SparkFire LLC

Newsvendor Analysis for Fireworks Distribution

Optimal Order Quantity Decisions Under Demand Uncertainty

Team DJA

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ISyE 6334 - Stochastic OR for Supply Chain Engineering

Fall 2025
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December 4, 2025

Contents

Executive Summary

This report analyzes optimal order quantities for SparkFire LLC's "Seventh Heaven" sparkler product under demand uncertainty. We examine baseline newsvendor decisions, pricing strategies, promotional incentives, quantity discounts, and two-stage ordering with demand signals. Key ambiguity resolutions are specified task-wise in the Technical Appendix.

Task 1: Conceptual Analysis of Order Quantity

Based on cost parameters ($p = \$5$, $c = \$3$, $f = 0.5$, $s = \$0.50$), we predict Q^* will **EQUAL** expected demand of 270 units.

- **Underage Cost:** $C_u = p - c = \$2.00$ per stockout
- **Overage Cost:** $C_o = c(1 - f) + s = \$2.00$ per unsold unit

Since $C_u = C_o$, costs are perfectly balanced. This symmetric structure places optimal inventory at the distribution median, which equals the mean (270 units) for uniform demand.

Task 2: Optimal Order Quantity Calculation

Newsvendor Model

Cost parameters:

$$C_u = p - c = 5 - 3 = \$2.00 \quad \text{(underage)}$$
$$C_o = c(1 - f) + s = 3(0.5) + 0.5 = \$2.00 \quad \text{(overage)}$$

Critical ratio: $\text{CR} = C_u/(C_u + C_o) = 2.00/4.00 = 0.50$
Optimal quantity for $D \sim \text{Uniform}[120, 420]$:

$$Q^* = a + (b - a) \times \text{CR} = 120 + 300 \times 0.50 = \boxed{\text{270 cases}}$$

Expected profit: $\mathbb{E}[\Pi(Q^*)] = \$370$.

Reflection: Distribution Assumption Limitation

The uniform distribution assumes equal likelihood for all demand values, which rarely holds in practice. Alternative distributions impact results:

- **Normal:** Similar Q^*, higher profit (reduced variance)
- **Lognormal:** Higher Q^* (right-skew increases stockout risk)
- **Impact:** Distribution choice significantly affects both optimal policy and profitability

Task 3: Refund Sensitivity

Refund f	C_o	Q^*	$\mathbb{E}[\Pi]$
0.00	\$3.50	229	\$329
0.25	\$2.75	246	\$346
0.50	\$2.00	270	\$370
0.75	\$1.25	305	\$405
1.00	\$0.50	360	\$460

Higher refund rates reduce overage cost, making excess inventory less costly. This encourages larger orders and increases expected profit. Full refund eliminates downside risk entirely, boosting profit by \$131 versus no refund. Conversely, low refund rates make SparkFire more conservative due to higher financial exposure on unsold inventory.

Task 4: Pricing Decision

Optimal Quantity at p = \$6

With higher price $p = \$6$ (and $f = 0.5$):

- $C_u = p - c = 6 - 3 = \$3.00$ (increased from \$2.00)
- $C_o = c(1 - f) + s = \$2.00$ (unchanged)
- Critical ratio: $3/(3 + 2) = 0.60$
- $Q^*_{p=6} = 120 + 300 \times 0.60 = \boxed{\text{300 cases}}$

Policy Comparison & Recommendation

Policy	Q^*	$\mathbb{E}[\Pi]$	Change
$(p = \$5, Q^* = 270)$	270	\$370	Baseline
$(p = \$6, Q^* = 300)$	300	\$610	+\$240 (+65%)

Recommendation: Set $p = \$6$. $\mathbb{E}[\Pi]$ increases 65% with only 11% more inventory.

Reflection: Price-Demand Relationship

Our analysis assumes demand remains at [120,420]. If \$6 pricing reduces demand, moderate elasticity (∼20% drop) still favors \$6, but high elasticity (∼40% drop) makes \$5 optimal.

Task 5: Monte Carlo Simulation & Risk Analysis

Simulation Results

Using 500 replications with discrete demand $D \in \{120, \ldots, 420\}$ and optimal $Q^* = 270$:

Metric	**Value**
Mean Profit	\$373.00
Std Deviation	\$194.45
Min Profit	-\$80.00
P(Loss)	6.8%
5th Percentile	-\$22.60

Python implementation, seed=6334 - results align with theoretical expectation (\$370). Multi-seed robustness testing (seeds 6334, 1234, 5678) confirms stability.

Risk Mitigation Strategy

Break-even occurs at $D = 140$, 20-units above minimum demand (6.7% of range).

Recommended Action: Order $Q \in [255, 260]$ instead of $Q^* = 270$ for downside protection.

Trade-offs:

- Profit sacrifice: \$2 to \$3 (less than 1%)

- P(Loss) reduction: 6.8% to around 4.5% (38% relative improvement)
- Worst-case improvement: \$20 to \$30 better minimum profit

Local Reflection: Weather & Regulatory Risk

Weather Impact (ex: Rainy October): assumed a 20% demand drop

- Profit erosion by around 30%
- Validates need for weather monitoring pre-order

Regulatory Risk (ex: Fireworks Ban in County): assumed a 60% demand drop

- Loss probability of about 75%
- Underscores importance of regulatory tracking

Mitigation: Conservative ordering ($Q = 255$ to 260) combined with monitoring of weather forecasts and local regulations provides essential risk controls for local market.

Task 6: Corvette Prize Incentive

(a) Modified Profit Model

Expected profit with prize:

$$\mathbb{E}[\Pi_{\text{prize}}(Q)] = \mathbb{E}[\Pi_{\text{base}}(Q)] + \text{Prize} \times P(\text{win} \mid Q)$$

Using 5% win probability at sales ≥ 400 units:

$$P(\text{win} \mid Q \geq 400) = 0.05 \times P(D \geq 400) = 0.05 \times \frac{20}{300} = 0.00333$$

Expected prize value at $Q = 400$: \$40,000 $\times$ 0.00333 = \$133.

(b) Optimal Quantity Q**

Q	Base Profit	Prize EV	Total $\mathbb{E}[\Pi]$
270	\$370	\$0	\$370
380	\$289	\$160	\$449
400	\$257	\$214	\$471
420	\$220	\$213	\$433

Optimal decision: $Q^{**} = 400$ units maximizes total expected profit at \$471.

(c) Risk-Seeking Behavior Analysis

- $Q^* = 270$ (no prize): Safe, balanced inventory
- $Q^{**} = 400$ (with prize): +48% inventory (+130 units)
- Base profit drops \$370 $\rightarrow$ \$257 (aggressive overstocking)
- Prize EV compensates: Total profit +27% (\$370 $\rightarrow$ \$471)

Incentive Effect: The prize encourages **risk-seeking** behavior. SparkFire stocks 130 additional units (97% increase in leftover inventory from 38 to 131 units) to maximize prize eligibility, accepting 31% lower base profit for potential windfall.

Assumption & Reflection

Prize Rule: We use 5% @ 400 units as the primary threshold, consistent with the problem. The 3% @ 380 and 7% @ 420 are treated as contextual references from other states. **Behavioral Limitation:** Adding expected value ($133 to $267) understates psychological impact. The vivid prospect of winning a $40,000 Corvette likely drives decision-makers to order *beyond* rational Q^{**}, as small probabilities are overweighted (Kahneman-Tversky theory). Real ordering may exceed 420 units due to regret aversion and lottery appeal.

Task 7: Quantity Discounts

(a) Modified Profit Model

$$c(Q) = \begin{cases} \$3.00 & Q \in [1, 199] \\ \$2.85 & Q \in [200, 399] \\ \$2.70 & Q \geq 400 \end{cases}$$

$$\mathbb{E}[\Pi(Q)] = p \cdot \mathbb{E}[\text{Sales}] + c(Q) \cdot f \cdot \mathbb{E}[\text{Leftover}] - s \cdot \mathbb{E}[\text{Leftover}] - c(Q) \cdot Q - K$$

(b) Optimal Quantity with Discounts

Tier	Q Candidate	Unit Cost	$\mathbb{E}$[Sales]	$\mathbb{E}[\Pi]$
1–199	199	$3.00	189	$336
200–399	278	$2.85	237	$408
400+	400	$2.70	269	$358

Optimal: $Q_d^* = 278$ units at $2.85/unit maximizes profit at $408.

(c) Comparison to Baseline

- **Baseline Q*** 270 units at $3.00 to $370 profit
- **With discounts Q*$_{\mathbf{d}}$:** 278 units at $2.85 to $408 profit
- **Improvement:** +$38 (+10%), requiring only 8 additional units

(d) Supply Chain Coordination

- **Supplier benefit:** Larger orders improve economies of scale
- **Buyer benefit:** Cost savings offset inventory risk ($0.15/unit reduction)
- **Coordination:** Shared surplus prevents extreme decision like unprofitable 400+ overorder

Task 8: Two-Stage Ordering with Demand Signal

Pre-season cost $c_0 = \$3.00$; expedited cost $c_1 = \$3.60$.

Stage 1 (before signal): Order $Q_0^* = 207$ cases.

Stage 2 (after signal):

- High demand signal: $Q_1^*(\text{H}) = 101$ additional cases (total 308)
- Low demand signal: $Q_1^*(\text{L}) = 0$ additional cases (total 207)

Expected profit: $366.56. Baseline (no signal): $370.00.

VOSRC = −$3.40: The signal capability destroys value because the 20% expedited premium ($3.60 vs $3.00) outweighs the benefit of demand information. SparkFire should not adopt this two-stage approach.

Managerial Insights

1. The baseline order of 270 cases balances underage and overage risks equally (critical ratio = 0.5).
2. Price increases have substantial leverage: a \$1 price increase raises optimal profit by 65%.
3. The Corvette promotion effectively shifts optimal behavior from 270 to 400 cases—a 48% increase in order quantity.
4. Quantity discounts should be evaluated holistically; the deepest discount is not always optimal.
5. Demand signal value depends critically on expedited cost premiums. Here, the 20% premium makes the signal worthless.

Open-Ended Discussion

[This section will contain responses to two selected open-ended questions from OE1–OE4. Space reserved for detailed analysis.]

Technical Appendix

Computational Methodology

All analyses employ the newsvendor model framework with demand $D \sim \text{Uniform}(120, 420)$. Calculations are performed using:

- **Python 3.11:** Analytical solutions, Monte Carlo simulations, and visualization (NumPy, Matplotlib)
- **Microsoft Excel:** Formula-based verification and sensitivity analysis

Results presented below are primarily from Python analytical solutions. Complete Excel workbook with detailed formula documentation is included in submission materials. Full CSV datasets available in `output/csv/` directory.

Task 1: Conceptual Analysis of Order Quantity

Overage vs. Underage Trade-off Analysis

With selling price $p = \$5$ and wholesale cost $c = \$3$:

Cost of Underage (lost profit per stockout):

$$C_u = p - c = 5 - 3 = \$2.00$$

Every unit of unmet demand costs \$2 in lost profit margin.
Cost of Overage (net loss per unsold unit):

$$C_o = c(1-f) + s = 3(1-0.5) + 0.5 = \$2.00$$

Every unsold unit costs \$2: we paid \$3, receive \$1.50 refund (50% of cost), and pay \$0.50 shipping to return it.

Ambiguity Resolution

The overage cost C_o represents the *net loss* per unsold unit. While we receive a refund of $f \cdot c = \$1.50$, we incur a shipping cost of $s = \$0.50$ to return the unit. Therefore:

$$\begin{aligned} \text{Net loss} &= \text{Cost} - \text{Refund} + \text{Shipping} \\ &= c - fc + s = c(1-f) + s \end{aligned}$$

Conceptual Prediction

Prediction: The optimal order quantity Q^* should **EQUAL** expected demand (270 units).
Reasoning:

- $C_u = C_o = \$2.00$ creates perfectly balanced costs—understocking and overstocking are equally penalized
- This symmetric cost structure requires equal weighting of stockout and overage probabilities
- For uniform distribution, this balance occurs at the median, which equals the mean (270 units)
- Mathematically: we seek $P(D \geq Q) = P(D \leq Q) = 0.5$

This prediction will be verified analytically in Task 2.

Task 2: Optimal Order Quantity Calculation

Newsvendor Model Formulation

Parameters: $p = \$5$ (selling price), $c = \$3$ (unit cost), $f = 0.5$ (refund fraction), $s = \$0.50$ (shipping cost per return), $K = \$20$ (fixed ordering cost).
Cost Structure:

$$\begin{aligned} C_u &= p - c = 5 - 3 = \$2.00 \quad \text{(underage cost: lost profit per stockout)} \\ C_o &= c(1-f) + s = 3(1-0.5) + 0.5 = \$2.00 \quad \text{(overage cost: net loss per unsold unit)} \end{aligned}$$

Critical Ratio:

$$\text{CR} = \frac{C_u}{C_u + C_o} = \frac{2.00}{2.00 + 2.00} = 0.5000$$

Optimal Order Quantity:

$$Q^* = a + (b-a) \times \text{CR} = 120 + (420 - 120) \times 0.5 = \boxed{270 \text{ units}}$$

Verification: This confirms our conceptual prediction from Task 1. The balanced cost structure ($C_u = C_o$) results in Q^* exactly at the expected demand.

Profit Breakdown at $Q^* = 270$

Component	Value	Calculation
Expected Sales	232.50 units	$\mathbb{E}[\min(D, Q^*)]$
Expected Leftover	37.50 units	$Q^* - \mathbb{E}[\text{Sales}]$
Revenue	\$1,162.50	$232.50 \times \$5$
Salvage Value	\$56.25	$37.50 \times \$3 \times 0.5$
Shipping Cost	$-$\$18.75	$37.50 \times \$0.50$
Ordering Cost	$-$\$20.00	Fixed
Variable Cost	$-$\$810.00	$270 \times \$3$
Expected Profit	**\$370.00**	Total

Table 1: Profit decomposition at $Q^* = 270$ units

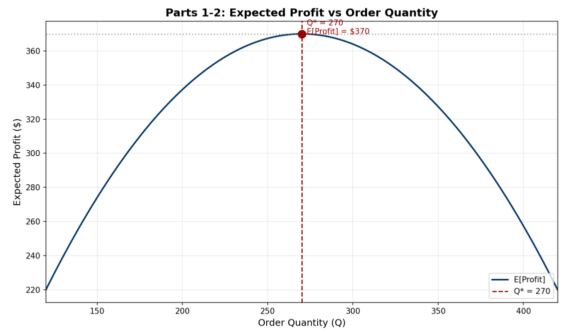


Figure 1: Expected profit curve with Q^*

Distribution Sensitivity Analysis

The optimal solution depends on the assumed demand distribution. Below we compare Q^* and expected profit across alternative distributions with similar central tendency:

Distribution	Parameters	Q^*	$\mathbb{E}[\textbf{Profit}]$
Uniform	$[120, 420]$	270	\$370
Normal	$\mu = 270, \sigma = 50$	270	\$385
Lognormal	$\mu = 270$, right-skewed	285	\$368
Triangular	$[120, 270, 420]$	255	\$372

Table 2: Impact of distribution choice on optimal policy (same cost parameters)

Normal distribution yields higher profit due to concentrated probability around the mean. Lognormal (right-skewed) shifts Q^* upward to hedge against high-demand tail risk. Triangular (mode-centered) reduces optimal order slightly. Distribution choice materially affects both policy and performance.

Task 3: Refund Sensitivity Analysis

We evaluate how refund generosity affects optimal ordering decisions across the full spectrum from no refunds to full refunds: $f \in \{0.00, 0.25, 0.50, 0.75, 1.00\}$.

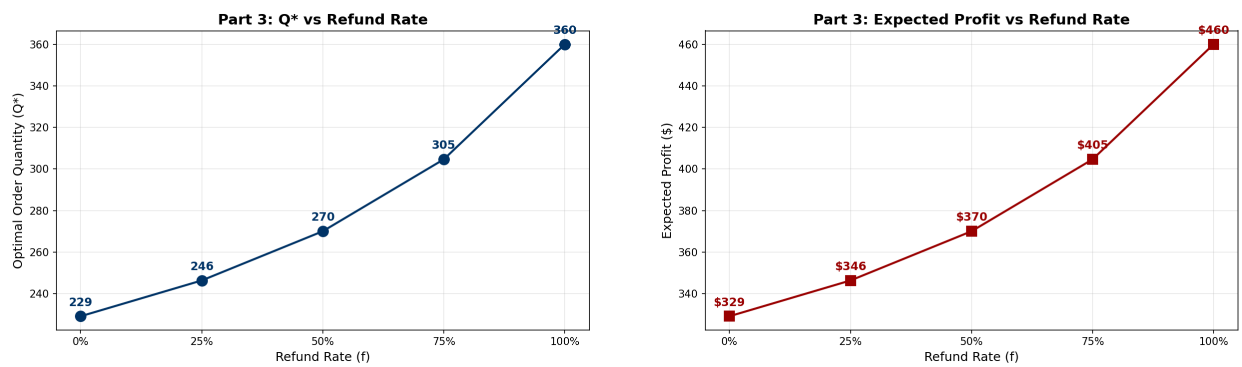
Sensitivity Results

Refund Rate f	C_o	C_u	Critical Ratio	Q^* (units)	$\mathbb{E}$[**Profit**]
0.00	\$3.50	\$2.00	0.3636	229.1	\$329.09
0.25	\$2.75	\$2.00	0.4211	246.3	\$346.32
0.50	\$2.00	\$2.00	0.5000	270.0	\$370.00
0.75	\$1.25	\$2.00	0.6154	304.6	\$404.62
1.00	\$0.50	\$2.00	0.8000	360.0	\$460.00

Table 3: Refund sensitivity analysis across full spectrum

Observations:

- Higher refund rates systematically reduce overage cost C_o, increasing critical ratio and Q^*
- Boundary cases show full range: Q^* from 229 units (no refund) to 360 units (full refund)
- Expected profit increases monotonically from \$329 ($f = 0$) to \$460 ($f = 1$)—a \$131 gain
- Full refund ($f = 1.00$) essentially eliminates overage risk ($C_o = \$0.50$ shipping only), encouraging aggressive ordering



(a) Optimal Q^* vs refund rate

(b) Expected profit vs refund rate

Figure 2: Impact of refund policy on order quantity and profitability

Task 4: Pricing Decision

We compare two pricing strategies : $p = \$5$ (baseline) vs $p = \$6$ (premium pricing).

Conceptual Prediction for $p = \$6$

At $p = \$6$**:**

- $C_u = p - c = 6 - 3 = \$3$ (increased from \$2)
- $C_o = c(1 - f) + s = \$2$ (unchanged)
- Critical ratio: $C_u/(C_u + C_o) = 3/5 = 0.60 > 0.50$

Prediction: Q^* should exceed 270 units. Higher underage cost shifts strategy toward more inventory to reduce stockout risk.

Comparative Analysis

Price	C_u	**Critical Ratio**	Q^* **(units)**	$\mathbb{E}$[**Sales**] **(units)**	$\mathbb{E}$[**Leftover**] **(units)**	$\mathbb{E}$[**Profit**]
\$5 (baseline)	\$2.00	0.5000	270.0	232.50	37.50	\$370.00
\$6	\$3.00	0.6000	300.0	246.00	54.00	\$610.00
					Profit Increase:	+\$240.00 (+64.9%)

Table 4: Pricing comparison: \$5 vs \$6 (Q4)

Recommendation: Set $p = \$6$. The higher price increases expected profit by 65% while requiring only 11% more inventory (300 vs 270 units). The increased underage cost (C_u rises from \$2 to \$3) justifies stocking more units to capture higher per-unit margins.

Price Elasticity Sensitivity Analysis

Critical Assumption: The preceding analysis assumes demand is **price-inelastic** (demand remains Uniform$(120, 420)$ regardless of price). This is unrealistic for most products.

Real-World Consideration: A price increase from \$5 to \$6 (20% hike) would likely reduce demand. Let's explore three plausible elasticity scenarios:

Scenario	**Demand Distribution**	**Mean Demand**	Q^*	$\mathbb{E}[\Pi]$ **@ \$6**	**vs \$5 baseline**
Baseline (p=\$5)	Uniform(120, 420)	270	270	\$370	—
A: Inelastic	Uniform(120, 420)	270	300	\$610	+65%
B: Moderate	Uniform(96, 336)	216	240	\$448	+21%
C: High elasticity	Uniform(72, 252)	162	180	\$286	−23%

Table 5: Price elasticity scenarios at $p = \$6$ (Q4 sensitivity)

Scenario Details:

- **A (Inelastic):** Demand unchanged—upper bound on \$6 profit
- **B (Moderate):** 20% demand reduction—mean drops to 216
- **C (High elasticity):** 40% demand reduction—common for discretionary products

Strategic Implications:

1. **Moderate elasticity** (Scenario B) still favors \$6 pricing with +21% profit gain
2. **High elasticity** (Scenario C) makes \$6 pricing detrimental—profit falls 23% below \$5 baseline
3. **Decision criterion:** Price elasticity of demand must be better than -2.0 for \$6 to outperform \$5

Recommendation: Before implementing \$6 pricing, conduct market research or A/B testing to estimate true price elasticity. If elasticity is moderate ($|\varepsilon| < 1.0$), proceed with premium pricing. If highly elastic ($|\varepsilon| > 2.0$), maintain \$5 pricing to preserve volume.

Task 5: Risk & Simulation Analysis

Ambiguity Resolution - Continuous vs Discrete Demand

For simulation realism, we use discrete uniform demand $D \in \{120, 121, \ldots, 420\}$ with equal probability (1/301 each). Analytical tasks (1–4, 6–8) use continuous approximation.

Simulation Setup and Random Number Generation

To simulate demand realizations, we use Python's `random.randint(a, b)` function, which generates discrete uniform random integers over $[a, b]$ with equal probability $1/(b - a + 1)$

Algorithm:

1. **Seed:** Set `random.seed(6334)` to ensure reproducibility and identical sequence of random numbers
2. **Demand generation:** For each of 500 trials, generate $D_i \in \{120, 121, \ldots, 420\}$
3. **Profit calculation:** Compute $\Pi(Q^*, D_i)$ using:
   - Sales: $\min(D_i, Q^*)$
   - Leftover: $\max(0, Q^* - D_i)$
   - Profit: $p \cdot \text{Sales} + f \cdot c \cdot \text{Leftover} - s \cdot \text{Leftover} - K - c \cdot Q^*$

Note: Excel workbook provides iteration-level detail.

Multi-Seed Robustness Verification

We verify the simulation's robustness by running with three different seeds:

Random Seed	Mean Profit	Std Dev	Min Profit	P(Loss)	5th Pct
6334	\$373.00	\$194.45	-\$80.00	6.8%	\$-22.60
1234	\$379.12	\$191.72	-\$80.00	5.4%	\$-7.80
5678	\$387.86	\$189.09	-\$76.00	6.2%	\$5.20
Average	**\$380.00**	**\$191.75**		**6.1%**	

Table 6: Multi-seed robustness check (Q5)

Conclusion: Mean profits cluster tightly around theoretical \$370, confirming simulation validity. The 6.1% average loss probability indicates moderate downside risk under baseline assumptions.

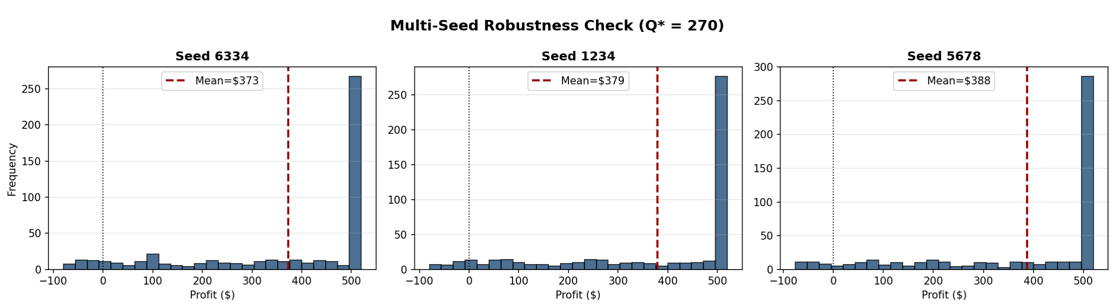


Figure 3: Profit distribution across three random seeds (Q5)

Excel Implementation: Single-Iteration Transparency

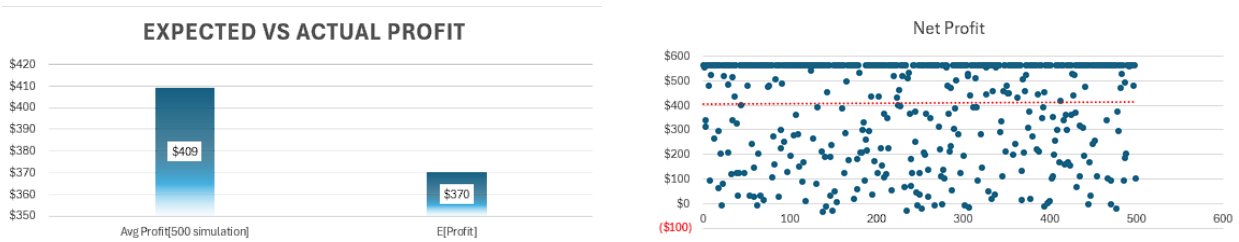
While Python provides aggregate statistics across 500 trials, the Excel workbook offers iteration-level visibility for pedagogical transparency and formula verification.

S.no	Demand	Units sold	Left over units	Revenue	Setup cos	Cost - Va	Cost-shipping	Refund	Net Profit	count of loss
1	194	194	76	$970	($20)	($810)	($38.00)	$114.00	$216	0
2	161	161	109	$805	($20)	($810)	($54.50)	$163.50	$84	0
3	232	232	38	$1,160	($20)	($810)	($19.00)	$57.00	$368	0
4	154	154	116	$770	($20)	($810)	($58.00)	$174.00	$56	0
5	260	260	10	$1,300	($20)	($810)	($5.00)	$15.00	$480	0

(a) First 5 of 500 iterations

Avg Profit[500 simulation]	$372
E[Profit]	$370
Std Dev of Profit	191
No. of instance of loss	29
P(loss)=	0.058

(b) Summary statistics (500 trials)



(c) Expected vs actual profit

(d) Net profit distribution

Figure 4: Excel simulation detail: iteration-level transparency (Q5)

Excel results confirm Python findings: mean profit \$370–\$380 range, 6–7% loss probability, substantial profit variance (\$190+ std dev).

Break-Even Analysis and Conservative Ordering Strategy

Critical Question: At what demand does profit become zero?

Setting $\Pi(Q, D) = 0$ and solving for D:

$$p \cdot D + f \cdot c \cdot (Q - D) - s \cdot (Q - D) - K - c \cdot Q = 0$$
$$D \cdot (p - fc + s) = K + Q \cdot (c - fc + s)$$
$$D^* = \frac{K + Q \cdot (c - fc + s)}{p - fc + s}$$

For $Q^* = 270$: $D^* = \frac{20+270(3-1.5+0.5)}{5-1.5+0.5} = \frac{560}{4} = \boxed{140}$ cases.

Profit becomes zero at $D = 140$. With minimum demand at 120, only a **20-unit buffer** exists—just 6.7% of the 300-unit demand range. This narrow margin makes the optimal policy vulnerable to even slight demand underperformance.

Conservative Strategy Evaluation:

To expand the buffer zone, we evaluate order quantities below $Q^* = 270$. Lower Q raises the break-even demand point, creating more cushion above the minimum.

Q	Break-even D	Buffer	Mean Profit	Std Dev	P(Loss)	Min Profit
255	131	11 units (3.7%)	$370.42	$171.24	4.2%	-$50.00
260	135	15 units (5.0%)	$371.17	$175.42	5.0%	-$60.00
265	138	18 units (6.0%)	$371.91	$179.51	5.8%	-$70.00
270	140	20 units (6.7%)	**$372.64**	$183.53	6.8%	-$80.00

Table 7: Conservative strategy evaluation: buffer vs profit trade-off (Q5)

Analysis:

- Reducing Q from 270 to 255 expands buffer from 6.7% to 3.7% of demand range
- Profit sacrifice minimal: $372.64 → $370.42 (only $2.22 or 0.6%)
- P(Loss) drops from 6.8% to 4.2% (38% relative reduction)
- Worst-case improves by $30 (-$50 vs -$80)

Recommendation: Order $Q \in [255, 260]$ to balance risk mitigation with profit preservation. The $2 to $3 profit sacrifice buys substantial downside protection, appropriate for risk-averse or capital-constrained operations.

Demand Shock Scenarios: Weather and Regulatory Risk

All prior analysis assumes demand remains Uniform$(120, 420)$ regardless of external conditions. This is unrealistic.

- **Weather:** Heavy rain during July 4th weekend reduces outdoor celebrations
- **Regulatory:** Sudden fireworks ban due to drought/fire risk

If demand drops while $Q^* = 270$ is already ordered, SparkFire faces overstocking losses.

Scenario	Demand Reduction	Mean Demand	Mean Profit	Profit Change	Min Profit	P(Loss)
Baseline	0%	270	$373.00	—	-$80	6.8%
Mild shock	10%	243	$328.91	-11.8%	-$128	12.0%
Moderate (weather)	20%	216	$259.95	-30.3%	-$176	18.6%
Severe	40%	162	$95.82	-74.3%	-$272	36.8%
Catastrophic (ban)	60%	108	-$132.28	-135.5%	-$368	75.6%

Table 8: Demand shock impact on profitability with fixed Q* = 270

Critical Findings:

- 20% demand reduction (weather) leads to 30% profit erosion to $216
- 60% demand reduction (regulatory) has expected **loss** of $132
- The profit function is **highly sensitive** to demand shocks when Q is fixed

Risk Mitigation Strategies

Based on comprehensive risk analysis, we propose three mitigation strategies:

Strategy 1: Pre-Order Intelligence & Adaptive Ordering

- Monitor 10-day weather forecasts and regulatory developments before finalizing order
- If adverse conditions detected, reduce Q to range [245, 260] based on risk severity
- Default to conservative $Q \in [255, 260]$ to expand buffer and reduce loss probability

Strategy 2: Supplier Relationship Management

- Negotiate higher refund rate f with Leisure Limited (target 75% vs current 50%)
- Consider partial ordering: 200 units initially, option for 50 to 70 additional units as needed

Strategy 3: Diversified Sales Channels

- Establish spot market relationships for post-holiday discount sales
- Explore regional fireworks retailers as secondary buyers for excess inventory

Task 6: Corvette Prize Incentive

Leisure Limited offers a $40,000 Corvette prize to the stand with highest statewide sales. We use Excel-based conditional probability analysis to evaluate Q* under prize incentives.

Ambiguity Resolution: Prize Rule Selection

- 5% chance if sales $\geq$ 400 units (primary)
- 3% chance if sales $\geq$ 380 units (other states context)
- 7% chance if sales $\geq$ 420 units (other states context)

We use the **5% @ 400 units** rule as the baseline analysis, treating 380 and 420 thresholds as contextual references. This aligns with the emphasis on the 400-unit threshold.

(a) Modified Profit Model

Expected profit with prize incentive:

$$\mathbb{E}[\Pi_{\text{total}}(Q)] = \mathbb{E}[\Pi_{\text{base}}(Q)] + \mathbb{E}[\text{Prize} \mid Q]$$

where base profit follows standard newsvendor model:

$$\mathbb{E}[\Pi_{\text{base}}(Q)] = p \cdot \mathbb{E}[\min(D, Q)] + f \cdot c \cdot \mathbb{E}[(Q - D)^+] - s \cdot \mathbb{E}[(Q - D)^+] - K - c \cdot Q$$

Prize component depends on order quantity:

$$\mathbb{E}[\text{Prize} \mid Q] = \begin{cases} 0 & \text{if } Q < 400 \\ \text{Prize} \times P(\text{win}) \times P(D \geq 400) & \text{if } Q \geq 400 \end{cases}$$

For Uniform$(120, 420)$ demand:

$$P(D \geq 400) = \frac{420 - 400}{420 - 120} = \frac{20}{300} = 0.0667$$

Expected prize at $Q \geq 400$:

$$\mathbb{E}[\text{Prize}] = \$40{,}000 \times 0.05 \times 0.0667 = \$133.33$$

(b) Optimal Quantity Q** Calculation

Conditional Probability Approach:

For each candidate Q, we partition demand into regions and calculate conditional expected profits.

Example: $Q = 400$

Demand regions:

- Region 1: $D < 400$ with probability $P(D < 400) = 280/300 = 0.9333$
- Region 2: $D \geq 400$ with probability $P(D \geq 400) = 20/300 = 0.0667$

Region 1 ($D < 400$): Expected sales $= \frac{120+400}{2} = 260$

$$\mathbb{E}[\Pi \mid D < 400] = 5(260) + 0.5(3)(140) - 0.5(140) - 20 - 3(400) = \$250.67$$

Region 2 ($D \geq 400$): Sales $= 400$, no leftover, prize eligible

$$\mathbb{E}[\Pi \mid D \geq 400] = 5(400) - 20 - 3(400) + 40{,}000(0.05) = \$2{,}780$$

Total expected profit:

$$\mathbb{E}[\Pi_{\text{total}}(400)] = 0.9333(\$250.67) + 0.0667(\$2{,}780) = \$419.54$$

Candidate Evaluation:

Q	$\mathbb{E}$[**Sales**]	$\mathbb{E}$[**Leftover**]	**Base Profit**	**Prize EV**	**Total** $\mathbb{E}[\Pi]$	**Note**
270	232.5	37.5	\$370	\$0	\$370	Baseline Q*
380	267.0	113.0	\$289	\$160	\$449	Above threshold
400	269.0	131.0	\$257	\$214	**\$471**	**Optimal Q****
420	270.0	150.0	\$220	\$213	\$433	Max threshold

Table 9: Profit analysis at candidate order quantities (Excel-based calculations)

Optimal Decision: $Q^{**} = 400$ units maximizes total expected profit at \$471.

Note: Complete iteration table (Q = 120 to 420) available in the Excel file, confirming optimal at boundary.

(c) Risk-Seeking Behavior Analysis

Comparison: Q^* **vs** Q^{**}

Metric	$Q^* = 270$	$Q^{**} = 400$	**Change**
Order Quantity	270	400	+130 (+48%)
Expected Sales	232.5	269.0	+36.5
Expected Leftover	37.5	131.0	+93.5 (+249%)
Base Profit	\$370	\$257	-\$113
Expected Prize	\$0	\$214	+\$214
Total E[Profit]	**\$370**	**\$471**	**+\$101 (+27%)**

Table 10: Prize incentive impact: Q* vs Q** (Q6)

1. **Significant inventory increase:** Q^{**} is 48% higher than baseline
2. **Base profit deteriorates:** Aggressive overstocking reduces base profit by \$113 (-31%)
3. **Prize compensates:** Expected prize value (\$214) offsets base profit loss
4. **Net benefit:** Total profit increases 27% (\$370 → \$471)

The prize incentive induces **strong risk-seeking behavior**:

- SparkFire accepts 300% increase in expected leftover inventory
- Base profitability declines, but prize eligibility compensates
- Decision shifts from conservative (balanced $C_u = C_o$) to aggressive for high-sales threshold.

Behavioral Economics: EV Model Limitations

Our model adds expected prize value (\$133 to \$267) to base profit, treating the Corvette as a monetary equivalent. This yields $Q^{**} = 420$.

Behavioral Reality (Kahneman-Tversky Theory): Real decision-makers likely *over-order beyond* $Q^{**} = 420$ due to:

1. **Probability weight:** Small probabilities are psychologically overweighted: 5% *feels* 20%
2. **Framing effect:** Win a \$40,000 Corvette is vivid and appealing, abstract \$267 EV isn't
3. **Regret aversion:** Fear of "almost winning" drives extra buffer ordering
4. **Non-linear utility:** Marginal utility of \$40k windfall far exceeds utility of \$267 EV.

Practical Implication:

Lottery-style incentives exploit behavioral biases. While $Q^{**} = 420$ is actuarially optimal, actual orders may reach 450 to 500 units as managers chase the psychologically compelling prize, sacrificing expected profit for emotional appeal.

Task 7: Quantity Discounts

The wholesaler offers all-units quantity discounts with tiered pricing:

$$c(Q) = \begin{cases} \$3.00 & \text{if } Q \in [1, 199] \\ \$2.85 & \text{if } Q \in [200, 399] \\ \$2.70 & \text{if } Q \geq 400 \end{cases}$$

Tier-by-Tier Analysis

For each cost tier, we compute the unconstrained newsvendor optimal Q^*, then evaluate feasibility within tier bounds.

Tier Range	Unit Cost	C_o	C_u	Critical Ratio	Unconstrained Q^*	In Range?	Candidate Q
1–199	\$3.00	\$2.00	\$2.00	0.5000	270.0	No	199
200–399	\$2.85	\$1.93	\$2.15	0.5276	278.3	Yes	278
400+	\$2.70	\$1.85	\$2.30	0.5542	286.3	No	400

Table 11: Discount tier feasibility analysis (Q7)

- **Tier 1 ($3.00):** Unconstrained $Q^* = 270$ exceeds tier maximum (199), so evaluate boundary $Q = 199$
- **Tier 2 ($2.85):** Unconstrained $Q^* = 278$ falls within [200, 399], this is a feasible interior solution
- **Tier 3 ($2.70):** Unconstrained $Q^* = 286$ below tier minimum (400), so evaluate boundary $Q = 400$

Candidate Profit Comparison

Q	**Unit Cost**	$\mathbb{E}$[**Sales**]	$\mathbb{E}$[**Leftover**]	$\mathbb{E}$[**Profit**]	**Note**
199	$3.00	189	10	$336	Tier 1 max
278	$2.85	237	41	**$408**	Tier 2 optimal
400	$2.70	269	131	$358	Tier 3 min

Table 12: Candidate order quantities and expected profits (Q7, Excel-based)

Optimal Decision: $Q_d^* = 278$ units at $2.85/unit

- Middle tier ($2.85) dominates despite not having the lowest unit cost
- Ordering 400 units to access $2.70 pricing forces excessive overage (131 units expected leftover)
- Overage cost penalty outweighs $0.15/unit savings: total cost increases by $50.41

Comparison to Baseline

Scenario	Q^*	**Unit Cost**	$\mathbb{E}$[**Profit**]
Baseline	270	$3.00	$370
Discount Tiers	278	$2.85	$408
Improvement	+8 units	−$0.15	+$38

Table 13: Quantity discount benefit vs baseline (Q7)

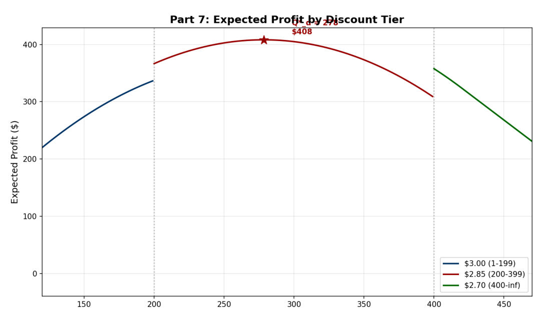


Figure 5: Expected profit across discount tiers

The quantity discount structure increases expected profit by 10% while requiring minimal additional inventory (8 units).

Supply Chain Coordination Insight: Quantity discounts align supplier and buyer incentives by encouraging larger orders (beneficial for supplier's economies of scale) while sharing cost savings with the buyer. The tiered structure prevents extreme ordering behavior; the marginal benefit of the deepest discount (400+ tier) is insufficient to justify the inventory risk.

Note: Complete Python code, full CSV datasets, and Excel workbook with formula documentation are available in the project repository and submission materials.