

Homework 1 - Linear Algebra

1. Normalize the vector $\mathbf{v} = [1, 2, 3]$. Round your answer to 2 decimal points.

$$|\vec{v}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{1+4+9} = \sqrt{14} = 3.74 \quad \left\{ \hat{v} = \left[\frac{1}{3.74}, \frac{2}{3.74}, \frac{3}{3.74} \right] \approx [0.27, 0.53, 0.80] \right.$$

2. Simplify and Find the Length of the Vector

$$(a) \quad 3 * [1, 1] + 2 * [-1, 1] \quad \begin{matrix} 3 \cdot [1, 1] = [3, 3] \\ 2 \cdot [-1, 1] = [-2, 2] \end{matrix} \quad \rightarrow [3, 3] + [-2, 2] = [1, 5]$$

$$(b) \quad \text{Find the length of the vector calculated in (a)} \quad |\vec{v}| = \sqrt{1^2 + 5^2} = \sqrt{1+25} = \sqrt{26} \approx 5.10$$

3. Calculate the Cross Product

$$\begin{array}{lll} (a) \quad [0, 1, 1] \times [1, 1, 0] & \begin{matrix} a) \quad x = 1 \cdot 0 - 1 \cdot 1 = -1 \\ y = 1 \cdot 1 - 0 \cdot 0 = 1 \\ z = 0 \cdot 1 - 1 \cdot 1 = -1 \end{matrix} & \begin{matrix} b) \quad x = 3 \cdot 0 - 4 \cdot 0 = 0 \\ y = 4 \cdot 1 - 2 \cdot 0 = 4 \\ z = 2 \cdot 0 - 3 \cdot 1 = -3 \end{matrix} \\ (b) \quad [2, 3, 4] \times [1, 0, 0] & \begin{matrix} [-1, 1, -1] \end{matrix} & \begin{matrix} [0, 4, -3] \end{matrix} \\ (c) \quad [0, 3, 4] \times [2, 2, 2] & & \begin{matrix} c) \quad x = 3 \cdot 2 - 4 \cdot 2 = -2 \\ y = 4 \cdot 2 - 0 \cdot 8 = 8 \\ z = 0 \cdot 2 - 3 \cdot 2 = -6 \end{matrix} \end{array}$$

4. Calculate the Dot Product

$$\begin{array}{ll} (a) \quad [1, 0, 1] \cdot [0, 1, 1] & a) = (1)(0) + (0)(1) + (1)(1) = 0 + 0 + 1 = 1 \\ (b) \quad [0, 3, 4] \cdot [1, 0, 0] & b) = (0)(1) + (3)(0) + (4)(0) = 0 \\ (c) \quad [2, 3, 4] \cdot [6, 4, 3] & c) = (2)(6) + (3)(4) + (4)(3) = 12 + 12 + 12 = 36 \end{array}$$

5. Consider a triangle formed by connecting the three points $\mathbf{p}_1 = (0, 0, 0)$, $\mathbf{p}_2 = (1, 0, 0)$ and $\mathbf{p}_3 = (1, 1, 1)$.

- (a) Find the area of the surface of this triangle.

$$= (0 \cdot 1 - 0 \cdot 1, 0 \cdot 1 - 1 \cdot 1, 1 \cdot 1 - 0 \cdot 1) = (0, -1, 1) \rightarrow \sqrt{0^2 + (-1)^2 + 1^2} = \sqrt{2} \rightarrow \text{Area} = \frac{1}{2} \sqrt{2} \approx 0.71$$

- (b) Find the vector which is perpendicular to the surface of this triangle, AND has a positive z-direction. $\vec{n} = \vec{a} \times \vec{b} = (0, -1, 1) \rightarrow [0, -1, 1]$

6. Calculate the Matrix

$$\begin{array}{ll} (a) \quad \begin{bmatrix} 1 & 2 & 5 \\ -1 & -1 & 1 \\ 4 & 4 & -2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} & \begin{matrix} 1 \cdot 1 + 2 \cdot 2 + 5 \cdot 3 = 1 + 4 + 15 = 20 \\ -1 \cdot 1 + 1 \cdot 2 + 1 \cdot 3 = -1 + 2 + 3 = 0 \\ 4 \cdot 1 + 4 \cdot 2 + (-2) \cdot 3 = 4 + 8 - 6 = 6 \end{matrix} \quad \begin{bmatrix} 20 \\ 0 \\ 6 \end{bmatrix} \\ (b) \quad \begin{bmatrix} 2 & 0 & 3 \\ 0 & -1 & 2 \\ 3 & 2 & -2 \end{bmatrix} \cdot \begin{bmatrix} -1 & -4 & 1 \\ 1 & -1 & 4 \\ 0 & 0 & 5 \end{bmatrix} & \begin{matrix} 2 \cdot (-1) + 0 \cdot 1 + 3 \cdot 0 = -2 \\ 0 \cdot (-1) + (-1) \cdot 1 + 2 \cdot 0 = -1 \\ 3 \cdot (-1) + 2 \cdot 1 + (-2) \cdot 0 = -1 \end{matrix} \quad \begin{matrix} 2 \cdot (-4) + 0 \cdot (-1) + 3 \cdot 0 = -8 \\ 0 \cdot (-4) + (-1) \cdot (-1) + 2 \cdot 0 = 1 \\ 3 \cdot (-4) + 2 \cdot (-1) + (-2) \cdot 0 = -14 \end{matrix} \quad \begin{matrix} 2 \cdot 1 + 0 \cdot 4 + 3 \cdot 5 = 2 + 0 + 15 = 17 \\ 0 \cdot 1 + (-1) \cdot 4 + 2 \cdot 5 = -4 + 10 = 6 \\ 3 \cdot 1 + 2 \cdot 4 + (-2) \cdot 5 = 3 + 8 - 10 = 1 \end{matrix} \end{array}$$

7. Consider two lines $y = \frac{4}{3}x - 1$ and $y = 0$:

- (a) Find the intersection point between the two lines (Draw the lines on a graph if stuck). $\frac{4}{3}x - 1 = 0 \Rightarrow \frac{4}{3}x = 1 \Rightarrow x = \frac{3}{4}, y = 0 \quad \left(\frac{3}{4}, 0 \right)$

- (b) Are these lines perpendicular?

$$\frac{4}{3} \cdot 0 = 0 \neq -1 \Rightarrow \text{No, they are not perpendicular!}$$

Homework 1 - Readings Ch1 & Ch2

1. Define OpenGL, OpenGL ES and WebGL. Describe their relationship.
2. What is the difference in the software architecture of a webpage and a webpage using WebGL. Describe all components involved in the two cases.
3. In HTML, what is the `<canvas>` tag?
4. In Javascript, what line of code one writes to retrieve a 2D rendering context?

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- 1) *OpenGL: a cross-language, cross-platform API for rendering 2D and 3D graphics. It's used in desktop applications, including games and simulations.*
OpenGL ES: a lightweight version of OpenGL designed for mobile devices and embedded systems. It removes some features of standard OpenGL to improve performance on less powerful hardware.
WebGL: a JavaScript API based on OpenGL ES that enables rendering 3D graphics within any compatible web browser without plugins. It brings GPU-accelerated graphics to the web.
Relationship: WebGL is based on OpenGL ES, which in turn is a subset of OpenGL.
OpenGL → OpenGL ES → WebGL
(Desktop) → (Mobile) → (Browser)
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- 2) *A standard webpage uses HTML, CSS, and JavaScript to render content through the browser's CPU. In contrast, a WebGL-enabled page uses the `<canvas>` element and the WebGL API to render interactive 2D/3D graphics using the GPU. This setup involves shaders and a graphics pipeline, allowing real-time, high-performance rendering in the browser.*
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- 3) *It creates a drawable area in HTML where graphics can be rendered using JavaScript, including 2D and 3D content via WebGL.*
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- 4) *var ctx = canvas.getContext("2d");*