

# Chapter 1

## Introduction

(TODO) Set the scenes. Explain why you are doing this work and why the problem being solved is difficult. Most importantly you should clearly explain what the aims and objectives of your work are.

(TODO) Structure of the thesis. Academic publications produced (if any), including any achievements/highlights

[1]

## Chapter 2

# Literature Review

### 2.1 Optimisation in Air Travel

#### 2.1.1 Airline Scheduling Optimisation

In [1], airline scheduling optimisation is highlighted as a critical driver of revenue for airlines. A significant challenge of airline scheduling optimisation toThis is primarily cause by improper aircraft assignment, one major particularly regarding capacity, can lead to significant inefficiencies. The complexity of this problem is further compounded by the extensive number of daily flights that an airline must manage.

Traditionally, airline scheduling has been approached as a standalone problem with simplified assumptions, such as fixed daily flight times and point forecasts for passenger demand. However, this traditional approach has significant limitations. The modern approach, in contrast, integrates the Fleet Assignment Problem (FAP) with other airline processes, such as maintenance and crew scheduling, leading to more accurate and flexible scheduling that better optimises resources and improves operational efficiency.

#### 2.1.2 crew scheduling problem

The Crew Scheduling Problem (CSP), as discussed in [2], is distinct from the broader set covering and partitioning problems. CSP involves assigning crews to a sequence of tasks, each with defined start and end times, with the primary objective of ensuring

that all tasks are covered while adhering to regulations on maximum working hours for crew members.

This problem is particularly critical for low-cost carriers, which rely heavily on optimised crew schedules to maintain competitiveness. Efficient crew scheduling is essential not only for cost minimisation but also for ensuring operational reliability and flexibility in response to unexpected disruptions [3].

### 2.1.3 disruption management

Disruptions in airline operations, as noted in [4], can occur due to various factors, including crew unavailability, delays from air traffic control, adverse weather conditions, or mechanical failures. Given that flight schedules are typically planned months in advance [5], effective disruption management is crucial to minimise the impact on passengers and overall airline operations.

The two main drivers of disruption management are aircraft recovery and crew recovery.

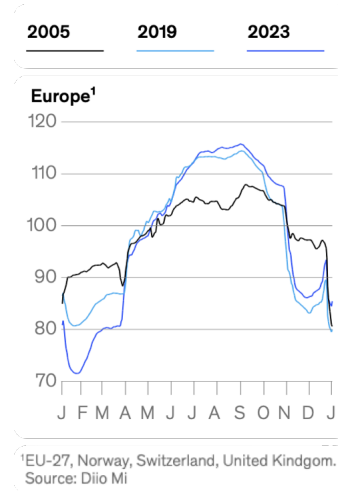
- **Aircraft recovery:** Reassigning aircraft to flights after disruption by taking care of schedules, airports availability, maintenance
- **Crew recovery:** reassigning crew members to flights by respecting all flights are staffed appropriately

### 2.1.4 Airline adaptation to new demand

Airlines must continuously adapt their schedules to meet evolving market demands, particularly with the growing dominance of leisure travel over business travel, which has introduced new patterns of demand seasonality, especially in Europe. This seasonality poses a challenge for airlines as they balance high demand during peak seasons with the risk of underutilisation during off-peak times.

The following are key strategies used by airlines to adapt to seasonal demand:

- **Summer Peak:** Airlines schedule 65% more seats in August than in February, deploying additional aircraft and crew to meet the heightened demand. Optimisation models help determine which routes to prioritise, the number of additional



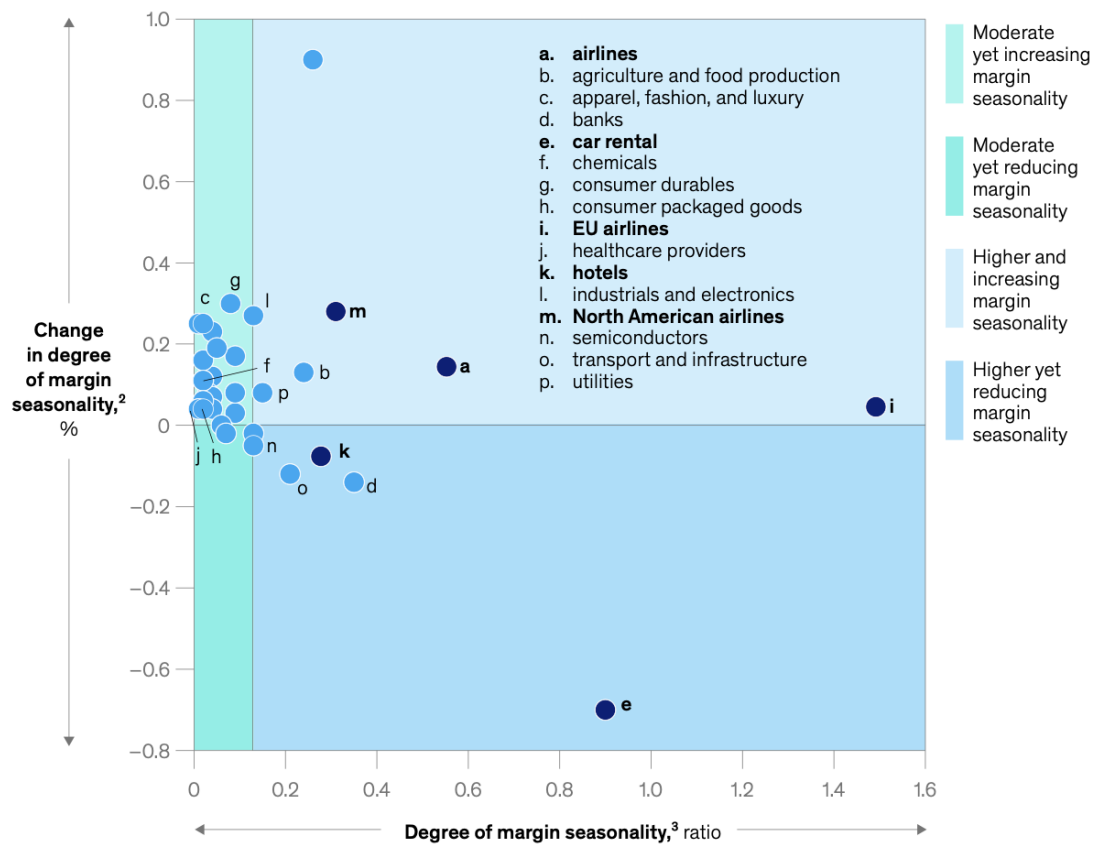
flights to add, and how to manage crew rotations to ensure smooth operations under high resource pressure.

- **Winter Through:** During winter, airlines face low demand, with many aircraft potentially sitting idle. To optimise operations, airlines may engage in ACMI leasing, temporarily reducing their owned fleet, and outsourcing capacity to adjust dynamically to lower demand. Maintenance activities are ramped up, and crew members are encouraged to take vacations or attend training, thereby optimising overall productivity.
- **Revenue Management:** Dynamic pricing algorithms adjust fares in real-time based on demand patterns. In summer, prices are optimised to capture the maximum willingness to pay from tourists, while in winter, prices might be adjusted to stimulate demand, filling otherwise empty seats and ensuring higher aircraft utilisation.

### 2.1.5 Flight connection challenges

The problems of flight connections have been in the limelight for the past two years owing to the apparent difficulties of coming up with fast, inexpensive multi-city itinerary solutions. Air travel being a significant mode of locomotion for most of the world's population, travelers are using online avenues to find the best options in flights.

One of the well-known challenges in this respect is the Flying Tourist Problem, which aims at constructing the best multicity journey, given user-defined criteria such as minimal cost and travel time and maximal convenience. It is not just a matter of finding

Link between margins and season, by sector<sup>1</sup>

<sup>1</sup>Analysis based on top 5,000 companies, by market capitalization. Operating-margin seasonality used as measure to look at quarterly operating-profit variability. Car rental based on Hertz and Avis Budget Group data.

<sup>2</sup>Year-over-year change in ratio of standard deviation of weighted average operating margin to simple average operating margin, Q1–Q3 2019 vs Q1–Q3 2023.

<sup>3</sup>Ratio of standard deviation of average operating margin to simple average operating margin, Q1–Q3 2019.

Source: S&P Capital IQ; The Airline Analyst

FIGURE 2.1: Compared with other sectors, airlines exhibit a significant and growing link between margins and seasons.

the cheapest flights, but working out an itinerary that efficiently joins a number of destinations, considering a variety of travel constraints.

The other major problem when dealing with the constraint of time-bound travel concerns travelers who need to adhere to certain schedules, such as spending some stipulated time at each or following the order of visits in some particular sequence. This adds a little complexity into planning, as it involves making sure that proper flight connections and timing are in place.

The more elaborate complications in the case of planning optimal flight routes have to do with the Air-Travelling Salesman Project. Here, it is required to find out the best possible sequence of flights between a set of airports with the constraint that most cases

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are there where there are no direct links from one arbitrary airport to another. That makes the problem hard to solve due to the requirement of intermediate stops, while the total traveling distance is at the same time desirable to be as small as possible.

These challenges underline the growing need for sophisticated methods of optimization to cope with the growing complexity of flight connections. With an increased number of airlines, destinations, and flights, efficient and user-friendly solutions for multi-city travel will certainly remain some of the most focused areas by researchers and the travel industry.

## 2.2 Traveling Salesman problem and its adaption

The Traveling Salesman Problem is a well known problem in the Operational Research and Computer Science area. The basic version of the TSP is to find the best roundtrip for a salesman that has to travel around a given number of cities while minimising the overall journey's distance. This problem is characterised as  $\mathcal{NP}$ -Hard [6]. This means that there is no known polynomial-time algorithm that can solve all instances of the problem efficiently. In terms of time complexity, if we were to solve it exploring all the possible solutions the time complexity would have been  $\mathcal{O}(\frac{(n-1)!}{2})$  where  $n$  represents the number of cities.

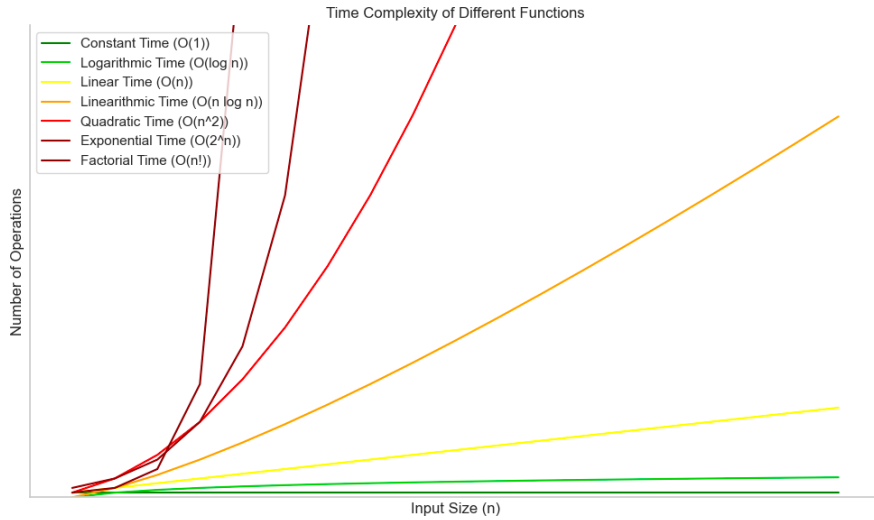


FIGURE 2.2: Time complexity of different functions [7]

On Figure 2.2, different time complexity are compared and the factorial time complexity is the worst. That means that these kinds of  $\mathcal{NP}$ -Hard problem are typically not solved exploiting all the search area but using heuristics algorithms. Heuristics solutions do not guarantee to find the absolute optimal solution but can find near-optimal solutions in a much more reasonable times.

The TSP has been studied a lot, especially because from the TSP il découle de nombreux variantes:

- Symmetric TSP (STSP): The distance between cities are symmetric, meaning that the distance to travel from city A to city B is the same as from city B to city A.

- **Assymmetric TSP (ATSP):** The distance between cities are assymmetric, meaning that the distance to travel from city A to city B is different than the distance to travel from city B to city A. [8]
- **Multiple TSP (mTSP):** Instead of one salesman, multiple salesman are starting from one city visit all the cities such that each city is visited exactly once. [9]
- **Time Window TSP (TWTSP):** Each city has to be visited in a defined time slot. [10]
- **Price-collection TSP (PCTSP):** Not all the cities have to be visited, the goal is to to minimise the overall traveler's distance while maximising the price collected earned when visiting a city. [11]
- **Stochastic TSP (STSP):** The distances between the cities or the cost of travels are stochastic (i.e random variables) rather than deterministic. [12]
- **Dynamic TSP (DTSP):** The problem can change over the times, that means that new cities can be added or distances between cities can change while the salesman has already started his journey. [13]
- **Generalised TSP (GTSP):** The cities are grouped into clusters, the goal is to visit exactly one city from each cluster. [14]
- **Open TSP (OTSP):** The traveler does not have to end his journey at the starting city. [15]

Multiple algorithms have been developed to address these TSP variants, we can classify them into two major categories:

- **Exact Algorithms:** These algorithms aim to find the optimal solution to the TSP by exploring all possible routes or by using mathematical techniques to prune the search space efficiently. Examples include:
  - **Branch and Bound:** This method systematically explores the set of all possible solutions, using bounds to eliminate parts of the search space that cannot contain the optimal solution. It is often used for smaller instances of TSP due to its computational intensity. [16]
  - **Cutting Planes:** This technique adds constraints (or cuts) to the TSP formulation iteratively to remove infeasible solutions and converge to the optimal solution. This approach is particularly effective for symmetric TSPs. [17]



- **Dynamic Programming:** Introduced by Bellman, this approach breaks down the TSP into subproblems and solves them recursively, which is highly effective for specific TSP variants, though its complexity grows exponentially. [18]
- **Approximation and Heuristic Algorithms:** These algorithms are designed to find near-optimal solutions within a reasonable time frame, especially for large-scale problems where exact methods are computationally infeasible. Examples include:
  - **Greedy Algorithms:** These algorithms make a series of locally optimal choices in the hope of finding a global optimum. An example is the Nearest Neighbor algorithm, which selects the nearest unvisited city at each step. [19]
  - **Genetic Algorithms:** Inspired by the process of natural selection, these algorithms evolve a population of solutions over time, using operations such as mutation and crossover to explore the solution space. [20]
  - **Simulated Annealing:** This probabilistic technique searches for a global optimum by allowing moves to worse solutions based on a temperature parameter that gradually decreases. It is particularly useful for escaping local optima. [21]
  - **Ant Colony Optimization:** This metaheuristic is inspired by the foraging behavior of ants and uses a combination of deterministic and probabilistic rules to construct solutions, gradually improving them through pheromone updates. [22]

Some TSP problems (or its variants) have been solved using other algorithms, hence they are less traditionnal than those mentionned.

## 2.3 The Monte Carlo Tree Search algorithm

The Monte Carlo Tree Search (MCTS) algorithm can be characterised less traditional than the previously enounced method in the previous subsection to solve TSP because MCTS is typically used in games. MCTS' (and its variants) have been successfully implemented across a range of games, such as Havannah [23], Amazons [24], Lines of Actions [25], Go, chess, and Shogi [26], establishing it as the state-of-the-art method for many of these games [27], [28], [29]. It is widely use in board games and has been really popular when Google DeepMind developed AlphaGo. AlphaGo is a software that was created to beat the best Go's player in the world.

Go is an ancient board game from China where two players take turns placing black or white stones on a grid. The goal is to capture territory by surrounding empty spaces or the opponent's stones. Despite its simple rules, Go is incredibly deep and complex, with countless possible moves and strategies. It is known for its balance between intuition and logic, hence it has been a significant focus of artificial intelligence research. [30]

In 2016, Lee Sedol [31] - the best Go's player in the world has been beaten by AlphaGo 4-1 [32].

MCTS with policy and value networks are at the heart of AlphaGo's decision-making process, enabling AlphaGo's to pick the optimal moves in the complex search of Go. [33]

### 2.3.1 Overview

The MCTS' process is conceptually straightforward. A tree is built in an incremental and assymatric manner (Figure 2.3). For every iteration, a selection policy is used to determine which node we have to select in the tree to perform simulations. The selection policy balances the exploration i.e. look into part of the tree that have not been visited yet and the exploitation i.e. look into part of the trees that appear to be promising. Once the node is selected, a simulation i.e. a sequence of available actions, based on a simulation policy, are applied from this node until a terminal condition is reached e.g no further actions are possible. [34]

To ensure that the reader understands the various stages of the Monte Carlo Tree Search Algorithm, we will begin by looking at a detailed example. This example will illustrate

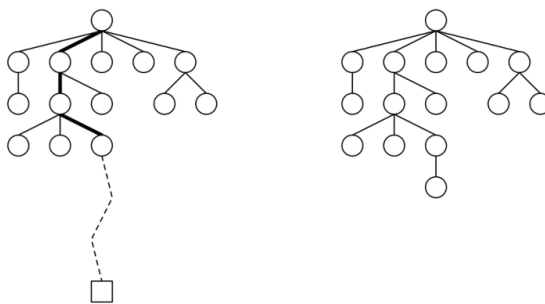


FIGURE 2.3: Assymetrical growth of MCTS - Simulation and Expansion - [35]

each component of the algorithm in action. We will then generalise the principles discussed, as the basic methodology of this paper is based on the application of the MCTS algorithm.

### 2.3.2 Example

Let's say we are given a maximisation problem. When starting the game, you have two possible actions  $a_1$  and  $a_2$  from  $S_0^{0,0}$  in the tree  $\mathcal{T}$ . Every node is defined like so:  $S_i^{n_i, t_i}$  where  $n_i$  represents the number of times node  $i$  has been visited,  $t_i$  the total score of this node. Furthermore, for every node - we can compute a selection metric, here the  $UCB1$  value:  $UCB1(S_i^{n_i, t_i}) = \bar{V}_i + 2\sqrt{\frac{\ln N}{n_i}}$  where  $\bar{V}_i = \frac{t_i}{n_i}$  represents the average value of the node,  $n_i$  the number of times node  $i$  has been visited,  $N = n_0$  the number of times the root node has been visited/the number of iterations.

Before the first iteration, none node have been visited -  $\forall i \in \mathcal{T}, S_i^{0,0}$ . At the beginning

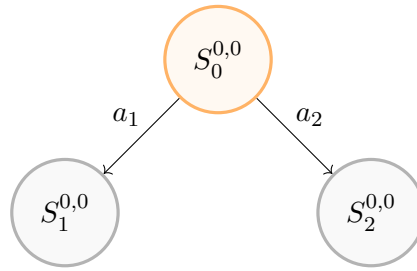
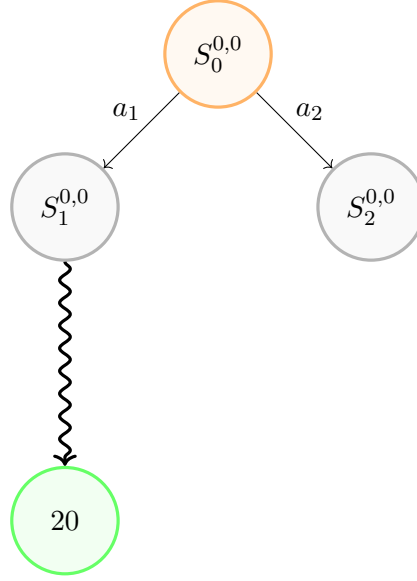


FIGURE 2.4: Selection -  $I1$

of  $I1$ , we then have to choose between these two child nodes (or choose between taking  $a_1$  or  $a_2$ ). We then have to calculate the  $UCB1$  value for these two nodes and pick the node that maximises the  $UCB1$  value (as we are dealing with a maximisation problem). In Figure 2.4, neither of these have been visited yet so  $UCB1(S_1^{0,0}) = UCB1(S_2^{0,0}) = \infty$ . Hence we decide to choose randomly  $S_1^{0,0}$ .

$S_1^{0,0}$  is a leaf node that has not been visited - then we can simulate from this node i.e. selecting actions from this node based on the simulation policy to a terminal state as shown on Figure 2.5:

FIGURE 2.5: Simulation -  $I1$ 

The terminal state has a value of 20, we can write that the rollout/simulation from node  $S_1^{0,0}$  node is  $\mathcal{R}(S_1^{0,0}) = 20$ . The final step of  $I1$  is backpropagation. Every node that has been visited in the iteration is updated. Let  $\mathcal{N}_{\mathcal{R},j}$  be the indexes of the nodes visited during the  $j$ -th iteration of the MCTS:

- Before backpropagation:

$$\forall i \in \mathcal{N}_{\mathcal{R},j}, S_{i,old}^{n_i,t_i} \quad (2.1)$$

- After backpropagation:

$$\forall i \in \mathcal{N}_{\mathcal{R},j}, S_{i,new}^{n_i+1,t_i+\mathcal{R}(S_{i,old}^{n_i,t_i})} \quad (2.2)$$

We can then define a backpropagation function:

$$\begin{aligned} \mathcal{B} : \mathcal{N}_{\mathcal{R},j} &\rightarrow \mathcal{N}_{\mathcal{R},j} \\ S_i^{n_i,t_i} &\mapsto S_i^{n_i+1,t_i+\mathcal{R}(S_i^{n_i,t_i})} \end{aligned}$$

Then, back to our example on Figure 2.6 we update the nodes  $\mathcal{B}(S_1^{0,0}) = S_1^{1,20}$  and  $\mathcal{B}(S_0^{0,0}) = S_0^{1,20}$ .

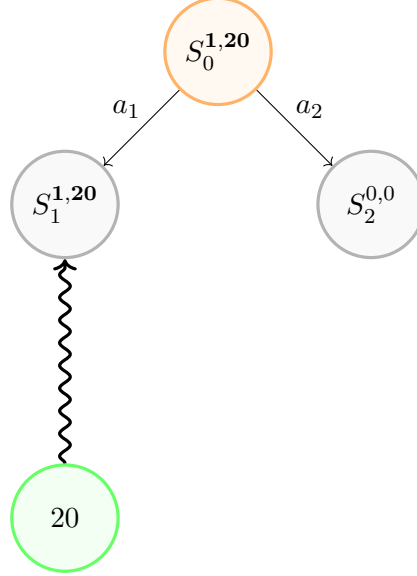


FIGURE 2.6: Backpropagation - I1

The fourth phase of the algorithm have been done for  $I1$ . We can then start the  $2^{nd}$  iteration  $I2$ . On Figure 2.7, we can either choose  $a_1$  or  $a_2$ . When a child node has not

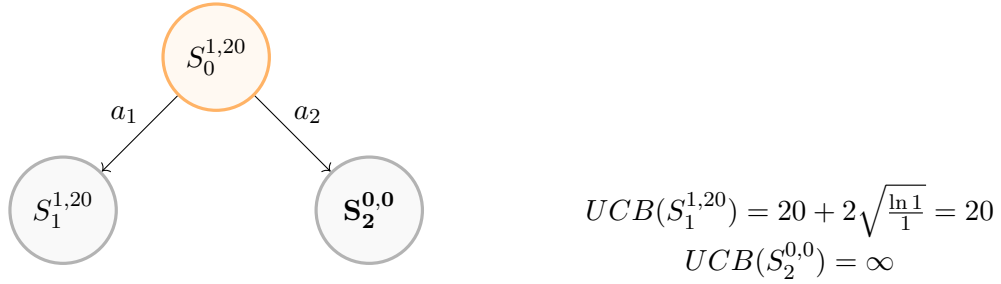


FIGURE 2.7: Selection - I2

been visited yet, you pick this node for the Selection or you can compute the  $UCB1$  value, it leads to the same conclusion.

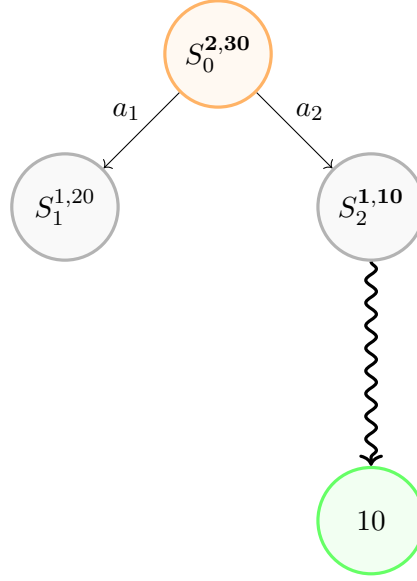


FIGURE 2.8: Simulation and Backpropagation - I2

We can then simulate (Figure 2.8) from the chosen node  $S_2^{0,0}$  and  $\mathcal{R}(S_2^{0,0}) = 10$  and backpropagate all the visited nodes:  $\mathcal{B}(S_2^{0,0}) = S_2^{1,10}$  and  $\mathcal{B}(S_1^{1,20}) = S_0^{2,30}$ . We now start the 3<sup>rd</sup> iteration, based on the *UCB1* score we decide to choose  $a_1$ .

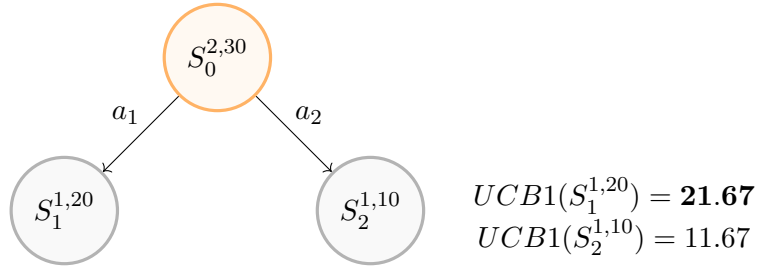


FIGURE 2.9: Selection - I3

$S_1^{1,20}$  is a leaf node and has been visited so we can expand this node.

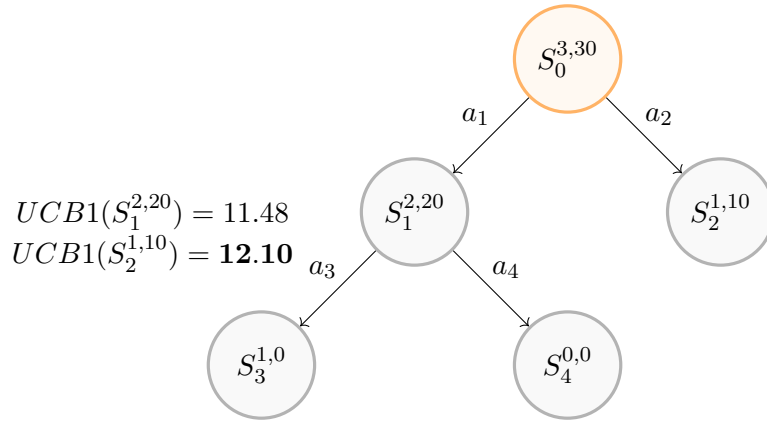


FIGURE 2.10: Selection and Expansion - I3

Based on  $UCB1$  score we decide to simulate from  $S_3^{0,0}$  on Figure 2.11

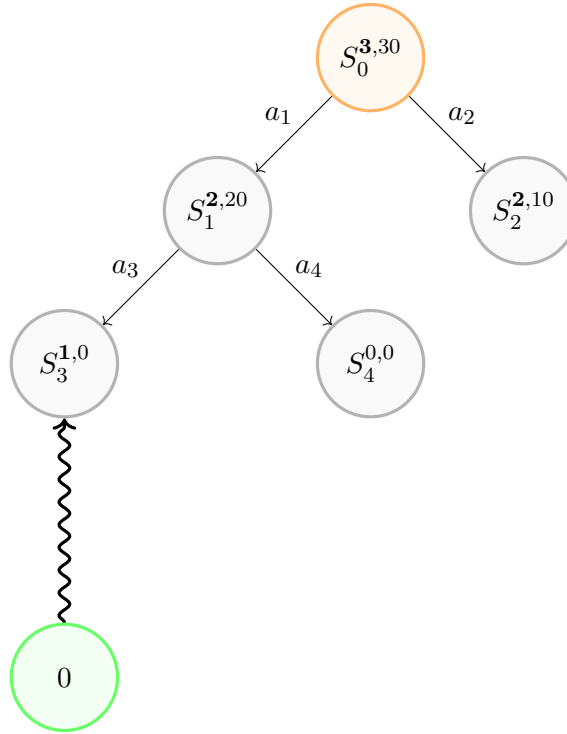


FIGURE 2.11: Simulation and Backpropagation - I3



Let's do the fourth iteration  $I4$  represented on Figure 2.12:

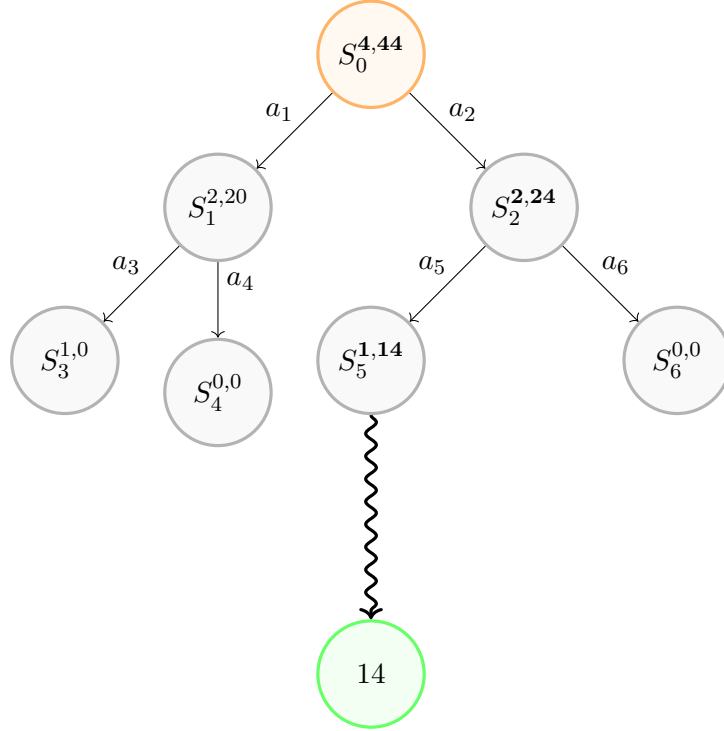


FIGURE 2.12: Selection - Simulation - Backpropagation - I4

If we were to stop at this stage of the algorithm, the best action to undertake is  $a_2$  because it has the higher average value:  $\bar{V}_1 = \frac{20}{2} \leq \bar{V}_2 = \frac{24}{2}$ .

### 2.3.3 The different parameters in the MCTS

#### 2.3.4 Selection policy

[36]

- **Upper Confidence Bound (UCB):**

- *Description:* UCB is a popular selection policy in MCTS that balances exploration and exploitation using a mathematical formulation that considers both the average reward of a node and the uncertainty of that reward.
- *Trade-offs:*
  - \* **Exploration vs. Exploitation:** UCB adjusts the balance between exploring less-visited nodes and exploiting nodes with high rewards.

- \* **Parameter Sensitivity:** The performance of UCB depends on the constant  $C_p$  in its formula, which controls the level of exploration.
- \* **Children's Count:** UCB can lead to varying numbers of children being explored, depending on the setting of  $C_p$ .

- **UCB1-Tuned:**

- *Description:* A variation of UCB that dynamically adjusts the exploration term based on the variance of rewards, making it more adaptive to different scenarios.
- *Trade-offs:*
  - \* **Adaptivity:** UCB1-Tuned can adapt to the variance in the rewards, potentially leading to better performance in complex environments.
  - \* **Computational Cost:** The added complexity in adjusting the exploration term may lead to higher computational costs.
  - \* **Children's Count:** This policy may result in fewer nodes being selected for expansion when the variance is low, focusing more on exploitation.

- **Single-Player (SP-MCTS):**

- *Description:* A variant of MCTS specifically designed for single-player scenarios, incorporating a third term in the UCB formula to account for the uncertainty in node values.
- *Trade-offs:*
  - \* **Exploration of Uncertainty:** SP-MCTS inflates the uncertainty term for less-visited nodes, which can lead to more thorough exploration in single-player settings.
  - \* **Focus on Strong Lines:** This policy tends to favor strong lines of play, potentially neglecting less promising but necessary exploratory paths.
  - \* **Children's Count:** Tends to explore a wider range of children nodes, balancing between known strong strategies and potential new solutions.

- **Bayesian UCT:**

- *Description:* Bayesian UCT integrates Bayesian statistics into the selection policy, allowing for a probabilistic approach to balancing exploration and exploitation.
- *Trade-offs:*

- \* **Probabilistic Exploration:** Bayesian UCT provides a more nuanced exploration strategy by considering prior knowledge and updating beliefs as more information is gathered.
- \* **Complexity:** The Bayesian approach can be computationally expensive, especially in large search spaces.
- \* **Children's Count:** The number of children explored under Bayesian UCT can be influenced by the prior distributions used, potentially leading to more focused exploration in areas of high uncertainty.

### 2.3.5 Simulation policy

#### 2.3.5.1 Different nodes

## Chapter 3

# Problem Description

### 3.1 Overview

Kiwi's traveler wants to travel in  $N$  different areas in  $N$  days, let's denote  $A$  the set of areas the traveler wants to visit:

$$A = \{A_1, A_2, \dots, A_N\}$$

where each  $A_j$  is a set of airports in area  $j$ :

$$A_j = \{a_{j,1}, a_{j,2}, \dots, a_{j,k_j}\}$$

where  $a_{j,k_j}$  being airports in area  $j$  and  $k_j$  is the number of airports in area  $j$ .

The traveler has to visit one area per day. He has to leave this area to visit a new area by flying from the airport he flew in. He leaves from a known starting airport and has to do his journey and come back to the starting area, not necessarily the starting airport. There are flight connections between different airports, with different prices depending on the day of the travel: we can write  $c_{ij}^d$  the cost to travel from  $city_i$  to  $city_j$  on day  $d$ . We do not necessarily have  $c_{ij}^d = c_{ji}^d$  neither  $c_{ij}^{d_1} = c_{ij}^{d_2}$  if  $d_1 \neq d_2$ . The problem can hence be characterised as assymetric and time dependant.

The aim of the problem is to find the cheapest route for the traveler's journey.

We can then formulate the problem more effectively:

- $\mathcal{A} = \{1, 2, \dots, N\}$ : Set of areas.

- $A_j = \{a_{j,1}, a_{j,2}, \dots, a_{j,k_j}\}$ : Set of airports in area  $j \in \mathcal{A}$ .
- $\mathcal{D} = \{1, 2, \dots, N\}$ : Set of days.
- $U_d \subseteq \mathcal{A}$ : Set of areas that have not been visited by the end of day  $d$ .

#### Parameters

- $c_{ij}^d$ : Cost to travel from airport  $i$  to airport  $j$  on day  $d \in \mathcal{D}$ .

#### Variables

- $x_{ij}^d$ : Binary variable which is 1 if the traveler flies from airport  $i$  to airport  $j$  on day  $d$ , and 0 otherwise.
- $v_j^d$ : Binary variable which is 1 if area  $j$  is visited on day  $d$ , and 0 otherwise.

#### Constraints

##### 1. Starting and Ending Constraints:

- The traveler starts at the known starting airport  $S_0$ .
- The traveler must return to an airport in the starting area on the final day  $N$ .

##### 2. Flow Constraints:

- The traveler must leave each area and arrive at the next area on consecutive days, the next area has not been visited yet.
- Ensure that the traveler can only fly into and out of the same airport within an area.
- Ensure each area is visited exactly once.
- Update the unvisited list as areas are visited.

## Objective Function

The goal is to minimise the journey's total travel cost:

$$\min \left( \sum_{d=2}^{N-1} \sum_{i \in \bigcup_{k=2}^{N-1} A_k} \sum_{j \in \bigcup_{k=3}^N A_k} c_{ij}^d x_{ij}^d + \sum_{j \in A_1} c_{S_0,j}^1 x_{S_0,j}^1 + \sum_{i \in A_N} \sum_{j \in A_1} c_{ij}^N x_{ij}^N \right)$$

## Constraints

- Starting at the known starting airport  $S_0$  at take an existing flight connection:

$$\sum_{j \in A_1} x_{S_0,j}^1 = 1$$

$$\forall d \in \mathcal{D}, c_{S_0,j}^d \in \mathbb{R}^{+*}$$

- Visit exactly one airport in each area each day:

$$\sum_{i \in A_d} \sum_{j \in A_{d+1}} x_{ij}^d = 1 \quad \forall d \in \{1, 2, \dots, N-1\}$$

- Ensure the traveler leaves from the same airport they arrived at the previous day:

$$\sum_{k \in A_d} x_{ik}^d = \sum_{k \in A_{d-1}} x_{ki}^{d-1} \quad \forall i \in \bigcup_{j=1}^N A_j, \forall d \in \{2, 3, \dots, N\}$$

- Return to an airport in the starting area on the final day with an existing flight connection:

$$\sum_{i \in A_N} \sum_{j \in A_1} x_{ij}^N = 1$$

$$\forall (i, j) \in A_N \times A_1, c_{i,j}^N \in \mathbb{R}^{+*}$$

- Ensure each area is visited exactly once:

$$\sum_{d \in \mathcal{D}} v_j^d = 1 \quad \forall j \in \mathcal{A}$$

- Update the unvisited list:

$$v_j^d = 1 \implies j \notin U_d \quad \forall j \in \mathcal{A}, \forall d \in \mathcal{D}$$

- Ensure a flight on day  $d$  between  $i$  and  $j$  exists only if the cost exists and  $j$  is in the unvisited areas on day  $d$ :

$$x_{ij}^d \leq c_{ij}^d \cdot v_j^d \quad \forall i, j \in \left(\bigcup_{j=1}^N A_j\right)^2, \forall d \in \mathcal{D}$$

$$x_{ij}^d \leq v_j^d \quad \forall j \in \bigcup_{j=1}^N A_j, \forall d \in \mathcal{D}$$

- Binary variable constraints:

$$x_{ij}^d \in \{0, 1\} \quad \forall (i, j) \in \left(\bigcup_{j=1}^N A_j\right)^2, \forall d \in \mathcal{D}$$

$$v_j^d \in \{0, 1\} \quad \forall j \in \mathcal{A}, \forall d \in \mathcal{D}$$

## 3.2 Instances

### 3.2.1 Description

We are given a set of 14 Instances  $I_n = \{I_1, I_2, \dots, I_{13}, I_{14}\}$  that we have to solve. Every instances has the same overall structure.

For example, the first few lines of  $I_4$  are:

13 GDN  
first  
WRO DL1  
second  
BZG KJ1  
third  
BXP LB1

That means that the Traveller will visit 13 different areas, he starts from airport **GDN**, that belongs to the starting area. Then we are given the list of airports that are in every zone. For example, the second zone is named **second** and has two airports: **WRO** and **DL1**.

After all the information regarding the areas and the airports we have the flight connections informations. In Table 3.1, few flights are displayed from  $I_6$  for illustrative purpose.

Departure from	Arrival	Day	Cost
KKE	BIL	1	19
UAX	NKE	73	16
UXA	BCT	0	141
UXA	DBD	0	112
UXA	DBD	0	128
UXA	DBD	0	110

TABLE 3.1: Flight connections sample I6

For every instance  $I_i$ , we know what connections exist between two airports for a specific day and the associated cost. There might be in some instances flights connections at day 0, this means these connections exist for every day of the journey at the same price. Furthermore, we could have same flight connections at a specific day but with different prices. We then have to consider only the more relevant connections i.e. the flight connection with the lowest fare.

### 3.2.2 General formulation

An instance  $I_i$  can be mathematically defined as follows:

$$I_i = (N_i, S_{i0}, A_i, F_i)$$

where:

- **Number of Areas  $N_i$ :**

$$N_i \in \mathbb{N}$$

The total number of distinct areas in instance  $I_i$ .



- **Starting Airport  $S_{i0}$ :**

$$S_{i0} \in \text{Airports}$$

The starting airport of the traveller.

- **Airports in Each Area:**

$$A_i = \{A_{i,1}, A_{i,2}, \dots, A_{i,N_i}\}$$

where each  $A_{i,j}$  is a set of airports in area  $j$  for instance  $i$ :

$$A_{i,j} = \{a_{i,j,1}, a_{i,j,2}, \dots, a_{i,j,k_j}\}$$

with  $a_{i,j,k_j}$  being airports in area  $j$  and  $k_j$  is the number of airports in area  $j$ .

- **Flight Connections:**

$$F_i = \{F_{i,0}, F_{i,1}, F_{i,2}, \dots, F_{i,N_i}\}$$

where each flight matrix  $F_{i,k}$  represents the flight information of instance  $i$  on day  $k$ :

$$F_{i,k} = \begin{pmatrix} a_{i,k,1}^d & a_{i,k,1}^a & f_{i,k,1} \\ a_{i,k,2}^d & a_{i,k,2}^a & f_{i,k,2} \\ \vdots & \vdots & \vdots \\ a_{i,k,l_{k,i}}^d & a_{i,k,l_{k,i}}^a & f_{i,k,l_{k,i}} \end{pmatrix}$$

- **Columns:**

- \* Departure Airport:  $a_{i,k,j}^d$  (Departure airport for the  $j$ -th flight on day  $k$ )
- \* Arrival Airport:  $a_{i,k,j}^a$  (Arrival airport for the  $j$ -th flight on day  $k$ )
- \* Cost:  $f_{i,k,j}$  (Cost of the  $j$ -th flight on day  $k$ ), where  $j \in [1, l_{k,i}]$

- **Rows:** Each row corresponds to a specific flight on day  $k$ . The number of rows  $l_{k,i}$  depends on the number of flights available on that day.

### 3.2.3 Kiwi's rules

When solving all the instances, Kiwi's defined time limits constraints based on the nature of the instance. We can summarise these constraints in the Table above:

Instance	nb areas	Nb Airports	Time limit (s)
Small	$\leq 20$	$< 50$	3
Medium	$\leq 100$	$< 200$	5
Large	$> 100$		15

TABLE 3.2: Time limits based on the number of areas and airports

All the useful information about the instances such as the starting airport, the associated area, the range of airports per area, the number of airports and the time limit constraints defined in Table 3.2.

Instances	Starting Area - Airport	N° areas	Min - Max airport per area	N° Airports	Time Limit (s)
I1	Zona_0 - AB0	10	1 - 1	10	3
I2	Area_0 - EBJ	10	1 - 2	15	3
I3	ninth - GDN	13	1 - 6	38	3
I4	Poland - GDN	40	1 - 5	99	5
I5	zone0 - RCF	46	3 - 3	138	5
I6	zone0 - VHK	96	2 - 2	192	5
I7	abfluidmorz - AHG	150	1 - 6	300	15
I8	atrdruwbz - AEW	200	1 - 4	300	15
I9	fcjsqtmccq - GVT	250	1 - 1	250	15
I10	eqlfrvhlwu - ECB	300	1 - 1	300	15
I11	pbggaeftjv - LIJ	150	1 - 4	200	15
I12	unnwaxhnoq - PJE	200	1 - 4	250	15
I13	hpkogdfpf - GKU	250	1 - 3	275	15
I14	jjewssxvsc - IXG	300	1 - 1	300	15

TABLE 3.3: Instances and their respective parameters

## Chapter 4

# Results and performance

(TODO) Present the results and discuss any differences between the findings and your initial predictions/hypothesis

(TODO) Interpret your experimental results - do not just present lots of data and expect the reader to understand it. Evaluate what you have achieved against the aims and objectives you outlined in the introduction

- Based on the different rollout policy, analyse the output - the greedy is deterministic whereas the 2 others are stochastic
- Maybe try to fine tune the selection policy - one idea could be instead of returning the average score is to return the min score
- Compare outputs with known solutions - Yaro + literature - efficiency in terms of time
- Ahmed - literature : <https://code.kiwi.com/articles/travelling-salesman-challenge-2-0-wrap-up/>

## Chapter 5

# Conclusion

(TODO) Explain what conclusions you have come to as a result of doing this work. Lessons learnt and what would you do different next time. Please summarise the key recommendations at the end of this section, in no more than 5 bullet points.

### 5.1 Summary of Work

### 5.2 Critics

### 5.3 Future Work

(TODO) The References section should include a full list of references. Avoid having a list of web sites. Examiners may mark you down very heavily if your references are mainly web sites.

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