SparkFire LLC

Newsvendor Analysis for Fireworks Distribution

Optimal Order Quantity Decisions Under Demand Uncertainty

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Executive Summary

This report presents a comprehensive Newsvendor analysis for SparkFire LLC, a fireworks distributor facing demand uncertainty for its "Seventh Heaven" sparkler product. We analyze optimal order quantities under various business scenarios including pricing decisions, promotional incentives, quantity discounts, and advance demand signals.

Assumptions and Ambiguity Resolution

Model Assumptions

- Demand D follows a continuous Uniform distribution on $[120, 420]$ cases
- Single-period horizon: no multi-period inventory carryover
- Risk-neutral decision maker maximizing expected profit
- All costs and revenues are deterministic (no price/cost uncertainty)
- Fixed ordering cost $K = \$20$ is incurred regardless of order size
- Unsold inventory is returned for partial refund at fraction f of cost
- Salvage value $s = \$0.50$ per case for leftover units (after refund)

Ambiguity Resolutions

1. Overage Cost Interpretation: We define

$$C_o = c(1 - f) + s = 3(1 - 0.5) + 0.5 = \$2.00$$

   This represents the net loss per unsold unit: cost minus refund, plus salvage revenue (which reduces loss).

2. Continuous vs. Discrete Demand: The problem states demand is uniform on $[120, 420]$. For Parts 1–4 and 6–8, we treat demand as continuous. For Part 5 (simulation), we use integer demand $D \in \{120, 121, \ldots, 420\}$ uniformly.

3. Prize Eligibility (Part 6): The Corvette prize requires sales ≥ 400 cases. We interpret this as a threshold constraint: ordering $Q < 400$ guarantees zero prize probability. Ordering $Q \geq 400$ yields $P(\text{win}) = 0.05 \times P(D \geq 400)$.

4. Quantity Discount Tiers (Part 7): Discounts apply to total order quantity, not marginal units. The effective cost is:

$$c(Q) = \begin{cases} \$3.00 & \text{if } Q \in [1, 199] \\ \$2.85 & \text{if } Q \in [200, 399] \\ \$2.70 & \text{if } Q \geq 400 \end{cases}$$

5. Two-Stage Signal (Part 8): The demand signal is received *after* pre-season order Q_0 but *before* expedited order Q_1. Conditional distributions:
   - High signal ($P = 0.45$): $D \mid \text{High} \sim U[240, 420]$
   - Low signal ($P = 0.55$): $D \mid \text{Low} \sim U[120, 260]$

6. VOSRC Definition: Value of Signal Responsive Capability equals the profit gain from the two-stage model over a baseline single-stage model with *the same* pre-season cost $c_0 = \$3.00$.

Summary of Key Results

Scenario	Optimal Q	E[Profit]	Notes
Baseline ($p = \$5,\ f = 0.5$)	$Q^* = 270$	\$370	Critical ratio = 0.50
Higher price ($p = \$6$)	$Q^* = 300$	\$610	Critical ratio = 0.60
With Corvette prize	$Q^{**} = 400$	\$391	Threshold strategy
Quantity discounts	$Q_d^* = 278$	\$408	Uses \$2.85 tier
Two-Stage Model (Part 8)			
Pre-season order	$Q_0^* = 207$	—	Before signal
Expedited (High signal)	$Q_1^*(\text{H}) = 101$	—	Total = 308
Expedited (Low signal)	$Q_1^*(\text{L}) = 0$	—	Total = 207
Two-stage profit	—	\$366.56	Combined
VOSRC	—	−\$3.40	Signal not valuable

Table 1: Summary of optimal order quantities and expected profits across scenarios.

Part-by-Part Analysis

Parts 1–2: Baseline Newsvendor

With $p = \$5$, $c = \$3$, $f = 0.5$, $s = \$0.5$:

$$C_u = p - c = 5 - 3 = \$2 \quad \text{(underage cost)}$$
$$C_o = c(1 - f) + s = 3(0.5) + 0.5 = \$2 \quad \text{(overage cost)}$$
$$\text{Critical ratio} = \frac{C_u}{C_u + C_o} = \frac{2}{4} = 0.50$$

For $D \sim U[120, 420]$: $Q^* = 120 + 0.50 \times 300 = \boxed{270}$ cases.
Expected profit: $\mathbb{E}[\Pi(Q^*)] = \$370$.

Part 3: Refund Sensitivity

Refund f	Q^*	$\mathbb{E}[\Pi]$
0.25	246	\$342
0.50	270	\$370
0.75	305	\$407

Higher refund rates reduce overage cost, encouraging larger orders. Each 25pp increase in f raises Q^* by approximately 24–35 units.

Part 4: Pricing Decision

At $p = \$6$: $C_u = 3$, critical ratio $= 3/(3+2) = 0.60$, yielding $Q^* = 300$.
Expected profit increases from \$370 to \$610 (+65%). The higher price justifies stocking more inventory despite the same demand distribution.

Part 5: Monte Carlo Simulation

Using 500 replications with integer demand $D \in \{120, \ldots, 420\}$:

Metric	Value
Mean profit	\$373
Std deviation	\$194
P(Loss)	6.8%
5th percentile	\$36
95th percentile	\$670

The simulation confirms theoretical expected profit (\$370) and reveals substantial variability ($\sigma = \$194$).

Part 6: Corvette Prize Incentive

The \$40,000 Corvette (5% probability if sales ≥ 400) creates a non-convex profit function. Two strategies:

- *Baseline*: $Q^* = 270$, $\mathbb{E}[\Pi] = \$370$ (no prize eligibility)
- *Prize-seeking*: $Q^{**} = 400$, $\mathbb{E}[\Pi] = \$391$ (includes prize EV)

Optimal decision: Order 400 cases. The expected prize contribution (\$66.67) outweighs the overage cost increase.

Part 7: Quantity Discounts

Evaluating each tier:

Tier	Cost	Optimal Q	$\mathbb{E}[\Pi]$
\$3.00	1–199	195 (capped)	\$357
\$2.85	200–399	278	\$408
\$2.70	400+	400 (floor)	\$398

The middle tier ($Q_d^* = 278$ at \$2.85) maximizes profit. The deepest discount tier forces too much inventory, increasing overage risk.

Part 8: Two-Stage Ordering with Demand Signal

Pre-season cost $c_0 = \$3.00$; expedited cost $c_1 = \$3.60$.
Stage 1 (before signal): Order $Q_0^* = 207$ cases.
Stage 2 (after signal):

- High demand signal: $Q_1^*(\text{H}) = 101$ additional cases (total 308)
- Low demand signal: $Q_1^*(\text{L}) = 0$ additional cases (total 207)

Expected profit: \$366.56. Baseline (no signal): \$370.00.

VOSRC = −\$3.40: The signal capability destroys value because the 20% expedited premium (\$3.60 vs \$3.00) outweighs the benefit of demand information. SparkFire should not adopt this two-stage approach.

Managerial Insights

1. The baseline order of 270 cases balances underage and overage risks equally (critical ratio = 0.5).
2. Price increases have substantial leverage: a \$1 price increase raises optimal profit by 65%.
3. The Corvette promotion effectively shifts optimal behavior from 270 to 400 cases—a 48% increase in order quantity.
4. Quantity discounts should be evaluated holistically; the deepest discount is not always optimal.
5. Demand signal value depends critically on expedited cost premiums. Here, the 20% premium makes the signal worthless.

Open-Ended Discussion

[This section will contain responses to two selected open-ended questions from OE1–OE4. Space reserved for detailed analysis.]