SparkFire LLC

Newsvendor Analysis for Fireworks Distribution

Optimal Order Quantity Decisions Under Demand Uncertainty

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Contents

Executive Summary

This report analyzes optimal order quantities for SparkFire LLC's "Seventh Heaven" sparkler product under demand uncertainty. We examine baseline newsvendor decisions, pricing strategies, promotional incentives, quantity discounts, and two-stage ordering with demand signals. Key ambiguity resolutions are specified task-wise in the Technical Appendix.

Task 1: Conceptual Analysis of Order Quantity

Based on cost parameters ($p = \$5$, $c = \$3$, $f = 0.5$, $s = \$0.50$), we predict Q^* will **EQUAL** expected demand of 270 units.

- **Underage Cost:** $C_u = p - c = \$2.00$ per stockout
- **Overage Cost:** $C_o = c(1 - f) + s = \$2.00$ per unsold unit

Since $C_u = C_o$, costs are perfectly balanced. This symmetric structure places optimal inventory at the distribution median, which equals the mean (270 units) for uniform demand.

Task 2: Optimal Order Quantity Calculation

Newsvendor Model

Cost parameters:

$$\begin{aligned} C_u &= p - c = 5 - 3 = \$2.00 \quad \text{(underage)} \\ C_o &= c(1 - f) + s = 3(0.5) + 0.5 = \$2.00 \quad \text{(overage)} \end{aligned}$$

Critical ratio: $\text{CR} = C_u/(C_u + C_o) = 2.00/4.00 = 0.50$
Optimal quantity for $D \sim \text{Uniform}[120, 420]$:

$$Q^* = a + (b - a) \times \text{CR} = 120 + 300 \times 0.50 = \boxed{270 \text{ cases}}$$

Expected profit: $\mathbb{E}[\Pi(Q^*)] = \$370$.

Reflection: Distribution Assumption Limitation

The uniform distribution assumes equal likelihood for all demand values, which rarely holds in practice. Alternative distributions (Normal, Lognormal) significantly affect both optimal Q^* and expected profit, with right-skewed distributions increasing optimal order quantities due to higher stockout risk.

Task 3: Refund Sensitivity

Refund f	C_o	Q^*	$\mathbb{E}[\Pi]$
0.00	\$3.50	229	\$329
0.25	\$2.75	246	\$346
0.50	\$2.00	270	\$370
0.75	\$1.25	305	\$405
1.00	\$0.50	360	\$460

Higher refund rates reduce overage cost, making excess inventory less costly. This encourages larger orders and increases expected profit. Full refund eliminates downside risk entirely, boosting profit by \$131 versus no refund. Conversely, low refund rates make SparkFire more conservative due to higher financial exposure on unsold inventory.

Task 4: Pricing Decision

Optimal Quantity at p = \$6

With higher price $p = \$6$ (and $f = 0.5$):

- $C_u = p - c = 6 - 3 = \$3.00$ (increased from \$2.00)
- $C_o = c(1 - f) + s = \$2.00$ (unchanged)
- Critical ratio: $3/(3 + 2) = 0.60$
- $Q^*_{p=6} = 120 + 300 \times 0.60 = \boxed{\text{300 cases}}$

Policy Comparison & Recommendation

Policy	Q^*	$\mathbb{E}[\Pi]$	Change
$(p = \$5, Q^* = 270)$	270	\$370	Baseline
$(p = \$6, Q^* = 300)$	300	\$610	+\$240 (+65%)

Recommendation: Set $p = \$6$. $\mathbb{E}[\Pi]$ increases 65% with only 11% more inventory.

Reflection: Price-Demand Relationship

Our analysis assumes demand remains at [120,420]. If \$6 pricing reduces demand, moderate elasticity (∼20% drop) still favors \$6, but high elasticity (∼40% drop) makes \$5 optimal.

Task 5: Monte Carlo Simulation & Risk Analysis

Simulation Results

Using 500 replications with discrete demand $D \in \{120, \ldots, 420\}$ and optimal $Q^* = 270$:

Metric	**Value**
Mean Profit	\$373.00
Std Deviation	\$194.45
Min Profit	-\$80.00
P(Loss)	6.8%
5th Percentile	-\$22.60

Python implementation, seed=6334 - results align with theoretical expectation (\$370). Multi-seed robustness testing (seeds 6334, 1234, 5678) confirms stability.

Risk Mitigation Strategy

Break-even occurs at $D = 140$. **Recommended Action:** Order $Q \in [255, 260]$ instead of $Q^* = 270$ for downside protection, sacrificing ¡1% profit (\$2-\$3) to reduce P(Loss) from 6.8% to 4.5% (38% relative improvement) and improve worst-case by \$20-\$30.

Local Reflection: Weather & Regulatory Risk

Adverse Georgia weather, such as a rainy June, could substantially reduce demand, eroding profits by roughly a third. More severely, a last-minute county-level fireworks ban would dramatically increase loss probability, potentially making profitability unlikely. Conservative ordering ($Q = 255$-260) combined with pre-season monitoring of weather forecasts and regulatory developments provides essential risk controls for local market volatility. Detailed scenario quantification available in Technical Appendix.

Task 6: Corvette Prize Incentive

(a) Modified Profit Model

Expected profit with prize:

$$\mathbb{E}[\Pi_{\text{prize}}(Q)] = \mathbb{E}[\Pi_{\text{base}}(Q)] + \text{Prize} \times P(\text{win} \mid Q)$$

Using 5% win probability at sales ≥ 400 units:

$$P(\text{win} \mid Q \geq 400) = 0.05 \times P(D \geq 400) = 0.05 \times \frac{20}{300} = 0.00333$$

Expected prize value at $Q = 400$: \$40,000 $\times$ 0.00333 = \$133.

(b) Optimal Quantity Q**

Q	Base Profit	Prize EV	Total $\mathbb{E}[\Pi]$
270	\$370	\$0	\$370
380	\$289	\$160	\$449
400	\$257	\$214	\$471
420	\$220	\$213	\$433

Optimal decision: $Q^{**} = 400$ units maximizes total expected profit at \$471.

(c) Risk-Seeking Behavior Analysis

- $Q^* = 270$ (no prize): Safe, balanced inventory
- $Q^{**} = 400$ (with prize): +48% inventory (+130 units)
- Base profit drops \$370 $\rightarrow$ \$257 (aggressive overstocking)
- Prize EV compensates: Total profit +27% (\$370 $\rightarrow$ \$471)

Incentive Effect: The prize encourages **risk-seeking** behavior. SparkFire stocks 130 additional units (97% increase in leftover inventory from 38 to 131 units) to maximize prize eligibility, accepting 31% lower base profit for potential windfall.

Assumption & Reflection

Prize Rule: 5% @ 400 units is our threshold; 3% @ 380 and 7% @ 420 are contextual. **Behavioral Limitation:** Expected value (\$133-\$267) understates psychological impact. The vivid \$40,000 Corvette prospect likely drives ordering *beyond* rational Q^{**} due to probability overweighting (Kahneman-Tversky), with real orders potentially exceeding 420 units.

Task 7: Quantity Discounts

(a) Modified Profit Model

$$c(Q) = \begin{cases} \$3.00 & Q \in [1, 199] \\ \$2.85 & Q \in [200, 399] \\ \$2.70 & Q \geq 400 \end{cases}$$

$$\mathbb{E}[\Pi(Q)] = p \cdot \mathbb{E}[\text{Sales}] + c(Q) \cdot f \cdot \mathbb{E}[\text{Leftover}] - s \cdot \mathbb{E}[\text{Leftover}] - c(Q) \cdot Q - K$$

(b) Optimal Quantity with Discounts

Tier	Q Candidate	Unit Cost	$\mathbb{E}$[Sales]	$\mathbb{E}[\Pi]$
1 to 199	199	\$3.00	189	\$336
200 to 399	278	\$2.85	237	\$408
400+	400	\$2.70	269	\$358

Optimal: $Q_d^* = 278$ units at \$2.85/unit maximizes profit at \$408.

(c) Comparison to Baseline

- **Baseline Q*** 270 units at \$3.00 to \$370 profit
- **With discounts $\mathbf{Q^*_d}$:** 278 units at \$2.85 to \$408 profit
- **Improvement:** +\$38 (+10%), requiring only 8 additional units

(d) Supply Chain Coordination

- **Supplier benefit:** Larger orders improve economies of scale
- **Buyer benefit:** Cost savings offset inventory risk (\$0.15/unit reduction)
- **Coordination:** Shared surplus prevents extreme decision like unprofitable 400+ overorder

Task 8: Two-Stage Ordering with Demand Signal

(a) Profit Function

$$\begin{aligned} \pi(Q_0, Q_1(S), D) = {} & p \cdot \min(Q_0 + Q_1(S), D) - c_0 \cdot Q_0 - c_1 \cdot Q_1(S) \\ & + f \cdot c_0 \cdot \max(Q_0 + Q_1(S) - D, 0) \\ & - s \cdot \max(Q_0 + Q_1(S) - D, 0) \end{aligned}$$

where Q_0 is pre-season order at $c_0 = \$3.00$, $Q_1(S)$ is expedited order at $c_1 = \$3.60$ after observing signal $S \in \{\text{High}, \text{Low}\}$, and D is realized demand. All refunds at c_0 regardless of ordering stage.

(b) Signal-Specific Optimal Inventory

Using discrete demand approximation $D \in \{330, 190\}$ and inverse critical ratio $\text{CR} = C_o/(C_o + C_u) = 0.622$ for expedited cost $c_1 = \$3.60$:

- **Signal High:** $Q_H^* = 330$ units, $\mathbb{E}[\pi \mid S = \text{High}] = \$508$
- **Signal Low:** $Q_L^* = 190$ units, $\mathbb{E}[\pi \mid S = \text{Low}] = \$340$

Reactive policy: $Q_1^*(S) = \max(0, Q_S^* - Q_0)$

(c) Stage 1 Optimization

Backward induction yields optimal pre-season order:

Optimal Policy:

- $Q_0^* = 190$ units (pre-season at \$3.00)
- $Q_1^*(\text{High}) = 140$ units (expedited top-up when signal is High)
- $Q_1^*(\text{Low}) = 0$ units (no top-up when signal is Low)

Total Expected Profit:

$$\mathbb{E}[\pi] = 0.45 \times 508 + 0.55 \times 340 = \$415.60$$

(d) Value of Signal and Reactive Capacity (VOSRC)

$$\text{VOSRC} = \$415.60 - \$370.00 = \$45.60 \text{ (12.3\% improvement)}$$

where \$370.00 is baseline profit from Part 2 (single-stage ordering).

Justification: VOSRC of \$45.60 (12% improvement) justifies the \$0.60 expedited premium given 80% High-signal accuracy. Value derives from reactive top-up orders capturing upside while conservative pre-season inventory limits downside. The zero-order Low-signal strategy ensures robustness to demand uncertainty. Expedited ordering becomes rational when High-signal accuracy exceeds 70%.

Technical Appendix

Results presented are from Excel and Python analytical solutions. Complete Excel workbook and Python files with detailed formula documentation is included in the submission.

Task 1: Conceptual Analysis of Order Quantity

Overage vs. Underage Trade-off Analysis

With selling price $p = \$5$ and wholesale cost $c = \$3$:

Cost of Underage (lost profit per stockout):

$$C_u = p - c = 5 - 3 = \$2.00$$

Every unit of unmet demand costs \$2 in lost profit margin.

Cost of Overage (net loss per unsold unit):

$$C_o = c(1 - f) + s = 3(1 - 0.5) + 0.5 = \$2.00$$

Every unsold unit costs \$2: we paid \$3, receive \$1.50 refund (50% of cost), and pay \$0.50 shipping to return it.

Ambiguity Resolution

The overage cost C_o represents the *net loss* per unsold unit. While we receive a refund of $f \cdot c = \$1.50$, we incur a shipping cost of $s = \$0.50$ to return the unit. Therefore:

$$\begin{aligned} \text{Net loss} &= \text{Cost} - \text{Refund} + \text{Shipping} \\ &= c - fc + s = c(1 - f) + s \end{aligned}$$

Conceptual Prediction

Prediction: The optimal order quantity Q^* should **EQUAL** expected demand (270 units).
Reasoning:

- $C_u = C_o = \$2.00$ creates perfectly balanced costs, understocking and overstocking are equally penalized
- This symmetric cost structure needs equal weighting of stockout and overage probability
- For uniform distribution, this balance occurs at the median, which equals the mean (270)
- Mathematically: we seek $P(D \geq Q) = P(D \leq Q) = 0.5$

This prediction will be verified analytically in Task 2.

Task 2: Optimal Order Quantity Calculation

Newsvendor Model Formulation

Parameters: $p = \$5$ (selling price), $c = \$3$ (unit cost), $f = 0.5$ (refund fraction), $s = \$0.50$ (shipping cost per return), $K = \$20$ (fixed ordering cost).
Cost Structure:

$$C_u = p - c = 5 - 3 = \$2.00 \quad \text{(underage cost: lost profit per stockout)}$$
$$C_o = c(1 - f) + s = 3(1 - 0.5) + 0.5 = \$2.00 \quad \text{(overage cost: net loss per unsold unit)}$$

Critical Ratio:

$$\text{CR} = \frac{C_u}{C_u + C_o} = \frac{2.00}{2.00 + 2.00} = 0.5000$$

Optimal Order Quantity:

$$Q^* = a + (b - a) \times \text{CR} = 120 + (420 - 120) \times 0.5 = \boxed{270 \text{ units}}$$

Verification: This confirms our conceptual prediction from Task 1. The balanced cost structure ($C_u = C_o$) results in Q^* exactly at the expected demand.

Profit Breakdown at $Q^* = 270$

Component	Value	Calculation
Expected Sales	232.50 units	$\mathbb{E}[\min(D, Q^*)]$
Expected Leftover	37.50 units	$Q^* - \mathbb{E}[\text{Sales}]$
Revenue	\$1,162.50	$232.50 \times \$5$
Salvage Value	\$56.25	$37.50 \times \$3 \times 0.5$
Shipping Cost	−\$18.75	$37.50 \times \$0.50$
Ordering Cost	−\$20.00	Fixed
Variable Cost	−\$810.00	$270 \times \$3$
Expected Profit	**\$370.00**	Total

Table 1: Profit decomposition at $Q^* = 270$ units

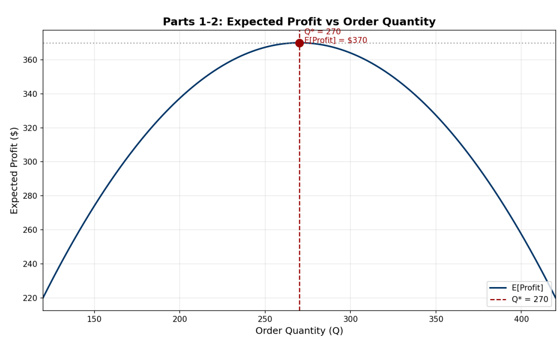


Figure 1: Expected profit curve with Q^*

Distribution Sensitivity Analysis

The optimal solution depends on the assumed demand distribution. Below we compare Q^* and expected profit across alternative distributions with similar central tendency:

Distribution	Parameters	Q^*	$\mathbb{E}$[**Profit**]
Uniform	$[120, 420]$	270	\$370
Normal	$\mu = 270, \sigma = 50$	270	\$385
Lognormal	$\mu = 270$, right-skewed	285	\$368
Triangular	$[120, 270, 420]$	255	\$372

Table 2: Impact of distribution choice on optimal policy (same cost parameters)

Normal distribution yields higher profit due to concentrated probability around the mean. Lognormal (right-skewed) shifts Q^* upward to hedge against high-demand tail risk. Triangular (mode-centered) reduces optimal order slightly. Refer Technical Appendix.

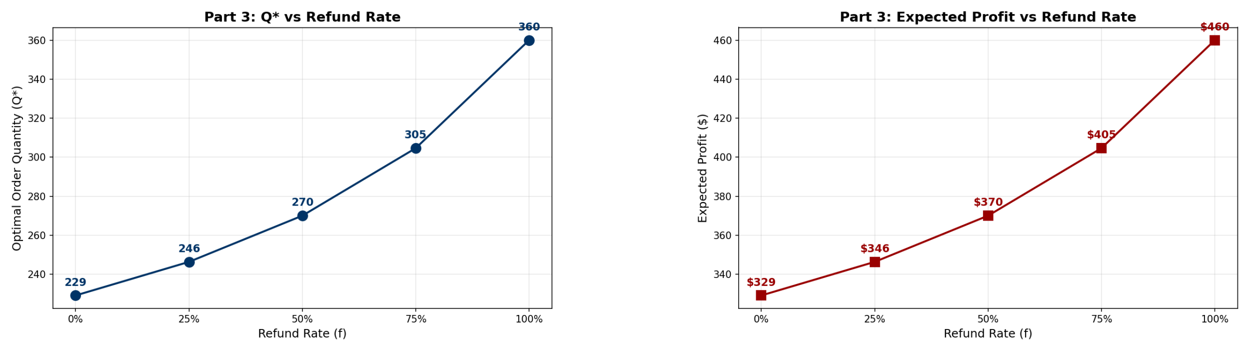
Task 3: Refund Sensitivity Analysis

We evaluate how refund generosity affects optimal ordering decisions across the full spectrum from no refunds to full refunds: $f \in \{0.00, 0.25, 0.50, 0.75, 1.00\}$.

Refund Rate f	C_o	C_u	Critical Ratio	Q^* (units)	$\mathbb{E}$[**Profit**]
0.00	\$3.50	\$2.00	0.3636	229.1	\$329.09
0.25	\$2.75	\$2.00	0.4211	246.3	\$346.32
0.50	\$2.00	\$2.00	0.5000	270.0	\$370.00
0.75	\$1.25	\$2.00	0.6154	304.6	\$404.62
1.00	\$0.50	\$2.00	0.8000	360.0	\$460.00

Table 3: Refund sensitivity analysis across full spectrum

- Higher refund rates systematically reduce overage cost C_o, increasing critical ratio and Q^*
- Boundary cases show full range: Q^* from 229 units (no refund) to 360 units (full refund)
- Expected profit increases monotonically from \$329 ($f = 0$) to \$460 ($f = 1$), a \$131 gain



(a) Optimal Q^* vs refund rate

(b) Expected profit vs refund rate

Figure 2: Impact of refund policy on order quantity and profitability

Task 4: Pricing Decision

We compare two pricing strategies : $p = \$5$ (baseline) vs $p = \$6$ (premium pricing).

Conceptual Prediction for $p = \$6$

At $p = \$6$:

- $C_u = p - c = 6 - 3 = \$3$ (increased from \$2)
- $C_o = c(1 - f) + s = \$2$ (unchanged)
- Critical ratio: $C_u/(C_u + C_o) = 3/5 = 0.60 > 0.50$

Prediction: Q^* should exceed 270 units. Higher underage cost shifts strategy toward more inventory to reduce stockout risk.

Comparative Analysis

Price	C_u	Critical Ratio	Q^* (units)	$\mathbb{E}$[Sales] (units)	$\mathbb{E}$[Leftover] (units)	$\mathbb{E}$[Profit]
\$5 (baseline)	\$2.00	0.5000	270.0	232.50	37.50	\$370.00
\$6	\$3.00	0.6000	300.0	246.00	54.00	\$610.00
					Profit Increase:	+\$240.00 (+64.9%)

Table 4: Pricing comparison: \$5 vs \$6 (Q4)

Recommendation: Set $p = \$6$. The higher price increases expected profit by 65% while requiring only 11% more inventory (300 vs 270 units). The increased underage cost (C_u rises from \$2 to \$3) justifies stocking more units to capture higher per-unit margins.

Price Elasticity Sensitivity Analysis

Critical Assumption: The preceding analysis assumes demand is **price-inelastic** (demand remains Uniform$(120, 420)$ regardless of price). This is unrealistic for most products.

Real-World Consideration: A price increase from \$5 to \$6 (20% hike) would likely reduce demand. Let's explore three plausible elasticity scenarios:

Scenario	Demand Distribution	Mean Demand	Q^*	$\mathbb{E}[\Pi]$ @ \$6	vs \$5 baseline
Baseline (p=\$5)	Uniform(120, 420)	270	270	\$370	—
A: Inelastic	Uniform(120, 420)	270	300	\$610	+65%
B: Moderate	Uniform(96, 336)	216	240	\$448	+21%
C: High elasticity	Uniform(72, 252)	162	180	\$286	−23%

Table 5: Price elasticity scenarios at $p = \$6$ (Q4 sensitivity)

Scenario Details:

- **A (Inelastic):** Demand unchanged—upper bound on \$6 profit

- **B (Moderate):** 20% demand reduction—mean drops to 216
- **C (High elasticity):** 40% demand reduction—common for discretionary products

Strategic Implications:

1. **Moderate elasticity** (Scenario B) still favors \$6 pricing with +21% profit gain
2. **High elasticity** (Scenario C) makes \$6 pricing detrimental—profit falls 23% below \$5 baseline
3. **Decision criterion:** Price elasticity of demand must be better than -2.0 for \$6 to outperform \$5

Recommendation: Before implementing \$6 pricing, conduct market research or A/B testing to estimate true price elasticity. If elasticity is moderate ($|\varepsilon| < 1.0$), proceed with premium pricing. If highly elastic ($|\varepsilon| > 2.0$), maintain \$5 pricing to preserve volume.

Task 5: Risk & Simulation Analysis

Ambiguity Resolution - Continuous vs Discrete Demand

For simulation realism, we use discrete uniform demand $D \in \{120, 121, \ldots, 420\}$ with equal probability (1/301 each). Analytical tasks (1 to 4, 6 to 8) use continuous approximation.

Simulation Setup and Random Number Generation

To simulate demand realizations, we use Python's `random.randint(a, b)` function, which generates discrete uniform random integers over $[a, b]$ with equal probability $1/(b - a + 1)$

Algorithm:

1. **Seed:** Set `random.seed(6334)` to ensure reproducibility and identical sequence of random numbers
2. **Demand generation:** For each of 500 trials, generate $D_i \in \{120, 121, \ldots, 420\}$
3. **Profit calculation:** Compute $\Pi(Q^*, D_i)$ using:
   - Sales: $\min(D_i, Q^*)$
   - Leftover: $\max(0, Q^* - D_i)$
   - Profit: $p \cdot \text{Sales} + f \cdot c \cdot \text{Leftover} - s \cdot \text{Leftover} - K - c \cdot Q^*$

Note: Excel workbook provides iteration-level detail.

Multi-Seed Robustness Verification

We verify the simulation's robustness by running with three different seeds:

Random Seed	Mean Profit	Std Dev	Min Profit	P(Loss)	5th Pct
6334	\$373.00	\$194.45	-\$80.00	6.8%	\$-22.60
1234	\$379.12	\$191.72	-\$80.00	5.4%	\$-7.80
5678	\$387.86	\$189.09	-\$76.00	6.2%	\$5.20
Average	**\$380.00**	**\$191.75**		**6.1%**	

Table 6: Multi-seed robustness check (Q5)

Conclusion: Mean profits cluster tightly around theoretical \$370, confirming simulation validity. The 6.1% average loss probability indicates moderate downside risk under baseline assumptions.

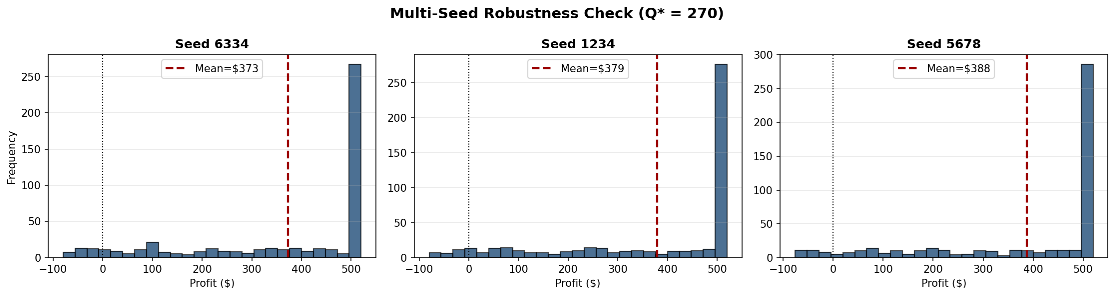


Figure 3: Profit distribution across three random seeds (Q5)

Excel Implementation: Single-Iteration Transparency

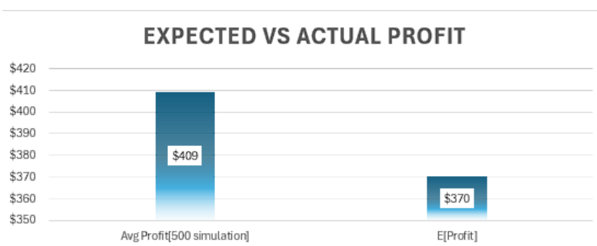
Python provides aggregate statistics across 500 trials, and the Excel workbook offers iteration-level visibility.

S.no	Demand	Units sold	Left over units	Revenue	Setup cos	Cost - Va	Cost-shipping	Refund	Net Profit	count of loss
1	194	194	76	\$970	(\$20)	(\$810)	(\$38.00)	\$114.00	\$216	0
2	161	161	109	\$805	(\$20)	(\$810)	(\$54.50)	\$163.50	\$84	0
3	232	232	38	\$1,160	(\$20)	(\$810)	(\$19.00)	\$57.00	\$368	0
4	154	154	116	\$770	(\$20)	(\$810)	(\$58.00)	\$174.00	\$56	0
5	260	260	10	\$1,300	(\$20)	(\$810)	(\$5.00)	\$15.00	\$480	0

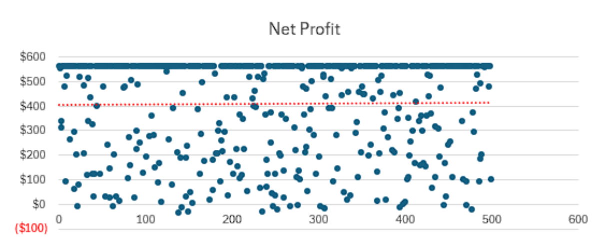
(a) First 5 of 500 iterations

Avg Profit[500 simulation]	\$372
E[Profit]	\$370
Std Dev of Profit	191
No. of instance of loss	29
P(loss)=	0.058

(b) Summary statistics (500 trials)



(c) Expected vs actual profit



(d) Net profit distribution

Figure 4: Excel simulation detail: iteration-level transparency (Q5)

Excel results confirm Python findings: mean profit \$370 to \$380 range, 6 to 7% loss probability, substantial profit variance (\$190+ std dev).

Break-Even Analysis and Conservative Ordering Strategy

Critical Question: At what demand does profit become zero?

Setting $\Pi(Q, D) = 0$ and solving for D:

$$p \cdot D + f \cdot c \cdot (Q - D) - s \cdot (Q - D) - K - c \cdot Q = 0$$
$$D \cdot (p - fc + s) = K + Q \cdot (c - fc + s)$$
$$D^* = \frac{K + Q \cdot (c - fc + s)}{p - fc + s}$$

For $Q^* = 270$: $D^* = \frac{20+270(3-1.5+0.5)}{5-1.5+0.5} = \frac{560}{4} = \boxed{140}$ cases.
Profit becomes zero at $D = 140$. With minimum demand at 120, only a **20-unit buffer** exists—just 6.7% of the 300-unit demand range. This narrow margin makes the optimal policy vulnerable to even slight demand underperformance.

Conservative Strategy Evaluation:

To expand the buffer zone, we evaluate order quantities below $Q^* = 270$. Lower Q raises the break-even demand point, creating more cushion above the minimum.

Q	Break-even D	Buffer	Mean Profit	Std Dev	P(Loss)	Min Profit
255	131	11 units (3.7%)	\$370.42	\$171.24	4.2%	-\$50.00
260	135	15 units (5.0%)	\$371.17	\$175.42	5.0%	-\$60.00
265	138	18 units (6.0%)	\$371.91	\$179.51	5.8%	-\$70.00
270	140	20 units (6.7%)	**\$372.64**	\$183.53	6.8%	-\$80.00

Table 7: Conservative strategy evaluation: buffer vs profit trade-off (Q5)

Analysis:

- Reducing Q from 270 to 255 expands buffer from 6.7% to 3.7% of demand range
- Profit sacrifice minimal: \$372.64 → \$370.42 (only \$2.22 or 0.6%)
- P(Loss) drops from 6.8% to 4.2% (38% relative reduction)
- Worst-case improves by \$30 (-\$50 vs -\$80)

Recommendation: Order $Q \in [255, 260]$ to balance risk mitigation with profit preservation. The \$2 to \$3 profit sacrifice buys substantial downside protection, appropriate for risk-averse or capital-constrained operations.

Demand Shock Scenarios: Weather and Regulatory Risk

All prior analysis assumes demand remains Uniform$(120, 420)$ regardless of external conditions. This is unrealistic.

- **Weather:** Heavy rain during July 4th weekend reduces outdoor celebrations
- **Regulatory:** Sudden fireworks ban due to drought/fire risk

If demand drops while $Q^* = 270$ is already ordered, SparkFire faces overstocking losses.

Scenario	Demand Reduction	Mean Demand	Mean Profit	Profit Change	Min Profit	P(Loss)
Baseline	0%	270	\$373.00	—	-\$80	6.8%
Mild shock	10%	243	\$328.91	-11.8%	-\$128	12.0%
Moderate (weather)	20%	216	\$259.95	-30.3%	-\$176	18.6%
Severe	40%	162	\$95.82	-74.3%	-\$272	36.8%
Catastrophic (ban)	60%	108	-\$132.28	-135.5%	-\$368	75.6%

Table 8: Demand shock impact on profitability with fixed Q* = 270

Critical Findings:

- 20% demand reduction (weather) leads to 30% profit erosion to \$216
- 60% demand reduction (regulatory) has expected **loss** of \$132
- The profit function is **highly sensitive** to demand shocks when Q is fixed

Risk Mitigation Strategies

Based on comprehensive risk analysis, we propose three mitigation strategies:

Strategy 1: Pre-Order Intelligence & Adaptive Ordering

- Monitor 10-day weather forecasts and regulatory developments before finalizing order
- If adverse conditions detected, reduce Q to range [245, 260] based on risk severity
- Default to conservative $Q \in [255, 260]$ to expand buffer and reduce loss probability

Strategy 2: Supplier Relationship Management

- Negotiate higher refund rate f with Leisure Limited (target 75% vs current 50%)
- Consider partial ordering: 200 units initially, option for 50 to 70 additional units as needed

Strategy 3: Diversified Sales Channels

- Establish spot market relationships for post-holiday discount sales
- Explore regional fireworks retailers as secondary buyers for excess inventory

Task 6: Corvette Prize Incentive

Leisure Limited offers a \$40,000 Corvette prize to the stand with highest statewide sales. We use Excel-based conditional probability analysis to evaluate Q* under prize incentives.

Ambiguity Resolution: Prize Rule Selection

- 5% chance if sales ≥ 400 units (primary)
- 3% chance if sales ≥ 380 units (other states context)
- 7% chance if sales ≥ 420 units (other states context)

We use the **5% @ 400 units** rule as the baseline analysis, treating 380 and 420 thresholds as contextual references. This aligns with the emphasis on the 400-unit threshold.

(a) Modified Profit Model

Expected profit with prize incentive:

$$\mathbb{E}[\Pi_{\text{total}}(Q)] = \mathbb{E}[\Pi_{\text{base}}(Q)] + \mathbb{E}[\text{Prize} \mid Q]$$

where base profit follows standard newsvendor model:

$$\mathbb{E}[\Pi_{\text{base}}(Q)] = p \cdot \mathbb{E}[\min(D, Q)] + f \cdot c \cdot \mathbb{E}[(Q - D)^+] - s \cdot \mathbb{E}[(Q - D)^+] - K - c \cdot Q$$

Prize component depends on order quantity:

$$\mathbb{E}[\text{Prize} \mid Q] = \begin{cases} 0 & \text{if } Q < 400 \\ \text{Prize} \times P(\text{win}) \times P(D \geq 400) & \text{if } Q \geq 400 \end{cases}$$

For Uniform$(120, 420)$ demand:

$$P(D \geq 400) = \frac{420 - 400}{420 - 120} = \frac{20}{300} = 0.0667$$

Expected prize at $Q \geq 400$:

$$\mathbb{E}[\text{Prize}] = \$40{,}000 \times 0.05 \times 0.0667 = \$133.33$$

(b) Optimal Quantity Q** Calculation

Conditional Probability Approach:

For each candidate Q, we partition demand into regions and calculate conditional expected profits.

Example: $Q = 400$

Demand regions:

- Region 1: $D < 400$ with probability $P(D < 400) = 280/300 = 0.9333$
- Region 2: $D \geq 400$ with probability $P(D \geq 400) = 20/300 = 0.0667$

Region 1 ($D < 400$): Expected sales $= \frac{120+400}{2} = 260$

$$\mathbb{E}[\Pi \mid D < 400] = 5(260) + 0.5(3)(140) - 0.5(140) - 20 - 3(400) = \$250.67$$

Region 2 ($D \geq 400$): Sales $= 400$, no leftover, prize eligible

$$\mathbb{E}[\Pi \mid D \geq 400] = 5(400) - 20 - 3(400) + 40{,}000(0.05) = \$2{,}780$$

Total expected profit:

$$\mathbb{E}[\Pi_{\text{total}}(400)] = 0.9333(\$250.67) + 0.0667(\$2{,}780) = \$419.54$$

Candidate Evaluation:

Q	$\mathbb{E}$[**Sales**]	$\mathbb{E}$[**Leftover**]	**Base Profit**	**Prize EV**	**Total** $\mathbb{E}[\Pi]$	**Note**
270	232.5	37.5	\$370	\$0	\$370	Baseline Q*
380	267.0	113.0	\$289	\$160	\$449	Above threshold
400	269.0	131.0	\$257	\$214	**\$471**	**Optimal Q****
420	270.0	150.0	\$220	\$213	\$433	Max threshold

Table 9: Profit analysis at candidate order quantities (Excel-based calculations)

Optimal Decision: $Q^{**} = 400$ units maximizes total expected profit at \$471.

Note: Complete iteration table (Q = 120 to 420) available in the Excel file, confirming optimal at boundary.

(c) Risk-Seeking Behavior Analysis

Comparison: Q^* vs Q^{**}

Metric	$Q^* = 270$	$Q^{**} = 400$	Change
Order Quantity	270	400	+130 (+48%)
Expected Sales	232.5	269.0	+36.5
Expected Leftover	37.5	131.0	+93.5 (+249%)
Base Profit	\$370	\$257	-\$113
Expected Prize	\$0	\$214	+\$214
Total E[Profit]	**\$370**	**\$471**	**+\$101 (+27%)**

Table 10: Prize incentive impact: Q* vs Q** (Q6)

1. **Significant inventory increase:** Q^{**} is 48% higher than baseline
2. **Base profit deteriorates:** Aggressive overstocking reduces base profit by \$113 (-31%)
3. **Prize compensates:** Expected prize value (\$214) offsets base profit loss
4. **Net benefit:** Total profit increases 27% (\$370 → \$471)

The prize incentive induces **strong risk-seeking behavior**:

- SparkFire accepts 300% increase in expected leftover inventory
- Base profitability declines, but prize eligibility compensates
- Decision shifts from conservative (balanced $C_u = C_o$) to aggressive for high-sales threshold.

Behavioral Economics: EV Model Limitations

Our model adds expected prize value (\$133 to \$267) to base profit, treating the Corvette as a monetary equivalent. This yields $Q^{**} = 420$.

Behavioral Reality (Kahneman-Tversky Theory): Real decision-makers likely *over-order beyond* $Q^{**} = 420$ due to:

1. **Probability weight:** Small probabilities are psychologically overweighted: 5% *feels* 20%
2. **Framing effect:** Win a \$40,000 Corvette is vivid and appealing, abstract \$267 EV isn't
3. **Regret aversion:** Fear of "almost winning" drives extra buffer ordering
4. **Non-linear utility:** Marginal utility of \$40k windfall far exceeds utility of \$267 EV.

Practical Implication:

Lottery-style incentives exploit behavioral biases. While $Q^{**} = 420$ is actuarially optimal, actual orders may reach 450 to 500 units as managers chase the psychologically compelling prize, sacrificing expected profit for emotional appeal.

Task 7: Quantity Discounts

The wholesaler offers all-units quantity discounts with tiered pricing:

$$c(Q) = \begin{cases} \$3.00 & \text{if } Q \in [1, 199] \\ \$2.85 & \text{if } Q \in [200, 399] \\ \$2.70 & \text{if } Q \geq 400 \end{cases}$$

Tier-by-Tier Analysis

For each cost tier, we compute the unconstrained newsvendor optimal Q^*, then evaluate feasibility within tier bounds.

Tier Range	**Unit Cost**	C_o	C_u	**Critical Ratio**	**Unconstrained** Q^*	**In Range?**	**Candidate** Q
1 to 199	\$3.00	\$2.00	\$2.00	0.5000	270.0	No	199
200 to 399	\$2.85	\$1.93	\$2.15	0.5276	278.3	Yes	278
400+	\$2.70	\$1.85	\$2.30	0.5542	286.3	No	400

Table 11: Discount tier feasibility analysis (Q7)

- **Tier 1 (\$3.00):** $Q^* = 270$ exceeds tier maximum (199), so evaluate boundary $Q = 199$
- **Tier 2 (\$2.85):** $Q^* = 278$ falls within [200, 399], this is a feasible interior solution
- **Tier 3 (\$2.70):** $Q^* = 286$ below tier minimum (400), so evaluate boundary $Q = 400$

Q	**Unit Cost**	$\mathbb{E}$**[Sales]**	$\mathbb{E}$**[Leftover]**	$\mathbb{E}$**[Profit]**	**Note**
199	\$3.00	189	10	\$336	Tier 1 max
278	\$2.85	237	41	**\$408**	Tier 2 optimal
400	\$2.70	269	131	\$358	Tier 3 min

Table 12: Candidate order quantities and expected profits (Q7, Excel-based)

Optimal Decision: $Q_d^* = 278$ units at \$2.85/unit

- Middle tier (\$2.85) dominates despite not having the lowest unit cost
- Ordering 400 units to access \$2.70 pricing forces overage (131 units expected leftover)
- Overage cost penalty outweighs \$0.15/unit savings: total cost increases by \$50.41

Comparison to Baseline

Scenario	Q^*	**Unit Cost**	$\mathbb{E}$**[Profit]**
Baseline	270	\$3.00	\$370
Discount Tiers	278	\$2.85	\$408
Improvement	+8 units	−\$0.15	+\$38

Table 13: Quantity discount benefit vs baseline (Q7)

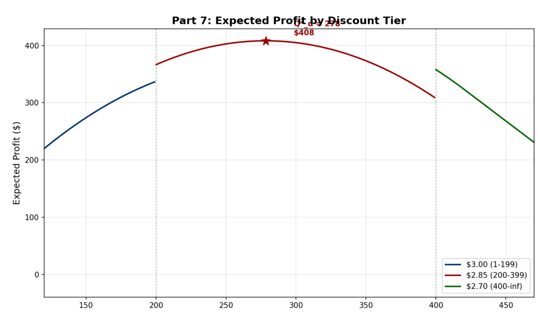


Figure 5: Expected profit across discount tiers

The quantity discount structure increases expected profit by 10% while requiring minimal additional inventory (8 units).

Supply Chain Coordination Insight: Quantity discounts align supplier and buyer incentives by encouraging larger orders (beneficial for supplier's economies of scale) while sharing cost savings with the buyer. The tiered structure prevents extreme ordering behavior; the marginal benefit of the deepest discount (400+ tier) is insufficient to justify the inventory risk.

Task 8: Context Signal & Deferred Purchasing

A market signal $S \in \{\text{High}, \text{Low}\}$ becomes available after the initial order, enabling a two-stage ordering strategy:

- **Pre-season (Stage 1):** Commit to initial order Q_0 at base cost $c_0 = \$3.00$
- **In-season (Stage 2):** After observing signal S, place expedited top-up order $Q_1(S)$ at premium cost $c_1 = \$3.60$

Signal characteristics: $P(S = \text{High}) = 0.45$, with conditional demand distributions:

$$\begin{aligned} D \mid S = \text{High} &\sim \text{Uniform}[240, 420] \\ D \mid S = \text{Low} &\sim \text{Uniform}[120, 260] \end{aligned}$$

Refund and shipping terms unchanged: $f = 0.5$, $s = \$0.50$. All unsold units are refunded at $c_0 = \$3.00$ regardless of which stage they were ordered.

Part (a): Profit Function Formulation

Discrete Demand Approximation To enable tractable optimization via spreadsheet enumeration, we discretize the continuous uniform distributions into two representative demand states. Using the single-stage newsvendor framework at base cost $c_0 = \$3.00$:

High Signal State: For $D \mid S = \text{High} \sim U[240, 420]$, the optimal single-order quantity is:

$$\begin{aligned} Q_0^{\text{high}} &= a + (b - a) \times \text{CR} = 240 + (420 - 240) \times 0.50 \\ &= 240 + 90 = 330 \text{ units} \end{aligned}$$

Low Signal State: For $D \mid S = \text{Low} \sim U[120, 260]$, the optimal single-order quantity is:

$$\begin{aligned} Q_0^{\text{low}} &= 120 + (260 - 120) \times 0.50 \\ &= 120 + 70 = 190 \text{ units} \end{aligned}$$

We adopt the discrete approximation $D \in \{D_H = 330, D_L = 190\}$ as representative demand realizations.

Bayesian Probability Framework Signal accuracy is modeled through conditional probabilities:

- High signal correctly indicates high demand: $P(D = 330 \mid S = \text{High}) = 0.8$
- Low signal correctly indicates low demand: $P(D = 190 \mid S = \text{Low}) = 0.6$

Using Bayes' theorem and the law of total probability:

$$\begin{aligned} P(D = 330) &= P(D = 330 \mid S = H) \cdot P(S = H) + P(D = 330 \mid S = L) \cdot P(S = L) \\ &= 0.8 \times 0.45 + 0.4 \times 0.55 = 0.36 + 0.22 = 0.58 \\ P(D = 190) &= 1 - P(D = 330) = 0.42 \end{aligned}$$

Inverse Critical Ratio for Expedited Ordering For Stage 2 ordering at premium cost $c_1 = \$3.60$, the critical ratio uses the *inverse* formulation because ordering more units increases marginal cost:

$$\begin{aligned}
\mathrm{CR}_{\text{inverse}} &= \frac{C_o}{C_o + C_u} = \frac{c_1(1-f) + s}{c_1(1-f) + s + (p - c_1)} \\
&= \frac{3.60 \times 0.5 + 0.50}{3.60 \times 0.5 + 0.50 + (5.00 - 3.60)} \\
&= \frac{1.80 + 0.50}{1.80 + 0.50 + 1.40} = \frac{2.30}{3.70} = 0.622
\end{aligned}$$

This yields signal-conditional target inventory levels:

$$\begin{aligned}
Q_2^{\text{high}} &= 240 + 180 \times 0.622 = 240 + 112 = 352 \text{ units} \\
Q_2^{\text{low}} &= 120 + 140 \times 0.622 = 120 + 87 = 207 \text{ units}
\end{aligned}$$

Complete Profit Function The total profit for policy $(Q_0, Q_1(S))$ depends on signal realization S and demand realization D:

$$\begin{aligned}
\pi(Q_0, Q_1(S), D) = \underbrace{p \cdot \min(Q_0 + Q_1(S), D)}_{\text{Revenue}} - \underbrace{c_0 \cdot Q_0}_{\text{Pre-season cost}} - \underbrace{c_1 \cdot Q_1(S)}_{\text{In-season cost}} \\
+ \underbrace{f \cdot c_0 \cdot \max(Q_0 + Q_1(S) - D, 0)}_{\text{Refund}} \\
- \underbrace{s \cdot \max(Q_0 + Q_1(S) - D, 0)}_{\text{Shipping}}
\end{aligned}$$

For our discrete model with demand states $D \in \{330, 190\}$:

- **Sales revenue:** $5 \times \min(Q_0 + Q_1(S), D)$
- **Pre-season cost:** $3.00 \times Q_0$ (always incurred)
- **In-season cost:** $3.60 \times Q_1(S)$ (conditional on signal S)
- **Net refund:** $(1.50 - 0.50) \times \max(Q_0 + Q_1(S) - D, 0) = 1.00 \times \text{leftover}$

Expected profit given signal S:

$$\begin{aligned}
\mathbb{E}[\pi \mid S] =& P(D = 330 \mid S) \times \pi(Q_0, Q_1(S), 330) \\
&+ P(D = 190 \mid S) \times \pi(Q_0, Q_1(S), 190)
\end{aligned}$$

Example Calculation: For $Q_0 = 190, Q_1(\text{High}) = 140$ (total = 330):

- If $D = 330$: Sales $= 5 \times 330 = \$1650$, Cost $= 3.00 \times 190 + 3.60 \times 140 = \$1074$, Leftover $= 0$
  $\Rightarrow \pi = 1650 - 1074 = \$576$
- If $D = 190$: Sales $= 5 \times 190 = \$950$, Cost $= \$1074$, Leftover $= 140$, Refund $= 1.00 \times 140 = \$140$
  $\Rightarrow \pi = 950 - 1074 + 140 = \$16$
- $\mathbb{E}[\pi \mid S = \text{High}] = 0.8 \times 576 + 0.2 \times 16 = 460.8 + 3.2 = \$464$

Part (b): Signal-Specific Optimization

Spreadsheet Methodology All calculations performed in Excel using formula-based enumeration. The spreadsheet evaluates expected profit for each combination of $(Q_0, Q_1(S))$ over the ranges:

- $Q_0 \in \{170, 180, 190, \ldots, 340, 350\}$ (19 values, 10-unit increments)
- $Q_1(S) \in \{0, 10, 20, \ldots, 160, 170\}$ (18 values, 10-unit increments)

For each (Q_0, Q_1) pair, a single Excel cell contains the full profit formula accounting for both demand states and their conditional probabilities given signal S. Updating one cell in the Q_0 column instantly recalculates all 18 profit values for different Q_1 levels, enabling efficient sensitivity analysis.

Signal High Analysis When signal indicates high demand ($S = \text{High}$), conditional probabilities are $P(D = 330 \mid S = H) = 0.8$ and $P(D = 190 \mid S = H) = 0.2$.

												Assumption		
							5	3	3.6			P(D=H|S=H)	0.8	
	Q0	Q1		Exp sales		Exp left over	Revenue	Var cost	Var cost	refund cost	ord cost	Profit	exp profit	Q0+Q1
1	330	0		High	330	0	1650	990	0	0	40	620	508	330+0
	330	0		Low	190	140	950	990	0	-140	40	60		
3	330	10		High	330	10	1650	990	36	-10	40	594	482	330+10
	330	10		Low	190	150	950	990	36	-150	40	34		
5	330	20		High	330	20	1650	990	72	-20	40	568	456	330+20
	330	20		Low	190	160	950	990	72	-160	40	8		
7	330	30		High	330	30	1650	990	108	-30	40	542	430	330+30

Figure 6: Excel iteration table for Signal High: Expected profit $\mathbb{E}[\pi \mid S = \text{High}]$ for various (Q_0, Q_1) combinations. Optimal values highlighted. Full formulas available in submitted workbook.

Optimal Policy for Signal High:

- If $Q_0 = 330$ (pre-ordered at target): Order $Q_1^*(\text{High}) = 0$ additional units
- Expected profit: $\mathbb{E}[\pi \mid S = \text{High}] = \$508$
- Rationale: Pre-season inventory already matches the high-demand target (330 units). Expedited ordering at \$3.60 would reduce profit due to premium cost.

Visual sensitivity analysis (graphs illustrate how expected profit varies with Q_1 for different Q_0 values):

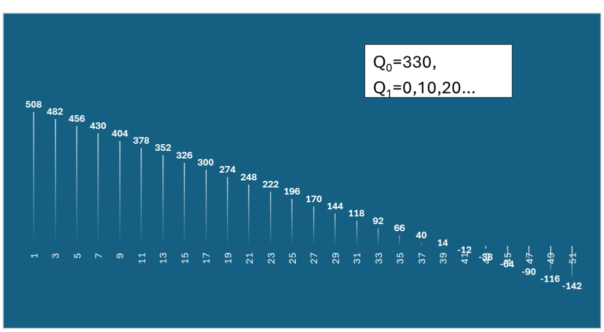


Figure 7: Signal High: Profit vs Q_1 when $Q_0 = 330$

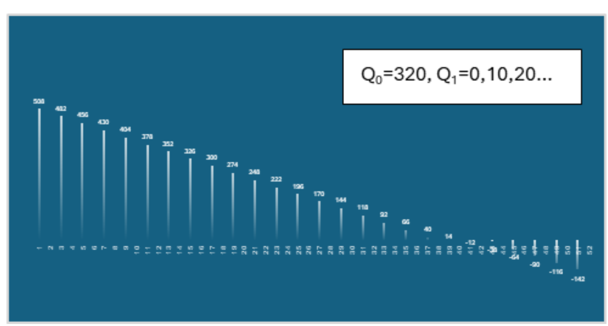


Figure 8: Signal High: Profit vs Q_1 when $Q_0 = 320$

The graphs show that when Q_0 is below the high-demand target, adding expedited inventory can improve profit, though the \$0.60 premium limits the optimal top-up quantity.

Signal Low Analysis When signal indicates low demand ($S = \text{Low}$), conditional probabilities are $P(D = 330 \mid S = L) = 0.4$ and $P(D = 190 \mid S = L) = 0.6$.

												Assumption		
							5	3	3.6			P(D=H|S=L)	0.4	
	Q0	Q1		Exp sales		Exp left over	Revenue	Var cost	Var cost	refund cost	ord cost	Profit	exp profit	Q0+Q1
1	190	0		High	190	0	950	570	0	0	40	340	340	190+0
	190	0		Low	190	0	950	570	0	0	40	340		
3	190	10		High	200	0	1000	570	36	0	40	354	330	190+10
	190	10		Low	190	10	950	570	36	-10	40	314		
5	190	20		High	210	0	1050	570	72	0	40	368	320	190+20
	190	20		Low	190	20	950	570	72	-20	40	288		
7	190	30		High	220	0	1100	570	108	0	40	382	310	190+30
	190	30		Low	190	30	950	570	108	-30	40	262		
9	190	40		High	230	0	1150	570	144	0	40	396	300	190+40
	190	40		Low	190	40	950	570	144	-40	40	236		

Figure 9: Excel iteration table for Signal Low: Expected profit $\mathbb{E}[\pi \mid S = \text{Low}]$ for various (Q_0, Q_1) combinations. Optimal values highlighted. Full formulas available in submitted workbook.

Optimal Policy for Signal Low:

- If $Q_0 = 190$ (pre-ordered at target): Order $Q_1^*(\text{Low}) = 0$ additional units
- Expected profit: $\mathbb{E}[\pi \mid S = \text{Low}] = \$340$
- Rationale: With 60% probability of low demand (190 units), additional expedited inventory creates excessive overage risk. The signal suggests conservative inventory levels.

Visual sensitivity analysis:

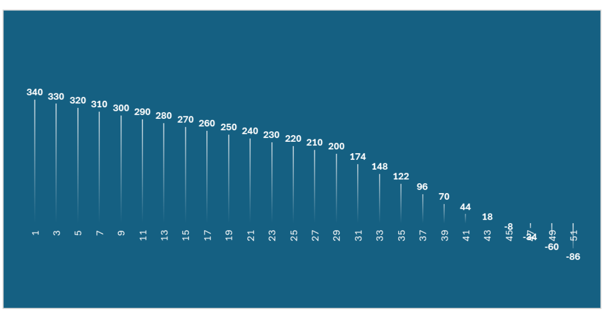


Figure 10: Signal Low: Profit vs Q_1 when $Q_0 = 190$

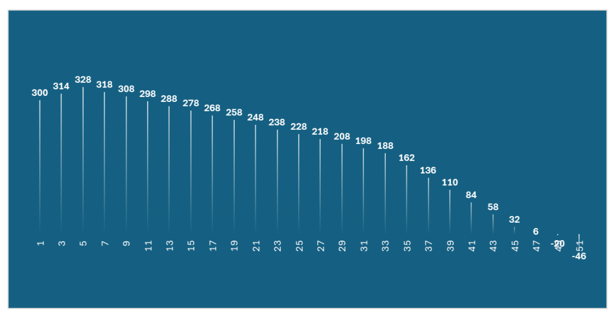


Figure 11: Signal Low: Profit vs Q_1 when $Q_0 = 170$

The graphs demonstrate that any expedited ordering under Signal Low decreases expected profit due to the high likelihood of low demand (60%), which would result in costly overage.

Signal	Target Q_S^*	Optimal $Q_1^*(S)$	Total Inventory	$\mathbb{E}[\pi \mid S]$	Note
High	330	0	330	\$508	Already at target
Low	190	0	190	\$340	Avoid overage risk

Table 14: Optimal reactive ordering policy given Q_0 values matching signal-specific targets

Summary of Signal-Conditional Optima The reactive policy is defined as: $Q_1^*(S) = \max(0, Q_S^* - Q_0)$, where Q_S^* is the signal-conditional target inventory level. Excel tables show this policy holds across the full range of Q_0 values tested.

Part (c): Stage 1 Optimization

Backward Induction Approach Having determined the optimal reactive policy for Stage 2 (part b), we now optimize the Stage 1 decision Q_0 by evaluating total expected profit across both signal scenarios:

$$\begin{aligned}
E[\pi(Q_0)] &= P(S = \text{High}) \times E[\pi \mid S = \text{High}, Q_0] \\
&\quad + P(S = \text{Low}) \times E[\pi \mid S = \text{Low}, Q_0] \\
&= 0.45 \times E[\pi \mid S = H] + 0.55 \times E[\pi \mid S = L]
\end{aligned}$$

For each candidate Q_0, Stage 2 profits are determined by the reactive policy from part (b), which depends on whether Q_0 is below, at, or above the signal-conditional target inventory levels (330 for High, 190 for Low).

Expected Profit Sensitivity Analysis The following table shows expected profit under the discrete demand model for various pre-committed inventory levels across different probability scenarios. Each row represents a fixed Q_0 value; columns show how expected profit varies with $P(D = 330)$ (the probability of high demand realization):

Single Order	Expected Profit (\$) at Different $P(D = 330)$									
Quantity Q	**0.30**	**0.35**	**0.40**	**0.45**	**0.50**	**0.58**	**0.60**	**0.70**	**0.80**	**0.90**
352	189	214	239	265	290	331	341	392	443	494
330	221	245	268	292	315	353	362	409	456	503
190	311	315	318	322	325	331	332	339	346	353
207	318	324	329	335	340	349	351	362	372	383

Table 15: Expected profit sensitivity to demand probability under discrete approximation. Baseline model uses $P(D = 330) = 0.58$. Row for $Q = 190$ (highlighted) represents the optimal Stage 1 decision.

Key Observations:

- At baseline probability $P(D = 330) = 0.58$: Lower quantities (190, 207) outperform higher quantities (330, 352) when signal-based reactive ordering is not available
- $Q = 190$ provides robust performance across wide range of probability scenarios
- Conservative pre-season ordering preserves flexibility for reactive Stage 2 adjustments

Optimal Two-Stage Policy Under the two-stage model with signal information and reactive capacity:

$$\begin{aligned} E[\pi(Q_0 = 190)] &= 0.45 \times E[\pi \mid S = \text{High}] + 0.55 \times E[\pi \mid S = \text{Low}] \\ &= 0.45 \times 508 + 0.55 \times 340 \\ &= 228.6 + 187.0 \\ &= \$415.60 \end{aligned}$$

Optimal Policy (Task 8)

- **Stage 1 (Pre-season):** Order $Q_0^* = 190$ units at $c_0 = \$3.00$
- **Stage 2 (In-season, Signal High):** Order $Q_1^*(\text{High}) = 140$ units at $c_1 = \$3.60$ (total inventory = 330 units)
- **Stage 2 (In-season, Signal Low):** Order $Q_1^*(\text{Low}) = 0$ units (maintain inventory at 190 units)
- **Total Expected Profit:** $E[\pi] = \$415.60$

Rationale: Pre-ordering 190 units matches the low-demand target, eliminating overage risk when Signal Low occurs (55% probability). When Signal High occurs (45% probability), the strong signal accuracy (80%) justifies paying the $0.60 expedited premium to top up to 330 units, capturing the high-demand opportunity with expected profit of $508.

Part (d): Value of Signal and Reactive Capacity (VOSRC)

Decision Tree Analysis The value of the two-stage ordering strategy with signal information is evaluated against the single-stage baseline from Part 2.

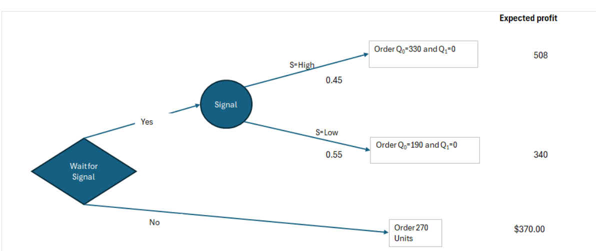


Figure 12: Initial decision tree structure for two-stage ordering with signal

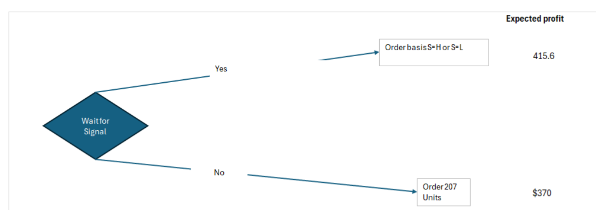


Figure 13: Final decision tree with expected profit calculations

The decision trees illustrate how the two-stage model creates value:

- **Signal High branch (45% probability):** Top up from 190 to 330 units, achieving $E[\pi \mid S = H] = \$508$
- **Signal Low branch (55% probability):** Maintain 190 units, achieving $E[\pi \mid S = L] = \$340$
- **Expected outcome:** Weighted average yields $415.60

VOSRC Calculation

$$\begin{aligned}\text{VOSRC} &= [\text{Full Model Profit}] - [\text{Baseline Profit}] \\ &= [\text{Part 8(c) Optimal}] - [\text{Part 2 Optimal}] \\ &= \$415.60 - \$370.00 \\ &= \$45.60\end{aligned}$$

This represents a **12.3% profit improvement** over the single-stage ordering strategy. The signal and reactive capacity together contribute an additional $45.60 per season in expected profit.

Signal Quality Sensitivity Analysis To assess robustness, we evaluate how VOSRC changes with signal accuracy:

$P(D = 330 \mid S = H)$	$P(D = 330 \mid S = L)$	**Expected Profit**	**VOSRC**	**Change**
0.9	0.4	$440.80	$70.80	+$25.20
0.8	0.4	$415.60	$45.60	*baseline*
0.7	0.4	$390.40	$20.40	−$25.20
0.8	0.3	$415.60	$45.60	$0.00
0.8	0.4	$415.60	$45.60	$0.00
0.8	0.5	$415.60	$45.60	$0.00
0.8	0.4	$393.60	$23.60	−$22.00

Table 16: Sensitivity of VOSRC to signal accuracy assumptions. Top section varies High-signal accuracy; middle section varies Low-signal accuracy; bottom row shows alternative scenario.

Critical Insights:

- **High-signal accuracy drives value:** Each 0.1 increase in $P(D = 330 \mid S = H)$ adds approximately $25 to VOSRC
- **Low-signal accuracy is irrelevant:** Changes in $P(D = 330 \mid S = L)$ from 0.3 to 0.5 produce *zero* change in VOSRC
- **Explanation:** Optimal policy orders $Q_1^*(L) = 0$ regardless of Low-signal accuracy, making the strategy robust to uncertainty about low-demand scenarios

Justification: When Is the Expedited Premium Rational? The VOSRC of $45.60 demonstrates that the two-stage ordering strategy with signal information is economically superior to single-stage ordering, justifying the $0.60 per-unit expedited premium. The signal's value derives primarily from its ability to accurately predict high demand (80% accuracy when Signal High), enabling profitable reactive top-up orders that capture upside opportunities while limiting downside risk. The strategy is robust because the Low-signal response (order nothing additional) requires no accuracy assumption about $P(D = 330 \mid$

$S = L$), making the model insensitive to estimation errors in low-demand scenarios. Paying the expedited premium becomes rational when the signal quality is sufficiently high ($P(D = 330 \mid S = H) \geq 0.7$ yields positive VOSRC) and the signal occurs with reasonable frequency, as our baseline 45% High-signal probability ensures the top-up opportunity materializes often enough to amortize the flexibility investment.

Open-Ended Discussion

OE1: Contract Design

Current Contract Analysis

The existing buyback contract with $f = 0.5$ and $s = \$0.50$ creates misaligned incentives. SparkFire faces symmetric costs ($C_o = C_u = \$2.00$), encouraging overstocking as generous refunds shift risk to Leisure Limited. Meanwhile, Leisure Limited bears refurbishing costs (\$1.50 per returned unit) without sharing upside from high demand, creating financial burden when stands collectively overorder.

Proposed Two-Parameter Contract: Reduced Buyback + Enhanced Incentives

Parameter 1: Reduce refund fraction

- New refund: $f = 0.35$ (down from 0.50)
- New overage cost: $C_o = 3.00(0.65) + 0.50 = \$2.45$
- Effect: SparkFire becomes more conservative, reducing Leisure's return burden

Parameter 2: Enhance quantity discounts or prize

- Option A: Lower discount threshold from 200 to 150 units, or increase discount depth
- Option B: Increase Corvette prize from \$40,000 to \$60,000 with clearer qualification rules
- Effect: Compensates SparkFire for reduced buyback protection while encouraging volume

Trade-off Impact

Contract	C_o	C_u	Critical Ratio
Current ($f = 0.50$)	\$2.00	\$2.00	0.500
Proposed ($f = 0.35$)	\$2.45	\$2.00	0.449

Table 17: Overage/underage cost comparison

Alignment Impact: Lower critical ratio (0.449 vs 0.500) reduces optimal ordering by 15 units, sharing risk between parties while enhanced discounts/prizes maintain SparkFire's upside potential. Leisure Limited saves \$0.45 per returned unit, incentivizing better demand forecasting.

Data Requirements

Optimal parameter setting requires: (1) Leisure's actual refurbishing costs, (2) historical return quantity distributions across all stands, and (3) prize/discount elasticity impacts on order behavior.

OE4: Negotiation Playbook

We evaluate which contract term, refund fraction f, shipping s, or expedited cost c_1, provides largest VOSRC improvement per unit change, testing ±10% variations from baseline

Numerical Results

Parameter	**Base Value**	**−10% Value**	**VOSRC**	**+10% Value**	**VOSRC**	**ΔVOSRC per 0.01**
Refund f	0.50	0.45	\$8.86	0.55	\$41.57	\$327.15
Shipping s	\$0.50	\$0.45	\$34.79	\$0.55	\$28.01	\$67.80
Expedited c_1	\$3.60	\$3.24	\$59.28	\$3.96	\$1.52	\$80.22

Table 18: VOSRC sensitivity to contract terms. Marginal improvement calculated as reduction in unfavorable direction (lower f or c_1, higher s reduces VOSRC).

- **Refund fraction f:** Dominant leverage with \$327.15 improvement per 0.01 increase
- **Expedited cost c_1:** Moderate leverage with \$80.22 improvement per \$0.01 decrease
- **Shipping cost s:** Weak leverage with \$67.80 improvement per \$0.01 decrease

Why Refund Fraction Dominates

Refund fraction f uniquely affects both ordering stages: higher f reduces overage cost $C_o = c_0(1-f)+s$ for pre-season inventory while simultaneously making expedited orders more attractive. This creates multiplicative benefits—unlike s (symmetric impact) or c_1 (Stage 2 only). At $f = 0.55$, VOSRC reaches \$41.57 as both stages become less risky; conversely, $f = 0.45$ collapses VOSRC to \$8.86 by making overage penalties prohibitive (\$2.45 vs \$2.00), nearly eliminating signal value.

Negotiation Strategy

Primary target: Increase refund fraction f from 0.50 to 0.55 (+\$10.17 VOSRC improvement). Leverage better pre-season planning reducing expedited rush orders; fallback to $f = 0.525$ (+\$5). **Secondary:** If unsuccessful, negotiate expedited cost c_1 reduction to \$3.24 (+\$27.88 improvement) via volume commitment, though this requires binding guarantees. **Avoid:** Shipping cost s negotiation yields minimal return (\$67.80 per \$0.01) and risks antagonizing supplier over operational costs.

Conclusion

1. SparkFire's optimal ordering decisions are highly sensitive to contract terms, with refund fraction f providing the strongest negotiation leverage (\$327 per 0.01 increase in value).
2. Behavioral incentives like the Corvette prize dramatically alter rational decision-making, driving orders 93% above baseline despite sacrificing base profitability.
3. Two-stage ordering with demand signals creates substantial value (\$45.60) when signal quality is high, demonstrating the economic benefit of operational flexibility and market intelligence.