

# Low Cost Portable Inverted Pendulum Built on a Toy Car

Ken Sasaki\*

\*Department of Human and Engineered Environmental Studies, The University of Tokyo  
Japan (e-mail: ksasaki@k.u-tokyo.ac.jp).

**Abstract:** A standalone inverted pendulum system for education was built on a toy car. A stick of approximately 50 cm was placed on a toy car with a potentiometer at its bottom to measure the tilt angle. A simple optical encoder of three pulses-per-revolution is attached to the wheel to measure the position of the car. A microcomputer runs on BASIC-like language which is easy enough to use with minimal knowledge of computer programming. It has a function to monitor values of variables in real time, which is very useful for showing the outputs of the sensors as well as for debugging. It is portable and compact enough to use it on a desktop in a classroom or on a small space of floor. Its low cost makes it easy to provide many sets within a low budget. This inverted pendulum car has been used in introductory courses on electro-mechanical systems for several years.

**Keywords:** control, education, microprocessor, programming, motor, sensor;

## 1. INTRODUCTION

Inverted pendulum is one of the popular mechanisms used for education of control theory and electro-mechanical systems (mechatronics). This is because an inverted pendulum is an unstable system which is difficult to stabilize without the knowledge of control theory. Adjustment of feedback gains of inverted pendulum requires a good tuning strategy because inverted pendulum is a 1-input-2-output system whose feedback gains affect both outputs. Feedback gains obtained by control theory show much better performance than gains adjusted by trial and error. One may never find the right combination of gains to balance the inverted pendulum without the knowledge of control theory. When students succeed in balancing the pendulum, they will experience sense of accomplishment and will begin to appreciate the usefulness of theory in practical applications. These experiences are important factors in education. Furthermore, inverted pendulum is just fun to watch.

Many inverted pendulum mechanism for education are built on a linear pendulum table or a rotary table (Kling et al., 2001). Although a stationary mechanism is reliable and easy to build its control system, there will be some restrictions on places to set up the system in classrooms. This paper describes a portable inverted pendulum built on a toy car that can be used on any horizontal flat surface with a size of an office table. A student can do experiments on a desk top, on the floor, or in a hallway. They can even take this inverted pendulum car home and try it at home. A similar cart-type inverted pendulum has been reported (F.L.Tan et al. 2002). The difference is that their control circuit is all analogue circuit and the design process of the circuit is part of the objective of the education. The inverted pendulum car reported in this paper utilizes a microcomputer for easy implementation of different feedback strategies and gains.

## 2. SYSTEM ARCHITECTURE AND COMPONENTS

### 2.1 System Architecture

Composition of the inverted pendulum built on a toy car is shown in Fig. 1. A stick of approximately 50 cm is used as the inverted pendulum. Bottom end of this stick is directly attached to the axis of a potentiometer that measures the tilt angle. An optical encoder disk with three slits is inserted in the front wheel axle to measure the position and the velocity of the car. A microcomputer reads the analogue voltage output of the potentiometer and counts the encoder pulses. Then the microcomputer calculates a current command as a linear sum of the four state variables, namely tilt angle and angular velocity of the stick, and position and velocity of the car. The motor current command is sent to a motor driver circuit and the motor drives the rear axle.

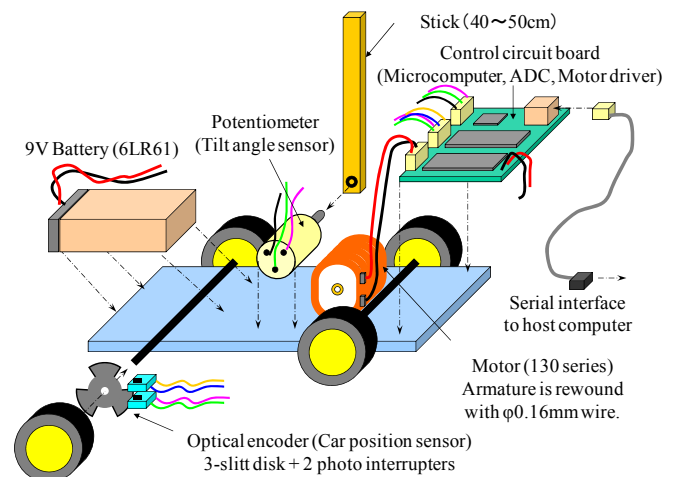


Fig. 1. Composition of inverted pendulum car

The micro computer has a serial link with a host computer to download programs and to upload values of variables in the program. The serial link cable can be disconnected after a program is downloaded. Since the motor and the microcomputer are both driven by a single 9 V battery, this inverted pendulum car runs without any cables attached to stationary experimental apparatuses.

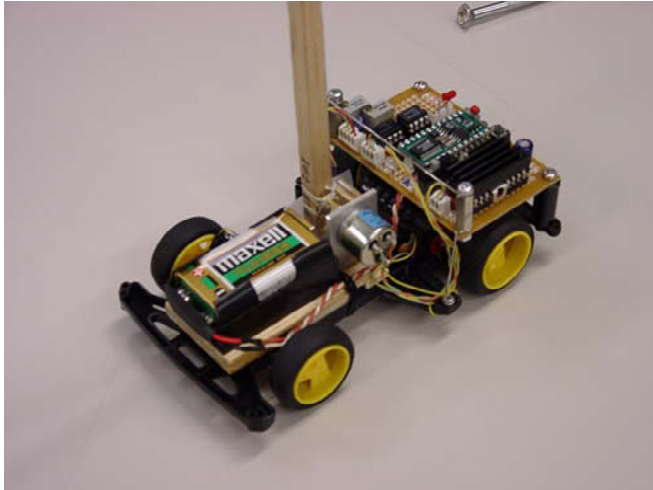


Fig. 2. Inverted pendulum built on a toy car

## 2.2 Toy Car

The toy car used for this portable inverted pendulum car is called “Mini-4WD” (registered trademark) manufactured by Tamiya Inc. This toy car kit has been very popular among children for many years in Japan. Many students, especially the students who are interested in engineering, have had played with them in their childhood. The kit consists of a plastic chassis and a body, gear train, wheels and axles. Motor is not included. The price of this kit is about 800 Japanese Yen which is approximately 8 US dollars.

## 2.3 Microcomputer

There are several features that must be considered in selecting a microcomputer for this portable inverted pendulum for education. Peripheral parts should be minimal. It should have simple interface with a host computer for programming. Programming and debugging environment should be simple and easy to understand. Free licence is preferable. Among the many microcomputers for embedded systems available on the market, “BASIC Stamp2” manufactured by Parallax Inc. matched best to these criterions.

Peripheral parts: Voltage regulator and serial interface with a host computer are both mounted on a 28pin DIP size board. Only AD convertor and motor driver had to be added for controlling the inverted pendulum car.

Programming language: It uses a BASIC-type language called PBASIC. Its syntax is intuitive and easy to understand. A student with minimal knowledge of computer programming can start using the language after about an hour. Since PBASIC is an interpreter language, its execution speed is slower than microcomputers that run on compiler

languages. Complicated control algorithms are difficult to program. Mathematical operation is limited to integers.

Runtime monitoring: It has instructions for monitoring values of variables while executing a program. This feature is extremely useful for education as well as for debugging. Although most microprocessor development tools have this monitoring function, it takes time to learn.

Memory space: There are only 32 bytes of RAM available for variables. This small memory space limits programming of complicated control algorithms.

## 2.4 AD Converter and Motor Driver

Twelve bit AD converter LTC1298 (Linear Technology) converts the analogue output of the potentiometer for tilt angle measurement. This device has serial interface which is compatible with BASIC Stamp2 microcomputer. Programming language PBASIC also has an instruction set for this interface.

A PWM motor driver TA7298P (Toshiba) controls the armature current with 4 bit resolution. The maximum current is 0.7 A for continuous current and 1.5 A for short period.

Two LEDs and one momentary switch are attached to the input/output pins of the microcomputer for simple display and control input.

Fig. 3 shows the control circuit board with motor driver, Basic Stamp2 microcomputer, AD converter, and a modular connector socket for serial interface with a host computer.

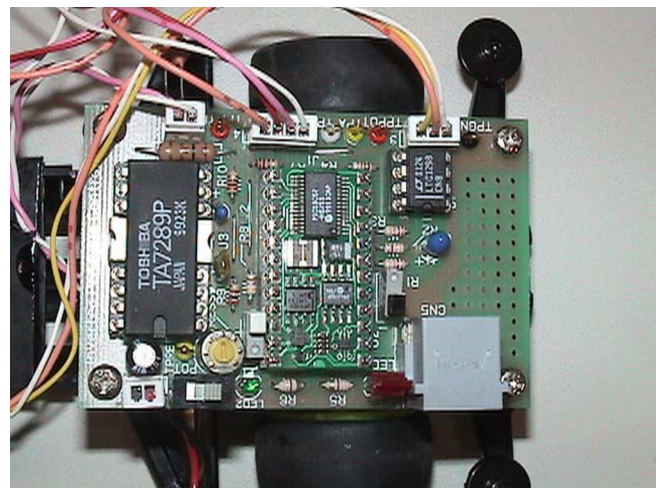


Fig. 3. Control circuit board

## 2.5 Tilt Sensor

A stick is directly attached to the axis of a potentiometer to measure the tilt angle. Rotational friction at the axis must be small enough to allow continuous linear control near the equilibrium point where moment caused by gravity becomes small. The stick should not be able to stand by itself without control. We selected CP-2UT (MIDORI Precision Co. Ltd.) that uses magnetoresistive sensors to eliminate Coulomb

friction at the contact brushes that are often used in low cost potentiometers. Measurement range is 90 degrees, which is sufficient for control of inverted pendulum.

## 2.6 Wheel Axle Encoder

A small disk with three slits at its circumference is inserted between the front wheel and the chassis as shown in Fig. 4. This encoder disk is cut out from a 1.5 mm thick aluminium sheet by electric discharge machining. It has a hexagonal hole at its center to match the axle of Mini-4WD that has a hexagonal cross section as shown in Fig. 5. This hexagonal fitting makes the disk turn with the wheel axle without using screws or other parts to fix the disk on the axle.

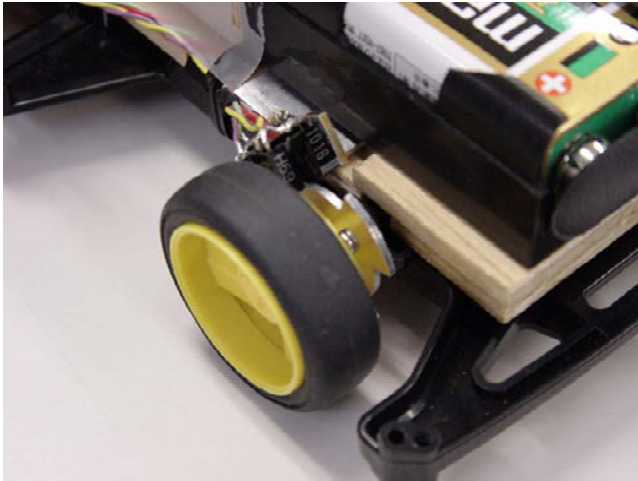


Fig. 4. Optical encoder for car position measurement

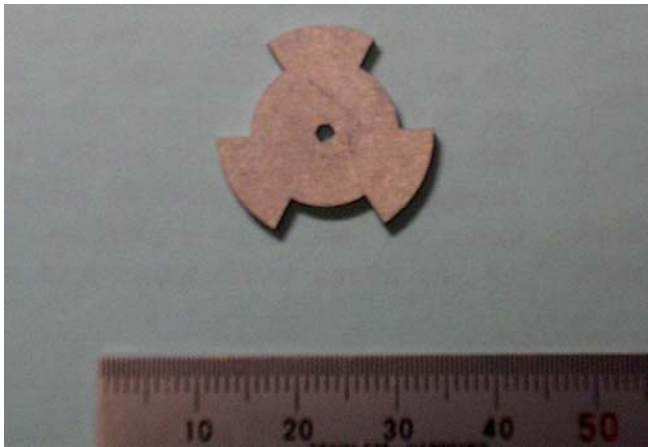


Fig. 5. Optical encoder disk with 3 slits

A pair of photo-interrupters (EE-SX1018, OMRON) sends pulses according to the rotary position of the disk. The control program counts these pulses by analyzing the transitions of zeros and ones of pulses. This scheme allows only one increment or decrement during one control cycle. Therefore, the shortest interval between the transitions of pulses must be longer than the control cycle, which is approximately 10 ms in this case. The maximum velocity of the car may reach up to 1 m/s (1000 mm/s), and the diameter of the wheel is 30 mm. Since there are four state transitions of encoder pulses for every slit of the encoder disk, the

maximum number of slits on the encoder disk that guarantees correct pulse counting by software, i.e. no more than one state transition within one control cycle, will be 2.356. This means that the encoder disk should have only two slits per revolution for correct pulse counting. However, we have chosen to use three slits per revolution disk instead to obtain better resolution at low speed. The maximum car velocity that guarantees correct pulse counting will be reduced to 0.785 m/s, which is still sufficient for balancing the inverted pendulum.

The period of the control cycle is dependent on the execution time of the program because there is no timer interrupt handling function in Basic Stamp2 microcomputer. If the transition of encoder pulses were analysed using “if-then” statements, then the execution time will vary depending on the transition sequence. In order to minimize this variation, table-jump function was utilized. Let A1 and B1 be the previous state of the two encoder pulses, and let A2 and B2 be the current state. A four bit number obtained by arranging A1, B1, A2 and B2, will represent all possible transition patterns of the encoder pulses, including patterns that are not permitted, such as change of both pulses during one sample period. Three procedures, increment, decrement, and null (do nothing), are selected according to this four bit number that corresponds to the transition pattern. We can easily select pulse edge detection methods, namely quadrature detection, double detection, and single detection, just by rewriting the table.

## 2.7 Rewinding Armature Coil

Standard motor for “Mini-4WD” is designed for supply voltage of 2.4V. In order to use the same 9 V battery used for the microcomputer, the armature coil winding of the motor, which uses 0.3 mm diameter wire, was rewound by a thinner 0.16 mm diameter wire. Derivation process of this new wire diameter is included in the handout materials because it involves basic knowledge of DC motor.

## 2.8 Drive Gear Train

Standard gear ratio of 1:5 was selected because the motion of the inverted pendulum is a rapid back and forth motion. Acceleration is more important than maximum speed. Since the tires slip during acceleration, the propeller shaft that drives the front axle from the rear axle is removed in order to avoid slippage of the front tire that measures the car position with the optical encoder attached to its axle.

## 2.9 Control Program

The control output, which is a 4-bit current command to the motor driver, is calculated as a linear weighted sum of four state variables, namely, tilt angle, tilt angular velocity, car position, and car velocity. The tilt angular velocity is obtained by taking the difference between the current tilt angle and the previous tilt angle. Car velocity is calculated differently. Since the increment and decrement of the optical encoder output is determined at every control cycle, the change will be only  $\pm 1$  or 0 at every control cycle. Therefore, in order to increase the dynamic range and resolution of the car velocity, the velocity is obtained by taking the total

increment or decrement of the past 8 cycles. The resolution of car velocity will be  $\pm 8$  pulses, which is still coarse but sufficient for controlling the inverted pendulum. This is one of the drawbacks of not using a hardware counter.

Another limitation of using BASIC Stamp2 is that the arithmetic operation is limited to integers, which limits the use of feedback gains obtained by design methods such as optimum regulator from which the gains are obtained as floating point numbers. This may be a limitation for using this car for learning advanced control algorithms. This inverted pendulum car is more suited for giving hands-on experiences to engineering students who have little experiences with mechanisms and electronic devices.

## 2.10 Parts Cost

Cost of experimental apparatuses is a very important factor in education. This inverted pendulum car costs approximately 15000 Japanese Yen which is equivalent to approximately 150 US dollars. The most expensive part is the potentiometer which is approximately 8000 Japanese Yen. Since friction of rotation affects the performance of the inverted pendulum, potentiometers that use contact brushes are not recommended. Although we have not yet tried, the potentiometer may be replaced by a photo-interrupter and a small opaque plate attached at the lower part of the stick. Small displacement of the plate caused by the tilt of the stick will change the amount of light reaching the photo transistor of the photo-interrupter. The analogue output of the photo transistor is fairly linear about the middle point and can be used to as a substitute of the potentiometer. This method will reduce the cost for tilt angle sensor to about several US dollars.

## 3. CONTENTS OF EXERCISES

This inverted pendulum toy car has been used mainly in introductory courses on mechatronics for freshmen and sophomores in the past several years at the University of Tokyo.

### 3.1 Programming Exercises

Programming exercises will begin from simple tasks such as input of a push button switch and turning on LEDs, and then proceed to AD conversion of potentiometer output, counting rotary encoder pulses, and motor control. Run-time monitoring of variables in the program is very useful for checking AD convertor output and encoder pulses. After writing and checking these elementary programs, a sample program of inverted pendulum control will be given to students.

### 3.2 Feedback Gain Adjustment

Since freshmen and sophomores do not have knowledge of state space methods and optimum control, the feedback gains will be adjusted by trial and error. First, the feedback gain for tilt angle will be adjusted by writing a control program that uses the tilt angle only. Next, the feedback of tilt angular velocity is added and its gain is adjusted. Students will loosely hold the tip of the stick and observe the motion and evaluate the performance. Next, feedbacks of car position and

velocity are added to the control program. If the tip of the stick is loosely held between two fingers at this stage, the car and the stick will be stable about a certain position and tilt angle. Students can check the polarity of the feedback gains for car position and velocity by moving the tip of the stick to the right or left. If the polarity of the feedback gains is correct, the stick should tilt farther to the left when the tip is moved to the right, and vice versa.

When students have knowledge of optimum linear quadratic regulator or pole placement method, the only unknown parameters will be the relationship between armature current command and the acceleration of the car. This relationship can be obtained experimentally by measuring velocity curves for different current commands.

### 3.3 Explanation on Polarity of Feedback Gains

One of the interesting features of inverted pendulum is that the polarity of the feedback gains for car position and velocity is the opposite of the gains for car position control without the stick. Although this can be easily shown by applying stability analysis in state space method, the following explanation is given to students have not studied the method yet.

First, assume that there is some offset in the tilt angle sensor. When only tilt angle and angular velocity are used in the feedback loop, i.e. car position and velocity are not used; the equilibrium state of this control will be a constant acceleration of the car that keeps the stick at this offset angle. The sensor output is zero at this state. This means that we can control the acceleration of the car, and hence the velocity and position of the car while balancing the stick, by adding an offset to the tilt angle sensor. Students will find that the polarity of the feedback gains of car position and velocity is opposite by examining the coefficients for the linear sum of the four state variables. Although this explanation is qualitative, it provides another way of understanding the dynamics of inverted pendulum.

## 4. CONCLUSION

Inverted pendulum build on a popular toy car presented in this paper utilizes low cost off-the-shelf parts and has basic functions for education of control engineering and mechatronics. It is easy to assemble and program, and it is portable. These features enable instructors to supply many sets to the students with low budget.

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