

Deep Neural Networks

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- 1 Introduction
- 2 First generation of Neural Networks
- 3 Modern Neural Networks
- 4 Convolutional Neural Networks
- 5 Regularizing Neural Networks

1 Introduction

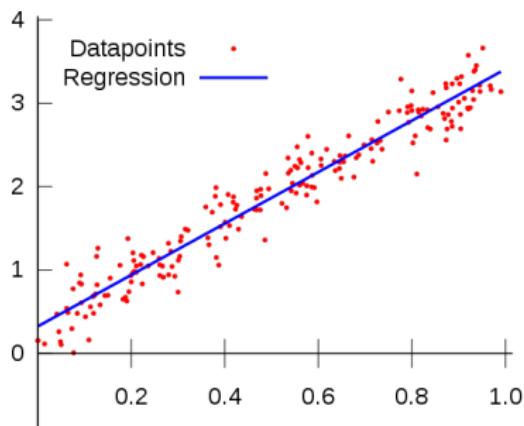
2 First generation of Neural Networks

3 Modern Neural Networks

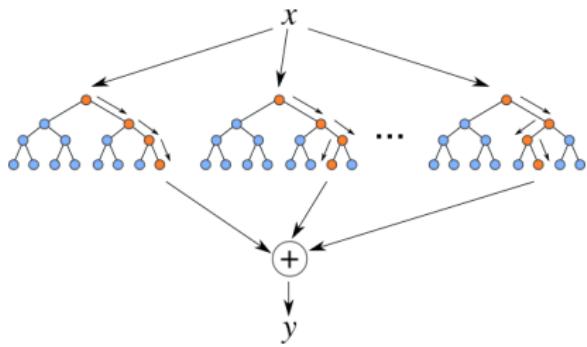
4 Convolutional Neural Networks

5 Regularizing Neural Networks

Linear models



Decision trees and ensembles

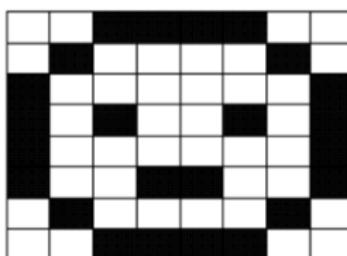


- U is a set of objects
- $X(U)$ is a set of features
- Y is a set of labels (classes, categories, etc.)
- For a new X we should define a corresponding class Y

U



$X(U)$



Y

Person
Bird
House



Please click on all the images that show cats:

The grid contains 12 images arranged in three rows of four. The images are as follows:
Row 1: A black and white cat sitting on a wire mesh, a brown cat lying on a patterned blanket, a dog standing next to a person, and an orange cat sitting on a wooden surface.
Row 2: A white and brown dog standing, a brown dog sitting on grass, a person petting a black cat, and a brown cat sitting in a cage.
Row 3: A white goat standing, a yellow cat sitting, a brown and white dog, and a black and white dog standing.

- 2006: accuracy 60%, $0.6^{12} = 0.00217$



Completed • Swag

Dogs vs. Cats

Wed 25 Sep 2013 – Sat 1 Feb 2014 (2 years ago)

Dashboard

Private Leaderboard - Dogs vs. Cats

This competition has completed. This leaderboard reflects the final standings.

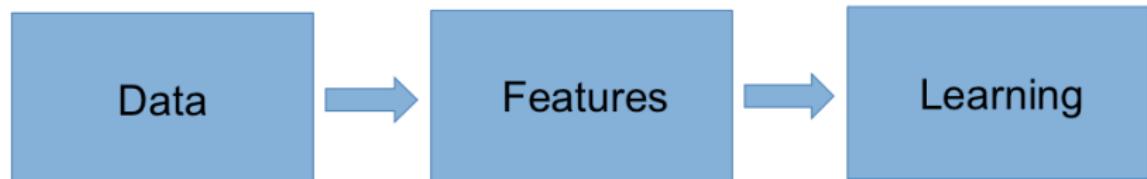
See someone using multiple accounts?

[Let us know.](#)

#	Rank	Team Name *in the money	Score ⓘ	Entries	Last Submission UTC (Best – Last Submission)
1	—	Pierre Sermanet *	0.98914	5	Sat, 01 Feb 2014 21:43:19 (-1.2h)
2	14	orchid *	0.98309	17	Sat, 01 Feb 2014 23:52:30
3	—	Owen	0.98171	15	Sat, 01 Feb 2014 17:04:40 (-1.3h)
4	—	Paul Covington	0.98171	3	Sat, 01 Feb 2014 23:05:20
5	13	Maxim Milakov	0.98137	24	Sat, 01 Feb 2014 18:20:58
6	11	we've been in KAIST 🇰🇷	0.98103	8	Sat, 01 Feb 2014 21:15:30

- $0.989^{12} = 0.875$

Old-school approach



Image



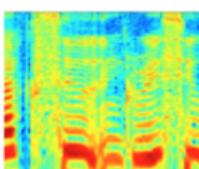
Features



Recognition



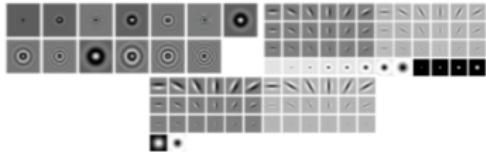
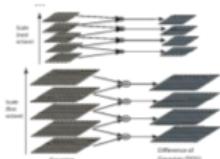
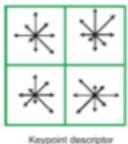
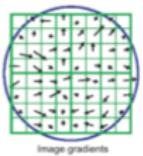
Sound



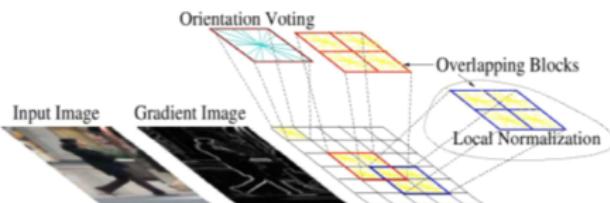
Features



Recognition

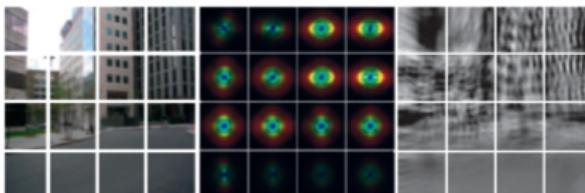


SIFT

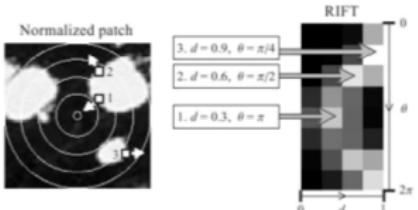


HoG

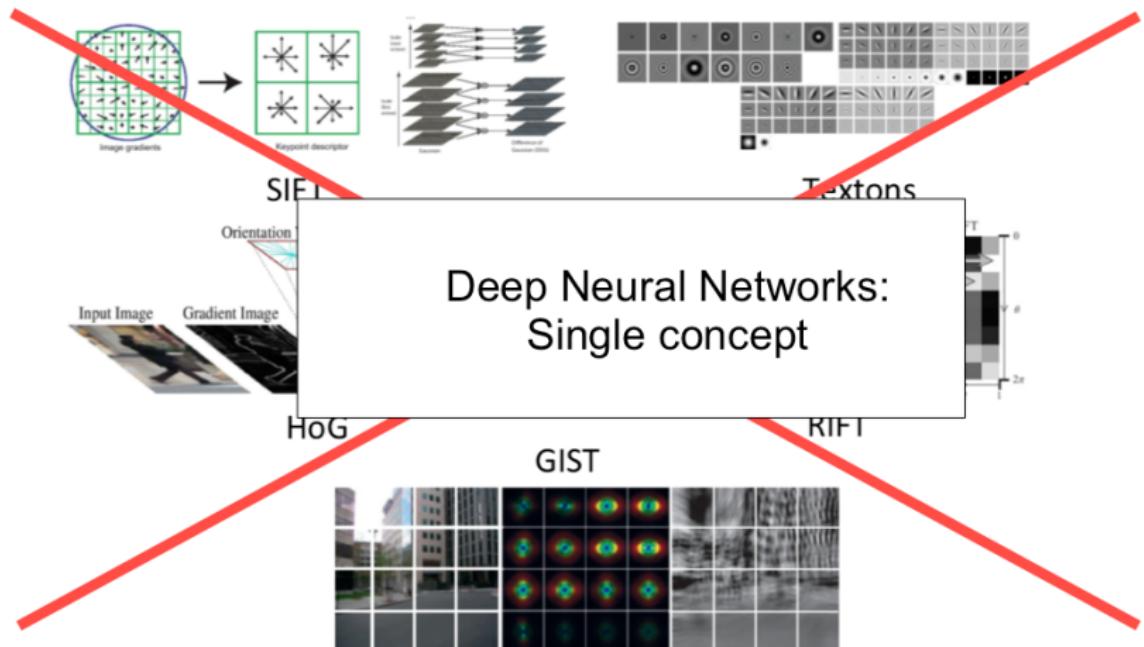
GIST



Textons

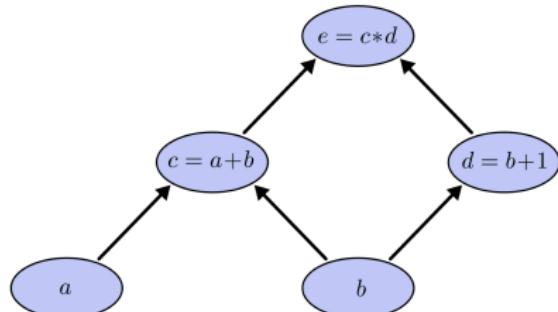


RIFT

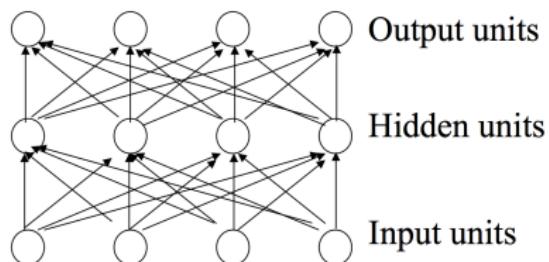


- Mathematically a neural network is a combination of simple transformations (linear/non-linear)
- Depth of an architecture
- Types of layers (convolutional, full-connected, etc.)
- Neural Networks are made up of **nodes** or **units**, connected by **links**, i.e. it is a kind of a computational graph
- Each link has an associated **weight** and activation level
- Each node has an **input function** (typically summing over weighted inputs), an **activation function**, and an **output**

Computational graph

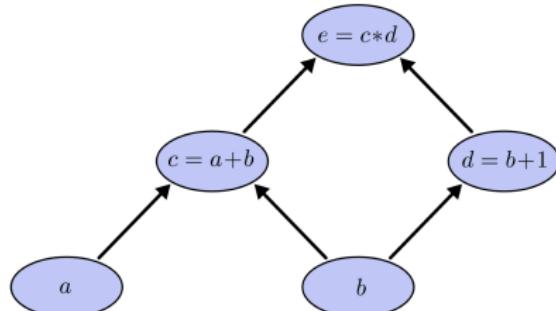


Fully-connected layered feed-forward network

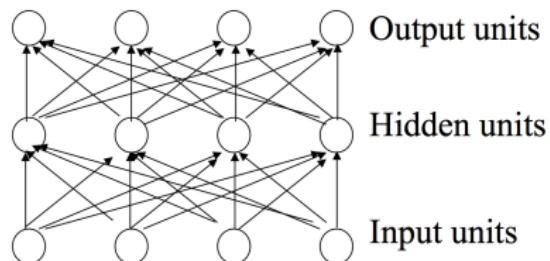


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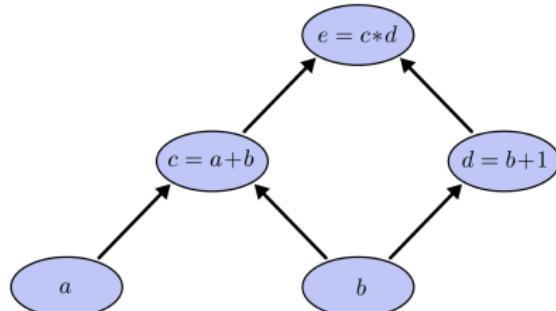


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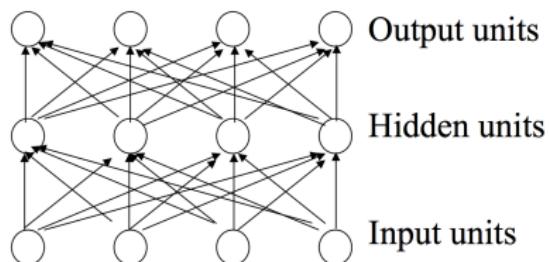


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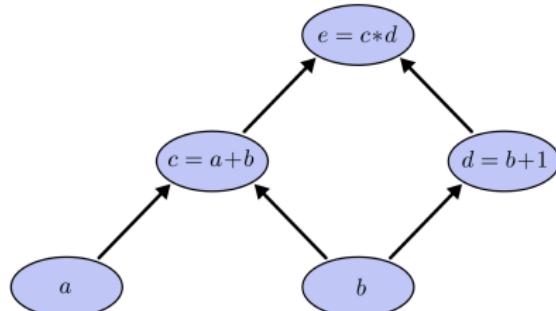


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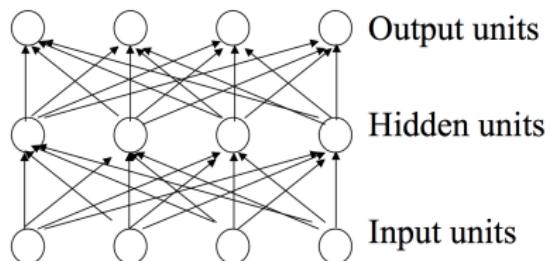


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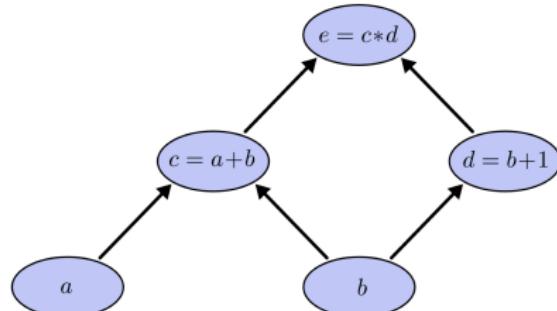


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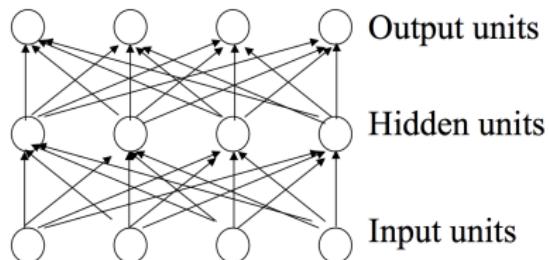


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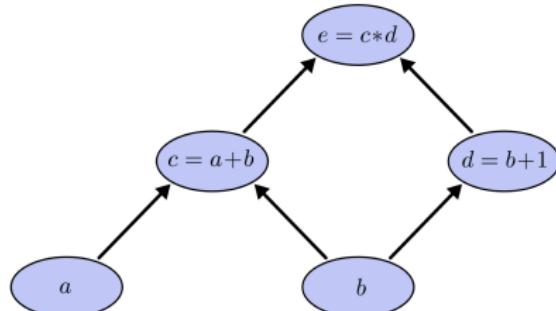


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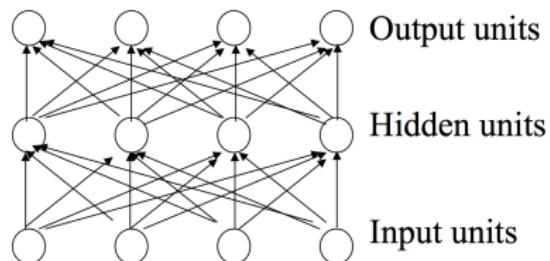


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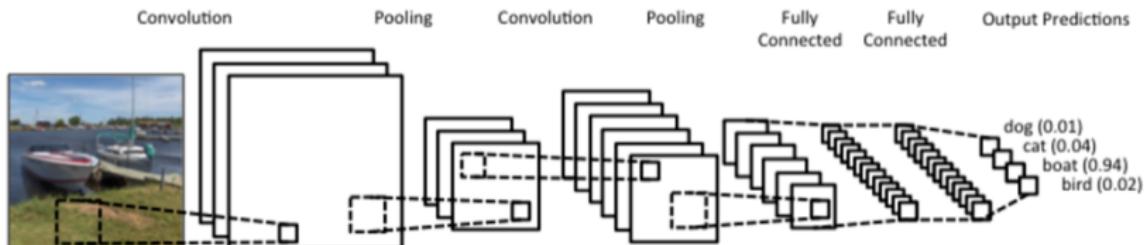
Computational graph



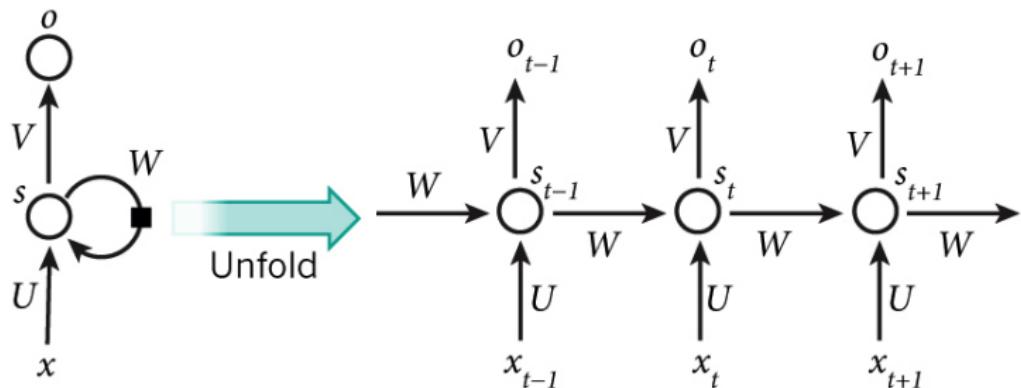
Fully-connected layered feed-forward network



Convolutional network



Recurrent neural network



Images,
video



Text



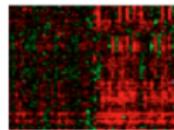
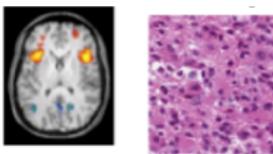
Social networks



Recommendation
systems



Genomics, biology



Example of application

- Image captioning

This bird has a yellow belly and tarsus, grey back, wings, and brown throat, nape with a black face



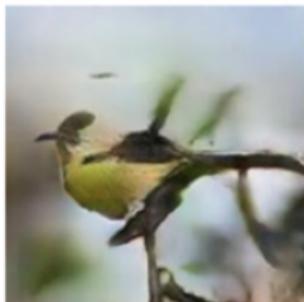
This bird is white with some black on its head and wings, and has a long orange beak



This flower has overlapping pink pointed petals surrounding a ring of short yellow filaments



(a) Stage-I images

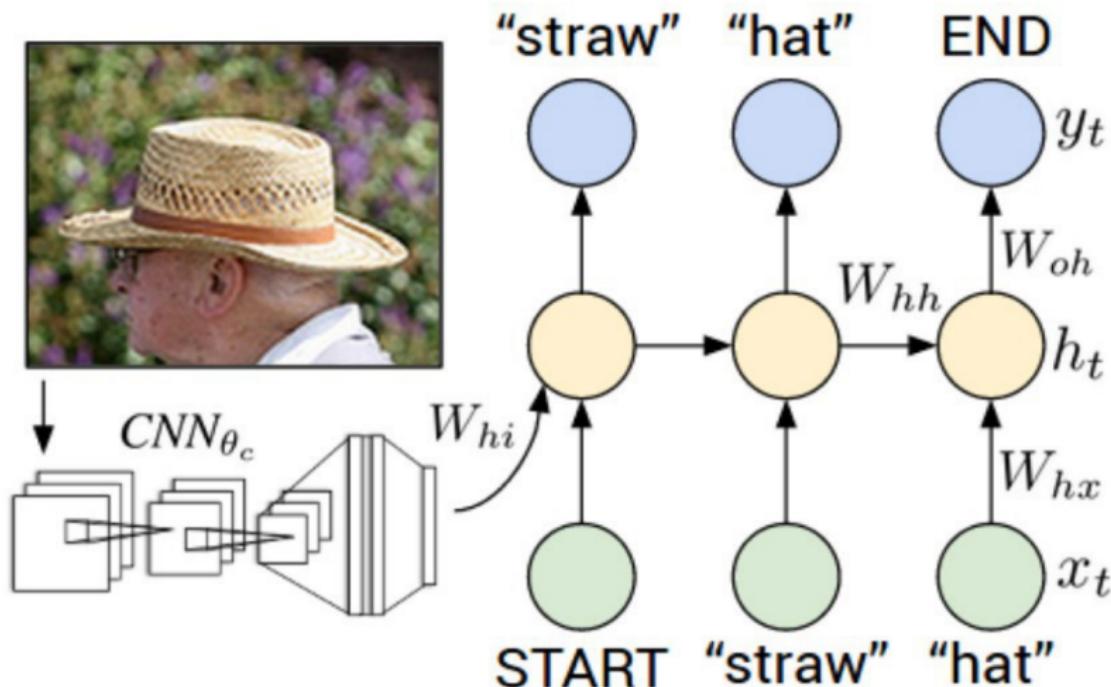


(b) Stage-II images



Example of application

- Image captioning



- Princeton, Stanford, Google
- More than 10 millions of images, manually labeled using Amazon Mechanical Turk



- E.g. Tensorflow is an open framework for deep neural networks (and not only!)
- More than 15 Google engineers develop it full-time
- ~ 700 thousands of code, ~ 600 participants of external open-source development
- Example: playground.tensorflow.org

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- Face recognition and identification from camera images
- Several-fold advantage from using Deep Learning (requirement to the number of “bad” loans — not more than 1 for 10 millions)
- Integration of the system in top Russian banks

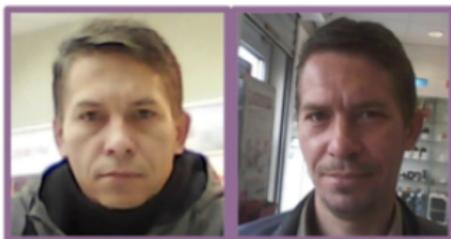
Are persons different? (from Khanin's presentation)



1



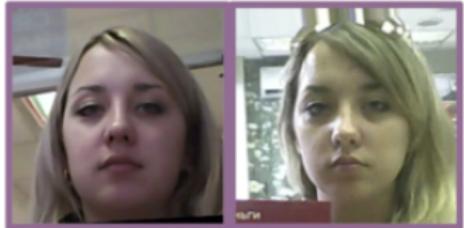
2



3



4



5



6



- www.vocord.ru
- Recognition of car number plates
- Record accuracy on large datasets



- Accelerates the search of new drugs
- Already discovered a new drug for multiple sclerosis and anti-virus tool aimed at slowing the spread of Ebola haemorrhagic fever

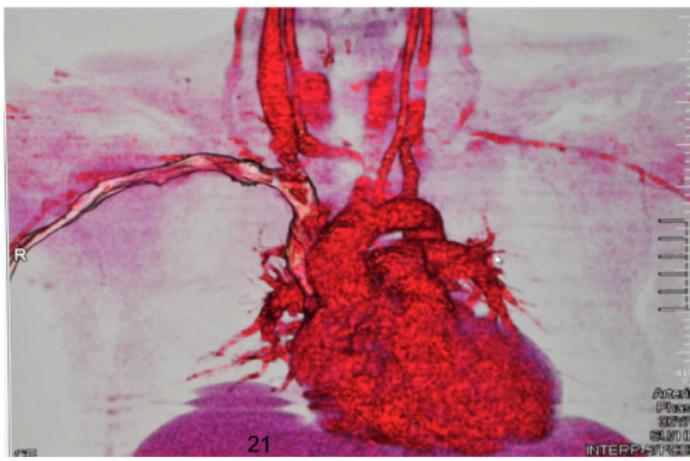
- Search by photos in social networks

The screenshot shows the FindFace search interface. At the top, there is a navigation bar with links for 'ПОИСК', 'ТУР ПО СЕРВИСУ', 'СИМПАТИИ', and 'ИСТОРИЯ ПОИСКА'. Below the navigation bar, there is a search bar with placeholder text 'Остались 29 поиска' and a button 'Как получить больше?'. To the right of the search bar are buttons for 'Показывать меня на 1-ом месте', 'Сортировка', and 'Результаты'. Further right are links for 'Получить премиум-аккаунт на месяц бесплатно' and 'Выйти из аккаунта'. The main search results area displays a large profile picture of a man with a beard and the text 'Найдено: 95 человек' and a 'Новый поиск' button. Below this are search filters: 'Пол: Мужской', 'Возраст: от 14 лет до 55+', 'Город: из города', and 'Статус: Все'. A checked checkbox says 'Отобразить всех пользователей'. Below these filters, a message reads 'Скорее всего в числе этих пользователей есть искомый человек' followed by 'Найдено: 12'. A grid of 12 user profiles is shown in two rows of six. Each profile includes a small photo, a name, an age, and a location. The names visible are Фома, Павел, Анатолий, Алексей, Илья, Денис, Николай, Никита, Нео, Дмитрий, Сергей, and Анатолий. Each profile has a 'Лайк' button.

- Styling images



- Startap Arterys, AI based on DL in the cloud
- The first FDA approval for the diagnosis of heart disease
- Accurate measurements of the volume of each ventricle allowing more precise assessment of health
- Precise quantification of blood flow through the heart Arterys takes an average of 15 seconds to produce a result for one case, which a professional human analyst would expect to spend between 30 minutes to an hour working on



Historical Background



Warren McCulloch



Walter Pitts

The first formal
neuron - 1943



Frank Rosenblatt

The first artificial
neural network - 1958

- What did cause the breakthrough?

The reasons for the breakthrough in the combination of factors!

- The increase in power and availability of hardware (GPU)
- Big volumes of open, labeled data
- Available frameworks: Tensorflow (Google), Torch (Facebook, Google), Theano (open-source), Caffe (Berkley), Veles (Samsung)
- The openness of research and of source code, realizing the most widely used frameworks
- Significant investments from big IT companies (Google, Facebook, Amazon, IBM, etc.)

As a result, the point “focusing” of research on a “limited” topic (> 50 research groups, > 1000 researchers)

1 Introduction

2 First generation of Neural Networks

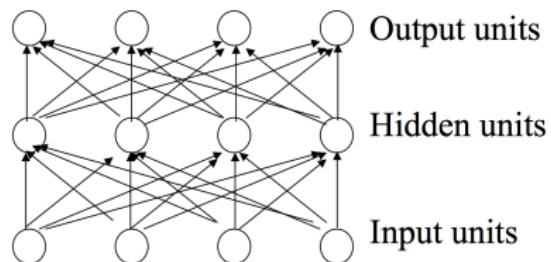
3 Modern Neural Networks

4 Convolutional Neural Networks

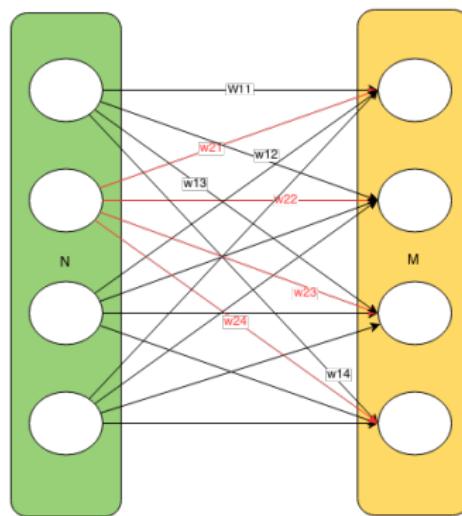
5 Regularizing Neural Networks

- Initially the most popular networks were fully-connected layered feed-forward networks

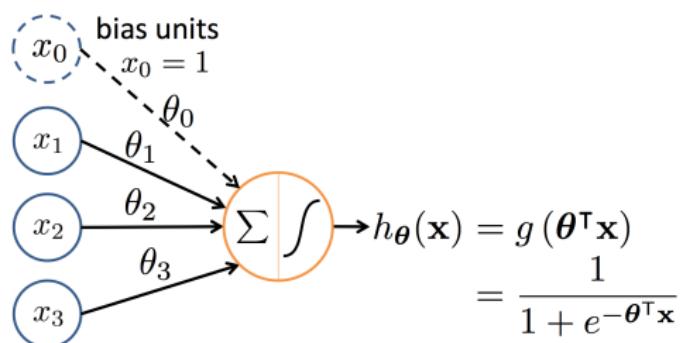
Fully-connected layered feed-forward network



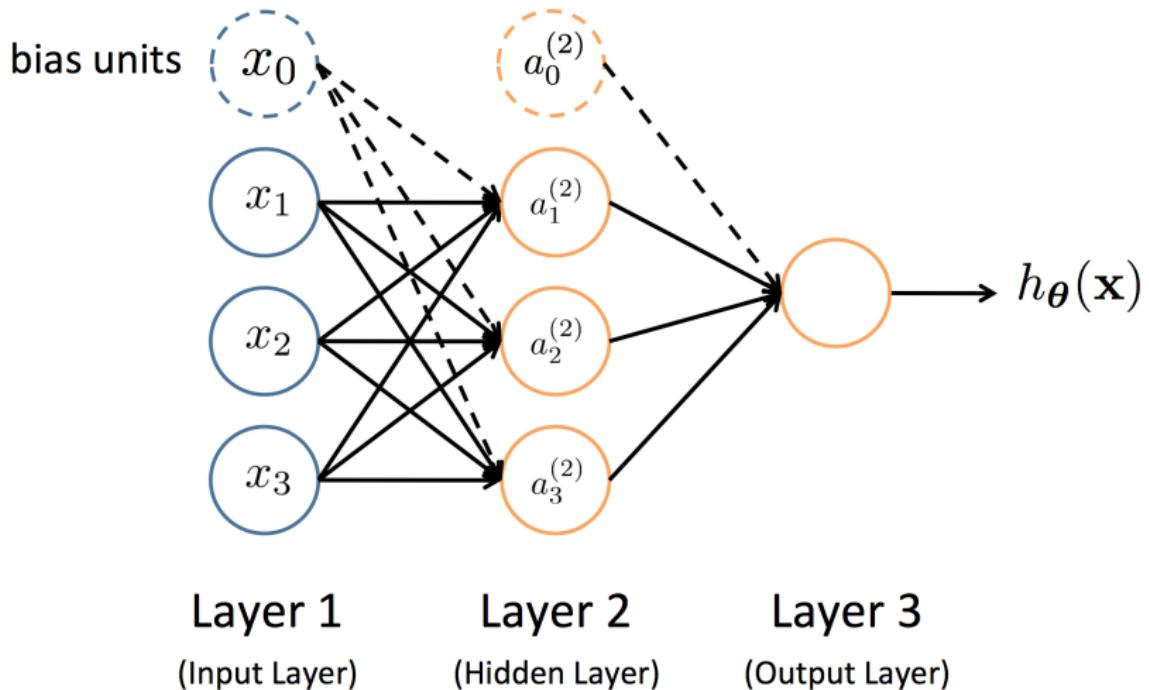
Fully connected layer



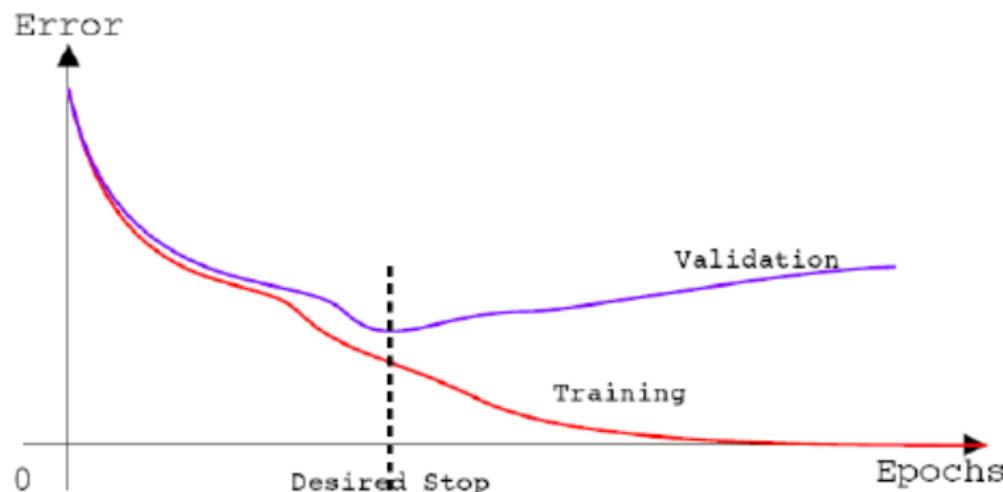
- Let us define $\mathbf{x} = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $\boldsymbol{\theta} = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$



- Sigmoid (logistic) activation function $g(z) = \frac{1}{1+e^{-z}}$



- Backpropagation for training: gradient descent + efficient computations (based on graph-like representation)
- Local minima
- Random initialization
- Random division into train and validation sets
- Second order optimization methods
- Early stopping



1 Introduction

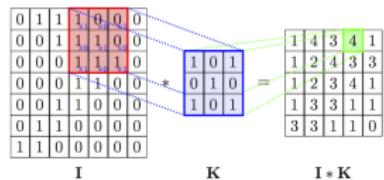
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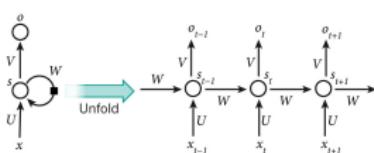
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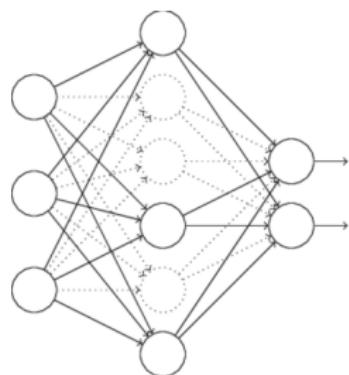
Convolutional layer



Recurrent layer



Dropout layer



- State-of-the-art quality for many computer vision and NLP problems
- Sometimes even superhuman performance!
- Key components are:
 - Computational power and GPUs
 - Lots of data (or transfer learning)
 - Stochastic optimization
 - Regularization techniques

Let $\phi(\cdot)$ be a nonconstant, bounded, and monotonically-increasing continuous function. Let I_m denote the m -dimensional unit hypercube $[0, 1]^m$. The space of continuous function on I_m is denoted by $C(I_m)$. Then, given any $\varepsilon > 0$ and any function $f \in C(I_m)$, there exist an integer p , real constants $v_i, b_i \in \mathbb{R}$ and real vectors $\mathbf{w}_i \in \mathbb{R}^m$, where $i = 1, \dots, p$, such that we may define:

$$F(\mathbf{x}) = \sum_{i=1}^p v_i \phi(\mathbf{w}_i^T \mathbf{x} + b_i)$$

as an approximate realization of the function f , where f is independent of ϕ ; that is,

$$|F(\mathbf{x}) - f(\mathbf{x})| < \varepsilon$$

for all $\mathbf{x} \in I_m$. In other words, functions of the form $F(\mathbf{x})$ are dense in $C(I_m)$.

- Deep nets can be trained and regularized effectively
- Deep nets can achieve better performance with same number of parameters
- Deep architectures are more intuitive and can be interpreted as a sequence of simple data transformations

- $f_j(\mathbf{x}, \mathbf{w}_j)$ is a j -th layer of the network
- $v_j(\mathbf{x}) = f_j(v_{j-1}(\mathbf{x}), \mathbf{w}_j)$
- $a(\mathbf{x}, \mathbf{w}) = v_n(\mathbf{x})$ is an output
- Optimization problem

$$Q(\mathbf{w}) = \sum_{i=1}^l L(y_i, a(\mathbf{x}_i, \mathbf{w})) \rightarrow \min_{\mathbf{w}}$$

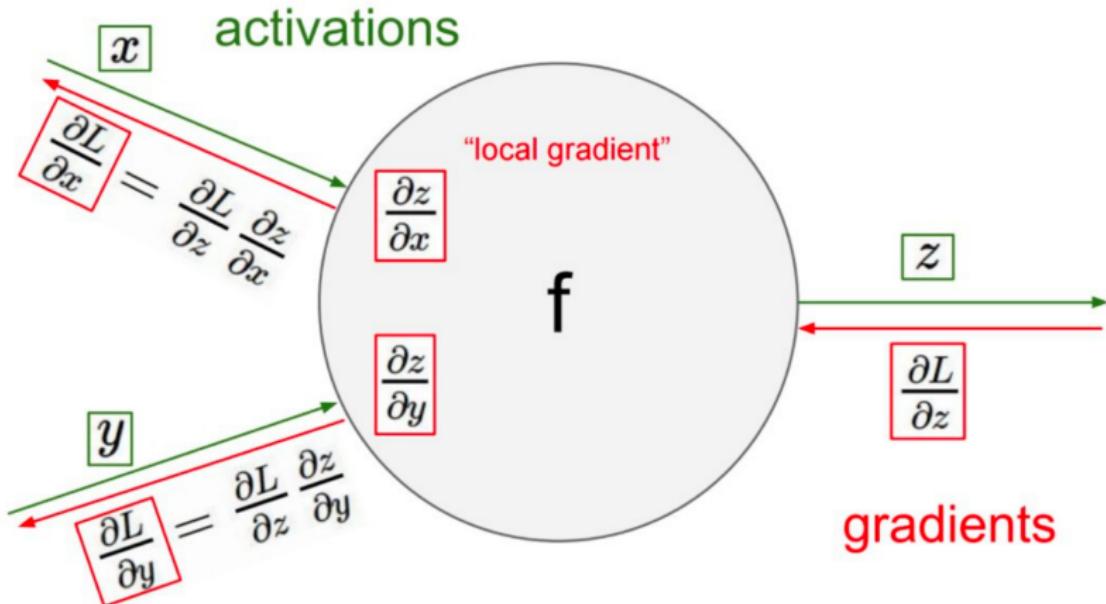
$$\frac{\partial Q}{\partial w_i} = \frac{\partial Q}{\partial f_{i+1}} \frac{\partial f_{i+1}}{\partial f_i} \frac{\partial f_i}{\partial w_i}$$

Comes from previous steps

Easy for differentiable layers

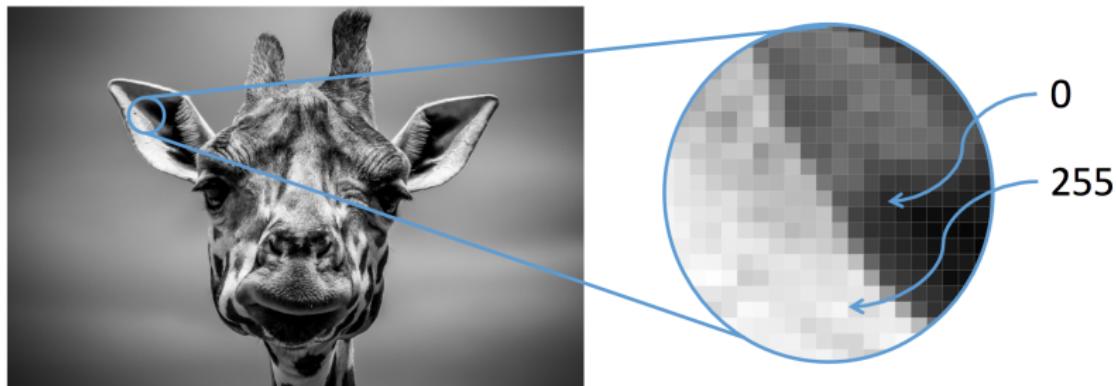
$$\frac{\partial Q}{\partial f_{ij}} = \sum_k \frac{\partial Q}{\partial f_{i+1,k}} \frac{\partial f_{i+1,k}}{\partial f_{ij}}$$

$$\frac{\partial Q}{\partial w_{ij}} = \sum_k \frac{\partial Q}{\partial f_{ik}} \frac{\partial f_{ik}}{\partial w_{ij}}$$

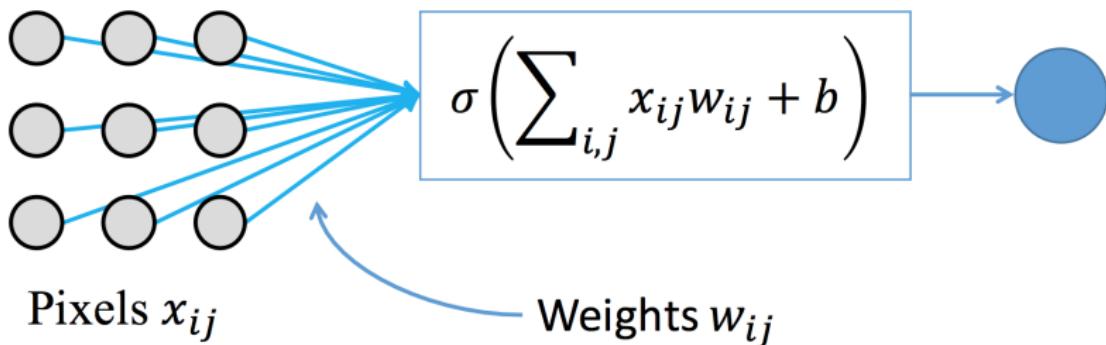


4 Convolutional Neural Networks

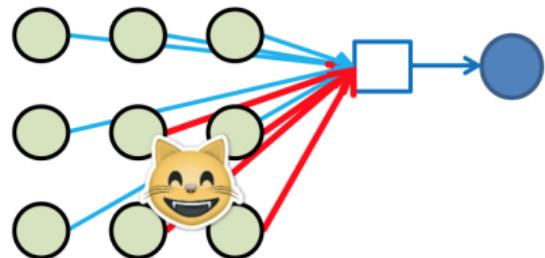
- Each pixel stores its brightness (or **intensity**) ranging from 0 to 255, 0 intensity corresponds to black color
- Color images store pixel intensities for 3 channels: **red**, **green** and **blue**



- Normalize input pixels: $\mathbf{x}_{\text{norm}} = \frac{\mathbf{x}}{255} - 0.5$
- Maybe MLP will work?

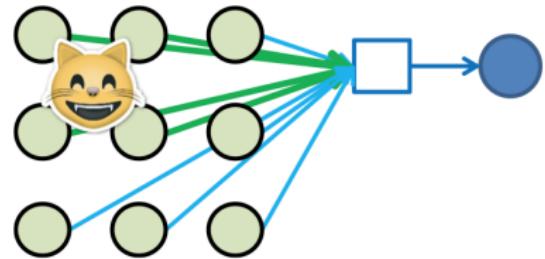


Fully-connected layered feed-forward network



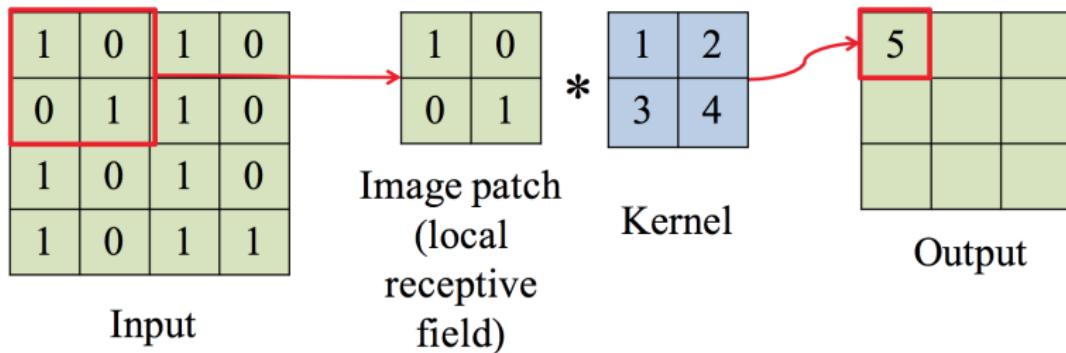
On this training image **red** weights
 w_{ij} will change a little bit to better
detect a cat

Fully-connected layered feed-forward network

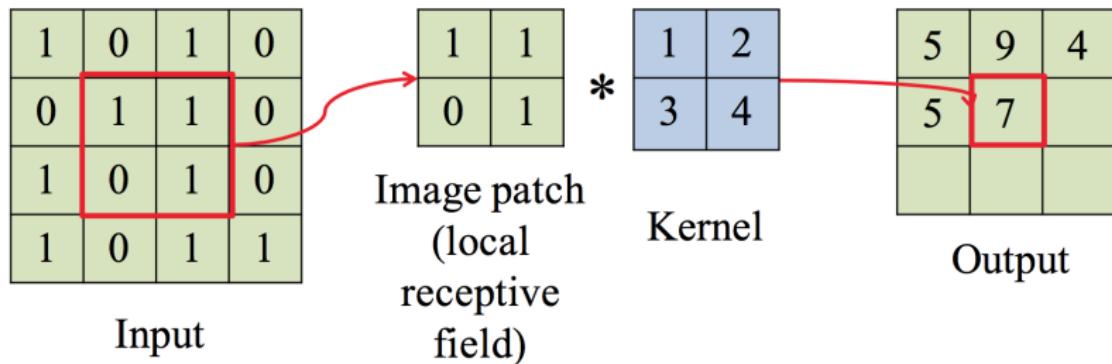


On this training image **green** weights
 w_{ij} will change a little bit to better
detect a cat

- Convolution is a dot product of a **kernel** (or filter) and a patch of an image (**local receptive field**) of the same size



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Original image

Kernel

$$\begin{matrix} -1 & -1 & -1 \\ * & \begin{matrix} -1 & 8 & -1 \\ -1 & -1 & -1 \end{matrix} & = \end{matrix}$$


Edge detection

$$\begin{matrix} * & \begin{matrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{matrix} & = \end{matrix}$$


Sharpening

$$\begin{matrix} * \frac{1}{9} & \begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix} & = \end{matrix}$$


Blurring

Convolution is similar to correlation

0	0	0	0
0	0	0	0
0	0	1	0
0	0	0	1

Input

$$\begin{matrix} & \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \end{matrix} * \begin{matrix} & = \end{matrix} \begin{matrix} & \begin{matrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{matrix} \end{matrix}$$

Kernel Output

Max = 2

0	0	0	0
0	0	0	0
0	0	0	1
0	0	1	0

Input

$$\begin{matrix} & \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \end{matrix} * \begin{matrix} & = \end{matrix} \begin{matrix} & \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{matrix} \end{matrix}$$

Kernel Output

Max = 1

Simple classifier

Convolution is translation invariant

0	0	0	0
0	0	0	0
0	0	1	0
0	0	0	1

Input

*

1	0
0	1

=

0	0	0
0	1	0
0	0	2

Output

Max = 2

1	0	0	0
0	1	0	0
0	0	0	0
0	0	0	0

Input

*

1	0
0	1

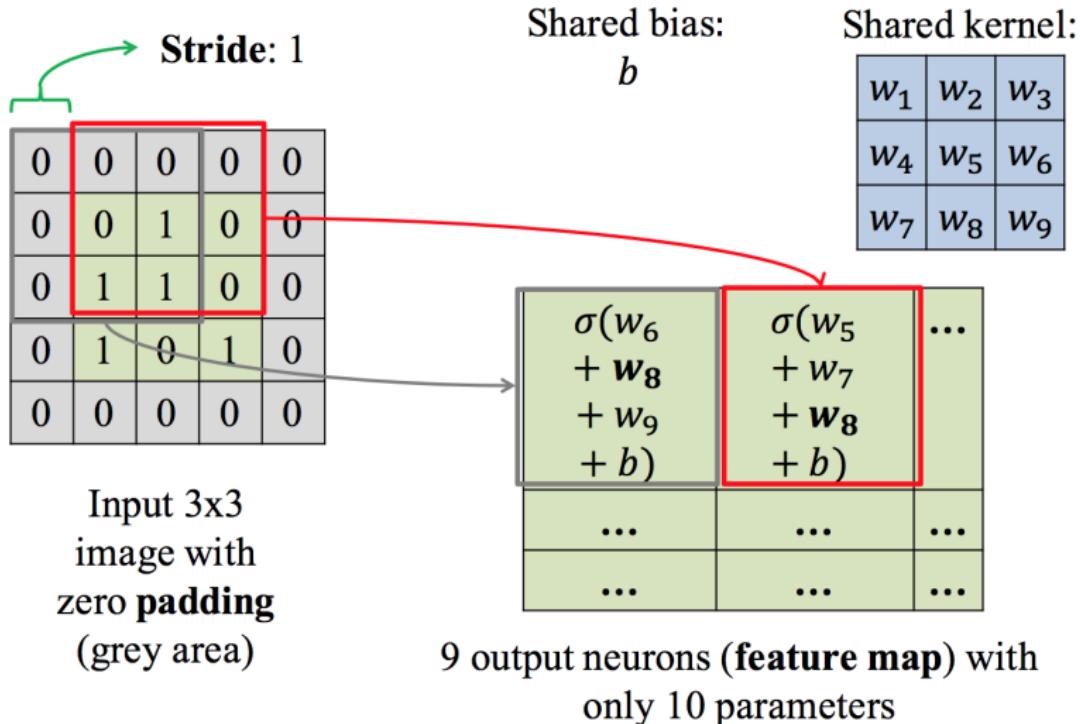
=

2	0	0
0	1	0
0	0	0

Output

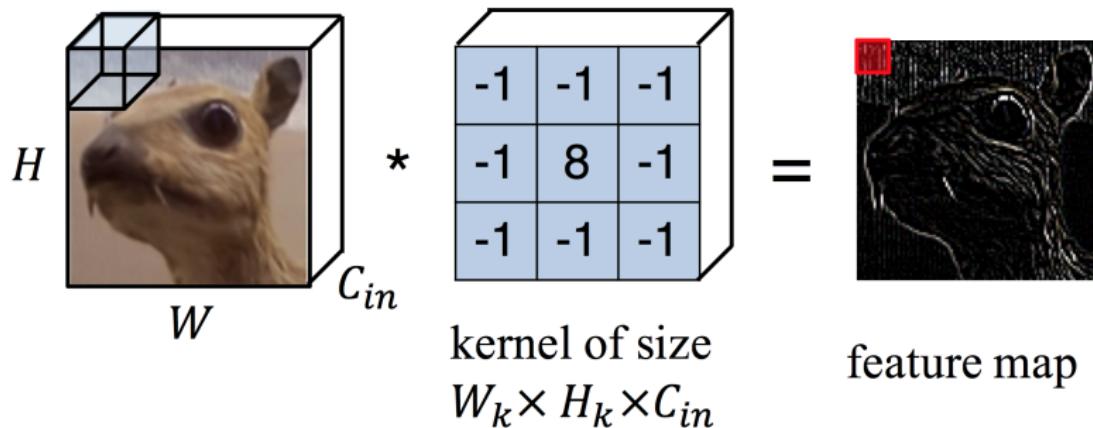
Max = 2

Didn't
change



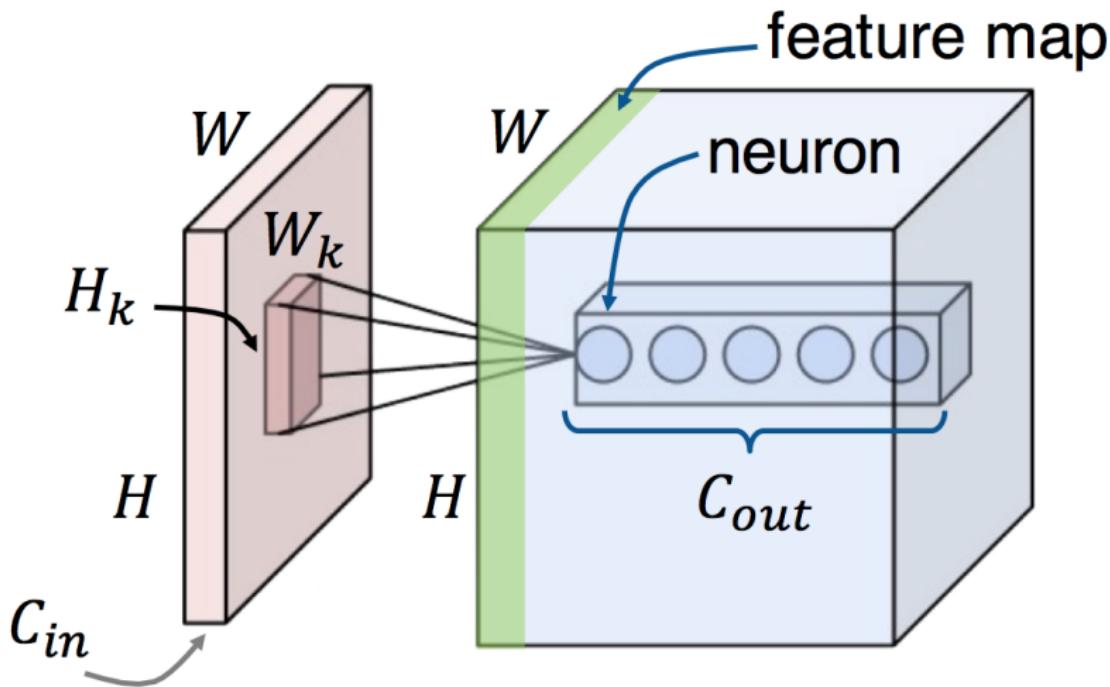
Let's say we have a color image as an input, which is $W \times H \times C_{in}$ **tensor** (multidimensional array), where

- W is an image width
- H is an image height
- C_{in} is a number of input channels (e.g. 3 **RGB** channels)



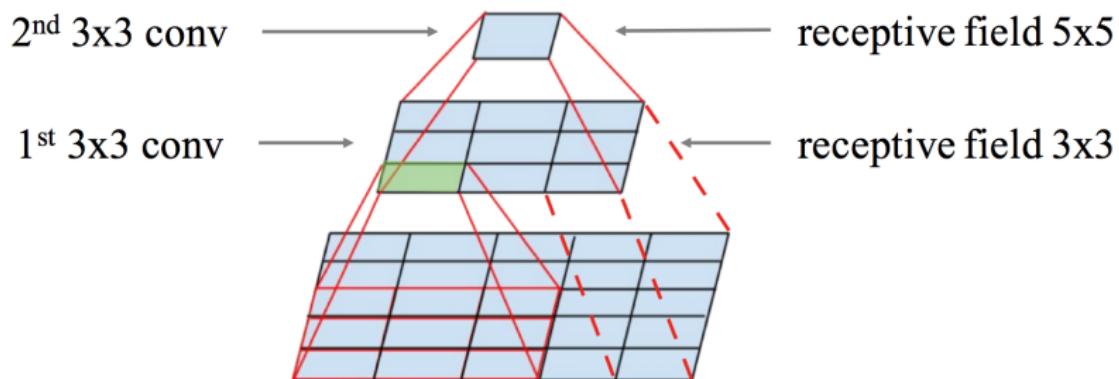
Convolutional layer

- We want to train C_{out} kernel of size $W_k \times H_k \times C_{in}$
- Having a stride of 1 and enough zero padding we can have $W \times H \times C_{out}$ output neurons



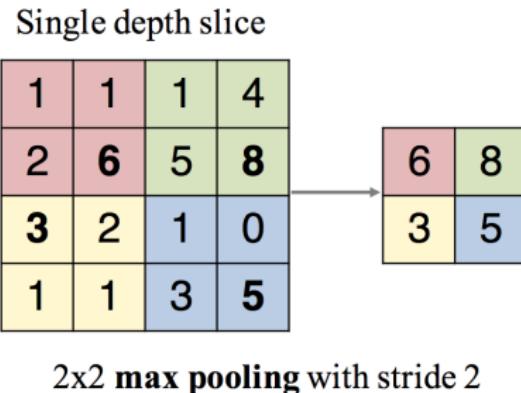
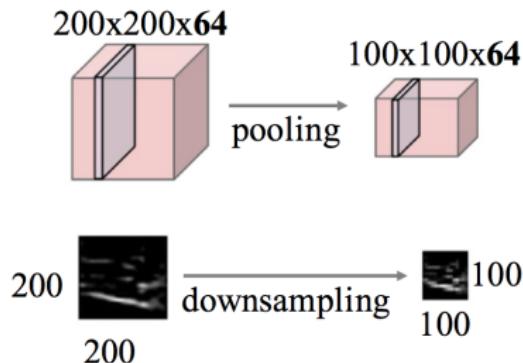
One convolutional layer is not enough!

- Let's say neurons of the 1-st convolutional layer look at the patches of the image of size 3×3
- What if an object of interest is bigger than that?
- We need a 2-nd convolutional layer on top of the 1-st!



Pooling layer

- This layer works like a convolutional layer but doesn't have kernel, instead it calculates **maximum** or **average** of input patch values



- Strictly speaking: maximum is not a differentiable function!

6	8
3	5

Maximum = 8

7	8
3	5

Maximum = 8

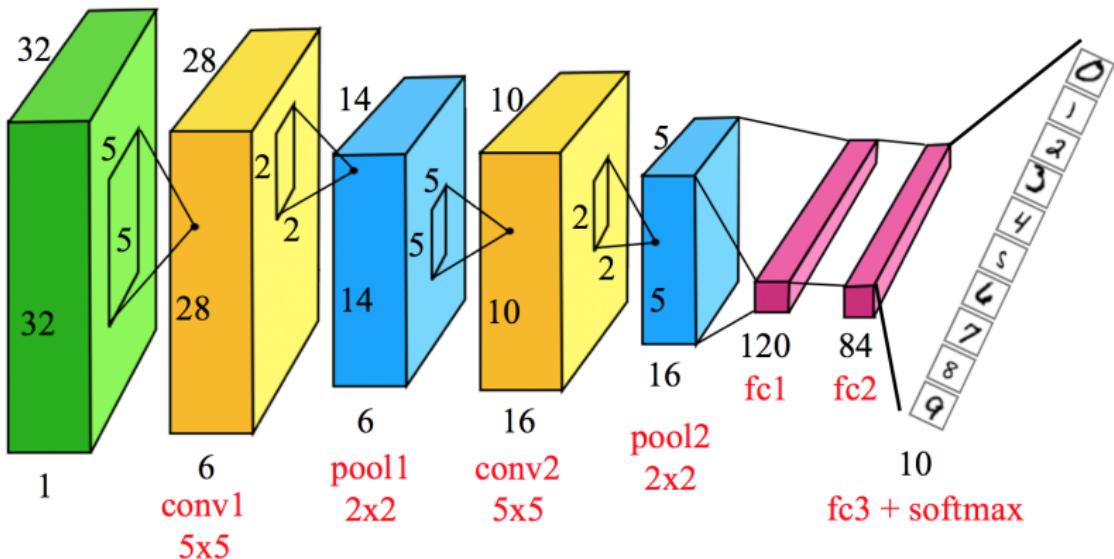
6	8
3	5

Maximum = 8

7	9
3	5

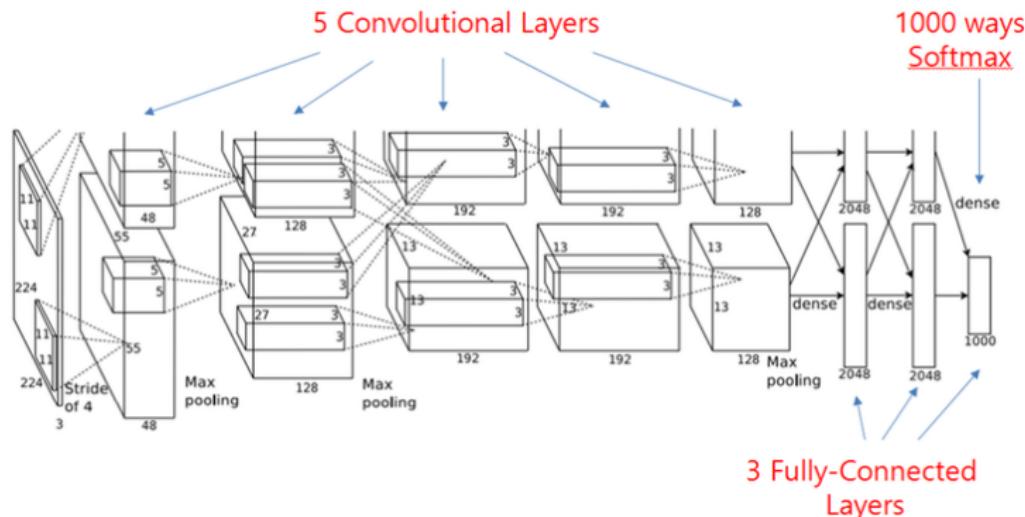
Maximum = 9

Convolutional network



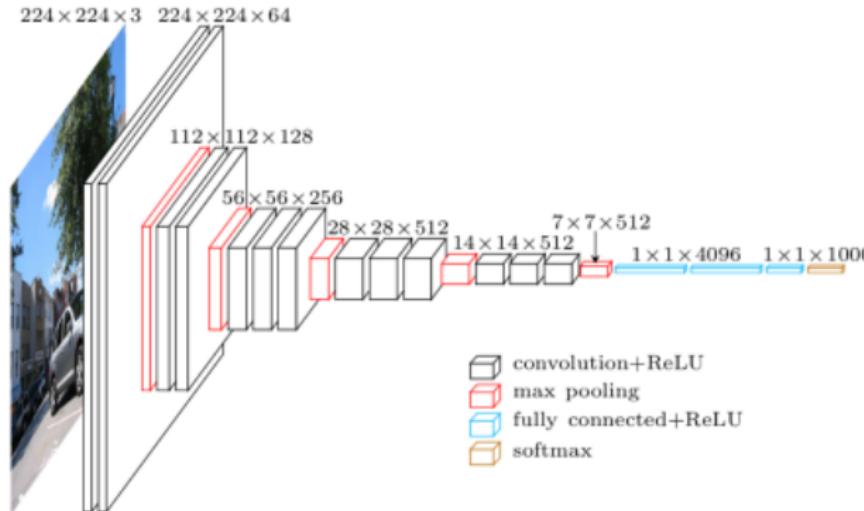
<http://yann.lecun.com/exdb/publis/pdf/lecun-98.pdf>

- First deep convolutional neural net for ImageNet
- Significantly reduced top 5 error from 26% to 15%



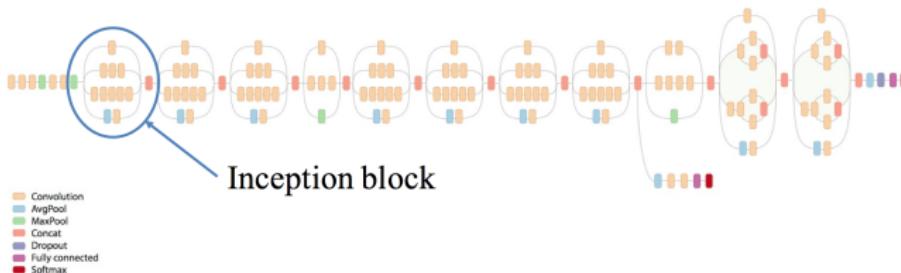
- 11 × 11, 5 × 5, 3 × 3 convolutions, max pooling, dropout, data augmentation, ReLU activations, SGD with momentum
- 60 million parameters
- Trains on 2 GPUs for 6 days

- Similar to AlexNet, only 3×3 convolutions, but lots of filters!
- ImageNet top 5 error: 8.0% (single model)



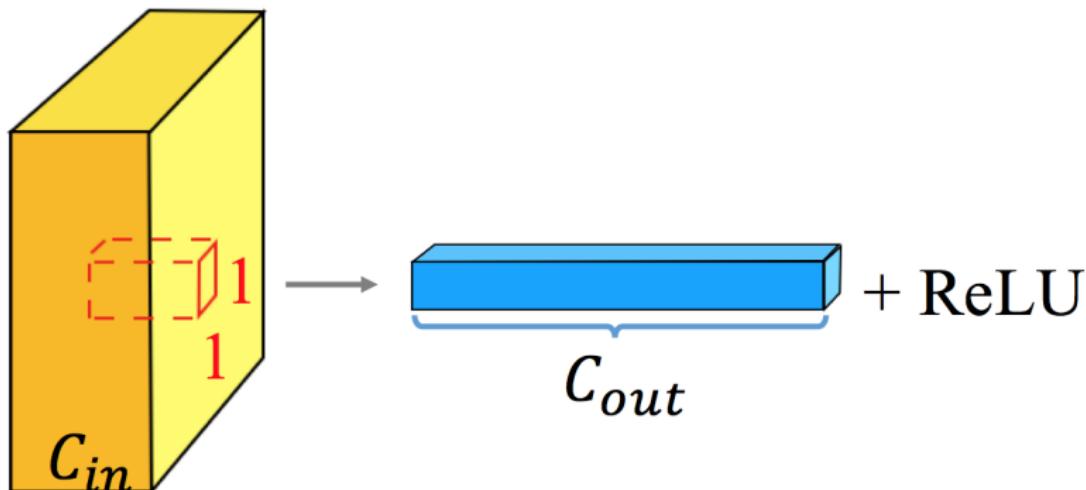
- Training similar to AlexNet with additional multi-scale cropping
- 138 million parameters
- Trains on 4 GPUs for 2 – 3 weeks

- Similar to AlexNet? Not quite, uses Inception block introduced in GoogLeNet (a.k.a. Inception V1)
- ImageNet top 5 error: 5.6% (single model), 3.6% (ensemble)

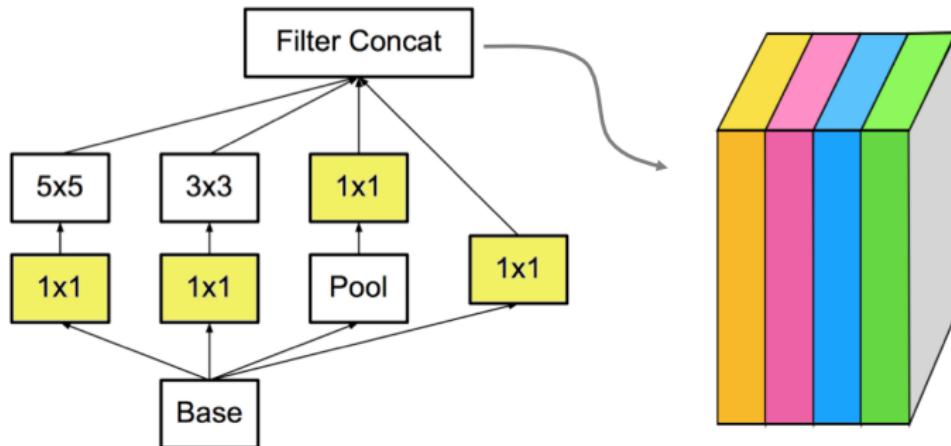


- Batch normalization, image distortions, RMSProp
- 25 million parameters!
- Trains on 8 GPUs for 2 weeks

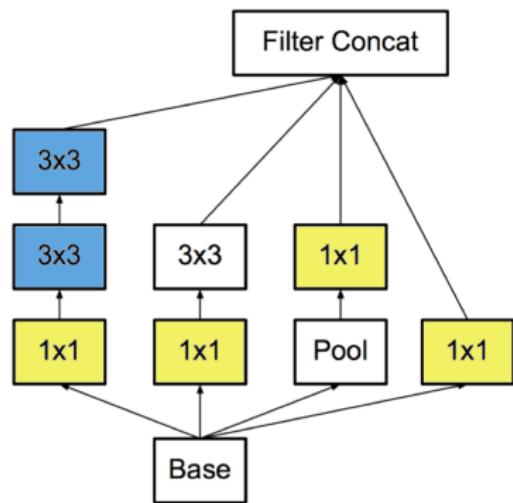
- Such convolutions capture interactions of input channels in one “pixel” of feature map
- They can reduce the number of channels not hurting the quality of the model, because different channels can correlate
- Dimensionality reduction with added ReLU activation



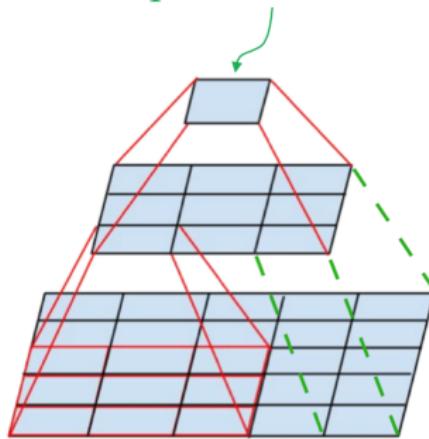
- All operations inside a block use stride 1 and enough padding to output the same spatial dimensions ($W \times H$) of feature map.
- 4 different feature maps are concatenated on depth at the end



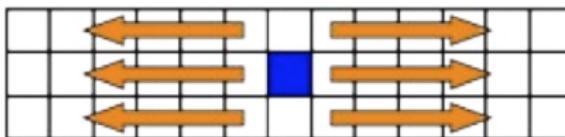
Christian Szegedy, <https://arxiv.org/pdf/1512.00567.pdf>



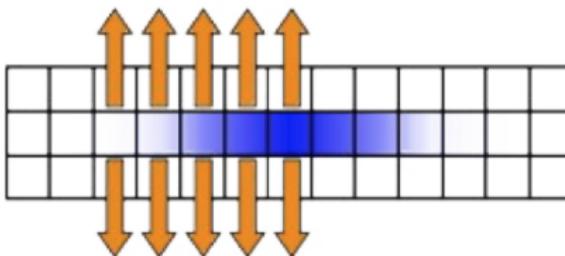
Receptive field 5×5



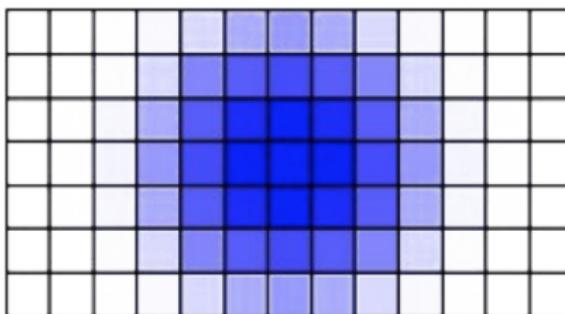
Christian Szegedy, <https://arxiv.org/pdf/1512.00567.pdf>



Blur the source horizontally

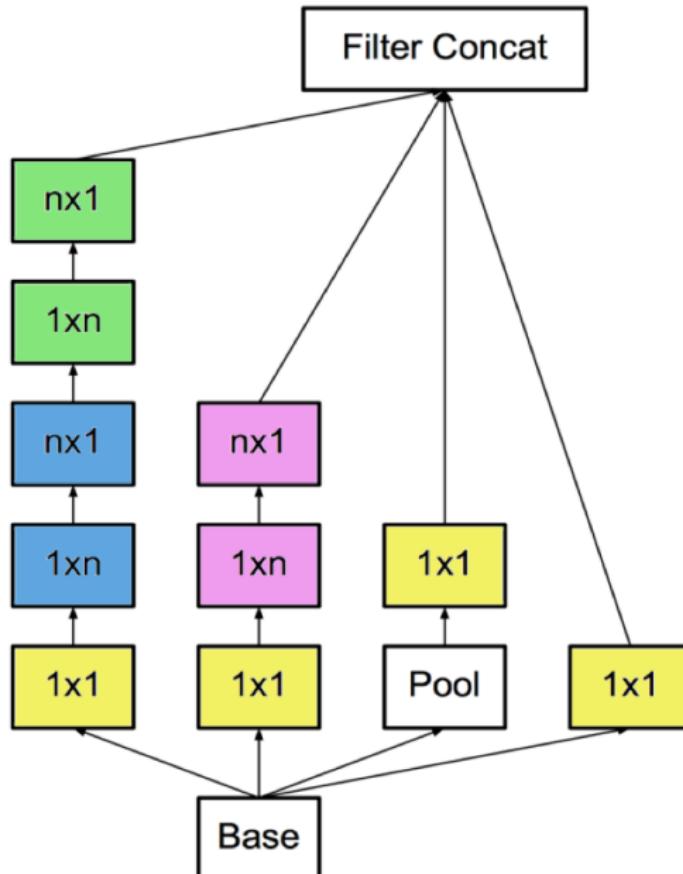


Blur the blur vertically

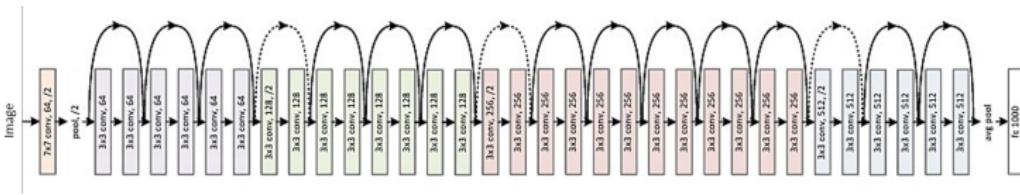


Result

Inception block

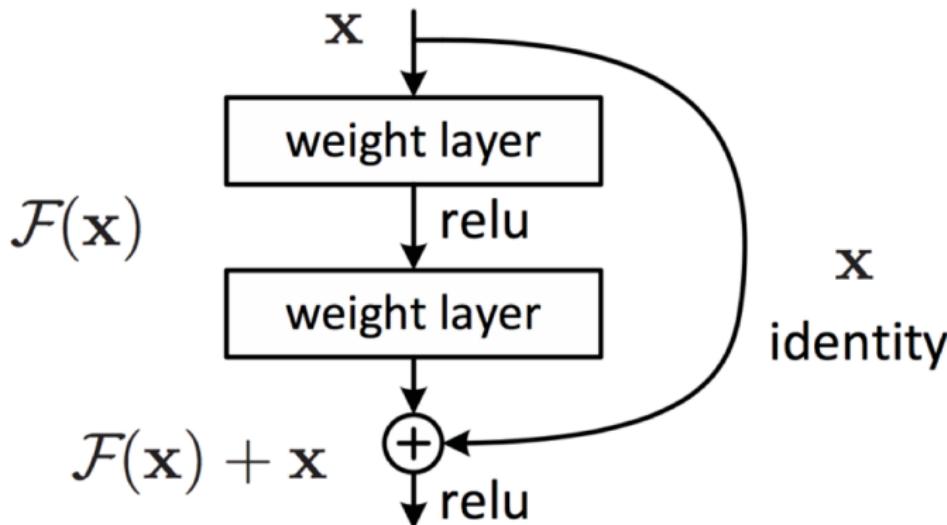


- Introduces residual connections
 - ImageNet top 5 error: 4.5% (single model), 3.5% (ensemble)



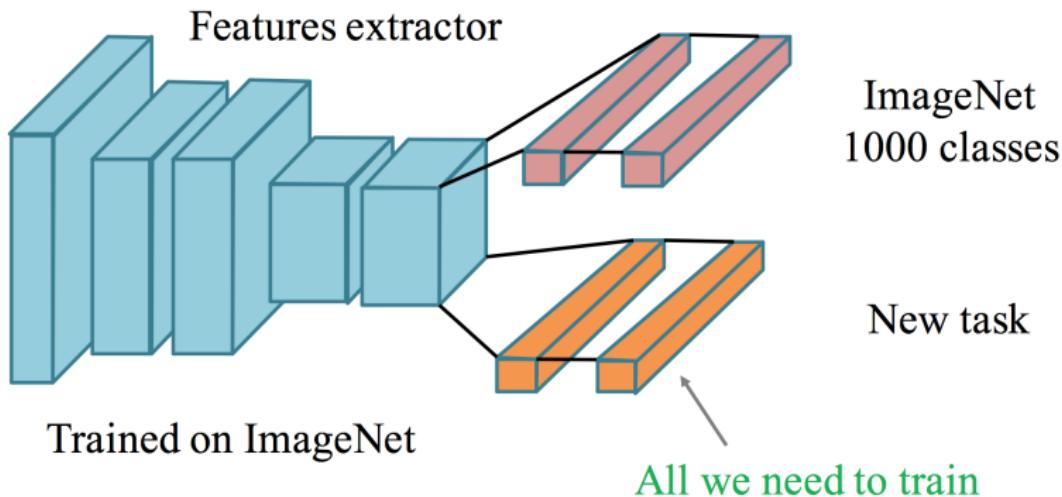
- 152 layers, few 7×7 convolutional layers, the rest are 3×3 , batch normalization, max and average pooling
 - 60 million parameters
 - Trains on 8 GPUs for 2 – 3 weeks

- We create output channels adding a small delta $F(x)$ to original input channels x



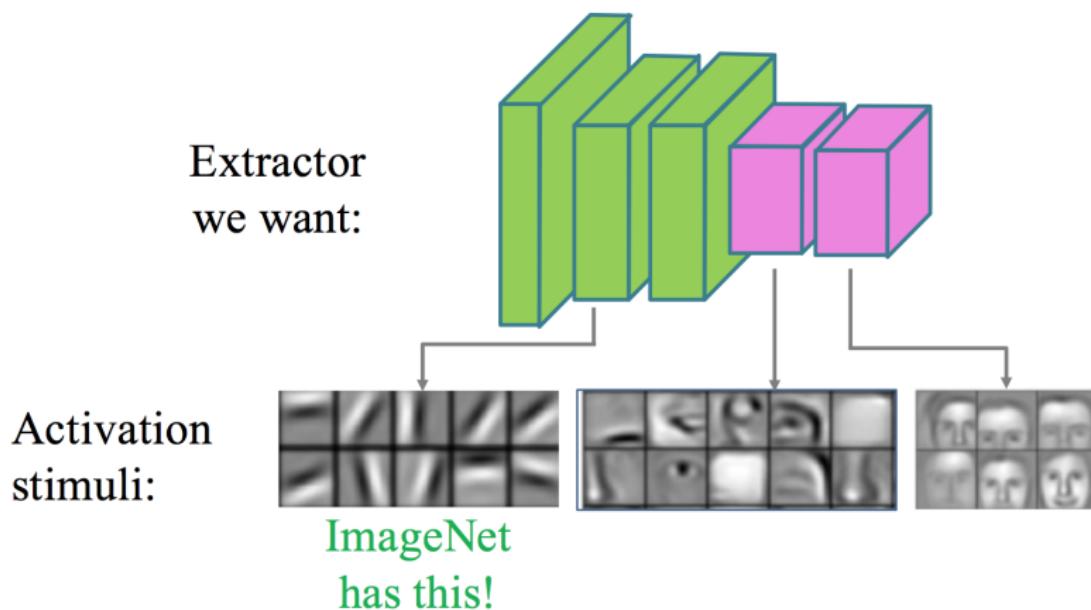
- This way we can stack thousands of layers and gradients do not vanish thanks to residual connections

- Deep networks learn complex features extractor, but we need lots of data to train it from scratch!
- What if we can reuse an existing features extractor for a new task?



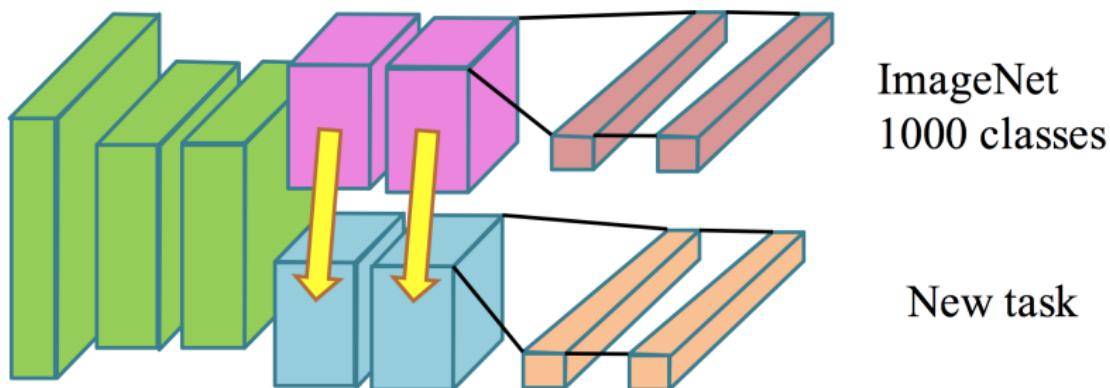
- You need less data to train (for training only final MLP)
- It works if a domain of a new task is similar to ImageNet's
- E.g. won't work for human emotions classification, ImageNet doesn't have people faces in the dataset!

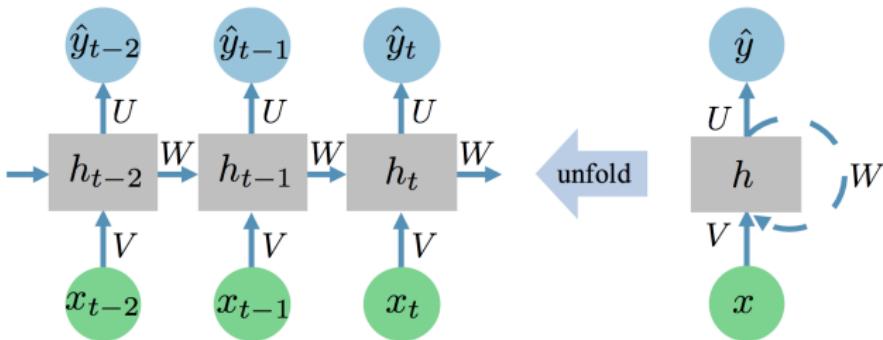
- But what if we need to classify human emotions?
- Maybe we can partially reuse ImageNet features extractor?



Honglak Lee, <http://web.eecs.umich.edu/~honglak/icml09-ConvolutionalDeepBeliefNetworks.pdf>

- You can initialize deeper layers with values from ImageNet
- This is called **fine-tuning**, because you don't start with a random initialization
- Propagate all gradients with smaller learning rate





- \mathbf{x} is an input
- \hat{y} is an output (prediction)
- h is a hidden state

- $h_t = f_h(V\mathbf{x}_t + Wh_{t-1} + b_h)$
- $\hat{y}_t = f_y(Uh_t + b_y)$

- Simple differentiable computational blocks can be combined into graphs and still efficiently trained
- Convolutional architectures achieve superhuman performance in some vision problems
- Graphs trained on large datasets can be reused for other tasks
- Recurrent neural networks are quite efficient for sequential data

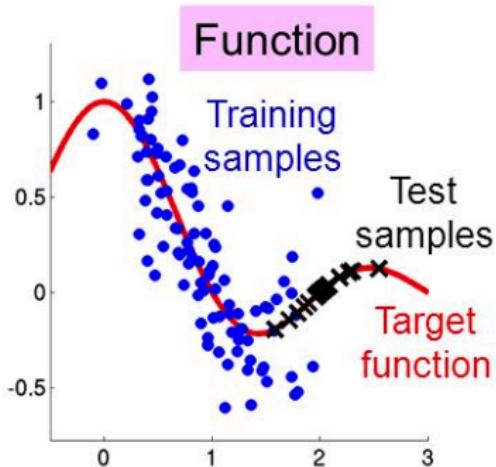
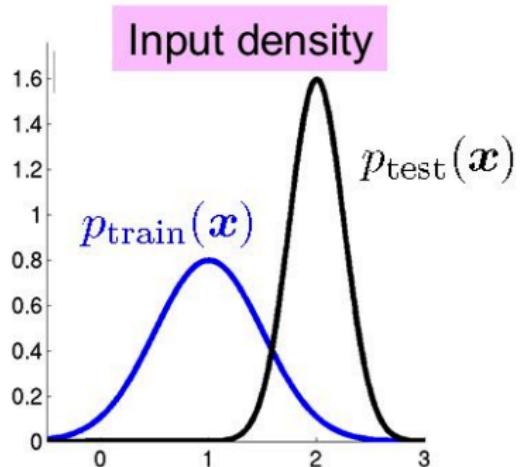
1 Introduction

2 First generation of Neural Networks

3 Modern Neural Networks

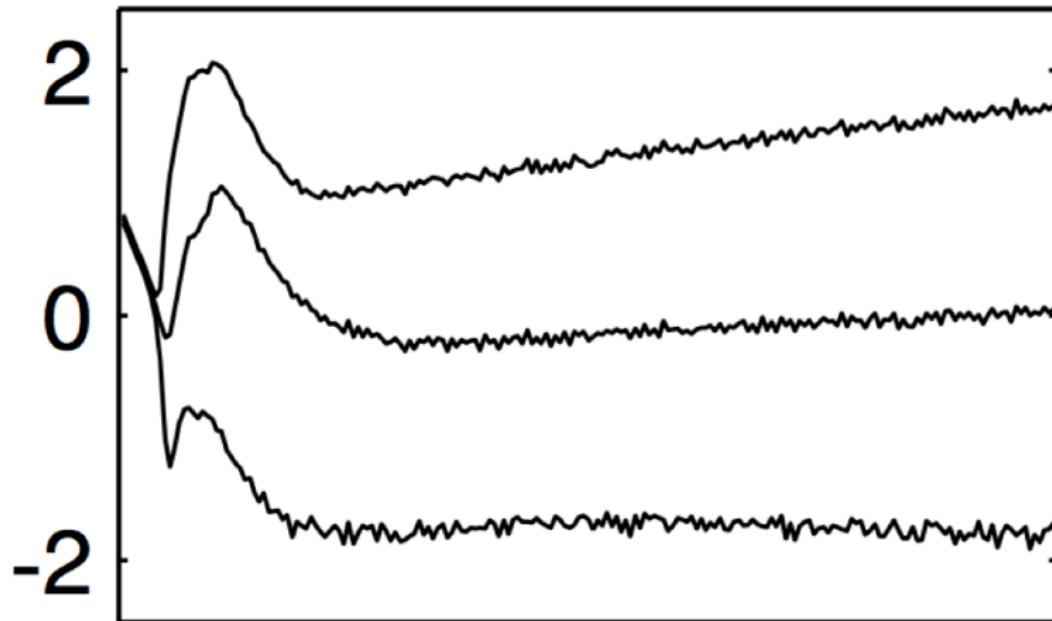
4 Convolutional Neural Networks

5 Regularizing Neural Networks



Input distribution in neural network

- Input distribution changes over the course of training
- Example: 3 fully connected layers, 100 activations each
- Strong internal covariate shift



- First idea: standardize output of a layer
- $f(\mathbf{x}; b) = \mathbf{x} + b - \mathbb{E}(\mathbf{x} + b)$
- If the backprop ignores dependence of sample average on b , then after step $b + g$:

$$f(\mathbf{x}; b) = \mathbf{x} + b + g - \mathbb{E}(\mathbf{x} + b + g) = \mathbf{x} + b - \mathbb{E}(\mathbf{x} + b)$$

- Parameter b can grow indefinitely
- Gradients should take normalization into account!

- Normalizations layer:

$$\hat{\mathbf{x}} = \text{Norm}(\mathbf{x}, \mathcal{X})$$

- We should be able to calculate gradients:

$$\frac{\partial \text{Norm}(\mathbf{x}, \mathcal{X})}{\partial \mathbf{x}} \quad \text{and} \quad \frac{\partial \text{Norm}(\mathbf{x}, \mathcal{X})}{\partial \mathcal{X}}$$

- Standardization requires inverse square root of covariance matrix:

$$\{\text{Cov}(\mathbf{x})\}^{-1/2} (\mathbf{x} - \mathbb{E}\mathbf{x})$$

- Normalize each layer activation independently:

$$\hat{x}^{(k)} = \frac{x^{(k)} - \mathbb{E}[x^{(k)}]}{\sqrt{\text{Var}[x^{(k)}]}}$$

- Estimate mean and variance of input based on current mini-batch

Input: Values of x over a mini-batch: $\mathcal{B} = \{x_1 \dots m\}$;

Parameters to be learned: γ, β

Output: $\{y_i = \text{BN}_{\gamma, \beta}(x_i)\}$

$$\mu_{\mathcal{B}} \leftarrow \frac{1}{m} \sum_{i=1}^m x_i \quad // \text{mini-batch mean}$$

$$\sigma_{\mathcal{B}}^2 \leftarrow \frac{1}{m} \sum_{i=1}^m (x_i - \mu_{\mathcal{B}})^2 \quad // \text{mini-batch variance}$$

$$\hat{x}_i \leftarrow \frac{x_i - \mu_{\mathcal{B}}}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}} \quad // \text{normalize}$$

$$y_i \leftarrow \gamma \hat{x}_i + \beta \equiv \text{BN}_{\gamma, \beta}(x_i) \quad // \text{scale and shift}$$

$$\frac{\partial \ell}{\partial \hat{x}_i} = \frac{\partial \ell}{\partial y_i} \cdot \gamma$$

$$\frac{\partial \ell}{\partial \sigma_B^2} = \sum_{i=1}^m \frac{\partial \ell}{\partial \hat{x}_i} \cdot (x_i - \mu_B) \cdot \frac{-1}{2} (\sigma_B^2 + \epsilon)^{-3/2}$$

$$\frac{\partial \ell}{\partial \mu_B} = \left(\sum_{i=1}^m \frac{\partial \ell}{\partial \hat{x}_i} \cdot \frac{-1}{\sqrt{\sigma_B^2 + \epsilon}} \right) + \frac{\partial \ell}{\partial \sigma_B^2} \cdot \frac{\sum_{i=1}^m -2(x_i - \mu_B)}{m}$$

$$\frac{\partial \ell}{\partial x_i} = \frac{\partial \ell}{\partial \hat{x}_i} \cdot \frac{1}{\sqrt{\sigma_B^2 + \epsilon}} + \frac{\partial \ell}{\partial \sigma_B^2} \cdot \frac{2(x_i - \mu_B)}{m} + \frac{\partial \ell}{\partial \mu_B} \cdot \frac{1}{m}$$

$$\frac{\partial \ell}{\partial \gamma} = \sum_{i=1}^m \frac{\partial \ell}{\partial y_i} \cdot \hat{x}_i$$

$$\frac{\partial \ell}{\partial \beta} = \sum_{i=1}^m \frac{\partial \ell}{\partial y_i}$$

- Average mean and variance estimates over all training batches:

for $k = 1 \dots K$ **do**

// For clarity, $x \equiv x^{(k)}$, $\gamma \equiv \gamma^{(k)}$, $\mu_{\mathcal{B}} \equiv \mu_{\mathcal{B}}^{(k)}$, etc.

Process multiple training mini-batches \mathcal{B} , each of size m , and average over them:

$$\mathbb{E}[x] \leftarrow \mathbb{E}_{\mathcal{B}}[\mu_{\mathcal{B}}]$$

$$\text{Var}[x] \leftarrow \frac{m}{m-1} \mathbb{E}_{\mathcal{B}}[\sigma_{\mathcal{B}}^2]$$

- Use these new estimates for inference:

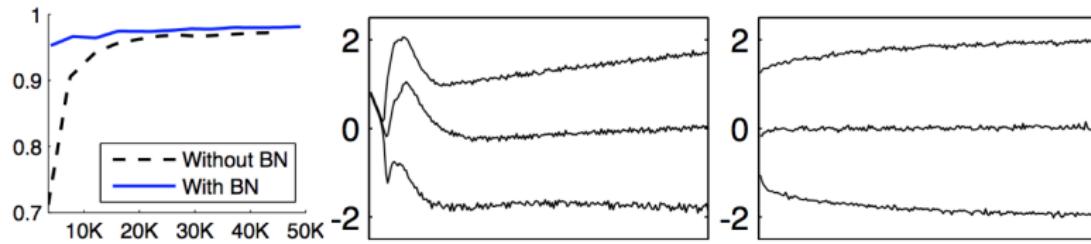
$$y = \frac{\gamma}{\sqrt{\text{Var}[x]+\epsilon}} \cdot x + \left(\beta - \frac{\gamma \mathbb{E}[x]}{\sqrt{\text{Var}[x]+\epsilon}} \right)$$

- Usually BatchNorm is inserted before nonlinearity
- Learning rate can be increased
- Optimization becomes robust to parameter scale:

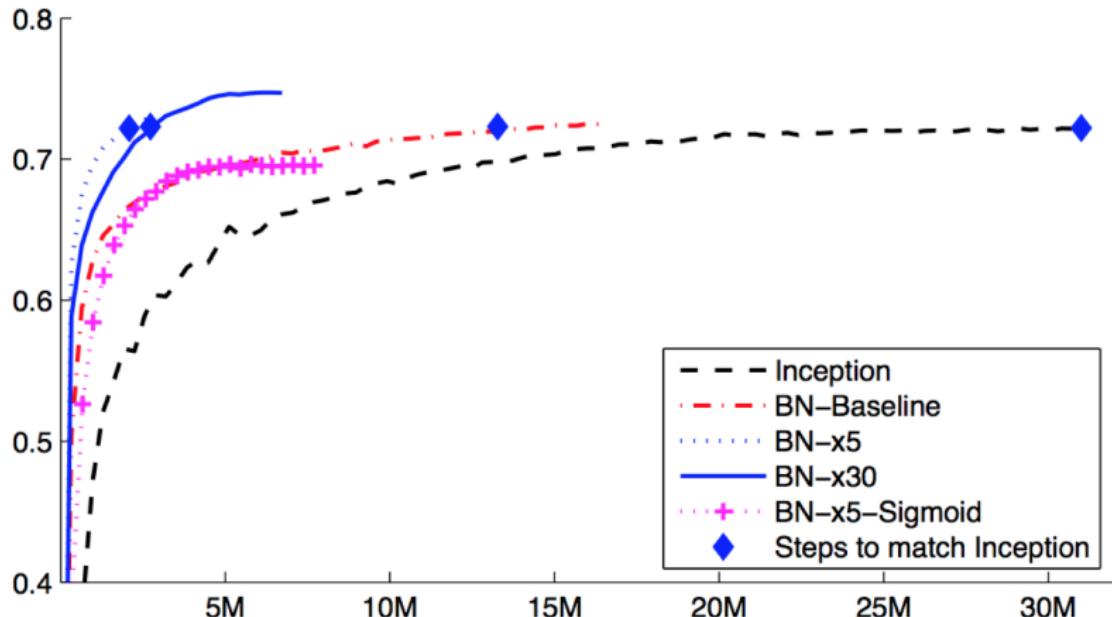
$$\frac{\partial \text{BN}((aW)u)}{\partial u} = \frac{\partial \text{BN}(Wu)}{\partial u}$$
$$\frac{\partial \text{BN}((aW)u)}{\partial (aW)} = \frac{1}{a} \cdot \frac{\partial \text{BN}(Wu)}{\partial W}$$

- Dropout layers can be removed
- Weight regularization can be reduced

BatchNorm example



BatchNorm example



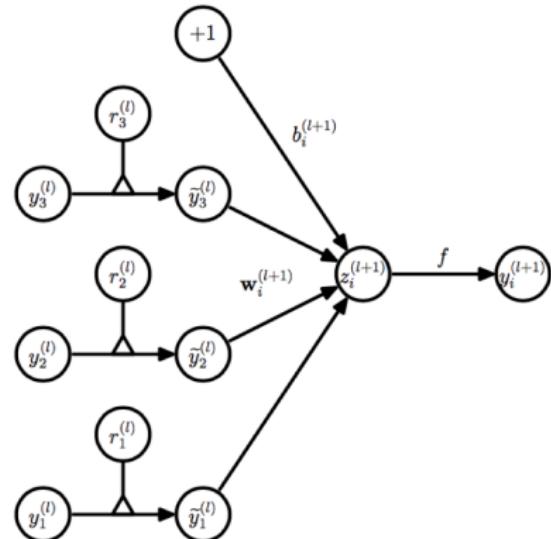
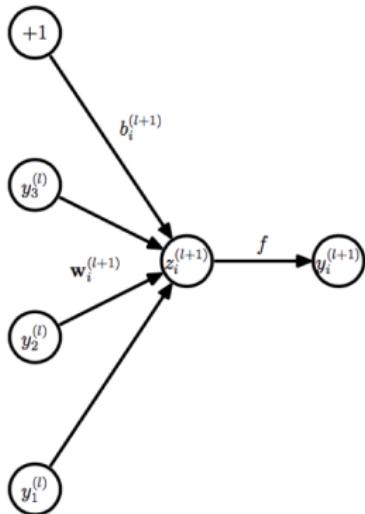
- Weights reparametrization:

$$\mathbf{w} = \frac{g}{\|\mathbf{v}\|} \mathbf{v}$$

- Fixes the Euclidean norm of \mathbf{w} to g
- Gradients:

$$\nabla_g L = \frac{\nabla_{\mathbf{w}} L \cdot \mathbf{v}}{\|\mathbf{v}\|}, \quad \nabla_{\mathbf{v}} L = \frac{g}{\|\mathbf{v}\|} \nabla_{\mathbf{w}} L - \frac{g \nabla_g L}{\|\mathbf{v}\|^2} \mathbf{v}$$

Model	Test Error
Maxout [6]	11.68%
Network in Network [17]	10.41%
Deeply Supervised [16]	9.6%
ConvPool-CNN-C [26]	9.31%
ALL-CNN-C [26]	9.08%
our CNN, mean-only B.N.	8.52%
our CNN, weight norm.	8.46%
our CNN, normal param.	8.43%
our CNN, batch norm.	8.05%
ours, W.N. + mean-only B.N.	7.31%



- Randomly disable outputs of a layer:

$$r_{jk} \sim \text{Bernoulli}(p)$$

$$v_{j+1}(\mathbf{x}) = f_{j+1}(v_j(\mathbf{x}) \otimes r_j; w_j)$$

- Prevents co-adaptation of neurons and makes them more robust to random perturbations
- Similar to training 2^n models with shared weights
- During inference: multiply weights of dropout layer by p
- Works well with max-norm regularization
- Variational dropout can learn separate dropout weight for each node

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- L_2 regularization: adds $\lambda \|\mathbf{w}\|^2$ to the loss function
- Penalizes peaky weight vectors
- Prevents neurons from focusing on one strong input
- Something similar to dropout
- Wager, Wang, Liang. Dropout Training as Adaptive Regularization:
“We show that the dropout regularizer is first-order equivalent to an L2 regularizer applied after scaling the features by an estimate of the inverse diagonal Fisher information matrix”

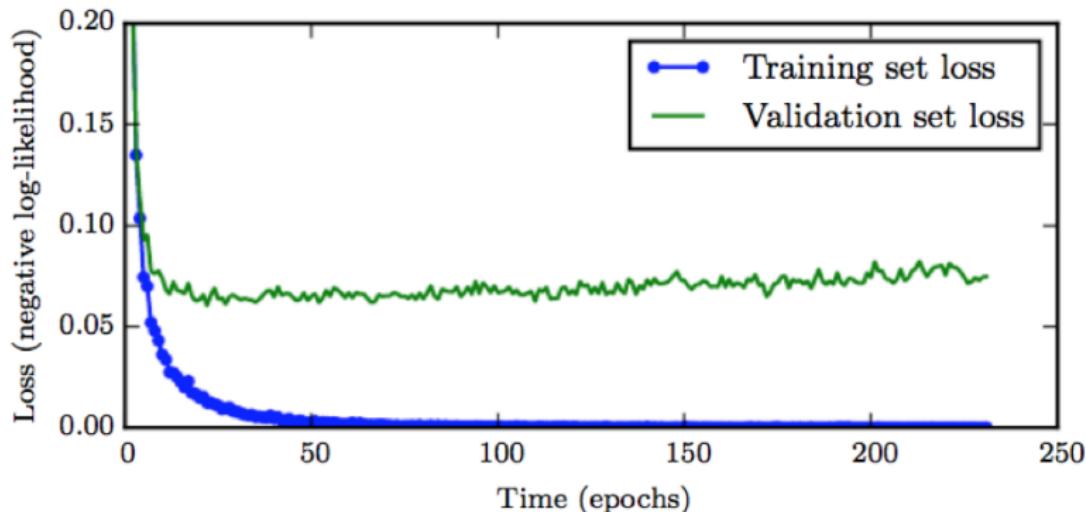
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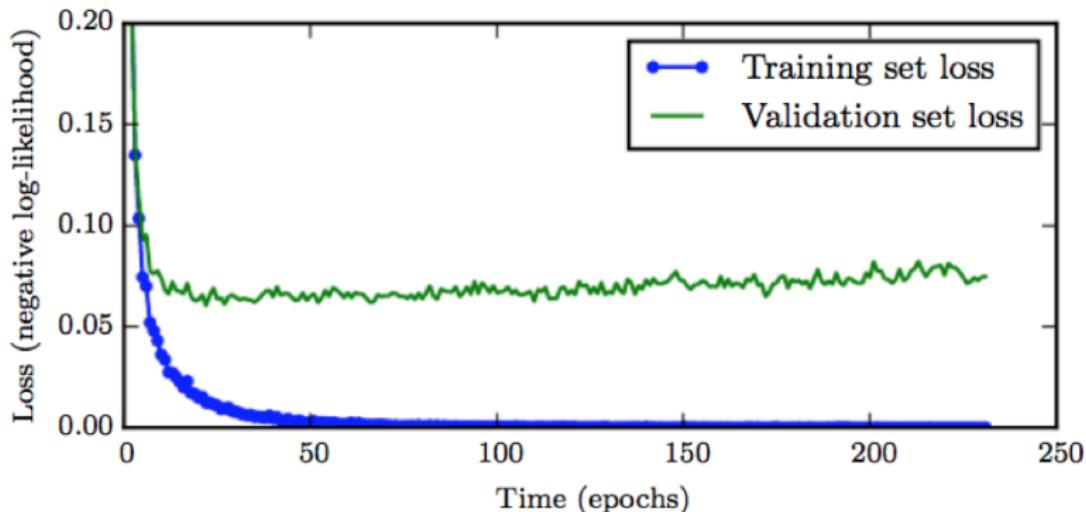
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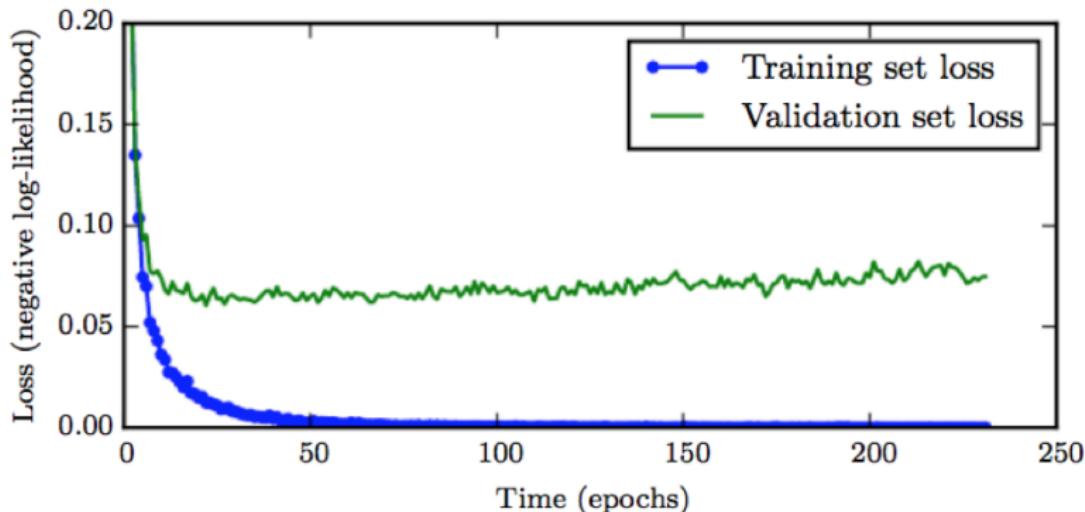
- Select number of optimization steps based on validation quality
- When the optimal number of steps is known, one can retrain the model both on training and validation data
- For a linear model with MSE loss and SGD optimizer, early stopping is equivalent to L_2 regularization



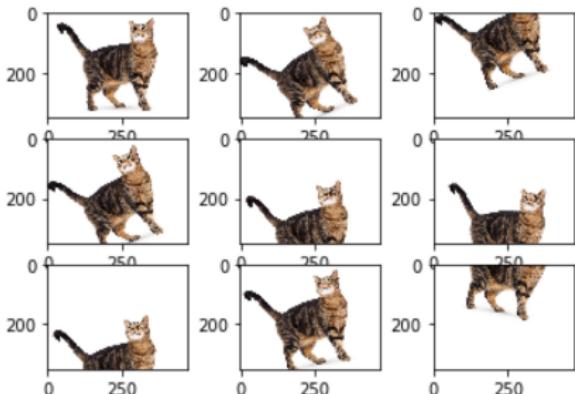
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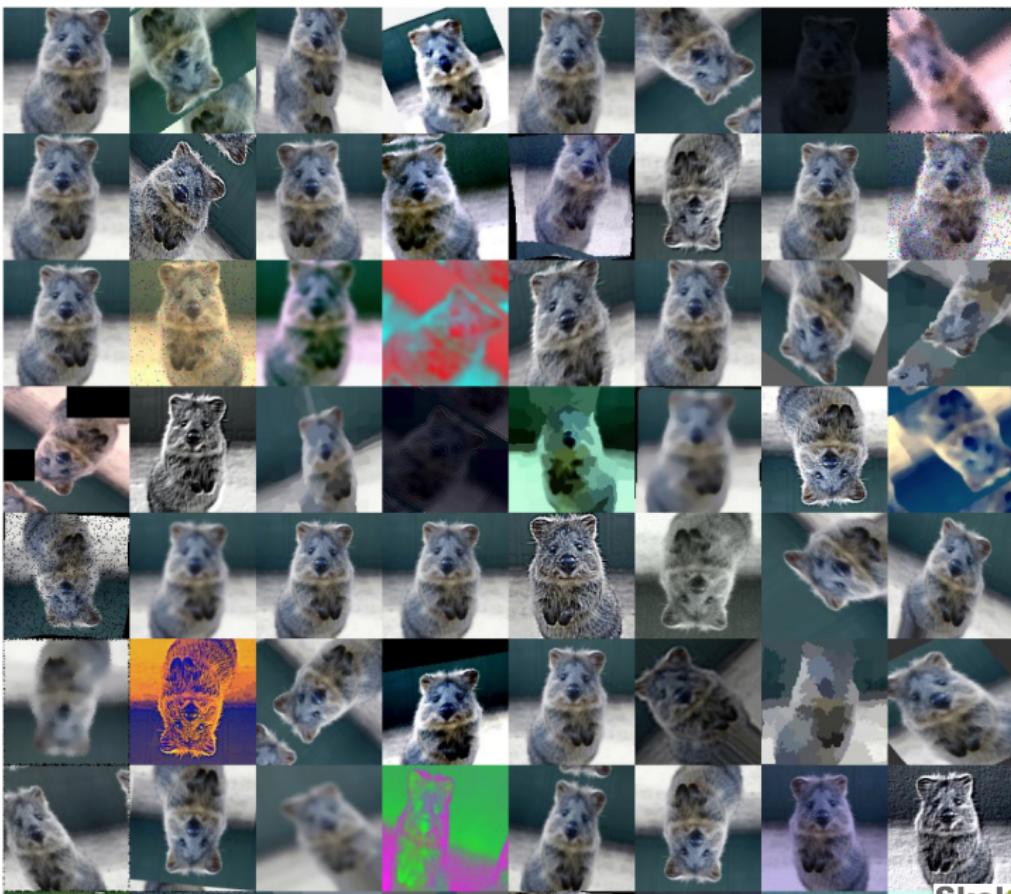
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Data augmentation



Data augmentation



- Shift
- Zooming in/out
- Rotation
- Flip
- Distortion
- Shade

- You can even use style transfer!



- Time stretching
- Pitch shifting
- Dynamic Range Compression
- Background noise
- ...

- Improves robustness of the model
- Useful for small samples
- Improves results on large datasets too

- Normalizations improve training both in terms speed and quality
- Regularizations (dropout, max-norm, L_2) prevent co-adaptation
- Data augmentation is useful to improve generalization
- Many other regularization techniques: multi-task learning, adversarial samples, sparsification etc.