ADARSH SINGH
04/20/2025

Homework 2 - Transformations

1. **Using the labels Translate, Rotation, Scale, or Reflect, label the transformation of point p when multiplied by matrix M (p' = Mp):**

   (a) Rotation (− 90° counterclockwise) $M = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

   (b) Rotation (− 45° clockwise) $M = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$

   (c) Scale $M = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

   (d) Reflect (about the y-axis) $M = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

   (e) Translate (by (3, 2)) $M = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

2. **Find the matrix for a 120° rotation about the axis defined by the vector r = (1,1,0).**

   $\vec{r} = (1,1,0)$ $\quad \hat{r} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$ $\quad x^2 = y^2 = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$ $\quad R = \begin{bmatrix} 1/4 & 3/4 & \sqrt{3}/2 \\ 3/4 & 1/4 & -\sqrt{3}/2 \\ -\sqrt{3}/2 & \sqrt{3}/2 & -1/2 \end{bmatrix}$

   $\|r\| = \sqrt{1^2 + 1^2 + 0^2} = \sqrt{2}$ $\quad xy = \frac{1}{2}, xz = yz = 0$

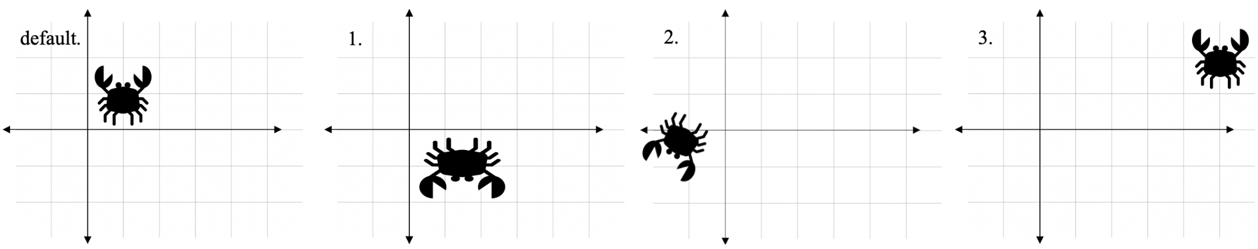
3. **Match the following 2D homogeneous matrices to the transformations in the image:**

   a. $\begin{bmatrix} -\frac{\sqrt{3}}{2} & -\frac{\sqrt{1}}{2} & 0 \\ \frac{\sqrt{1}}{2} & -\frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 1.5 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ c. $\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

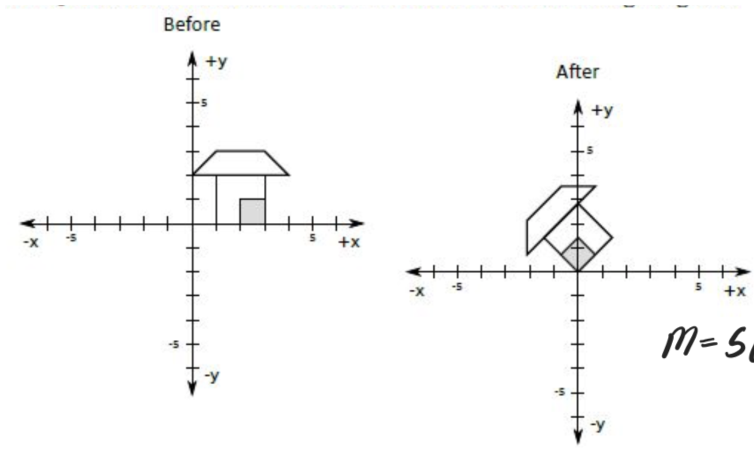
   Rotation → 150° counter-clockwise

   Scale + Reflect ×by 1.5, over x-axis

   Translate (by (4,1))



4. **Describe a sequence of Translate(x,y), Rotate(degrees), and/or Scale (by a value of) that when multiplied describe the below transformation to go from the 'Before' to 'After'.**

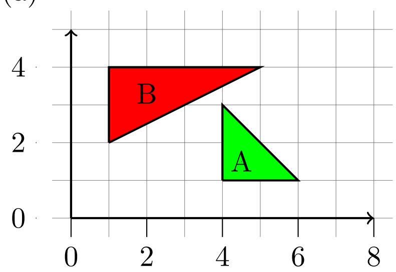


Translate (-2.5, -2.5)
→ Rotate (-90°)
→ Scale (0.5, 0.5)

$M = S(0.5, 0.5) \cdot R(-90°) \cdot T(-2.5, -2.5)$

5. (a) **Describe (using a sequence of (3 x 3) matrix multiplications) the transformations needed to transform the triangle from A to B in the figure below:**

(a)



$$\begin{bmatrix}x'\\y'\\w'\end{bmatrix} = \begin{bmatrix}1&0&2\\0&1&0\\0&0&1\end{bmatrix} * \begin{bmatrix}1&0&0\\0&-1&0\\0&0&1\end{bmatrix} * \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} * \begin{bmatrix}x\\y\\w\end{bmatrix}$$

1) $R = \begin{bmatrix}1&0&0\\0&-1&0\\0&0&1\end{bmatrix}$

2) Translate by (2,0) to move it right Matrix.

$T = \begin{bmatrix}1&0&2\\0&1&0\\0&0&1\end{bmatrix}$

$P' = T \cdot R \cdot p$

(b) **Using the transformations found in part (a), multiply the following points with the matrix.**

P_1.**(4, 1)**P_2.**(4, 3)**

$$M = \underbrace{\begin{bmatrix}1&0&2\\0&1&0\\0&0&1\end{bmatrix}}_{\text{Translate }(2,0)} \cdot \underbrace{\begin{bmatrix}1&0&0\\0&-1&0\\0&0&1\end{bmatrix}}_{\text{Reflect x-axis}} \cdot \underbrace{\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}}_{\text{Identity}} = \begin{bmatrix}1&0&2\\0&-1&0\\0&0&1\end{bmatrix}$$

$$\begin{bmatrix}x\\y\\1\end{bmatrix} = \begin{bmatrix}4\\1\\1\end{bmatrix} \qquad M \cdot P_1 = \begin{bmatrix}1&0&2\\0&-1&0\\0&0&1\end{bmatrix} \cdot \begin{bmatrix}4\\1\\1\end{bmatrix} = \begin{bmatrix}1\cdot4+0+2=6\\0-1\cdot1+0=-1\\1\end{bmatrix} \Rightarrow \underline{(6,-1) = P_1'}$$

$$\begin{bmatrix}4\\3\\1\end{bmatrix} \Rightarrow M \cdot P_2 = \begin{bmatrix}1\cdot4+0+2=6\\0-1\cdot3+0=-3\\1\end{bmatrix} \Rightarrow \underline{(6,-3) = P_2'}$$

ADARSH SINGH
04/20/2025

Homework 2 - Modeling

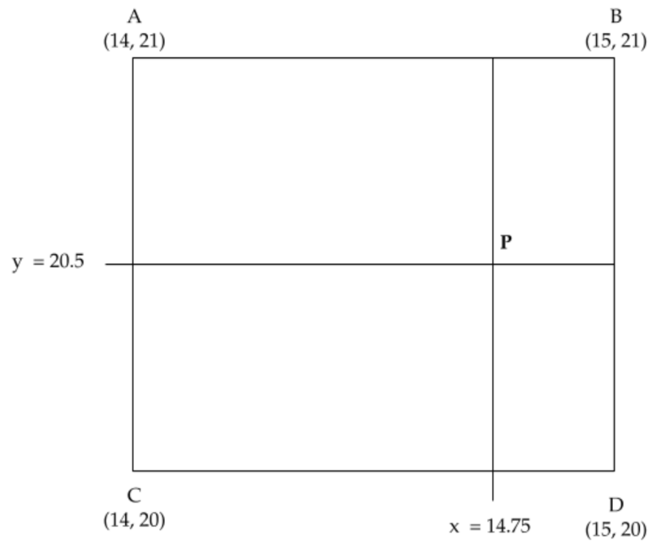
Homework Answers on Next Page

1. **Define uniform, attribute and varying variables.**
2. **What type is the shader and describe its function?**

(a)
```
attribute vec2 a_Position;
main() {
    gl_Position = vec4(0.5 * a_Position, 0.0, 1.0);
}
```

(b)
```
main() {
    gl_FragColor = vec4(0.0, 0.0, 1.0, 1.0);
}
```

3. **Using linear interpolation, calculate the color of the following points that are on the line defined by the 2D points p1 = [3,5] with color [255,0,0] and p2 = [10, 5] with color [0,0,255].**
   (a) [4.8, 5] (b)[7.5, 5] (b)[9, 5]
4. **Using barycentric coordinates, calculate the color for each of the following points inside the triangle defined by points p1 = [0,0,0], p2 = [3,5,0] and p3 = [6,0,0] with respective RGB colors c1 = [255, 0, 0], c2 = [0, 255, 0], c3 = [0, 0, 255].**
   (a) [1, 1, 0] (b) [3, 4, 0] (c) [5, 0.25, 0]
5. **Calculate the RGB color value for point P in the diagram below using bilinear interpolation. Show the intermediate values that you compute for interpolating along one of the two axes. The colors of the points A, B, C and D are the following:**
   A = RGB(0, 0, 0) B = RGB(200, 200, 200)
   C = RGB(200, 0, 0) D = RGB(200, 0, 0)



6. **Given a polygon with vertices P1, P2, P3, each represented by a 3D vector [x y z], what formula calculates the face's normal?**

1. Uniform: a variable that says constant for all processed vertices of fragments during a single draw call and is set by the CPU.

Attribute: a per-vertex input variable passed from the CPU to the vertex shader, used to supply data like posistion or color.

Varying: a variable used to pass interpolated data from the vertex shader to the fragment shader.

2. a) Vertex; transforms 2D vertex posistion a_posistion by calling it by 0.5 and converting it into a 4D homogenous coordinate for rendering.

b) Fragment; outputs a constant blue color for every fragment rendered.

3. a) $p=[4.8, 5]$ $\quad t=\frac{4.8-3}{10-3}=0.2571$

$Color=(1-t)\cdot[255,0,0]+t\cdot[0,0,255]=[255\cdot(1-0.2571),0,255\cdot0.2571]\approx[189.5,0,65.5]$

b) $p=[7.5,5]$ $\quad t=\frac{7.5-3}{7}=\frac{4.5}{7}\approx0.6429$

$Color=[255\cdot(1-0.6429),0,255\cdot0.6429]\approx[90.05,0,163.95]$

c) $p=[9,5]$ $\quad t=\frac{9-3}{7}=\frac{6}{7}\approx0.8571$

$Color=[255\cdot(1-0.8571),0,255\cdot0.8571]\approx[36.43,0,218.57]$

4. a) Point: $[1,1,0]$, $Area_{total}=\frac{1}{2}\left|(3(0-0)+6(5-0)+0(0-0))\right|=\frac{1}{2}\cdot30=15$

$\alpha=7/15$, $\beta=0.1$, $\gamma=6.5/15$, $Color=0.4667\cdot[255,0,0]+0.1\cdot[0,255,0]+0.4333\cdot[0,0,255]$

$=[119,25.5,110.5]$

b) Point: $[3,4,0]$

$\alpha=1/6$, $\beta=5/6$, $\gamma=0$, $Color=\frac{1}{6}\cdot[255,0,0]+\frac{5}{6}[0,255,0]=[42.5,212.5,0]$

c) Point: $[5,0.25,0]$

$\alpha=0.0417$, $\beta=0.0833$, $\gamma=0.875$, $Color=0.0417[255,0,0]+0.0833\cdot[0,255,0]+0.875\cdot[0,0,255]$

$\approx[10.6,21.25,223.125]$

5. A to B: $P_{AB}=(1-0.3)\cdot A+0.3\cdot B=0.7\cdot[0,0,0]+0.3\cdot[200,200,200]=[60,60,60]$

C to D: $P_{CD}=(1-0.3)\cdot C+0.3\cdot D=0.7\cdot[200,0,0]+0.3\cdot[200,0,0]=[200,0,0]$

$y=0.6: P=(1-0.6)\cdot P_{AB}+0.6\cdot P_{CD}=0.4\cdot[60,60,60]+0.6\cdot[200,0,0]=[144,24,24]$

6. $n=(P_2-P_1)\times(P_3-P_1)$

Where $\times$ denotes the cross product.

ADARSH SINGH
04/20/2025

Homework 2 - Readings Ch3 (91-113) & Ch4

1. Given a point $p(x, y, z)$ and the point $p'(x', y', z')$ where $p'(x', y', z')$ is the point $p(x, y, z)$ translated by Tx, Ty, Tz:

   (a) Show the **equations** to compute p' from p.

   $x' = x + T_x \quad , \quad y' = y + T_y \quad , \quad z' = z + T_z$

   (b) Show the 4x4 **transformation matrix** to compute p' from p.

   $$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

2. Given a point $p(x, y, z)$ and the point $p'(x', y', z')$, where $p'(x', y', z')$ is the β degree rotated point of $p(x, y, z)$ around the z-axis:

   (a) Show the **equations** to compute p' from p.

   $x' = x \cos\beta - y \sin\beta \quad , \quad y' = x \sin\beta + y \cos\beta \quad , \quad z' = z$

   (b) Show the 4x4 **transformation matrix** to compute p' from p.

   $$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\beta & -\sin\beta & 0 & 0 \\ \sin\beta & \cos\beta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3. Given a point $p(x, y, z)$ and the point $p'(x', y', z')$, where $p'(x', y', z')$ is the point $p(x, y, z)$ scaled by Sx, Sy, Sz :

   (a) Show the **equations** to compute p' from p.

   $x' = x \cdot S_x \quad , \quad y' = y \cdot S_y \quad , \quad z' = z \cdot S_z$

   (b) Show the 4x4 **transformation matrix** to compute p' from p.

   $$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

4. What is a model matrix? Write a simple vertex shader that uses a model matrix. Explain your shader.

A model matrix is a 4x4 transformation matrix that converts object (local) coordinates into world coordinates, applying transformations like translation, rotation, scaling to posistion an object in the scene.

Vertex Shader:

```
attribute vec4 a_Posistion;
uniform mat4 u_ModeMatrix;

void main() {
    gl_Posistion = u_ModelMatrix * a_Posistion;
}
```