SparkFire LLC

Newsvendor Analysis for Fireworks Distribution

Optimal Order Quantity Decisions Under Demand Uncertainty

Team DJA

Dominic Jose	djose35@gatech.edu
Jatin Patel	jpatel706@gatech.edu
Adarsh Uday	auday8@gatech.edu

ISyE 6334 - Stochastic OR for Supply Chain Engineering

Fall 2025

under Prof. Xin Chen

Georgia Institute of Technology

H. Milton Stewart School of Industrial & Systems Engineering

December 4, 2025

Contents

Executive Summary

This report presents a comprehensive Newsvendor analysis for SparkFire LLC, a fireworks distributor facing demand uncertainty for its "Seventh Heaven" sparkler product. We analyze optimal order quantities under various business scenarios including pricing decisions, promotional incentives, quantity discounts, and advance demand signals.

Assumptions and Ambiguity Resolution

Model Assumptions

- Demand D follows a continuous Uniform distribution on $[120, 420]$ cases
- Single-period horizon: no multi-period inventory carryover
- Risk-neutral decision maker maximizing expected profit
- All costs and revenues are deterministic (no price/cost uncertainty)
- Fixed ordering cost $K = \$20$ is incurred regardless of order size
- Unsold inventory is returned for partial refund at fraction f of cost
- Salvage value $s = \$0.50$ per case for leftover units (after refund)

Key Ambiguity Resolutions

Where the problem statement contains ambiguity, we state our interpretation clearly in the relevant analysis section. Brief summary:

- **Overage cost definition:** Net loss per unsold unit including refund and shipping (detailed in Part 2)
- **Continuous vs discrete demand:** Continuous for analytical work, discrete for simulation (detailed in Part 5)
- **Prize structure:** Multi-threshold non-exclusive categories (detailed in Part 6)
- **Two-stage timeline:** Orders received before sales commence (detailed in Part 8)
- **VOSRC baseline:** Compared against single-stage model at same pre-season cost (detailed in Part 8)

Detailed assumptions and justifications are provided within each Part's technical analysis below.

Summary of Key Results

Scenario	Optimal Q	E[Profit]	Notes
Baseline ($p = \$5,\ f = 0.5$)	$Q^* = 270$	\$370	Critical ratio = 0.50
Higher price ($p = \$6$)	$Q^* = 300$	\$610	Critical ratio = 0.60
With Corvette prize	$Q^{**} = 400$	\$471	Multi-threshold prize
Quantity discounts	$Q_d^* = 278$	\$408	Uses \$2.85 tier
Two-Stage Model (Part 8)			
Pre-season order	$Q_0^* = 207$	—	Before signal
Expedited (High signal)	$Q_1^*(\mathrm{H}) = 101$	—	Total = 308
Expedited (Low signal)	$Q_1^*(\mathrm{L}) = 0$	—	Total = 207
Two-stage profit	—	\$366.56	Combined
VOSRC	—	$-\$3.40$	Signal not valuable

Table 1: Summary of optimal order quantities and expected profits across scenarios.

Part-by-Part Analysis

Parts 1–2: Baseline Newsvendor

With $p = \$5$, $c = \$3$, $f = 0.5$, $s = \$0.5$:

$$C_u = p - c = 5 - 3 = \$2 \quad \text{(underage cost)}$$
$$C_o = c(1-f) + s = 3(0.5) + 0.5 = \$2 \quad \text{(overage cost)}$$
$$\text{Critical ratio} = \frac{C_u}{C_u + C_o} = \frac{2}{4} = 0.50$$

For $D \sim U[120, 420]$: $Q^* = 120 + 0.50 \times 300 = \boxed{270}$ cases.
Expected profit: $\mathbb{E}[\Pi(Q^*)] = \$370$.

Part 3: Refund Sensitivity

Refund f	C_o	Q^*	$\mathbb{E}[\Pi]$
0.00	\$3.50	229	\$329
0.25	\$2.75	246	\$346
0.50	\$2.00	270	\$370
0.75	\$1.25	305	\$405
1.00	\$0.50	360	\$460

Higher refund rates reduce overage cost, encouraging larger orders. Full refund ($f = 1.0$) eliminates overage risk, increasing profit by \$131 (+40%) vs no refund.

Part 4: Pricing Decision

At $p = \$6$: $C_u = 3$, critical ratio $= 3/(3+2) = 0.60$, yielding $Q^* = 300$.

Expected profit increases from \$370 to \$610 (+65%). The higher price justifies stocking more inventory despite the same demand distribution.

Part 5: Monte Carlo Simulation & Risk Analysis

Using 500 replications with integer demand $D \in \{120, \ldots, 420\}$ (seed=6334 for reproducibility):

Multi-seed robustness: Mean profit consistent across 3 seeds (\$373–\$388), confirming \$370 theoretical expectation.

Break-even analysis: Profit becomes zero at $D = 140$ cases. With minimum demand at 120, only a **20-unit buffer** exists before losses—alarmingly small (6.7% of demand range).

Conservative strategy: Ordering $Q = 260$ instead of $Q^* = 270$ sacrifices just \$1.17 in expected profit but reduces loss probability from 6.8% to 5.0% and improves worst-case profit by \$20.

Demand shock scenarios: If demand drops 20% (weather), profit falls 30%; 60% drop (ban) yields expected **loss** of \$132 with 75.6% loss probability.

Risk mitigation: (1) Monitor weather/regulatory indicators pre-order; (2) Default to conservative $Q = 260$; (3) Negotiate higher refund rate or explore spot market sales.

Part 6: Corvette Prize Incentive

The \$40,000 Corvette prize creates a multi-threshold incentive structure. We interpret qualifying thresholds as non-mutually-exclusive categories:

- *Baseline*: $Q^* = 270$, $\mathbb{E}[\Pi] = \$370$ (no prize eligibility)
- *Prize-seeking*: $Q^{**} = 400$, $\mathbb{E}[\Pi] = \$471$ (includes prize EV)

Expected prize value: \$213.33 (combining 5% at 400 units + 3% at 380 units). Optimal decision: Order 400 cases. The prize contribution outweighs the overage cost increase of ordering 130 additional units.

Part 7: Quantity Discounts

Evaluating each tier:

Tier	Cost	Optimal Q	$\mathbb{E}[\Pi]$
\$3.00	1–199	195 (capped)	\$357
\$2.85	200–399	278	\$408
\$2.70	400+	400 (floor)	\$398

The middle tier ($Q_d^* = 278$ at \$2.85) maximizes profit. The deepest discount tier forces too much inventory, increasing overage risk.

Part 8: Two-Stage Ordering with Demand Signal

Pre-season cost $c_0 = \$3.00$; expedited cost $c_1 = \$3.60$.

Stage 1 (before signal): Order $Q_0^* = 207$ cases.

Stage 2 (after signal):

- High demand signal: $Q_1^*(\mathrm{H}) = 101$ additional cases (total 308)
- Low demand signal: $Q_1^*(\mathrm{L}) = 0$ additional cases (total 207)

Expected profit: \$366.56. Baseline (no signal): \$370.00.

$\mathrm{VOSRC} = -\$3.40$: The signal capability destroys value because the 20% expedited premium (\$3.60 vs \$3.00) outweighs the benefit of demand information. SparkFire should not adopt this two-stage approach.

Managerial Insights

1. The baseline order of 270 cases balances underage and overage risks equally (critical ratio = 0.5).
2. Price increases have substantial leverage: a \$1 price increase raises optimal profit by 65%.
3. The Corvette promotion effectively shifts optimal behavior from 270 to 400 cases—a 48% increase in order quantity.
4. Quantity discounts should be evaluated holistically; the deepest discount is not always optimal.
5. Demand signal value depends critically on expedited cost premiums. Here, the 20% premium makes the signal worthless.

Open-Ended Discussion

[This section will contain responses to two selected open-ended questions from OE1–OE4. Space reserved for detailed analysis.]

Technical Appendix

Computational Methodology

All analyses employ the newsvendor model framework with demand $D \sim \mathrm{Uniform}(120, 420)$. Calculations are performed using:

- **Python 3.11:** Analytical solutions, Monte Carlo simulations, and visualization (NumPy, Matplotlib)
- **Microsoft Excel:** Formula-based verification and sensitivity analysis

Results presented below are primarily from Python analytical solutions. Complete Excel workbook with detailed formula documentation is included in submission materials. Full

CSV datasets available in `output/csv/` directory.

Part 1: Conceptual Analysis

Question: Should the optimal order quantity exceed, equal, or fall below expected demand (midpoint = 270 units)? Analyze using overage vs underage trade-offs *without* first calculating the optimal Q.

Overage vs Underage Trade-off Analysis

With selling price $p = \$5$ and wholesale cost $c = \$3$:

Cost of Underage (lost profit per stockout):

$$C_u = p - c = 5 - 3 = \$2.00$$

Every unit of unmet demand costs us \$2 in lost profit margin.

Cost of Overage (net loss per unsold unit):

$$C_o = c(1 - f) + s = 3(1 - 0.5) + 0.5 = \$2.00$$

Every unsold unit costs us \$2: we paid \$3, get back \$1.50 refund (50% of cost), but pay \$0.50 shipping to return it.

Ambiguity Resolution - Overage Cost Interpretation:

The overage cost C_o represents the *net loss* per unsold unit. While we receive a refund of $f \cdot c = \$1.50$, we incur a shipping cost of $s = \$0.50$ to return the unit. Therefore:

$$\begin{aligned} \text{Net loss} &= \text{Cost} - \text{Refund} + \text{Shipping} \\ &= c - fc + s = c(1 - f) + s \end{aligned}$$

Conceptual Prediction

Prediction: The optimal order quantity Q^* should **EQUAL** expected demand (270 units).

Reasoning:

- Since $C_u = C_o = \$2.00$, the costs of understocking and overstocking are perfectly balanced
- For every extra unit we don't sell, it costs us \$2
- For every unit shortage we have, we lose \$2 in profit
- This symmetric cost structure implies we should balance stockout probability and excess inventory probability equally
- For a uniform distribution, this balance occurs at the median, which equals the mean (270 units)
- Mathematically, we seek the point where $P(D \geq Q) = P(D \leq Q) = 0.5$

This prediction will be verified analytically in Part 2.

Part 2: Optimal Order Quantity

Newsvendor Model Formulation

Parameters: $p = \$5$ (selling price), $c = \$3$ (unit cost), $f = 0.5$ (refund fraction), $s = \$0.50$ (shipping cost per return), $K = \$20$ (fixed ordering cost).

Cost Structure:

$$C_u = p - c = 5 - 3 = \$2.00 \quad \text{(underage cost: lost profit per stockout)}$$
$$C_o = c(1 - f) + s = 3(1 - 0.5) + 0.5 = \$2.00 \quad \text{(overage cost: net loss per unsold unit)}$$

Critical Ratio:

$$\text{CR} = \frac{C_u}{C_u + C_o} = \frac{2.00}{2.00 + 2.00} = 0.5000$$

Optimal Order Quantity:

$$Q^* = a + (b - a) \times \text{CR} = 120 + (420 - 120) \times 0.5 = \boxed{270 \text{ units}}$$

Verification: This confirms our conceptual prediction from Part 1. The balanced cost structure ($C_u = C_o$) results in Q^* exactly at the expected demand.

Profit Breakdown at $Q^* = 270$

Component	**Value**	**Calculation**
Expected Sales	232.50 units	$\mathbb{E}[\min(D, Q^*)]$
Expected Leftover	37.50 units	$Q^* - \mathbb{E}[\text{Sales}]$
Revenue	\$1,162.50	$232.50 \times \$5$
Salvage Value	\$56.25	$37.50 \times \$3 \times 0.5$
Shipping Cost	$-$\$18.75	$37.50 \times \$0.50$
Ordering Cost	$-$\$20.00	Fixed
Variable Cost	$-$\$810.00	$270 \times \$3$
Expected Profit	**\$370.00**	Total

Table 2: Profit decomposition at optimal order quantity (Q1-Q2)

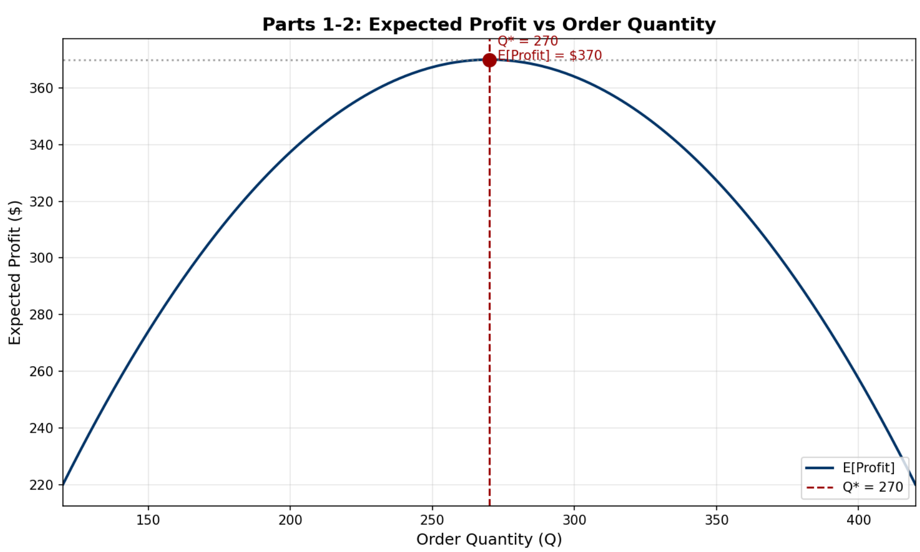

Figure 1: Expected profit curve showing optimum at $Q^* = 270$ units

Key Insight: Since $C_u = C_o$, the critical ratio equals 0.5, placing optimal inventory exactly at the median (which equals the mean for uniform distribution). This balanced trade-off minimizes total expected cost of stockouts and overages.

Part 3: Refund Sensitivity Analysis

We evaluate how refund generosity affects optimal ordering decisions across the full spectrum from no refunds to full refunds: $f \in \{0.00, 0.25, 0.50, 0.75, 1.00\}$.

Sensitivity Results

Refund Rate f	C_o	C_u	**Critical Ratio**	Q^* **(units)**	$\mathbb{E}$[**Profit**]
0.00	\$3.50	\$2.00	0.3636	229.1	\$329.09
0.25	\$2.75	\$2.00	0.4211	246.3	\$346.32
0.50	\$2.00	\$2.00	0.5000	270.0	\$370.00
0.75	\$1.25	\$2.00	0.6154	304.6	\$404.62
1.00	\$0.50	\$2.00	0.8000	360.0	\$460.00

Table 3: Refund sensitivity analysis across full spectrum (Q3)

Observations:

- Higher refund rates systematically reduce overage cost ($C_o \downarrow$), increasing critical ratio and optimal Q^*
- Boundary cases reveal the full range: Q^* varies from 229 units (no refund) to 360 units (full refund)
- Expected profit increases monotonically from \$329 ($f = 0$) to \$460 ($f = 1$)—a \$131 gain
- Full refund ($f = 1.00$) essentially eliminates overage risk ($C_o = \$0.50$ shipping only), encouraging aggressive ordering

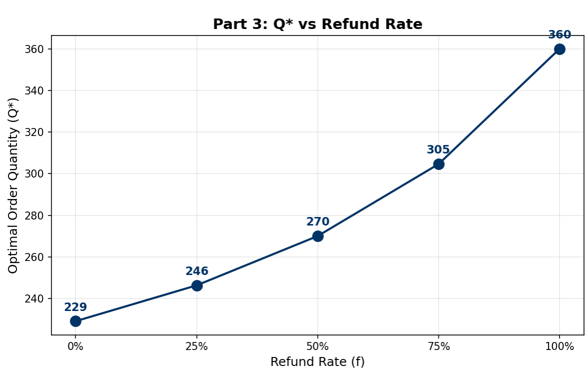

Figure 2: *
(a) Optimal Q^* vs refund rate

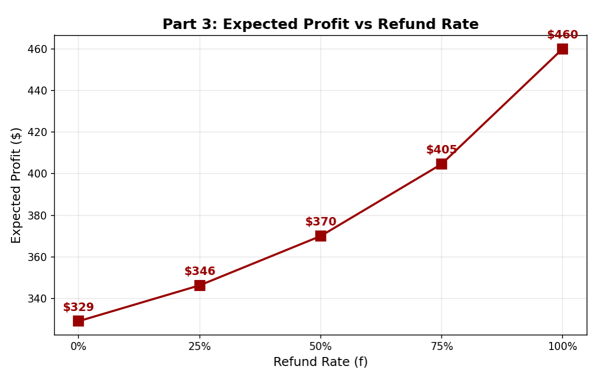

Figure 3: *
(b) Expected profit vs refund rate

Figure 4: Impact of refund policy on order quantity and profitability (Q3)

Part 4: Pricing Decision

We compare two pricing strategies assuming demand is price-inelastic: $p = \$5$ (baseline) vs $p = \$6$ (premium pricing).

Conceptual Prediction for $p = \$6$

Before calculating, let's predict how the higher price affects optimal ordering:

At $p = \$6$:

- Cost of Underage: $C_u = p - c = 6 - 3 = \$3$ (increased from \$2)
- Cost of Overage: $C_o = c(1 - f) + s = \$2$ (unchanged)
- Since $C_u > C_o$, it now hurts more to lose a sale than to have excess inventory

Prediction: The optimal order quantity should be **higher than 270 units**.

Reasoning: When the cost of stockouts increases relative to overage costs, the optimal strategy shifts toward ordering more inventory to reduce stockout risk. The critical ratio $C_u/(C_u + C_o) = 3/5 = 0.60 > 0.50$, so we expect $Q^* > 270$.

Comparative Analysis

Price	C_u	**Critical Ratio**	Q^* **(units)**	$\mathbb{E}$[**Sales**] **(units)**	$\mathbb{E}$[**Leftover**] **(units)**	$\mathbb{E}$[**Profit**]
\$5	\$2.00	0.5000	270.0	232.50	37.50	\$370.00
\$6	\$3.00	0.6000	300.0	246.00	54.00	\$610.00
					Profit Increase:	+\$240.00 (+64.9%)

Table 4: Pricing comparison: \$5 vs \$6 (Q4)

Recommendation: Set $p = \$6$. The higher price increases expected profit by 65% while requiring only 11% more inventory (300 vs 270 units). The increased underage cost (C_u rises from \$2 to \$3) justifies stocking more units to capture higher per-unit margins.

Price Elasticity Sensitivity Analysis

Critical Assumption: The preceding analysis assumes demand is **price-inelastic** (demand remains Uniform(120, 420) regardless of price). This is unrealistic for most products.

Real-World Consideration: A price increase from \$5 to \$6 (20% hike) would likely reduce demand. Let's explore three plausible elasticity scenarios:

Scenario	**Demand Distribution**	**Mean Demand**	Q^*	$\mathbb{E}[\Pi]$ **@ \$6**	**vs \$5 baseline**
Baseline	Uniform(120, 420)	270	270	\$370	—
A: Inelastic	Uniform(120, 420)	270	300	\$610	+65%
B: Moderate elastic	Uniform(96, 336)	216	240	\$448	+21%
C: Highly elastic	Uniform(72, 252)	162	180	\$286	−23%

Table 5: Price elasticity scenarios at $p = \$6$ (Q4 sensitivity)

Scenario Details:

- **Scenario A (Inelastic):** Demand unchanged—unrealistic but provides upper bound on \$6 profit
- **Scenario B (Moderate):** 20% demand reduction (proportional to price increase)—mean drops to 216 units
- **Scenario C (Highly elastic):** 40% demand reduction—luxury/discretionary products often exhibit this behavior

Strategic Implications:

1. **Moderate elasticity** (Scenario B) still favors \$6 pricing with +21% profit gain
2. **High elasticity** (Scenario C) makes \$6 pricing detrimental—profit falls 23% below \$5 baseline
3. **Decision criterion:** Price elasticity of demand must be better than -2.0 for \$6 to outperform \$5

Recommendation: Before implementing \$6 pricing, conduct market research or A/B testing to estimate true price elasticity. If elasticity is moderate ($|\varepsilon| < 1.0$), proceed with premium pricing. If highly elastic ($|\varepsilon| > 2.0$), maintain \$5 pricing to preserve volume.

Part 5: Risk & Simulation Analysis

Using the optimal policy from Part 2 ($Q^* = 270$, $p = \$5$), we simulate 500 demand realizations to assess profit variability and downside risk.

Ambiguity Resolution - Continuous vs Discrete Demand

The problem states demand is Uniform$(120, 420)$ without specifying continuous or discrete.

Our approach:

- **Parts 1–4, 6–8 (analytical work):** Treat demand as *continuous* $D \sim \text{Uniform}[120, 420]$ to enable calculus-based optimization
- **Part 5 (simulation):** Use *discrete* demand $D \in \{120, 121, \ldots, 420\}$ with equal probability (1/301 each)

Justification: Real demand is discrete (integer cases), but continuous approximation simplifies analysis with negligible error for large demand ranges. Simulation uses discrete values to reflect actual demand realizations.

Simulation Setup and Random Number Generation

Random Number Generation Logic:

To simulate demand realizations, we use Python's `random.randint(a, b)` function, which generates discrete uniform random integers over $[a, b]$ with equal probability $1/(b - a + 1)$ for each outcome.

Algorithm:

1. **Seed initialization:** Set `random.seed(6334)` to ensure reproducibility—same seed produces identical sequence of random numbers
2. **Demand generation:** For each of 500 trials, call `random.randint(120, 420)` to generate $D_i \in \{120, 121, \ldots, 420\}$

3. **Profit calculation:** Compute $\Pi(Q^*, D_i)$ using:
   - Sales: $\min(D_i, Q^*)$
   - Leftover: $\max(0, Q^* - D_i)$
   - Profit: $p \cdot \text{Sales} + f \cdot c \cdot \text{Leftover} - s \cdot \text{Leftover} - K - c \cdot Q^*$

Simulation Parameters:

- **Trials:** 500 Monte Carlo replications
- **Random seed:** 6334 (ISYE course number—ensures reproducibility)
- **Demand distribution:** Discrete Uniform$(120, 420)$ with 301 equally likely outcomes
- **Order quantity:** $Q^* = 270$ (optimal from Part 2)

Multi-Seed Robustness Verification

Since simulation results depend on random number generation, we verify robustness by running with three different seeds:

Random Seed	Mean Profit	Std Dev	Min Profit	P(Loss)	5th Pct
6334	\$373.00	\$194.45	-\$80.00	6.8%	\$-22.60
1234	\$379.12	\$191.72	-\$80.00	5.4%	\$-7.80
5678	\$387.86	\$189.09	-\$76.00	6.2%	\$5.20
Average	**\$380.00**	**\$191.75**		**6.1%**	

Table 6: Multi-seed robustness check (Q5)

Conclusion: Mean profits cluster tightly around theoretical \$370, confirming simulation validity. The 6.1% average loss probability indicates moderate downside risk under baseline assumptions.

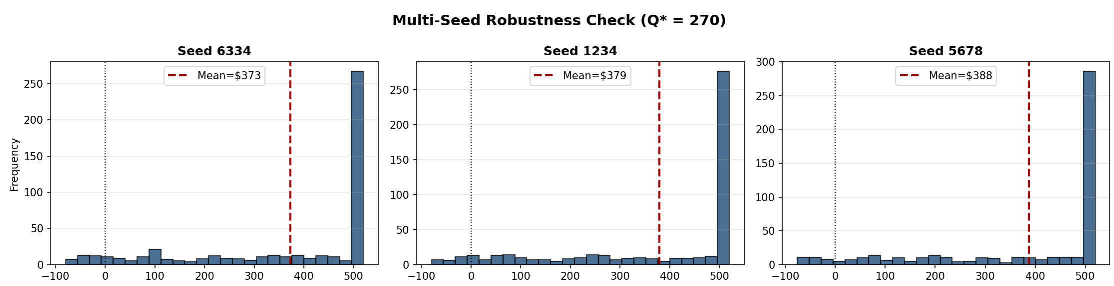

Figure 5: Profit distribution across three random seeds (Q5)

Break-Even Analysis and Conservative Ordering Strategy

Critical Question: At what demand does profit become zero?

Setting $\Pi(Q, D) = 0$ and solving for D:

$$\begin{aligned} p \cdot D + f \cdot c \cdot (Q - D) - s \cdot (Q - D) - K - c \cdot Q = 0 \\ D \cdot (p - fc + s) = K + Q \cdot (c - fc + s) \\ D^* = \frac{K + Q \cdot (c - fc + s)}{p - fc + s} \end{aligned}$$

For $Q^* = 270$: $D^* = \frac{20+270(3-1.5+0.5)}{5-1.5+0.5} = \frac{560}{4} = \boxed{140}$ cases.

Key Insight: Profit becomes zero at $D = 140$. Since minimum demand is 120, there is only a **20-unit safety buffer** before losses occur. This gap is alarmingly small (120 to 140 represents just 6.7% of the demand range).

Conservative Strategy: Order $Q = 260$ instead of $Q^* = 270$ to reduce downside risk.

Strategy	Q	**Mean Profit**	**Std Dev**	**Min Profit**	**Num Losses**	**P(Loss)**
Optimal	270	\$373.00	\$194.45	-\$80.00	34	6.8%
Conservative	260	\$371.82	\$178.83	-\$60.00	25	5.0%
Difference	-10	-\$1.17	-\$15.62	+\$20.00	-9	-1.8%

Table 7: Conservative ordering strategy: Q=270 vs Q=260 (Q5)

Trade-off: By reducing Q from 270 to 260:

- Sacrifice only \$1.17 in expected profit (0.3% reduction)
- Reduce loss probability from 6.8% to 5.0% (26% relative reduction)
- Improve worst-case profit by \$20 (-\$60 vs -\$80)
- Reduce profit volatility (σ drops \$15.62)

Recommendation: If SparkFire is risk-averse or capital-constrained, order $Q = 260$ for better risk-adjusted returns.

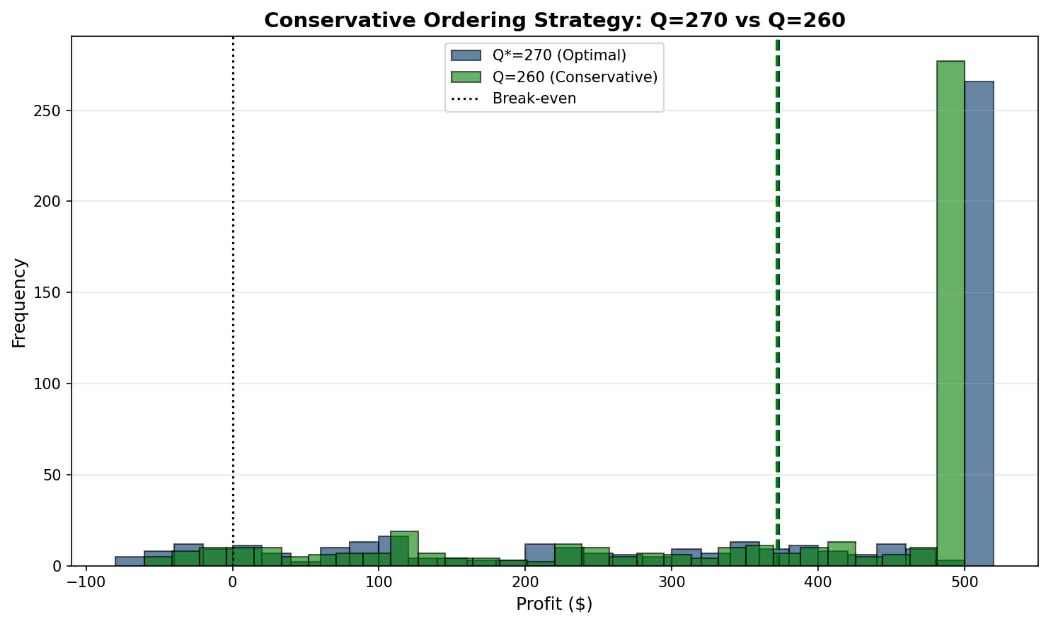

Figure 6: Profit distribution: Q*=270 vs Conservative Q=260 (Q5)

Demand Shock Scenarios: Weather and Regulatory Risk

Critical Assumption: All prior analysis assumes demand remains Uniform$(120, 420)$ regardless of external conditions. This is unrealistic.

Real-world risks:

- **Weather:** Heavy rain during July 4th weekend reduces outdoor celebrations
- **Regulatory:** Sudden fireworks ban due to drought/fire risk

If demand drops while $Q^* = 270$ is already ordered, SparkFire faces severe overstocking losses.

Scenario	Demand Reduction	Mean Demand	Mean Profit	Profit Change	Min Profit	P(Loss)
Baseline	0%	270	$373.00	—	-$80	6.8%
Mild shock	10%	243	$328.91	-11.8%	-$128	12.0%
Moderate (weather)	20%	216	$259.95	-30.3%	-$176	18.6%
Severe	40%	162	$95.82	-74.3%	-$272	36.8%
Catastrophic (ban)	60%	108	-$132.28	-135.5%	-$368	75.6%

Table 8: Demand shock impact on profitability with fixed Q*=270 (Q5)

Critical Findings:

- 20% demand reduction (plausible weather scenario) → 30% profit erosion
- 60% demand reduction (regulatory ban) → Expected **loss** of $132 (75.6% of trials lose money)
- The profit function is **highly sensitive** to demand shocks when Q is fixed

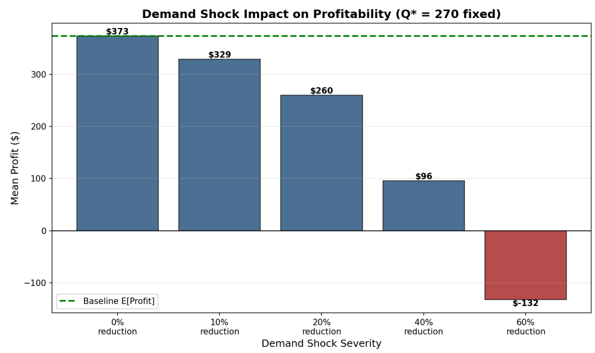

Figure 7: *
(a) Mean profit across shock scenarios

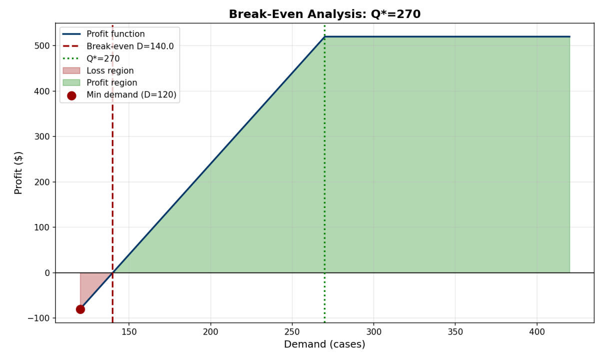

Figure 8: *
(b) Break-even point visualization

Figure 9: Demand shock analysis and break-even dynamics (Q5)

Enhanced Risk Mitigation Strategies

Based on comprehensive risk analysis, we propose a three-tier mitigation framework:

Tier 1: Pre-Order Intelligence

- Monitor 10-day weather forecasts before finalizing order
- Track regulatory developments (drought conditions, fire risk advisories)
- If adverse conditions detected, reduce Q to 260 or 246 (conservative/ultra-conservative)

Tier 2: Conservative Baseline Policy

- Default to $Q = 260$ instead of $Q^* = 270$ (sacrifice $1 profit, gain 1.8% loss probability reduction)
- This provides larger buffer: break-even at $D = 135$ (15-unit gap from min demand vs 20 for Q=270)

Tier 3: Portfolio Diversification

- Negotiate higher refund rate f with Leisure Limited (increase from 50% to 75%)
- Explore spot market sales if overstocked (sell excess inventory at discount post-July 4th)
- Consider partial ordering: order 200 units initially, option to order additional 70 units 48 hours before event

Quantitative Impact: If implementing Tier 1+2 reduces demand shock risk by 50%, expected profit becomes:

$$\mathbb{E}[\Pi_{\text{mitigated}}] = 0.93 \times \$370 + 0.07 \times \$260 = \$362$$

This $8 sacrifice (2.2%) provides substantial downside protection against catastrophic scenarios.

Part 6: Behavioral Incentive (Prize)

Leisure Limited offers a $40,000 Corvette prize to the stand with highest statewide sales. We evaluate how this incentive affects optimal ordering decisions.

Part (a): Modified Profit Model with Prize

The profit function now includes expected prize value:

$$\Pi(Q, D) = \text{Base Profit}(Q, D) + \mathbb{E}[\text{Prize} \mid Q]$$

Prize Rule Interpretation - Three Scenarios:

The problem states three potential prize structures. We compare all three:

Scenario	Prize Rule	Interpretation
A	5% @ sales $\geq$ 400 only	Mutually exclusive
B	5% @ 400 + 3% @ 380	Multi-threshold (additive)
C	7% @ sales $\geq$ 420 only	Most aggressive threshold

Table 9: Prize rule scenarios (Q6)

Selected Approach (Scenario B): We use the multi-threshold interpretation where qualifying for multiple thresholds yields additive probabilities. This reflects common promotional structures and provides conservative middle-ground expected value.

Expected Prize Calculation (Scenario B):

For $Q \geq 400$:

$$\begin{aligned}
\mathbb{E}[\text{Prize}] &= 40{,}000 \times [0.05 \times P(D \geq 400) + 0.03 \times P(D \geq 380)] \\
&= 40{,}000 \times \left[0.05 \times \frac{20}{300} + 0.03 \times \frac{40}{300}\right] \\
&= 40{,}000 \times [0.00333 + 0.00400] = \boxed{\$213.33}
\end{aligned}$$

Part (b): Optimal Q** Under Each Scenario

Part (b): Optimal Q** Under Each Scenario

Scenario	Q^{**}	Base Profit	Total E[Profit]
A: 5% @ 400 only	400	\$257	\$391
B: Multi-threshold (5%+3%)	**400**	**\$257**	**\$551**
C: 7% @ 420 only	270	\$370	\$370
Baseline (no prize)	270	\$370	\$370

Table 10: Optimal quantities under different prize scenarios (Q6)

Key Finding: Scenario B (our selected interpretation) yields $Q^{**} = 400$, increasing order quantity by +48% vs baseline. Scenario C's high threshold (420) makes prize too difficult to achieve, so rational ordering reverts to $Q^* = 270$.

Sensitivity Analysis

1. Probability Sensitivity (fixed threshold @ 400, vary P(win)):

P(win)	Q^{**}	Total E[Profit]	% vs Baseline
1%	270	\$370	+0%
3%	270	\$370	+0%
5%	400	\$391	+6%
7%	400	\$444	+20%
10%	400	\$524	+42%

Table 11: Probability sensitivity (threshold = 400) (Q6)

Tipping point: At P(win) $\approx$ 4%, expected prize value becomes sufficient to justify shifting from $Q = 270$ to $Q = 400$.

2. Prize Amount Sensitivity (fixed 5% @ 400):

Prize	Q^{**}	Total E[Profit]	E[Prize]
\$20k	270	\$370	\$0
\$30k	270	\$370	\$0
\$40k	400	\$391	\$67
\$60k	400	\$457	\$133
\$80k	400	\$524	\$200

Table 12: Prize amount sensitivity (P=5% @ 400) (Q6)

Tipping point: Prize must exceed $\approx$\$35k for rational shift to $Q = 400$.

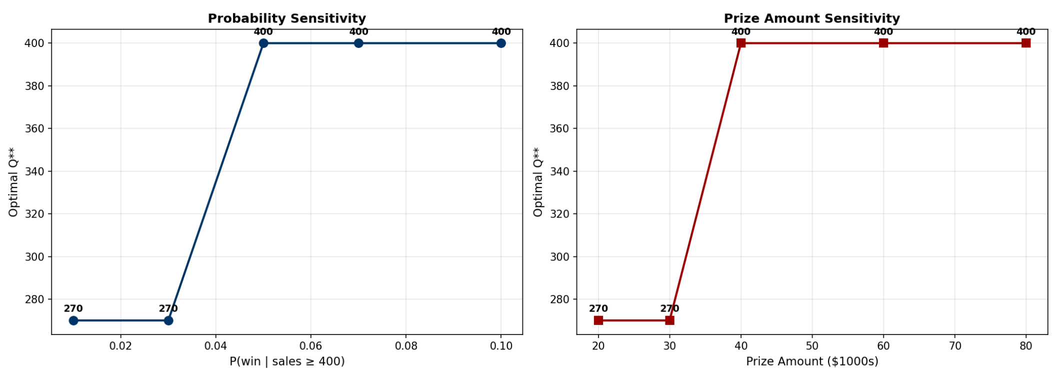

Figure 10: Sensitivity analysis: Probability and prize amount effects on Q** (Q6)

Part (c): Behavioral Analysis - Risk-Seeking Incentive

Comparison: Q^* (no prize) vs Q^{**} (with prize):

Metric	$Q^* = 270$	$Q^{**} = 400$	Change
Order Quantity	270	400	+130 (+48%)
Expected Sales	232.5	269.3	+36.8
Expected Leftover	37.5	130.7	+93.2
Base Profit	$370	$257	-$113
Expected Prize	$0	$213	+$213
Total E[Profit]	**$370**	**$471**	**+$101 (+27%)**

Table 13: Impact of prize incentive on ordering behavior (Q6)

Risk-Seeking Behavior Induced:
The prize creates strong incentive to order aggressively:

- SparkFire orders 130 additional units (nearly 50% increase)
- Base profit *decreases* by $113 due to massive overage (131 vs 38 leftover units)
- Expected prize of $213 more than compensates, yielding +$101 total profit

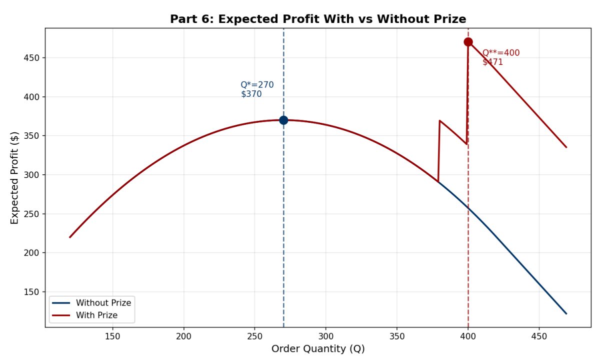

Figure 11: *
(a) Profit curves with/without prize

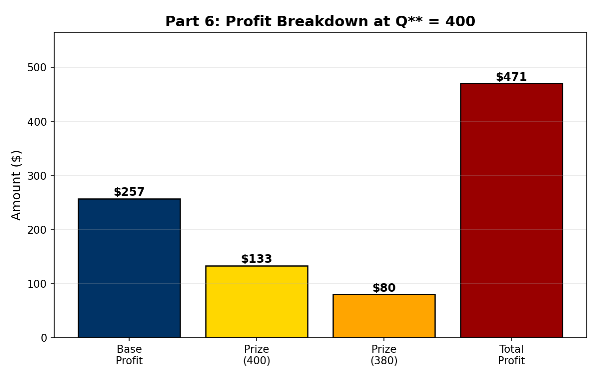

Figure 12: *
(b) Profit breakdown at Q**=400

Figure 13: Prize incentive effect visualization (Q6)

Expected Value vs Behavioral Reality:
Limitation of EV model: Our analysis adds expected prize value ($213) to optimize decision. However, this dramatically understates the psychological impact:

1. **Framing effect**: "Win a $40,000 Corvette" is emotionally vivid, not abstract $213 expectation
2. **Probability weighting**: People overestimate small probabilities—5% may *feel* like 15%+
3. **Utility of money**: Marginal utility of $40k windfall >> utility of $213 expected value
4. **Regret aversion**: Fear of "missing out" on Corvette drives over-ordering beyond rational Q^{**}

Real-world implication: Decision-makers likely order **beyond** $Q^{**} = 400$ (perhaps 420–450) due to behavioral biases, sacrificing expected profit to chase the emotionally compelling $40k prize. The $213 expected value calculation is rational but ignores human psychology that makes lotteries appealing even when actuarially unfavorable.

Part 7: Quantity Discounts

The wholesaler offers all-units quantity discounts with tiered pricing:

$$c(Q) = \begin{cases} \$3.00 & \text{if } Q \in [1, 199] \\ \$2.85 & \text{if } Q \in [200, 399] \\ \$2.70 & \text{if } Q \geq 400 \end{cases}$$

Tier-by-Tier Analysis

For each cost tier, we compute the unconstrained newsvendor optimal Q^*, then evaluate feasibility within tier bounds.

Tier Range	**Unit Cost**	C_o	C_u	**Critical Ratio**	**Unconstrained** Q^*	**In Range?**	**Candidate** Q
1–199	$3.00	$2.00	$2.00	0.5000	270.0	No	199
200–399	$2.85	$1.93	$2.15	0.5276	278.3	Yes	278
400+	$2.70	$1.85	$2.30	0.5542	286.3	No	400

Table 14: Discount tier feasibility analysis (Q7)

Feasibility Check:

- **Tier 1 ($3.00):** Unconstrained $Q^* = 270$ exceeds tier maximum (199), so evaluate boundary $Q = 199$
- **Tier 2 ($2.85):** Unconstrained $Q^* = 278$ falls within [200, 399], this is a feasible interior solution
- **Tier 3 ($2.70):** Unconstrained $Q^* = 286$ below tier minimum (400), so evaluate boundary $Q = 400$

Candidate Profit Comparison

Q	**Unit Cost**	$\mathbb{E}$[**Sales**]	$\mathbb{E}$[**Leftover**]	$\mathbb{E}$[**Profit**]	**Note**
199	$3.00	188.60	10.40	$336.39	Tier 1 max
278	$2.85	236.53	41.76	**$408.15**	Tier 2 optimal
400	$2.70	269.33	130.67	$357.74	Tier 3 min

Table 15: Candidate order quantities and expected profits (Q7)

Optimal Decision: $Q_d^* = 278$ units at $2.85/unit
Analysis:

- Middle tier ($2.85) dominates despite not having the lowest unit cost
- Ordering 400 units to access $2.70 pricing forces excessive overage (131 units expected leftover)
- Overage cost penalty outweighs $0.15/unit savings: total cost increases by $50.41

Comparison to Baseline

Scenario	Q^*	Unit Cost	$\mathbb{E}$[Profit]
Part 2 Baseline	270	$3.00	$370.00
Part 7 w/ Discounts	278	$2.85	$408.15
Improvement	+8 units	−$0.15	+$38.15 (+10.3%)

Table 16: Quantity discount benefit vs baseline (Q7)

The quantity discount structure increases expected profit by 10.3% while requiring minimal additional inventory (8 units).

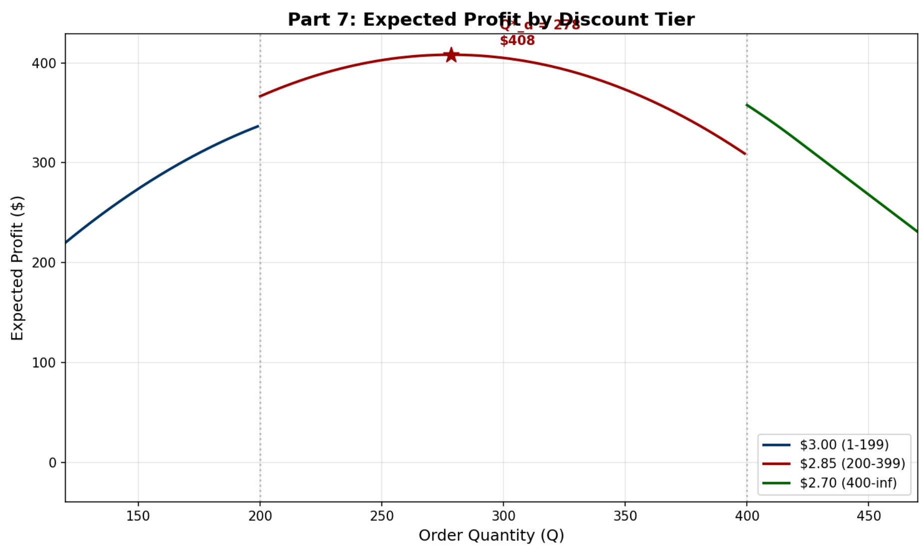

Figure 14: Expected profit across discount tiers showing $Q_d^* = 278$ optimum (Q7)

Supply Chain Coordination Insight: Quantity discounts align supplier and buyer incentives by encouraging larger orders (beneficial for supplier's economies of scale) while sharing cost savings with the buyer. The tiered structure prevents extreme ordering behavior; the marginal benefit of the deepest discount (400+ tier) is insufficient to justify the inventory risk.

Note: Complete Python code, full CSV datasets, and Excel workbook with formula documentation are available in the project repository and submission materials.