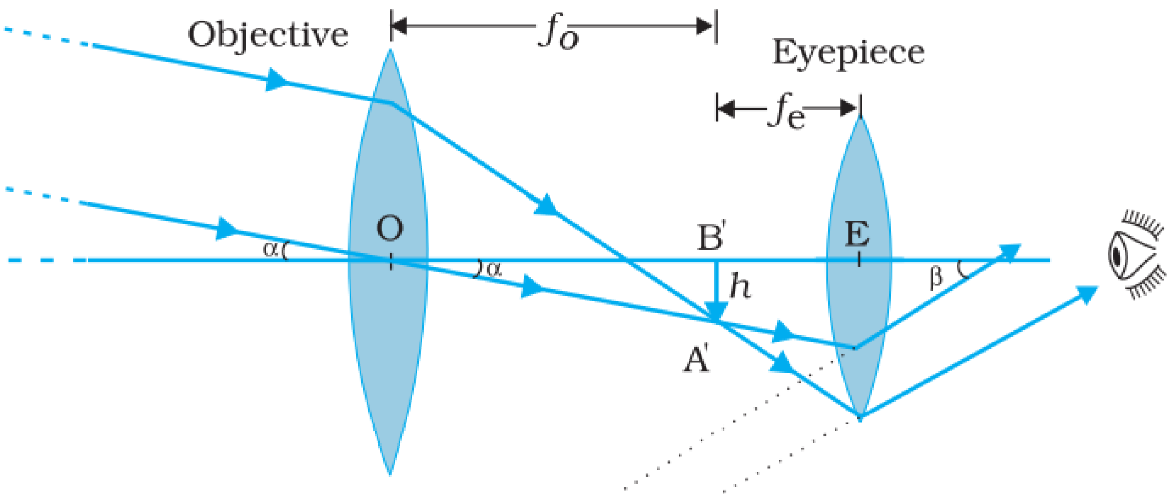
When the final image is formed at the near point,

$$m_e = 1 + \frac{D}{f_e}$$

$$\mathbf{m = \frac{L}{f_0}\left(1 + \frac{D}{f_e}\right)}$$

Telescope

Refracting Telescope



The telescope is used to provide angular magnification of distant objects . It also has an objective and an eyepiece. The objective has a large focal length and a much larger aperture than the eyepiece. Light from a distant object enters the objective and a real image is formed in the tube at its second focal point. The eyepiece magnifies this image producing a final inverted image.

The magnifying power m is the ratio of the angle β subtended at the eye by the final image to the angle α which the object subtends at the lens or the eye.

$$m \approx \frac{\beta}{\alpha} \approx \frac{h}{f_e} \cdot \frac{f_o}{h} = \frac{f_0}{f_e}$$

$$\mathbf{m = \frac{f_0}{f_e}}$$

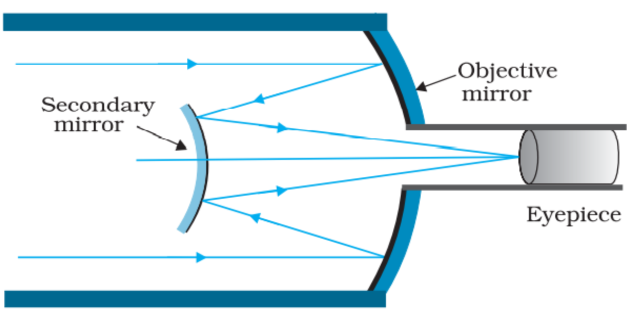
Terrestrial telescopes have, in addition, a pair of inverting lenses to make the final image erect. Refracting telescopes can be used both for terrestrial and astronomical observations.

For high resolving power, optical telescopes should have objective of large diameter. Such big lenses are very heavy and it is rather difficult and expensive to make such large sized lenses which form images that are free from any kind of chromatic aberration and distortions.So modern telescopes use a concave mirror rather than a lens for the objective.(Reflecting Telescope)

Reflecting Telescope

Telescopes with mirror objectives are called reflecting telescopes. They have several advantages.

- First, there is no chromatic aberration in a mirror.
- Second, if a parabolic reflecting surface is chosen, spherical aberration is also removed. Mechanical support is much less of a problem since a mirror weighs much less than a lens of equivalent optical quality.



A reflecting telescope is that the objective mirror focusses light inside the telescope tube. A convex secondary mirror focusses the incident light, which passes through a hole in the objective primary mirror. It has the advantages of a large focal length in a short telescope.

Chapter 10
Wave Optics

10.1 Introduction

In 1678, the Dutch physicist Christiaan Huygens put forward the wave theory of light .The wave model could satisfactorily explain the phenomena of reflection , refraction, interference, diffraction and polarisation .

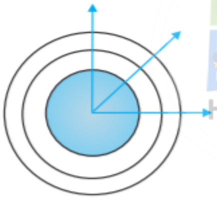
Wavefront

A locus of points, which oscillate in phase is called a wavefront; thus a wavefront is defined as a surface of constant phase.

The speed with which the wavefront moves outwards from the source is called the speed of the wave. The energy of the wave travels in a direction perpendicular to the wavefront.

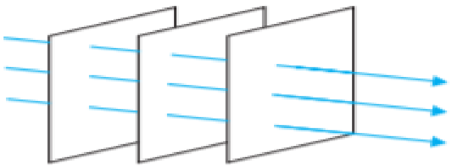
Spherical Wavefront

For a point source emitting waves uniformly in all directions, the wavefronts will be spherical .



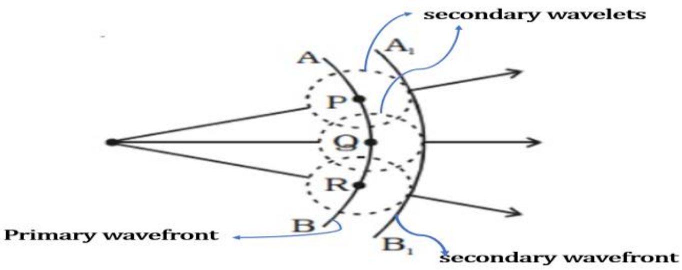
Plane Wavefront

At large distance from a source, a small portion of the sphere can be considered as a plane and is known as a plane wavefront.



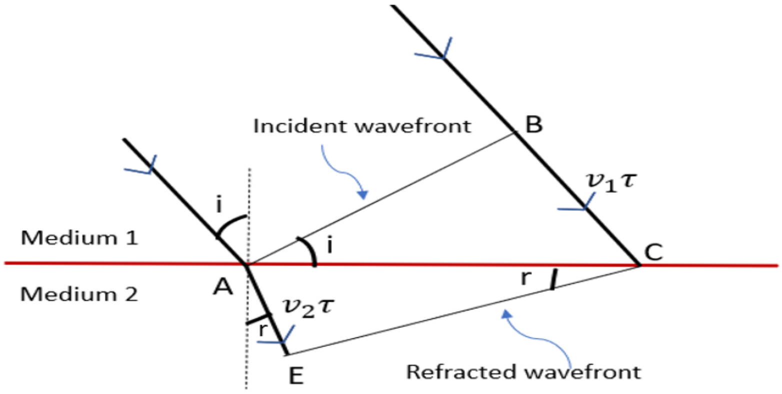
10.2 Huygens Principle

According to Huygens principle, each point of the wavefront acts as a source secondary wavelets and if we draw a common tangent to all these secondary wavelets, we obtain the new position of the wavefront at a later time.



10.3 Refraction and Reflection of Plane Waves Using Huygens Principle

Refraction of a Plane Wave



AB is the incident wavefront and EC is the refracted wavefront. Let v_1 and v_2 be the velocity of wave in medium 1 and 2 respectively.

$$\sin i = \frac{BC}{AC} = \frac{v_1\tau}{AC} \text{ ----------(1)}$$

$$\sin r = \frac{AE}{AC} = \frac{v_2\tau}{AC} \text{ ----------(2)}$$

$$\text{eqn}\frac{(1)}{(2)} \qquad \frac{\sin i}{\sin r} = \frac{v_1}{v_2} \text{ --------------(3)}$$

Refractive index of first medium $n_1 = \frac{c}{v_1}$

Refractive index of second medium $n_2 = \frac{c}{v_2}$

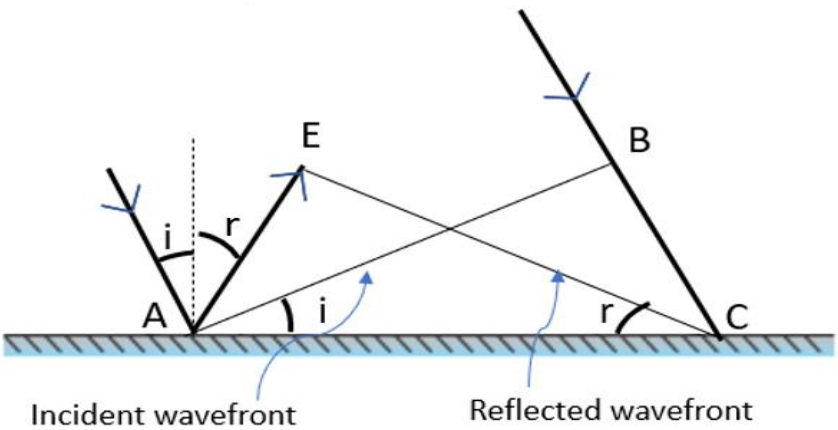
$$\frac{n_2}{n_1} = \frac{v_1}{v_2}$$

Substituting in eqn (3)

$$\boldsymbol{\frac{\sin i}{\sin r} = \frac{n_2}{n_1}}$$

This is the Snell's law of refraction.

Reflection of a plane wave by a plane surface



AB is the incident wavefront and EC is the reflected wavefront.

Let v be the velocity of the wave ,then

$$AE = BC = v\tau$$

$$AC = AC \text{ (common side)}$$

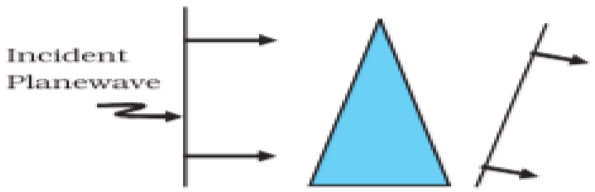
So the triangles EAC and BAC are congruent . Therefore

i=r

Angle of incidence=Angle of reflection

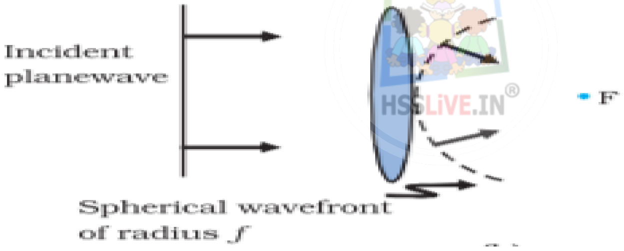
This is the law of reflection.

Refraction of a plane wave by a thin prism



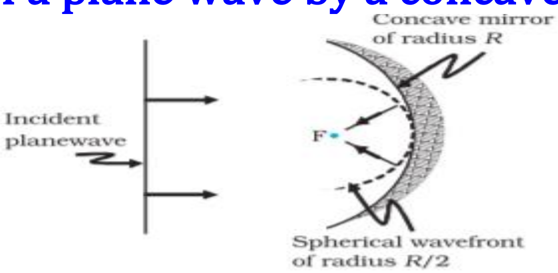
The emerging wavefront is also plane wavefront,but tilted.

Refraction of a plane wave by a convex lens



The emerging wavefront is spherical and converges to the point F which is known as the focus.

Reflection of a plane wave by a concave mirror



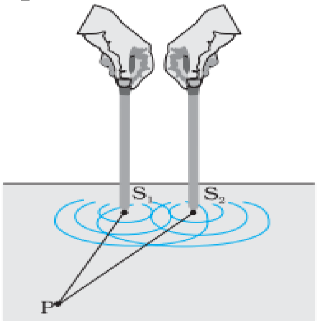
The reflected wavefront is a spherical converging to the focal point F.

Superposition Principle

According to superposition principle , the resultant displacement produced by a number of waves in a medium is the vector sum of the displacements produced by each of the waves.

10.4 Coherent sources

Two sources are said to be coherent if they emit light waves of same frequency and same phase or constant phase difference.



Two needles oscillating in phase in water represent two coherent sources.

Interference

Interference is the phenomenon in which two waves superpose to form a resultant wave of greater or lower amplitude.

The interference can be constructive or destructive.

Coherent and Incoherent Addition of Waves

Coherent Addition of Waves

Consider two coherent sources S_1 and S_2 . The displacement produced by the source S_1 and S_2 at the point P has a phase difference ϕ.

$$y_1 = a \cos \omega t$$

$$y_2 = a \cos (\omega t + \phi)$$

The resultant displacement

$$y = y_1 + y_2 = a [\cos \omega t + \cos (\omega t + \phi)]$$

$$= 2 a \cos (\phi /2) \cos (\omega t + \phi/2)$$

The amplitude of the resultant displacement is $2a \cos (\phi/2)$

$$\text{Intensity} \propto (\text{amplitude})^2$$

$$I \propto (2a \cos (\phi /2))^2$$

$$I \propto 4a^2 \cos^2 (\phi /2)$$

$$I \propto 4I_0 \cos^2 (\phi /2)$$

If $\phi = 0, \pm 2\pi, \pm 4\pi, \ldots$

We will have constructive interference leading to maximum intensity.

If $\phi = \pm \pi, \pm 3\pi, \pm 5\pi \ldots$

We will have destructive interference leading to zero intensity.

As the two sources are coherent, the phase difference ϕ at any point will not change with time and we will have a stable interference pattern; i.e., the positions of maxima and minima will not change with time.

Incoherent Addition of Waves

If the two sources are incoherent then the phase difference ϕ between the two vibrating sources changes rapidly with time .The positions of maxima and minima will also vary rapidly with time and we will see a "time-averaged" intensity distribution.

$$\langle I \rangle = 4I_0 \langle \cos^2 (\phi /2) \rangle$$

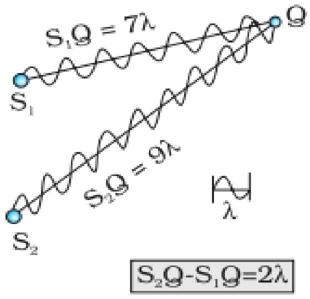
$$= 4I_0 \times \frac{1}{2}$$

$$= 2I_0$$

i.e. ,the intensities just add up.

This is indeed what happens when two separate light sources illuminate a wall.

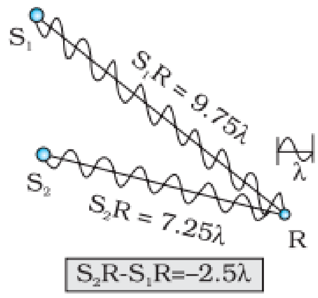
Condition for constructive interference



If the path difference at a point is an integral multiple of λ, there will be constructive interference and a bright fringe is formed at that point

$$S_2P - S_1P = n\lambda \qquad \text{where } (n = 0, 1, 2, 3, \ldots)$$

Condition for destructive interference



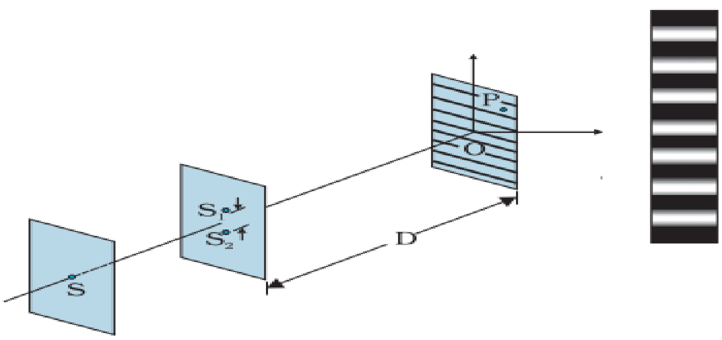
If the path difference at a point is an odd integral multiple of λ/2 , there will be destructive interference and a dark fringe is formed at that point

$$S_2P - S_1P = = \left(n+\frac{1}{2}\right)\lambda \qquad \text{where } (n = 0, 1, 2, 3, \ldots)$$

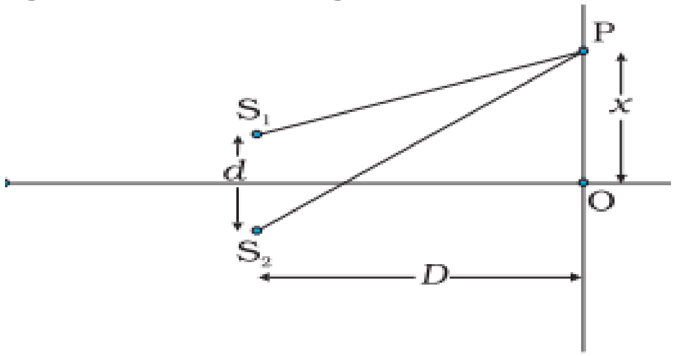
Two sodium lamps illuminating two pinholes cannot produce interference fringes. Why?

If we use two sodium lamps illuminating two pinholes we will not observe any interference fringes. This is because the light wave emitted from an ordinary source (like a sodium lamp) undergoes abrupt phase changes .Thus the light waves coming out from two independent sources of light will not have any fixed phase relationship and would be incoherent and cannot produce interference pattern.

10.5 Interference of Light Waves and Young's Experiment



The British physicist Thomas Young made two pinholes S_1 and S_2 (very close to each other) on an opaque screen. These were illuminated by another pinholes which is illuminated by a bright source. Light waves spread out from S and fall on both S_1 and S_2. S_1 and S_2 then behave like two coherent sources because light waves coming out from S_1 and S_2 are derived from the same original source and interference pattern with altermate bright and dark fringes is formed on the screen.

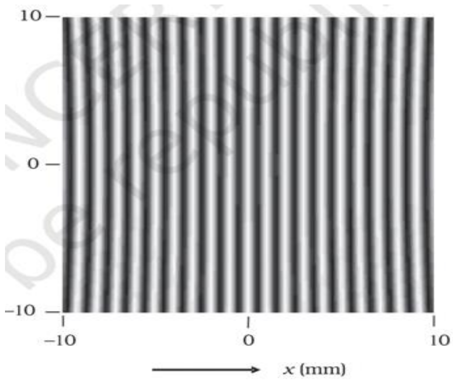


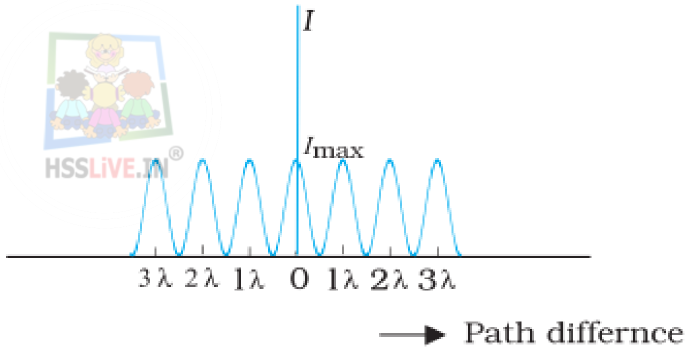
For bright band path difference , $\frac{xd}{D} = n\lambda$

$$x_n = \frac{n\lambda D}{d} \quad , \quad n=0, \pm1, \pm2,....$$

For dark band path difference, $\frac{xd}{D} = (n+\frac{1}{2})\,\lambda$

$$x_n = (\mathbf{n} + \tfrac{\mathbf{1}}{\mathbf{2}})\,\frac{\lambda D}{d} \quad , \quad n=0, \pm1, \pm2,....$$





Dark and bright bands appear on the screen are called fringes. Dark and bright fringes are equally spaced.

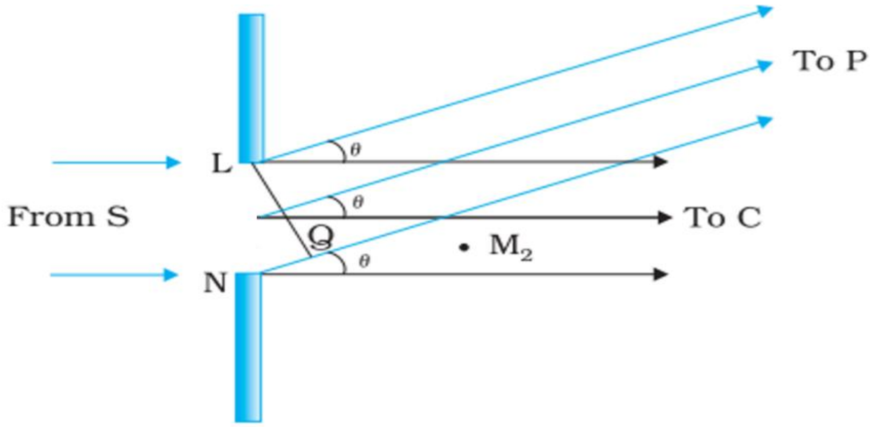
10.6 Diffraction

Diffraction is the phenomenon of bending of light around the corners of an obstacle , into the region of geometrical shadow of the obstacle.

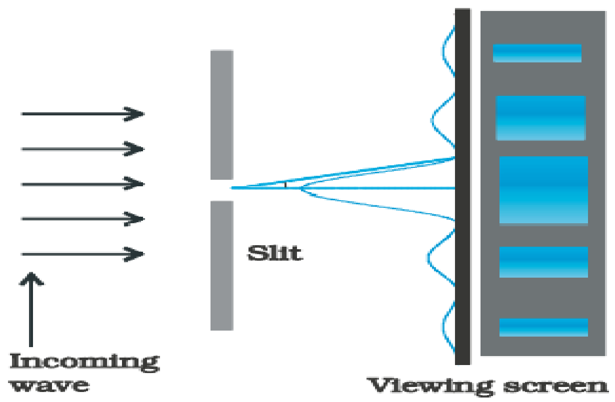
If we look clearly at the shadow cast by an opaque object, close to the region of geometrical shadow, there are alternate dark and bright regions just like in interference. This happens due to the phenomenon of diffraction. Diffraction is a general characteristic exhibited by all types of waves, be it sound waves, light waves, water waves or matter waves.

- Since the wavelength of light is much smaller than the dimensions of most obstacles; we do not encounter diffraction effects of light in everyday observations.
- The resolving power of our eye ,telescopes and microscopes are limited due to diffraction.
- The colours seen on CD are due to diffraction.

The single slit



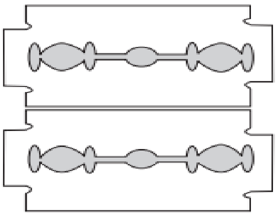
When a single narrow slit is illuminated by a monochromatic source, a broad pattern with a central bright region (central maximum) is seen on the screen. On both sides, there are alternate dark and bright regions(secondary maxima and secondary minima), the intensity becoming weaker away from the centre. This is diffraction pattern.



For central maximum, angle $\theta = 0$

For secondary maxima, $\theta = (n+\frac{1}{2})\frac{\lambda}{a}$ where $n = \pm1, \pm2, \pm3, ...$

For secondary minima, $\theta = n\frac{\lambda}{a}$ where $n = \pm1, \pm2, \pm3, ...$



Holding two blades to form a single slit. A bulb filament viewed through this shows clear diffraction bands.

Cosistency with Principle of Conservation of Energy.

In interference and diffraction, light energy is redistributed. If it reduces in one region, producing a dark fringe, it increases in another region, producing a bright fringe. There is no gain or loss of energy, which is consistent with the principle of conservation of energy.

10.7 Polarisation

A wave propagating in x direction in a horizontally string ,with displacement in y direction can be represented as

$$y(x,t) = a \sin(kx - \omega t)$$

It is referred to as a y-polarised wave.

Since each point on the string moves on a straight line, the wave is also referred to as a linearly polarised wave.

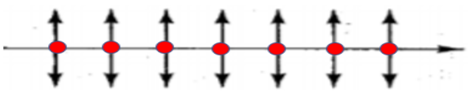
As the string always remains confined to the x-y plane , it is also referred to as a plane polarised wave.

Polarisation of Light

The phenomenon of restricting the electric field vibrations of light to one plane is called polarisation.

Unpolarised Light

For an unpolarised light the vibrations of electric vector takes all possible directions in the transverse plane. Natural light, e.g., from the sun is unpolarised.



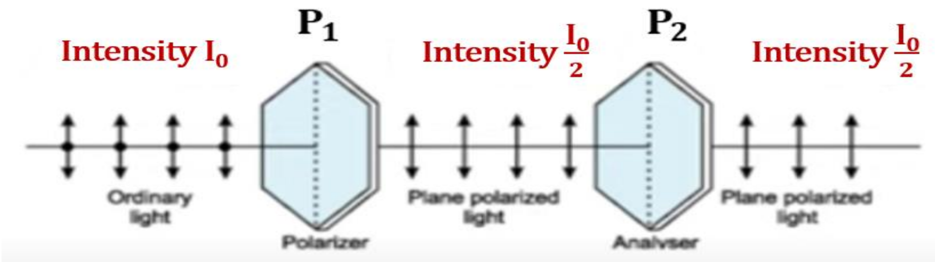
Plane Polarised Light

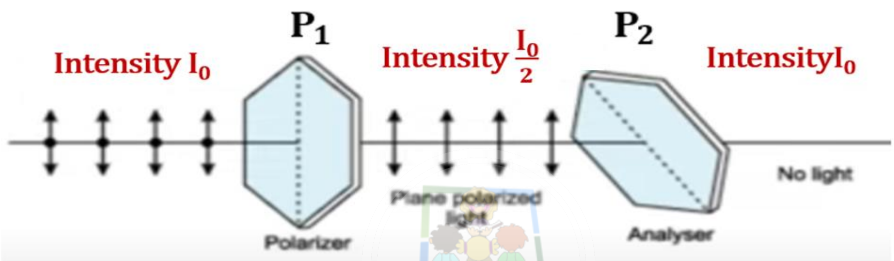
For a plane polarised light the vibrations of electric field vector are restricted in one direction .

Polaroids

Polaroids are thin plastic like sheets, which consists of long chain molecules aligned in a particular direction. The electric vectors along the direction of the aligned molecules get absorbed. Thus, if an unpolarised light wave is incident on a polaroid ,it transmits only one component of electric field vectors which are parallel to its pass axis and the resulting light is c linearly polarised or plane polarised.

Polaroids are used in sunglasses, wind screens in trains and aeroplanes, in 3D cameras.





Malus' Law

When an unpolarised light is passed through two polaroids P_1 and P_2 and if the angle between the polaroids is varied from 0º to 90º, the intensity of the transmitted light will vary as:

$$I = I_0 \cos^2\theta$$

where I_0 is the intensity of the polarized light after passing through P_1.
This is known as Malus' law.

Chapter 11
Dual Nature of Radiation and Matter

11.1 Introduction

- It was found that at low pressure ,when an electric field is applied to the gas in the discharge tube, a fluorescent glow appeared on the glass opposite to cathode. These cathode rays were discovered, in 1870, by William Crookes who later, in 1879, suggested that these rays consisted of streams of fast moving negatively charged particles.
- By applying mutually perpendicular electric and magnetic fields across the discharge tube, J. J. Thomson determined experimentally the speed and the specific charge [charge to mass ratio (e/m)] of the cathode ray.
- In 1887, it was found that certain metals, when irradiated by ultraviolet light, emitted negatively charged particles having small speeds. Also, certain metals when heated to a high temperature were found to emit negatively charged particles. The value of e/m of these particles was found to be the same as that for cathode ray particles.

These observations thus established that all these particles, although produced under different conditions, were identical in nature. J. J. Thomson, in 1897, named these particles as electrons, and suggested that they were fundamental, universal constituents of matter.In 1913, the American physicist R. A. Millikan performed oil-drop experiment and measured the charge of electron as 1.602×10^{-19} C. Millikan's experiment established that electric charge is quantised.

11.2 Electron emission

If an electron attempts to come out of the metal, the metal surface acquires a positive charge and pulls the electron back to the metal. The electron can come out of the metal surface only if it has got sufficient energy to overcome the attractive pull.

Work Function

The minimum energy required to eject an electron from the metal surface is called work function. The work function is denoted by ϕ_0 .

- Work function is measured in electron volt (eV).
- ϕ_0 depends on properties of metal and nature of its surface.

- One electron volt is the energy gained by an electron when it has been accelerated by a potential difference of 1 volt.

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J.}$$

TABLE 11.1 WORK FUNCTIONS OF SOME METALS

Metal	Work function ϕ_o (eV)	Metal	Work function ϕ_o (eV)
Cs	2.14	Al	4.28
K	2.30	Hg	4.49
Na	2.75	Cu	4.65
Ca	3.20	Ag	4.70
Mo	4.17	Ni	5.15
Pb	4.25	Pt	5.65

The work function of platinum is the highest ($\phi_0 = 5.65$ eV) while it is the lowest ($\phi_0 = 2.14$ eV) for caesium.

The minimum energy required for the electron emission from the metal surface can be supplied to the free electrons by any one of the following physical processes:

(i)Thermionic emission

By suitably heating, the free electrons will get sufficient thermal energy to escape from the metal surface.

(ii)Field Emission

By applying a very strong electric field (of the order of 10^8 V/m) to a metal, electrons will get sufficient energy to escape from the metal, as in a spark plug.

(iii) Photo-electric emission

When light of suitable frequency incident on a metal surface, electrons are emitted from the metal surface. These photo(light)-generated electrons are called photoelectrons.

11.3 Photoelectric Effect

Hertz's observations

The phenomenon of photoelectric emission was discovered in 1887 by Heinrich Hertz (1857-1894).

He observed that when light falls on a metal surface, the electrons escaped from the surface of the metal into the surrounding space.

Hallwachs' and Lenard's observations

Lenard (1862-1947) observed that when ultraviolet radiations were allowed to fall on the emitter plate of an evacuated glass tube enclosing two electrodes (metal plates), current flows in the circuit.
Hallwachs, in 1888, connected a negatively charged zinc plate to an electroscope and found that negatively charged particles were emitted from the zinc plate under the action of ultraviolet light.

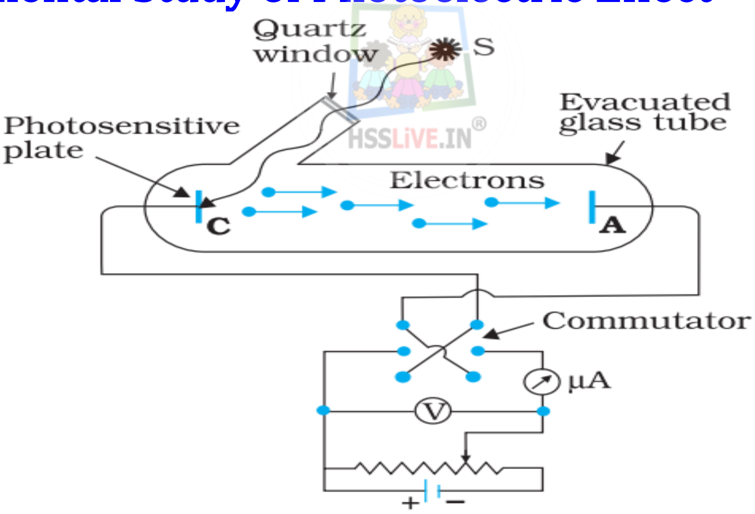
It was found that zinc, cadmium, magnesium, etc., responded only to ultraviolet light, having short wavelength, to cause electron emission from the surface.

However, some alkali metals such as lithium, sodium, potassium, caesium and rubidium were sensitive even to visible light.

Photoelectric Effect

The phenomenon of emission of electrons when photosensitive substances are illuminated by light of suitable frequency is called photoelectric effect.

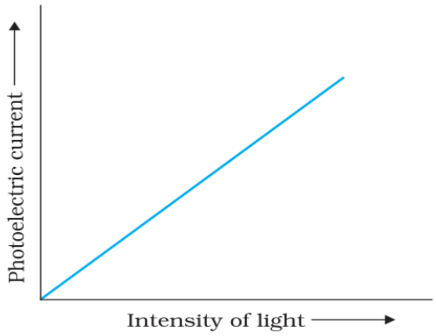
11.4 Experimental Study of Photoelectric Effect



Experimental arrangement consists of an evacuated glass/quartz tube having a photosensitive plate C and another metal plate A. Monochromatic light from the source S of sufficiently short wavelength passes through the window W and falls on the photosensitive plate C (emitter). A transparent quartz window permits ultraviolet radiation to pass through it and irradiate the photosensitive plate C. The electrons are emitted by the plate C and are collected by the plate A (collector), by the electric field created by the battery. The polarity of the plates C and A can be reversed by a commutator. When the collector plate A is positive with respect to the emitter plate C, the electrons are attracted to it. The emission of electrons causes flow of electric current in the circuit.

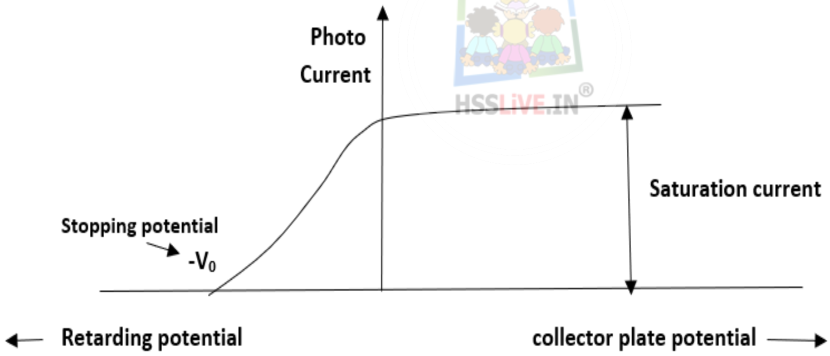
The photoelectric current can be increased or decreased by varying the potential of collector plate A with respect to the emitter plate C. The intensity and frequency of the incident light can also be varied.

1.Effect of intensity of light on photocurrent



When intensity of incident radiation is increased(keeping the frequency of the incident radiation and the accelerating potential fixe), the number of photoelectrons emitted per second increases and hence the photoelectric current also increases.
i.e., the photocurrent increases linearly with intensity of incident light.

2.Effect of potential on photoelectric current



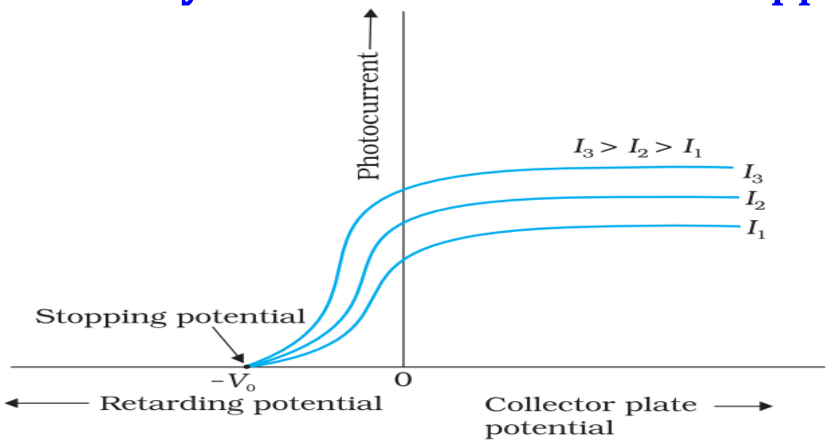
When the positive potential of collector (A) is increased the photoelectric current increases until all the electrons are collected by the collector(A). Then the photocurrent becomes maximum and is called saturation current.

Now the collector is made negative with respect to emitter C. Then the photocurrent decreases with increases in negative potential and finally becomes zero. The minimum negative potential of emitter plate A for which the photocurrent stops or bocomes zero is called the cut off potential or stopping potential (V_0)
At stopping potential,

$$K_{max} = e\,V_0$$

$$\frac{1}{2}mv_{max}^2 = e\,V_0$$

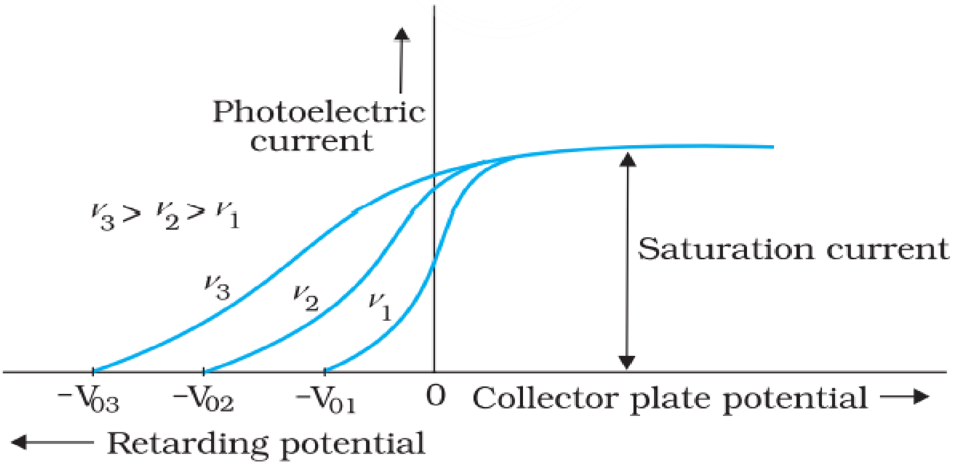
3.Effect of Intensity of incident radiation on stopping potential



The experiment is repeated with incident radiation of the same frequency but different intensities I_1, I_2 and I_3 ($I_3 > I_2 > I_1$).When the intensity of incident radiation is increased ,number of photo electrons emitted per second increases and hence the the saturation current increases. But as the kinetic energy of photoelectrons remains constant and the stopping potential also remains constant.

i.e., for a given frequency of incident radiation, the stopping potential is independent of intensity of radiation.

4.Effect of frequency of incident radiation on stopping potential



The experiment is repeated at same intensity of light radiation but differenr frequencies v_1, v_2 *and* v_3 such that $v_1 > v_2 > v_3$. When the frequency of incident radiation increases, the kinetic energy of photoelectrons increases and hence the stopping potential also increases. But as the intensity does not change , the saturation current will be the same for different frequencies.

i.e., the stopping potential increases with increase in frequency of incident radiation.

Laws of Photoelectric Effect

i. For a given photosensitive material and frequency of incident radiation, the photoelectric current is directly proportional to the intensity of incident light.

ii. For a given photosensitive material and frequency of incident radiation, saturation current is found to be proportional to the intensity of incident radiation whereas the stopping potential is independent of its intensity .

iii. For a given photosensitive material, there exists a certain minimum cut-off frequency of the incident radiation, called the threshold frequency(ν_0) below which no emission of photoelectrons takes place, no matter how intense the incident light is. Above the threshold frequency, the stopping potential or equivalently the maximum kinetic energy of the emitted photoelectrons increases linearly with the frequency of the incident radiation, but is independent of its intensity

iv. The photoelectric emission is an instantaneous process without any apparent time lag.

Threshold Frequency

Threshold frequency is the minimum cut-off frequency of the incident radiation, below which photo emission is not possible, no matter how intense the incident light is.

11.5 Photoelectric Effect and Wave Theory of Light

The phenomena of interference, diffraction and polarisation were explained by the wave picture of light. But the wave picture is unable to explain the most basic features of photoelectric emission.

- According to the wave picture of light, the free electrons at the surface of the metal absorb the radiant energy continuously. The greater the intensity of radiation, the greater should be the energy absorbed by each electron. This is contradictory to the observations of photoelectric effect.
- As large number of electrons absorb energy, the energy absorbed per electron per unit time turns out to be small. It can take hours or more for a single electron to pick up sufficient energy to overcome the work function and come out of the metal. This is contrast to observation that the photoelectric emission is instantaneous.

11.6 Einstein's Photoelectric Equation: Energy Quantum of Radiation

Einstein explained photoelectric effect based on Planck's quantum theory of radiation. When a photon incident on a metal surface, a part of its energy is used as work function and the remaining part is used to give kinetic energy to emitted photoelectrons.

Energy of photon =work function + KE of electrons

$$h\nu = \phi_0 + K_{max}$$

$$K_{max} = h\nu - \phi_0 \text{ ----------------(1)}$$

This is known as Einstein's photoelectric equation.

At stopping potential V_0

$$K_{max} = e V_0$$

$$e V_0 = h\nu - \phi_0 \text{ ----------------(2)}$$

At threshold frequency , $\nu = \nu_0$, KE = 0 , $\phi_0 = h\nu_0$

$$K_{max} = h\nu - h\nu_0 \text{ -------------(3)}$$

Since K_{max} is must be non negative, the photo emission is possible only if $h\nu > \phi_0$, $h\nu > h\nu_0$, $\nu > \nu_0$

where, $\nu_0 = \frac{\phi_0}{h}$

Greater the work function ,greater the threshold frequency.
Below threshold frequency ,photoemission is not possible.

but $K_{max} = \frac{1}{2} m {v_{max}}^2$

$$\frac{1}{2} m {v_{max}}^2 = h(\nu - \nu_0) \text{ ------------(4)}$$

$c = \nu\lambda$ then $\nu = \frac{c}{\lambda}$, $\nu_0 = \frac{c}{\lambda_0}$

$$\frac{1}{2} m {v_{max}}^2 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) \text{ ----------(5)}$$

where λ_0 is called threshold wavelength.

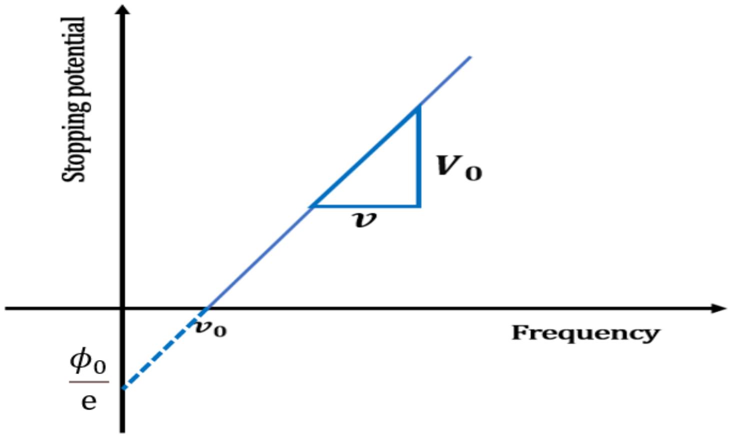
Variation of stopping potential with frequency of incident radiation

At stopping potential V_0 Einstein's photoelectric equation ,

$$eV_0 = h\nu - \phi_0$$

$$V_0 = \frac{h}{e}\nu - \frac{\phi_0}{e}$$

This equation shows that the graph between stopping potential V_0 and frequency ν is a straight line with slope $\frac{h}{e}$ which is a constant independent of nature of material.



From graph , slope $= \frac{V_0}{\nu} = \frac{h}{e}$

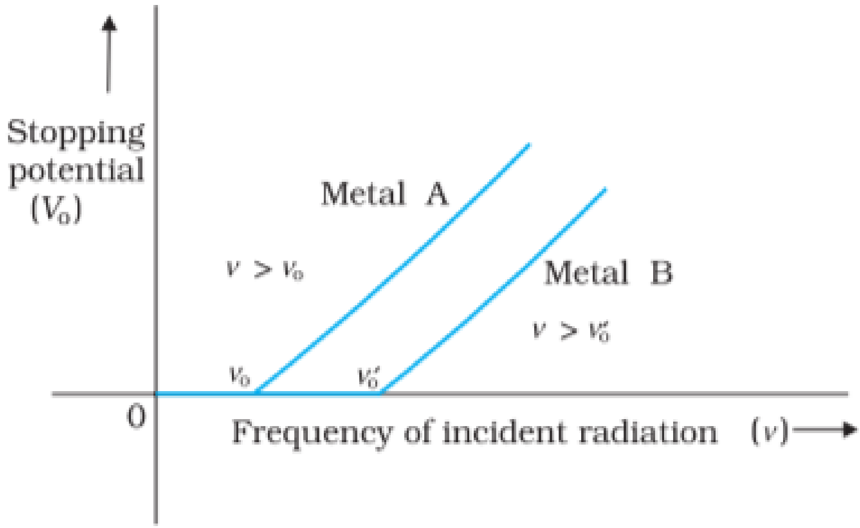
The y- intercept $= \frac{\phi_0}{e}$

The graph shows that

(i) the stopping potential V_0 varies linearly with the frequency of incident radiation for a given photosensitive material.

(ii) there exists a certain minimum cut-off frequency ν_0 for whicthe stopping potential is zero

For two metals A and B these graphs metal A metal B will be parallel straight lines



11.7 Particle Nature of Light –The Photon

1) In the interaction of light with matter , light behaves as if it is made up of particles called photon.

2) Each photon has energy, E=hν and momentum p= hν/c and speed c= 3x 10^8m/s

3) All photons of light of a particular frequency ν, or wavelength λ, have the same energy and momentum p, whatever the intensity of radiation may be.

4) When intensity of light is increased only the number of photons increases, but the energy of photon is independent of intensity of light.

5) Photons are electrically neutral. They are not deflected by electric and magnetic fields.

6) In photon-particle collision total energy and total momentum are conserved. However, the number of photons may not be conserved in a collision. The photon may be absorbed or a new photon may be created.

Example

Monochromatic light of frequency 6.0×10^{14} Hz is produced by a laser. The power emitted is 2.0×10^{-3} W.

(a) What is the energy of a photon in the light beam?

(b) How many photons per second, on an average, are emitted by the source?

(a) Each photon has an energy $E = h\nu = 6.63 \times 10^{-34} x 6.0 \times 10^{14}$ Hz

$$= 3.98 \times 10^{-19} \text{ J}$$

(b) $N = \frac{P}{E} = \frac{2x10^{-3}}{3.98x10^{-19}} = 5 \text{ x} 10^{15}$ photons per second

Example

The work function of a metal is 6eV. If two photons each having energy 4 eV strike the metal surface. Will the emission be possible? Why?

No, photo emission is not possible.

Photo emission is possible only if $h\nu > \phi_0$

Here energy of incident photon is less than work function and hence photo emission is not possible.

Example

The work function of caesium is 2.14 eV.

a) Find the threshold frequency for caesium.

b) the wavelength of the incident light if the photocurrent is brought to zero by a stopping potential of 0.60 V.

a) $$v_0 = \frac{\phi_0}{h}$$

ϕ_0 =2.14 eV =2.14 x1.6x10^{-19} J

h=6.63 x10^{-34}Js

$$v_0 = \frac{2.14\ x 1.6 x 10^{-19}}{6.63\ x 10^{-34}} = 5.16 \text{ x}10^{14}\text{ Hz}$$

b) $$e\,V_0 = hv - \phi_0$$

$$hv = e\,V_0 - \phi_0$$

$$h\frac{c}{\lambda} = e\,V_0 - \phi_0$$

$$\lambda = \frac{\text{hc}}{\text{eV}_0 - \Phi_0}$$

$$= \frac{6.63\text{ x}10^{-34}\text{x}3\text{ x}10^{8}}{1.6\text{ x}10^{-19}\text{x}0.6 - 2.14\text{ x}1.6\text{x}10-19} = 454\text{ nm}$$

11.8 Wave Nature of Matter

- The wave nature of light shows up in the phenomena of interference, diffraction and polarisation. On the other hand, in photoelectric effect and Compton effect which involve energy and momentum transfer, radiation behaves as if it is made up of particles – the photons.
- The gathering and focussing mechanism of light by the eye-lens is well described in the wave picture. But its absorption by the rods and cones (of the retina) requires the photon picture of light.

A natural question arises: If radiation has a dual (wave-particle) nature, might not the particles of nature (the electrons, protons, etc.) also exhibit wave-like character?

Louis Victor de Broglie argued that moving particles of matter should display wave-like properties under suitable conditions.

As nature is symmetrical , the two basic physical entities of nature – matter and energy, must have symmetrical character. If radiation shows dual aspects, matter should also exhibit dual nature.

de Broglie Relation -Wavelenth of matter wave

De Broglie proposed that the wave length λ associated with a particle of momentum p is given as

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

where m is the mass of the particle and v its speed.
λ is called de Broglie wavelength.

The dual aspect of matter is evident in the de Broglie relation. **Here λ is a wave attribute while the momentum p is a particle attribute. Planck's constant h relates the two attributes.**

Why macroscopic objects in our daily life do not show wave-like properties?

The de Broglie wavelength of a ball of mass 0.12 kg moving with a speed of 20 m s^{-1} is ,

$$\lambda = \frac{h}{mv} = \frac{6.6\,x10^{-34}}{0.12\,x20} = 2.76x10^{-34}\text{ nm}$$

This wavelength is so small that it is beyond any measurement. This is the reason why macroscopic objects in our daily life do not show wave-like properties. But in the sub-atomic domain, the wave character of particles is significant and measurable.

Example

What is the de Broglie wavelength associated with an electron moving with a speed of 5.4×10^{6} m/s?

$$\lambda = \frac{h}{mv} = \frac{6.6\,x10^{-34}}{9.1\,x\,10^{-31}x\,5.4\,x10^{6}} = 0.135\text{ nm}$$

This wavelength is measurable. i.e., in the sub-atomic domain, the wave character of particles is significant and measurable.

Chapter 12
Atoms

12.1 Introduction

Thomson Model of Atom- (plum pudding model)

The first model of atom was proposed by J. J. Thomson in 1898.

- According to this model, the positive charge of the atom is uniformly distributed throughout the volume of the atom .
- The negatively charged electrons are embedded in it like seeds in a watermelon.

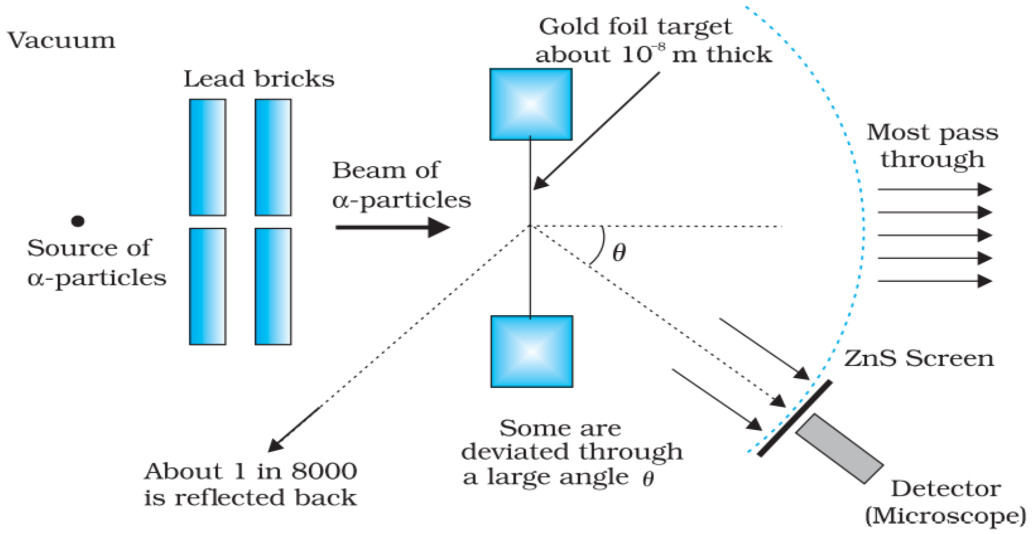
This model is also called plum pudding model of the atom.

Alpha-Particle Scattering and Rutherford's Nuclear Model of Atom

Ernst Rutherford , a former research student of J. J. Thomson, proposed a classic experiment of scattering of these α-particles by atoms to investigate the atomic structure. The explanation of the results led to the birth of Rutherford's planetary model of atom (also called the nuclear model of the atom).

12.2 Alpha-Particle Scattering and Rutherford's Nuclear Model of atom

At the suggestion of Ernst Rutherford, in 1911, H. Geiger and E. Marsden performed scattering experiment.



Alpha-particles emitted by a $^{214}_{83}Bi$ radioactive source were collimated into a narrow beam by passing through lead bricks. The beam was allowed to fall on a thin foil of gold of thickness 2.1×10^{-7} m. The scattered alpha-particles were observed through a rotatable detector consisting of zinc sulphide screen and a microscope.

Observations

- Many of the α-particles pass through the foil. It means that they do not suffer any collisions.
- Only 0.14% of the incident α-particles scatter by more than 1º.
- About 1 in 8000 of incident α-particles deflect by more than 90º.

Rutherford argued that , greater part of the mass of the atom and its positive charge were concentrated tightly at its centre. When the incoming α-particle make a close encounter with the positive charge ,that would result in a large deflection.

Rutherford's nuclear model of the atom

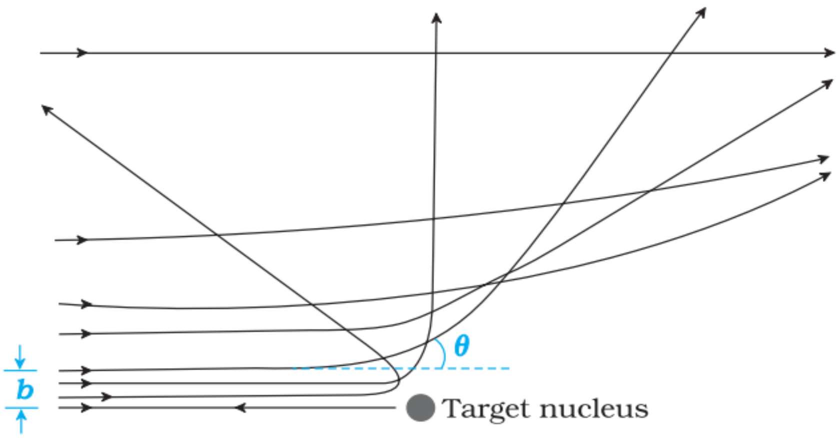
- Most of an atom is empty space.
- The entire positive charge and most of the mass of the atom are concentrated in the nucleus with the electrons some distance away.
- The electrons would be moving in orbits about the nucleus just as the planets do around the sun.
- The size of the nucleus to be about 10^{-15} m to 10^{-14} m.
- The electrostatic force of attraction, between the revolving electrons and the nucleus provides the centripetal force to keep them in their orbits.

Impact Parameter (b)

Impact parameter is the perpendicular distance of the initial velocity vector of the α particle from the centre of the nucleus.

Alpha-particle trajectory

The trajectory traced by an α-particle depends on the impact parameter, b of collision.



- For an α-particle close to the nucleus , impact parameter is small and it suffers large scattering.
  - For head on collision, the impact parameter b=0 and α particle rebounds back ie,angle of scattering $\theta = 180^0$.
  - For large impact parameter, the angle of scattering will be small ($\theta \approx 0^0$) and such α particles go undeviated.

Electron orbits

The electrostatic force of attraction(F_e), between the revolving electrons and the nucleus provides the centripetal force (F_c) to keep them in their orbits.

$$F_c = F_e$$

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

The kinetic energy (K) of electron

$$\mathbf{K = \frac{1}{2}mv^2 = \frac{e^2}{8\pi\varepsilon_0 r}}$$

The potential energy (U) of electron

$$\mathbf{U = \frac{-e^2}{4\pi\varepsilon_0 r}}$$

(The negative sign in U signifies that the electrostatic force is in the –r direction.)

Thus the total energy E of the electron in a hydrogen atom is

$$E = K+U$$

$$E = \frac{e^2}{8\pi\varepsilon_0 r} - \frac{e^2}{4\pi\varepsilon_0 r}$$

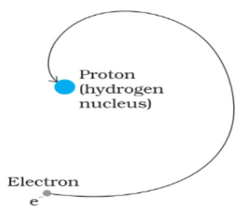
$$\mathbf{E = \frac{-e^2}{8\pi\varepsilon_0 r}}$$

The total energy of the electron is negative. This implies the fact that the electron is bound to the nucleus. If E were positive, an electron will not follow a closed orbit around the nucleus.

Limitations of Rutherford Model

Rutherford nuclear model has two main difficulties in explaining the structure of atom:

(a) Rutherford model could not explain stability of matter. The accelerated electrons revolving around the nucleus loses energy and must spiral into the nucleus. This contradicts the stability of matter.



(b) It cannot explain the characteristic line spectra of atoms of different elements.

12.3 Atomic Spectra

Each element has a characteristic spectrum of radiation, which it emits. There are two types of spectra-Emission spectrum and Absorption spectrum.

Emission Spectrum

When an atomic gas or vapour is excited at low pressure, by passing an electric current through it, the emitted radiation has a spectrum which contains certain specific wavelengths only. A spectrum of this kind is termed as emission line spectrum and it consists of bright lines on a dark background. Study of emission line spectra of a material is used for identification of the gas.

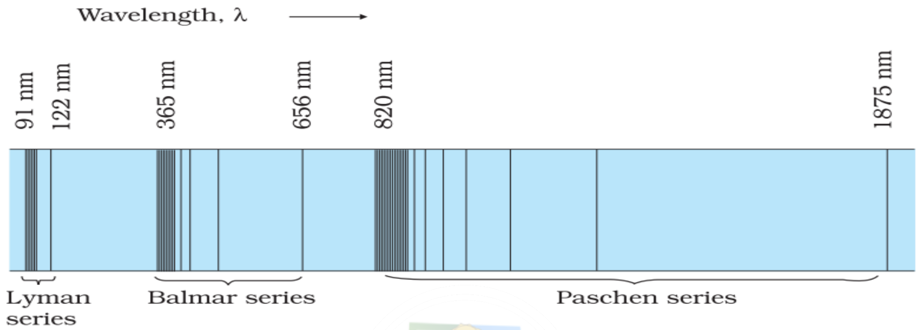


FIGURE 12.5 Emission lines in the spectrum of hydrogen.

Absorption Spectrum

When white light passes through a gas and we analyse the transmitted light using a spectrometer we find some dark lines in the spectrum. These dark lines correspond precisely to those wavelengths which were found in the emission line spectrum of the gas. This is called the absorption spectrum of the material of the gas.

12.4 Bohr Model of Hydrogen Atom

Niels Bohr made certain modifications in Rutherford's model using the ideas of quantum hypothesis. Bohr combined classical and early quantum concepts and gave his theory in the form of three postulates.

1) Bohr's first postulate states that an electron in an atom revolves in certain stable orbits without the emission of radiant energy.

2) Second postulate states that the electron revolves around the nucleus only in those orbits for which the angular momentum is an integral multiple of h/2π where h is the Planck's constant

$$L = mvr = \frac{nh}{2\pi} \quad ,\text{where } n = 1,2,3.........$$

n is called principal quantum number

3) Third postulate states that when an electron make a transition from higher energy level to lower energy level a photon is emitted having energy equal to the energy difference between the initial and final states. The frequency of the emitted photon is then given by

$$h\nu = E_i - E_f$$

Energy of Hydrogen Atom

Total energy of n^{th} energy level

$$E_n = \frac{-e^2}{8\pi\varepsilon_0 r_n}$$

The radius of hydrogen atom

$$r_n = \frac{n^2h^2\varepsilon_0}{\pi me^2}$$

Substituting for r_n from eqn(3)

$$E_n = \frac{-e^2}{8\pi\varepsilon_0\left(\frac{n^2h^2\varepsilon_0}{\pi me^2}\right)}$$

$$E_n = \frac{-me^4}{8n^2\varepsilon_0{}^2h^2}$$

$$E_n = \frac{-13.6}{n^2}\ \text{eV}$$

$$E_n \propto \frac{1}{n^2}$$

The negative sign of the total energy of an electron moving in an orbit means that the electron is bound with the nucleus.

Energy levels

The energy of an atom is the least (largest negative value) when its electron is revolving in an orbit closest to the nucleus for $n = 1$. The energy is progressively larger in the outer orbits.

Ground State

The lowest energy state of an atom is called the Ground State, with the electron revolving in the orbit of smallest radius, the Bohr radius, a_0.

For ground state n=1

$$E_1 = \frac{-13.6}{1^2}\ \text{eV} = -13.6\ \text{eV}$$

At room temperature most of the Hydrogen atoms are in ground state.

Excited States

When Hydrogen atom receives energy by the process such as collisions, the atoms may acquire sufficient energy to raise the electrons to higher energy states. Then atom is said to be in an excited state.

For first excited state (second energy level)

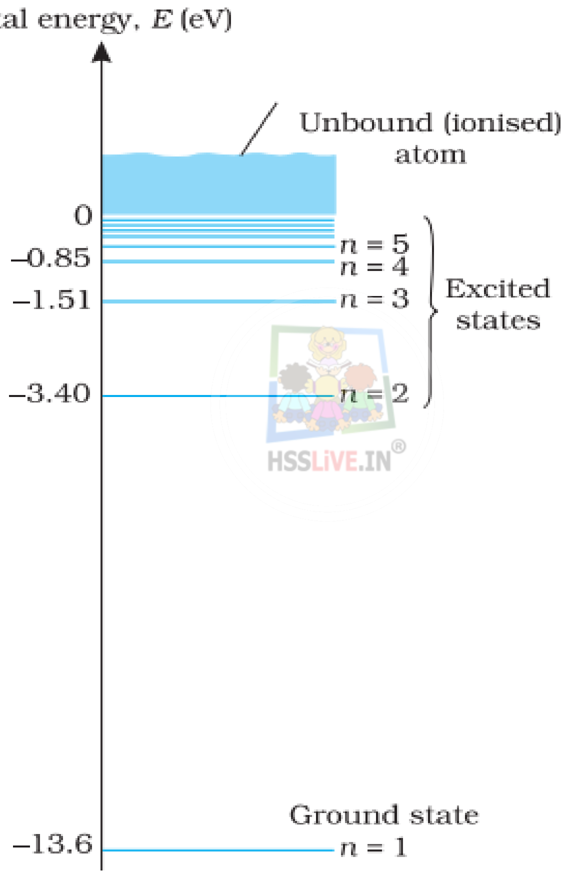
$$n = 2, \qquad E_2 = \frac{-13.6}{2^2} \text{ eV} = -3.4 \text{ eV}$$

For second excited state (third energy level)

$$n = 3, \qquad E_3 = \frac{-13.6}{3^2} \text{ eV} = -1.51 \text{ eV}$$

And so on..

The energy level diagram for the hydrogen atom

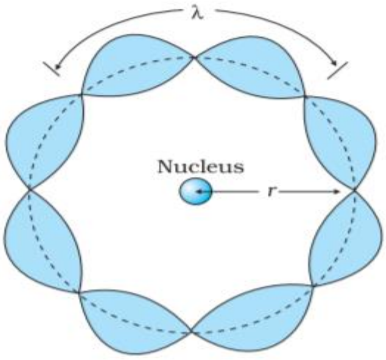


12.5 The Line spectra of Hydrogen Atom

The various lines in the atomic spectra are produced when electrons jump from higher energy state to a lower energy state and photons are emitted. These spectral lines are called emission lines. But when an atom absorbs a photon that has precisely the same energy needed by the electron in a lower energy state to make transitions to a higher energy state, the process is called absorption. Thus if photons with a continuous range of frequencies pass through a rarefied gas and then are analysed with a spectrometer, a series of dark spectral absorption lines appear in the continuous spectrum. The dark lines indicate the frequencies that have been absorbed by the atoms of the gas.

12.6 De Broglie's Explanation of Bohr's second postulate of Quantisation

De Broglie argued that electron in its circular orbit behaves as a particle wave. The particle wave can produce standing wave under resonant condition.



For $\boldsymbol{n^{th}}$ orbit of radius r_n , the resonant condition is

$$2\,\pi\, r_n = n\,\lambda \text{-----------} (1) \quad \text{where } n=1,2,3.....$$

But by de Broglie hypothesis , for matter waves

$$\boldsymbol{\lambda = \frac{h}{mv}} \text{---------------}(2)$$

Substituing eqn (2) in eqn (1),

$$2\,\pi\, r_n = n\,\frac{h}{mv}$$

$$\mathbf{mv\, r_n = \frac{nh}{2\,\pi}} \qquad \text{where } n=1,2,3......$$

This Bohr's second postulate of Quantisation.

Limitations of Bohr Atom Model

(i) The Bohr model is applicable to hydrogenic atoms. It cannot be extended two or more electron atoms. Difficulty lies in the fact that each electron interacts not only with the positively charged nucleus but also with all other electrons.

(ii) While the Bohr's model correctly predicts the frequencies of the light emitted by hydrogenic atoms, the model is unable to explain the intensity variations of the frequencies in the spectrum.

Chapter 13
Nuclei

13.1 Introduction

The volume of a nucleus is about 10^{-12} times smaller than the volume of the atom. In other words, an atom is almost empty. If an atom is enlarged to the size of a classroom, the nucleus would be of the size of pinhead. Nevertheless, the nucleus contains most (more than 99.9%) of the mass of an atom.

13.2 Atomic Masses and Composition of Nucleus

The mass of an atom is very small. Kilogram is not a very convenient unit to measure such small quantities. Therefore, a different mass unit is used for expressing atomic masses. This unit is the atomic mass unit (u).

Atomic Mass Unit (u)

Atomic mass unit (u) is defined as 1/12th of the mass of the carbon (^{12}C) atom.

$$1u = \frac{\textbf{mass of the one C}-\textbf{12 atom}}{\textbf{12}}$$

$$= \frac{1.992647\ x\ 10^{-26}}{12}$$

$$= 1.660539\ 10^{-27}\ \text{kg}$$

Accurate measurement of atomic masses is carried out with a mass spectrometer.

Composition of Nucleus

The composition of a nucleus can be described using the following terms and symbols:

Z - atomic number = number of protons = number of electrons
N - neutron number = number of neutrons=A-Z
A - mass number = Z + N = total number of protons and neutrons .

Protons and neutrons are also called nucleons. Thus the number of nucleons in an atom is its mass number A.

- All the electrons of an atom are outside the nucleus.
- The total charge of the atomic electrons is (–Ze)
- The total cherge of the nucleus is (+Ze).
- **Atom is electrically neutral**
- The mass of proton , m_p =1.00727 u = 1.67262 10^{-27} kg
- The mass of neutron , m_n = 1.00866 u = 1.6749×10^{-27} kg

- Neutron was discovered by James Chadwick.
- Chadwick was awarded the Nobel Prize in Physics in 1935 for his discovery of the neutron.

Isotopes

Isotpes are different types of atoms of the same element, with same atomic number ,but different mass number .They exhibit the same chemical properties, but differ in mass.

- Chlorine has two isotopes having masses 34.98 u and 36.98 u. The relative abundances of these isotopes are 75.4 and 24.6 per cent, respectively. Thus, the average mass of a chlorine atom is obtained by the weighted average of the masses of the two isotopes, which works out to be

$$= \frac{75.4 \text{ x}34.98 + 24.6\text{x } 36.98}{\mathbf{100}}$$

$$= 35.47 \text{ u}$$

which agrees with the atomic mass of chlorine.

- The lightest element, hydrogen has three isotopes ,

Proton($^1_1$H) - contains one proton only

Deuterium($^2_1$H) - contains one proton and one neutron.

Tritium($^3_1$H) - contains one proton and two neutrons.

Tritium nuclei, being unstable, do not occur naturally and are produced artficially in laboratories.

- The element gold has 32 isotopes, ranging from

A =173 to A = 204.

Isobars

All nuclides with same mass number A , but with different atomic number are called isobars.

For example, the nuclides ($^3_1$H) and ($^3_2$He)are isobars.

Isotones

Nuclides with same neutron number N but different atomic number Z are called isotones.

For example $^{198}_{80}$Hg $^{197}_{79}$Au are isotones.

13.3 Size of The Nucleus

By performing scattering experiments in which fast electrons, instead of α-particles, are projectiles that bombard targets made up of various elements, the sizes of nuclei of various elements have been accurately measured.

Radius of nucleus

A nucleus of mass number A has a radius

$$R = R_0 A^{1/3} \qquad \text{where } R_0 = 1.2 \times 10^{-15} \text{ m.}$$

Volume of nucleus

$$V = \frac{4}{3}\pi R^3$$
$$= \frac{4}{3}\pi \left(\mathbf{R_0 A^{1/3}}\right)^{\mathbf{3}}$$
$$= \frac{4}{3}\pi (\mathbf{R_0})^{\mathbf{3}}\mathbf{A}$$

The volume of the nucleus is proportional to A

Density of nucleus

$$\text{Nuclear density} = \frac{\text{mass}}{\text{volume}}$$
$$= \frac{Am_p}{\frac{4}{3}\pi(\mathbf{R_0})^3 \mathbf{A}}$$
$$= \frac{m_p}{\frac{4}{3}\pi(\mathbf{R_0})^3} = \text{constant}$$

Thus the density of nucleus is a constant, independent of A, for all nuclei.

Example

Given the mass of iron nucleus as 55.85u and A=56, find the nuclear density?

$$m_{Fe} = 55.85 \text{ u} = 9.27 \times 10^{-26} \text{ kg}$$
$$\text{Nuclear density} = \frac{m_{Fe}}{\frac{4}{3}\pi(\mathbf{R_0})^3 A}$$
$$= \frac{9.27 \times 10{-}26}{\frac{4}{3}\pi(1.2 \times 10{-}15)^3 \; x \; 56}$$
$$= 2.29 \times 10^{17} \text{ kg m}^{-3}$$

13.4 Mass- Energy and Nuclear Binding Energy

Mass – Energy

Einstein showed that mass is another form of energy and one can convert mass-energy into other forms of energy, say kinetic energy and vice-versa.

Einstein gave the famous mass-energy equivalence relation

$$E = mc^2$$

c is the velocity of light in vacuum.c= 3×10^8 m s^{-1}.

Experimental verification of the Einstein's mass-energy relation has been achieved in the study of nuclear reactions. In a reaction the conservation law of energy states that the initial energy and the final energy are equal provided the energy associated with mass is also included.

Example

Calculate the energy equivalent of 1 g of substance.

$$\begin{aligned} E &= mc^2 \\ &= 1x10^{-3} \times (3 \times 10^8)^2 \\ &= 10^{-3} \times 9 \times 10^{16} \\ &= 9 \times 10^{13}\ J \end{aligned}$$

Thus, if one gram of matter is converted to energy, there is a release of enormous amount of energy.

Example

Find the energy equivalent of one atomic mass unit, first in Joules and then in MeV.

$$1u = 1.6605 \times 10^{-27}\ kg$$

$$\begin{aligned} E &= mc^2 \\ &= 1.6605 \times 10^{-27} \ x (3\ x\ 10^8)^2 \end{aligned}$$

$$\mathbf{E = 1.4924 \times 10^{-10}\ J}$$

Energy equivalent in MeV.

$$1eV = 1.602\ x10^{-19} J$$

$$E = \frac{1.4924 \times 10{-}10}{1.602\ x10{-}19}$$

$$= 0.9315 \times 109\ eV$$

$$\mathbf{= 931.5\ MeV}$$

Mass Defect and Binding Energy

Mass Defect (ΔM)

The nucleus is made up of neutrons and protons. Therefore it may be expected that the mass of the nucleus is equal to the total mass of its individual protons and neutrons.

The nuclear mass M is always less than the total mass, of its constituents (protons and neutrons). The difference in mass of a nucleus and its constituents is called the mass defect.

$$\Delta M = \left[Z\, m_p + (A - Z)m_n\right] - M$$

For example, let us consider ${}^{16}_{8}O$; a nucleus which has 8 neutrons and 8 protons.

$$\text{Mass of 8 neutrons} = 8 \times 1.00866 \text{ u}$$
$$\text{Mass of 8 protons} = 8 \times 1.00727 \text{ u}$$
$$\text{The expected mass of } {}^{16}_{8}O \text{ nucleus} = 16.12744 \text{ u}$$

The atomic mass of ${}^{16}_{8}O$ from mass spectroscopy experiments= 15.99493 u

Substracting the mass of 8 electrons (8×0.00055 u)

The mass of ${}^{16}_{8}O$ nucleus =15.99493 u -(8×0.00055 u)= 15.99053 u

$$\text{Mass defect } \Delta M = 16.12744 - 15.99053 \text{ u}$$
$$= 0.13691 \text{u}$$

Binding Energy

The energy equivalent of mass defect is called binding energy.

$E_b = \Delta Mc^2$

- If we separate a nucleus into its nucleons, we would have to supply a total energy equal to E_b, to those particles.
- If a certain number of neutrons and protons are brought together to form a nucleus of a certain charge and mass, an energy E_b will be released .

Binding Energy Per Nucleon

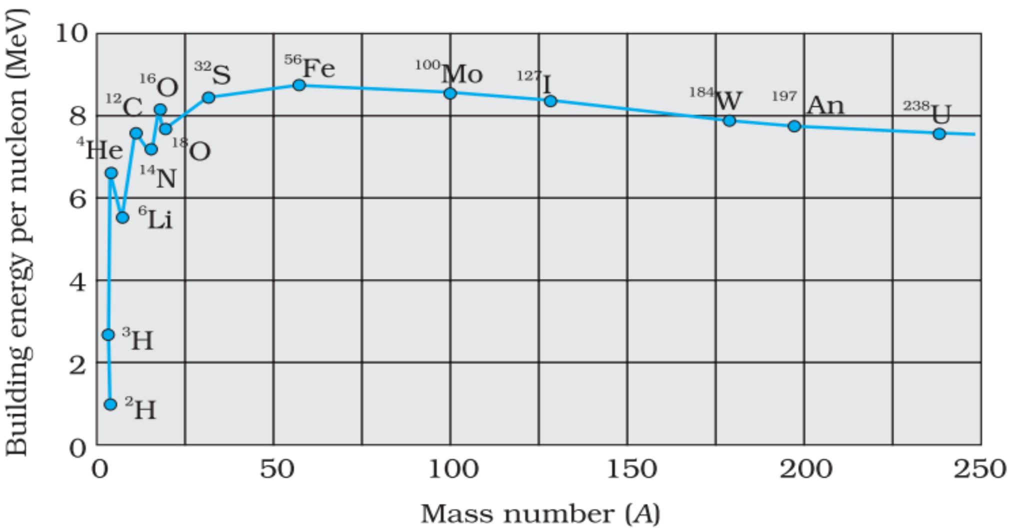
A more useful measure of the binding between the constituents of the nucleus is the binding energy per nucleon, E_{bn},

The binding energy per nucleon, E_{bn}, is the ratio of the binding energy Eb of a nucleus to the number of the nucleons, A, in that nucleus.

$$E_{bn} = E_b / A$$

It is the average energy per nucleon needed to separate a nucleus into its individual nucleons.

The plot of binding energy per nucleon versus mass number



Observations:

(i) In the mass number range A = 30 to 170 ($30 < A < 170$), the binding energy per nucleon is nearly constant, about 8 MeV/nucleon.

(ii) The maximum of about 8.75 MeV for A = 56 i,e,.for ^{56}Fe nucleus.

(iii) E_{bn} is lower for both light nuclei with $A < 30$ and for heavy nuclei with $A > 170$.

We can draw some conclusions from these two observations:

(i) The nuclear force is attractive and sufficiently strong to produce a binding energy of a few MeV per nucleon.

(ii) The constancy of the binding energy in the range $30 < A < 170$ indicates that the nuclear force is short-ranged. A nucleon influences only nucleons close to it and this property is called saturation property of the nuclear force.

(iii) A very heavy nucleus, say A = 240, has lower binding energy per nucleon .Such a heavy nucleus breaks into two lighter nuclei, thereby increasing the binding energy per nucleon and the nucleons get more tightly bound. Energy would be released in the process and this is an implication of through fission.

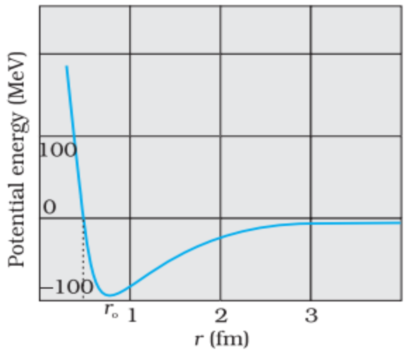
(iv) Two very light nuclei ($A \leq 10$) have lower binding energy per nucleon .They join to form a heavier nucleus , thereby increasing the binding energy per nucleon and the nucleons get more tightly bound. Energy would be released in such a process and this is an implication of through of fusion.

13.5 Nuclear Force

The nuclear force binds the nucleons together inside the nucleus.

(i) The nuclear force is much stronger than the Coulomb repulsive force between protons inside the nucleus and the gravitational force between the masses.

(ii) The nuclear force between two nucleons falls rapidly to zero as their distance is more than a few femtometres. The force is attractive for distances larger than 0.8 fm and repulsive if they are separated by distances less than 0.8 fm.



A rough plot of the potential energy between two nucleons as a function of distance. The potential energy is a minimum at a distance r_0 of about 0.8 fm.

(iii) The nuclear force between neutron-neutron, proton-neutron and proton-proton is approximately the same. The nuclear force does not depend on the electric charge.

13.6 Radioactivity

A .H. Becquerel discovered radioactivity in 1896. Radioactivity is a nuclear phenomenon in which an unstable nucleus undergoes a decay.This is referred to as radioactive decay.

Three types of radioactive decay occur in nature :

α-decay in which a helium nucleus (He) is emitted;
β-decay in which electrons or positrons (particles with the same mass as electrons, but with a charge exactly opposite to that of electron) are emitted;
γ-decay in which high energy (hundreds of keV or more) photons are emitted.

13.7 Nuclear Energy

Energy then can be released if less tightly bound nuclei are transmuted into more tightly bound nuclei. Two such processes, are fission and fusion.

For the same quantity of matter, nuclear sources will give a million times larger energy than conventional sources. One kilogram of coal on burning gives 10^7 J of energy, whereas 1 kg of uranium, which undergoes fission, will generate on fission 10^{14} J of energy.

Nuclear Fission

Nuclear fission is the process in which a heavier nucleus splits into lighter nuclei with the release of large amount of energy.

When a neutron was bombarded on a uranium target, the uranium nucleus broke into two nearly equal fragments releasing great amount of energy.

Example

$$\mathbf{{}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{236}_{92}U \rightarrow {}^{144}_{56}Ba + {}^{89}_{36}Kr + 3\,{}^{1}_{0}n}$$

Fission does not always produce barium and krypton. A different pair can be produced,

$${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{236}_{92}U \rightarrow {}^{133}_{51}Sb + {}^{99}_{41}Nb + 4\,{}^{1}_{0}n$$

$${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{236}_{92}U \rightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + 2\,{}^{1}_{0}n$$

The energy released (the Q value) in the fission reaction of nuclei like uranium is of the order of 200 MeV per fissioning nucleus.

The enormous energy released in an atom bomb comes from uncontrolled nuclear fission.

Nuclear Fusion – Energy Generation in Stars

Nuclear fusion is the process in which two light nuclei combine to form a single larger nucleus, with the release of a large amount of energy.
Examples are

$${}^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + e^{+} + \nu + 0.42\ \text{MeV}$$

$${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{2}_{3}He + n + 3.27\ \text{MeV}$$

$${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{1}H + {}^{1}_{1}H + 4.03\ \text{MeV}$$

Thermonuclear fusion

For nuclear fusion to occur in bulk matter the temperature of the material is to be raised until the particles have enough energy to penetrate the coulomb barrier. This process is called thermonuclear fusion.
for thermonuclear fusion to take place, extreme conditions of temperature and pressure are required, which are available only in the interiors of stars including sun.

The energy generation in stars takes place via thermonuclear fusion.

The fusion reaction in the sun is a multi-step process called the proton-proton (p, p) cycle.

$$ {}_1^1H + {}_1^1H \rightarrow {}_1^2H + e^+ + \nu + 0.42\ MeV $$

$$ e^+ + e^- \rightarrow \gamma + \gamma + 1.02\ MeV $$

$$ {}_1^2H + {}_1^1H \rightarrow {}_3^2He + \gamma + 5.49\ MeV $$

$$ {}_2^3H + {}_2^3H \rightarrow {}_2^4He + {}_1^1H + {}_1^1H + 12.86\ MeV $$

The combined reaction is

$$ 4\,{}_1^1H + 2\,e^- \rightarrow {}_2^4He + 2\nu + 6\gamma + 26.7\ MeV $$

$$ \text{Or } (4\,{}_1^1H + 4\,e^-) \rightarrow ({}_2^4He + 2e^-) + 2\nu + 6\gamma + 26.7\ MeV $$

Thus, four hydrogen atoms combine to form an 4 2He atom with a release of 26.7 MeV of energy.

In about 5 billion years, however, the sun's core, which by that time will be largely helium, will begin to cool and the sun will start to collapse under its own gravity. This will raise the core temperature and cause the outer envelope to expand, turning the sun into what is called a red giant.

Chapter 14
Semiconductor Electronics:
Materials, Devices and Simple Circuits Introduction

14.1 Introduction

Semiconductors are the basic materials used in the present solid state electronic devices like diode, transistor, ICs, etc. Lattice structure and the atomic structure of constituent elements decide whether a particular material will be insulator, metal or semiconductor.

14.2 Classification of Metals, Conductors and Semiconductors

On the basis of conductivity:

On the basis of the relative values of electrical conductivity (σ) or resistivity ($\rho = 1/\sigma$), the solids are broadly classified as:

(i) **Metals:** They possess very low resistivity (or high conductivity).

$$\rho \sim 10^{-2} – 10^{-8}\ \Omega\ m$$
$$\sigma \sim 10^{2} – 10^{8}\ S\ m^{-1}$$

(ii) **Semiconductors:** They have resistivity or conductivity intermediate to metals and insulators.

$$\rho \sim 10^{-5} – 10^{6}\ \Omega\ m$$
$$\sigma \sim 10^{5} – 10^{-6}\ S\ m^{-1}$$

(iii) **Insulators:** They have high resistivity (or low conductivity).

$$\rho \sim 10^{11} – 10^{19}\ \Omega\ m$$
$$\sigma \sim 10^{-11} – 10^{-19}\ S\ m^{-1}$$

Semiconductors which could be:

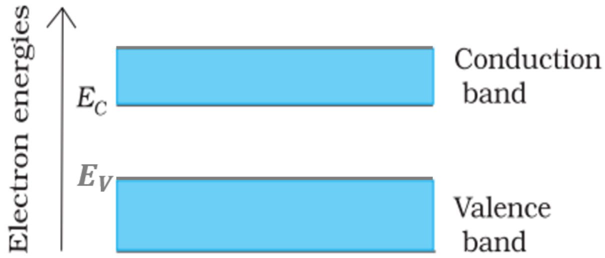
(i) Elemental semiconductors: Si and Ge

(ii) Compound semiconductors: Examples are:

- Inorganic: CdS, GaAs, CdSe, InP, etc.
- Organic: anthracene, doped pthalocyanines, etc.
- Organic polymers: polypyrrole, polyaniline, polythiophene, etc.

Most of the currently available semiconductor devices are based on elemental semiconductors Si or Ge and compound inorganic semiconductors.

Energy Bands In Solids

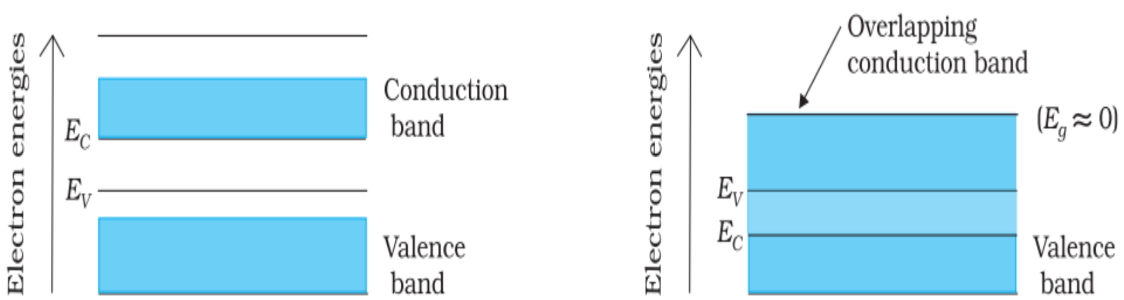


- Inside the crystal each electron will have a different energy level. These different energy levels with continuous energy variation form **energy bands.**
- The energy band which includes the energy levels of the valence electrons is called the **valence band.**
- The energy band which includes the energy levels of conduction electrons is called the **conduction band.**
- The conduction band is above the valence band .Normally the conduction band is empty and valence band is occupied.
- The gap between the top of the valence band and bottom of the conduction band is called the **energy band gap (Energy gap E_g)**. It is measured in **electron volt.**

Classification of Metals, Conductors and Semiconductors

On the basis of energy bands

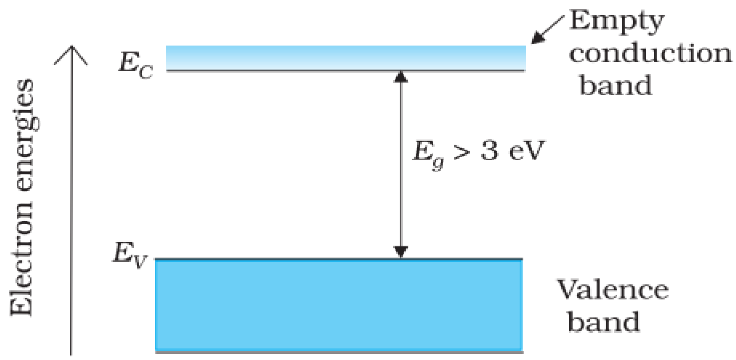
(i) metals



(i) (ii)

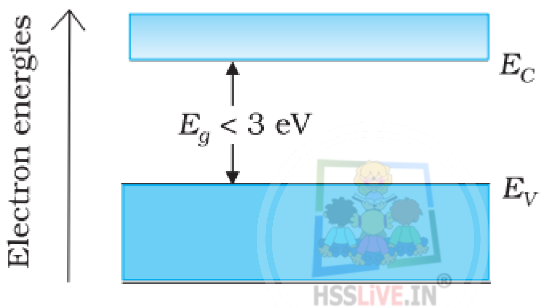
In some metals, the conduction band is partially filled and the valence band is partially empty with small energy gap and in some others the conduction and valance bands overlap. When there is overlap electrons from valence band can easily move into the conduction band. Therefore, the resistance of such materials is low or the conductivity is high.

(ii)Insulators



In insulators a large band gap , $E_g > 3$ eV. There are no electrons in the conduction band, and therefore no electrical conduction is possible. The energy gap is so large that electrons cannot be excited from the valence band to the conduction band by thermal excitation.

(iii)Semiconductors

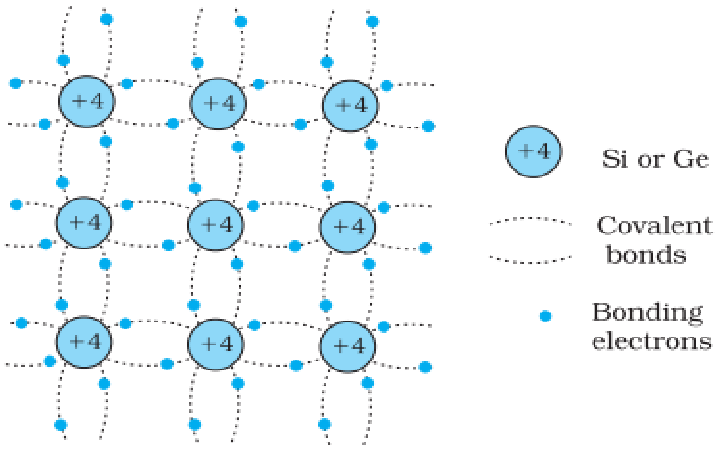


In semiconductors a finite but small band gap ($E_g < 3$ eV) exists. Because of the small band gap, at room temperature some electrons from valence band can acquire enough energy to cross the energy gap and enter the conduction band. These electrons (though small in numbers) can move in the conduction band. Hence, the resistance of semiconductors is lower than that of insulators.

When the electrons from valence band move to the conduction band vacant energy levels will be created in the valence band . This vacancy of electrons is called **hole**. Other valence electrons can move to this hole thereby producing **hole current.**

14.3 Intrinsic Semiconductor

Pure semiconductors are called 'intrinsic semiconductors'.
Si and Ge have four valence electrons. In a pure Si or Ge crystal ,each atom make covalent bond with four neighbouring atoms and share the four valence electrons.

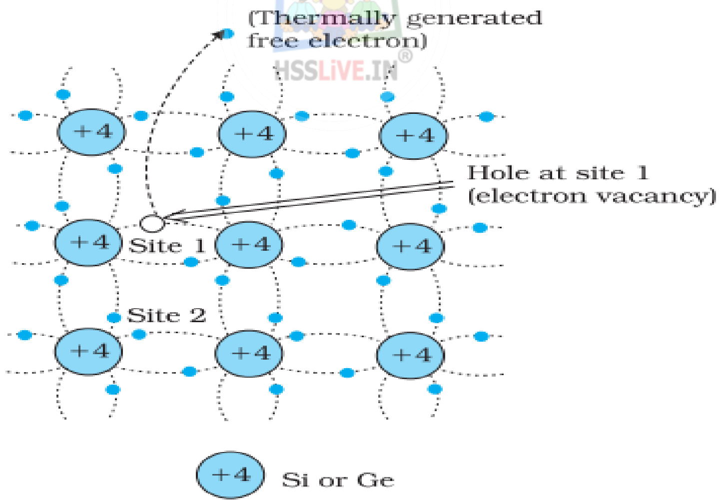


As the temperature increases, these electrons get more thermal energy , break–away the covalent bonds and become free electrons contributing to conduction. These free electrons (with charge –q) leaves a vacancy with an effective charge (+q). This vacancy with the effective positive electronic charge is called a hole.

In intrinsic semiconductors, the number of free electrons, n_e is equal to the number of holes, n_h.

$$n_e = n_h = n_i$$

where n_i is called intrinsic carrier concentration.

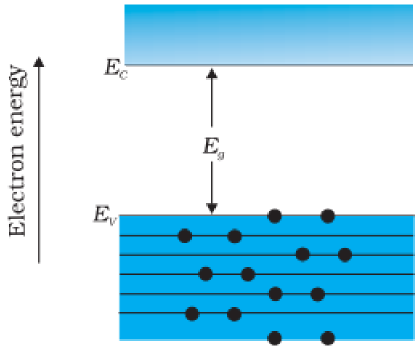


The free electrons move as conduction electron and gives rise to an electron current, I_e under an applied electric field. Under the action of an electric field, the holes move towards negative potential giving the hole current, I_h. The total current, I is thus the sum of the electron current I_e and the hole current I_h:

$$I = I_e + I_h$$

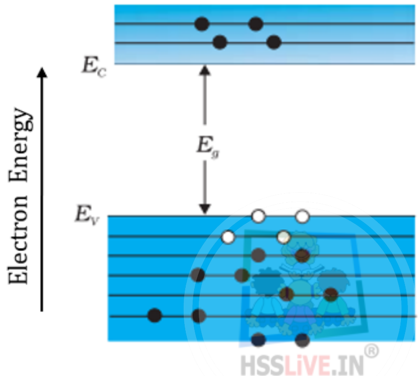
Energy-Band Diagram of an Intrinsic Semiconductor at T=0K

An intrinsic semiconductor will behave like an insulator at T = 0 K .



Energy-Band Diagram of an Intrinsic Semiconductor at T > 0K

At temperatures (T > 0K), some electrons are excited from the valence band to the conduction band, leaving equal number of holes there.



14.4 Extrinsic Semiconductor

When a small amount of a suitable impurity is added to the pure semiconductor, the conductivity of the semiconductor is increased . Such materials are known as **extrinsic semiconductors or impurity semiconductors.**

The deliberate addition of a desirable impurity is called doping and the impurity atoms are called dopants. Such a material is also called a doped semiconductor.

There are two types of dopants used in doping Si or Ge:

(i)Pentavalent (valency 5)

Eg: Arsenic (As), Antimony (Sb), Phosphorous (P), etc.

(ii)Trivalent (valency 3)

Eg: Indium (In), Boron (B), Aluminium (A*l*), etc.

Depending on the type of impurities added, there are two types of semiconductors –

(i) n-type semiconductor
(ii) p-type semiconductor

n-type semiconductor

n-type semiconductor is obtained by doping Si or Ge with pentavalent atoms (donors) like As, Sb, P, etc. The four valence electrons of pendavalent impurity atom bond with the four silicon neighbours ,while the fifth one is free to move in the lattice of the semiconductor ,at room temperature. Thus, the pentavalent dopant is donating one extra electron for conduction and hence is known as donor impurity.

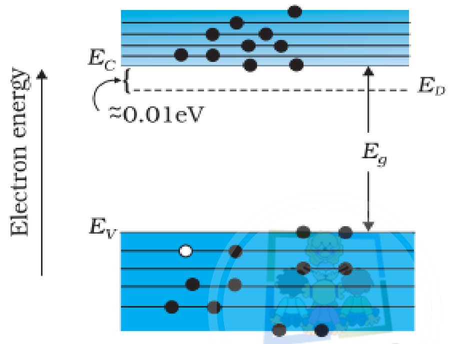
For n-type semiconductors, $n_e >> n_h$

Here electrons become the majority carriers and holes the minority carriers.

The electron and hole concentration in a semiconductor in thermal equilibrium is given by

$$n_e n_h = n_i^{\ 2}$$

Energy bands of n-type semiconductor at T > 0K



(a) T > 0K

For n-type Si semiconductor ,the donor energy level E_D ,is slightly below the bottom E_C of the conduction band .The electrons from this level move into the conduction band with very small supply of energy.

p-type semiconductor

p-type semiconductor is obtained when Si or Ge is doped with a trivalent impurity like A*l*, B, In, etc. The dopant has only 3 valence electrons and can form covalent bonds with neighbouring three Si atoms but does not have any electron to offer to the fourth Si atom. This vacancy of electron creates a hole. As the pendavalent impurities creates holes ,which can accept electrons from neighbouring atom, these impurities are called acceptor impurities.

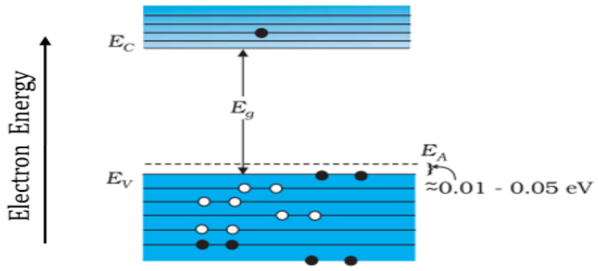
For p-type semiconductors, $n_h >> n_e$

Here holes become the majority carriers and electrons the minority carriers.

The electron and hole concentration in a semiconductor in thermal equilibrium is given by

$$n_e n_h = n_i^{\ 2}$$

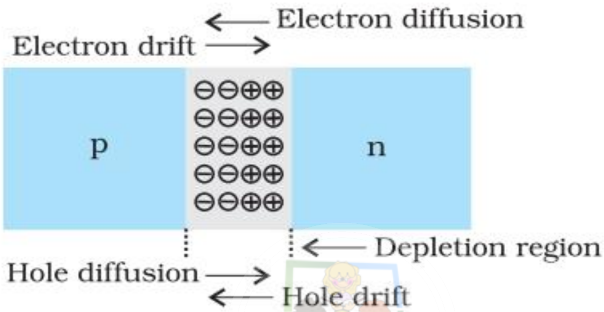
Energy bands of p-type semiconductor at T > 0K



(b) $T > 0K$

For p-type semiconductor, the acceptor energy level E_A is slightly above the top E_V of the valence band . With very small supply of energy an electron from the valence band can jump to the level E_A and ionise the acceptor .
negatively.

14.5 p-n junction



A p-n junction can be formed by adding a small quantity of pendavalent impurity to a p-type semiconductor or by adding a small quantity of trivalent impurity to an n-type semiconductor.

Two important processes occur during the formation of a p-n junction: diffusion and drift.

1.Diffusion

The holes diffuse from p-side to n-side (p → n) and electrons diffuse from n-side to p-side (n → p). This motion of charge carriers give rise to Diffusion current across the junction.
Due to diffusion, a layer of positive charge (or positive space-charge region) is developed on n-side of the junction and a layer of negative charge (or negative space-charge region) is developed on the p-side of the junction .

Depletion region (Depletion layer)

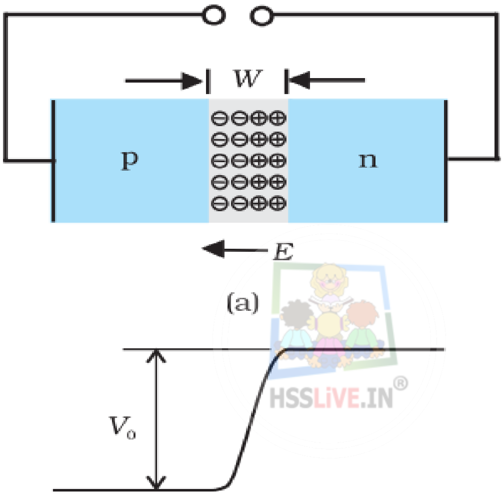
The space-charge region on either side of the junction together is known as depletion region. The depletion layer consist of immobile ion-cores and no free electrons or holes. This is responsible for a junction potential barrier.

2.Drift

The positive charge on n-side of the junction and negative charge on p-side of the junction develops an electric field. Due to this field, an electron(minority carrier) on p-side of the junction moves to n-side and a hole(minority carrier) on n- side of the junction moves to p-side. The motion of charge carriers due to the electric field is called drift.

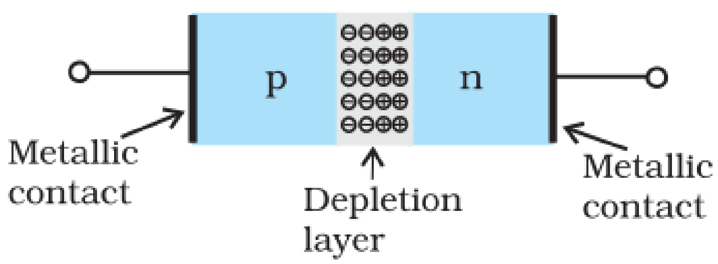
Initially, diffusion current is large and drift current is small. As the diffusion process continues, the electric field strength increases and hence drift current also increases. This process continues until the diffusion current equals the drift current.. **Thus in a p-n junction under equilibrium there is no net current.** .

Barrier Potential



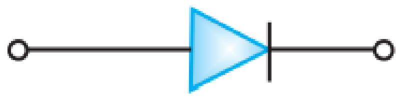
The loss of electrons from the n-region and the gain of electron by the p-region causes a difference of potential across the junction of the two regions. Since this potential tends to prevent the movement of electron from the n region into the p region, it is often called a barrier potential. **The barrier potential of a Ge diode is 0.2Vand that of a Si diode is 0.7V.**

14.6 Semiconductor Diode

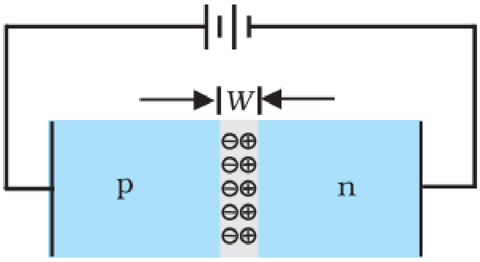


A semiconductor diode is basically a p-n junction with metallic contacts provided at the ends for the application of an external voltage. It is a two terminal device.

Symbol of a p-n junction Diode



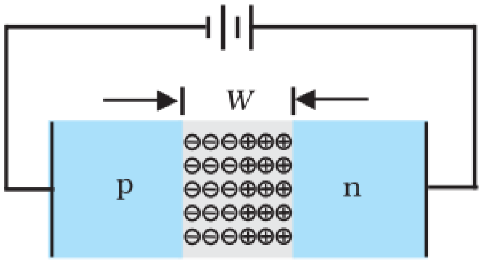
p-n junction diode under forward bias



If p-side of the diode is connected to the positive terminal and n-side to the negative terminal of the battery, it is said to be forward biased.

- The direction of the applied voltage (V) is opposite to barrier potential V_0. As a result, the depletion layer width decreases and the barrier height is reduced.
- The effective barrier height under forward bias is (V_0 – V).
- At high applied voltage, electrons from n-side cross the depletion region and reach p-side . Similarly, holes from p-side cross the junction and reach the n-side.
- This motion of majority carriers on either side gives rise to diffusion current.
- The magnitude of this current is usually in mA.

p-n junction diode under reverse bias

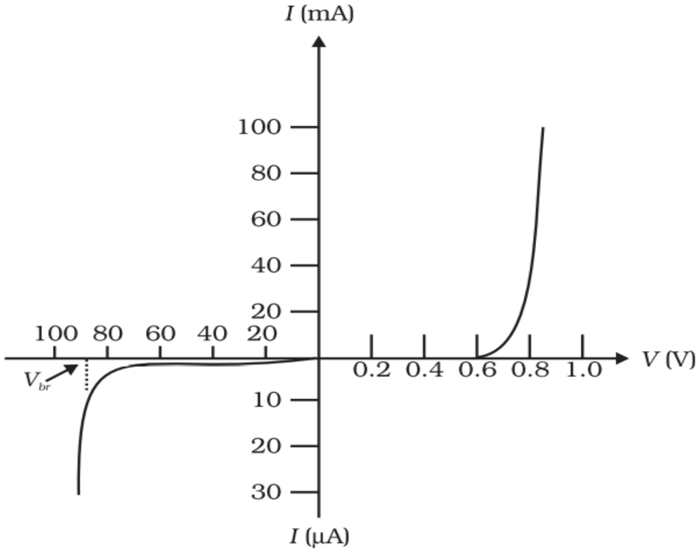


If n-side of the diode is connected to the positive terminal and p-side to the negative terminal of the battery, it is said to be reverse biased.

- The direction of the applied voltage (V) is same as barrier potential V_0. As a result, the depletion layer width increases and the barrier height is incresaed.
- The effective barrier height under reverse bias is (V_0 + V).
- The flow of electrons from n → p and holes from p → n is suppressed. Thus, diffusion current, decreases enormously compared to the diode under forward bias.

- The electricfield of the junction is such that the minority carriers are drifted to majority zone which gives rise to drift current.
- The drift current is of the order of a few μA.

V-I characteristics of a silicon diode.



- In forward bias, the current first increases very slowly, till the voltage across the diode crosses a certain value. . This voltage is called the **threshold voltage or cut-in voltage** (0.2V for germanium diode and 0.7 V for silicon diode).
- After threshold voltage, the diode current increases significantly , even for a very small increase in the diode bias voltage.
- For the diode in reverse bias, the current is very small (~μA) and almost remains constant with change in bias. It is called reverse saturation current. However, at very high reverse bias called **break down voltage V_{br},** the current suddenly increases. The general purpose diode are not used beyond the reverse saturation current region.

Threshold Voltage

The forward voltage beyond which the diode current increases significantly is called threshold voltage or cut-in voltage.

Break down Voltage

The reverse voltage at which the reverse current increases suddenly is called break down voltage.

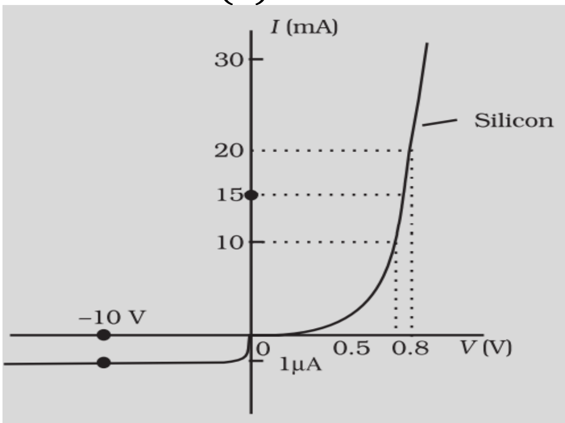
Dynamic Resistance(r_d)

Dynamic resistance is defined as the ratio of small change in voltage ΔV to a small change in current ΔI.

$$r_d = \frac{\Delta V}{\Delta I}$$

Example

The V-I characteristic of a silicon diode is shown in the Figure. Calculate the resistance of the diode at (a) $I_D = 15$ mA and (b) $V_D = -10$ V.



(a) From the curve, at I = 20 mA, V = 0.8 V

I = 10 mA, V = 0.7 V

$$r_{\text{forwrd bias}} = \frac{\Delta V}{\Delta I} = \frac{0.1}{10\text{x}10^{-3}} = 10\ \Omega$$

(b) From the curve at V = −10 V, I = −1 μA,

$$r_{\text{reverse bias}} = \frac{10}{1\text{x}10^{-6}} = 1.0 \times 10^{7}\ \Omega$$

14.7 Application of Junction Diode as a Rectifier

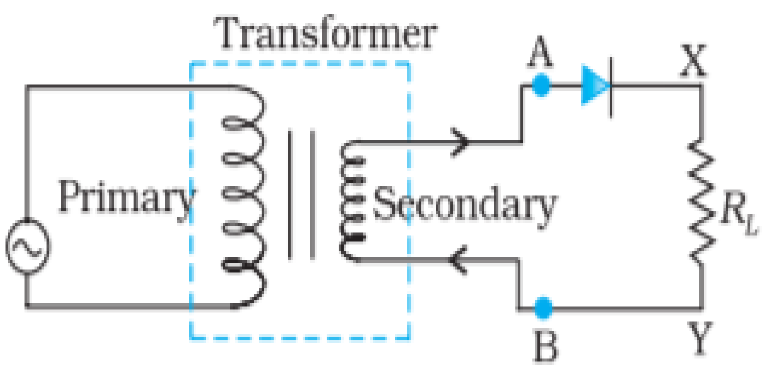
The diode allows current to pass only when it is forward biased.

If an alternating voltage is applied across a diode the current flows only in that part of the cycle when the diode is forward biased. This property is used to rectify alternating voltages .

Rectifier

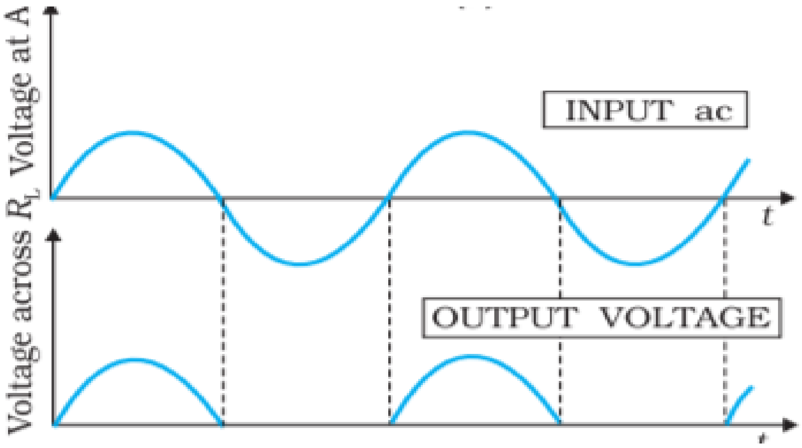
The process of conversion of ac voltage to dc voltage is called rectification and the circuit used for rectification is called rectifier.

Half wave Rectifier

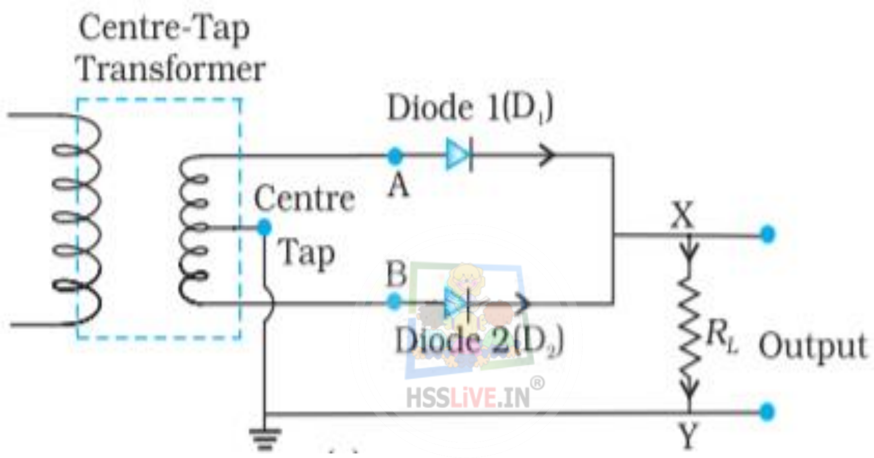


In the positive half-cycle of ac there is a current through the load resistor R_L and we get an output voltage, whereas there is no current in the negative half cycle. Since the rectified output of this circuit is only for half of the input ac wave it is called as half-wave rectifier.

Input ac voltage and output voltage waveforms from the rectifier circuit.

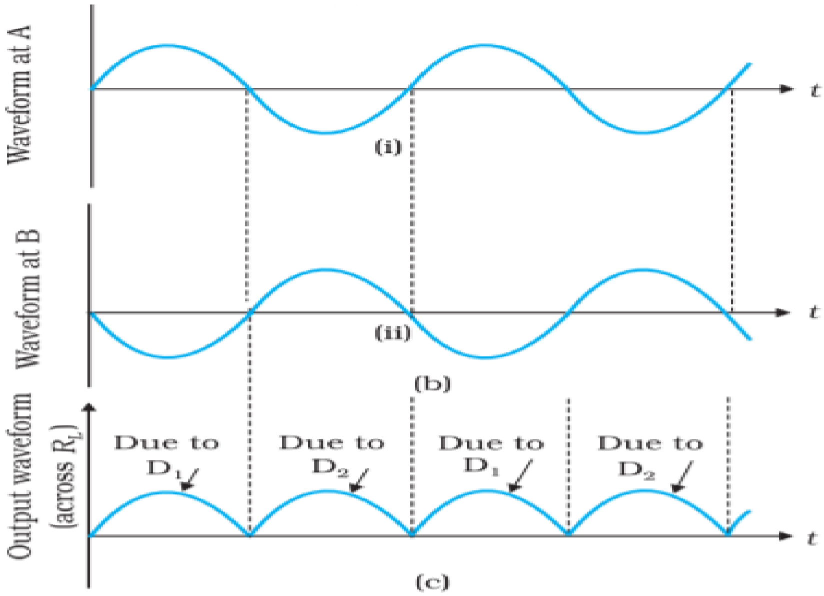


Full wave rectifier



- For a full-wave rectifier the secondary of the transformer is provided with a centre tapping and so it is called centre-tap transformer.
- During this positive half cycle, diode D_1 gets forward biased and conducts ,while D_2 being reverse biased is not conducting. Hence we get an output current and a output voltage across the load resistor R_L.
- During negative half cycle, diode D_1 would not conduct but diode D_2 conducts, giving an output current and output voltage across R_L in the same directionas in positive half.
- Thus, we get output voltage during both the positive as well as the negative half of the cycle. This is a more efficient circuit for getting rectified voltage or current than the halfwave rectifier.

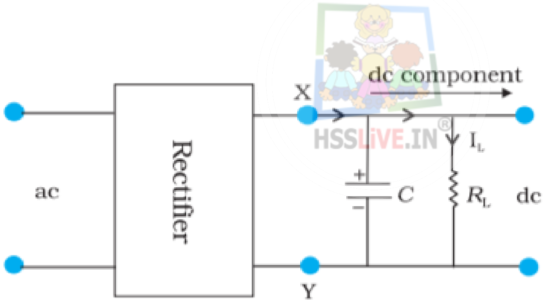
Input ac voltage and output voltage waveforms from the rectifier circuit.

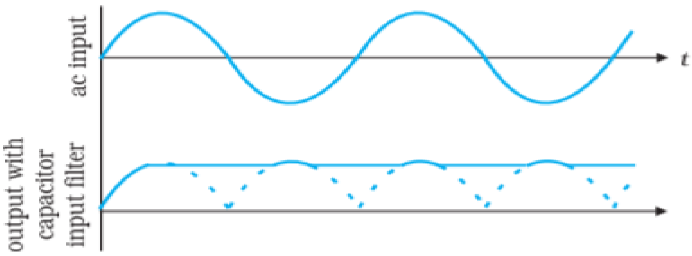


Filters

To get steady dc output from the pulsating voltage a capacitor is connected parallel to the output terminals.

The cicuits that filter out the ac ripple and give a pure dc voltage are called filters.





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