

# The Geometry Processing Pipeline

**Geometric Computer Vision**

GCV v2021.1, Module 1

Alexey Artemov, Spring 2021

# Lecture Outline

## §1. The geometry processing pipeline [45 min]

1.1. Goals of 3D/geometric computer vision systems

1.2. Common stages of geometry processing

*1.3. Scanning [next video]*

1.4. Registration

1.5. Reconstruction and meshing

*1.6. Postprocessing [next videos]*

# Lecture Outline

## **§2. 3D representations in computer vision/graphics [15 min, Friday]**

2.1. Directly measurable: multiple-view images, range-images, point clouds, volumes

2.2. Derived: surface meshes, implicit functions

2.3. Higher-level: CAD, shape programs

# §1. The geometry processing pipeline

# Goals of 3D/geometric computer vision systems

# 1.1. Goals of 3D/geometric computer vision systems

## §1. The geometry processing pipeline

- Two main aspects:
- **Construct** 3D geometry representations suitable for various tasks from raw data (range images, volumetric CT and MRI, LIDAR)
  - Usually involves multiple steps going from low to high level
  - As an intermediate tool, requires analysis, e.g. segmentation
  - E.g.: Points → meshes → parametrized patch layout
- **Manipulate and analyze** geometry:
  - Deformations, boolean operations, comparisons, physically-based deformations (related to CAE)

# 1.1. Goals of 3D/geometric computer vision systems

## §1. The geometry processing pipeline

- **The geometry processing pipeline:** a highly modular sequence of interrelated stages for manipulations with 3D data, commonly for 3D reconstruction and understanding
  - Convenient concept of conversions between 3D representations
  - Flow: going from low-level to higher-level representations/properties
  - Modularity: injecting methods/models easier

# 1.1. Goals of 3D/geometric computer vision systems

## §1. The geometry processing pipeline

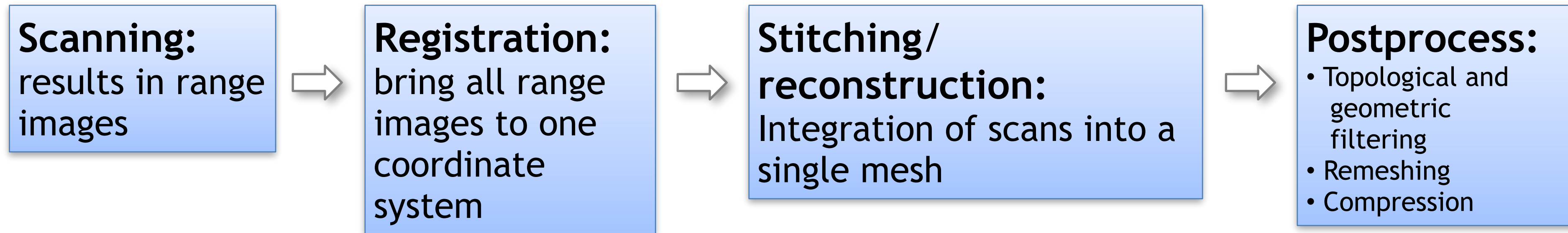
- Why need to study an entire pipeline for 3D processing? Don't 3D scanners have it all?
  - Most hardware systems for 3D acquisition: **standard/proprietary algorithms, no customization, limited conversion options**
  - Being able to intervene at any stage: flexibility, “debugging”, performance gains
- **Today: go over the “standard” reconstruction pipeline for 3d scanning**
- Consider two types of problems with existing techniques (how these can be addressed by ML-based methods?):
  - “Low-level”: related to local surface properties, e.g., noise, normals, curvature, outliers,
  - “High-level”: involve object semantic (e.g., high level part segmentation)



# Common stages of geometry processing

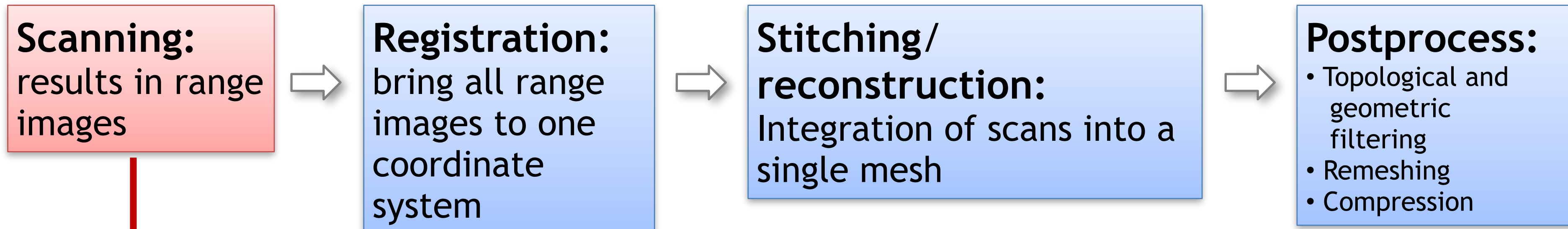
# 1.2. Common stages of geometry processing

## §1. The geometry processing pipeline



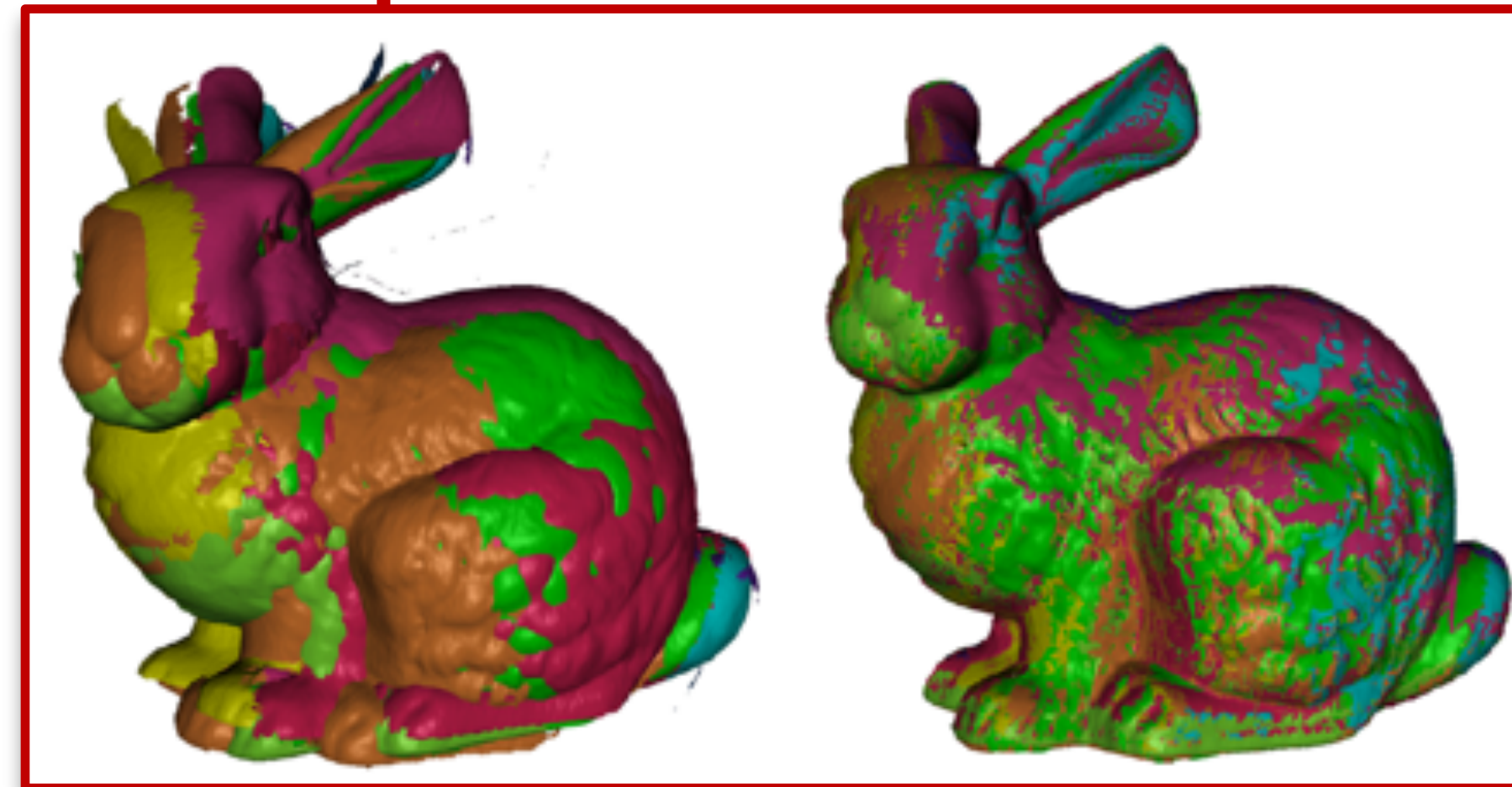
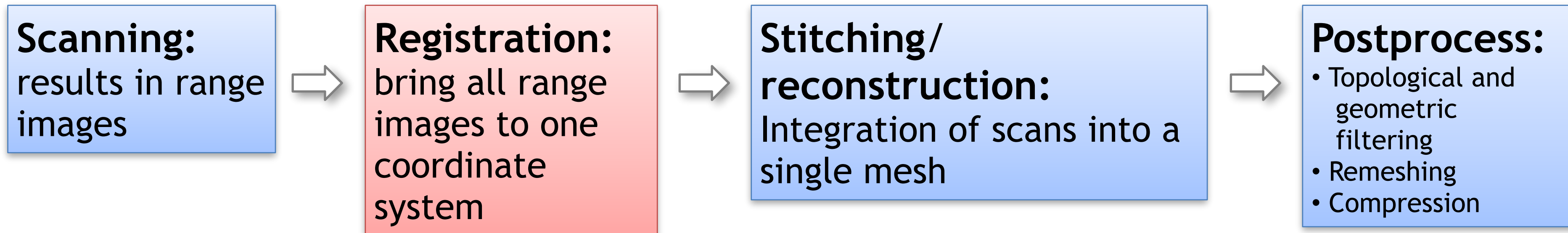
# 1.2. Common stages of geometry processing

## §1. The geometry processing pipeline



# 1.2. Common stages of geometry processing

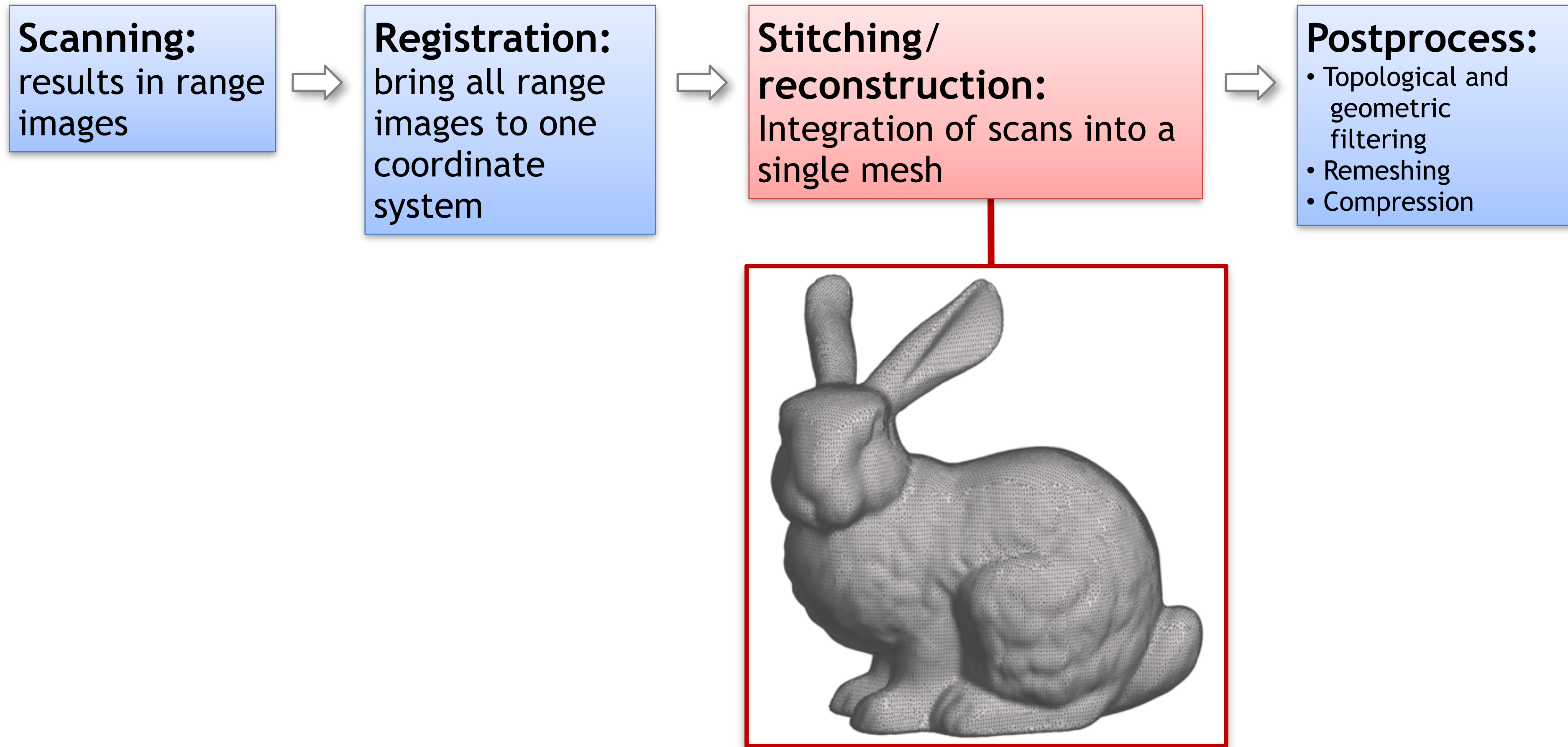
## §1. The geometry processing pipeline





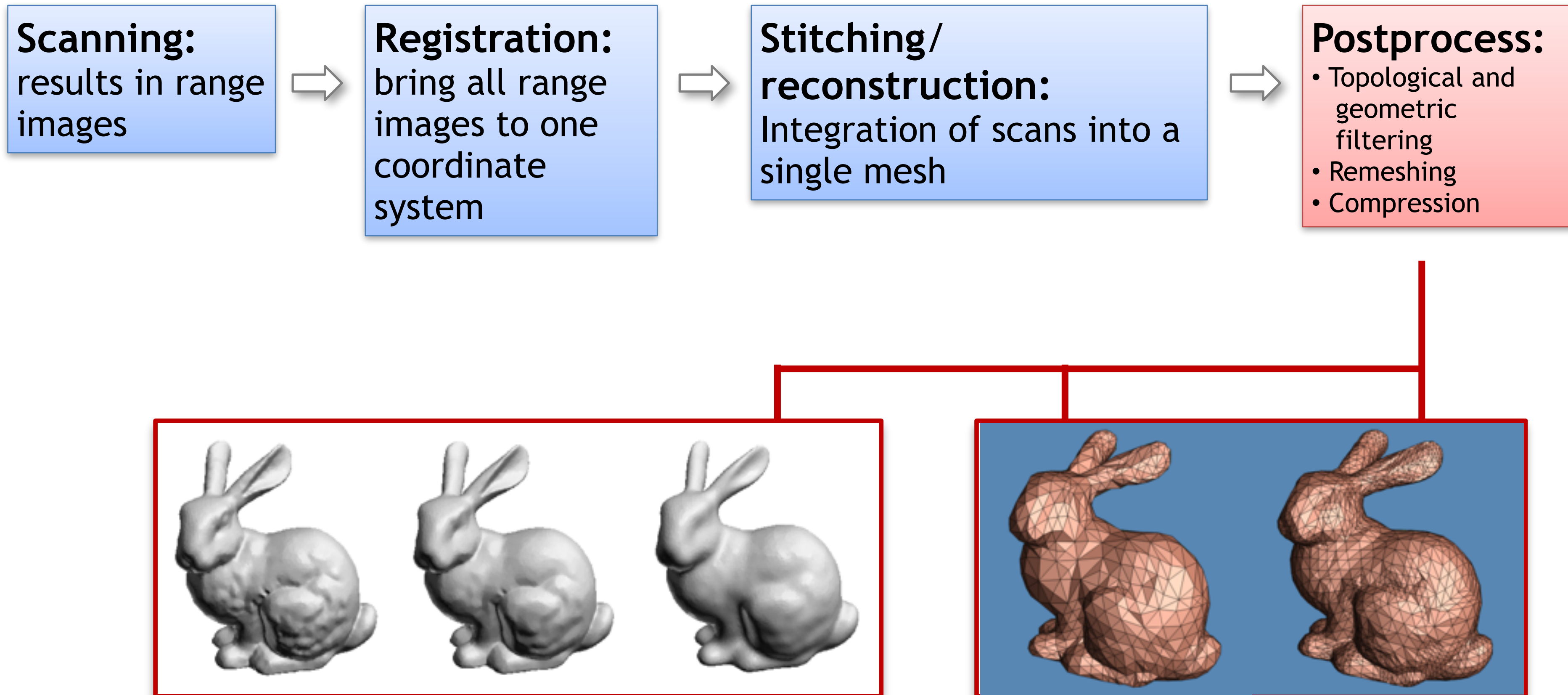
# 1.2. Common stages of geometry processing

## §1. The geometry processing pipeline



# 1.2. Common stages of geometry processing

## §1. The geometry processing pipeline



# 1.2. Common stages of geometry processing

## §1. The geometry processing pipeline





# 1.2. Common stages of geometry processing

## §1. The geometry processing pipeline

- The “standard” geometry pipeline for 3d scanning:
  - Scanning → registration → reconstruction → postprocessing
- From low-level to higher-level representations
- Natural modularity, allows extension/injection of stages



# Scanning

# 1.3. Scanning

## §1. The geometry processing pipeline

- Analyze a real-world object or environment to collect data on its shape/appearance
  - Many technologies: contact, optical, computed tomography, structured light...
- **Today: do not consider the first step in detail (obtaining depth data)**
- Assume depth images/range scans are available
- Focusing on the next steps
- **Next week:** detailed lecture about depth acquisition

# Registration

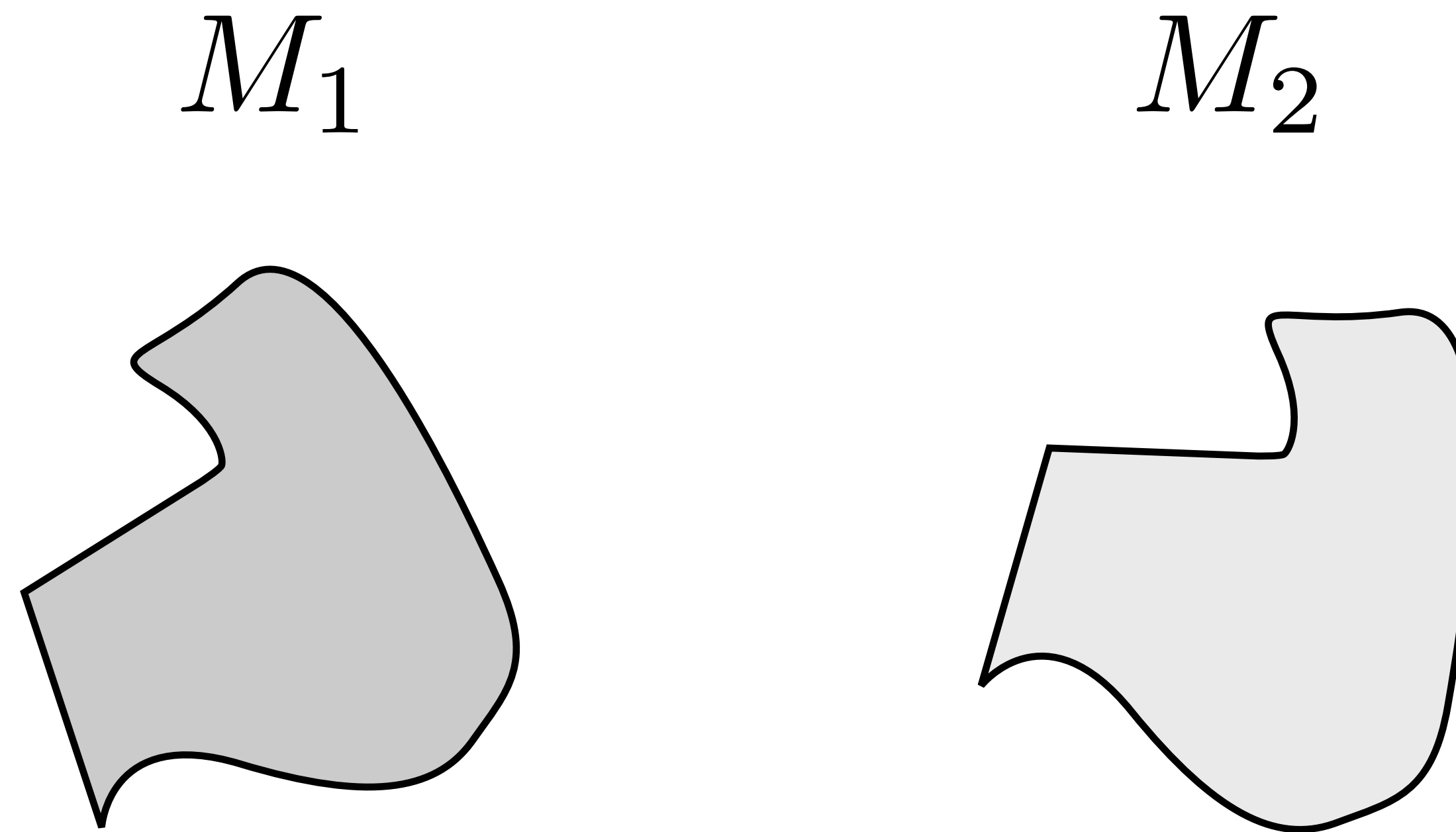
# 1.2. Registration: context

## §1. The geometry processing pipeline



# 1.2. Registration: problem statement

## §1. The geometry processing pipeline

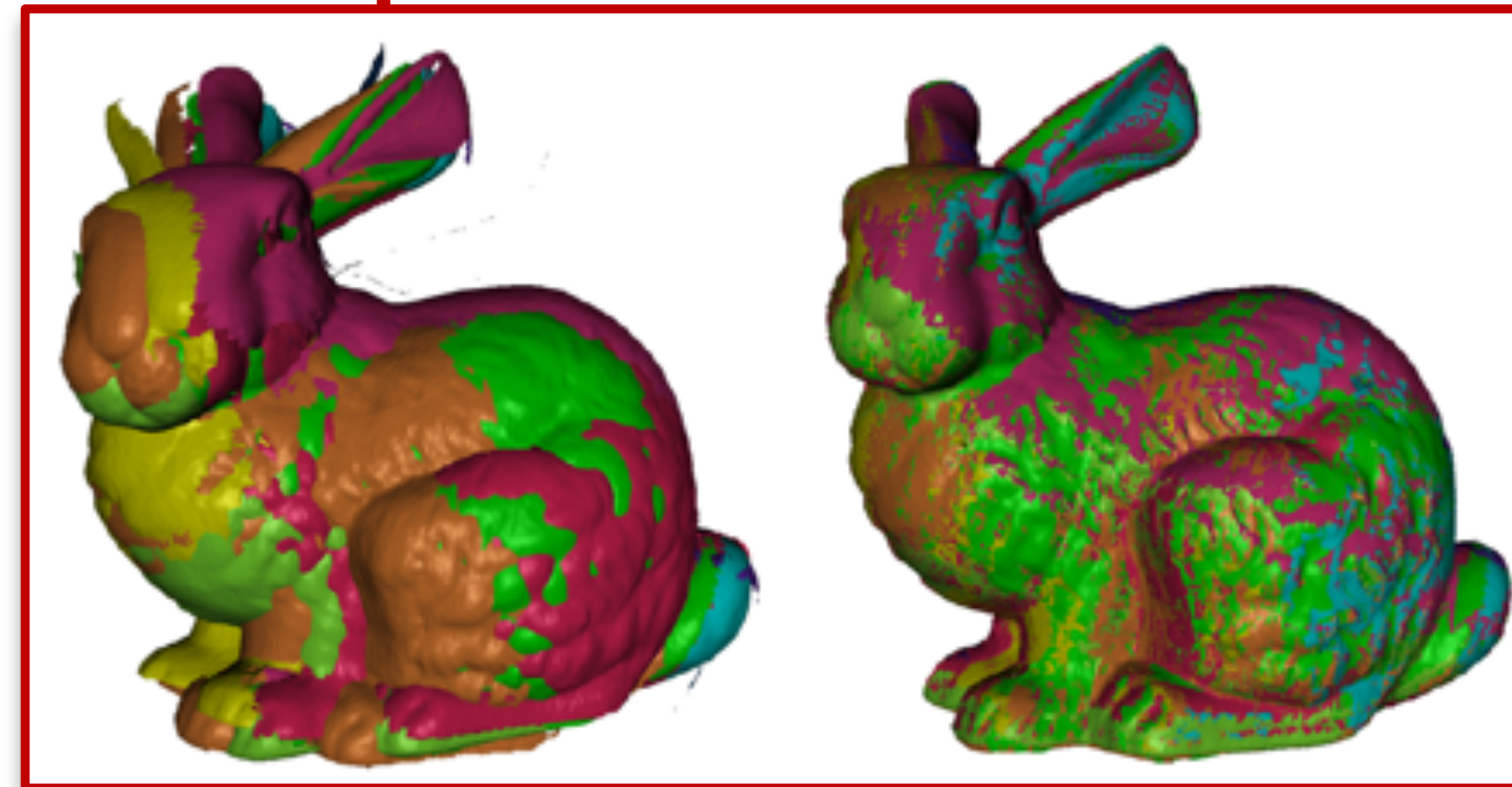
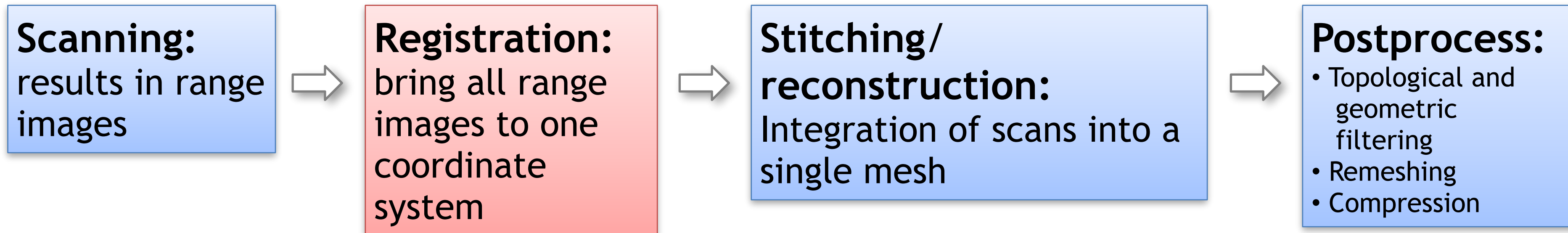


$$M_1 \approx T(M_2)$$

T: Translation + Rotation

# 1.2. Registration: context

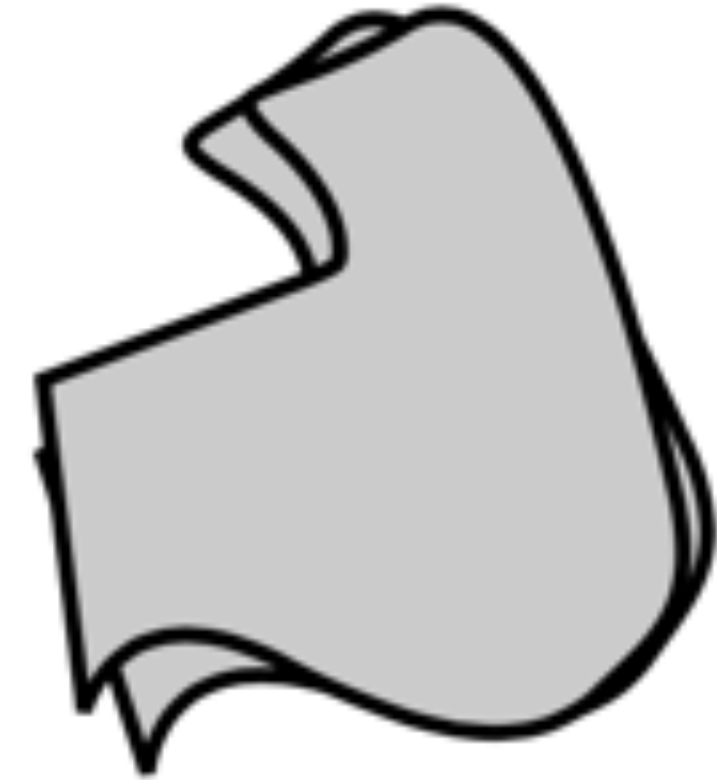
## §1. The geometry processing pipeline





# 1.2. Registration: local vs global

## §1. The geometry processing pipeline



### Global Registration

Arbitrary Transformation

### Local Registration

“Small” Transformation

Given  $M_1, \dots, M_n$ , find  $T_2, \dots, T_n$  such that

$$M_1 \approx T_2(M_2) \cdots \approx T_n(M_n)$$

# 1.2. Registration: correspondences

## §1. The geometry processing pipeline

- How many points are needed to define a unique rigid transformation?
- The first problem is finding corresponding pairs!

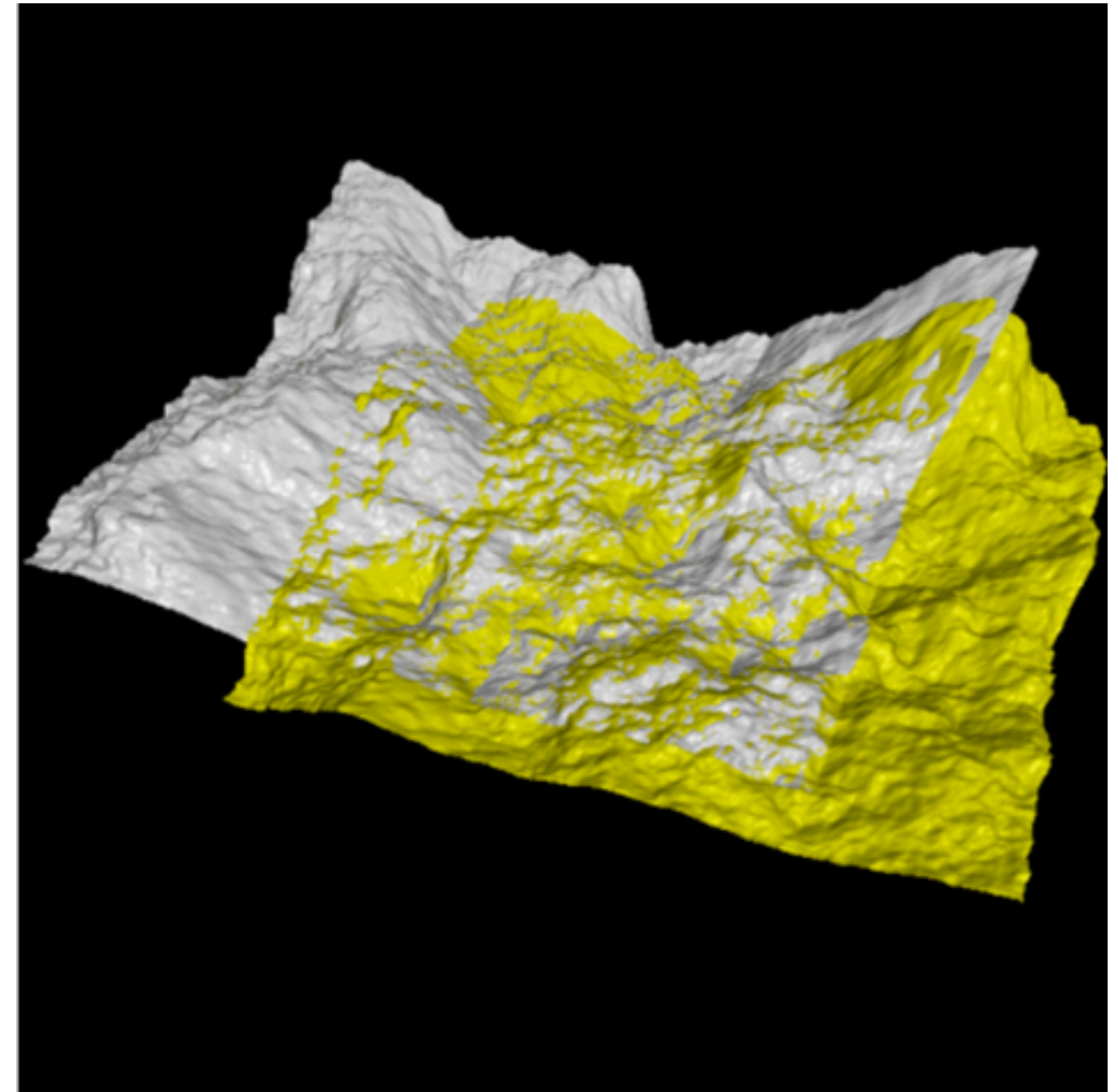
Let  $\mathbf{p}_i, \mathbf{q}_i$  define points on  $M_1$  and  $M_2$

$$\mathbf{p}_1 \rightarrow \mathbf{q}_1$$

$$\mathbf{p}_2 \rightarrow \mathbf{q}_2$$

$$\mathbf{p}_3 \rightarrow \mathbf{q}_3$$

$$R\mathbf{p}_i + t \approx \mathbf{q}_i$$

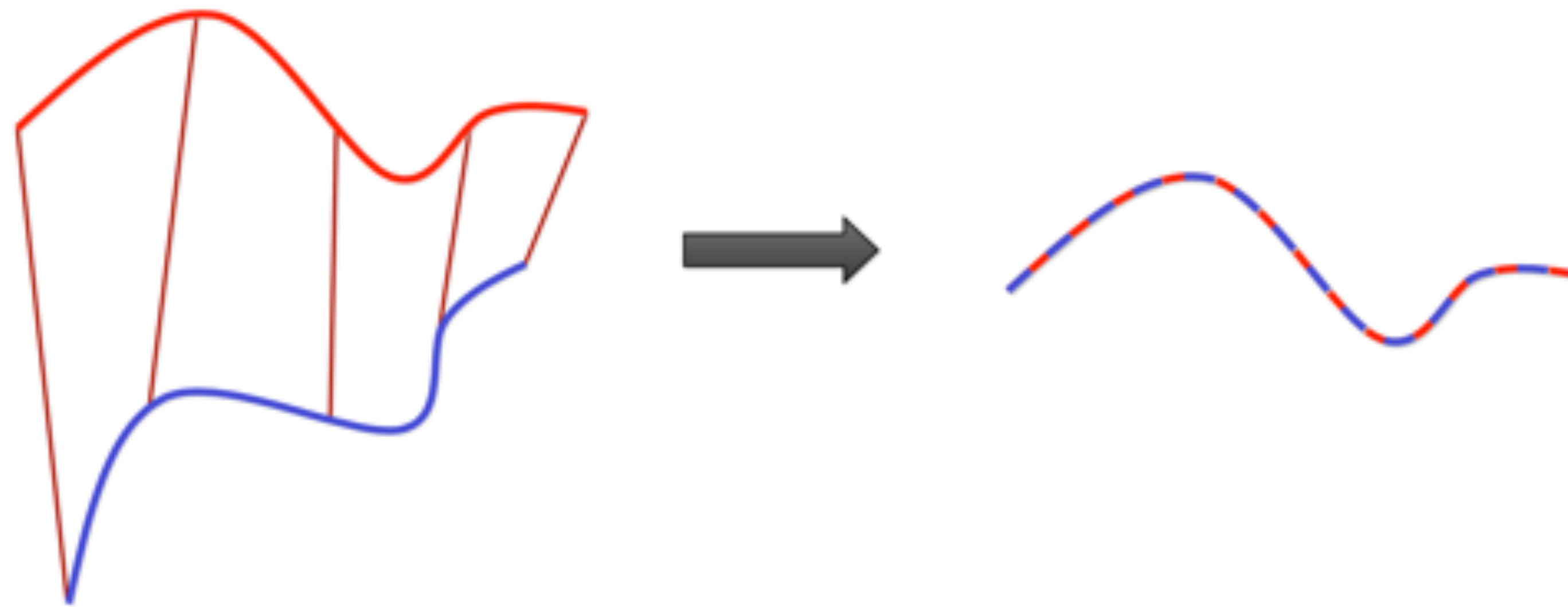




# 1.2. Registration via ICP: Iterative Closest Point

## §1. The geometry processing pipeline

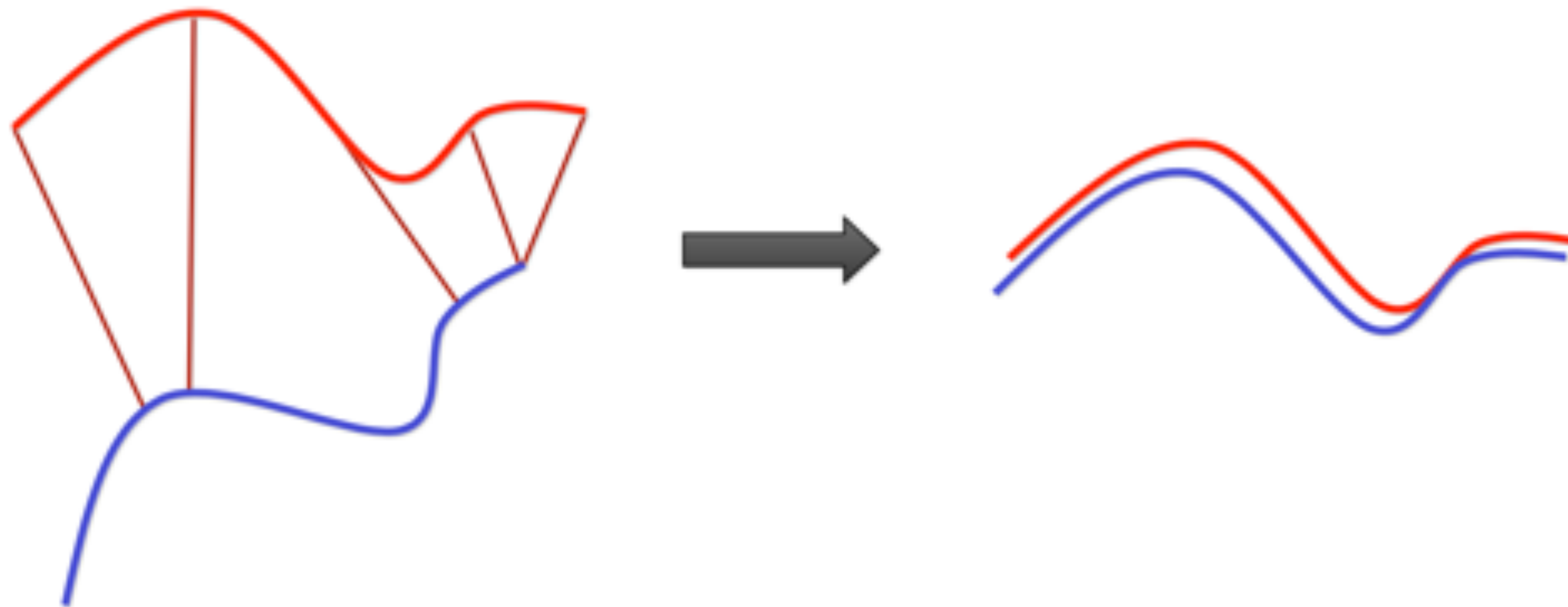
- Idea: Iteratively (1) find correspondences and (2) use them to find a transformation
- Intuition: If you have the right correspondences, then the problem is easy



# 1.2. Registration via ICP: Iterative Closest Point

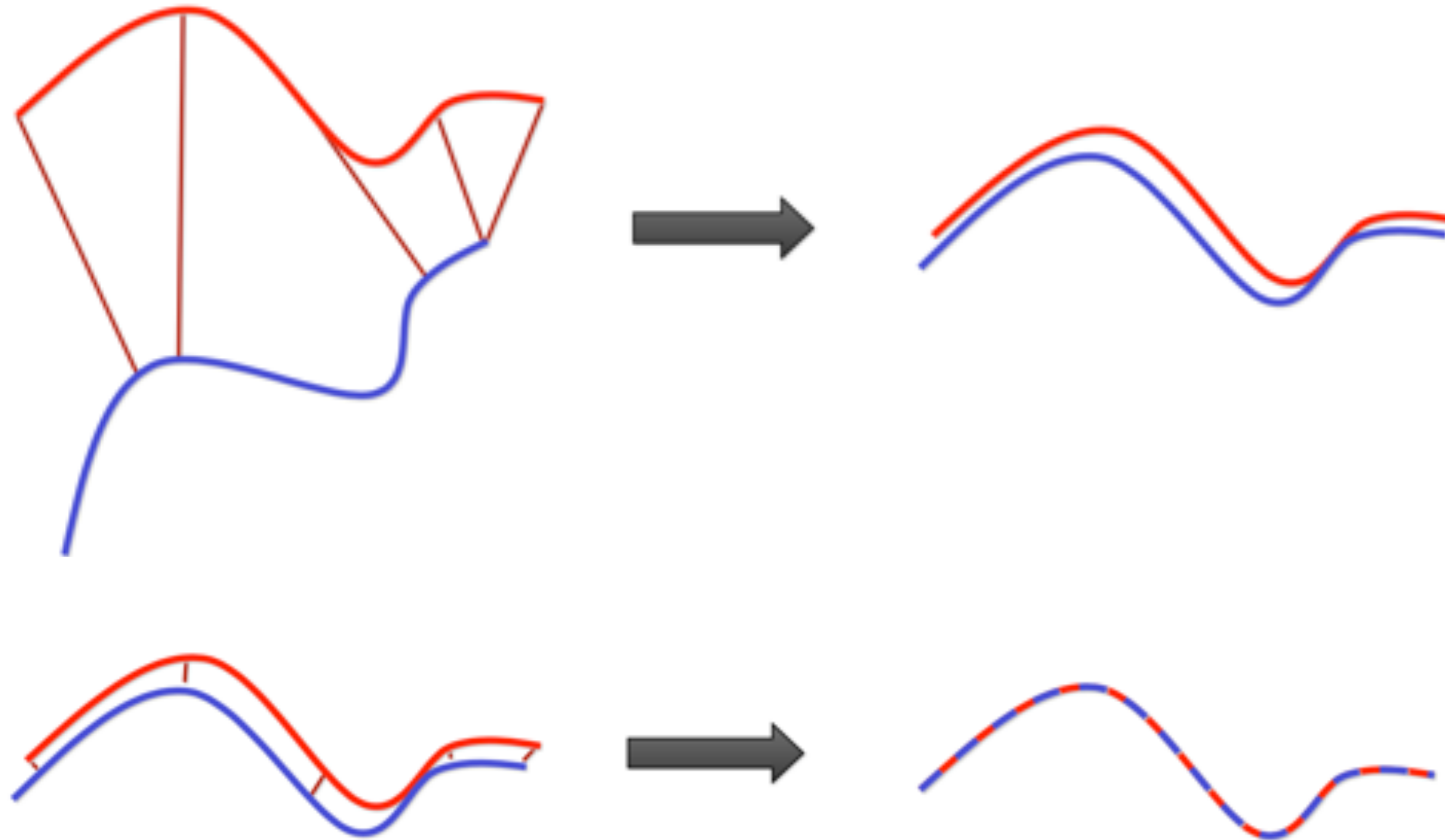
## §1. The geometry processing pipeline

- Idea: Iteratively (1) find correspondences and (2) use them to find a transformation
- Intuition: If you don't have the right correspondences, you still can make progress



# 1.2. Registration via ICP: Iterative Closest Point

## §1. The geometry processing pipeline



This algorithm converges to the correct solution only  
if the starting scans are “close enough”

# 1.2. Registration via ICP: basic algorithm

## §1. The geometry processing pipeline

- **Select** (e.g., 1000) random points
- **Match** each to closest point on other scan, using data structure such as  $k$ -d tree
- **Reject** pairs with distance  $> k$  times median

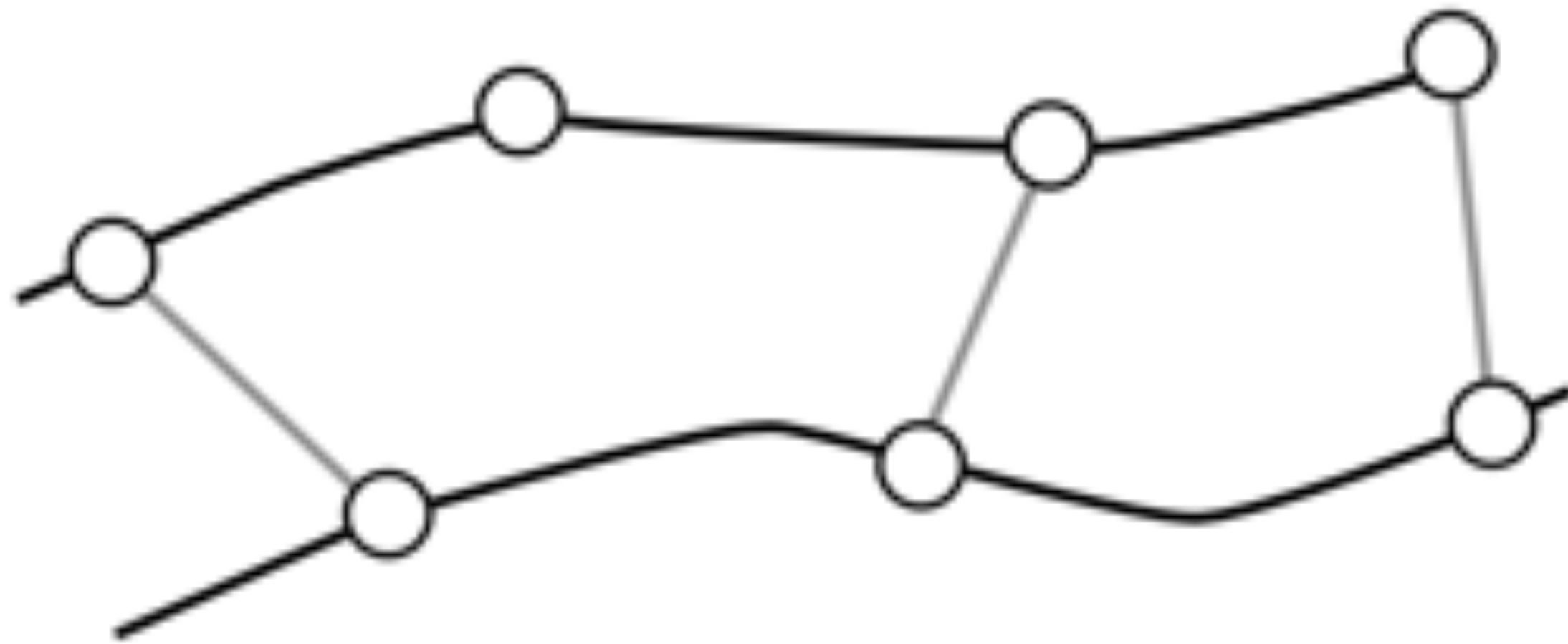
- Construct **error function**:

$$E := \sum_i (R\mathbf{p}_i + t - \mathbf{q}_i)^2$$

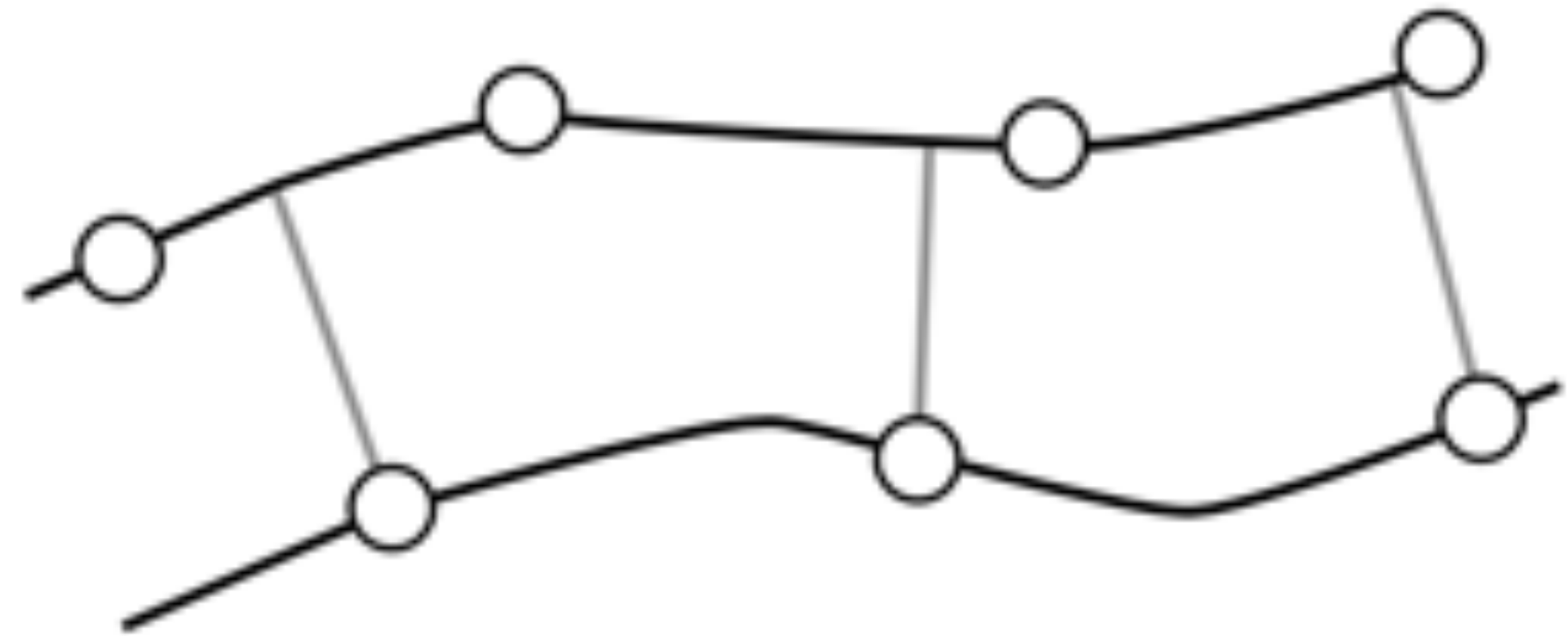
- **Minimize** (closed form solution  
comparison of four major algorithms", <http://dl.acm.org/citation.cfm?id=250160>)

# 1.2. Registration via ICP: important variant

## §1. The geometry processing pipeline



Point-to-Point



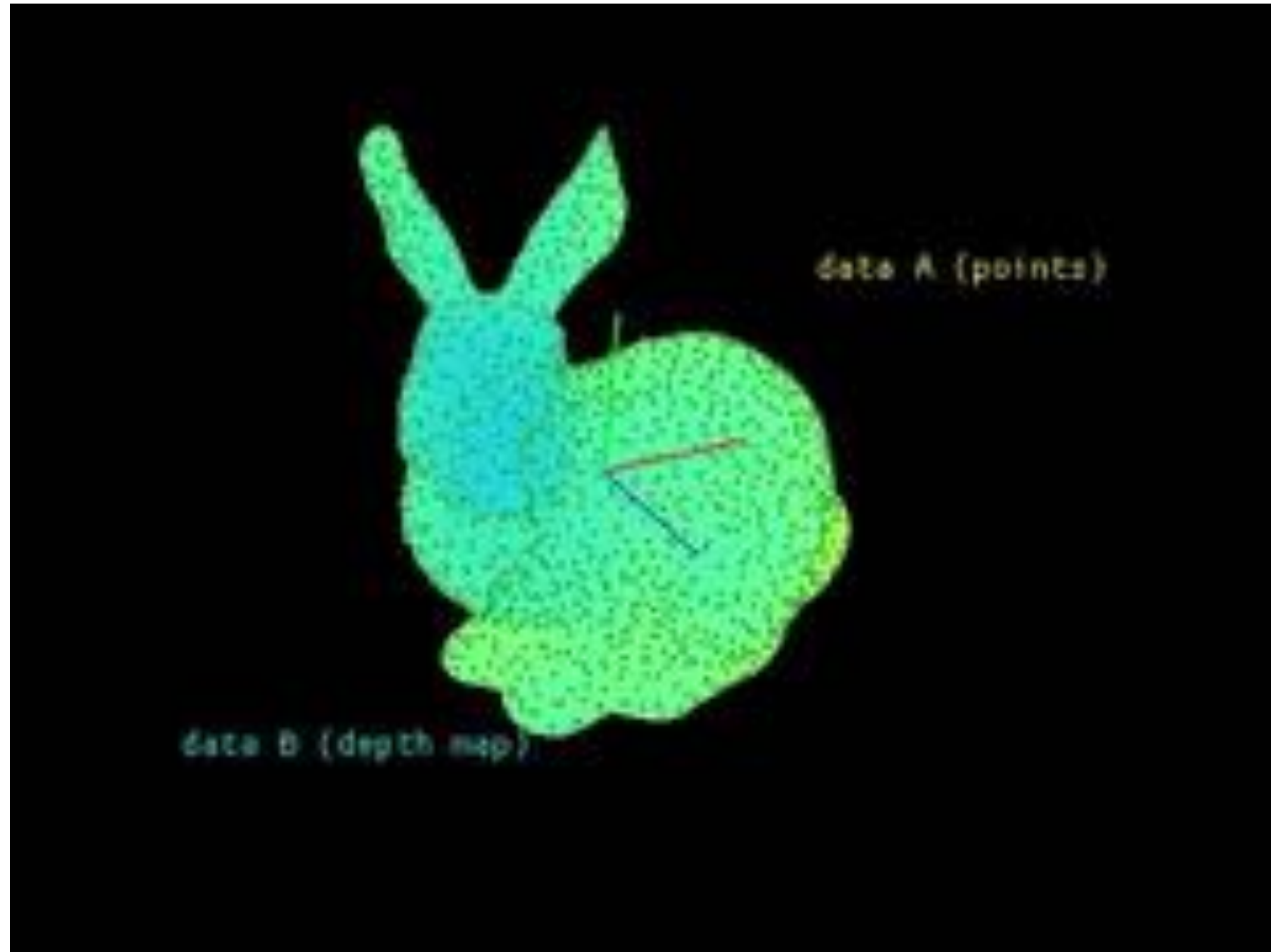
Point-to-Plane

See [http://resources.mpi-inf.mpg.de/deformableShapeMatching/EG2012\\_Tutorial/](http://resources.mpi-inf.mpg.de/deformableShapeMatching/EG2012_Tutorial/) for details



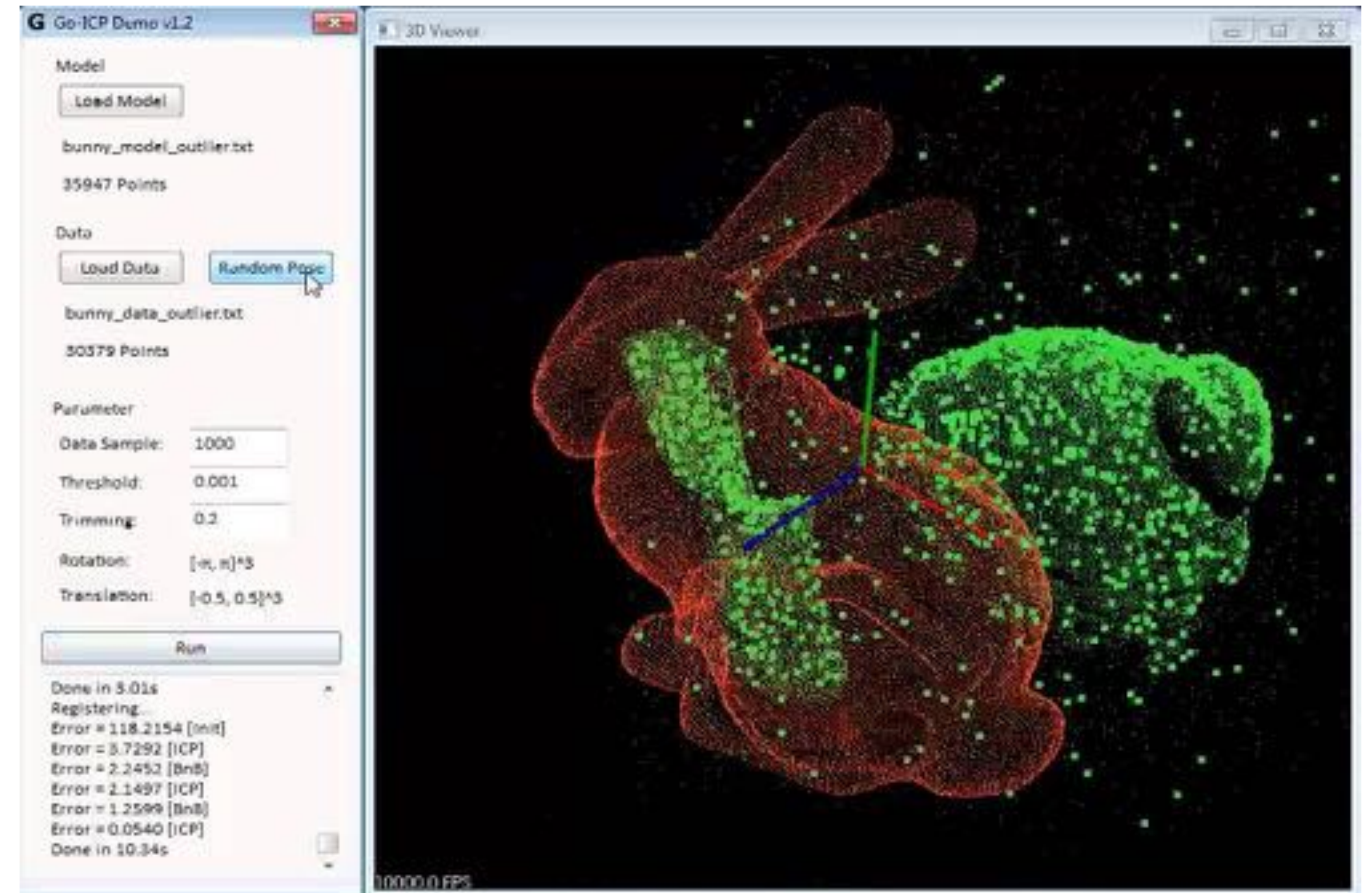
# 1.2. Registration via ICP: example impl.

## §1. The geometry processing pipeline



# 1.2. Registration via ICP: example impl.

## §1. The geometry processing pipeline



# 1.2. Registration via ICP: Related Work

## §1. The geometry processing pipeline

- Original ICP:  
[http://graphics.stanford.edu/courses/cs164-10-spring/Handouts/paper\\_icp.pdf](http://graphics.stanford.edu/courses/cs164-10-spring/Handouts/paper_icp.pdf)
- Commonly used improvement:  
[http://www8.cs.umu.se/research/ifor/dl/fasticp\\_paper.pdf](http://www8.cs.umu.se/research/ifor/dl/fasticp_paper.pdf)
- Global registration, one of initial reliable methods:  
[http://vecg.cs.ucl.ac.uk/Projects/SmartGeometry/global\\_registration/paper\\_docs/global\\_registration\\_sgp\\_05.pdf](http://vecg.cs.ucl.ac.uk/Projects/SmartGeometry/global_registration/paper_docs/global_registration_sgp_05.pdf)
- Recent paper, with a few refs:  
<http://vladlen.info/publications/fast-global-registration/> (Talk by V. Koltun: [http://videolectures.net/eccv2016\\_koltun\\_global\\_registration/](http://videolectures.net/eccv2016_koltun_global_registration/))



# 1.2. Registration via ICP: Problems

## §1. The geometry processing pipeline

- In reality, registration needs to be non-rigid: e.g. range scans are usually warped
  - matters only for high quality
  - no direct ground truth may be available
  - data: for a set of objects, a collections of warped scans for each
  - learn an alignment/dewarping transformation
- Extension of ICP:  
[http://gfx.cs.princeton.edu/pubs/Brown\\_2007\\_GNA/global\\_tps.pdf](http://gfx.cs.princeton.edu/pubs/Brown_2007_GNA/global_tps.pdf)
- Related, more difficult (in general form) problem that received a lot of attention:  
**non-rigid reconstruction**

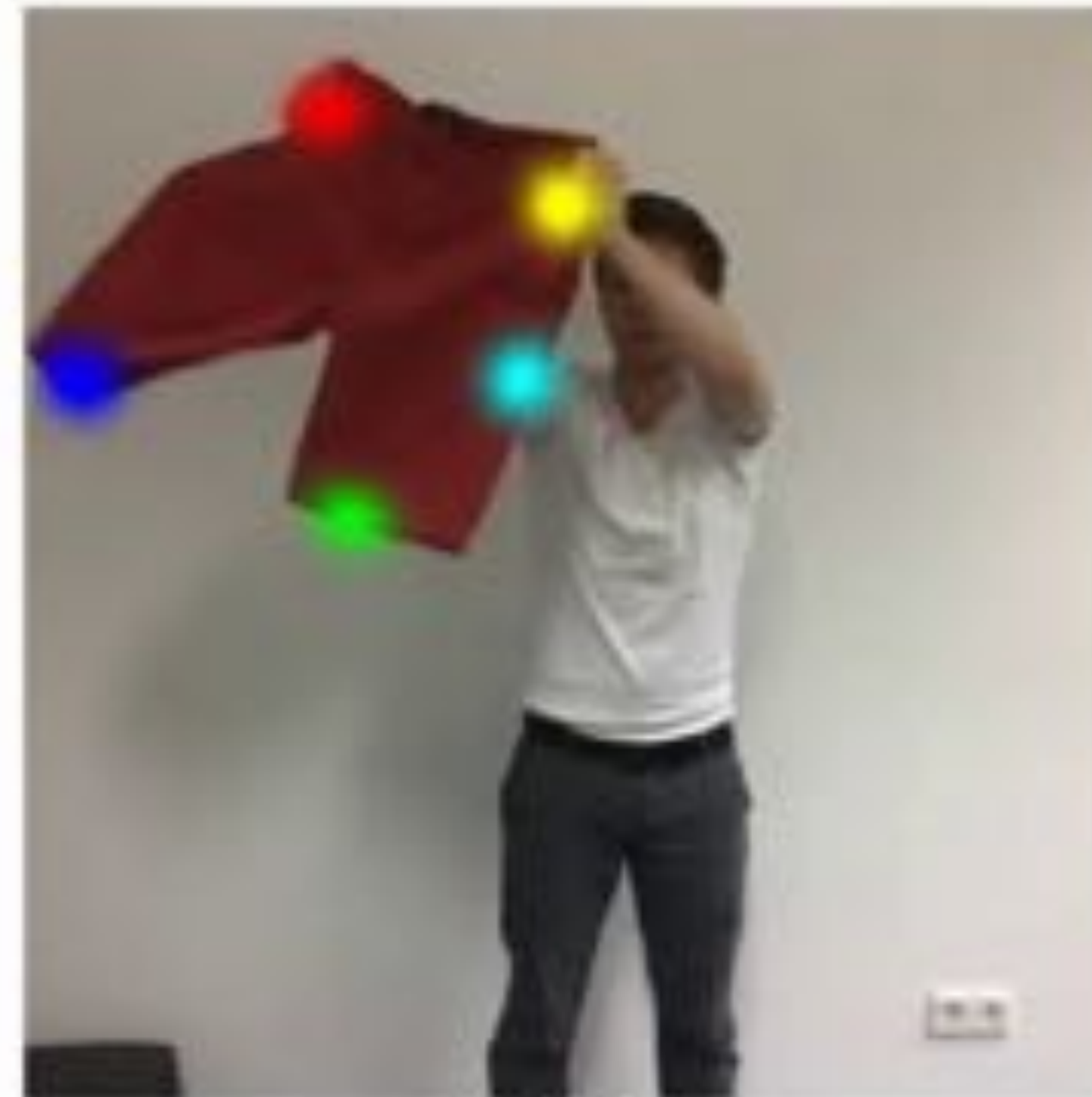
# 1.2. Registration: Non-Rigid

## §1. The geometry processing pipeline

Results: Correspondence Matching



Reference



Predictions

# Reconstruction



# 1.5. Reconstruction: Digital Michelangelo Project

## §1. The geometry processing pipeline



1G sample points → 8M triangles



4G sample points → 8M triangles

# 1.5. Reconstruction: Problem statement

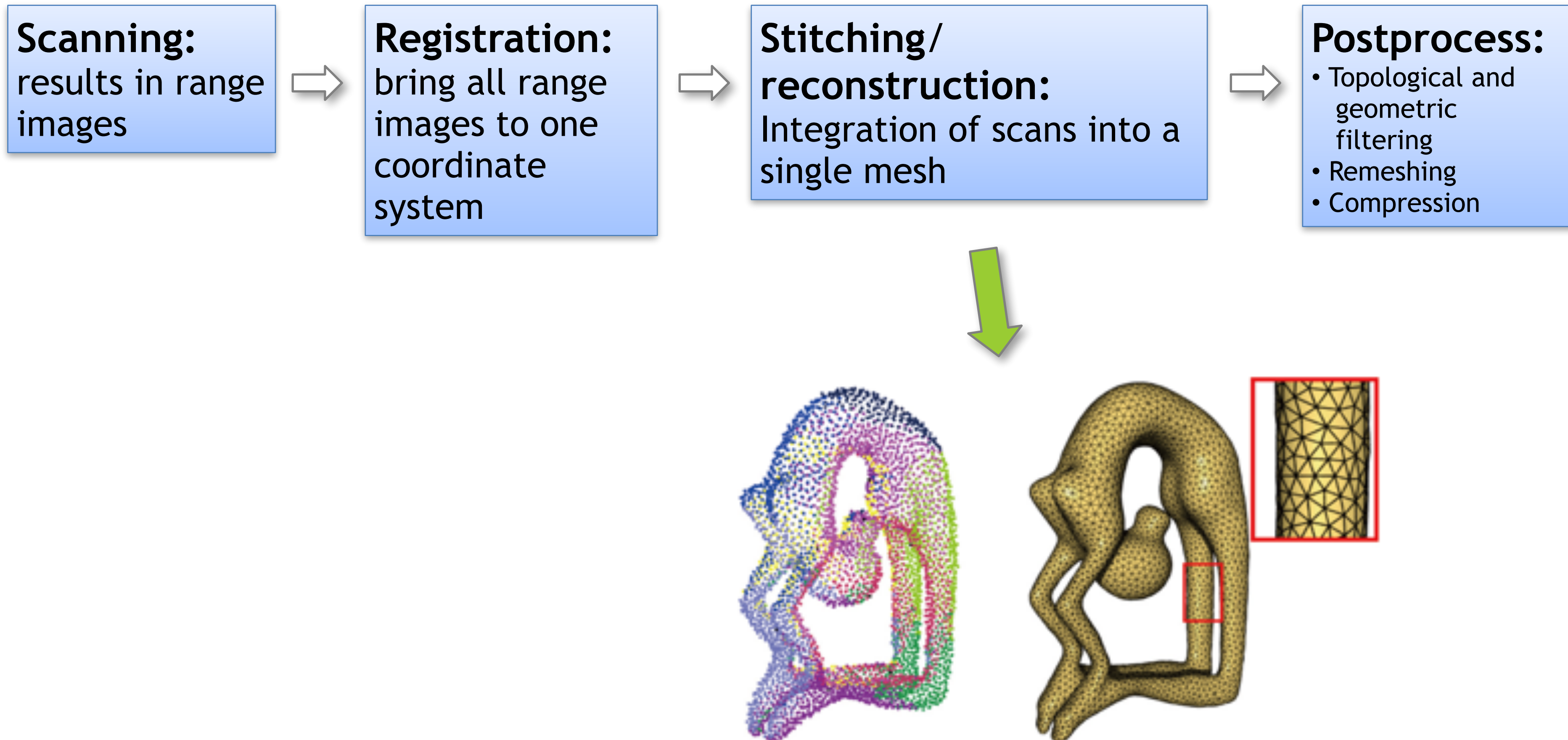
## §1. The geometry processing pipeline

- Given **partial information of an unknown surface**, construct, to the extent possible, a **compact representation of the surface** (Hoppe et al., 1992)
  - Commonly (in this course): use multiple viewpoints and range data
- Surface: compact, connected, orientable 2D manifold, possibly with boundary, embedded in  $\mathbb{R}^3$ 
  - *Closed surface*: a surface without a boundary, *bordered surface*: non-empty boundary
  - *Simplicial surface*: piecewise linear surface with triangular faces
- **Goal**: given samples  $X = \{\mathbf{x}_1, \dots, \mathbf{x}_n\}$  on or near an unknown surface  $M$ , recover  $M' \approx M$



# 1.5. Reconstruction

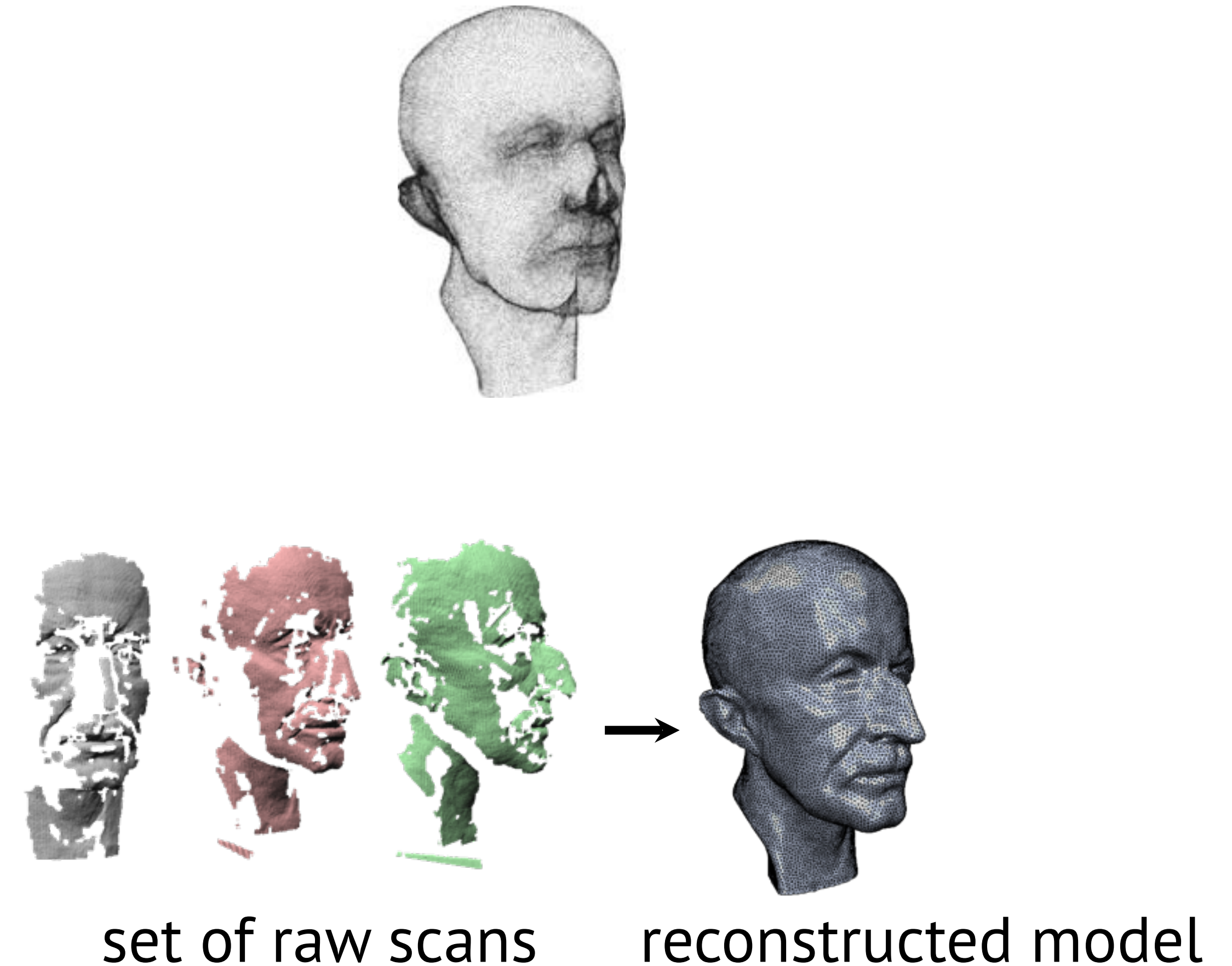
## §1. The geometry processing pipeline



# 1.5. Reconstruction: Input to Process

## §1. The geometry processing pipeline

- Input option 1:  
just a set of 3D points, irregularly spaced
  - Need to estimate normals
- Input option 2:  
normals come from the range scans

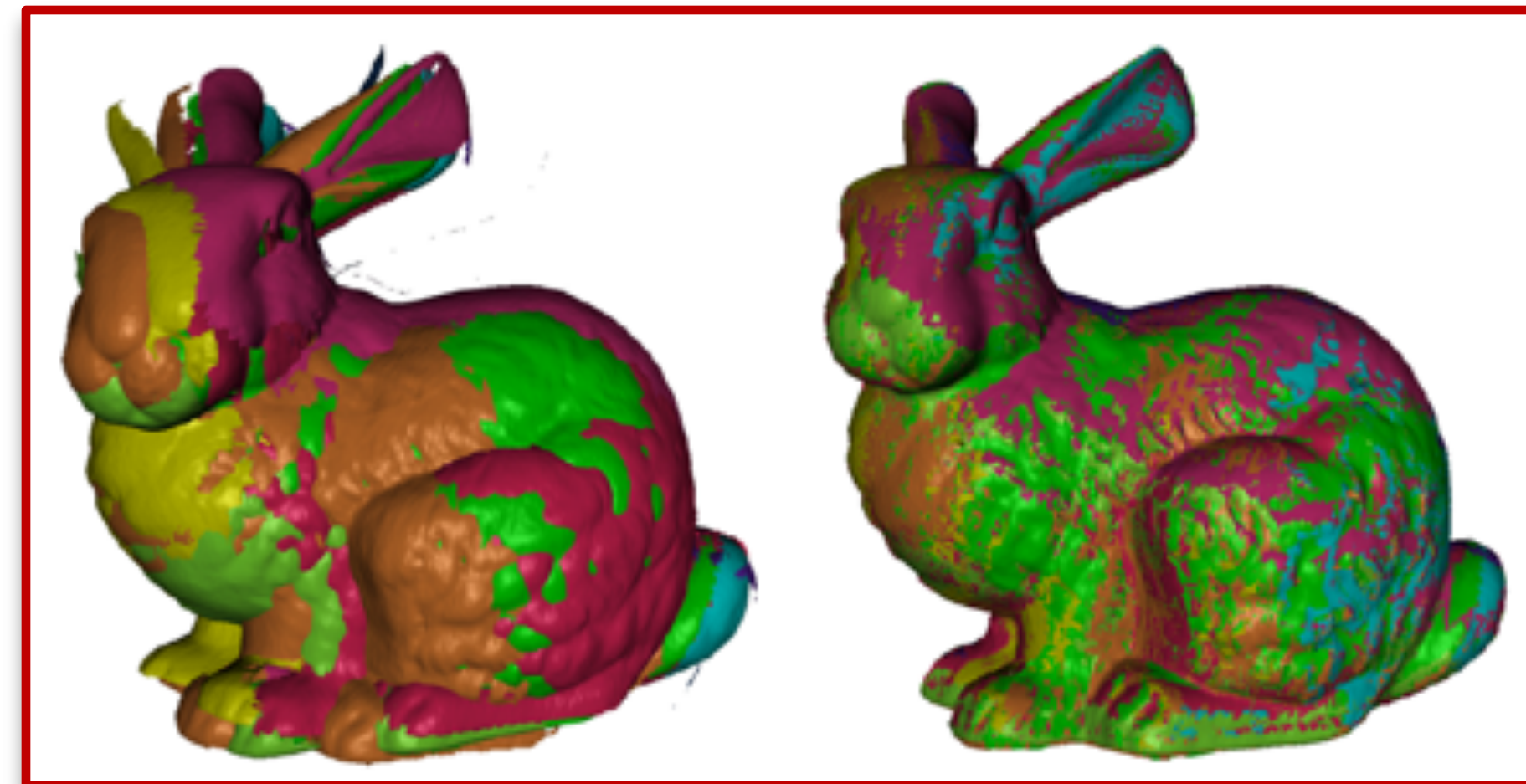
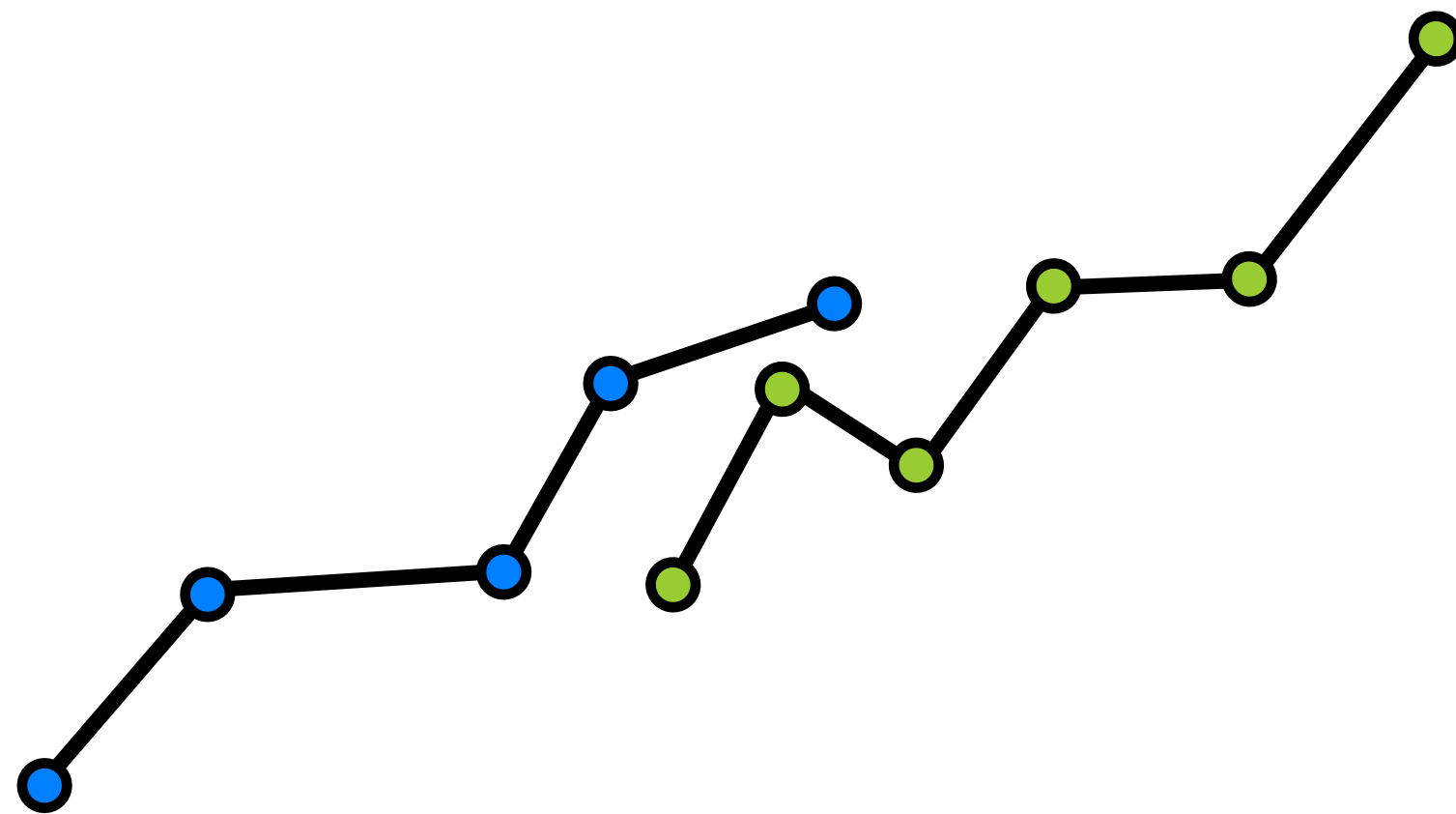




# 1.5. Reconstruction: How to Connect the Dots?

## §1. The geometry processing pipeline

- **Explicit reconstruction:**  
stitch the range scans together



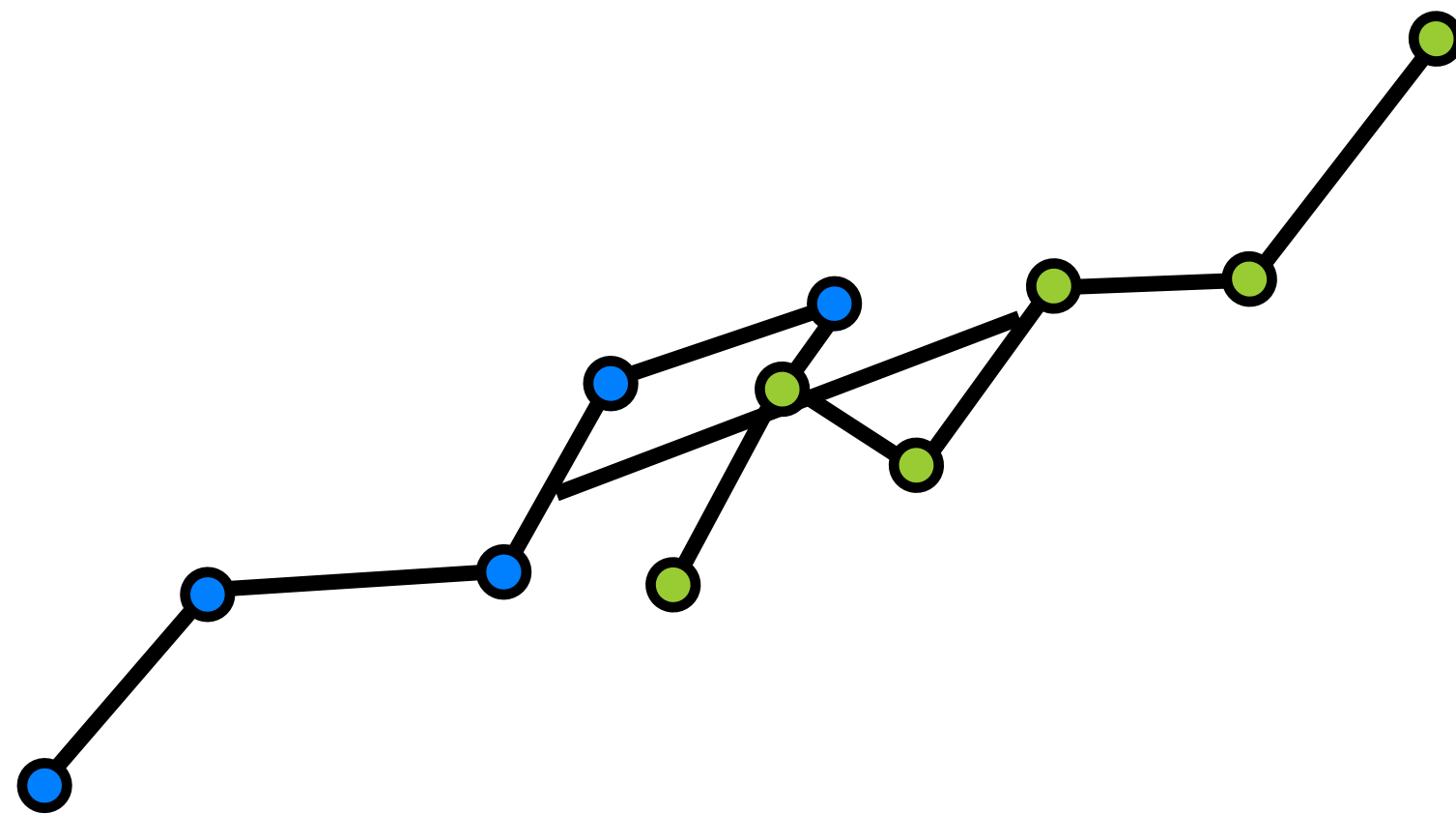
“Zippered Polygon Meshes from Range Images”, Greg Turk and Marc Levoy, ACM SIGGRAPH 1994



# 1.5. Reconstruction: How to Connect the Dots?

## §1. The geometry processing pipeline

- **Explicit reconstruction:**  
stitch the range scans together



- Connect sample points by triangles
- Exact interpolation of sample points
- Bad for noisy or misaligned data
- Can lead to holes or non-manifold situations

# 1.5. Reconstruction: Range-Image to Mesh

## §1. The geometry processing pipeline

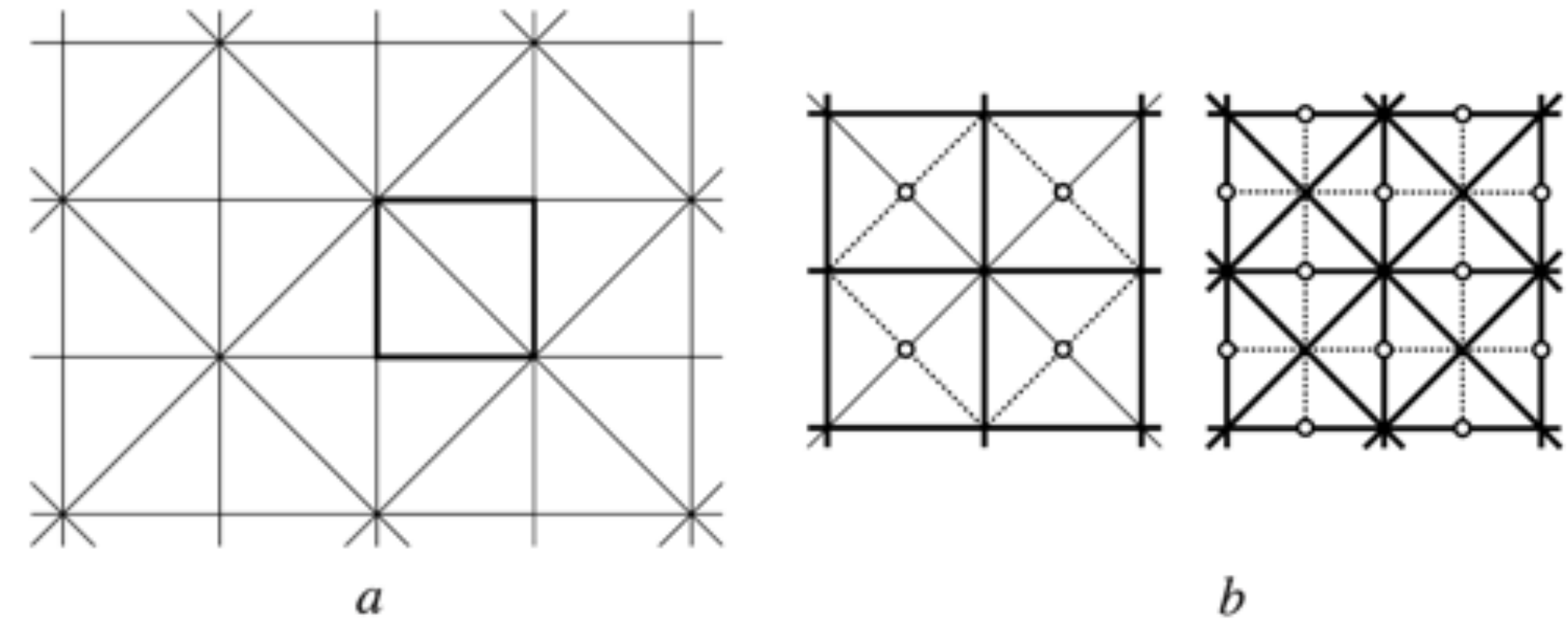
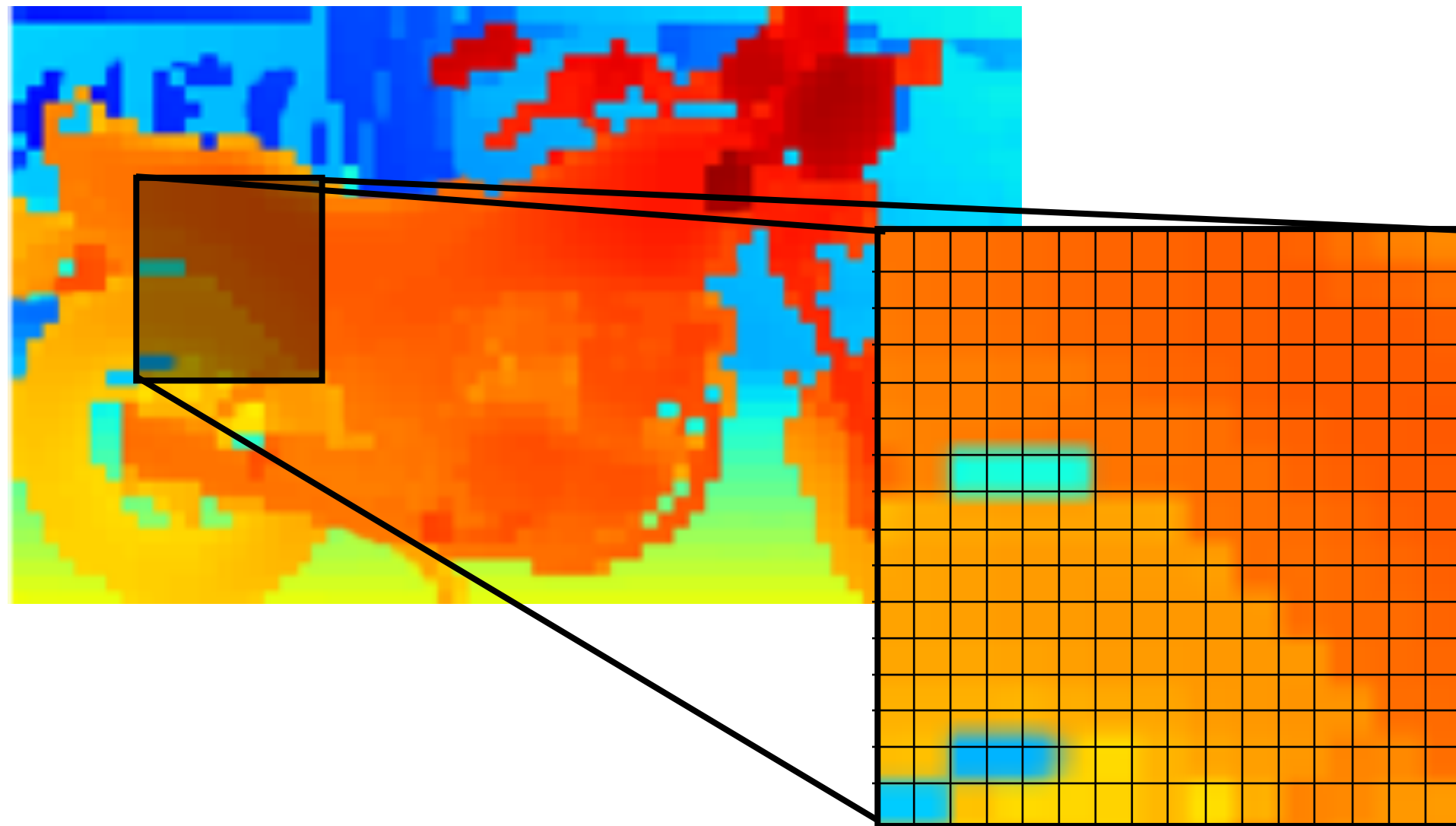


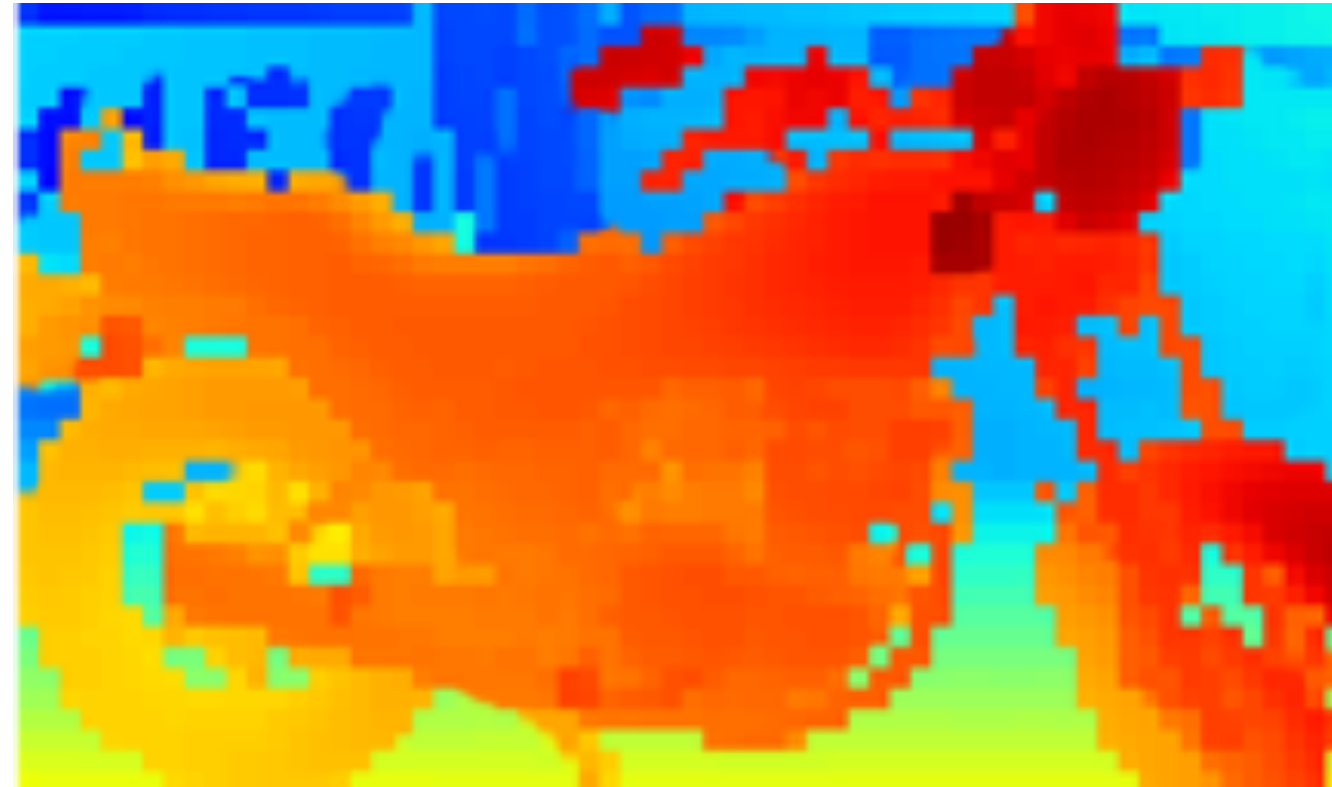
Fig. 1. a. Laves [4.8<sup>2</sup>] tiling with one of the basic blocks outlined. b. Two bisection refinement steps are equivalent to a face split. Vertices inserted at each step are shown as circles, new edges are shown as dotted lines.

Velho, Luiz, and Denis Zorin. "4-8 Subdivision." *Computer Aided Geometric Design* 18.5 (2001): 397-427.

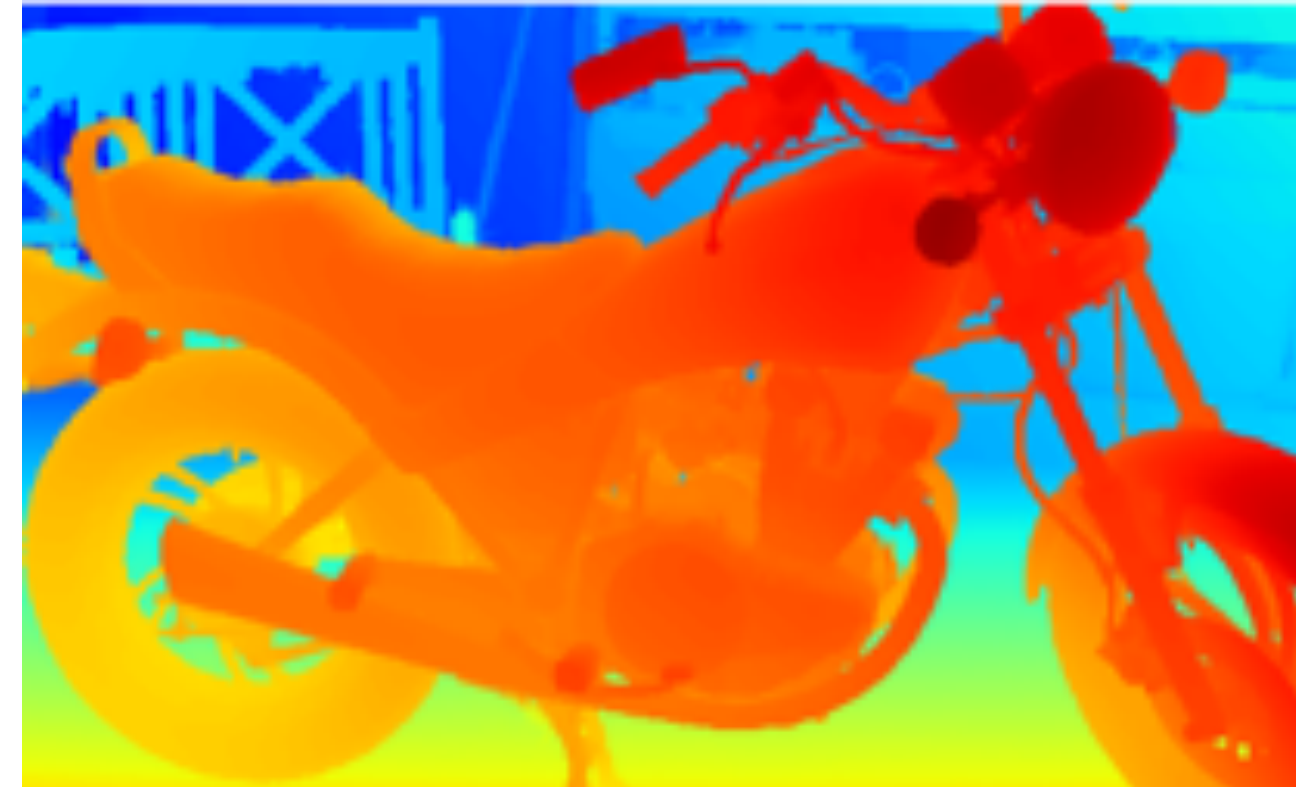
# 1.5. Reconstruction: Range-Image to Mesh

## §1. The geometry processing pipeline

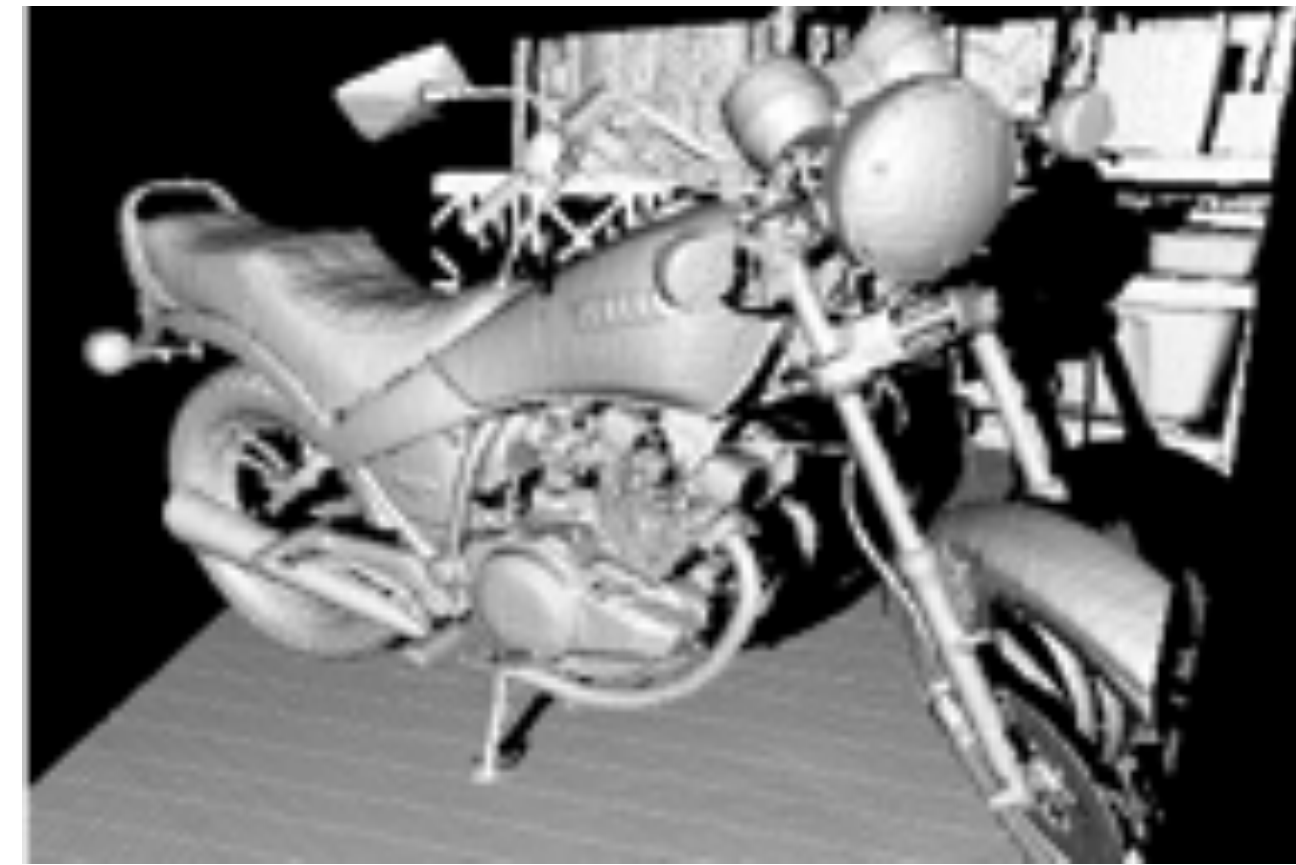
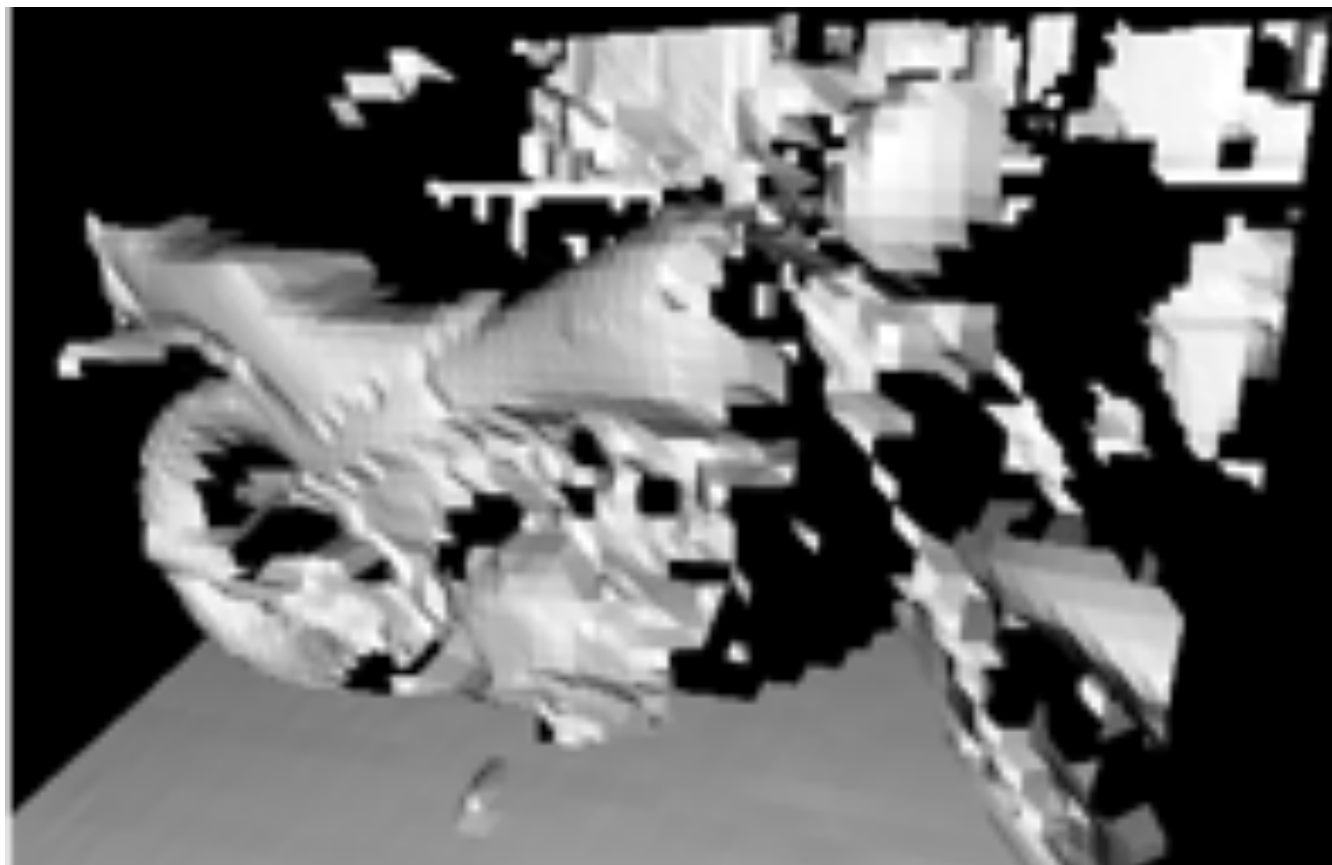
Low resolution



High resolution



Range-images

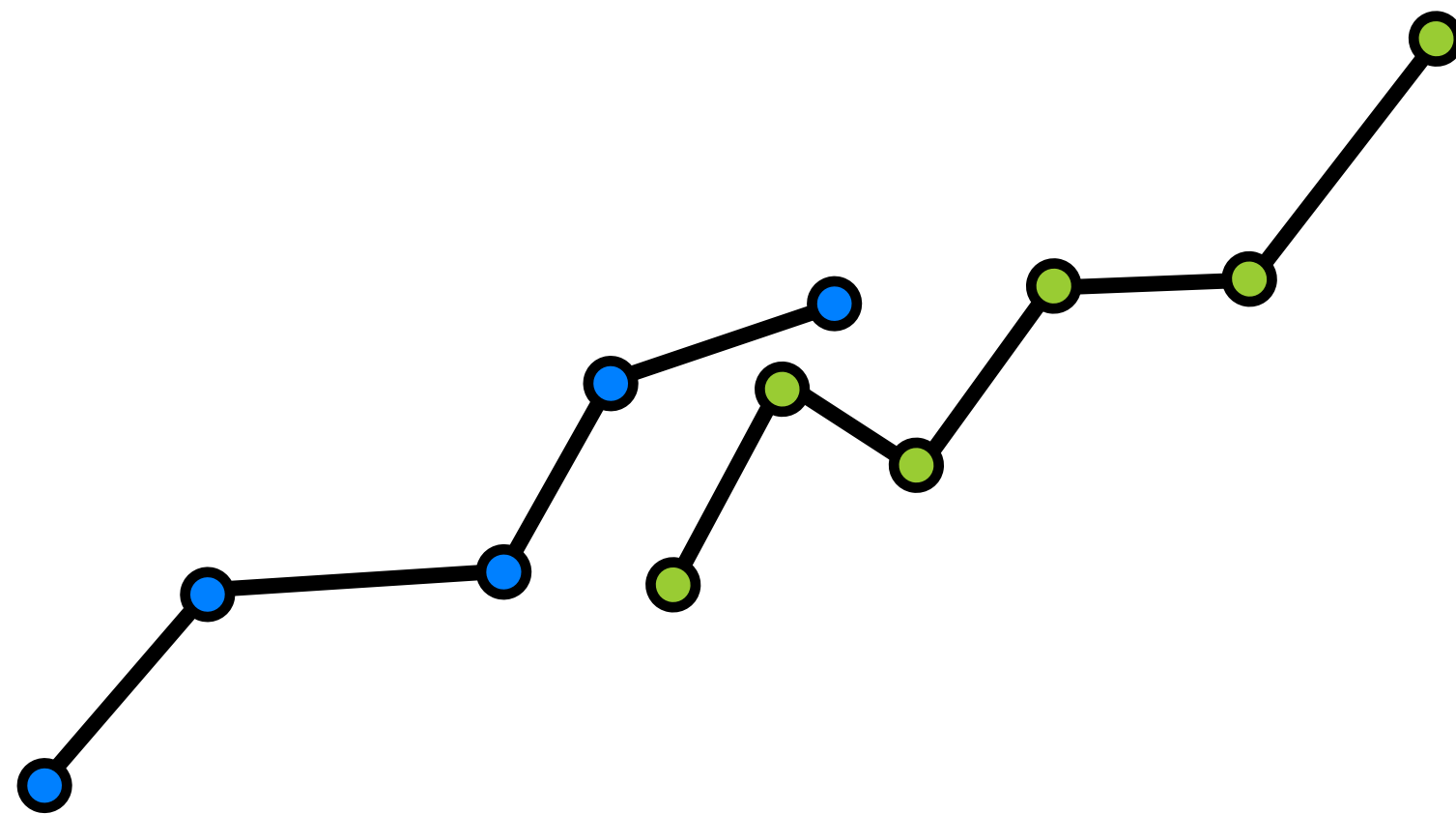


Explicit  
reconstruction

# 1.5. Reconstruction: How to Connect the Dots?

## §1. The geometry processing pipeline

- **Implicit reconstruction:** estimate a signed distance function (SDF); extract 0-level set

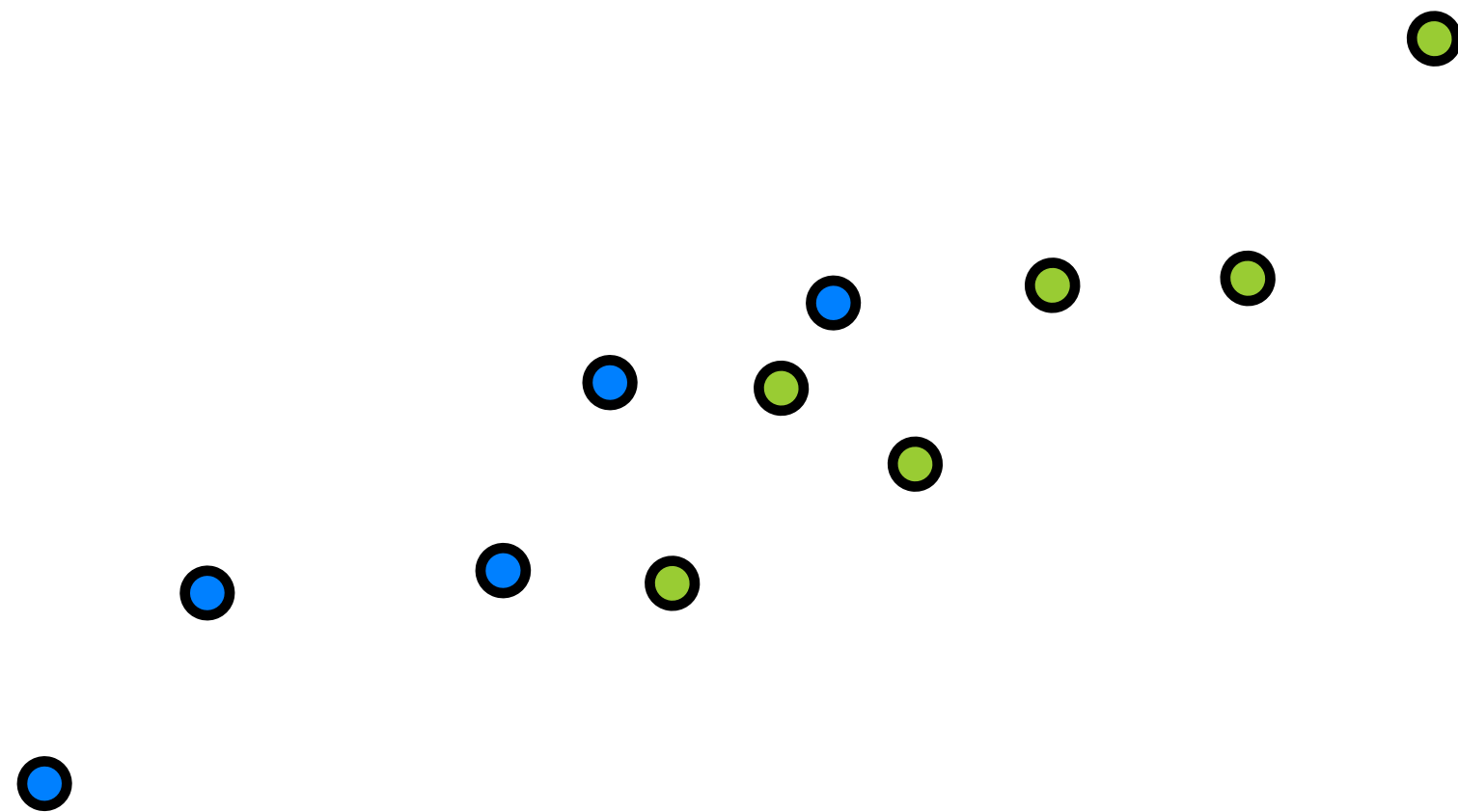




# 1.5. Reconstruction: How to Connect the Dots?

## §1. The geometry processing pipeline

- **Implicit reconstruction:** estimate a signed distance function (SDF); extract 0-level set

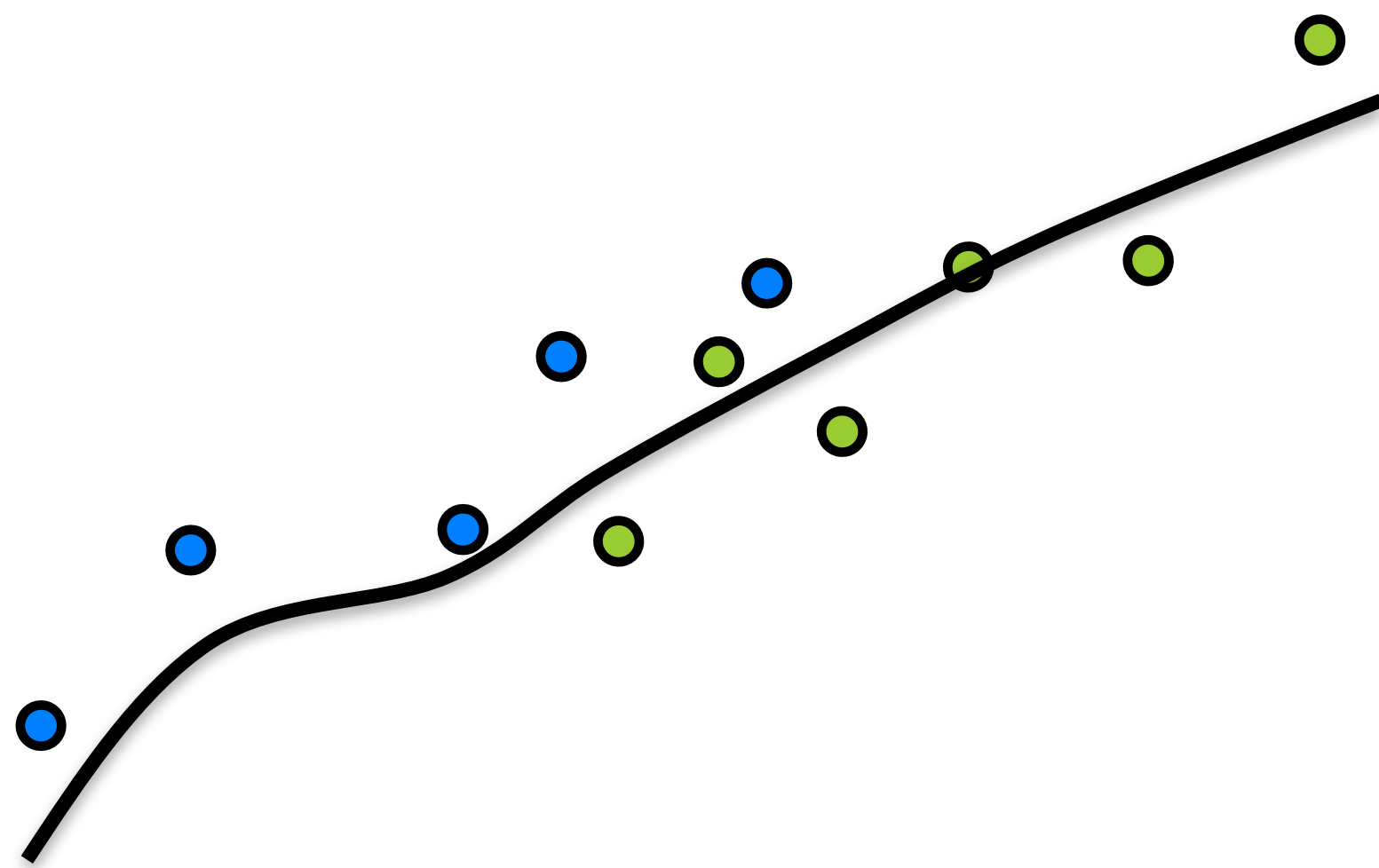




# 1.5. Reconstruction: How to Connect the Dots?

## §1. The geometry processing pipeline

- **Implicit reconstruction:** estimate a signed distance function (SDF); extract 0-level set

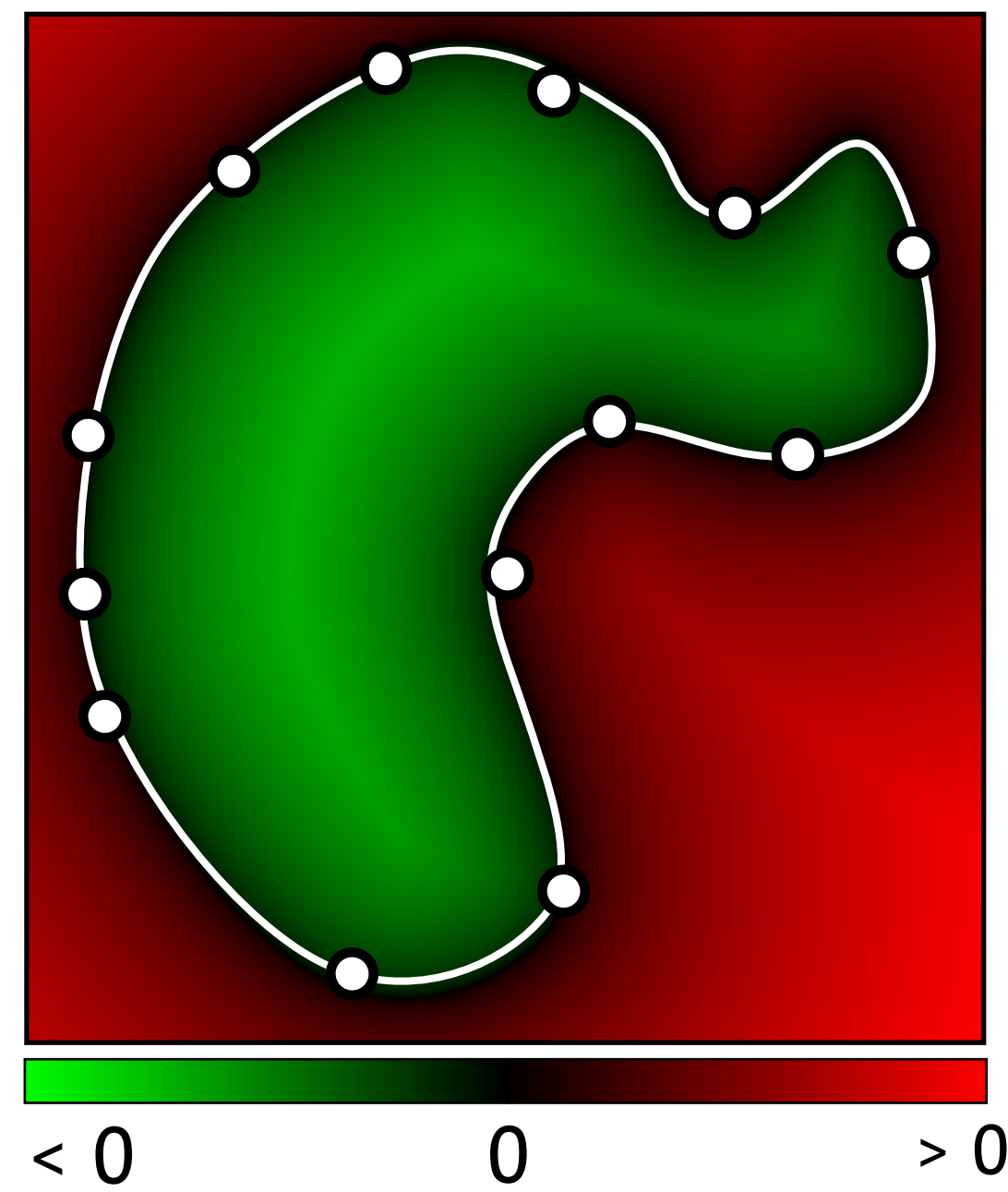


- Approximation of input points
- Watertight manifold results by construction

# 1.5. Reconstruction: How to Connect the Dots?

## §1. The geometry processing pipeline

- **Implicit reconstruction:** estimate a **signed distance function (SDF)**; extract 0-level set



- Assumes the existence of a function

$$f : \mathbb{R}^3 \rightarrow \mathbb{R}$$

with value  $> 0$  outside the shape  
and  $< 0$  inside

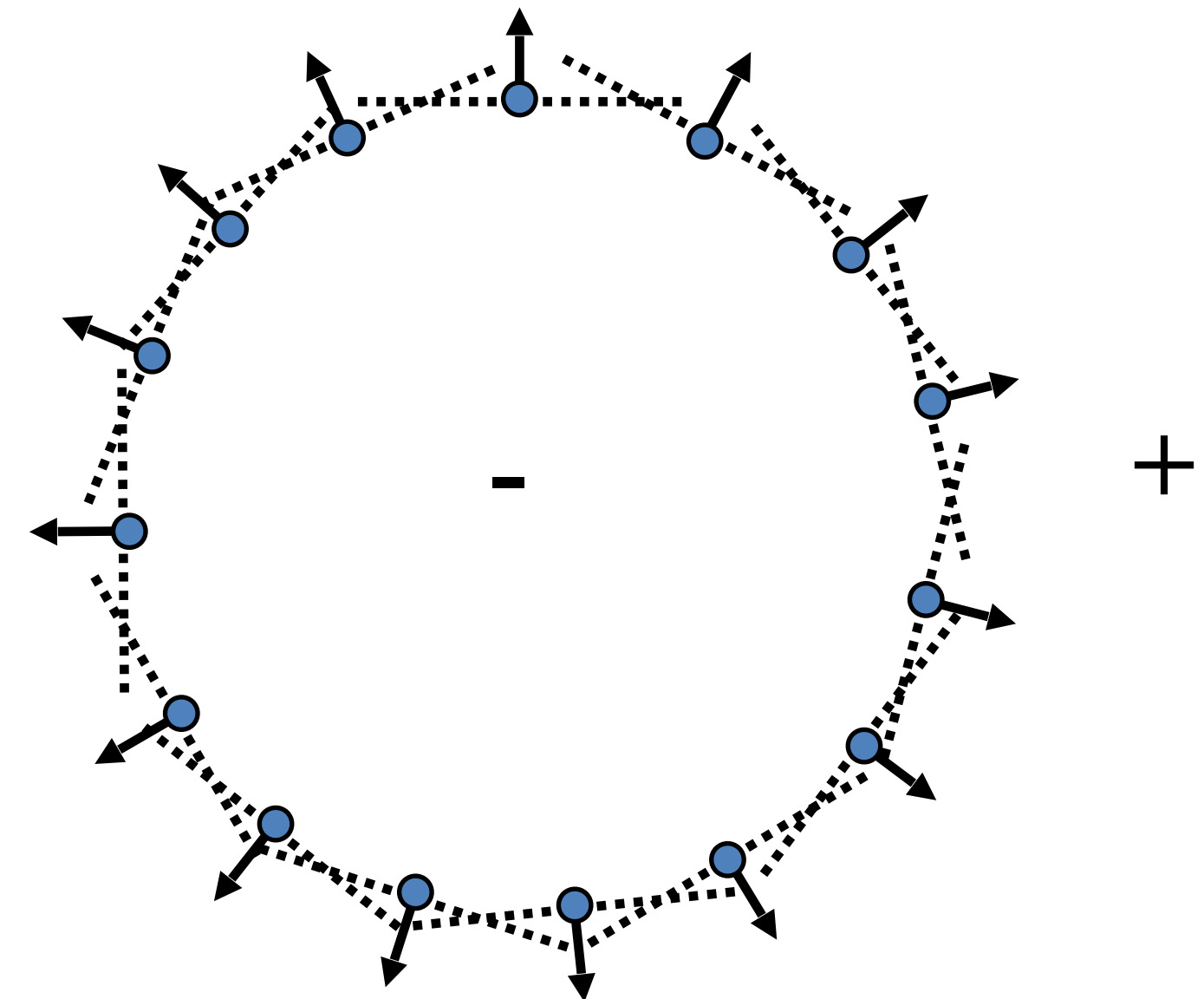
- Extract zero-level set

$$\{\mathbf{x} : f(\mathbf{x}) = 0\}$$

# 1.5. Reconstruction: SDF from Points and Normals

## §1. The geometry processing pipeline

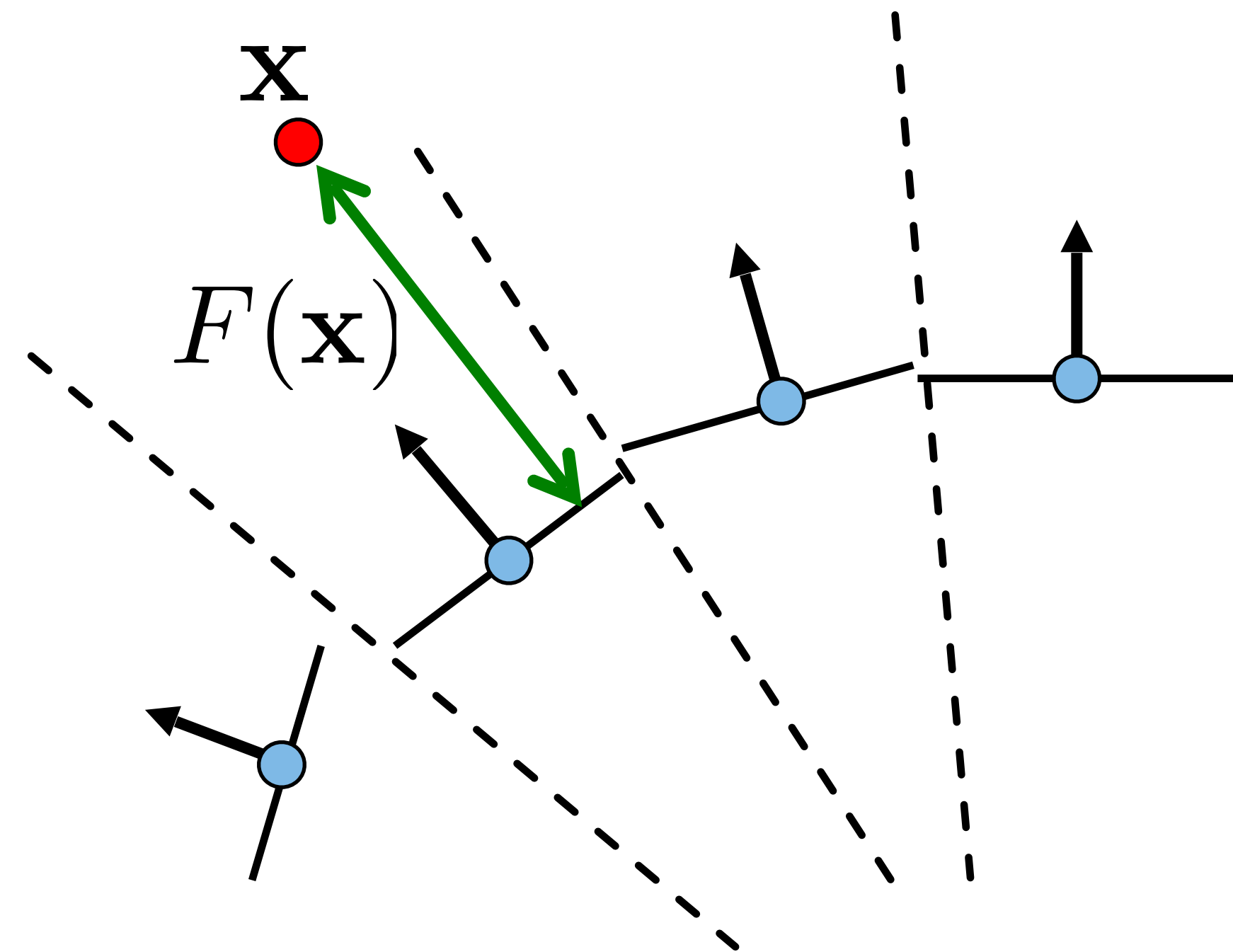
- Compute **signed distance function (SDF)** to the tangent plane of the closest point
- Normals help to distinguish between inside and outside
- “Surface reconstruction from unorganized points”, Hoppe et al., ACM SIGGRAPH 1992 <http://research.microsoft.com/en-us/um/people/hoppe/proj/recon/>



# 1.5. Reconstruction: SDF from Points and Normals

## §1. The geometry processing pipeline

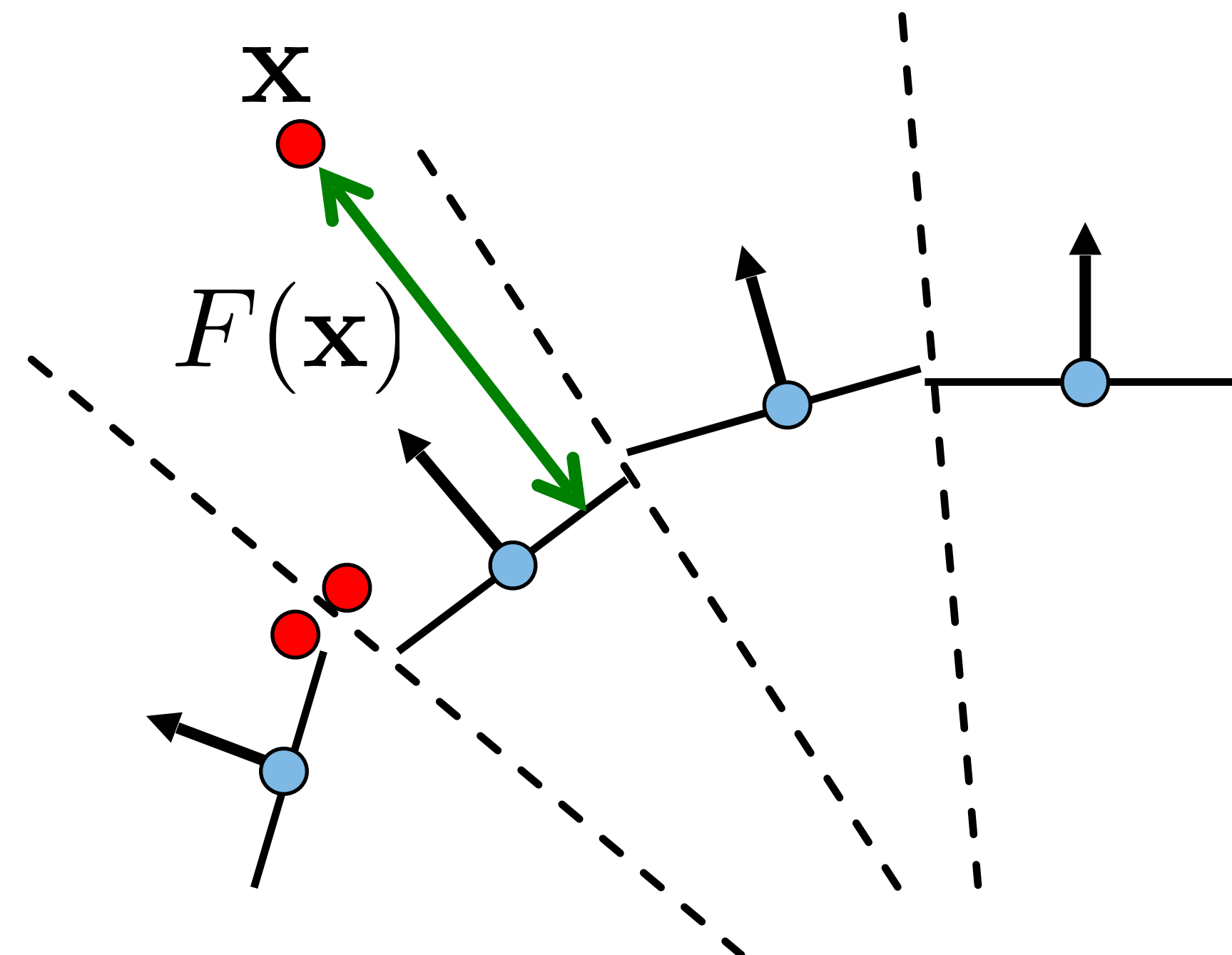
- Compute **signed distance function (SDF)** to the tangent plane of the closest point
- Problem??



# 1.5. Reconstruction: SDF from Points and Normals

## §1. The geometry processing pipeline

- Compute **signed distance function (SDF)** to the tangent plane\* of the closest point
- The function will be discontinuous



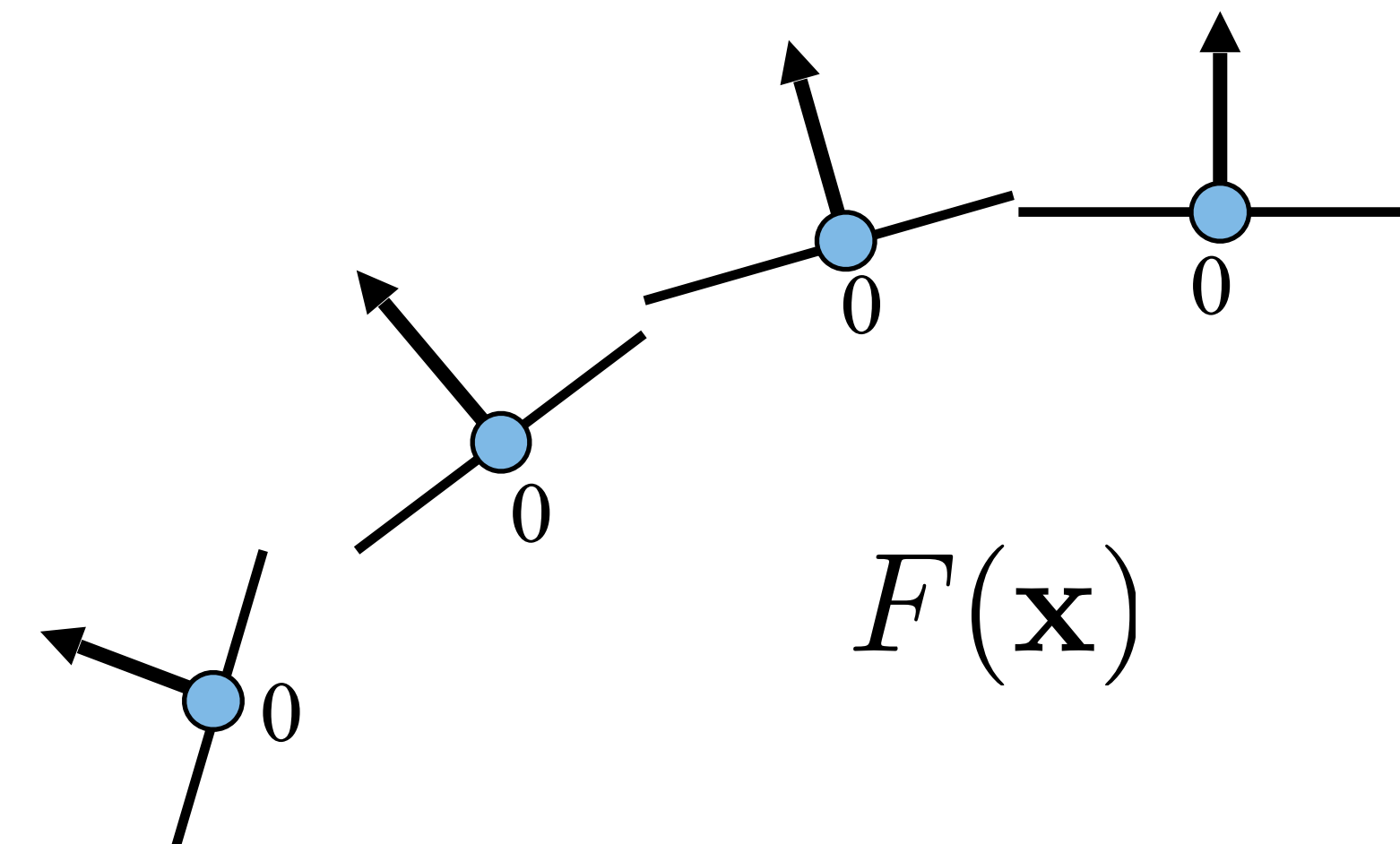
\* The Hoppe92 paper computes the tangent planes slightly differently (by PCA on k-nearest-neighbors of each data point, see next class), but the consequences are still the same.



# 1.5. Reconstruction: Smooth SDF

## §1. The geometry processing pipeline

- Instead find a smooth formulation for  $F$ .
- Scattered data interpolation:
  - $F(\mathbf{p}_i) = 0$
  - $F$  is smooth
  - Avoid trivial  $F \equiv 0$



“Reconstruction and representation of 3D objects with radial basis functions”, Carr et al., ACM SIGGRAPH 2001

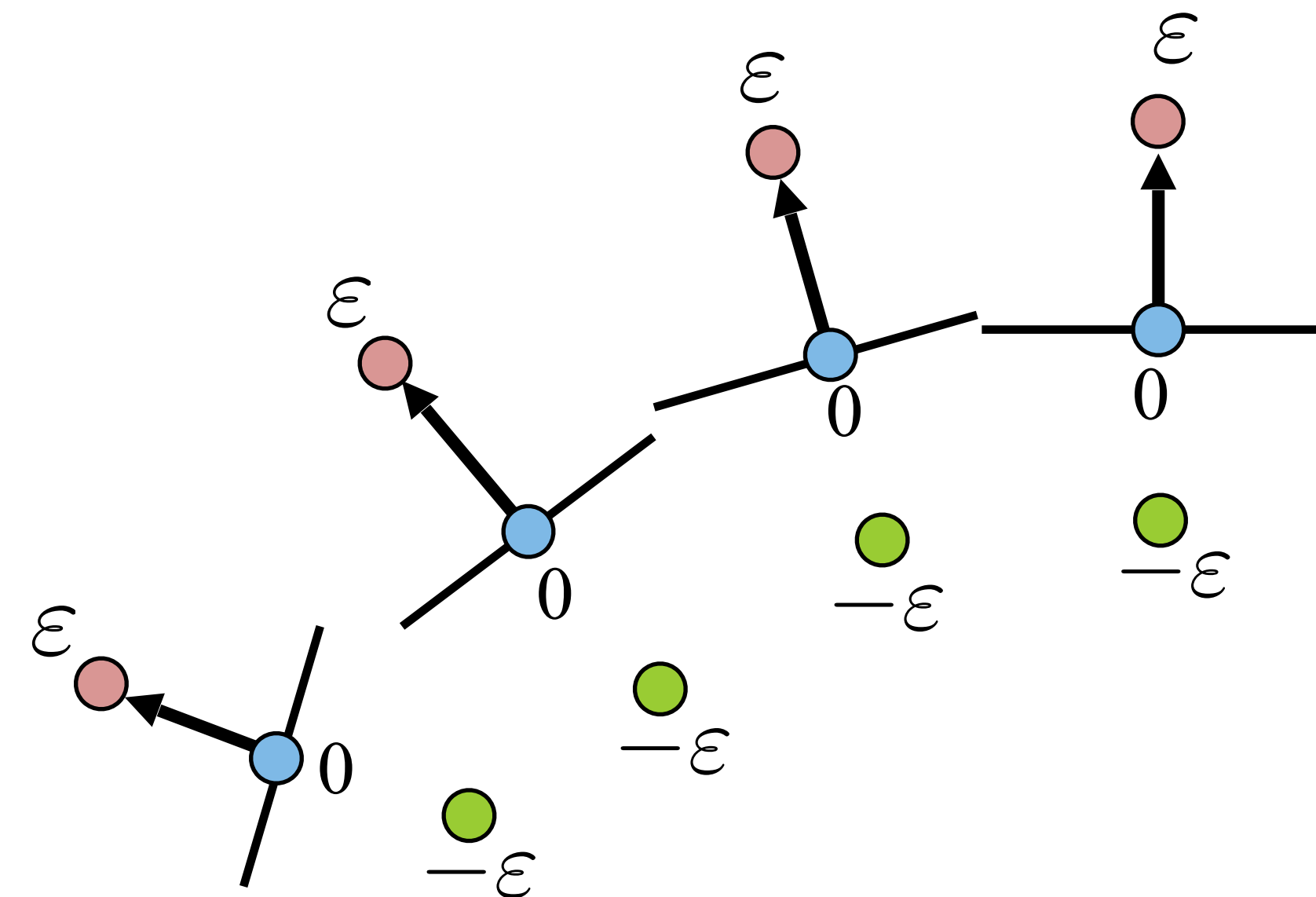
# 1.5. Reconstruction: Smooth SDF

## §1. The geometry processing pipeline

- Scattered data interpolation:

- $F(\mathbf{p}_i) = 0$
- $F$  is smooth
- Avoid trivial  $F \equiv 0$

- Add off-surface constraints



$$F(\mathbf{p}_i + \epsilon \mathbf{n}_i) = \epsilon$$

$$F(\mathbf{p}_i - \epsilon \mathbf{n}_i) = -\epsilon$$

“Reconstruction and representation of 3D objects with radial basis functions”, Carr et al., ACM SIGGRAPH 2001

# 1.5. Reconstruction: Radial Basis Function Interpolation

## §1. The geometry processing pipeline

- **RBF**: Weighted sum of shifted, smooth kernels

Spline  $\varphi(r) = r^2 \log r$

Gaussian  $\varphi(r) = \exp\{-cr^2\}$

$$F(\mathbf{x}) = \sum_{i=0}^{N-1} w_i \varphi(\|\mathbf{x} - \mathbf{c}_i\|)$$

Scalar weights  
**Unknowns**

Smooth kernels  
(basis functions)  
centered at constrained  
points.

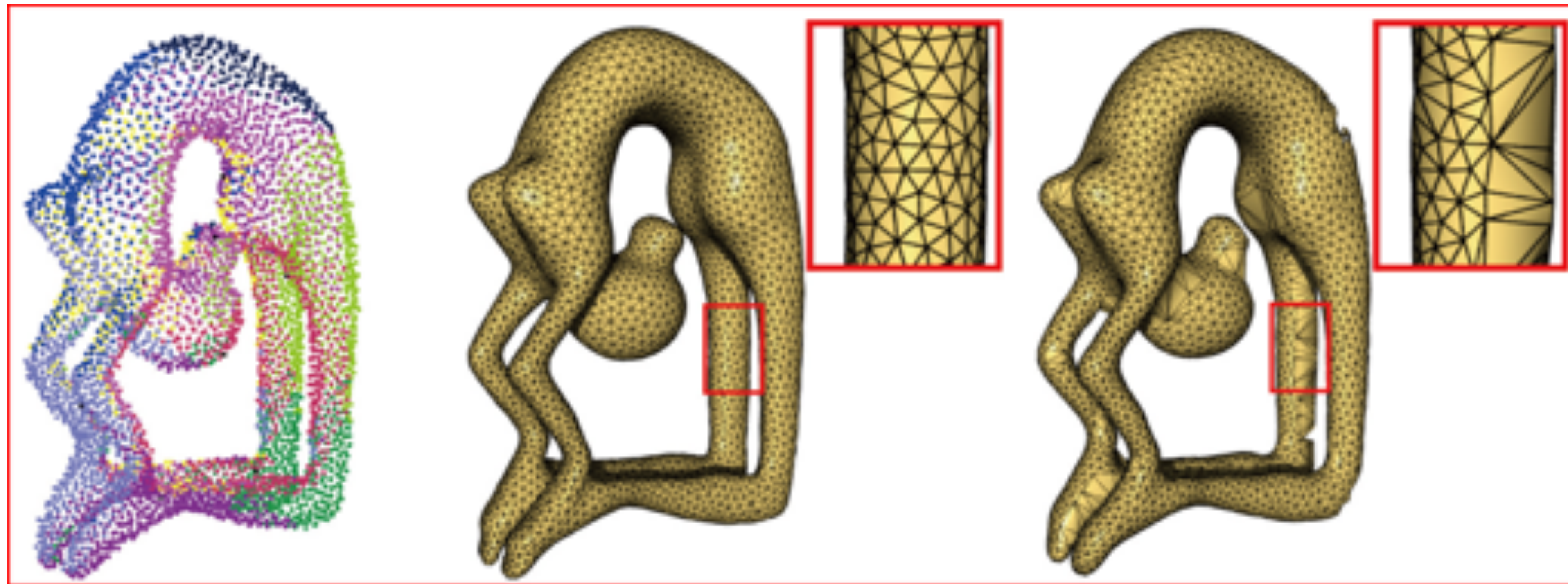
For example:  
 $\varphi(r) = r^3$

$$N = 3n$$



# 1.5. Reconstruction: Implicit vs. Explicit

## §1. The geometry processing pipeline



Input

Implicit

Explicit

# 1.5. Reconstruction: Summary

## §1. The geometry processing pipeline

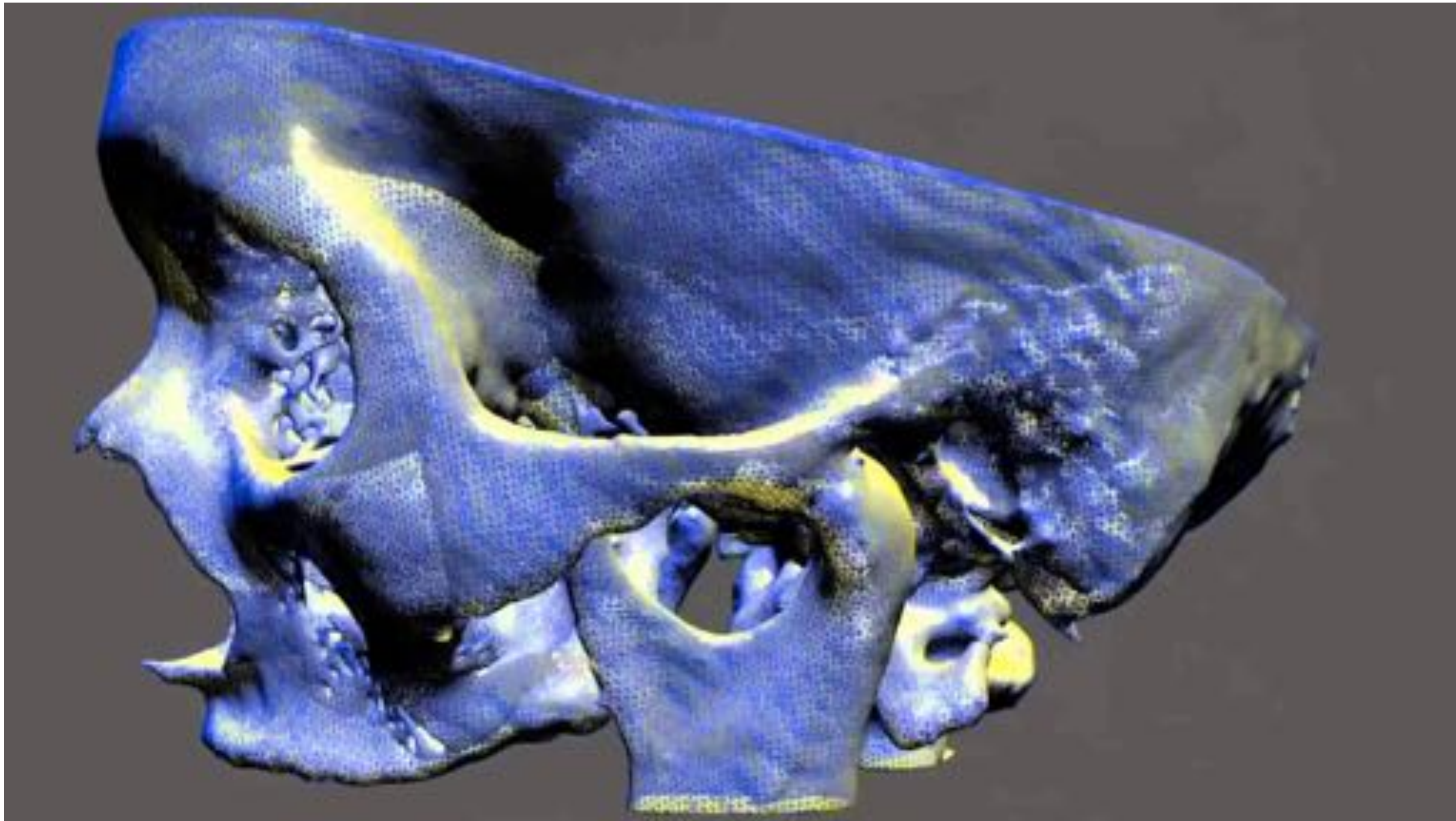
- **Surface reconstruction:** create a surface representation from sparse input points
  - **Explicit:** directly create connectivity by linking close points together
  - **Implicit:** recover a signed distance function (SDF) with values  $< 0$  inside the shape and  $> 0$  outside, then extract level set (next section)
- State-of-the-art reconstruction algorithm: Poisson Surface Reconstruction (more in ~Lecture 6)



# Meshing

# 1.5. Meshing: Motivation

## §1. The geometry processing pipeline





# 1.5. Meshing: Extracting the Surface

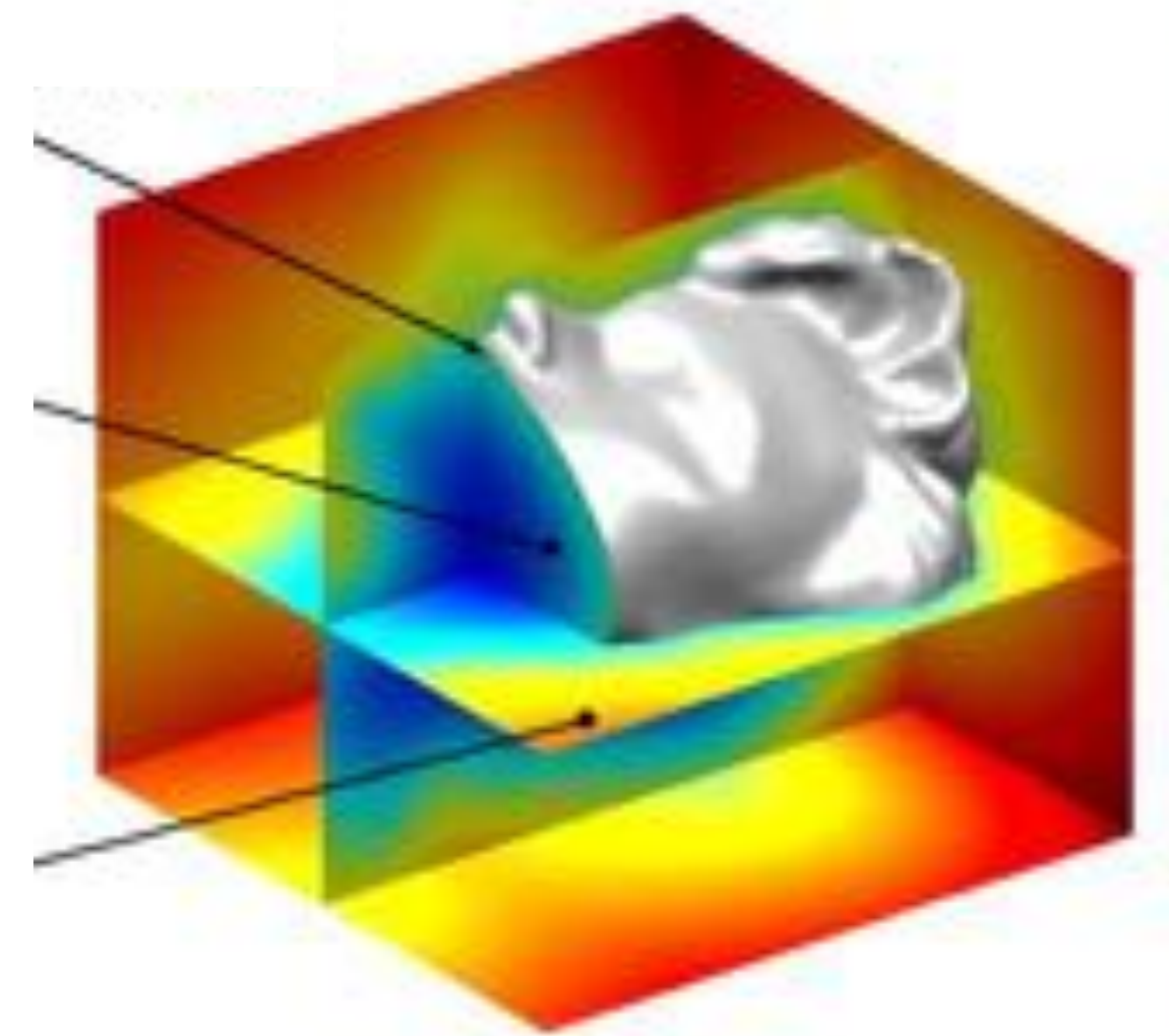
## §1. The geometry processing pipeline

- Wish to compute a manifold mesh of the level set
- **Mesh:** a subdivision of a continuous geometric space into discrete geometric and topological cells (often simplicial surface is constructed)
- **Meshing:** implicit surface  $\rightarrow$  simplicial surface

$F(\mathbf{x}) = 0 \rightarrow$   
surface

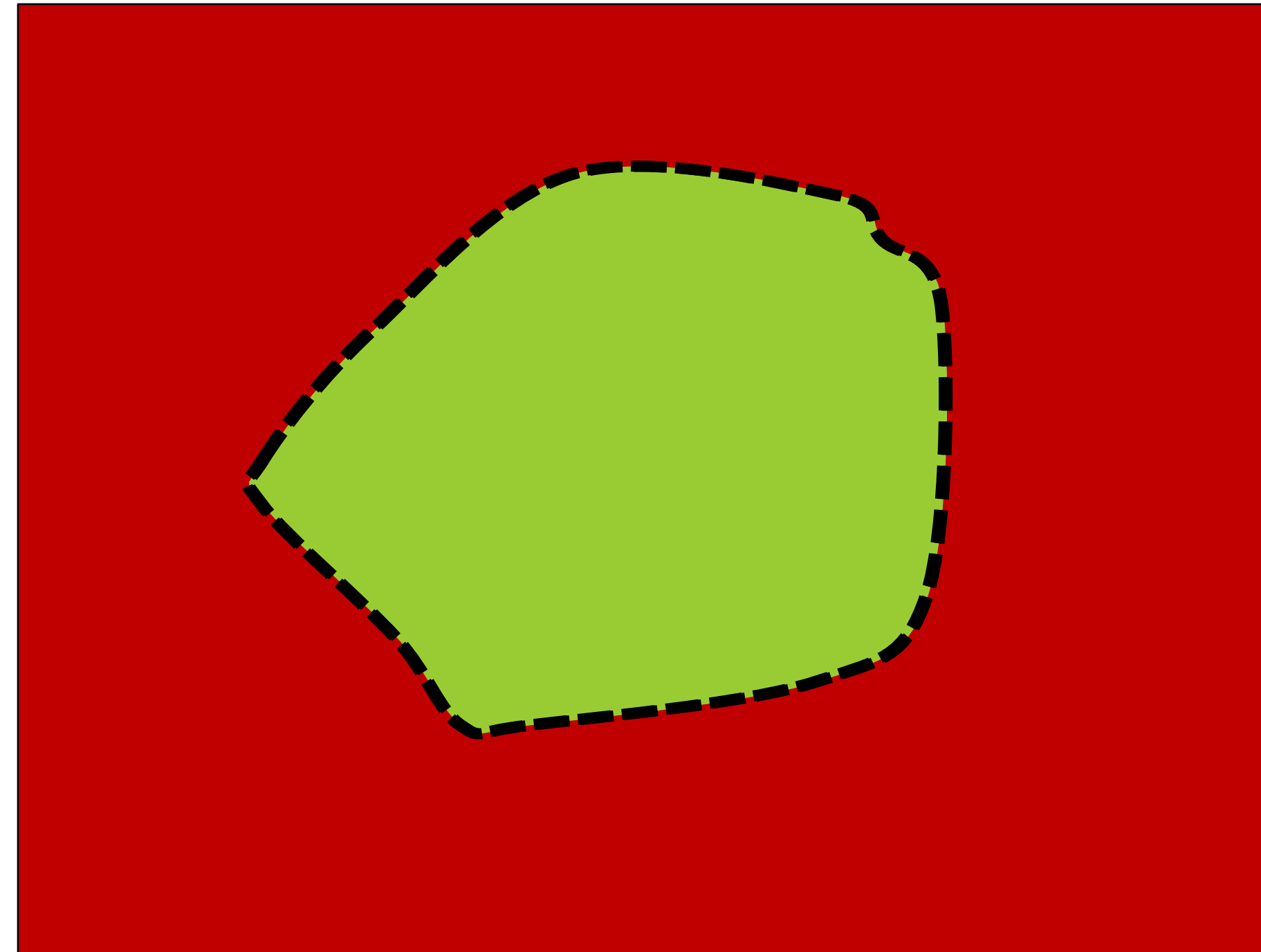
$F(\mathbf{x}) < 0 \rightarrow$   
inside

$F(\mathbf{x}) > 0 \rightarrow$   
outside



# 1.5. Meshing: Sample the SDF

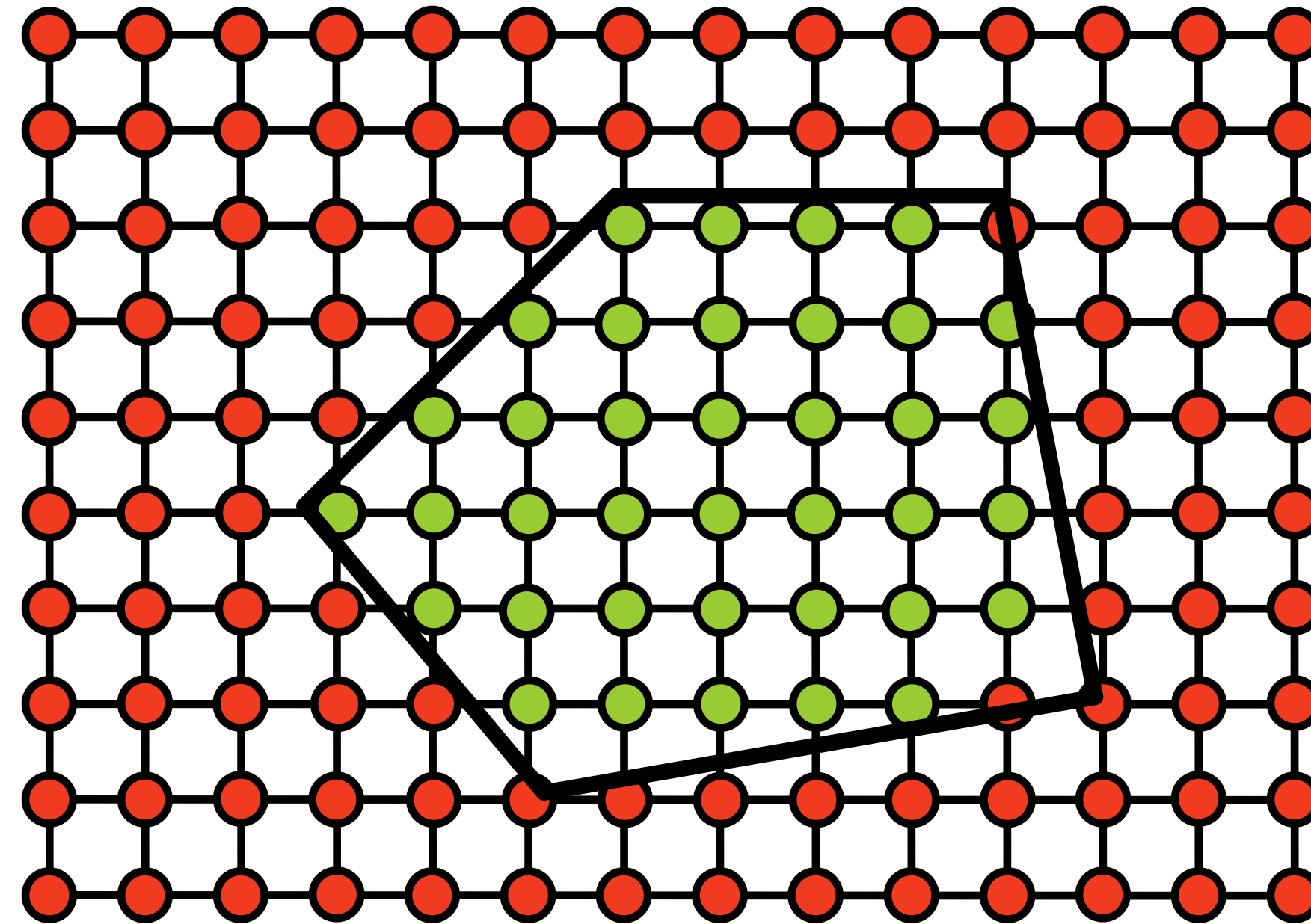
## §1. The geometry processing pipeline





# 1.5. Meshing: Sample the SDF

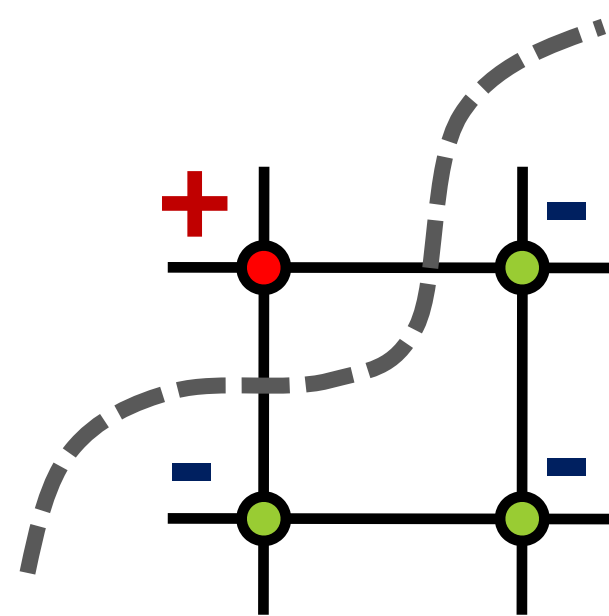
## §1. The geometry processing pipeline



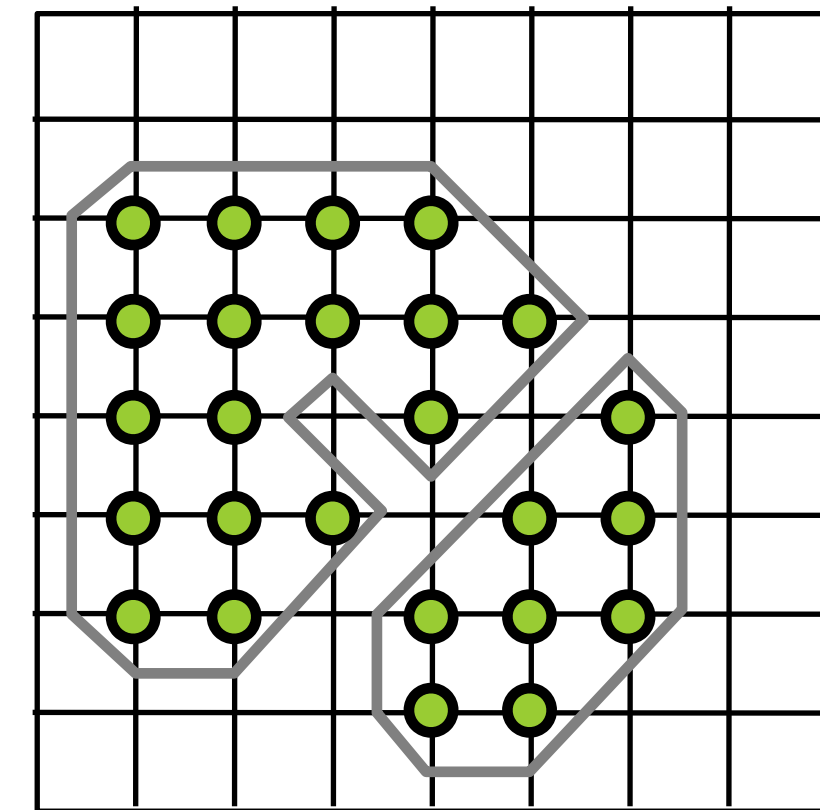
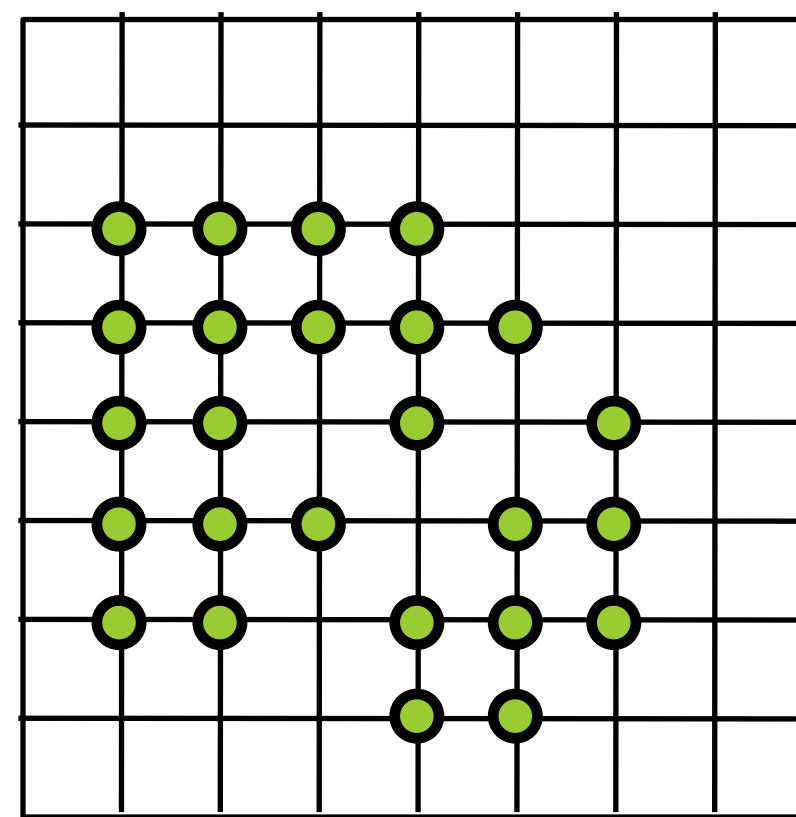
# 1.5. Meshing: Tessellation

## §1. The geometry processing pipeline

- Want to approximate an implicit surface with a mesh
- Can't explicitly compute all the roots
  - Sampling the level set is difficult (root finding)
- Solution: find approximate roots by trapping the implicit surface in a grid (lattice)



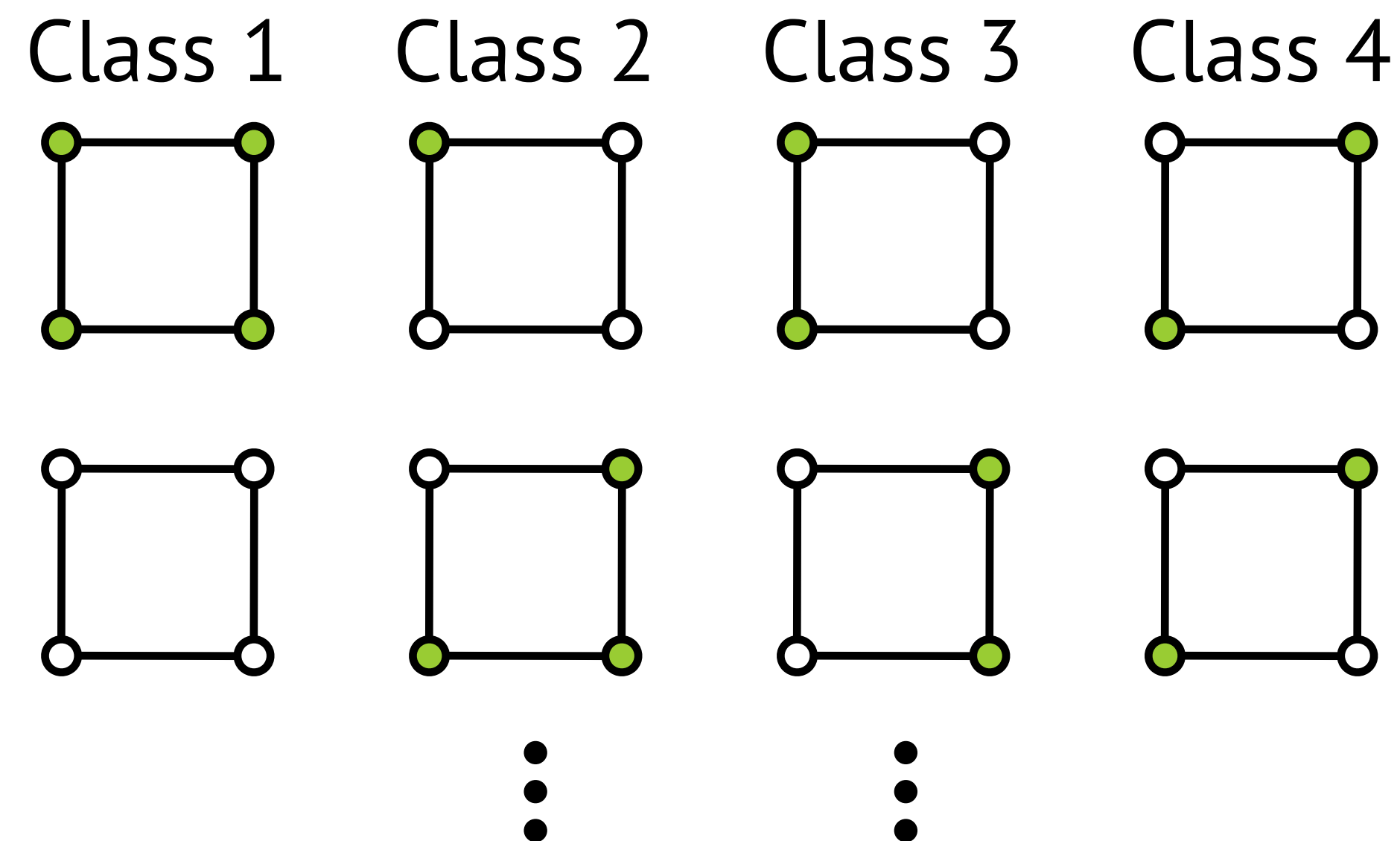
•  $F(\mathbf{x}) < 0$



# 1.5. Meshing: Marching Squares

## §1. The geometry processing pipeline

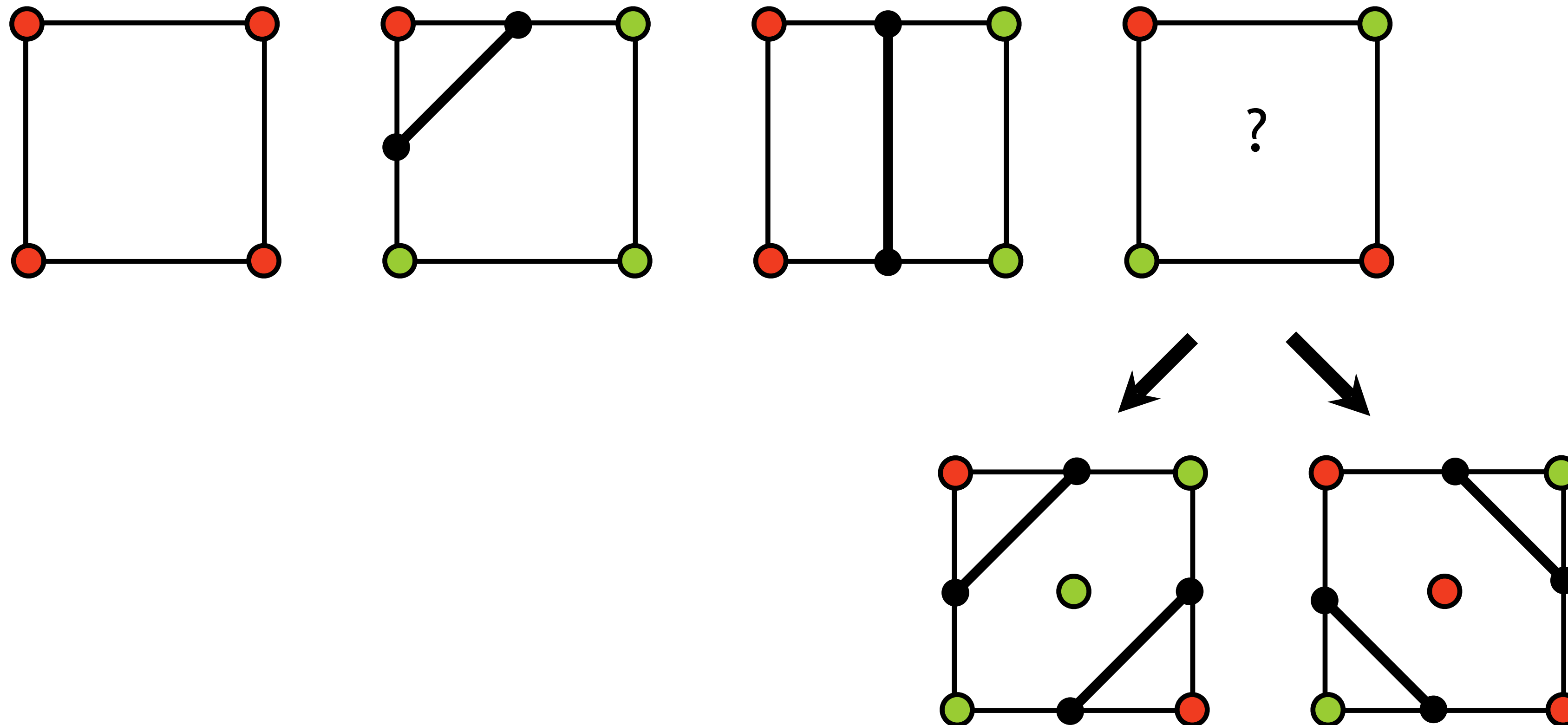
- 16 different configurations in 2D
- 4 equivalence classes (up to rotational and reflection symmetry + complement)



# 1.5. Meshing: Tessellation in 2D

## §1. The geometry processing pipeline

- 4 equivalence classes (up to rotational and reflection symmetry + complement)

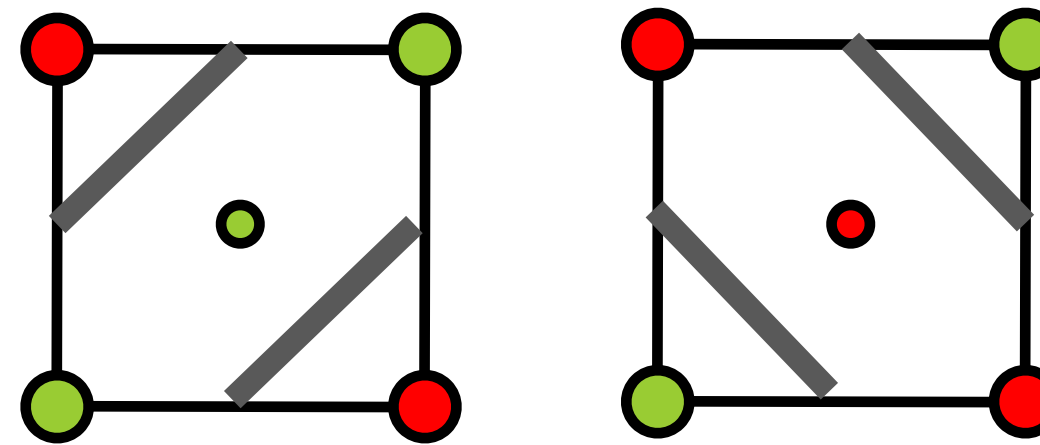




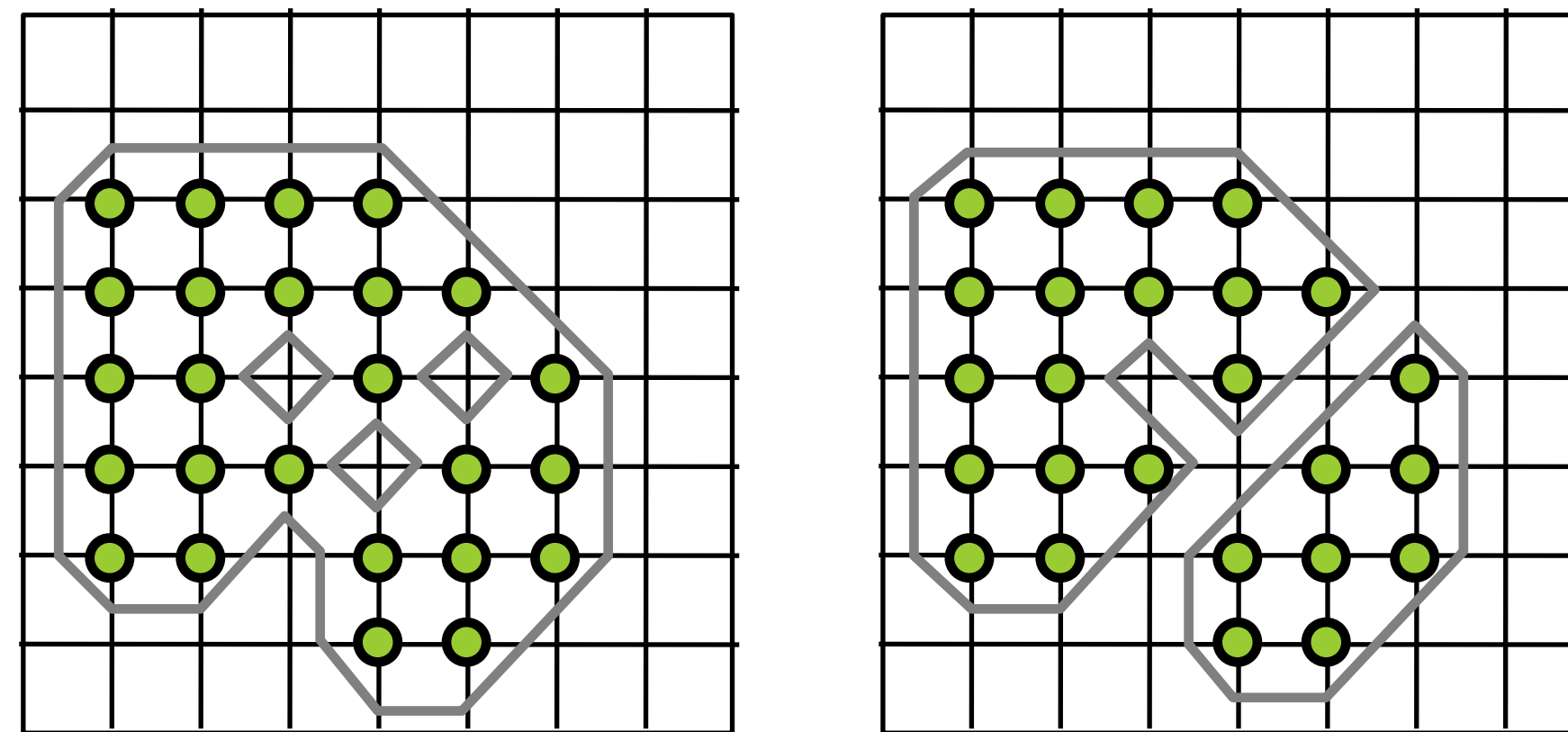
# 1.5. Meshing: Tessellation in 2D

## §1. The geometry processing pipeline

- Case 4 is ambiguous:



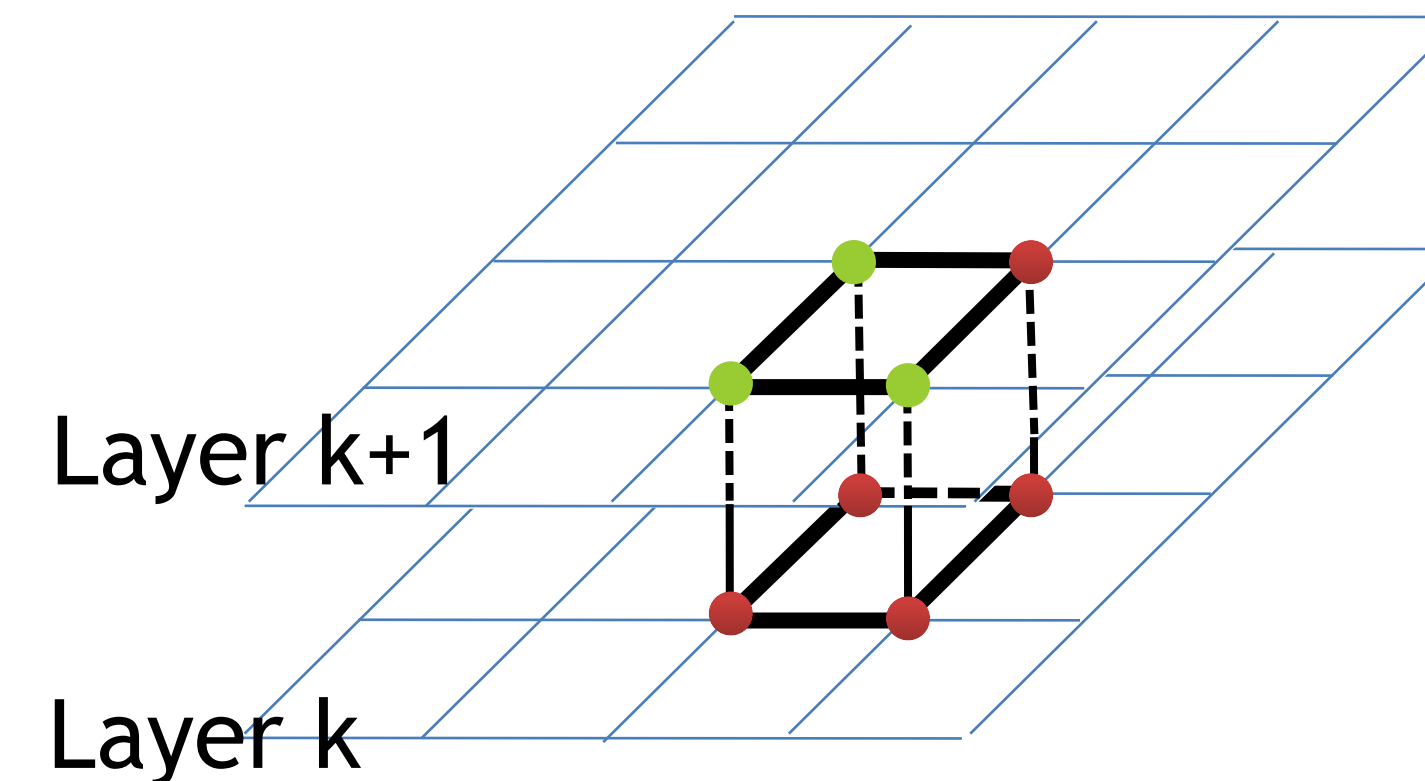
- Always pick consistently to avoid problems with the resulting mesh



# 1.5. Meshing: 3D Marching Cubes

## §1. The geometry processing pipeline

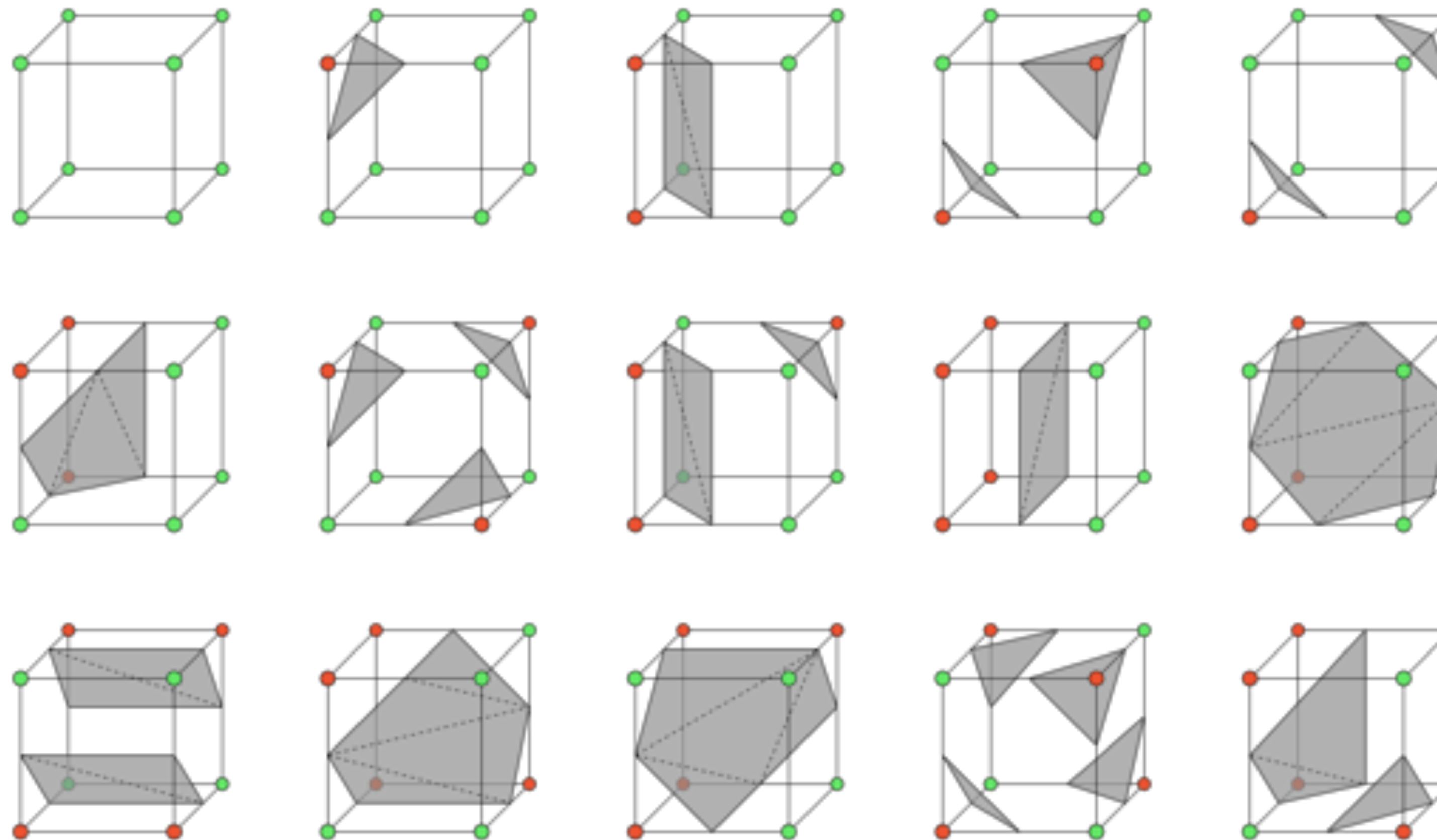
- Marching Cubes (Lorensen and Cline 1987)
  1. Load 4 layers of the grid into memory
  2. Create a cube whose vertices lie on the two middle layers
  3. Classify the vertices of the cube according to the implicit function (inside, outside or on the surface)



# 1.5. Meshing: 3D Marching Cubes

## §1. The geometry processing pipeline

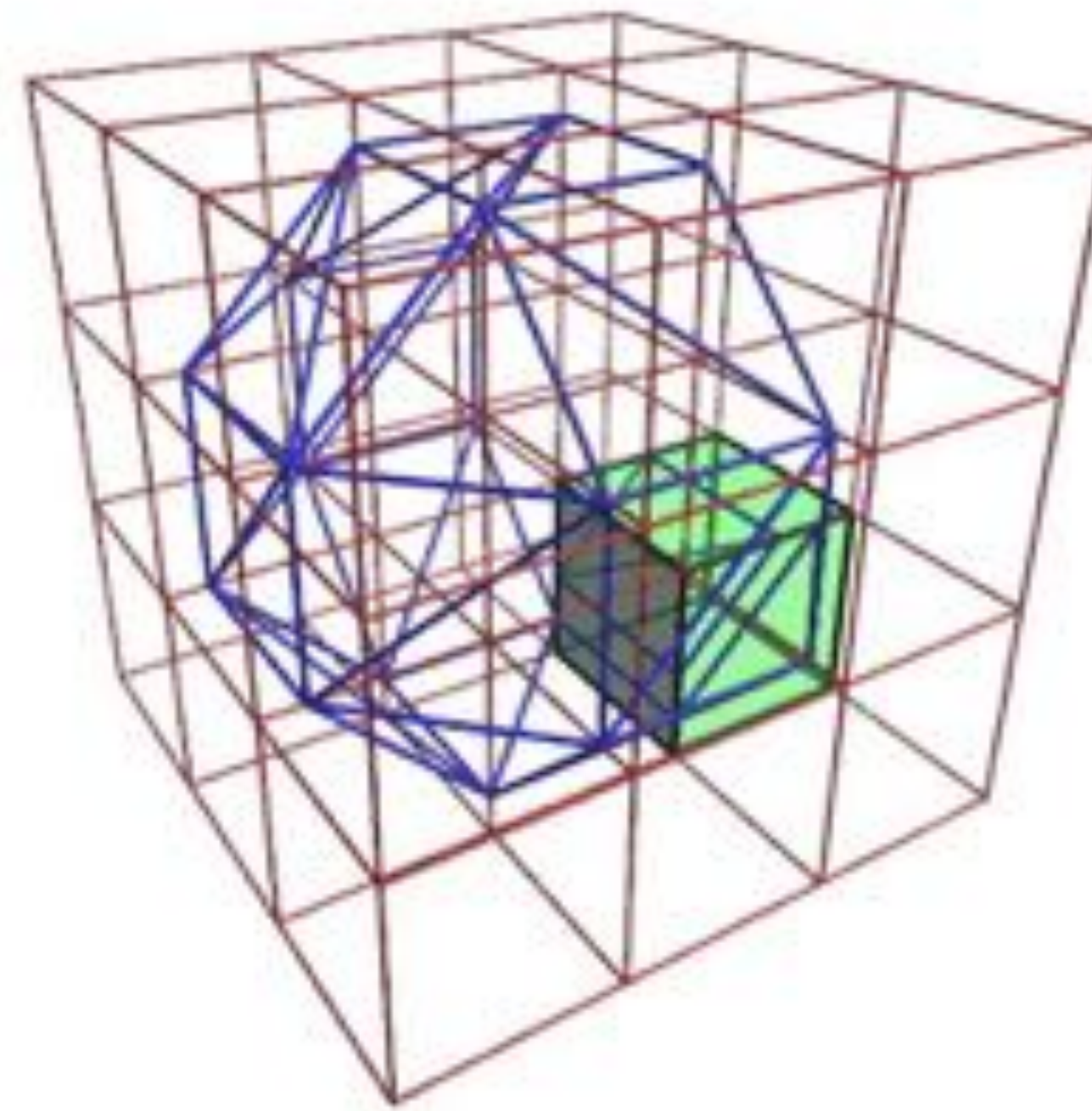
- Unique cases (by rotation, reflection and complement)



# 1.5. Meshing: 3D Marching Cubes

## §1. The geometry processing pipeline

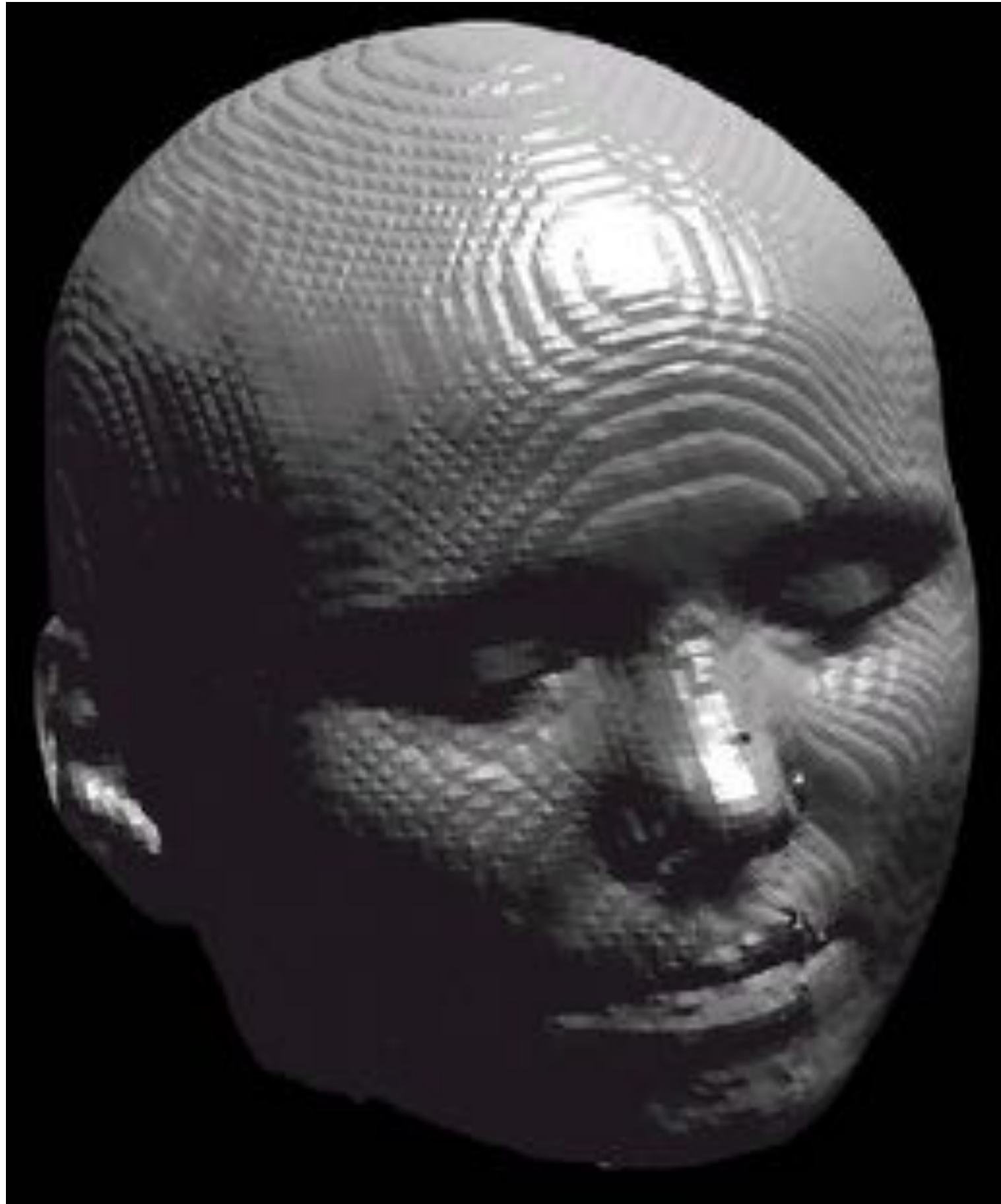
Implementation





# 1.5. Meshing: Marching Cubes – Problems

## §1. The geometry processing pipeline



Output aliasing artefacts

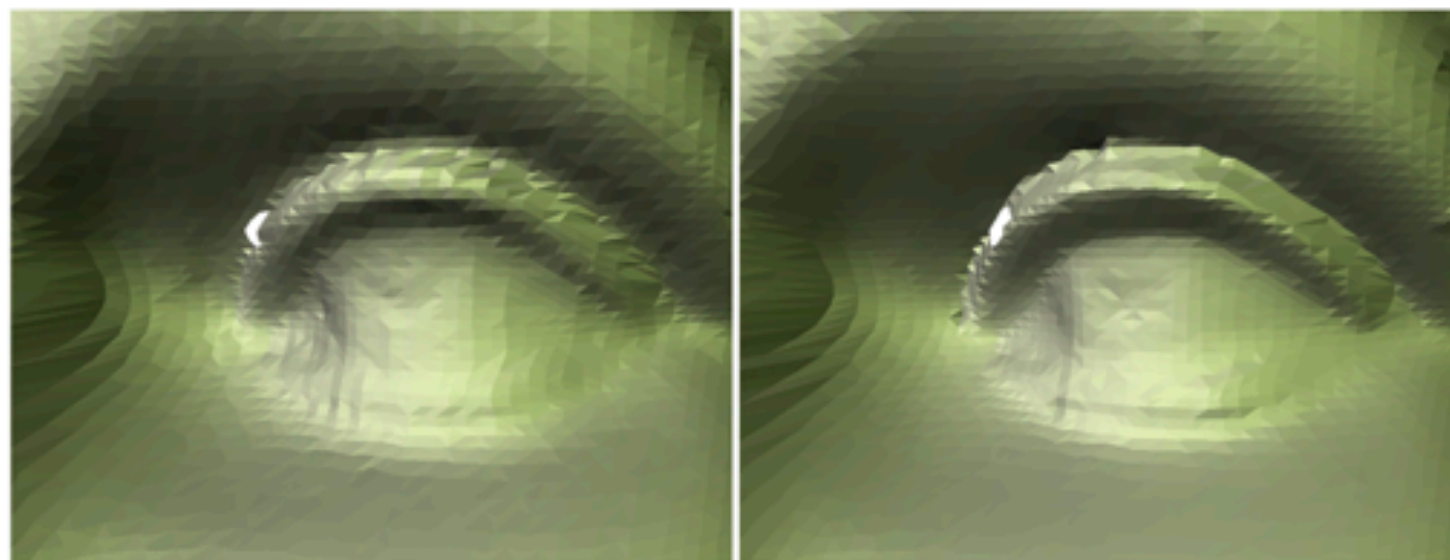
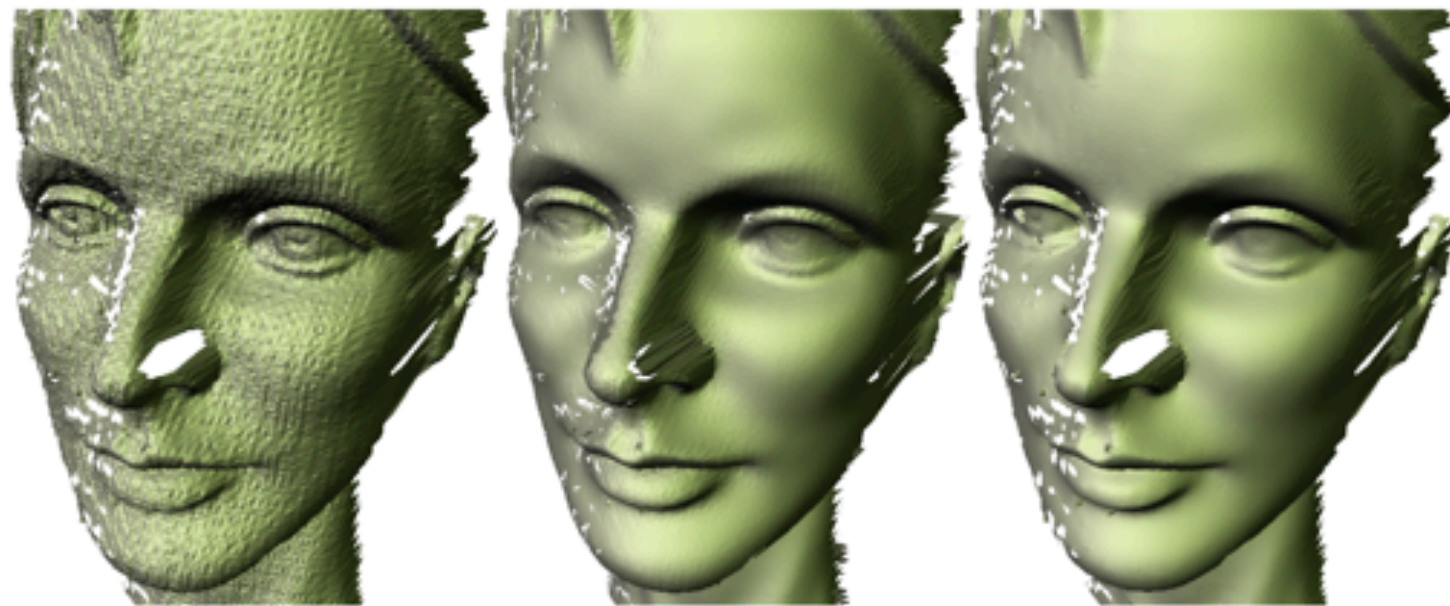
# Postprocessing



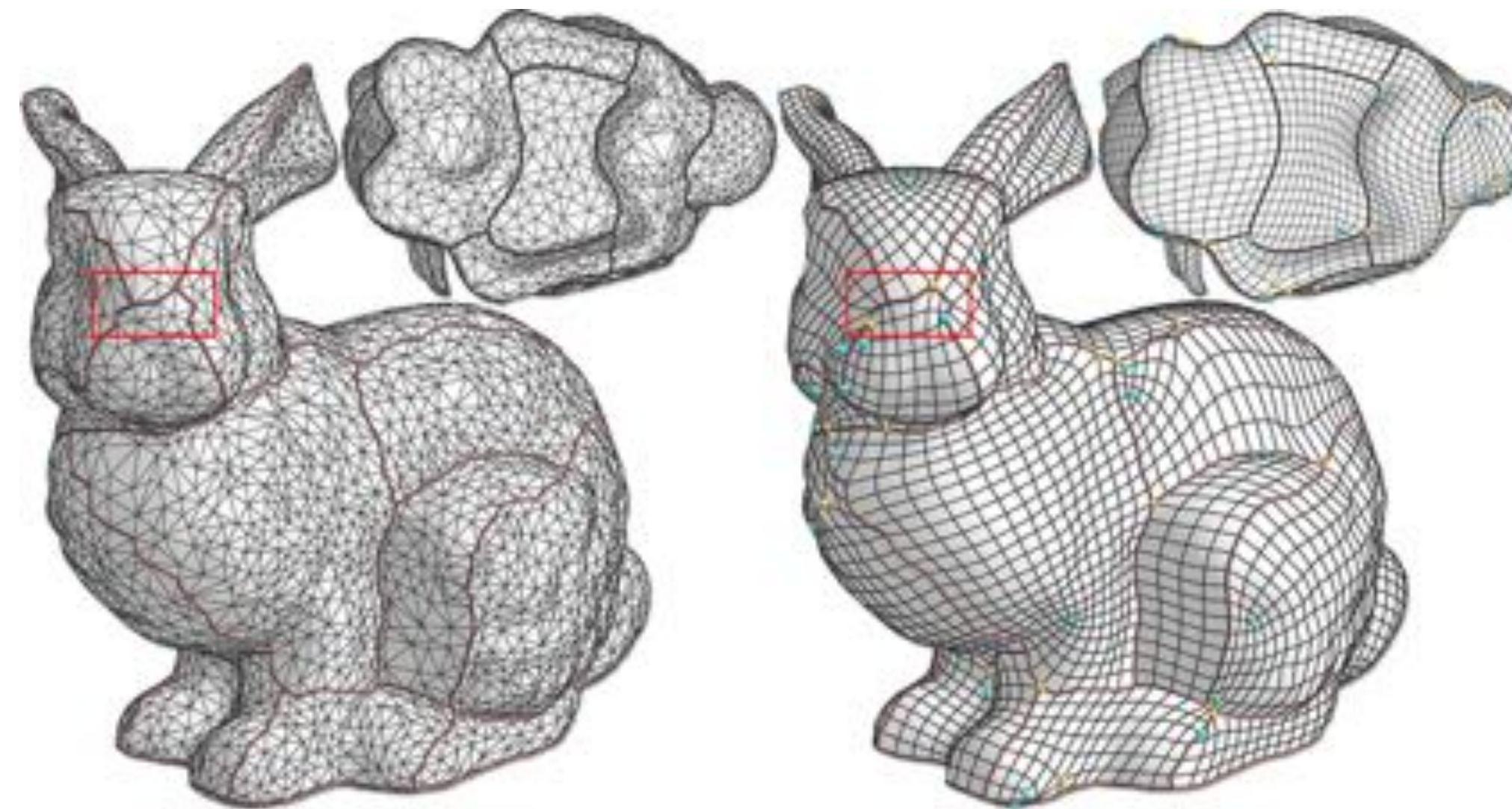
# 1.6. Postprocessing

## §1. The geometry processing pipeline

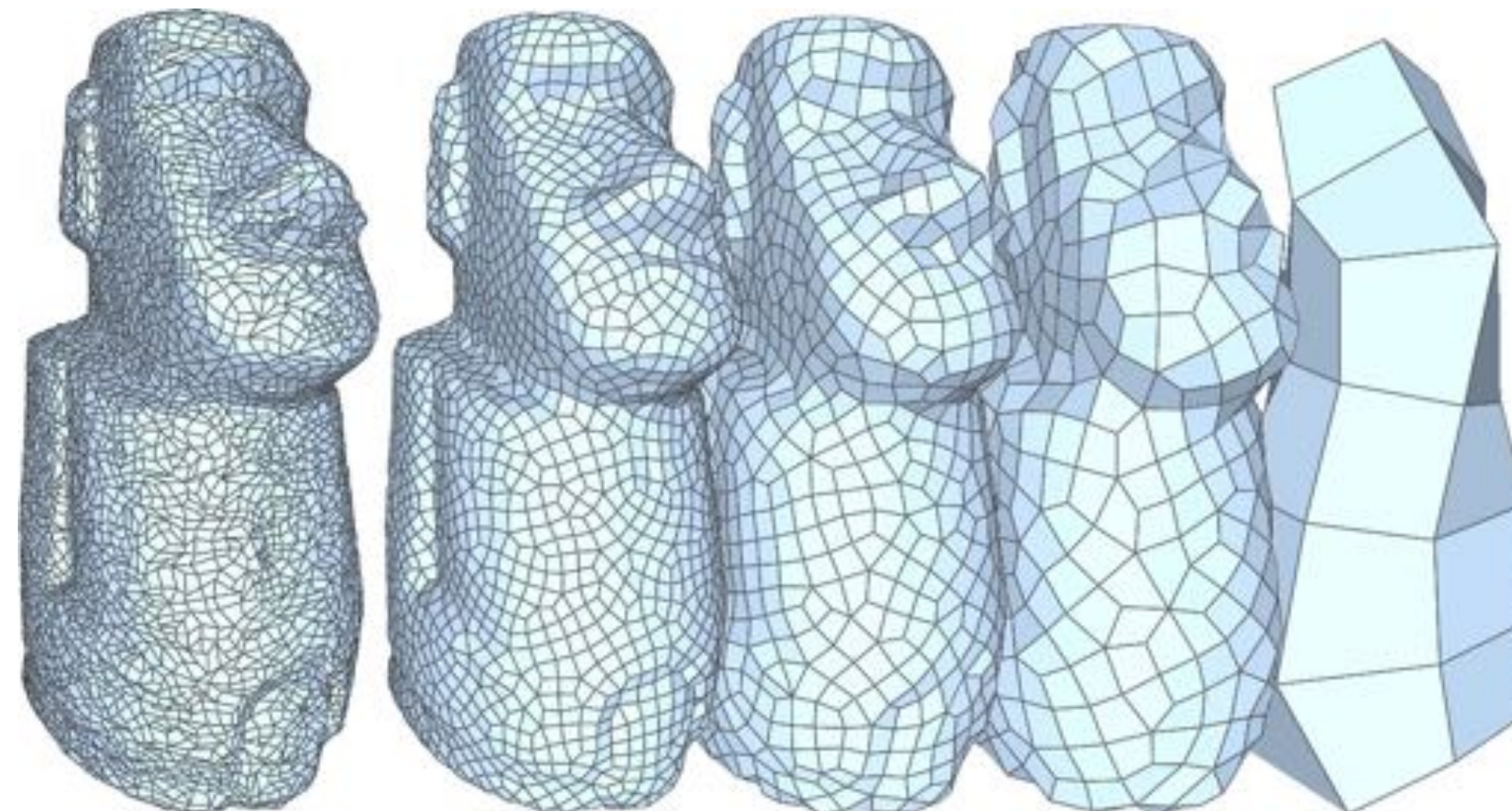
- Highly application-dependent



Mesh smoothing/de-noising



Re-meshing/quadrangulation



Simplification,  
compression



# **§2. 3D representations in vision and graphics [Friday Tutorial]**



# References

1. Botsch, M., Kobbelt, L., Pauly, M., Alliez, P., & Lévy, B. (2010). *Polygon mesh processing*. CRC press.

