



# Disruption management in the airline industry—Concepts, models and methods

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## ABSTRACT

This paper provides a thorough review of the current state-of-the-art within airline disruption management of resources, including aircraft, crew, passenger and integrated recovery. An overview of model formulations of the aircraft and crew scheduling problems is presented in order to emphasize similarities between solution approaches applied to the planning and recovery problems. A brief overview of research within schedule robustness in airline scheduling is included in the review, since this proactive measure is a natural complement to disruption management.

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## 1. Introduction

The airline industry is one of the most successful examples of applying operations research methods and tools for the planning and scheduling of resources. Optimization-based decision support systems have proven to be efficient and cost-saving for the scheduling of aircraft and crew, not to mention the short term re-scheduling problems, where modifications to the initial plans are required before the final schedules can be executed.

On the day of operation carefully planned crew and aircraft schedules can become infeasible due to external disruptions and internal failures. To date, no planning tools have been able to cope with the complexity of re-planning all airline operations at the same time during disruptions. Despite the increasing power of hardware and sophisticated solution methods, there is still a gap between the reality faced in airlines' operations control and the decision support offered by the commercial IT-systems targeting the recovery of aircraft, crew and passenger itineraries in one integrated system. However, substantial achievements have been made in developing solution methods that support the stand-alone recovery of aircraft and crew since the mid 1980s, and a few prototype systems for integrated airline recovery have been presented in the operations research literature. The majority of the mathematical models and solution methods for solving the airline recovery problems are similar to the methods applied for planning purposes. Tools for planning as well as for recovery are, in most research cases, based on a network representation that describes how flights can be sequenced either in a rotation or in a crew pairing. In the remainder of this section

we present an overview of the most commonly used network models for airline optimization problems and a short description of the planning process used by major airlines today. Section 2 describes aircraft, crew, and integrated and passenger recovery as presented in the literature, while Section 3 briefly discusses robustness in relation to disruption management. Finally, Section 4 contains discussions of future prospects for disruption management systems in the airline industry.

### 1.1. Airline planning process

Prior to the departure of an aircraft, a sequential planning approach takes place. First, the flight schedule is determined, based on forecasts of passenger demand, available slots at the airports and other relevant information. Thereafter, specific types of aircraft are assigned to individual flights in the schedule, and sequences of flights are generated within each fleet—these processes are called fleet assignment and aircraft routing, respectively. Aircraft rotations must respect various types of constraints as e.g. maintenance and night curfews. In the subsequent crew scheduling phase, flight crew and cabin crew are assigned to all flights based on the already determined aircraft rotations. Individual flights are grouped to form anonymous crew pairings. Each pairing starts and ends at the same crew base and has a typical length of three–four days. Afterwards, pairings are grouped to form personnel rosters, which are lines of work typically for 14 days or one month, including rest periods, vacations and training. Finally, physical aircraft from a given fleet are assigned to flights in the tail assignment process. The complete planning process is illustrated in Fig. 1.

The planning process is very complex since numerous restrictions and rules have to be considered. For aircraft, rules on maintenance, differences between various aircraft types, etc. must be taken into

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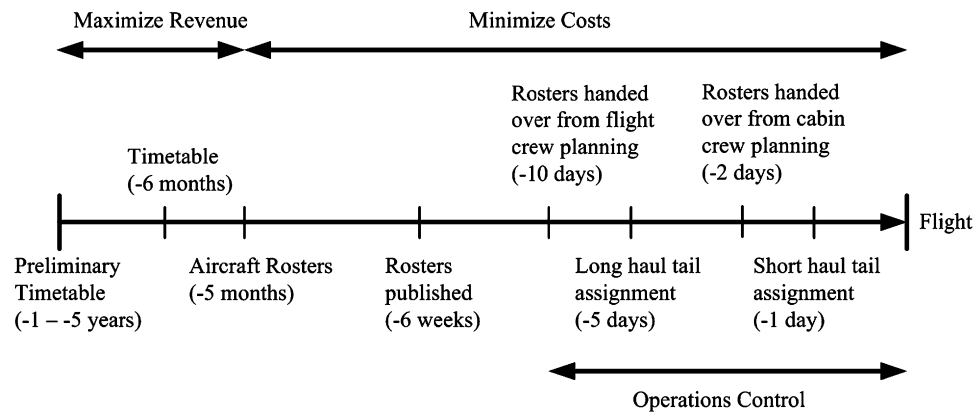


Fig. 1. The time line for the operation of a major European airline.

the planning. Also, characteristics of each individual airport have to be respected. For crew, there are regulations on flying time, off-time, etc., based on international and national rules, as well as regulations originating in agreements with unions, specific to each airline. Changes in plans due to e.g. crew sickness, aircraft breakdowns and changes in passenger forecasts take place in the tracking phase of the planning process. This phase normally resides with the planning department of the airline.

The plans for aircraft and crew assignments are handed over from the planning department to the operations control center (OCC) a few days ahead of the day of operation. It now becomes the responsibility of the OCC to maintain all resources so that the flight schedule is feasible as an integrated entity. Events like acute crew unavailabilities and delayed flights have to be handled. Not only the immediately affected flights, but also knock-on effects in other parts of the schedule can cause serious problems. Generally, a disrupted situation (often just denoted a disruption) is a state during the execution of the current operation, where the deviation from the plan is sufficiently large to impose a substantial change. This is not a very precise definition; however, it captures the important point that a disruption is not necessarily the result of one particular event.

The generation of recovery plans is a complex task, since many resources (crew, aircraft, passengers, slots, catering, cargo etc.) have to be re-planned. When a disruption occurs on the day of operation, large airlines usually react by solving the problem in a sequential fashion with respect to the problem components. First, infeasibilities in the aircraft schedule are resolved, then crewing problems are addressed. Afterwards, ground problems are tackled, and finally, the impact on passengers is evaluated. Sometimes, the process is iterated with all stakeholders until a feasible plan for recovery is found and can be implemented. As a rule, determining the quality of a recovery option is a difficult task. The objective function can be composed of several conflicting and sometimes non-quantifiable goals. Examples of objectives are minimizing the number of passenger delay minutes, returning to the plan as quickly as possible, minimizing passenger dissatisfaction, minimizing the cost of the recovery operation, etc. In most airlines, controllers performing the recovery have only limited IT-based decision support to help them construct recovery options or evaluate the quality of the recovery action they are about to implement. Often, controllers are content with only producing one viable recovery plan since there is no time to consider alternatives.

## 1.2. Models for airline optimization problems

The majority of airline recovery models are formulated and solved similar to the corresponding planning problems, using the same

Table 1

A sample schedule for Sample Air with aircraft rotations.

Aircraft	Flight	Origin	Destination	Departure	Arrival	Flight time
AC1	11	OSL	CPH	14:10	15:20	1:10
	12	CPH	AAR	16:00	16:40	0:40
	13	AAR	CPH	17:30	18:10	0:40
	14	CPH	OSL	18:50	20:00	1:10
AC2	21	CPH	WAV	14:30	15:30	1:00
	22	WAV	CPH	15:50	16:50	1:00
	23	CPH	WAV	17:30	18:30	1:00
	24	WAV	CPH	18:50	19:50	1:00
AC3	31	AAR	OSL	15:00	16:20	1:20
	32	OSL	AAR	17:00	18:20	1:20

network representations to model the schedules. However, there are also some differences between the modelling approaches. In order to draw a parallel between recovery models and optimization problems occurring during the planning phase, we briefly present the aircraft routing and the crew scheduling problem formulations, as well as their substantial differences from the recovery models.

### 1.2.1. Network representations

The three most commonly used network representations for airline planning and recovery problems are time-line networks, connection networks and time-band networks. In order to illustrate the networks, consider a small flight schedule of an artificial airline Sample Air shown in Table 1, where flights connecting Copenhagen (CPH), Oslo (OSL), Aarhus (AAR), and Warsaw (WAV) are given. Assume that the turn-around-time for an aircraft is 40 min in CPH and OSL and 20 min in AAR and WAV.

A *connection network* is an activity-on-node network, where flight legs correspond to nodes in the network and connections between flight legs correspond to directed edges (arcs) between the nodes. A flight leg is given by its origin, destination, departure time and date and arrival time and date. A node  $i$ , representing the flight leg  $l_i$ , is connected by a directed edge  $(i, j)$  to a node  $j$ , which represents the flight leg  $l_j$ , if it is feasible to fly  $l_j$  immediately after  $l_i$  using the same aircraft with respect to turn-around-times and airport. In addition, there is a set of origin and destination nodes indicating possible positions of aircraft in a fleet at the beginning and at the end of the planning horizon, respectively. A path in the network from an origin to a destination node corresponds to a sequence of flights feasible as part of a rotation. Schedule information is not represented explicitly in the network, but is used when generating the nodes in the network. Maintenance restrictions can be easily incorporated through the concept of a maintenance feasible path,

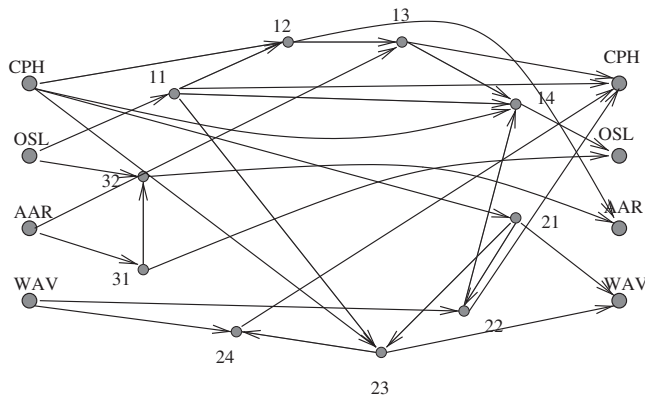


Fig. 2. The sample schedule shown as a connection network. The rotation for AC1 shown in Table 1 corresponds to the path OSL-11-12-13-14-OSL.

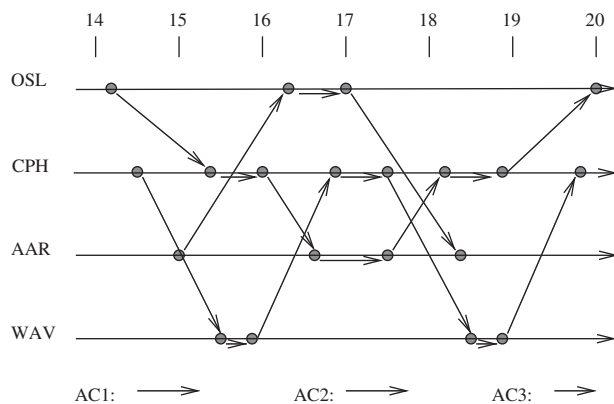


Fig. 3. The sample schedule shown as a time-line network. The rotation for AC1 shown in Table 1 corresponds to the AC1 path.

which is a path providing sufficient extra time with the required intervals at a node corresponding to a station, where maintenance can take place. Note that the number of feasible paths may be very large—it grows exponentially with the planning time horizon. The Sample Air flight schedule represented as a connection network is shown in Fig. 2.

The idea of a *time-line network* is to represent the possible schedules in a natural way from the time-and-station point of view, which is not possible when using a connection network. A time-line network has a node for each event, an event being an arrival or a departure of an aircraft at a particular station. Time-line networks are activity-on-edge networks, where directed edges correspond to activities of an aircraft, and schedule information is represented explicitly by the event nodes. All event-nodes of a particular station are located on a time line corresponding to that station. The length of the time line corresponds to the planning horizon. There is a directed edge from one event-node to another, if the two events may follow each other in a sequence in a schedule of the same aircraft. Edges connecting nodes on the time lines for different stations correspond to flights feasible with respect to flying time, while edges connecting nodes on the time line for a particular station correspond to grounded aircraft. In the same way as for the connection network, a direct path is possible rotation for an aircraft. The time-line network for Sample Air is shown in Fig. 3. Notice that ground arcs that are not used in the aircraft schedule presented in Table 1 are omitted from the network for simplicity.

When network representations are used in the recovery context, a network is usually built for shorter time periods, beginning at a

time of disruption and limited by the time when the schedule is expected to be recovered. The source nodes in the network represent the exact positions of the aircraft at the time of disruption, while the sink nodes represent the expected positions of the aircraft at the end of the recovery. The schedules within the recovery time window are then re-planned in order to repair infeasibilities caused by disruptions, while the schedules outside of the recovery time window are not changed.

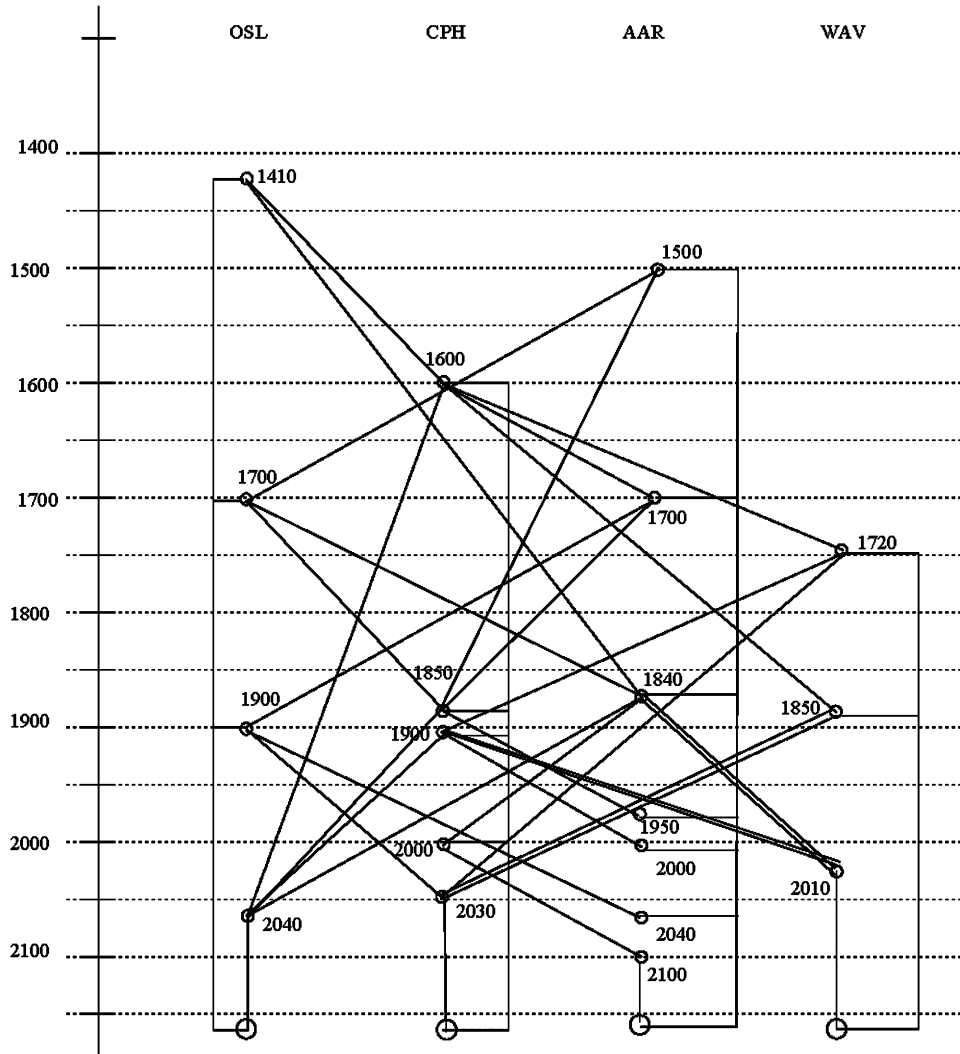
A *time-band network* is proposed by Argüello [7] in order to model the aircraft schedule affected by disruptions, and is used in the context of aircraft recovery. The network can be constructed dynamically as disruptions occur, for a certain recovery time period. There is a set of station-time nodes and a set of station-sink nodes. A station-time node represents activities at a particular airport aggregated within a certain discrete time interval, called a time band. The time label of a station-time node corresponds to the availability time (the arrival time plus the turn-around time) of the first available aircraft in the time band. A station-sink nodes represent the end of the recovery period at each station. The edges in the network represent the flights. A scheduled flight from station A to station B has an emanating edge for each A-time node, in which there is an aircraft available, and for which the flight can be flown within the recovery period. Each of these edges will end in the B-time node corresponding to the time when the aircraft becomes available at B. The number of emanating edges is the same for all station-time nodes corresponding to the same station. Finally, there are edges connecting each station-time node to the station-sink node for the relevant airport. A recovery solution corresponds to a flow in the network. Edges of the originally scheduled flights, which carry no flow, correspond to cancelled flights, and re-timings of flights correspond to the flow on the “new” flight edges, indicating that flights are flown at a later time than scheduled. Fig. 4 shows the time-band network model for the Sample Air schedule, where aircraft AC2 is out of service from 14:00 to 21:00 due to an unexpected maintenance, and with time bands of 30 min. The network is constructed in a stepwise fashion in order to avoid generating time-station nodes with no aircraft availability. Two flows in this network, one starting in OSL and another in AAR, and ending in either OSL or AAR, determine the way to use the two remaining aircraft, AC1 and AC3.

### 1.2.2. Aircraft routing

An *aircraft routing problem* (also called *aircraft rotation problem*) determines the optimal set of routes flown by all aircraft in a given fleet, given that the fleet assignment is already performed. There are two general formulations of the aircraft routing problem: a set partitioning model and a multicommodity network flow model. The connection network and the time-line network can both be used to represent the schedule.

In a multicommodity network flow formulation of the aircraft routing problem non-negative integer decision variables  $x_{ij}$  represent the flow on arc  $(i,j)$  of the network, each unit of flow representing one aircraft in a given fleet. Flow balance constraints of the problem at each node of the network ensure that each flight leg is covered by exactly one aircraft and that the balance of grounded aircraft at each station is ensured. This also ensures that the number of rotations in the network is less than or equal to the number of aircraft in a given fleet.

The aircraft routing problem can also be formulated as a set partitioning problem. Let  $F$  be the set of available aircraft in a fleet. For each aircraft  $f \in F$ , an origin  $o^f$  and a destination  $d^f$  relative to the planning horizon is given. Given a connection network with a set of flight nodes  $N$ , origins  $o^f$  and destinations  $d^f$ ,  $P^f$  denotes the set of feasible paths between  $o^f$  and  $d^f$  in the network. If maintenance is to be taken into account, only maintenance feasible paths are considered. The relations between the flights and the paths are given



**Fig. 4.** The time-band network of the sample schedule. The aircraft AC2 is out of service from 14:00 to 21:00. Time bands are 30 min. A feasible recovery rotation for AC1 is OSL-11-21(delayed 1.5 h)-22(delayed 1.5 h)-14(delayed 10 min)-OSL. This rotation corresponds to the path OSL(1400-1429)-CPH(1600-1629)-WAV(1700-1729)-CPH(1900-1929)-OSL(2030-2059)-OSL(sink).

by binary parameters  $a_{ip}$ , which are equal to one if flight leg  $i$  is on path  $p$ . To determine which aircraft are to fly the scheduled flights, we define binary decision variables  $x_p^f$ , which are equal to one if and only if the flight legs on the path  $p$  with cost  $c_p^f$  are flown by aircraft  $f$ . The constraints of the problem ensure that each flight leg is contained in exactly one of the selected paths, and that only one path must be chosen for each aircraft:

$$\begin{aligned}
 &\text{Minimize} && \sum_{f \in F} \sum_{p \in P^f} c_p^f x_p^f \\
 &\text{subject to} && \sum_{f \in F} \sum_{p \in P^f} a_{ip} x_p^f = 1 \quad \forall i \in N, \\
 &&& \sum_{p \in P^f} x_p^f = 1 \quad \forall f \in F, \\
 &&& x_p^f \in \{1, 0\} \quad \forall f \in F, p \in P^f.
 \end{aligned}$$

The aircraft recovery model can be formulated similar to the above aircraft routing problem, with extra binary decision variables determining if flight  $f$  is to be cancelled or not in the recovery solution, and expressing the costs of delays and cancellations in the objective function.

### 1.2.3. Crew scheduling

On passenger aircraft there are two types of crew: flight (cockpit) crew responsible for flying the aircraft and cabin crew who service the passengers. Each of the crew groups are further divided by rank. A crew will typically get a plan of work for a two or four-week period. The task of assigning crew to itineraries is generally very complex. It is therefore split into two stages: *crew pairing* and *crew assignment* (also known as *crew rostering*). Both problems are usually formulated as generalized set partitioning or set covering problems with one constraint for each task to be performed. In the crew pairing problem the task is a flight to be covered and in the crew assignment problem the task is a pairing/other work to be covered. For an exhaustive description of airline crew scheduling problems and solution methods refer to Barhnart et al. [12].

The objective of the crew pairing problem is to find a minimum cost subset of feasible pairings such that every flight is covered by exactly one selected pairing. Let  $F$  be the set of flights to be covered and  $P$  the set of all feasible pairings. Decision variable  $y_p$  is equal to one if pairing  $p$  is included in the solution and zero otherwise. The relation between pairing  $p$  and flight  $i$  is given by a parameter  $a_{ip}$ , which is equal to one if  $p$  contains  $i$  and zero otherwise. The cost



of a pairing is denoted  $c_p$  and includes allowances, hotel and meal costs, ground transport costs and paid duty hours.

$$\begin{aligned} & \text{Minimize} \quad \sum_{p \in P} c_p y_p \\ & \text{subject to} \quad \sum_{p \in P} a_{ip} y_p = 1 \quad \forall i \in F, \\ & \quad y_p \in \{1, 0\} \quad \forall p \in P. \end{aligned}$$

Generation of pairings can be done using one of the two network representations presented earlier: the flight connection network (mainly used for domestic and short-haul operations) or the duty time-line network (mainly appropriate for international and long-haul operations). A pairing is a path from the source to the sink, usually represented by crew bases. However, not all paths represent legal pairings since duty rules, like maximum flying hours, etc., are not explicitly expressed in the network. These rules must be checked for each path in order to ensure legality.

In order to solve the crew pairing problem one possibility is to construct all legal pairings. The challenge is that the number of legal pairings can be extremely large, typically varying from 500,000 for a minor airline to billions of pairings for major airlines. For smaller problems all legal pairings can be generated a priori. For larger problems, a limited a priori generation can be used as a heuristic, finding a good solution without guaranteeing optimality. Another approach is to generate the pairings as they are needed in a dynamic column generation process. The problem of generating the pairings then becomes a variant of the shortest path problem.

The crew assignment (rostering) problem is solved for each crew type, i.e. captain, first officer, etc. Each crew member should be assigned to exactly one work schedule, while each pairing from the crew pairing solution must be contained in the appropriate number of selected work schedules, depending on how many crew members of each type are required for a given pairing. Let  $K$  be the set of crew members of a given type and let  $P$  be the set of pairings to be covered. For each crew member  $k$  the set of feasible work schedules is denoted  $S^k$ .  $n_p$  is the minimum number of crew members needed to cover pairing  $p$  and  $\gamma_p^s$  is equal to one if pairing  $p$  is included in schedule  $s$  and zero otherwise.  $c_s^k$  is the cost of schedule  $s$  for crew  $k$ . Decision variables are  $x_s^k$ , taking the value of one if schedule  $s \in S^k$  is assigned to crew  $k \in K$  and zero otherwise.

$$\begin{aligned} & \text{Minimize} \quad \sum_{k \in K} \sum_{s \in S^k} c_s^k x_s^k \\ & \text{subject to} \quad \sum_{k \in K} \sum_{s \in S^k} \gamma_p^s x_s^k \geq n_p \quad \forall p \in P, \\ & \quad \sum_{s \in S^k} x_s^k = 1 \quad \forall k \in K, \\ & \quad x_s^k \in \{1, 0\} \quad \forall s \in S^k, k \in K. \end{aligned}$$

The network representation for the crew assignment problem is similar to the pairing problem, but instead of defining a path of flights as in the pairing problem the path consists of pairings. The problem can be solved with the same solution methods as the crew pairing problem, e.g. column generation.

The crew recovery problem formulations presented in the Operations Research literature are similar to the crew scheduling models, but often other decision variables are added, representing the decisions to be taken in order to recover disrupted situations. For instance, a binary decision variable  $z_i$  can determine if flight leg  $i$  is cancelled or not, or an integer decision variable  $s_i$  can decide the number of crew deadheading (flying as a passenger for re-positioning reasons) on flight leg  $i$ . By introducing a cost in the objective function corresponding to decision variables responsible for recovery and adding the variables to the problem constraints, an optimal

re-scheduling solution can be found with respect to the objectives specified for recovering a particular disruption.

## 2. Disruption management

Clarke [20] provides the first overview of the state-of-the-practice in operations control centers in the aftermath of irregular operations. The overview is based on field studies at several airlines. The author provides an extensive review of the literature within the airline disruption management and proposes a decision framework that addresses how airlines can re-assign aircraft to scheduled flights after a disruptive situation. Kohl et al. [33] provide a general introduction to the airline disruption management and include a description of the planning processes in the airline industry. The paper reports on the experiences obtained during the large-scale airline disruption management research and development project DESCARTES, supported by European Union. A survey incorporating issues from the point of view of airports can be found in Filar et al. [25], and a small section devoted to disruption management is included in Yu and Yang [77].

The book by Yu and Qi [76] considers disruption management from a more general perspective. It includes chapters on disruption management for flight and crew scheduling for airlines as well as chapters on disruption management for a number of other applications, e.g. machine scheduling and supply chain coordination. Due to the general view on disruption management taken by the authors, the chapters on disruption management for airlines are not particularly detailed with respect to methodology. Ball et al. [10] give insight into the infrastructure and constraints of airline operations, as well as the air traffic flow management methods and actions. Simulation and optimization models for aircraft, crew and passenger recovery are also discussed. Furthermore, the authors give an excellent survey of the airline schedule robustness as a proactive alternative to recovery, including model descriptions and a literature review.

### 2.1. Aircraft recovery

The initial research within disruption management focused on aircraft recovery, possibly due to the fact that the number of aircraft is much smaller than the number of crew members, and the rules for aircraft scheduling are less complex. Teodorović and Guberinić [63] are the pioneers of the aircraft recovery research, their research is extended by Teodorović and Stojković [64,65]. Since the complexity and the size of problem instances are not as challenging as for crew, many solution approaches to aircraft recovery are to a larger extent based on the original planning models. Jarrah et al. [31], Rakshit et al. [48], Mathaisel [44], Yan and Yang [73], Cao and Kanafani [14,15] formulate the aircraft recovery problem as a minimum cost network flow problem and use network flow algorithms to solve it. Argüello et al. [8,9], Løve et al. [40], Andersson [5], and Liu et al. [37,38] apply metaheuristics, while the vast majority of the publications use integer programming solution methods to solve the aircraft recovery problem. We group the latter by the network representations used by the authors for formulating the IP models, i.e. the time-line network, the time-band network and the connection network. Finally, Table 2 gives a summary of the aircraft recovery literature in a chronological order.

#### 2.1.1. Initial efforts

One of the first studies of the airline recovery problem is presented in the paper from 1984 by Teodorović and Guberinić [63]. Here, one or more aircraft are unavailable and the objective is to minimize the total passenger delays by reassigning and re-timing the flights. The authors devise a heuristic that sequentially constructs the chain of flights to be flown by each aircraft. Their solution assumes a single fleet type and ignores all maintenance constraints.

**Table 2**

Overview of proposed methods for the aircraft recovery problem.

Authors	Year	Network	Functionality			Data	Dimensions			Solution time	Objectives
			Cancel	Retime	Multi-fleet		AC	Fleets	Flights		
Teodorović and Guberinić [63]	1984	CN	No	Yes	No	G	3	1	8	NA	Delay minutes
Teodorović and Stojković [64]	1990	CN	Yes	Yes	No	G	14	1	80	180	Canx and delay minutes
Jarrah et al. [31], Rakshit et al. [48]	1993/6	TLN	Yes	Yes	No	RL	NA	9	NA	0–30	Delay, swap and ferrying
Teodorović and Stojković [65]	1995	CN	Yes	Yes	No	G	NA	1	80	140	Canx and delay minutes
Mathaisel [44]	1996	TLN	Yes	Yes	No	NA	NA	NA	NA	NA	Revenue loss, operating cost
Talluri [61]	1996	CN	No	No	Yes	G	NA	NA	NA	10	Swaps when changing AC type
Yan and Yang [73]	1996	TLN	Yes	Yes	No	RL	17	1	39	49	Costs minus revenue
Clarke [18,19]	1997	CN	Yes	Yes	Yes	RL	177	4	612	NA	Costs minus revenues
Yan and Tu [72]	1997	TLN	Yes	Yes	Yes	RL	273	3	3	1800	Costs minus revenue
Cao and Kanafani [14,15]	1997	TLN	Yes	Yes	No	G	162	1	504	869	Revenue minus costs
Luo and Yu [41]	1997	NA	No	Yes	NA	RL	NA	NA	71	15	Number of delayed flights
Argüello et al. [8]	1997	TBN	Yes	Yes	Yes	RL	16	1	42	2	Route cost and cancellation cost
Luo and Yu [42]	1998	NA	No	Yes	NA	RL	NA	NA	71	15	Delayed flights
Thengval et al. [66]	2000	TLN	Yes	Yes	No	RL	27	1	162	6	Revenue minus cost
Thengvall et al. [67,68]	2001/3	TLN	Yes	Yes	Yes	RL	332	12	2921	1490	Revenue minus cost
Bard et al. [11]	2001	TBN	Yes	Yes	No	RL	27	1	162	750	Delay and canx
Rosenberger et al. [50]	2003	CN	Yes	Yes	No	G	96	1	407	16	Delay and canx
Andersson and Värbrand [6]	2004	CN	Yes	Yes	Yes	RL	30	5	215	10–1100	Cancellations, swap and fleet swap
Løve et al. [40]	2005	TLN	Yes	Yes	No	RL	80	1	340	6	Revenue minus costs
Andersson [5]	2006	–	Yes	Yes	Yes	RL	30	5	215	10 <sup>a</sup>	Cancellations, swap and fleet swap
Liu et al. [37,38]	2006/8	–	No	Yes	No	RL	7	1	70	NA	Delay, cancellations and assignment
Eggenberg et al. [23]	2007	TBN	Yes	Yes	No	RL	10	1	240	29	Flight, delay, plus maintenance cost
Zhao and Zhu [80]	2007	–	Yes	Yes	No	G	6	1	20	NA	Cost

Model types: connection network (CN), time-line network (TLN) or time-band network (TBN). Data types: generated (G) or real-life (RL) instances. Solution times are in seconds. Luo and Yu [43] is not mentioned since it is identical to Luo and Yu [41]. Yan and Lin [71] and Yan and Young [74] are not mentioned since the papers are very similar to Yan and Young [73] and Yan and Tu [72].

<sup>a</sup>This is the running time of the tabu search which is superior to the simulated annealing algorithm of the same paper.

The authors present a very simple example with only eight flights. Teodorović and Stojković [64] extend this work to also consider airport curfews. The described method is tested on a small example of 14 aircraft and 80 flights. Teodorović and Stojković [65] further extend their model to also include crew considerations. The proposed method is tested on 240 different randomly generated numerical examples.

### 2.1.2. Solution approaches based on network flow algorithms

Jarrah et al. [31] present two network flow models for solving the aircraft recovery problem: one for cancellation and one for re-timing. The models are based on the successive shortest path method presented by Gershkoff [28]. The major disadvantage of their approach is that the methods do not allow for a trade-off between cancelling and delaying in a single decision process. To evaluate the cost of delaying or cancelling the aircraft the authors construct a disutility function, which depends on the total number of passengers, the number of passengers with a down-line connection, lost crew time and disruption of maintenance. The three test scenarios in the paper are based on United Airlines' B737 fleet and a regional subdivision of the United States. In both cases, running times of the models are sufficiently small to allow their use in a real-time implementation. The solution method was successfully implemented in a decision support system at United Airlines. The impact of this implementation is reported in Rakshit et al. [48]. The papers by Cao and Kanafani [14,15] are basically extensions of this work. A quadratic zero-one programming model is presented in which the flight revenue subtracted swap and delay costs are maximized. Their model allows for a solution combining delays and cancellations. Furthermore they also take into account the issues of ferrying (flying an empty aircraft to an airport to cover open flights from that airport) as well as multiple aircraft type swapping. The algorithm is tested on a set of randomly generated scenarios with 20–50 airports, 30–150 aircraft, 5–12 surplus aircraft, 65–504 flights, and approximately 25% delayed aircraft. The work by Løve and Sørensen [39], in which a reproduction of the

results is attempted, suggests that the description of the model is not complete.

Mathaisel [44] describes the business process as well as the IT challenges faced in the design and implementation of a decision support system for airline disruption management. The system described is based on a network of workstations; one of them working as a server, the remaining ones as clients. The author mentions that several optimization methods are embedded in the environment. The network flow model for aircraft rerouting in case of disruptions is presented, and the out-of-kilter network flow algorithm on a time-line network is used to solve the problem. The model is capable of using cancellation as well as re-timing. However, the paper does not discuss multiple types of aircraft, crew considerations, or solution times.

### 2.1.3. Solution approaches based on time-line networks

Another approach that has received significant attention is the representation introduced by Yan and Yang [73]. The framework is based on the classical time-line network with flight arcs, ground arcs, and overnight arcs. The final, and most general, model is derived step-by-step, so the paper essentially encompasses four models. Arcs for ferrying are added to the model. Furthermore, in order to allow for delays, time-shifted copies of the "original" flights are also added to the network. An extra set of constraints is added to the model in order to make sure that at most one of the copies is used in a solution. The model is based on a single fleet set up with no maintenance or crew scheduling considered. The authors consider the case where only one aircraft is disrupted. While the first two models of the paper are pure network flow models, the latter two are the network flow models with side constraints which are difficult to solve. In order to obtain solutions fast, all side constraints are relaxed, and the resulting model is solved using Lagrangian relaxation with the subgradient method. A feasible solution is derived from this using a Lagrangian heuristic. Near-optimal solutions were generated within a few minutes on practical problems of a considerable size. Yan and

Tu [72] describe similar methods (much of the text in the papers is in fact identical), except the models are extended to multi-fleet problems. Yan and Lin [71] looks at the case of the temporary closure of airports, but the paper remains very similar to Yan and Yang [73] and Yan and Tu [72]. Finally, Yan and Young [74] possesses identical text to the three aforementioned papers and merely adds multi-stop flights to the method. Though the modelling framework and the solution methods are identical, the proposed strategies for solving the perturbation problem are slightly different.

Thengvall et al. [66] use the model proposed in Yan and Yang [73] and adds protection arcs as well as through-flight arcs. In the evaluation of a proposed recovery schedule, such arcs make it possible to prioritize the deviation from the original schedule by giving special emphasis to flying all legs in a flight with several stops by the same aircraft. Like the previous time-line network models, this model can handle swaps, delays and cancellations. However, it does not take crew nor maintenance into consideration. The LP relaxation of the integer programming model is solved. If the solution is fractional, a heuristic is used to produce an integer solution based on the LP relaxation optimum. The approach is tested on real-life data from Continental Airlines (B757 schedule with 16 aircraft and 13 stations, and B737-100 with 27 aircraft and 30 stations). Results indicate that the approach clearly allows the construction of different recovery schedules corresponding to changes in priorities between delay minute costs, cancellation costs and the cost of deviation from the original schedule. Computing times are sufficiently small to allow it to be used in real-time. Thengvall et al. [67,68] extend the work of Thengvall et al. [66] to consider the closure of a hub, as well as multiple fleets. Three mixed-integer programming models are introduced: two so-called preference models, which are based on time-line networks for every subfleet, and a model based on time-bands, as introduced in Argüello [7].

#### 2.1.4. Solution approaches based on time-band networks

Bard et al. [11] present an aircraft recovery model based on the *time-band network*, introduced in the Ph.D. thesis of Argüello [7]. The idea is to represent the schedule on a time-line network, leaving out all arcs except those corresponding to the flights of the schedule. No ground arcs are included. The resulting model is an integer minimum cost flow model with additional constraints that ensure each flight is either cancelled or flown by a unique aircraft. The model is also described by the same authors in Argüello et al. [9]. During the initialization step of the solution method the time-band network is generated using the original flight schedule and the predetermined time-bands. The integer programming formulation is derived from the network. Based on the optimal LP-solution, an integer-valued solution that represents the final schedule is derived. The cost is calculated and compared to the lower bound provided by the LP-relaxation. The approach is tested on a Continental Airlines B737-100 fleet schedule with 162 flights covering 30 stations and serviced by 27 aircraft. Four hundred and twenty seven test cases are reported: 27, in which one aircraft is grounded, and 100 cases for each case of two, three, four and five aircraft grounded. The time-bands are varied from 5 to 30 min, and this also allows variations between hub and spoke stations. Using the lower bounds derived and the actual cost of the solutions, the quality of the solutions can be assessed. The results depend on the time-band resolution, and are generally encouraging with respect to quality.

Eggenberg et al. [23] use a decomposed problem structure of the aircraft recovery problem, where a generalized set partitioning problem is the master problem and a resource constrained shortest path problem is a subproblem. An independent recovery network is constructed for each aircraft. Having independent time-band networks for each aircraft makes it easy to incorporate maintenance constraints through the introduction of maintenance arcs in the

network for a given aircraft. In order to keep the problem small only a subset of recovery plans are considered. Data from a practical instance with up to 10 aircraft and a recovery period up to 7 days produce instances with up to 250 flights. Only larger instances require branching, while the remaining are solved in the root node. Running times suggest that the method is able to recover the proposed disruption scenarios.

#### 2.1.5. Set partitioning models formulated on connection networks

Another approach is to formulate the aircraft recovery problem as a set partitioning model on a connection network, traditionally used for tactical planning problem formulation. Rosenberger et al. [50] formulate the aircraft recovery problem as a set partitioning model with additional time slot and capacity constraints, ensuring the airport capacity restrictions during irregular operations. The objective of the recovery is to minimize the cost of cancellations and re-routing of aircraft, and it is the responsibility of the controllers to define the parameters accordingly. For each disrupted aircraft, a preprocessing heuristic determines a number of non-disrupted aircraft with routes allowing a swap with the disrupted aircraft. The legs of these routes are included in the route generation procedure. The generated routes form the columns in the set partitioning model, which is solved with CPLEX 6.0. This approach results in running times between 6 and 16 s for three real-size problem instances. The paper reports an impressive testing using SimAir, Rosenberger et al. [49], simulating 500 days of operations for the three fleets ranging in size from 32 to 96 aircraft servicing 139–407 flights.

Using the connection network as the underlying network, Andersson and Värbrand [6] base their approach on the set packing problem with generalized upper bound (GUB) constraints, which ensure that each aircraft is assigned exactly one route. The problem is solved with a Lagrangian relaxation-based heuristic and a method based on the Dantzig–Wolfe decomposition. Two of the three approaches implemented in Andersson [4]. The subproblem is a shortest path problem with time windows and linear node costs. To ensure fast convergence a heuristic is developed to solve the subproblem. Computational results are based on data from a domestic Swedish carrier that operates five fleets with a total of 30 aircraft. Instances consist of 98–215 flights and 19–32 airports. Smaller instances can be solved with the solution method based on the Dantzig–Wolfe decomposition, while the running times are excessive on the larger instances. Comparable results are also achieved for the Lagrangian relaxation-based heuristic.

It should be mentioned that the working papers by Clarke [18,19] also propose a column generation model based on a generalized set partitioning model, and a substantial number of extra constraints is added to incorporate crew availability, slot allocation and maintenance. The objective sums the cost associated with reassigning flights, operating costs, predetermined passenger revenue spill costs and operating revenue. A tree-search heuristic and a set packing-based optimal solution method are proposed. Each of the developed methods is based on a three-phase procedure: first, potential flight sequences adhering to all operational constraints are generated, second, the sequences are assigned to operating aircraft, and finally, the structure of the problem is revised. The case studies have multiple aircraft types, 35–177 aircraft, 180–612 flights and 15 or 37 airports.

#### 2.1.6. Metaheuristic approaches

As the field of aircraft recovery became more popular, contributions based on metaheuristics began to appear. Argüello et al. [8] describe a heuristic approach based on a Greedy Randomized Adaptive Search Procedure (GRASP) for the reconstruction of aircraft routes when one or several aircraft are grounded. Maintenance is not considered and the method is only made for a single fleet recovery



situation. An initial solution based on the cancellation of the affected flight is altered using three different neighborhood operations: flight route augmentation, partial route exchange and simple circuit cancellation. The method is tested on B757 fleet data from Continental Airlines with 16 aircraft and 42 flights. The results obtained by the proposed method are clearly superior to just cancelling the flights serviced by the grounded aircraft.

Optimization methods based on local search are presented in Løve et al. [40], based on the master thesis by Løve and Sørensen [39]. The heuristics are based on a network formulation, where nodes are either aircraft or flights. Assigning an aircraft to a given flight is done by selecting the edge connecting the aircraft and flight for the solution. Based on this representation the existing solution is altered by swaps that exchange flights between two aircraft. Using the so-called “ghost aircraft”, ferrying and cancellations can be incorporated. The actions are weighted in the objective function, which makes it possible to obtain solutions with different characteristics by changing the weights accordingly. Although “true” weights are difficult to assess, this approach has been used by several researchers, e.g. Andersson [5]. For more on the estimation of passenger delay costs, fuel burn, etc. see Cook et al. [22]. The data used in Løve et al. [40] are randomly generated. However, a feasibility study on real data from British Airways with 80 aircraft, 44 airports, and 340 flights has been carried out as part of the DESCARTES project described in Kohl et al. [33]; this confirmed the results from the randomly generated data.

The issue of generating multiple solutions with different characteristics is embedded in both the tabu search and simulated annealing metaheuristics presented in Andersson [5]. Both metaheuristics are based on a local search subroutine that allows flights to be delayed or cancelled and planes to be swapped. The representation of the solution is quite similar to Argüello et al. [8]. The neighborhood is defined by first selecting two aircraft and then adding their flights to the cancelled flights to form a pool. From this pool a new route for each aircraft is generated. The heuristics in the paper produce a set of ranked, structurally different (non-dominated) solutions. Tests are carried out on instances originating from a Swedish domestic carrier. Both methods produce encouraging results, although the tabu search consistently produces better results for the same computing resources. For both methods good solutions are often found in less than 60 s, sometimes even below 15 s.

Liu et al. [37] use multi-objective evolutionary algorithms to construct new feasible aircraft routings. The method only considers a single fleet. The options are to swap flights between aircraft or delay flights. Cancellation is not allowed, nor is ferrying. The objective function consists of three terms: delay costs, swap costs and a cost of assigning a given aircraft to a specific flight. The chromosome in the algorithm represents the allocation of flights to specific aircraft. The developed algorithm was tested on the flight schedule of a Taiwan domestic airline consisting of seven aircraft and 70 flights. No running times are given. In Liu et al. [38], the approach is extended to a multi-fleet airline. It is, however, difficult to extract the extensions made to accommodate multiple fleets. In the flight schedule used as a test case, the problem is decomposed into separate problems for each fleet.

### 2.1.7. Special cases

The problem of optimizing under the Ground Delay Program (GDP) of the US aviation authorities (FAA) addressed in Luo and Yu [41–43] can be considered a special case of the aircraft recovery. The problem can be defined as follows. Given a set of arriving flights and a set of landing slots provided by the FAA, the landing of the incoming flight must be adjusted in order to minimize the maximum delay of outgoing flights. The problem is modelled as an assignment problem with side constraints. The authors develop valid inequalities to further strengthen the formulation, and a heuristic based on solving

the landing assignment problem is developed. The two papers Luo and Yu [41,43] are identical.

The paper Talluri [61] investigates a special case of changing the assignment of equipment type for a specified leg while maintaining feasibility of the schedule. Central to the approach is the *swap opportunity* which is defined as change of equipment type where the new assignment is also valid. The method is based on classifying swap opportunities based upon the number of overnight equipment changes involved in the swap. The paper proposes a polynomial time algorithm for solutions with same-day swaps. Testing is very limited and only documented by a single instance.

A grey programming approach is presented by Zhao and Zhu [80]. Grey programming is a part of the grey system theory, where a system is called “white” when the system information is fully known, “black” when the system information is unknown, and “gray” when the system information is partially known. The authors transform the model of Jarrah et al. [31] to the concepts of grey programming. The approach includes surplus aircraft and the possibility of delays and cancellations. Testing is only done on one very small case of six crew members and 20 flights. Two schedule solutions to the test problem are presented, without describing the essence of the disruption or the computational details.

A view which is seldom adopted in the recovery literature is the view of the airport. The paper by Filar et al. [26] describes techniques that enhance the utilization of airport capacities. In addition, methods that limit damage or provide recovery in disruptive situations are reviewed. The paper describes methods involving the traffic management, airport authorities and airlines.

## 2.2. Crew recovery

The majority of publications formulate the crew recovery problem under assumption that the flight schedule is recovered before the crew re-scheduling decisions are made, thereby following the hierarchical structure of the disruption recovery in practice. These publications include Wei et al. [69] and Song et al. [57], which are almost identical, Stojković et al. [60], Guo [29], Nissen and Haase [47], and Medard and Sawhney [45]. When the flight schedule is fixed, the crew recovery problems can be formulated as tactical crew pairing or rostering models. Other authors extend the classical crew scheduling formulation of the recovery problem by adding a set of decision variables, which allow to cancel flight legs. This formulation is presented in Johnson et al. [32], Lettovsky et al. [36] and Yu et al. [78]. Finally, problem formulations of the crew recovery problem, which explicitly account for departure delays, are reported in Stojković and Soumis [58,59], Abdelghany et al. [1] and Zhao et al. [81]. Each minute of departure delay is given a cost in the objective function, while the flight precedences and delay limitations are ensured by constraints in the models.

The real-time nature of the recovery problems requires short computation times, which can be achieved by reducing the dimensions of optimization problems. Two general methods for reducing the problem space are used by various authors. First, the *time window technique* is applied. A part of the flight schedule used in the recovery problem is limited by a time window, which spans from the time the disruption occurs up to a certain number of hours into the future. The length of the time window varies from a few hours until the rest of the day of operation. Second, the number of crew members included into the recovery is limited by only including the *affected crew members* and a number of selected *candidate crew members*. The set of candidate crew members is necessary in order to expand the solution space and ensure that a high quality solution can be found. An example of reducing the recovery problem space is shown in Fig. 5, following the notation of Medard and Sawhney [45]. The authors refer to the fixed part of the schedule before and after



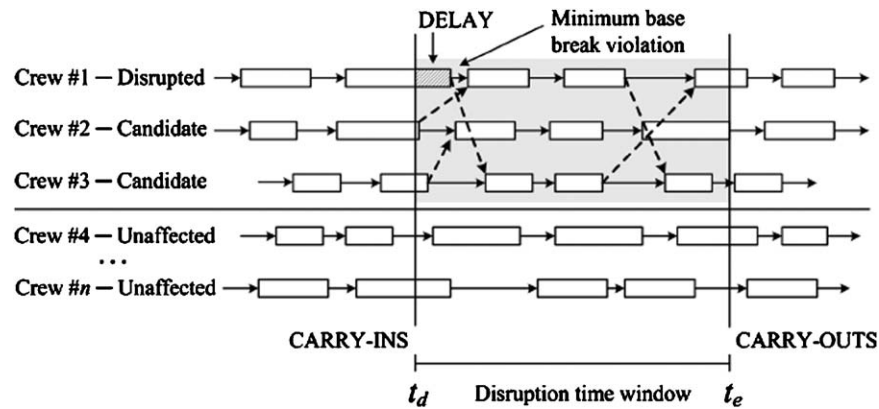


Fig. 5. Reducing the problem space of a crew recovery problem.

the disruption as the *carry in* and the *carry out* flights, respectively. In the given example a single crew member is delayed on the second flight leg of the day. The disruption leaves the schedule infeasible, since the rule that specifies the minimum duration of a break at the base station is violated. The time window is then defined from the time of the disruption,  $t_d$ , until the end of the day of operation,  $t_e$ . Two candidate crew members are used to construct the recovery solution, whereas the remaining crew rosters are left unaffected by the disruption.

In the remaining part of this section we review literature contributions dedicated to the recovery of crew resources, with or without a possibility to alter the flight schedule by delays or cancellations. The proposed methods are summarized in Table 3, where the publications are listed in order of appearance in the literature.

### 2.2.1. Crew recovery with fixed flight schedules

Wei et al. [69] and Song et al. [57] model the crew pairing repair problem as an integer multicommodity network flow problem on a connection network. The challenge is to repair the pairings that are broken and the objective is to return the entire system to the original schedule as soon as possible while minimizing the operational cost. The authors use the time window technique including only a fraction of the full schedule into the recovery problem. The problem is formulated as a generalized set covering problem, which is solved using a depth-first branch-and-bound search algorithm.

Stojković et al. [60] use the time window technique and the set of crew candidates in order to limit the solution space of the crew recovery. The problem is formulated as an integer nonlinear multicommodity network flow problem, which is decomposed into a set partitioning master problem and a shortest path with resource constraints subproblem. A column generation procedure within a branch-and-bound tree is used to obtain integer solutions, and an early branching strategy is applied to accelerate the solution process. Solutions to the one-day period test scenarios were found within half a minute, while the seven-days period test scenarios took between 4 and 20 min to resolve.

Medard and Sawhney [45] stress that the crew recovery problem is structurally different from the crew pairing and rostering problems because contrary to the planning phase, these two subproblems have to be solved at the same time in the recovery phase. This means that both rules on the pairing and the rostering level have to be respected. Thus, the recovery challenge is to merge the pairing characteristics into a rostering problem which is modelled at the flight level. The authors propose an optimization model which is the flight-based equivalent to the original pairing-based rostering model, where the re-assigned flights replace the pairings. The optimization model is formulated as a set covering model, which

is solved using column generation. The columns are generated by finding shortest paths using either a depth-first search strategy or a reduced cost column generator. The latter approach performs worse than the depth-first search method due to a large overhead for setting up the duty network, and the authors conclude that the column generation framework has to be refined. Single base test problems are resolved within approximately one and a half minute, while the multi-base test problems could take several minutes to be resolved.

Nissen and Haase [47] propose a duty-based formulation for the crew recovery problem, which is different from the earlier published modelling approaches. This modelling approach is especially well suited for solving the crew disruption for European airlines, as these, contrary to the North American airlines, employ fixed monthly crew rates, which should be taken into consideration when solving a crew disruption. The duty-based approach means that the disruption is resolved within each duty period. This implies shorter recovery horizons which leads to a reduction of the problem size. The authors propose a branch-and-price-based solution method using a set covering formulation as the master problem and a resource constrained shortest path as the pricing problem. The approach is tested on a number of scenarios covering realistic disruptions from the delay of single flight to a several hour long closure of a hub airport. The authors conclude that the running times are acceptable for operational environment for the best choice of carefully tested model parameters, e.g. the length of the recovery period.

Guo [29] presents a decision support framework for recovering airline crew rosters, which is also reported in Guo [30]. The crew recovery problem is formulated as a set partitioning problem, aimed at minimizing the modifications from the planned schedule. Two solution methods are implemented, a standard column generation with LP relaxation of the set partitioning problem and a heuristic method based on a hybrid of a genetic algorithm with a local search. At the preprocessing step, a solution algorithm is chosen using the *strategy mapping*, which is the main focus of the article. The strategy mapping provides a method to prioritize alternative solution methods for solving the crew recovery problem by evaluating different criteria, such as additional cost for recovering the schedule, solution time, the number of crew members that need to be notified, the period of time starting from the first updated flight to the last one, and the number of disturbances to crew. A solution strategy is a combination of solution methods (column generation and genetic algorithm) and relevant parameters, as for instance the length of the recovery period. A case study containing data from a European airline with several home bases is presented. The disruption involves two delayed flights, one cancelled flight, two new flights, 188 crew members and 85 daily flights on average during a five days recovery period. The authors present the way to compare three chosen

**Table 3**

Overview of proposed methods for the crew recovery problem.

Authors	Year	Functionality			Data	Dimensions		Solution time	Objectives
		Canx	Retime	Indiv. roster		Crew	Flights		
Johnson et al. [32]	1994	Yes	No	No	NA	NA	NA	NA	Pairing, stand-by, deadheading costs
Wei et al. [69]	1997	No	No	No	G	18	51	6	Pairing cost
Stojković et al. [60]	1998	No	No	Yes	RL	32	210	1200	Pairing, deadheading, undercovering costs
Letovsky et al. [36]	2000	Yes	No	No	RL	38	122	97	Pairing, cancel flight costs
Stojković and Soumis [58]	2001	No	Yes	Yes	RL	59	190	13	Modifications, uncovered flights, flight delays.
Yu et al. [78]	2003	Yes	No	No	RL	NA	40	321	Deadheading, modifications, uncovered flight costs
Guo [29]	2004	No	No	Yes	NA	NA	NA	NA	Stand-by, modifications, operating costs
Abdelgahny et al. [1]	2004	No	Yes	Yes	RL	121	NA	2	Deadheading, stand-by, swap, flight delay costs
Stojković and Soumis [59]	2005	No	Yes	Yes	RL	177	190	5105	Modifications, uncovered flights, flight delays
Nissen and Haase [47]	2006	No	No	Yes	RL	NA	860	345	Modifications to original schedule
Medard and Sawhney [45]	2007	No	No	Yes	NA	885	NA	840	Illegal crew, uncovered flights, and affected crew
Castro and Oliveira [16]	2007	No	No	Yes	RL	NA	NA	25	Crew cost
Zhao et al. [81]	2007	No	Yes	Yes	G	6	20	NA	Crew, flight delay cost

Solution times are in seconds.

criteria in order to choose between two solution method strategies, and conclude that in the presented case study the genetic algorithm solution method is preferred to the column generation method, and produces an acceptable solution within approximately 3 min.

An implementation of the distributed multi-agent system (MAS) represents the operations control center of an airline is presented in two very similar publications by Castro and Oliveira [16,17]. The MAS includes a crew recovery agent, an aircraft recovery agent and a passenger recovery agent. The papers are focused on the architecture and test experiments of the crew recovery agent. A monitoring agent class of the crew recovery agent is responsible for monitoring crew events (e.g. non-assignments for some flights) and reporting to the crew finder agent class. The crew finder collects a list of solutions to the problem from the algorithmic agent classes and chooses the cheapest one using the crew payroll information. The authors do not mention what kind of algorithms and heuristics are used in the algorithmic agent classes to find solutions to the recovery problem. Only one test scenario was reported, where 15 crew members with different ranks were set to be absent from their duties at the same base. The results produced by the MAS were compared to the results obtained by a human operator, comparing the solution times and the costs of solutions expressed through the crew payment. The MAS recovery agent came up with a cheaper solution in 25 s compared to the one and a half minutes, which took the operator to find a solution.

### 2.2.2. Crew recovery with flight cancellations

To our knowledge, the 1994 paper of Johnson et al. [32] is the first published work regarding the airline crew recovery. The problem is formulated as a set covering problem with decision variables allowing flight cancellations, determining the number of deadheading crew on a flight leg and forcing crew to stay at base in the recovery solution. The authors consider the recovery of pilot pairings when a single flight is delayed at a single airport. The approach for identifying crew to be involved in the recovery solution is proposed. Experiments are conducted based on data files supplied by Northwest Airlines. All pairings for the crew recovery problem are generated a priori from a time-line network and the set covering problem is solved using MINTO [46]. Three small test scenarios are described, but the running times are not presented. This research laid the ground for the work by Letovsky et al. [36], who use the same problem formulation. Preprocessing techniques are used to extract a subset of the schedule for rescheduling. A fast crew pairing generator constructs feasible continuations of partially flown crew trips, and an efficient tree-based data structure is used for storing generated pairings. The crew recovery model is solved with LP relaxation

and a branch-and-bound. A three-step branching strategy is used, resolving first cancellation variables, then deadheading variables, and finally performing constraint branching. Test results based on data from a US carrier demonstrate that the applied techniques can be used for managing medium-sized disruptions. The authors conclude that further research is required to handle large-scale disruptions.

Yu et al. [78] report a successful implementation of a crew recovery decision support system CrewSolver in Continental Airlines. The system is interconnected with other systems of the airline. The optimization engine of CrewSolver uses the depth-first search procedure developed by Wei et al. [69], and can generate several solutions to give the operator a flexibility to choose the most suitable recovery solution. Reported test problems with up to 20 affected flights are solved within 1 min, while it takes between 3 and 5 min on average to resolve larger instances.

### 2.2.3. Crew recovery with departure delays

Stojković and Soumis [58] extend the crew recovery problem of Stojković et al. [60] with a possibility to delay scheduled flights explicitly through the problem formulation. Some flights have fixed departure times, some others have more flexible times in terms of a flight specific time window. The problem is formulated as a multi-commodity network flow with additional constraints, and is solved using column generation with a master problem and a subproblem per pilot. The solution may include the use of reserve pilots, treated as extra artificial commodities in the problem. The model and solution method has been tested on three problems. The largest problem has 59 pilots and 190 flights, of which 52 are originally delayed. All problems are tested with and without reserve pilots, allowing delays of flights and with a fixed flight schedule. The results are encouraging, both in terms of quality and in terms of computing times. Stojković and Soumis [59] builds on the model derived in Stojković and Soumis [58], but extends this to work with multiple crew members. This makes the situation addressed more realistic. The extension is achieved by using a number of copies of each flight corresponding to the number of crew required. A set of constraints ensure that the departure times for all copies of each flight are added to the model. The solution process is similar to that described in Stojković and Soumis [58]. Three different models are tested: one corresponding to that from the previous work with strict flight covering constraints, one in which there is a linear cost for missing crew members, and one with a cost for each flight with missing crew. It is demonstrated that using both the second and the third model, substantial improvements compared to the initial situation can be obtained. However, the solution times experienced for large problems are prohibitive in an on-line situation (more than an hour).

Abdelgahny et al. [1] address the problem of flight crew recovery for an airline with a hub-and-spoke network structure. Several preprocessing steps are applied, including shifting the problem occurrences from the spokes to hubs, adding undisturbed crew to the recovery in order to cover open flights, grouping flights into resource-independent sets, etc. The assignment of crew members to flights is formulated as a mixed integer program, where linear variables for flight departure times allow to minimize the total flight delay in the objective function, while the assignment variables take care of the minimum cost crew assignment to the flights. The recovery horizon is divided into a set of consecutive stages, and the crew recovery problem is solved at each recovery stage in a sequence, using CPLEX Callable Library solver. One disruption scenario from the operations of a major US airline is used as a test case with 18 disrupted crew members and 121 candidate crew members. The number of affected flights is not listed in the paper. The recovery problem was solved within 2 min. The problem formulation of Abdelgahny et al. [1] is transformed into a grey programming model by Zhao et al. [81]. Linear decision variables for departure and arrival times of flights and the linear parameter defining the ready time for crew in the model of Abdelgahny et al. [1] are defined as grey variables in Zhao et al. [81]. A local search heuristic is applied to solve the problem. The basic framework of the implementation is the same as Zhao and Zhu [80], which considers the aircraft recovery. The authors present the same small test case as in Zhao and Zhu [80] without any further details.

### 2.3. Integrated and passenger recovery

Integrating the recovery of several resources (aircraft, pilots, flight attendants) in the same system is a difficult task, and only a few attempts to integrate resources has been presented in the operations research literature. The 1997 Ph.D. thesis of Lettovsky [35] is the first presentation of a truly integrated approach, although only parts of it are implemented. The thesis presents a linear mixed-integer mathematical problem that maximizes total profit to the airline while capturing availability of the three most important resources: aircraft, crew and passengers. The formulation has three parts corresponding to each of the resources, that is, crew assignment, aircraft routing and passenger flow. In a decomposition scheme these three parts are “controlled” by a master problem denoted the Schedule Recovery Model (SRM). The solution algorithm is derived by applying Benders decomposition. The SRM determines a plan for cancellation, delays and equipment assignment considering landing restrictions. Then for each equipment type  $f$  the ARM <sub>$f$</sub>  (aircraft recovery model) is solved, and for each crew group  $c$  the CRM <sub>$c$</sub>  (crew recovery model) is solved returning Benders feasibility or optimality cuts to the SRM. Finally, the PFM (passenger flow model) evaluates the passenger flow. In this way the built-in hierarchy of the framework to a large extent resembles the present manual process at many airlines.

Another work on integrated recovery is reported by Abdelghany et al. [2]. The authors address the situation, where a Ground Delay Program is issued by the US authorities, often due to anticipated adverse weather conditions. A proactive schedule recovery tool DSTAR aims at integrating aircraft, cockpit crew and cabin crew. The recovery process is divided into separate stages in a rolling horizon. Every stage is limited by a number of independent flights that cannot share resources and can therefore be re-assigned to aircraft and crew at the same stage without “competing” for the same resources. At each stage, a simulation model produces the list of disrupted flights for the rest of the horizon, and an optimization solver makes minimum cost resource assignments. The authors present a mixed integer program similar to the formulation of Abdelgahny et al. [1], but where several resources can be rescheduled and flight legs can be cancelled. The authors describe an application scenario with 522

cockpit crew, 1360 pilots, and 2040 cabin crew, where DSTAR saves 8.7% of the total delay. Also, a systematic experimental investigation is reported, which shows that DSTAR offers short computing times and gives savings of approximately 5%. The approach of the paper is very promising when considering larger disruptions, which are foreseeable a number of hours ahead.

The next two papers focus on passenger recovery. Recovering passenger itineraries is an important issue for the airlines, not only due to the operational costs related to passenger delays, but also because continuous flight delays and cancellations can lead to major passenger dissatisfaction and a potential loss of goodwill. When recovering passenger itineraries, it is, however, important to ensure feasibility in the aircraft and crew schedules. Hence, we consider the passenger recovery being an integrated recovery task.

Bratu and Barnhart [13] present two passenger recovery models, which allow to delay or cancel flight departures, and assign reserve crew and aircraft to the flight legs. The models are based on the flight schedule network, where every flight is represented by several arcs, one for each departure time, in order to incorporate the re-timing of flight departures. The same technique is used in e.g. Andersson [4] and Thengvall et al. [66]. While crew regulations for the reserve crew are incorporated into the models, recovery of the disrupted crew is not considered. In the passenger delay model (PDM) delay costs are modelled more exactly by explicitly modelling disruptions, recovery options and delays costs, whereas in the disrupted passenger metric (DPM) model delay costs are only approximate. Based on the single instance for which both methods are tested, the execution time for PDM is roughly a factor 25 higher than for DPM. An operations control center simulator is developed in order to test the models, and data from the domestic operations of a major US airline are provided. The data set contains 302 aircraft divided into four fleets, 74 airports and three hubs. Furthermore, 83,869 passengers on 9925 different passenger itineraries per day are used. Three different scenarios with different levels of disruption are presented. Execution times ranges from 201 to 5042 s. Due to its excessive execution times the PDM is considered unfit for operational use. For all scenarios the DPM generate solutions with noticeable reductions in passenger delays and disruptions.

Another recent work on the passenger recovery problem is reported by Zhang and Hansen [79]. The authors introduce ground transportation modes as an alternative to the passenger recovery by air during disruptions in hub-and-spoke networks. An integer model with a nonlinear objective function allows to substitute flight legs with other form for transportation, respecting the ground transportation times. The objectives of the model are aimed at minimizing passenger costs due to delay, cancellation or substitution, as well as at minimizing the operating cost of the transportation. The problem is solved by first relaxing the integrality constraints and then solving the nonlinear program. After fixing all flight decision variables with the value greater than 0.5 to one and the rest to zero, the original problem is solved in order to find the values for decision variables representing the number of passengers on departure flights. A numerical example includes 40 flights in a 4 h time window. The authors present test results for a disrupted situation, where the capacity of the hub airport drops to half the normal level for 5 h. If the ground transportation substitution is not allowed, 90 passengers are disrupted. With intermodal substitution the number of disrupted passengers decreases to 14. The running time for solving the problem is approximately 1000 s.

### 3. Disruption management by robustness

An interesting research topic closely related to disruption management is *robust planning* or *schedule robustness*. The goal in robust planning is to make flight and crew schedules and aircraft rotations

less sensitive to disruptions. Robustness can be seen as a pro-active way of handling disruptions. The central idea is to incorporate the possibility to either absorb disruptions and remain feasible or facilitate an easy recovery in case of a disruption by enhancing the possibilities of different types of recovery actions. A report from the US Government Accountability Office [27] mentions examples of already established manual methods used by airlines to improve robustness. Among those there are adding buffers in the schedules, having standby reserve crew, partitioning aircraft or crew schedules into sections in order to keep delays occurred in one section from spreading to the remaining flight network.

The first type of robustness is *absorption robustness*. The general goal is to keep the plans feasible in case of smaller disruption events and avoid the knock-on effects. The most obvious way of introducing absorption robustness is to build time buffers into the schedule. However, buffers require a trade-off between robustness and cost: the larger the buffers are, the more robust and costly the operation is in general. There are two important issues when applying the absorption robustness to the schedule. Firstly, how much buffer time is an airline willing to invest in order to increase robustness of the operation, and secondly, exactly where should the buffer times be placed in the schedule? Another way to introduce absorption robustness is to avoid short turn-around times when building the schedules. It should be mentioned that the absorption robustness alone is not able to handle severe disruptions. In case of major disruption good recovery measures are required in order to restore the operation.

*Recovery robustness* aims at designing rotations and schedules so that the plans fit well to the existing recovery strategies when a disruption occurs. The most common recovery strategies are crew and aircraft swaps, delaying flight departures and cancelling flights covered by the same aircraft rotation cycle. The schedules with incorporated recovery robustness have many possibilities for crew swapping or many short cycles in the aircraft rotations. These plans are more expensive than cost-optimal plans. However, considering the cost of recovery from disruptions, the recovery robust schedules turn out to be cheaper than cost-optimal schedules. Again, there is a trade-off between the cost and the robustness of the schedule.

Robust planning is a difficult task, particularly due to the unpredictability of disruptions. Recently there have been efforts to set up a theoretical framework to evaluate and quantify robustness. The situation, however, resembles that of approximation algorithms: if mathematical proofs are to be given, the results turn out to be weak, and the methods and strategies are far from those known to be most efficient in practice. The need for simulation tools is therefore apparent, and a number of such tools have been built, the most well known and complete being SimAir developed by Rosenberger et al. [49]. More and more publications on robust scheduling appear in the literature. Rosenberger et al. [51], Smith and Johnson [55] consider robust fleet assignment, Ageeva [3], Wu [70], Lan et al. [34] deal with robust aircraft schedules, and Ehrgott and Ryan [24], Schaefer et al. [53], Schaefer and Nemhauser [52], Yen and Birge [75], Shebalov and Klabjan [54], Sohoni et al. [56], Tekiner et al. [62] study robustness in crew schedules, among others. For a comprehensive review of robust planning publications, terminology and concepts, refer to Clausen and Rezanova [21].

#### 4. Discussion and further research

The field of disruption management in the airline industry has been increasingly active over the last decade. In the last years commercial tools for disruption management have also become available. The majority of the publications concentrate on dedicated crew or aircraft recovery, and only a very few researchers integrate recovery of resources during the day of operation. As it is today, the airlines' requirements for recovery decision support systems are still

substantially different from the services offered by commercial tools and from most of the prototype tools proposed in the literature.

The optimization problem formulations of recovery models are more or less identical to the formulations of the tactical scheduling problems, like multicommodity network flow problems or set covering/partitioning problems with side constraints. Researchers apply solution methods similar to the state-of-the-art within solution approaches for the tactical planning problems, while reducing the problem space by limiting the recovery time period and the number of crew members or aircraft included into recovery.

The concept of robust planning and methods for achieving robustness in schedules has received an increasing interest over the last years, and promising results have begun to appear. Robustness can be seen as the proactive counterpart of recovery, and we believe that these two concepts will be central in the process of minimizing the effect of disruptions on the daily operation of airline companies.

There is a large number of subjects for further research within the field of disruption management. We mention just a few of these here: quality versus computing time for both dedicated and integrated recovery methods, disruption management versus robustness, disruption management and robustness for other transportation industries, e.g. the railway industry. Therefore we expect that disruption management will be a very active research area over the coming years, both in the context of transportation, and more generally within other areas of logistics, e.g. as a part of supply chain management optimization.

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